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SMART LIMBED VEHICLES FOR NAVAL APPLICATIONS PART I. PERFORMANCE ANALYSIS

Andrew Weisberg and Lowell Wood

MASTER

30 September 1976

Prepared for The Naval Systems Division, Office of Naval Research
Arlington, Virginia
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SMART LMBED VEHICLES FOR NAVAL APPLICATIONS

PART I - PERFORMANCE ENVELOPES

Interim Report on Research Work on
Smart Vehicle Concepts for Military Use on the Ocean Surface

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FOREWORD

This is an account of research work in smart, unmanned limbed vehicles for naval warfare applications performed during the latter part of FY76 and FY76T by the Special Studies Group of the LLL Physics Department for the Office of Naval Research, under ONR Order #N00014-76-F-0019, along the lines specified in LLL Phys. Prop. 76-100, which was submitted to ONR in March 1976. This document reports satisfactory completion of the first of two items constituting this proposal's Work Statement; completion of the second item is reported in the second volume of this report.

The staff of the LLL Smart Vehicles Project during the reporting period included Hon Wah Chin, Donald Duerre, William Newman, Joseph Simpson, Andrew Weisberg and Lowell Wood, of the Special Studies Group, who were assisted in specialty functions by other staff members of the Laboratory.

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WHY RNV?

Remotely Navigated Vehicles, or RNV's, are ocean-going vehicles that operate without onboard human control. The single concept that defines the small universe of machines designated RNVs is that they all function without using the proximate capabilities of a human being.

Homo sapiens is, as far as vehicles are concerned, almost the ultimate guidance system. He possesses a variety of sensory equipment, and can make decisions based on his vast flux of sensory data far more quickly than can essentially all the products of modern technology. In a half second, a human can perceive and recognize a complex feature in his visual environment and act thereon. No aspect of Man's technology has even come close to this level of performance.

Besides pure input/output capability, the human mind, when used as a controller, has extraordinary scope and generality. One Homo sapiens can recognize millions of distinct patterns as easily as he recognizes one. The timing required simply to hit a baseball requires precision on a millisecond time scale. The entire human body is capable of very precise and repeatable complex motions (ballet, competition figure skating, etc.). Not only in precision and complexity, but in sheer speed of complex control (as, for example, in various of the Japanese martial arts, where individuals execute over a dozen complex and stylized distinct offensive and defensive motions per second), the human being remains unequalled by his mechanical creations.

The capabilities just outlined, plus the fact that a single human is capable of performing a large number of very different control functions, make Homo sapiens the best attainable control system element for any problem within his range. Obviously, there are systems both too fast or too slow for a human to control, but for the class of control problems involved in vehicular

control, the human is seldom mismatched to the required timescales.

With all of the aforementioned going for human vehicular control, why would one want to build RNVs? There are two central reasons for not using human beings as onboard controllers for all sea-going military vehicles. The first is mechanical in nature. The human mind and senses may constitute a versatile guidance system, but the package they come in has a relatively narrow region of applicability. Human vehicle pilots not only take up much space and need bulky supporting hardware (to provide air, temperature, food, etc.), but they also impose strict mechanical limits.

A human being cannot be expected to function at over 10g acceleration, and in many cases permanent damage can result above only 3g. Likewise, beyond the range from 0°C to 70°C a properly protected human might survive, but with severely impaired ability to provide apt guidance to a vehicle. Again, the ambient pressure range over which a human can function is small, and, worse yet, human performance drops rapidly with the slightest injury.

Indeed, often a good pilot can bring a severely damaged airplane home in wartime on a wing and a prayer, but how many otherwise functional airplanes would return if the pilot and copilot were damaged? The unrestricted military effort it takes to render a modern military vehicle inoperative is generally substantially larger than the effort it takes to seriously degrade its human guidance. One can well imagine that an automobile would often be quite functional after being barraged with an 8-gauge shotgun, but not so if any of that shot hit its driver.

So it becomes evident that the human pilot/controller is often the weakest component of a modern military vehicle. Unless his environment is precisely maintained, his performance drops (e.g., how well does a pilot perform when his oxygen level is 30% low?). Also, human

performance as a controller can be erratic for incidental and unpredictable physiological or psychological reasons.

Besides these liabilities as a controller, the human pilot often imposes some strong limitations on vehicle design. Not only is there a limit at about 3g acceleration, there is a lower limit on volume (somewhere near 20 feet³) and 5 feet in one linear dimension. A vehicle can't be lighter than its pilot, and often needs extra mass and volume to support the load of his weight. Therefore, human-piloted vehicles have a definite smallest size, approached in the space program.

Also, there are more subtle and pervasive limits to conventional vehicle design. Complex motions at 1 gravity accelerations are apt to induce pilot nausea, ruling out rapid evasive maneuvers. Rapid altitude changes upset the human sense of balance. Heavy sea states make conventional ocean surface vehicles of small size uncontrollable, again primarily due to human controller nausea. The great reliance of humans on vision often requires large windshields, which are obviously one of the weakest points in a conventional vehicle.

Thus, despite the vast advantage of the human pilot/controller in the performance of control tasks, there are evidently substantial reasons for investigating the systems advantages of RNVs. The second line of justification for RNVs has little to do with vehicle performance, but is involved with the logistics of using human capabilities wisely. These sort of arguments are perhaps even more germane to prospective military exploitation of the RNV concept.

Humans may make great controllers, but they require substantial amounts of time and money to train. In general, no human is suited to a given control application without some training. Training is not only costly (in dollars and effort), but it ties up

some of the available vehicles for training purposes, and, depending on the type of control task, may end up in destroying several vehicles before this training is complete.

It may be argued that the eventual performance available from human controllers makes this all worthwhile. In fact, depending on how one costs the alternatives, a year of training to pilot a particular vehicle may not infrequently cost less than the lower-performance electronics guidance alternative. But economy is not the only logistic consideration.

