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Stress Analyses of Perforated Flat Plates Under In-Plane Loadings

J. P. Callahan
J. W. Bryson

MASTER

OAK RIDGE NATIONAL LABORATORY

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STRESS ANALYSES OF PERFORATED FLAT PLATES
UNDER IN-PLANE LOADINGS

J. P. Callahan
J. W. Bryson

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FOREWORD

The work reported here was done in support of the ORNL Design Criteria for Piping and Nozzles Program being sponsored by the Division of Reactor Safety Research, Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission; E. K. Lynn of the Metallurgy and Materials Branch, Division of Reactor Safety Research (RSR) is the cognizant RSR engineer. S. E. Moore, Reactor Division, ORNL, is the program manager. Activities under the Design-Criteria program are coordinated with other safety-related piping and pressure-vessel research through the Design Division, Pressure Vessel Research Committee (PVRC) of the Welding Research Council and through the ASME Boiler and Pressure Vessel Code Committee.

The study of stresses in perforated flat plates described in this report is part of a continuing effort to confirm the adequacy of present code and standard rules, stress analysis methods and design criteria, and/or the development of such rules and methods, for assessing the safety of reinforced nozzle designs in nuclear pressure vessels. These studies are coordinated through the PVRC Subcommittee on Reinforced Openings and External Loadings, W. L. Greenstreet, Chairman. Results are used by appropriate regulatory and ASME Code groups as technical justification for drafting new or improved rules.

Other reports in the series of studies on flat plates are:

Mathews, Experimental Stress Analysis of Clusters of Holes of Equal Diameters in Flat Plates, Part I, A Single Hole, Report EM 69-2, The University of Tennessee, Knoxville (April 1969).

L. E. Hulbert, A. T. Hopper, and E. F. Rybicki, Second Annual Report on the Stress Analysis of Perforated Plates and Plates with Single and Clustered Nozzles, Battelle-Columbus Laboratories, Columbus, Ohio (October 9, 1970).

A. Mathews and R. B. Michaels, Experimental Stress Analysis of Clusters of Holes of Equal Diameters in Flat Plates, Part II, Two Holes, Report EM 70-2, The University of Tennessee, Knoxville (June 1970).

A. Mathews, R. B. Michaels, and R. J. Warmack, *Experimental Stress Analysis of Clusters of Holes of Equal Diameters in Flat Plates, Part III, Three Holes*, Report EM 70-3, The University of Tennessee, Knoxville (October 1970).

A. Mathews, R. B. Michaels, and R. J. Warmack, *Experimental Stress Analysis of Clusters of Holes of Equal Diameters in Flat Plates, Part IV, Five Holes*, Report EM 71-3, The University of Tennessee, Knoxville (June 1971).

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R. L. Battiste, W. H. Peters, W. F. Ranson, and W. F. Swinson, *Stress Analyses of Flat Plates with Attached Nozzles, Vol. 2. Experimental Stress Analyses of a Flat Plate with One Nozzle Attached*, ORNL-5044, Vol. 2 (July 1975).

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STRESS ANALYSES OF PERFORATED FLAT PLATES UNDER IN-PLANE LOADINGS

J. P. Callahan J. W. Bryson

ABSTRACT

Strain-gaged flat plates having one, two, three, and five closely spaced holes were tested under uniaxial and 1:1 biaxial loading conditions. The experimental results of these tests were compared with corresponding calculated values obtained using the computer program TABLES, which was developed at Battelle-Columbus Laboratories. The study was conducted (1) to obtain highly reliable experimental data that can be used in the design of penetrations in vessels having large radii of curvature, (2) to evaluate the capability of the computer program TABLES for accurately predicting the stresses in perforated flat plates, (3) to complete the initial step leading toward the planned development of a method of analysis for clusters of nozzles attached to pressure vessels, and (4) to investigate the stress states in regions of closely spaced holes.

The four models employed in the study were $91.44 \times 91.44 \times 0.95$ -cm ($36 \times 36 \times 0.375$ -in.) flat plates having one, two, three, or five 6.668-cm-diam (2.625-in.-diam) closely spaced holes. The plates were subjected to in-plane 1:0 and 0:1 uniaxial loadings and to 1:1 biaxial loadings.

The comparisons of calculated and experimental stresses were in good agreement, and the method of analysis was found to be capable of accurately predicting the complex state of stress in the vicinity of the closely spaced holes.

Keywords: Stress analysis, perforated plates, piping components, flat plates, penetrations, nozzles, nuclear piping, pressure vessels, nozzles program.

1. INTRODUCTION

1.1 Purpose and Scope

The design of nuclear-reactor pressure vessels for commercial power plants must allow for access to the vessel through clusters of closely spaced reinforced openings. Behavior of these openings is complicated by the presence of complex stress interactions, the nature of which are

not well understood. The Oak Ridge National Laboratory has therefore undertaken an extensive investigation to develop a better understanding of the stress distributions around vessel openings.

