**Relationship Between Length Variations
of HTGR Fuel Rod Stack Lengths
and Individual Rod Lengths**

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RELATIONSHIP BETWEEN LENGTH VARIATIONS OF HTGR FUEL
ROD STACK LENGTHS AND INDIVIDUAL ROD LENGTHS

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RELATIONSHIP BETWEEN LENGTH VARIATIONS OF HTGR FUEL
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C. K. Bayne* and D. R. Johnson

ABSTRACT

A mathematical relationship between the length variation of individual HTGR fuel rods and the length tolerance of a stack of fuel rods has been derived. The variations in fuel rod lengths were assumed to be distributed normally with known mean and standard deviation. If individual fuel rod lengths are within a length $L \pm \theta$, $(1 - \alpha)\%$ of the time and a stack of 15 rods is within $15 L \pm \delta$, $(1 - \alpha)\%$ of the time then $\theta = (0.258) \delta$ for all values of α . The specification for δ will likely be 0.25 in.; which corresponds to $\theta = 0.060$ in. However, we can manufacture rods by the slug injection process with an actual θ of 0.018 in. Thus, out-of-specification rod stacks will be extremely rare; so development of special equipment to recycle the out-of-specification stacks will not be necessary.

INTRODUCTION

The fuel for the High-Temperature Gas-Cooled Reactor consists of coated fuel particles that are bonded together with pitch and graphite powder to form molded cylindrical fuel rods. The fuel rods are stacked into blind holes in a prismatic graphite fuel block (Fig. 1).

As the graphite block shrinks relative to the fuel column during irradiation, it is important that an adequate axial void be provided between the fuel column and the graphite block. Otherwise, the differential contraction might result in mechanical damage to the fuel. The size of the space between fuel column and graphite block depends upon whether the void is at one end of the column or distributed among the fuel rods. The tolerance on the total void in a fuel column is expected to be $\sim \pm 0.25$ in.

In the manufacturing of HTGR fuel rods, this void requirement can be expressed as the tolerance on the length of a stack of 15 individual

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Fig. 1. HTGR Fuel Components.

fuel rods, nominally 2 in. long; the distribution of the variation in the fuel rod lengths is approximately normal (Fig.2). Figure 2 shows a normal probability plot for fuel rods manufactured to a fuel rod length of 1.99 in., the mean fuel rod length can be changed at will by adjusting the fuel rod molding equipment. The present work is a method to calculate the required dimensional specifications for individual fuel rods when the tolerance for the stack of 15 fuel rods is known.

Fig. 2. Probability Plot of Lengths of Slug Injected Fuel Rods.

DERIVATION

A number of definitions must be established:

Let l_i , $i = 1, 2, \dots, n$, be the observed lengths of n individual fuel rods.

Let the expected value of each individual rod length be L .

Let the standard deviation of the individual lengths be σ .

Let TL be the total length of n fuel rods, $TL = \sum_{i=1}^n l_i$.

The expected value of TL will be nL and the standard deviation of TL will be $\sqrt{n}\sigma$.

Then given the requirement that with $(1 - \alpha)$ probability, TL is within the interval $nL \pm \delta$,

$$Pr(nL - \delta < TL < nL + \delta) = 1 - \alpha, \quad (1)$$

we wish to derive an expression for θ as a function of δ such that

$$Pr(L - \theta < \ell_i < L + \theta) = 1 - \gamma. \quad (2)$$

The fuel rod lengths which are assumed to be independently normally distributed can be transformed by $Z_i = (\ell_i - L)/\sigma$ to independently normally distributed random variables with mean 0 and standard derivation 1. Tables of the percentage points of the normal random variable Z [i.e., the number Z_β such that $Pr(Z > Z_\beta) = \beta$] can be found in most statistical handbooks.

In the inequality (2) one can subtract L and divide by σ to obtain

$$Pr[-\theta/\sigma < (\ell_i - L)/\sigma < \theta/\sigma] = 1 - \gamma, \quad (3)$$

Then

$$\theta/\sigma = Z_{\gamma/2}, \quad (4)$$

and

$$\sigma = \theta/Z_{\gamma/2}. \quad (5)$$

By transforming the normally distributed variable TL from Eq. (1), we can follow a similar argument to derive an expression relating σ and δ :

$$Pr[-\delta/\sqrt{n}\sigma < (TL - nL)/\sqrt{n}\sigma < \delta/\sqrt{n}\sigma] = 1 - \alpha, \quad (6)$$

Then

$$\delta/\sqrt{n}\sigma = Z_{\alpha/2} , \quad (7)$$

and

$$\sigma = \delta \sqrt{n} Z_{\alpha/2} . \quad (8)$$

From (5) and (8) it is apparent that

$$\theta/Z_{\gamma/2} = \delta/\sqrt{n} Z_{\alpha/2} , \quad (9)$$

or

$$\theta = \delta Z_{\gamma/2} / \sqrt{n} Z_{\alpha/2} . \quad (10)$$

Equation (10) is our desired result.

DISCUSSION OF RESULTS

Suppose we are going to make $n = 15$ fuel rods with each fuel rod $2 \pm \theta$ in. in length. On the average only $\gamma\%$ ($\gamma = 5\%, 1\%, 0.1\%$) of the fuel rods will be rejected if they have lengths greater than $2 + \theta$ in. or less than $2 - \theta$ in. We would like to find the values of θ such that $\Pr(30 - \delta < \sum_{i=1}^n l_i < 30 + \delta) = 1 - \alpha$, when $\alpha = 0.05, 0.01$, and 0.001 .

The percentile values 1.96 (for $\alpha = 0.05$), 2.58 (for $\alpha = 0.01$), and 3.29 (for $\alpha = 0.001$) may be used with (Eq. 10) with different values of γ .

The θ values are listed in Table 1 for any value of δ . We should note that if $\alpha = \gamma$ then the value of θ is $\theta = \delta/\sqrt{15} = 0.258\delta$, which is independent of the value of α or γ . Under the normality assumption, if there is a high degree of confidence that the lengths of the fuel rod will be within the tolerance range of $2 \pm 0.258\delta$ then there should be the same high degree of confidence that the total length will be within

the tolerance range $30 \pm \delta$. For example, if $\delta = 0.25$ in. and $\gamma = 0.001$ and $\alpha = 0.001$ then the value of θ is $\theta = 0.258$ (0.25 in.) = 0.060 in. That is, if 99.9% of the fuel rods are manufactured within 2.000 ± 0.060 in., then the stack will be within 30.00 ± 0.25 in. 99.9% of the time. Recent data for 92 fuel rods manufactured by the slug injection process yielded a standard deviation of 0.0056 in. Thus, the 3σ limit for the individual rod lengths is ± 0.018 in., much better than the ± 0.060 in. required for the void requirements. These considerations led to the conclusion that out-of-specification stacks of fuel rods will be extremely rare. This conclusion is important to the HTGR Fuel Refabrication Development Program as it indicates that the development of equipment for handling out-of-specification stacks will not be required. In the remote refabrication of HTGR fuel, the most efficient handling of the occasional out-of-specification fuel rod stacks may be to scrap them rather than develop systems to recombine the rods with other rods to form acceptable stacks.

Table 1. Values of θ for Various Values of α and γ

γ	θ for each α value		
	0.05	0.01	0.001
0.05	0.258 δ	0.196 δ	0.154 δ
0.01	0.340 δ	0.258 δ	0.203 δ
0.001	0.433 δ	0.329 δ	0.258 δ

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