Often it is found that not all people are suitable to a given control task. Few are suitable as aircraft pilots; very, very few are suitable for astronauts; and even a large fraction of military personnel are unsuited to drive jeeps. This exclusion includes not only those who are incapable of the desired control function, but must also include the very frequent exclusion of those whose contributions are more in demand elsewhere. People who can think unusually well are put to best use as executives, researchers, physicians, administrators, etc. People with relatively great mechanical ability are needed to maintain the ever-expanding number of mechanical contrivances which support military operations. One could continue with these examples, but the general thrust of this argument is clear: The vast majority of service personnel are more aptly employed doing other things rather than becoming vehicle pilot/controllers.

Being a pilot or navigator (or other type of controller element) in the military is a career that lasts only as long as it takes for new talents (usually administrative or leadership ones) to develop, at which time optimal use of human resources dictates promotion to some other task. Thus, although the fraction of the available population for pilot/controller applications may not be the same in the armed services as it is in the general population, it is similarly limited.

What's worse, the military sector is also faced with the spectre of combat casualties that may rapidly depopulate the ranks of available, trained pilot/controllers in wartime. The more complex the vehicle, the longer the training of a replacement takes. If, in fact, a military organization has optimal manpower usage prior to combat attrition, such casualties will also deoptimize the system by requiring replacements who may have been better used elsewhere.

To some extent, net combat losses of trained pilot/controllers may be avoided by a military commander, though at the cost of larger losses in other manpower sectors. Not only does vehicle controller replacement required removal of talented manpower from other sectors, it introduces a time lag due to training time, implying that costly hardware will be underutilized during a military crisis. The alternative to the possibility of large time lags for replacements is to cut down on training, or to maintain an over-optimal pool of trained manpower during long intervals between periods of conflict. With or without shortening training periods, it is certain that when a vehicle pilot/controller must be replaced, there must be at least one new trainee available. This man will not have the same level of experience as his predecessor, and thus the net talent of the pool of men available to pilot/control the given set of vehicle has necessarily decreased, due to that casualty.

This is a more serious loss than the deoptimization of the service which has suffered the casualties, since a combat vehicle with a less trained pilot/controller is corresponding more likely to be lost due to hostile action. Since the vehicle is destroyed when its operator perishes, a less-trained operator is more likely to be lost in combat, and to take an expensive vehicle with him. If effectiveness of any fighting force is considered probabilistically, combat losses of pilot/controllers at least initially decrease

the expected performance of pilot/controllers, which in turn increases the personnel casualty rate and the loss rate of expensive machinery. This downward spiral, unless reversed by suppression of enemy capability or increasing combat effectiveness of remaining/replaced personnel, clearly leads to an untenable situation. The Battle of Britain a third of a century ago quite clearly illustrated this point.

Of course, a modern armed service is capable of dealing with such manpower problems under casualty-rate conditions up to some maximum level, but one of the salient advantages of the RNV concept is that personnel replacement problems are eliminated. If an RNV is lost, no trained human is lost. This personnel damage elimination, coupled with the fact that the presence of a human pilot/navigator is one of the most limiting elements in military vehicle performance, constitutes much of the case for RNVs.

Another portion of the case for RNVs concerns the likely constraints on future U.S. military capabilities. The question motivating this consideration is: How is the future of the U.S. military system going to differ from the present, and how will this shift from present circumstance impact the applicability of RNVs which will only be available sometime hence? The answer seems to be that the RNV is one of the few viable long-term alternatives for the U.S. military, in the following scenario.

First, simply assume that the world does not change to major extents over the next decade. However, trained pilot/controllers resign from the service or are promoted to other jobs. The new generation of such personnel that replace them must be trained, at considerable expense. If one replaced this cycle for a given mission set by using an RNV, retraining of officer-level personnel is never needed, but simply that of enlisted level maintenance personnel, whose average time-in-grade is substantially longer than that of vehicle controller-officers. Service for the hardware that replaces the

pilot/controller is likely to require less manpower than the equivalent pilot/controller training program, and the "training level" of RNVs does not degrade with time after the last major exercise, unlike the problems encountered with human controllers.

This is one aspect of the future; consider now the longer term outlook.

Smart RNV technology will continue to advance with time, implying sustained production of new vehicles. The cost of the RNV components and thus entire systems will therefore drop according to a composite learning curve for the relevant set of technologies. Of course, this economic gain will be offset to some extent because the newer RNVs will tend to be improved ones, and thus will cost relatively more. But as the amount available for a given complement of such vehicles rises slowly with time, one can be rather certain that the unit price of a representative RNV will rise slowly, vehicle performance will rise very markedly, and the price-to-performance ratio will drop sharply.

So, if one is willing to posit near term extension of the present pattern of technological progress, the RNV will become ever more cost effective, relative to the oft-replaced, constant-capability human controller. It is also well to note that a single RNV potentially replaces several human controllers in wartime, since it can operate 24 hours a day.

Next, one may inquire about the future political effectiveness of the RNV concept. The American public, it appears, is becoming increasingly sensitive to casualties incurred in military operations. Recent experience indicates that public willingness to accept even relatively very small numbers of military casualties without very broad consensus regarding the strategic issues requiring military action is presently small, and is likely to remain so for sometime. While the American public may be willing to make substantial military-related sacrifices in the future,

it may well be much less tolerant of military casualties seen to be avoidable in any major sense. RNV employment in a military engagement might reduce casualties sharply, simply because there would be no friendly personnel at risk associated with RNV usage. As such, RNVs might well be increasingly popular with the American public, due at least to the reduced casualty rates sustained by RNV-supported U.S. forces.

This last point is perhaps the most important of the future-directed ones, and the one with the least quantifiable supporting evidence at present, since it deals implicitly with the military effectiveness of RNVs. The U.S. has interests to defend in almost every corner of the globe. As demonstrated very recently in the Angolan crisis, the nation is not now willing to spend troops or money on conflicts perceived to be only marginally important. A sizable RNV force is not only one alternative for providing the better grade of defense that the future will require, but it is also a casualty-free way to effectively (and possibly even covertly) assert an interest in this sort of marginal conflict.

In addition, the U.S. may also look forward to a better grade of defensive capability through RNVs. Armed conflict presently involves both men and machines. Of these two, the capabilities of the machinery has increased sharply with time, covering orders of magnitude in many instances over the last few decades, while those of their human operators have not done so to discernable extents. One can therefore forecast (if one doesn't observe it already) that any weapons technology's advance in time to come will be limited by the capabilities of the men operating it. Already, technological advances in nuclear weapons systems have necessitated the removal of humans from many controller functions in strategic force structures to decrease strike/strike response times.