In the case of relatively small nozzles attached to large-diameter vessels, vessel curvature can be assumed to have a negligible effect on the stresses at the intersection. Consequently, flat plates are convenient models for studying the behavior of clustered openings in relatively large-diameter vessels. In addition, the planned systematic development of a satisfactory method of analysis for reinforced nozzle-to-vessel attachments employs a series of models of increasing complexity, beginning with perforated flat plates and then considering flat plates having attached nozzles, perforated vessels, and finally vessels having attached nozzles. A more-detailed description of the overall program can be found in Refs. 1 and 2. This report contains the results of the first step toward developing analysis methods for closely spaced reinforced openings.

Comparisons between experimental and analytical stress distributions are given for in-plane uniaxial and biaxial tensile loadings of a series of four flat-plate models having one, two, three, and five closely spaced holes. Comparisons are given for 1:0 and 0:1 uniaxial loadings, and for 1:1 biaxial loadings. Comparisons for 2:1, 1:2, and other biaxial loading ratios can be obtained by superposition of the results presented in this report.

The experimental data used herein were obtained from studies conducted under subcontract by the Department of Engineering Science and Mechanics, The University of Tennessee; the principal investigator was A. Mathews. A complete tabulation of data was published in Refs. 3, 4, 5, and 6.

The computer analysis employed for this study was developed under subcontract at Battelle-Columbus Laboratories under the direction of L. E. Hulbert. A user's manual and a detailed description of the computer program are given in Ref. 7.

The report summarizes the findings of the experimental studies conducted at The University of Tennessee and of the analytical studies made at ORNL using the Battelle-Columbus computer program. The trial series and boundary conditions used in the analysis for each loading, as well

as plots of calculated and experimental stress distributions along and normal to the various model gage lines, are included.

1.2 Nomenclature

The following nomenclature is used in this report:

$A_{i,j}$; $B_{i,j}$; $C_{i,j}$ = i,j th coefficients in the Airy stress-function series

C = normalizing stress = average normal stress in an unpierced plate either in the x or y direction

E = Young's modulus

i = integer index in trial series

i = subscript denoting i th hole

$\log$ = natural logarithm

r = radial distance measured from origin of coordinates

r_i = radial distance measured from i th hole center

R = hole radius

S = normal stress, representing either σ_x or σ_y

x = rectangular coordinate

y = rectangular coordinate

θ = angle between the x axis and the radius vector to a point in the plate from the origin of coordinates

θ_i = angle between the x axis and the radius vector to a point in the plate from the i th hole center

ν = Poisson's ratio

σ_x = stress in the x direction

σ_y = stress in the y direction

τ_{xy} = shear stress

ϕ = Airy stress function

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ = Laplacian operator

2. EXPERIMENTAL STUDY

2.1 Model Fabrication

The basic $91.44 \times 91.44 \times 0.95$ -cm ($36 \times 36 \times 0.375$ -in.) steel flat plate and the one-, two-, three-, and five-hole geometries are shown in Fig. 1. In addition to these models, two instrumented, unpierced flat plates of the same size and type shown in the figure were tested to evaluate the loading system. One of the unpierced plates is shown mounted in the loading system in Fig. 2.

The plates were machined from a thicker piece of hot-rolled low-carbon steel in order to maintain flatness to within 0.005 cm (0.002 in.). The centers of all holes were located with a tolerance of 0.013 cm (0.005 in.).

Twelve tensile test specimens were obtained from the reference material, with six oriented parallel to the rolling direction and six transverse to the rolling direction. The modulus of elasticity and Poisson's ratio were determined from tensile tests of 11 of the specimens; the results from the twelfth were rejected because of apparent severe bending during testing. The results indicated that the average modulus of elasticity was 2.068×10^5 Pa (30.0×10^6 psi), with a 1.96 MPa (0.284 ksi) standard deviation; the average Poisson's ratio was 0.283 with 0.00186 standard deviation. No significant differences could be discerned between specimens representing the two basic orientations in the plates. One specimen was tested beyond the elastic range to determine the yield point, which was 240 MPa (34,800 psi).

2.2 Strain-Gage Layout

Each specimen was instrumented with electrical-resistance strain gages on one side of the plate in the immediate vicinity of the hole clusters. Single-element-type gages having 0.318-cm (0.125-in.) gage length (Budd gage No. C-6-121A) were oriented circumferentially along the edge of the holes, and three-element, 60° delta rosettes were used at other locations. The rosettes were oriented so that the No. 1 gage was parallel to the edge of a specific hole. The gage layouts used for the

Fig. 1. Hole-cluster pattern and details for flat-plate models. All plates were 0.95 cm (3/8 in.) thick. Dimensions in drawings are in inches; 1 in. = 2.54 cm.

