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END STOPPERING IN MIRROR MACHINES

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August 24, 1976

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Prepared for U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.

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END STOPPERING IN MIRROR MACHINES

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ABSTRACT

Like toroidal confinement, confinement by a potential barrier offers the possibility of unlimited Q if the barrier is good enough. For a barrier height ϕ , approximately

$$\tau \approx \tau_{\text{collision}} \exp \phi/T_i$$

These notes review previous ideas about how to establish a potential barrier at the ends of a mirror machine and we also discuss a different possibility using mirror confinement itself to establish a barrier utilizing the positive ambipolar potential characteristic of mirror machines. One example consists of three mirrors in tandem, the two at the ends having a much smaller volume. Then mirror confinement in the end cells would be used to establish ambipolar barriers that stopper up the ends of the central, large mirror cell. An example is discussed in which, by approximate calculation, it appears that ignition might occur in a center cell 10 to 20 meters long and the overall Q is 5 or more, including the power required to sustain the end cells by neutral injection.

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DISCUSSION OF THE CONCEPT:

Like toroidal confinement, confinement by a potential barrier offers the possibility of unlimited Q if the barrier is good enough. For a barrier of height ϕ , approximately

$$\tau \approx \tau_{\text{collision}} \exp \phi/T_i, \quad (1)$$

A combination of a mirror machine with potential barriers at the ends offers the additional advantage of $\beta \approx 1$ in the main volume. The principal disadvantage of potential barriers thus far has been the power investment to sustain the barrier itself.

These notes review previous ideas and discuss a different possibility of using mirror confinement itself as a barrier. One example of the latter would consist of three mirrors in tandem, the two at the ends having a much smaller volume. Then mirror confinement in the end cells would be used to establish ambipolar barriers that stopper up the ends of the central, large mirror cell.

Electrostatic or other potential barrier confinement is better for ions than for electrons - this shows up as a difference in barrier heights for the two to get the same $n\tau$:

$$\frac{\phi_{\text{elec}}}{T_e} \approx \frac{\phi_{\text{ion}}}{T_i} + \log \sqrt{M_i/M_e} \cdot (T_i/T_e)^{3/2}$$

where the ϕ 's are the respective barrier heights. (This is a crude approximation . . . for electron confinement in the "standard mirror" it gives too high values by almost a factor of 2 as compared to classical Fokker-Planck calculations.)

Consequently, for a potential barrier the overall investment in energy per escaping ion-electron pair is

$$\sim \frac{\text{Max}}{\text{of}} \left\{ \begin{array}{c} \phi \\ \text{elec} \\ \text{or} \\ \phi_{\text{ion}} \end{array} \right\} + (T_e + T_i) \approx (4 \text{ to } 8) + 2 \approx (6 \text{ to } 10) T \quad \text{if } T_e = T_i$$

For toroidal confinement, the escaping energy per ion-electron pair is $2 \left(\frac{3}{2} T \right) = 3T$. That is, the energy leakage per pair is 2 to 3 times greater for a potential barrier. Put differently, the energy confinement is poorer than particle confinement for barriers and to a greater degree than for toroidal machines. On the other hand, potential barriers are (probably) less subject to unstable transport.

Aside from gravity, all potential barriers yet proposed, whether electrostatic grids, r.f. pressure or ambipolar barriers, require an investment of power. On the other hand, just as a dam can hold back a much greater volume of water, a barrier will expend a smaller fraction of the total power the less volume it occupies. An attractive way to minimize the barrier volume is to utilize a magnetic mirror field to contain the plasma except for potential barriers at the ends. The fact that the magnetic lines converge to a smaller cross sectional area at the mirror throat further reduces the barrier volume. The remaining design objective is to minimize the barrier volume and power density input necessary to sustain it. Secondly, one may seek also to "re-use" the barrier power in heating the plasma to whatever extent that is possible.