The RNV may well be a similar "out" for tactical

force structures. RNV technology will advance beyond the fixed capabilities of humans as vehicle controllers. Indeed, it is not unreasonable to expect that mobile military hardware that exploits RNV technology will be much superior to that which does not, two decades hence: if we expect technological advance to continue, RNVs will eventually emerge as superior to any manned vehicular technology.

If one is willing to accept this prediction, based as it is on a not-improbable future, then we immediately can see why RNV development and utilization is one of the few optimistic approaches for the U.S. military in the intermediate-to-distant future. Not only will the RNVs save lives in defense, reduce the cost of offense, and generally outperform other mobile military hardware, but they are also uniquely suited to American deployment. The U. S. does not have a monopoly on the electronic technologies that make the RNV development possible at present, but it does have a substantial lead, particularly in large scale production capabilities in high technologies.

Indeed, the U. S. is the only country that could produce a militarily useful RNV today. Furthermore, the American approach to production is thoroughly oriented toward high-technology batch production, as well as ultra-low lead times into such production. Several of the key component technologies for RNVs are practiced only in the U.S. At present and with the commercial competition-induced pace of advance, it is to be expected that we would have at least half a decade lead over Europe in RNV technology, and more than one decade's lead over Russia and China.

Once the U.S. enters the RNV business, it's not simply a matter of waiting for everyone else to catch up. In any fast learning-curve endeavor (which semiconductor electronics has been and RNVs will be), a five-year head start constitutes effectively cornering the market. Having a lead, one gets more into production first and moves more money into the new endeavor; more talented people join the new field, and progress

continually accelerates. In high technology industrial competition, this simply implies that is an initial five-year gap widens, as it indeed has in international military competition. One would expect that strenuous efforts might be made by others to narrow such gaps by nationally-sponsored crash programs. Recent technological history suggests that one can expect that only those willing to make huge expenditures would be at all successful in such programs. The Soviet Union, the competition most capable of affording such programs, would pay one of the highest premiums, because of the wider initial gap, and due to its relative unfamiliarity with large-scale production of most high technologies. China is even less prepared to close such an RNV gap, either technologically or temperamentally.

The U.S. can realistically expect to gain only a temporary advantage in RNV-centered military capability from its current technological posture. Knowledge is easy to transfer, technology can be bought, and the gap will eventually narrow. Aside from the advantage the U.S. would enjoy from such a lead in the interim, a permanent advantage for the U.S. will result: The introduction of the RNV will change the basic nature of tactical-scale warfare to one that the United States will be relatively more suited to waging.

As the RNV advances, its effectiveness against armed humans will increase. Using humans against RNV-equipped forces will become increasingly futile. Replacing destroyed RNVs is just a question of mass production, an endeavor to which America is far better suited to the replacing of trained personnel. In time to come, the U.S. will very probably find it ever more difficult to send its citizens into combat, anticipating substantial casualties; this has already occurred to a considerable extent. RNVs not only can substitute for such an ultimate levy on the citizenry, but they can provide the sort of extraordinarily potent conflict capability America has always lacked: the solitary suicide attack, the wave of expendable

attackers, and the one-way mission pilot/navigator. As such, our nation, which will have ever fewer people to expend as casualties in the future, may nonetheless be able to maintain tactical parity or superiority with respect to adversaries who hold human life cheap, due to the ascendancy of U.S. RNV technology.

WHY LIMBED VEHICLES?

The primary purpose of this document is to consider RNVs for use by the U.S. Navy. The term "vehicles" is, at first, hard to apply to naval craft because, in general, the word invokes images of transportation, rather than armament. When the RNV concept is adapted to naval requirements, however, such a vehicle becomes simply a mobile piece of hardware whose major purpose is military effectiveness at the task for which was designed.

Thus, an attempt to fit the current naval definition of a vehicle to present hardware produces the simplistic equation that a vehicle for naval use is a boat. It could be anything from a submarine to a hovercraft, but every naval vehicle in existence at present whose medium is the ocean is a hulled craft of substantial size.

The relatively novel thesis of this document is that the optimal type of vehicle for naval RNV applications will most often be a limbed vehicle. By limbed vehicle (sometimes abbreviated as LV), we mean a vehicle whose movement is effected by the movement of limbs.

Limbs themselves are well-defined objects, being collections of rigid mechanical elements interconnected by rotating or sliding "joints." The definition is not meant to include non-rigid structures (an octopus has 8 legs but no limbs), or complicated interconnections such as 4-bar linkages or lazy-susans. A limb is simply a set of rigid segments connected end-to-end, with some sort of joint where motion is allowed between each segment, and the whole limb serving to connect the bulk of the vehicle (the body) with a "foot" that produces thrust (see Figure 2).

Once one forms an adequate picture of what is meant by a limb, one integrates several such to describe a limbed vehicle. Such a vehicle has a central body with

Figure 1

ALTERNATIVE LIMBED-VEHICLE DESIGNS

Biped

Wide
stance

Medium
number
of legs
(4 or 6)

Can be in Bug or
Horse configuration;
vehicle could be
convertable

Tall stance

Many legs

Omnidirectional
Radially
Symmetric

OR

Long Body
Can lose many legs
and still operate

Figure 2

CONCEIVABLE TYPES OF LIMBS

several limbs (or "legs") attached to it. These legs can be moved in response to control signals originating in the body. Since the legs are composed of rigid segments, the only motion that results is the changing configuration of the joints (see Figure 3). The jointed nature of the legs dictates that they will have a definite, restricted range of achievable geometries, which can be attained upon receipt of control signals from the body via the action of tractor-pressor-equivalent devices, initially to be located in the limb joints.