Fig. 2. Biaxial tensile testing machine.

one-, two-, three-, and five-hole plates are shown in Figs. 3, 4, 5, and 6, respectively. The radial gage lines are designated as planes in the figures, and their respective angular orientations are indicated relative to the plate centerline. The basic sets of gage positions are indicated in the figures with distances from the center of the hole indicated in multiples of the hole radius [$R = 3.3338 \text{ cm (1.3125 in.)}$]. The single-element type of strain gage was used only at position 1; the three-element rosettes were used at all the other positions indicated in the figures. The number of gages used for each model was as follows:

Model	Number of Gages	
	Single Element	Rosette
One-hole	9	29
Two-hole	12	38
Three-hole	23	72
Five-hole	21	60

The gages were monitored using a Budd Model TC-22 strain indicator in combination with a 20-channel switch and balance unit, a printer-control unit, and a printer.

2.3 Test Description

The biaxial tensile testing machine shown in Fig. 2 consists of a heavy, rigid, steel framework mounted horizontally on its base and a system of thirty-two $1.78 \times 10^5\text{-N}$ (20-ton) pull-type hydraulic rams attached in sets of eight along each side of the plate. The loading rams on opposing sides of the plate were connected to the same pressurization system so that uniaxial tensile loadings could be independently applied to the plate in either direction. The applied hydraulic pressures were measured using a Bourdon gage; a precision dead-weight tester was added to the system for testing the three- and five-hole models. The $6.89 \times 10^7\text{-Pa}$ (10,000-psi) maximum operating pressure of the system was capable of exceeding the yield strength of the test specimens.

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POSITION	LOCATION	GAGE TYPE
1	1.00 <i>R</i>	SINGLE ELEMENT
2	1.07 <i>R</i>	DELTA ROSETTE
3	1.5 <i>R</i>	DELTA ROSETTE
4	2.0 <i>R</i>	DELTA ROSETTE
5	3.0 <i>R</i>	DELTA ROSETTE
6	4.0 <i>R</i>	DELTA ROSETTE

Fig. 3. Location of strain gages for single-hole model.

POSITION	LOCATION	GAGE TYPE
1	1.00 <i>R</i>	SINGLE ELEMENT
2	1.07 <i>R</i>	DELTA ROSETTE
3	1.5 <i>R</i>	DELTA ROSETTE
4	2.0 <i>R</i>	DELTA ROSETTE
5	3.0 <i>R</i>	DELTA ROSETTE
6	4.0 <i>R</i>	DELTA ROSETTE

Fig. 4. Location of strain gages for two-hole model.

POSITION	LOCATION	GAGE TYPE
1	1.00 R	SINGLE ELEMENT
2	1.07 R	DELTA ROSETTE
3	1.5 R	DELTA ROSETTE
4	2.0 R	DELTA ROSETTE
5	3.0 R	DELTA ROSETTE
6	4.0 R	DELTA ROSETTE

Fig. 5. Location of strain gages for three-hole model.

POSITION	LOCATION	GAGE TYPE
1	1.00 R	SINGLE ELEMENT
2	1.07 R	DELTA ROSETTE
3	1.5 R	DELTA ROSETTE
4	2.0 R	DELTA ROSETTE
5	3.0 R	DELTA ROSETTE
6	4.0 R	DELTA ROSETTE

Fig. 6. Location of strain gages for five-hole model.

Each hydraulic ram was load calibrated separately. The rams were found to give load efficiencies of 96% or greater.

Preliminary tests were conducted using two unpierced plates, which were instrumented with strain gages, to determine the capability of the system to produce the required uniaxial and biaxial stress states. In general, the strain field was uniform over a 90.3-cm^2 (14-in.^2) test area at the center of the plate. However, it was concluded that each plate should be tested both with the instrumented surface facing upward and downward, and the data averaged for the two positions. This procedure tended to eliminate any bending effects produced either by the weight of the plate or by eccentricities in the loading system.

The average uniaxial tensile stress per 6.89×10^6 Pa (1,000 psi) of hydraulic pressure was calculated from the strain-gage readings to be 22.7 MPa (3,290 psi) with a 6.4% maximum variation. Relatively small compressive strains were measured normal to the direction of loading. The system was also found to produce the desired biaxial loadings, as confirmed using superposition of the uniaxial test data.

The one-, two-, three-, and five-hole plates were each tested in the "normal position," with the gages facing upward, and in the "inverted position," with the gages facing downward. Each model was tested using 1:0 uniaxial loading, 0:1 uniaxial loading (when differences in geometry existed in the two directions), and 1:1 and 2:1 biaxial loadings to simulate stress distributions produced by internal pressure loading of large-diameter spherical and cylindrical vessels.

3. ANALYTICAL STUDY

The computer program TABLES (Thermoelastic Analysis by Boundary Point Least Squares) was used to analyze the experimental models. The program was developed at Battelle-Columbus Laboratories for the plane-stress analysis of multiholed flat plates. TABLES is one of the basic building blocks in the ongoing development of a method of analysis for clusters of nozzles attached to flat plates.