Several kinds of barriers have been discussed in the literature. One is a ponderomotive force applied by electromagnetic fields, either R.F. at frequencies exceeding gyrofrequencies, or various schemes of adiabatic resonance that attempt to reverse the scattering process. The difficulty has generally been three-fold: (1) the power loss at walls, etc., is high; (2) the field required is so high that breakdown is a problem; and (3) the fields required to hold back the pressure tend to be so high that excessive heating of the plasma occurs. (In principle the latter effect can be diluted by a very large plasma volume; this is an example of "re-using" some of the power invested in making the barrier.)

Conceivably, since plasma waves tend to be reabsorbed by the plasma, there might be some scheme for generating an R.F. barrier in situ. On fundamental grounds the R.F. pressure could not exceed the pressure of the plasma that generates it, at least in an average sense, but there might be some eigenmode that concentrates R.F. energy so as to create a local barrier.

An example of a "ballistic" barrier is the purely electrostatic scheme in which energetic electrons injected into a sphere establish virtual potential barriers to hold ions. Again, the problem is mainly the power investment and losses.

Electrostatic grids mainly suffer from power loss to the grid itself. This appears to be fundamental, since if one species is repelled by the first grid, the species of opposite charge is attracted to it. If the first grid stops the ions, magnetic lines passing through the grid can channel many electrons through but still the impacted area is relatively large. Effectively, the lifetime for the electrons would be the time of flight divided by $(1-P)$ where P is the grid penetration factor for the electrons. One might imagine moving grids, liquid metal jets, etc., but these too require energy.

Finally, we can consider ambipolar barriers. The idea is this.

In the standard mirror concept, hot ions confined by the mirrors establish a positive ambipolar barrier that confines the electrons electrostatically. Consider instead the arrangement of three mirrors in tandem, as in Fig. 1. The two end mirrors would be filled with hot ions in the usual manner. This now creates two positive barriers at the ends that serve as electrostatic barriers for colder ions trapped in the central mirror region. Electrons, confined by the overall positive potential end to end, penetrate the central region just to the extent necessary to maintain charge neutrality. The power necessary to sustain the end barriers is that needed to maintain the hot ion distribution in the mirrors. If the outermost mirror fields are strongest, these hot ions leak preferentially into the center mirror before finally escaping and contribute to heating the central plasma. Thus the barrier power is "re-used".

It is conceivable that hot plasma escaping out of the center cell would trap itself by instability generated in flowing through the end cells. The appearance of fluctuations at the ion gyrofrequency and some degree of self-trapping has been observed in 2XIIB when a warm plasma stream is injected into one end of the machine. To act as an automatic means of establishing ambipolar barriers in the end cells, it would be necessary that this self-trapping process be able to cumulate in the end cells a density higher than that in the center cell. If all this works out, it would offer the design option of applying all power at the center, and, in a device large enough to achieve ignition (if that proves possible) it would not be necessary to apply any external power to maintain the barriers.

The idea is sketched in Fig. 1. The electron density n_{II} in the center cell is related to the barrier density n_I by

$$n_{II} = n_I \exp \left(- \Delta\Phi/T_e \right). \quad (2)$$

From equation (1), the lifetime for the Maxwellian ions confined between the ambipolar barriers is given by

$$\tau_i = \tau_{ii} \exp Z\Delta\Phi/T_i \quad (3)$$

where T_i is their temperature, τ_{ii} is the ion-ion collision time and Z is the charge ($Z = 1$ for hydrogen). Combining Eqs. (2) and (3) gives

$$\tau_i = \tau_{ii} \left(n_I/n_{II} \right)^{ZT_e/T_i} \quad (4)$$

It would also be necessary that the time for transverse diffusion across the magnetic field exceed τ_i , but this presents little problem.

If τ_i is large enough, the Maxwellian plasma can burn or even ignite, or it can serve as a target plasma in two-component arrangement together with hot ions injected into the center cell confined by the mirrors. In either case, the subsidiary condition in order that the power to sustain the barriers be negligible is approximately

$$2V_I \frac{n_I E}{\tau_I} < V_{II} \frac{n_{II}}{\tau_i} \left(\Phi + T_e \right). \quad (5)$$

where V_i is the plasma volume in an end cell and V_{II} is the volume in the center cell.