Thus, all that remains of the conceptual design task is to envision a reasonable configuration (see Figure 4) for an LV and to postulate that the proper motion of the limbs can cause the whole vehicle to walk, hop, or scuttle, much like familiar motions of animals. Obviously, some of the most rudimentary configurations of limbs can effect locomotion. One-legged amputees can often locomote well a second leg, and people can walk on unjointed stilts. However, it is evident with a little reflection that some configurations are better than others for a given application (e.g., one can hardly imagine hopping with stilts).

So, the discussion of the attributes of such a limbed vehicle could begin with a rather useful simplification: Assume that limbed vehicles can be made to do any sort of limbed locomotion that is familiar from biological examples. (This assumption is necessary in order to discuss the possible behavior of LV's, because so few such have been built, and those have been of such relatively poor performance, that it has hardly demonstrated yet that biological-type performance is presently technologically attainable.)

More fundamentally, since Nature has done it, already we know limbed locomotion can be effectively accomplished; one wonders first if it can be done with non-biological devices and, second, if there are not perhaps ways to accomplish limbed locomotion better than that done by biological systems.

FIGURE 3

BACKSLIDING RUN ON WATER

Only two legs shown for simplicity

⊕ indicates fixed inertial reference point

Note how feet backslip substantially compared to forward progress of vehicle whenever one attempts to obtain thrust from them

In the absence of hard experimental proof of capabilities, one can assert a positive answer to the first of these two questions as an assumption which will be discussed in great detail later. Assume that present technology is capable of achieving all biological limbed-system locomotions. The second question will be ignored for the present, since it hardly pays to worry about improving the sorts of motion until one is sure that they can be done at all. Thus, for this section we will assume that limbed vehicles can be made to move by the motion of their joints in much the same way that animals derive locomotion from their limbs.

However, even assuming one could build an LV and get it to move like a dog or a man or a lobster, why would it be desirable to do so? As will become evident later, the control of all of those joints required to get just a few desirable vehicular motions is not all all a simple matter. How could an LV possibly make a better naval vehicle than those built today?

There are a number of arguments which have been used to make the LV concept look sensible in contrast, for instance, to a wheeled vehicle for an all-terrain vehicle. However, when it comes to operation at the air-water interface, the LV has the potentiality of out-performing a comparable hulled vehicle by a wide margin. A limbed vehicle walks on the water, like a water-strider or a water-bug (see Figure 4). (Of course, it must use buoyant feet rather than surface tension to support its weight on the water surface.) With pontoon feet, a water-traversing limbed vehicle (WLV) can move about on the surface of the ocean, similarly to the way a man moves over powder snow with snowshoes.

When a hulled naval surface vessel moves in the water, it necessarily does so against the drag of its (large)hull in the water. Likewise, an LV waterbug experiences drag on its pontoon feet, but the velocity of the LV is not the same as the velocity of its feet, because the LV can lift its feet out of the water.

ON FLEET PICKET DUTY

Indeed, when limbed creatures like humans walk on land, they move at some given velocity but their feet do not move at all with respect to the ground while in contact with it. Likewise, if the pontoon feet of the LV had infinite drag with respect to horizontal motion in the water, they would not move when placed in the water, and yet the whole LV could progress at a velocity limited only by the force it could generate at its joints.

Thus, limited walking on water appears conceptually attractive, if the feet don't move. (Indeed, one could envision well-engineered, ultra-high drag foot designs that approximate this limiting case, but still the feet would move somewhat. Fortunately, we know that it is quite possible to get locomotion from feet that don't have a solid "grip." Both the water-bug and the human ice skater demonstrate this sort of motion.)

Such analogies strongly suggest that if a water-bug-type LV were thrusting against its feet, which were moving backward at the turbulence onset threshold of the order of 30 knots, then the entire LV could certainly progress forward at well over 30 knots. Drag of a vehicle's buoyant surfaces, the enemy of high speed naval transport for millenia, thus potentially becomes its ally, the more so as the drag is increased.

Speed is not the only advantage of limbed water-surface motion. Indeed, the same arguments would hold for wheeled systems since a high-drag wheel would slip very little with respect to the water as its axle translated forward. Thus, a wheeled vehicle might also seem to make substantial sense as an RNV, if velocity on a plane were the only concern. Unfortunately, the ocean surface is not flat, and waves would tend to immerse one wheel more than another from moment to moment, increasing drag in an erratic pattern would increase thrust on the more immersed wheel and push the vehicle off course. Thus, the trajectory of a multi-wheel vehicle is corrupted (eventually to distraction) by the action of ocean waves.

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FIGURE 5

HIGH SPEED DASH

As RNVs increase in capability following initial deployment, they may come to be employed as unmanned analogs of PT boats, and be used as expendable platforms for both reconnaissance and attack purposes during naval engagements.

The comparative failure of the single-wheel design serves to point out several of the relative advantages of the LV concept. A buoyant, gyro-stabilized single-wheel could roll over the waves without immersion depth or cruise velocity changing. But such a vehicle would be possessed of a large amount of rotational kinetic energy, and, as such, would be hard to stop or turn. A multi-wheel version of such a vehicle would have few such problem (though it would have others) and could stop by applying force between the wheels and its axle, while the single-wheel craft is singularly unmaneuverable. In comparison to either, however, the limbed vehicle is highly maneuverable. Its legs are relatively light and can move much more swiftly than can the body. For instance, braking from 100 knot speeds is accomplished simply by ceasing to run, putting all feet in the water in a balanced fashion, and letting drag do the deceleration. Likewise, turning is only a question of gait modulation. There is no intrinsic or preferred direction of motion, since a limbed vehicle could be designed to move equally readily sideways and backward, as well as forward.

In short, the statement that RNV velocity is not substantially limited by water drag is as valid for other motions as it is for the "forward trot". Thus, one could expect a WLV to be able to aptly dodge and weave, and to turn and stop. It could be expected to walk on waves, adjusting for variable foot immersion and prejudging the time-varying slope of the wave to be navigated. In comparison to any other craft that has ever traversed the ocean surface, the WLV could indeed be termed "light-footed."