The principal assumption employed in the analysis is that uniaxial and biaxial loadings produce a state of plane stress in the plates. The solution to the plane-stress problem is obtained by determining the Airy stress function ϕ satisfying the homogeneous biharmonic equation

$$\nabla^4 \phi = \nabla^2 (\nabla^2 \phi) = 0 \quad (3.1)$$

and boundary conditions that are expressed in terms of normal, tangential, and shear stress components. A series of functions, each of which satisfies Eq. (3.1) exactly as well as the prescribed boundary conditions, constitutes the solution to a particular problem. The stress and strains, as well as other quantities of interest, can be related to ϕ and, hence, determined.

The general series solution to Eq. (3.1) in polar coordinates (r, θ) is given by

$$\begin{aligned} \phi(r, \theta) = & A_{0,1} \log r + A_{0,2} r^2 + A_{0,3} r^2 \log r + A_{0,4} r^2 \theta + A_{0,5} \theta \\ & + A_{0,6} + A_{0,7} r \cos \theta + A_{0,8} r \sin \theta \\ & + A_{1,1} [r \theta \sin \theta + (\frac{1-\nu}{2}) r \log r \cos \theta] \\ & + (A_{1,2} r^3 + A_{1,3} r^{-1}) \cos \theta \\ & + A_{1,5} [r \theta \cos \theta + (\frac{1-\nu}{2}) r \log r \sin \theta] \\ & + (A_{1,6} r^3 + A_{1,7} r^{-1}) \sin \theta \\ & + \sum_{i=2}^{\infty} (A_{i,1} r^i + A_{i,2} r^{i+2} + A_{i,3} r^{-i} + A_{i,4} r^{-i+2}) \cos i\theta \\ & + \sum_{i=2}^{\infty} (A_{i,5} r^i + A_{i,6} r^{i+2} + A_{i,7} r^{-i} + A_{i,8} r^{-i+2}) \sin i\theta, \end{aligned} \quad (3.2)$$

where ν is Poisson's ratio. The presence or absence of singularities, symmetry and antisymmetry considerations, and the boundary conditions dictate the terms to be used.

The numerical method that TABLES uses is the boundary-point least-squares method, also known as the collocation or point-matching least-squares method.⁸ This method fits a truncated series solution to the boundary conditions of the problem yielding an exact solution for the governing differential equation that approximately (in a least-squares sense) satisfies the prescribed boundary conditions. The final step in the solution procedure is to choose a set of boundary points, at which the boundary conditions will be written, as well as a set of interior points, at which the stresses and other quantities of interest will be calculated.

4. COMPARISON OF THEORY AND EXPERIMENT

This chapter gives comparisons between experimental and analytical stress distributions for each of the four perforated flat-plate models under the various in-plane uniaxial and biaxial loadings. The trial-series stress function is given for each loading condition, and plots of normalized stress vs radial distance are included for selected gage lines or planes. Polar plots of normalized stresses vs angular position around a specified hole of each cluster are also included. The stresses are normalized using the average equivalent stress in an unperforated plate; the linear distance is expressed in multiples of the hole radius. The qualitative degree of agreement for these comparisons is discussed in Chap. 5.

4.1 One-Hole Model

This model was subjected to only one uniaxial loading and to a 1:1 biaxial loading because of symmetry.

4.1.1 Uniaxial loading

The trial series employed for the uniaxial loading depicted in Fig. 7 was

$$\phi = A_{0,1} \log r + A_{0,2} r^2 + \sum_{i=1}^6 (A_{2i,1} r^{2i} + A_{2i,2} r^{2i+2} + A_{2i,3} r^{-2i} + A_{2i,4} r^{-2i+2}) \cos 2i\theta$$

with boundary conditions

$$z = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$z = \pm 18: \sigma_y = 0, \tau_{xy} = 0 ;$$

$$\text{hole edge: } \sigma_r = 0, \tau_{r\theta} = 0 .$$

The equations were written for points located along the edge of the hole every 4.5° from $\theta = 0$ to $\theta = \pi/2$ and at 1.27-cm (0.5-in.) intervals along the edge boundaries of the plate's upper-right quadrant. Comparisons of analytical and experimental results are shown in Figs. 8 and 9

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Fig. 7. Uniaxial loading of plate with single hole.

Fig. 8. Flat plate with one hole under uniaxial loading; tangential and radial stresses for gage line normal to loading direction. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 9. Flat plate with one hole under uniaxial loading; tangential and radial stresses for gage line parallel to loading direction. Note: 1 in. = 2.54 cm; 1 psi = $6.89 \times 10^3 \text{ Pa}$.

for gage lines normal to and parallel to the direction of loading, respectively. Comparisons for tangential stresses vs angular position θ are shown in Fig. 10.