As can be seen from Eq. (4), the lifetime of ions in the center can be enhanced over τ_{ii} by an arbitrary amount, but either at a sacrifice of n_{II} (hence a lower power density in the center) or in a regime with $ZT_e \gg T_i$. From Eq. (5) we see that the permissible volume ratio V_I/V_{II} must be small if $\tau_i \gg \tau_{ii}$, and smaller still if also $n_I \gg n_{II}$. Thus the idea is most useful if $ZT_e \gg T_i$ so that n_{II} can be an appreciable fraction of n_I .

These requirements are now being evaluated. From preliminary results, it appears that, for both two-component and thermal burn, $V_I/V_{II} \sim 1\%$ is necessary to reach $Q = 1$ and V_I/V_{II} must be much less (0.1% or less) to get high Q .

The requirement for small V_I/V_{II} would be most easily met if the net mirror ratio is high (high beta at the center) and if the end cells could be very short. The data in 2XIIB at very high beta indicates that a high density plasma can be confined with very little mirror ratio (a few percent). Taking advantage of this, it may be possible to create a suitable tandem mirror structure by relatively minor modifications of a single mirror cell. Namely, it may be possible to create two shallow mirror cells in the high field regions near the main mirrors by adding small windings to the main coil. The fact that the field is high at the ends means n_I can be larger. Neutral beam access in the end cells may be a problem, and also MHD stability. Also, though still small, the mirror ratio at the ends must be greater than that for a single cell; it is the outer gradient in the vacuum field that absorbs the outward pressure that the Maxwellian ions exert against the ambipolar barriers.

No actual field design meeting these criteria has yet been carried out. Speaking qualitatively, one might hope to gain one factor of ten reduction in V_I/V_{II} from the effective mirror ratio at high beta. Noting that the 2XIIB plasma is some 40 cm in length, perhaps $V_I/V_{II} = 1\%$ could be obtained in a magnet the size of the MX magnet. Moreover, since the end mirror cells that actually contain anisotropic plasmas would be very short, the shear-Alfvén and convective instabilities would be strongly suppressed. The center cell would be Maxwellian (or at least a monotonic-decreasing function of energy if hot ions are also present) and hence microinstability would not be an issue there.

Evidently the magnet design problem would be easiest in a simple mirror with line tying, if that system can be made stable. The present scheme with its large throughput of current to sustain the Maxwellian plasma has some advantages in that regard. Also, it may yet prove possible to stabilize a simple mirror by means of a cold plasma blanket requiring little power to sustain it.

AN EXAMPLE

The following is an example of the above idea employing ambipolar barriers based on simple, approximate models. It is neither optimum nor exact, but it gives some feeling for the idea. Better calculations using the Fokker-Planck codes are in progress.

As noted above, the system tends to favor $T_e \gg T_i$. Regimes with $T_e = T_i$ exist but only for very large systems. A necessary condition in order that $T_e > T_i$ is that the ions carry away at least as much power as they receive from the electrons. We express this by the approximate condition

$$\frac{n^2}{n\tau} (\Delta\phi + W_i) \geq \frac{n^2}{K_{ie}} \frac{W_e - W_i}{W_e^{3/2}} \quad (6)$$

where $W = 3/2 T$ and $K_{ie} = 10^{12}$ with energies in Kev. All quantities refer to the center cell, region II. For $n\tau$ we take the following modification of Eq. (3) chosen to fit certain computer data:

$$n\tau = 5 \times 10^{10} W_i^{1/2} \Delta\phi \left(1 + \frac{W_i}{\Delta\phi}\right) \exp \frac{\Delta\phi}{T_i} \quad (7)$$

The factor $1 + W_i/\Delta\phi$ improves the agreement with the computer for small $\Delta\phi/W_i$. With a little work we can find that Eq. (6) is satisfied for all $W_e \geq W_i$ if the amplification factor $\exp \Delta\phi/T_i < 50$ (with the equality holding for this value of $\Delta\phi/T_i$ and with $T_e = 3T_i$). For larger amplification factors, Eq. (6) requires either that $T_e \approx T_i$, or T_e is very much larger than T_i . So for our example we choose $T_e = 3T_i$ and $\Delta\phi/T_i = 4$ for which case $\exp \Delta\phi/T_i = 50$.