Having developed some basic features of the WLV concept, we consider now some description of what it can't do, and why it nonetheless constitutes a highly useful RNV. If one did not carefully note the scale of the drawings in Figure 4 and 5, one could well conceive that the whole idea of WLVs was ridiculous. Relative to the scale of conventional Naval craft, a WLV must be very small. The reasons for this are structural (as will be discussed

later). A water-bug 40 feet tall is all but unattainable, due to the L^3 growth in structure weight. Very ponderous WLV's with perhaps 30-foot legs may be possible, but the required pontoon feet would be so large that it would be very difficult to move them in and out of the water sufficiently rapidly to achieve acceptable WLV performance.

Likewise, a WLV must be light if it is to realize the speed and maneuverability inherent in the concept, realized with present technologies. Thus, between size and speed limitations, it appears that human pilots would markedly increase the difficulty of achieving any notable WLV performance with present materials and power handling technologies. On the other hand, if a WLV could be operated as an RNV, the desired size and weight of the vehicle could be brought back within highly practical limits.

Thus, to realize any sort of WLV that would be of military use in the foreseeable future, one must satisfactorily address the RNV concept. It should be noted, however, that exchanging a human pilot for effectively equivalent on-board electronics is a step which implies a most challenging miniaturization problems for electronics packaging technology: that of packing several computers into a few pound package. Because these problems apparently involve only the full application of extant technologies, it may be claimed the choice of RNV is a viable one, at least with respect to control systems.

Considering the mating of concepts from the viewpoint of RNVs, the combined WLV-RNV concept seems sufficiently natural that one might claim the WLV was the ideal implementation of the RNV for naval applications. The major reason for such a statement derives from an examination of Navy RNV applications. It seems evident that the prime early applications of RNVs will be in those missions in which there is a high risk to friendly personnel. These include fleet picket duty, search-and-destroy activities (on rivers and among

WLRNV DEPLOYMENT AT MISSION COMMENCEMENT

WLRNVs may be most aptly deployed from ships via passenger aircraft escape-type flexible slides extending to the ocean surface from either ship upper or lower decks (e.g., through hull ports).

islands), wide area deployment of detection/surveillance gear under generally hostile conditions, amphibious landings, and deep penetration reconnaissance.

With such applications in mind, the ideal vehicle would seemingly be one which could move at speeds of at least 50 knots, was amphibious and was capable of rapid evasion, e.g. had WLV capabilities. The RNV scheme not only insures that there will be no risk to friendly personnel but its WLV implementation contributes the attributes of great hardness against hostile fire, low radar cross-section and suppressed vehicle signature, as well as a viable long loitering operating mode.

Consider each of these in turn: RNVs may be substantially harder than small conventional vehicles, because there are no soft-but-crucial humans on board whose severe injury would imply mission abort and probable vehicle loss. In comparison, the WLV is much harder than a hulled RNV. If buoyant feet support the WLV, there are few practical ways to sink it. If its control system were sufficiently versatile, one or more of its legs could be damaged or shot away entirely, and the WLV could still limp through most aspects of its total mission envelope.

A limbed vehicle could be made into a very hard RNV indeed. Not only could it be expected to readily withstand, for instance, 20g buffetting from a near miss, but, with several legs, the vehicle might well be able to sustain one or more direct hits with only slight loss of mission effectiveness. Given a few legs to spare, the only critical functions that might result in failure of the entire vehicle when hit would be the control computer system and the vehicle power supply. As the RNV technology advances, increased miniaturization will allow increases in reliability and hardness in these respects as well, primarily by positioning several redundant guidance computer packages at various locations throughout the vehicle body. This leaves the primary power supply as the most vulnerable function, and it can

FIGURE 7

AIRBORNE DEPLOYMENT

FIGURE 8

WLRNV DEPLOYMENT BAY IN RECOVERY PHASE NEAR THE END OF A SUCCESSFUL MISSION

While the relatively very low cost of WLRNVs is likely to make them very attractive for one-way missions (particularly when return capability may be traded off for augmented sensor or ordnance loads), recovery of those vehicles which survive a mission, to the extent that they can return to their originating ship, may often be desirable. Such recovery may consist simply of the inverse of the deployment procedures.

be hardened to very great extents with only small increase in total vehicle weight.

Thus, one can form the picture of the archetypical WLRNV which is released from a Navy ship (or even dropped from a plane) and goes trotting off over the waves, zig-zagging abruptly in a pseudo-random pattern to avoid being detected or, if detected and fired upon, being hit. If, by some chance, the enemy has weaponry sufficiently perceptive and responsive enough to detect and track the rapid, erratic motions of these small, wave-walking targets, and of sufficient caliber to do more than glance off its bulletproof fabric shell, then a direct hit will be strong enough to amputate, knock over, and possibly destroy it.

However, being wounded and knocked over does not mean being neutralized. If the stricken vehicle has enough equipment working, it may right itself out of the water and continue its mission. If the mission can be characterized as delivery (of explosives, surveillance gear, etc.) then reduced speed, possibly awkward motion ("limping") to the drop point may well be sufficient. If too little locomotive capability remains to be able to effect some gait, the damaged WLRNV may still move toward mission completion by a paddling motion on the ocean surface. In most any event, it may be able to radio back the fact and location of enemy fire, which in many military environments (e.g. nuclear war at sea) may compromise its (much more expensive) attacker with usefully high likelihood.

This brings up a facet of WLRNV utility that will be discussed in more detail in the next Section: there is no necessity for the WLRNV to return from its mission. If there's enough of the vehicle left for cost-effective salvage or reuse, then the RNV should strive to return to a pre-specified point for pickup, if it has the on-board power and energy available. Obviously, one would like to recover all RNVs from a low-hazard mission, but even in such cases the choice may be made to enhance vehicle hardness (or other parameters, such as ordnance

Figure 9

WLRNV MODES OF OPERATION

I. LOW-SPEED PATROL

Feasible even in heavy sea

Can be accompanied by weaving and jerking to avoid counteraction

II. LOITERING

Hides from detection on water surface;
can loiter without locomotive power drain

Floats without drifting

LIFT (Takes weight off feet)

III. LIFTING RUN

High velocity delivery in moderate sea

Can zig-zag for evasion, but cannot perform high-acceleration to avoid counteraction, due to aerodynamics

load) by spending mass that would otherwise be devoted to energy storage for return to the pickup point.