4.1.2 1:1 biaxial loading

The trial series employed for the single-hole model subjected to 1:1 biaxial loading (see Fig. 11) was

$$\begin{aligned} \phi = & A_{0,1} \log r + A_{0,2} r^2 + \sum_{j=1}^4 (A_{4j,1} r^{4j} + A_{4j,2} r^{4j+2} \\ & + A_{4j,3} r^{4j} + A_{4j,4} r^{4j+2}) \cos 4j\theta \end{aligned}$$

with boundary conditions

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 1, \tau_{xy} = 0 ;$$

$$\text{hole edge: } \sigma_r = 0, \tau_{r\theta} = 0 .$$

The boundary-point set was the same as that used for the uniaxial loading condition.

Figure 12 shows comparisons between experimental and calculated stresses for gage lines parallel to each of the loading directions; Fig. 13 shows tangential stresses around the hole edge for the 1:1 biaxial loading.

4.2 Two-Hole Model

The two-hole plate was subjected to two different uniaxial loadings, one parallel to the plane passing through the axes of both holes, and one normal to this plane. A 1:1 biaxial loading was also applied.

Fig. 10. Tangential stresses around hole of one-hole plate under uniaxial loading.

Fig. 11. 1:1 biaxial loading of plate with single hole.

4.2.1 Uniaxial loading parallel to plane passing through hole axes

The trial series used for the two-hole plate subjected to 1:0 uniaxial loading (see Fig. 14) was

$$\begin{aligned} \phi = & [A_{0,2}r^2 + \sum_{i=1}^6 (A_{2i,1}r^{2i} + A_{2i,2}r^{2i+2}) \cos 2i\theta] \\ & + [B_{0,1} \log r_1 + \sum_{i=1}^{15} (B_{i,3}r_1^{-i}) \cos i\theta_1 + \sum_{i=2}^{15} (B_{i,4}r_1^{-i+2}) \cos i\theta_1], \end{aligned}$$

where (r, θ) and (r_1, θ_1) represent polar coordinates measured from the origin and hole center, respectively.

The boundary conditions are

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 0, \tau_{xy} = 0 ;$$

$$\text{hole edge: } \sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0 .$$

Fig. 12. Flat plate with one hole under 1:1 biaxial loading; tangential and radial stresses for gage lines 5 and 9. Note: 1 in. = 2.54 cm.

Fig. 13. Tangential stresses around the edge of the hole for the one-hole plate under 1:1 biaxial loading.

Fig. 14. 1:0 uniaxial loading of plate with two holes.

The equations were written for points located every 4.5° from $\theta \approx 0$ to $\theta = \pi$ around the hole edge and at 1.27-cm (0.5-in.) intervals along the edge boundaries of the upper-right quadrant of the plate. The calculated and experimental results are shown in Figs. 15 and 16 for gage lines 5 and 9, respectively, and in Fig. 17 for tangential stresses around the edge of the hole.

4.2.2 1:1 biaxial loading

The trial series used for the 1:1 biaxial loading shown schematically in Fig. 18 was

$$\begin{aligned} \phi = & [A_{0,2}r^2 + \sum_{i=1}^6 (A_{2i,1}r^{2i} + A_{2i,2}r^{2i+2}) \cos 2i\theta] \\ & + [B_{0,1} \log r_1 + \sum_{i=1}^{15} (B_{i,3}r_1^{-i}) \cos i\theta_1 + \sum_{i=2}^{15} (B_{i,4}r_1^{-i+2}) \cos i\theta_1] \end{aligned}$$

with boundary conditions

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 1, \tau_{xy} = 0 ;$$

$$\text{hole edge: } \sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0 .$$

Fig. 15. Stresses along gage line 5 for flat plate with two holes under 1:0 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 16. Stresses along gage line 9 for flat plate with two holes under 1:0 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 17. Tangential stresses around hole No. 1 of two-hole plate under 1:0 uniaxial loading.

Fig. 18. 1:1 biaxial loading of plate with two holes.

The boundary-point set is the same as that used for the 1:0 uniaxial loading described in Sect. 4.2.1.

Figures 19 and 20 compare analytical and experimental results for gage lines 5 and 9, respectively, while Fig. 21 shows the tangential stress distribution at the edge of the hole.

4.2.3 Uniaxial loading normal to plane passing through hole axes

The analytical results for the loading shown in Fig. 22 were obtained by superposition of results from the 1:0 uniaxial load described in Sect. 4.2.1 and the 1:1 biaxial load described in Sect. 4.2.2 (see Fig. 23). Refer back to these sections for the series terms, boundary conditions, and boundary-point sets.

The theoretical and experimental comparisons for gage lines 5 and 9 are shown in Figs. 24 and 25, respectively. Figure 26 compares tangential stresses at the edge of the hole.