We now wish to evaluate the power gain in the center cell denoted by

Q_0 and given by:

$$Q_0 = \frac{1}{4} \frac{(n\tau) \overline{\sigma v} E_{nuc}}{\phi + W_i + W_e} \quad (8)$$

Note that Q_0 is the ratio of fusion power to escaping power. The denominator arises from the fact that, as viewed from the center cell, electrons carry away roughly an energy $\phi - \Delta\phi + W_e$ and ions an energy $\Delta\phi + W_i$. If $Q_0 \geq 5$, the alphas can ignite the center cell.

To evaluate Q_0 we must know ϕ , the overall electron barrier. Neglecting electrons injected into the end cells, this is determined by the barrier $\phi - \Delta\phi$ as seen from the center and is given by equating the electron escape to $n\tau$ for the ions. Again we use for the electrons an expression similar to Eq. (3) but adjusted to fit computer data:

$$(n\tau)_{\text{electron}} = 8.5 \times 10^8 W_e^{1/2} (\phi - \Delta\phi) \exp \frac{\phi - \Delta\phi}{T_e} = (n\tau)_{\text{ion}} \quad (9)$$

Substituting $(n\tau)_{\text{ion}}$ from Eq. (7) and solving for the case $\Delta\phi/T_i = 4$ and $T_e = 3T_i$ gives $\phi - \Delta\phi = 6.1 T_e$, or $\phi = 22.3 T_i$.

We shall choose $T_i = 20$ keV, or $W_i = 30$ keV, in which case $\overline{\sigma v} = 4.4 \times 10^{-16} \text{ cm}^3/\text{sec}$ for DT. Then, using the above results we find

$$Q_0 = 5. \quad (10)$$

That is, the center might ignite. Continuing, from Eq. (2) and taking the above values we find

$$n_I = 3.7 n_{II} \quad (11)$$

Parameters determined thus far are:

$$T_i = 20 \text{ keV}$$

$$T_e = 60 \text{ keV}$$

$$\phi = 440 \text{ keV}$$

$$\Delta\phi = 80 \text{ keV}$$

$$n_I = 3.7 n_{II}$$

$$Q_0 = 5.$$

To complete a design we now choose the injection energy in the end cell ($\sim$ mean energy there) as $E = 600 \text{ keV} = 10 T_e$, approximately as in "standard" mirror results. The potential $\phi = 440 \text{ keV}$ is higher than usual, but we count on the fact that the high beta plasma in the end cell can create for itself a large mirror ratio as in 2XIIB; this will suppress the energy of ions ejected by the ambipolar potential to $\phi/R-1 \ll E$. Thus we assume the end cell operates "normally" and take there

$$(n\tau)_I = 10^{10} E^{3/2} = 1.5 \times 10^{14}. \quad (12)$$

That the end cell energy $E = 600 \text{ keV}$ is too high for optimum burning does not matter, since most of the burning is in the center cell. By proper choice of V_I versus V_{II} , the 600 keV ions slowing down in the end cells can heat the electrons to ignition, or supplement the α 's in steady state. Heating of the D-T in the center by the electrons is assured by having chosen parameters that satisfy Eq. (6) as an equality.