Now that a picture of the possible hardness of the WLRNV has been sketched, one can consider one of the strongest single motivations for the WLV concept: low vehicle cross-section and signatures. More important, perhaps, than the ability to survive a hit is the ability to avoid being hit. Unlike conventional naval craft, the body of the WLV can dodge enemy fire at several gravities of acceleration, allowing it to escape a hit by small projectiles or small missiles, if they are detected at adequately great distance. Even more fundamentally, though, the WLV provides a very small, possibly masked target for its adversaries to locate and track in order to fire at it.

Firstly, the WLRNV is small. Properly coated, it would probably be hard to visually located against the ocean several hundred feet away, under even the best lighting conditions. Waste heat can be kept to a minimum by employing energy storage instead of power generation in areas likely to be IR-monitored. In this way, the small WLRNV leaves almost no infrared signature; if necessary, none of the waste heat it does produce may be rejected to the atmosphere, but rather may be injected under the ocean surface, so as to leave no IR scar in the air or on the air-water interface.

The mode of locomotion thus gives the WLV a distinct no-traces operation. Since it moves considerably faster than its feet, those feet disturb less water than would hulled craft of equal weight in moving the same distance. Since there is no propeller effect, kinetic energy is imparted to the water to effect vehicle motion only sporadically, and there will thus be essentially no wake behind a WLRNV.

Besides being small and hard to spot by optical or IR methods, the nature of the motion an RNV executes on a mission makes it very difficult to track. The WLRNV can skitter up and down across the waves. Part of its

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FIGURE 10

PASSIVE TRAILING OF AN ENEMY FLEET

The relatively low detectability of WLRNVs suggests that they may be ideal for monitoring enemy ship movements, following airborne or submarine emplacement near their trailing targets. Intrinsically capable of sustained 50 knot speeds and 1000 mile ranges, they could shadow enemy ships for periods of a few days to several weeks, covertly reporting as appropriate.

FIGURE 11

NIGHTTIME SURVEILLANCE OF AN ENEMY FLEET

WLRNV surveillance of enemy ships may be especially effective at night, when human-directed countermeasures may be of much lower efficiency, but night-vision-equipped WLRNV performance may be as high as during daylight hours.

evasive maneuvers may include hiding behind large waves. With sufficiently perfected vehicle control and guidance systems, the WLRNV could operate in very heavy sea states and be rendered virtually untrackable due to the obsuring effect of waves much higher than the vehicle.

But of what use is all this, one might well ask, in the face of radar? Firstly, radar is scattered by water, and ocean waves in even moderate sea states are large enough to effectively block and diffract WLRNV-seeking radar beams. Thus, a large 6-foot high WLRNV could manage to avoid a good side-looking radar fix in a 3-foot sea. Unless the enemy radar employs a quite unconventional sweep scheme, the use of numerous, swift WLRNVs can make effective targeting of them a difficult task in the face of simple electronic countermeasures.

Even better than intrinsically low radar detectability, the WLRNV could well be designed specifically for radar evasion. There is no a priori reason why the RNV need be mostly made of metal. It seems likely that full exploitation of modern fiber-composite, ceramics and plastics technologies would result in a WLRNV that employed no metal except as motor windings and electrical connections, which could be jacketed with RF-absorbing material.

Going to a low-metal design could reduce the radar cross-section against a sea water background by several orders of magnitude. Even better, use of proper coatings in the WLRNV body and limb shells could effectively match the radar reflection from the vehicle to that of sea water, so that it would be difficult to tell the WLRNV from an irregularity on a wave. It also means that it would be essentially fruitless to attempt to find the WLRNV from an airplane with down-looking radar. The low intrinsic angular resolution of radar would be hard put to differentiate the RNV from a wave, so long as the vehicle's reflectance did not make it stand out against the sea-water background.

At this point one might ask: what is wrong with a

hulled RNV that employs all these tricks? Indeed, one could well make a plastic hulled RNV small enough to hide behind waves, etc. But how well could one expect a small hulled vehicle to operate in a sea where the waves were considerably larger than the craft? Not only is the small hulled vehicle too unmaneuverable and intrinsically underpowered to readily negotiate the heavy seas, but it is slow and moreover can't bob and weave. As such, it is relatively easy prey to repeated radar scanning and the correlation of many low probability-of-detection scans to get a good position fix for counter fire direction.

The WLRNV thus seems to be a relatively singular concept, in that it can frustrate modern detection or tracking on the ocean surface. This, plus the fact that if one can walk on water one can very probably also walk on land, implies that the WLV concept is well-suited for all of the potential near-term applications of the RNV concept. The combined concept is attractive not only because RNVs are presently becoming mighty cost-effective and WLV's appear to be the best way to implement RNV's, but also because the combination may become extremely effective militarily.

FIGURE 12

ELECTROMAGNETIC SURVEILLANCE
(LONG DURATION LOITERING MODE)

WLRNVs, operating in long duration loitering mode, may be highly effective electromagnetic surveillance platforms, particularly if their covert deployment, evasive maneuvering and programmable self-destruction potentialities are fully exploited.

FIGURE 13

SKY SURVEILLANCE IN THE FORWARD AREA
(LONG DURATION LOITERING MODE)

Optical/IR sky surveillance in hazardous or distant locations may be most aptly accomplished by covertly deployed platforms such as WLRNVs that can re-deploy themselves and have substantial on-board data storage and processing capabilities, so that data may be obtained from continually re-optimized locations and reported in cogent and timely fashions.

WLRNV-AIDED ASW IN THE FORWARD AREA:
DEPLOYING A PASSIVE ACOUSTIC PROBE
(LONG DURATION LOITERING MODE)

WLRNVs may aptly serve as self-re-deploying sonobuoys in distant or hazardous areas. Their extensive on-board data processing and storage capabilities admit the option of internally controlled re-deployment to stay with an acoustic source, as well as to merely classify it and report its transit. Several such WLRNVs could work together to act as a self-towing acoustic array (active or passive) to stay with a target for 1000 mile distances without external assistance, even if the target were to have high peak speed capability.