Fig. 19. Stresses along gage line 5 for flat plate with two holes under 1:1 biaxial loading. Note: 1 in. = 2.54 cm; 1 psi = $6.89 \times 10^3 \text{ Pa}$.

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Fig. 20. Stresses along gage line 9 for flat plate with two holes under 1:1 biaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 21. Tangential stresses around the edge of hole No. 1 for the two-hole plate under 1:1 biaxial loading.

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Fig. 22. 0:1 uniaxial loading of plate with two holes.

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Fig. 23. 0:1 uniaxial loading on plate by superposition.

4.3 Three-Hole Model

The three-hole plate was also tensile edge loaded with two uniaxial loadings and one biaxial loading.

Fig. 24. Stresses along gage line 5 for flat plate with two holes under 0:1 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 25. Stresses along gage line 9 for flat plate with two holes under 0:1 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 26. Tangential stresses around hole No. 1 of two-hole plate under 0:1 uniaxial loading.

4.3.1 Uniaxial loading parallel to plate-symmetry plane

The trial series used for the 1:0 uniaxial loading of the three-hole plate depicted schematically in Fig. 27 was

$$\begin{aligned} \phi = & [A_{0,2}r^2 + \sum_{i=2}^{16} (A_{i,1}r^i) \cos i\theta + \sum_{i=1}^{16} (A_{i,2}r^{i+2}) \cos i\theta] \\ & + [B_{0,1} \log r_1 + \sum_{i=1}^{16} (B_{i,3}r_1^{-i}) \cos i\theta_1 + \sum_{i=2}^{16} (B_{i,4}r_1^{-i+2}) \cos i\theta_1] \\ & + [C_{0,1} \log r_2 + \sum_{i=1}^{16} (C_{i,3}r_2^{-i}) \cos i\theta_2 + \sum_{i=2}^{16} (C_{i,4}r_2^{-i+2}) \cos i\theta_2 \\ & + \sum_{i=1}^{16} (C_{i,7}r_2^{-i}) \sin i\theta_2 + \sum_{i=2}^{16} (C_{i,8}r_2^{-i+2}) \sin i\theta_2] \end{aligned}$$

with boundary conditions

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 0, \tau_{xy} = 0 ;$$

$$\text{hole edges: } \sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0 ,$$

$$\sigma_{r_2} = 0, \tau_{r_2\theta_2} = 0 .$$

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Fig. 27. 1:0 uniaxial loading of plate with three holes.

The equations were written for points located every 7.5° around both the hole whose center lies on the symmetry line (from $\theta = 0$ to $\theta = \pi$) and the hole in the half-plate-symmetry section whose center does not lie on the symmetry line (from $\theta = 0$ to $\theta = 2\pi$). Along the plate edges, the equations were written for points located at 3.81-cm (1.5-in.) intervals around the upper half-plate. Comparisons between analytical and experimental results for gage line 8 are shown in Fig. 28, and tangential stress distributions around the instrumented hole lying off the plate-symmetry line are shown in Fig. 29.

4.3.2 1:1 biaxial loading

The same trial series was used for the 1:1 biaxial loading (shown schematically in Fig. 30) of the three-hole plate as that used for the 1:0 uniaxial loading (see Sect. 4.3.1). Boundary conditions for the 1:1 biaxial loading were

$$x = \pm 18, \sigma_x = 1, \tau_{xy} = 0,$$

$$y = \pm 18, \sigma_y = 1, \tau_{xy} = 0;$$

$$\text{hole edges: } \sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0,$$

$$\sigma_{r_2} = 0, \tau_{r_2\theta_2} = 0.$$

The same boundary-point set as before was also used. The calculated and test results are shown in Figs. 31 and 32 for gage lines 8 and 12, respectively. Tangential stresses around the edges of the holes are shown in Fig. 33 for the hole located on the plate symmetry line and in Fig. 34 for the hole located away from the symmetry line.

4.3.3 Uniaxial loading normal to plate-symmetry plane

The analytical results for the 0:1 uniaxial loading condition (see Fig. 35) were obtained by superposition of the results from the 1:1 biaxial loading of the previous section and the 1:0 uniaxial loading of Sect. 4.3.1 (see Fig. 36). Refer back to these sections for the series terms, boundary conditions, and boundary-point sets.

Fig. 28. Stresses along gage line 8 for flat plate with three holes under 1:0 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = $6.89 \times 10^3 \text{ Pa}$.

Fig. 29. Tangential stresses around hole lying off symmetry line of three-hole plate under 1:0 uniaxial loading.

Fig. 30. 1:1 biaxial loading of plate with three holes.

Comparisons between analytical and experimental results for gage line 12 are presented in Fig. 37. Comparisons between the theoretical and experimental tangential stresses around one of the two identical holes located off the plate-symmetry line are given in Fig. 38.

4.4 Five-Hole Model

Because of the geometric symmetry along both the x and y axes, the plate with a five-hole cluster was subjected to only one uniaxial and one biaxial loading.