To complete the design, we choose $B = 100 \text{ kg}$ at the end cell. The vacuum mirror ratio is weak there. At 600 keV and $\beta \approx 1$,

$$n_I = 3.7 \times 10^{14}/\text{cc} \quad (13)$$

and, by Eq. (11),

$$n_{II} = 10^{14}/\text{cc}. \quad (14)$$

If $\beta \approx 0.9$ in the center cell, $B = 25$ kg at the center will contain the Maxwellian plasma. Thus R_{vac} in the center cell is $100 \text{ kg (at end)}/25 \text{ kg} = 4$ and at $\beta = .9$, $R = R_{\text{vac}} (1 - \beta)^{-1/2} = 12$. A subsidiary condition that cross field diffusion be compatible with n_τ seems to be well satisfied, since the larmor radius of a 30 keV triton is only 1.6 cm at the central field of 25 kg (compared to $R_p = 150$ cm) and 0.4 cm at the ends where $B = 100$ kg. To satisfy Eq. (5), we rewrite it as

$$\frac{2V_I}{V_{II}} = 2 \frac{\Delta}{L} \frac{1}{R} < \left(\frac{n_{II}}{n_I} \right)^2 \frac{(n_\tau)_I}{n_\tau} \frac{\phi + T_e}{E} \quad (15)$$

where L is the length of region II and Δ is the length of one of the end cells. Using the above parameters gives

$$\frac{\Delta}{L} \ll \frac{1}{20}. \quad (16)$$

Hence if $\Delta \sim 40 - 50$ cm as in 2XIIB, L could be 10 to 20 meters. The plasma radius in the end cell would also be ~ 40 cm and that in the center would be $R_p \sim \sqrt{R} 40_{\text{cm}} \sim 150 \text{ cm} = 1.5 \text{ meter}$. At n_{II} above, the power density is

$$p = \frac{1}{4} n_{II}^2 \overline{\sigma v} E_{\text{nuc}} = 3 \text{ watt/cc} \quad (17)$$

or, at $R_p = 1.5$ meter, the linear thermal power density is

$$P/L \approx \pi R_p^2 \cdot p \sim 20 \text{ MW/meter} \quad (18)$$

Thus, in a 10 to 20 meter length the total power is 200 to 400 MW thermal.

The power required to maintain the end cells is the total divided by Q_0 , or less if the longer length is chosen. The electrical output would be 75 to 100 MWe.

Thus we have arrived at a rough design for a steady-state d.c. device producing an overall power output of 100 MWe or less in a linear machine only 10 to 20 meters in length. The center region possibly ignites and the overall Q of the system including the power to sustain the end cells is probably 5 or greater.

Depending on how simple mirrors turn out, it may be necessary to construct the end cells as small, high field Yin Yangs with low mirror ratio and circularizers to join to a long solenoid connecting them. Because of the greater energy density in the min-B end cells, average min-B should maintain MHD stability despite bad curvature where the coils join. The main part of the machine where burning takes place is a simple solenoid at only 25 kg. In that region the coil structure and blanket could be purely cylindrical, modular and relatively easier to maintain. Parameters are summarized in Table I.

ACKNOWLEDGMENT:

The concept of ambipolar barriers discussed in this paper has recently been arrived at independently by B. Grant Logan. I am grateful to him for a critical reading of this manuscript. A joint publication on the same subject is in preparation.

TABLE I
PARAMETERS FOR A TANDEM MIRROR
REACTOR WITH AMBIPOLAR BARRIERS

Power Out	200 - 400 MW Thermal
Power to Sustain Barriers	40 MW
L (Length of Center Solenoidal Cell)	10 - 20 Meters
R_{plasma} (Plasma Radius in Center Cell)	1.5 Meters
Δ (Length of Plasma in End Cell)	40 - 50 cm
B_{center} (Field at Solenoid)	25 kg
B_{end} (Field at End Cell)	100 kg
n_{II} (Center)	10^{14} cm^{-3}
n_{I} (End Cell)	$3.7 \times 10^{14} \text{ cm}^{-3}$
T_{i} (Center)	20 keV
T_{e}	60 keV
E (End Cell Ion Energy)	600 keV
Q_0 (Center Cell)	5

Fig.1 Tandem mirrors with ambipolar barriers at the ends

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