REPORTING IN TO A SATELLITE MONITOR
FROM THE FORWARD AREA
(LONG DURATION LOITERING MODE)

WLRNVs may aptly report gathered, pre-processed and -compacted data to earth satellites from forward areas, particularly if reporting to friendly ships or planes would unduly jeopardize the latter. A foldable, steerable antenna might be employed by such WLRNVs to facilitate two-way communication of this type, thereby permitting interrogation/re-programming of the WLRNV from very distant command points.

GENERAL SYSTEM DESCRIPTION

The previous sections have dealt with the concepts and possibilities of smart, water-traversing, limbed, remotely navigated vehicles (SWLRNVs) in a fairly general fashion. The arguments offered therein mostly represent futuristic thinking based upon the more concrete evaluations of the subsequent sections. The fact remains that smart WLRNV's are not available today, and considerable work will have to be done to realize even the first generation ones. This document thus addresses a two-sided requirement: a near-term feasibility analysis and a projection of the longer term potential of the SWLRNV concept.

The future possibilities have already been sketched. Now it remains to consider in detail and analyse the types of vehicles outlined previously. Only when this analysis is completed can the potential of these vehicles, and the feasibility of the above-described future possibilities, be assessed in detail. Thus there will be a substantial diversion in the next few sections to describe the analysis of SWLRNV's performance using near-term concepts and technology.

Thereafter the assessment of potential capabilities can be made in a quantitative matrix provided by the near-term vehicle analysis. Thus, the method and meaning of the calculations performed in the Assessment sections will be succinctly described for those readers who wish to follow the calculations in the earlier analytic sections. For the less technically oriented reader, there will be sufficient interpretation of the results in these later sections so that the intermediate analytic sections need only be skimmed.

In the meantime, for those readers who intend to follow the technical analysis, and for those non-technical readers who are curious as to the details of a SWLRNV such as is being considered by the Livermore Project, this chapter was written. It serves as an

introductory framework for the subsequent analysis, but also explains in relatively non-technical terms the elements in a near-term RNV design and their integration into a functioning system. However, consideration of specific ordnance or other armament that could be included into SWLRNVs are beyond the scope of this document, and as such there is no further suggestions provided about specific potential military capabilities of SWLRNVs.

A RNV must be able to operate remote from support by larger craft, and to operate at considerable ranges from the vehicle that launched it. The greater the range (without assistance, such as refuelling) of the RNV, the more useful the RNV becomes in its function of keeping friendly personnel and ships out of high risk areas. Thus substantial free-running range is one of the fundamental desirable parameters of an RNV scheme.

The way that the WLV concept aims at attaining a good range is via sustained high velocity capability. To move a WLV a given distance across the water requires less energy than would the equivalent distance for a comparable hulled vehicle, since the large area of the WLV's body moves through air, as compared to a hull, which provides much greater drag loss due to its motion through water. The pontoon feet -- the only part of the WLV having substantial drag -- also do not travel in water a large fraction (perhaps 2" of wet motion for each 12" of vehicle travel) of the distance the whole vehicle traverses. (Quantification of the relative power efficiency of WLV and hulled vehicle propulsion at various vehicle speeds is beyond the scope of the present study period, but is to be included in the next one.)

So, for an equal-sized vehicle, the WLV would go further on an equal amount of energy (fuel) than would any conventional hulled craft. This simply illustrates the efficiency of WLV motion, but is of little use for real comparison, because the WLV is much smaller than conventional craft and may not have the same ratio of

fuel mass to vehicle mass as do such craft.

For a realistic comparison of vehicle schemes, it will be shown that the range of the craft is a strong function of one parameter only: the fuel mass energy content divided by the vehicle weight. This will appear in its $\frac{\text{mass}}{\text{mass}}$ dimensionless form as one of the major design choices when performing the system design for a WLRNV.

Since most of the motion of the WLW is in air and moreover at substantial speed, the body can be streamlined for improved aerodynamic performance. Thus, a crucial parameter of a given concept for a WLW is the vehicle mass, since vehicle surface areas are just a function of the mass in a given design.

Mass is one of two possible candidates for a one-parameter classification for WLWs the other being limb length. These two characterizations both describe the same object for a given design, since the limb slenderness is a design parameter and vehicle density is essentially constant for a choice of materials. Thus, often a vehicle can often be well-characterized by its mass or limb length, yet these two terms are both in use because they introduce different sorts of limits on performance.

The contribution of vehicle mass enters vehicle performance through the critical parameter specific energy, β , the ratio of on-board power to vehicle mass, which also indexes both the velocities and accelerations that the vehicle can achieve. Increasing the mass of a vehicle does not directly decrease its performance (i.e., add sluggishness), except if the available power for vehicle motion is not allowed to increase proportionally.

The contribution of limb length does not enter the energy viewpoint, but when given the limb-slenderness design parameter, it gives a structural picture of the limb. Thus limb length is the vehicle-characterizing parameter that most immediately addresses the structural strength issues.

Before continuing, it should be noted that there are two phases of the design problem. The first question is to pick a size range that makes sense. The second question is to pick a design philosophy in terms of slenderness ratios, mass ratios, gear ratios, etc. that likewise is reasonable.

The two parameters just described are descriptors of the size-range questions. The other design-characterizing parameters must be specified in order to get a meaningful description of the variation of performance with size. Obviously size is a very important and useful parameter, which must be scrutinized closely, since multi-ton or insect-sized vehicles will hardly provide the sort of mass-producibility that was a central motivation for the WLRNV.

These design parameters can be best explained by a subsystem breakdown of a given RNV design. The subdesigns can be listed: power, locomotive-mechanical, structural and control designs. Power, the first of these subsystems, has an evident purpose: the power it takes to move the vehicle has to come from somewhere onboard. Thus there must be an energy storage technology, a power generation technology, and a power transduction/distribution technology specified.

Besides the decisions on technology, there are a number of design decisions in the power subsystem. There are characteristic times, the ratios of energy storage capacities to the power generation limits of the devices which extract the stored energy as power. There are power ratios which can best be thought of as the fraction of the available onboard power a given system or component can take from the onboard supply. In short, there are key questions of how much power to allocate to and store in various vehicle components.