4.4.1 Uniaxial loading

The 1:0 uniaxial loading of the five-hole plate is depicted schematically in Fig. 39, and the trial series used for it was

$$\begin{aligned} \phi = & [A_{0,1} \log r + A_{0,2} r^2 + \sum_{i=1}^6 (A_{2i,1} r^{2i} + A_{2i,2} r^{2i+2} \\ & + A_{2i,3} r^{-2i} + A_{2i,4} r^{-2i+2}) \cos 2i\theta] \\ & + [B_{0,1} \log r_1 + \sum_{i=1}^{11} (B_{i,3} r_1^{-i}) \cos i\theta_1 + \sum_{i=2}^{11} (B_{i,4} r_1^{-i+2}) \cos i\theta_1 \\ & + \sum_{i=1}^{11} (B_{i,5} r_1^{-i}) \sin i\theta_1 + \sum_{i=2}^{11} (B_{i,6} r_1^{-i+2}) \sin i\theta_1] \end{aligned}$$

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Fig. 31. Stresses along gage line 8 for flat plate with three holes under 1:1 biaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

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Fig. 32. Stresses along gage line 12 for flat plate with three holes under 1:1 biaxial loading. Note: 1 in. = 2.54 cm; 1 psi = $6.89 \times 10^3 \text{ Pa}$.

Fig. 33. Tangential stresses around hole lying on symmetry line of three-hole plate under 1:1 biaxial loading.

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Fig. 35. 0:1 uniaxial loading of plate with three holes.

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Fig. 36. 0:1 uniaxial loading of three-hole plate by superposition.

Fig. 37. Stresses along gage line 12 for flat plate with three holes under 0:1 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 38. Tangential stresses around hole lying off symmetry line of three-hole plate under 0:1 uniaxial loading.

Fig. 39. 1:0 uniaxial loading of plate with five holes.

with boundary conditions

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 0, \tau_{xy} = 0 ;$$

$$\text{hole edges: } \sigma_r = 0, \tau_{r\theta} = 0 ,$$

$$\sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0 .$$

The equations were written for points located every 7.5° around both the center hole (from $\theta = 0$ to $\theta = \frac{\pi}{2}$) and the outer hole (from $\theta_1 = 0$ to $\theta_1 = 2\pi$) of the plate quarter-section, and for points at 3.81-cm (1.5-in.) intervals along the quarter-plate edges.

Figures 40 and 41 show comparisons between calculated and experimental results for gage lines 9 and 17, respectively. Figures 42 and 43 show tangential-stress comparisons around the hole edge for the center quarter-hole and outer hole, respectively.

Fig. 40. Stresses along gage line 9 for flat plate with five holes under 1:0 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 41. Stresses along gage line 17 for flat plate with five holes under 1:0 uniaxial loading. Note: 1 in. = 2.54 cm; 1 psi = $6.89 \times 10^3 \text{ Pa}$.

Fig. 42. Tangential stresses around hole at center of five-hole plate under 1:0 uniaxial loading.

Fig. 43. Tangential stresses around outer hole of five-hole plate under 1:0 uniaxial loading.

4.4.2 1:1 biaxial loading

The following terms were used in the trial series for 1:1 biaxial loading of the five-hole plate (see Fig. 44):

$$\begin{aligned} \phi = & [A_{0,1} \log r + A_{0,2} r^2 + \sum_{i=1}^5 (A_{4i,1} r^{4i} + A_{4i,2} r^{4i+2} \\ & + A_{4i,3} r^{-4i} + A_{4i,4} r^{-4i+2}) \cos 4i\theta] \\ & + [B_{0,1} \log r_1 + \sum_{i=1}^{16} (B_{i,3} r_1^{-i}) \cos i\theta_1 + \sum_{i=2}^{16} (B_{i,4} r_1^{-i+2}) \cos i\theta_1 \\ & + \sum_{i=1}^{16} (B_{i,7} r_1^{-i}) \sin i\theta_1 + \sum_{i=2}^{16} (B_{i,8} r_1^{-i+2}) \sin i\theta_1] . \end{aligned}$$

The boundary conditions were

$$x = \pm 18: \sigma_x = 1, \tau_{xy} = 0 ;$$

$$y = \pm 18: \sigma_y = 1, \tau_{xy} = 0 ;$$

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Fig. 44. 1:1 biaxial loading of plate with five holes.

$$\text{hole edges: } \sigma_r = 0, \tau_{r\theta} = 0,$$

$$\sigma_{r_1} = 0, \tau_{r_1\theta_1} = 0.$$

The equations were written for points located every 7.5° around both the quarter-hole at the center of the plate (from $\theta = 0$ to $\theta = \pi/2$) and the outer hole of the quarter-plate-symmetry section (from $\theta_1 = 0$ to $\theta_1 = 2\pi$). Around the quarter-plate boundary, points located at 3.81-cm (1.5-in.) intervals were used.

Figure 45 shows the calculated and experimental stresses for gage lines 9 and 13 for an outer hole, while Fig. 46 shows calculated and experimental comparisons for the center-hole gage line 17. Figures 47 and 48 compare tangential stresses around the hole edge for the center quarter-hole and outer hole, respectively.

5. CONCLUSIONS

The discussion of qualitative degrees of agreement below is divided into a discussion of the figures showing calculated vs experimental results for selected gage lines and a discussion of the series of polar plots of the tangential stresses for the gage position located nearest specific holes.

5.1 Selected Gage Lines

Table 1 provides qualitative ratings of the radial and tangential stress comparisons for each plate geometry (shown in Fig. 49), gage-line orientation, and loading condition. The arbitrarily assigned ratings of excellent, good, and fair represent relative differences between calculated and experimental normalized stresses of approximately 5% or less, 5%-10%, and 10%-20%, respectively. However, it should be emphasized that, without exception, all of the theoretically determined curves correctly represented the basic state of stress in the experimental models, even the complex stress state between holes in the five-hole plate for a gage line originating at the central hole of the cluster. (See Figs. 41 and 46.)

Fig. 45. Stresses along gage lines 9 and 13 for the flat plate with five holes under 1:1 biaxial stress. Note: 1 in. = 2.54 cm; 1 psi = 6.89×10^3 Pa.

Fig. 46. Stresses along gage line 17 for the flat plate with five holes under 1:1 biaxial stress.

Fig. 47. Tangential stresses around the edge of the center hole of the five-hole plate under 1:1 biaxial loading.

Fig. 48. Tangential stresses around the edge of the outer hole of the five-hole plate under 1:1 biaxial loading.

Table 1. Qualitative evaluation^a of calculated and experimental stresses for perforated flat plates subjected to uniaxial and biaxial in-plane loadings

Geometry ^b	Figure number	Type of stress and proximity to hole						
		Radial stress			Tangential stress			
		Close	Distant	Overall	Close	Distant	Overall	Overall
1X9	8	E	E	E	E	E	E	E
1X5	9	E	E	E	F	G	G	G
1XY9	12	E	E	E	E	G	G	G
2X5	15	E	E	E	F	G	G	G
2X9	16	E	E	E	E	E	E	E
2Y5	24	E	E	E	E	E-G	E	E
2Y9	25	E	E	E	F	G-F	G-F	G-F
2XY5	19	E	E	E	G	G	G	G
2XY9	20	E	E	E	F	G	G	G
3X8	28	E	G	E	F	G	G	G
3Y12	37	E	E	E	G	G	G	G
3XY8	31	E	G	E	F	F	F	F
3XY12	32	E	G	E	F	F	F	F
5X9	40	E	E	E	G	G	G	G
5X17	41	E	G	E	E	G	G	G
5XY9	45	E	E	E	G	G	G	G
5XY17	46	E	E	E	F	E	E	E

^aE = Excellent; differences between calculated and experimental results ≤5%. G = Good; differences between calculated and experimental results =5-10%. F = Fair; differences between calculated and experimental results =10-20%.

^bThe first number in these designations refers to the number of holes in the plate, the letters note the directions of tensile loading, and the last numbers denote the gage line under consideration.

Fig. 49. Plate geometries evaluated in Table 1.

The agreement between calculated and experimental radial stresses was consistently good for all plate geometries and loading conditions. Although reasonably good agreement was obtained for the tangential stresses, the greatest disagreement was observed for biaxial loadings on the two- and three-hole plates (Figs. 19, 20, 31, and 32) and for uniaxial loadings on the two-hole plate (Figs. 15 and 25). One possible explanation for this observed disagreement is that a true, plane stress state may not exist in the experimentally tested plates due to their finite [0.95-cm (3/8-in.)] thickness. Many of the points of greatest disagreement were in regions of steep stress gradients, and these differences might be attributed to the average values that are obtained experimentally using a strain gage having a 0.318-cm (0.125-in.) gage length. Any minor error in the position of a gage in these regions would compound the problem.

5.2 Tangential Stresses Around Holes

Comparisons between calculated and experimental tangential stresses for the gage position nearest a hole's edge were shown earlier in Figs. 10, 13, 17, 21, 26, 29, 33, 34, 38, 42, 43, 47, and 48 for the various geometries. In general, the comparisons are good.

The plots of tangential stresses are quite useful in assessing the capability of the analysis to identify the complex stress interactions of adjacent holes in perforated flat plates. It was concluded from these results that the analysis method accurately predicts the complex stress interactions, and in cases where some disagreement was seen, the analysis tended to be conservative with respect to the experimentally measured values. As a further check, theoretical results for the one-hole plate obtained using values contained in Ref. 9 (Savin) are shown on Fig. 10 for the uniaxial loading. These agreed exactly with the results obtained using the computer program TABLES.

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