This brings back issues of mass which, for a given choice of technologies, specifies power because power-to-mass ratios are quite characteristic descriptions of power component technologies. In effect, if we know how

the mass in the vehicle is allocated, we know the power distribution as well, from the performance of the component technologies.

For example, N grams of mass allocated for Motor #17 corresponds to 4.5 N watts of maximum power dissipation for A.C. electric motors, 3.7 N watts for 3000 psi fluidic motors, etc. There is motor mass and its distribution, primary and secondary energy storage mass, generator mass, etc, all requiring specification. Also there are critical non-power components whose masses must be determined: structure, mechanical, hardening, control computer, and payload masses.

In attempting to sort all of these into coherent subsystems that have independent problems, Power is only the first isolatable subsystem. Locomotion is the second, and is to some extent governed by available total power. This subsystem includes motors, fluidic cylinders, gearboxes, clutches, bearings, and all the other paraphernalia required to effect motion.

The third subsystem is vehicle structure, which, when allotted a certain mass fraction of the vehicle, must do its best to provide vehicle strength and hardness. This subsystem acts with locomotion to determine the physical capabilities of a WLTV. This subsystem differs from the others in that the difficulties of power, locomotion, and control are all directly governed by the rate of vehicle function, while some structural properties are rate-independent, and others depend on the rate squared.

The fourth subsystem, control, does not appear prima facie as critical as the other three in determining vehicle performance. Unfortunately, this is not the case: the control problems for this sort of vehicle are so complex that the computers which solve the real time vehicle control and navigation problems are by no means insignificant. Size and weight reduction of this electronic subsystem are perhaps the crucial unknowns of this type of smart RNV.

More important in these basic considerations than

the final size/weight of the control subsystem that supervises vehicle operation is a fundamental trade-off between controllable rates of vehicle phenomena and system complexity, and thus mass. This subsystem will doubtless remain the critical one for WLRNV reliability for sometime to come.

Two sections of this subsystem almost merit addention as subsystems in their own right: communication and guidance. The communications problems for those few signals that are absolutely required to move to/from the vehicle in the presence of heavy jamming/ usurpation efforts by a clever and determined enemy will be relatively novel. However, in comparison to the major subsystems, the required communication technology is close to an off-the-shelf item.

The same could be said of guidance. Whole inertial platform, radio location, and radar detection technologies exist which can be plugged into a WLRNV system design; the near term availability of the Global Positioning System will simplify this area still further. In contrast, the four major vehicle subsystems really deserve distinct design efforts because they involve technological pioneering. The components of power, locomotion, structure, computer technologies that must be put into the SWLRNV already exist; no more basic discoveries or inventions are needed. But the integration of these components into functioning systems is a challenging task.

The several thousand different components that would go into such a vehicle have never before been combined. The resultant whole will resemble no high-technology device ever built before by man. There is a high degree of confidence that this integration can be done, there is the requisite technological capability, but one can be just as certain there will be problems encountered in so doing that will be both novel and non-trivial.

VEHICLE MOTION

Before the questions involved in the design may be addressed, it would be well to understand the sort of motion that the power, locomotion, structure, and control systems must provide. The WLRNV has so far been depicted as a vehicle that walks water bug-like over the waves, with some intimation from the speeds discussed that it can also run. In fact, no reason has thus far been given as why such a vehicle would not be better configured for its purpose if it resembled a grasshopper, a centipede, or a man.

Each of these zoological forms has a different method of deriving mechanical locomotion from a limb. Whole classes of others, from snails to snakes, are similar but do not afford the mechanical advantage of the WLNV when called on to operate on the ocean surfaces. Thus one could well say that another fundamental design question concerns the mode of limbic motion. One thus inquires as to the design parameters "number-of-legs" and location of "shoulders" on the body.

On closer inspection, the 'mode of motion' question is a different sort of design problem than the others just described. To be sure, in order to build a moving WLRNV, one must specify the sort of motion it undergoes. But the "design" of this motion is a problem that exists quite independently from the previous problem of putting together a system that works.

Fortunately one can be fairly sure that if a system is found that does the job of locomoting one way, it will be able to perform at a similar rate in another way. Since there are a great variety of arbitrary leg motions (e.g. those which could be taken from analysis of motion pictures of animals) that would locomote a similar vehicle, there is confidence that several productive locomotions exist. This being the case, system design issues can be addressed temporarily by ignoring the exact motions the design must undergo to function.

On the other hand, the motion problem is the mathematical one of finding the best among those motions that exist. In almost every way it remains an unsolved problem, due to lack of interest in this complex and previously academic problem. It is, however, a problem whose solution will have to be found, perhaps by making a few nominal choices among the set of near-optimal ones, based on practical computational constraints, before a system design can be completed.

Unfortunately the solution to the motion problem depends on some mechanical design parameters, while the full solutions of the design problem require a knowledge of the motion to be performed. Two very difficult problems are thereby coupled, and by a large number of parameters. In general, there will be an optimal solution for both motion and vehicle design in terms of some performance criterion that depends on the desired mission. Obviously, the optimal solution for a fast-running WLV mission will look somewhat different from that of an omni-directional-lunging WLV whose purpose is to evade enemy ordinance while providing an accurate beacon for a tactical missile.

In the next few sections the nature and the proposed solution to the motion problem will be discussed. For the present, however, one must recall that often optimization procedures are the creatures of the process they are meant to optimize. Thus optimal motion for a five legged, 4-joint-per-leg vehicle will be found by a computer program that is substantially different from one for an 8-joint-per-leg hopper.

In some sense the number of legs and the configuration of their joints are discrete parameters posing barriers across which the optimizers for the smooth motion problem can't operate. Since the capability to do any high level optimization for a variety of different joint arrangements appears rather removed in time, it would seem reasonable to make apt estimates of these discrete parameters. Such decisions (really no more valid than considered rationalizations) should

PARAMETERS TO OPTIMIZE

MOTION PARAMETERS

- $\theta_i(t)$ all controllable degrees of freedom to get efficient motion
- V_0 maximize velocity for given vehicle
- $a(V_0)$ maximize acceleration for given maneuver at given speed
- B.R. backsliding ratio for forward propulsion

HARDWARE DESIGN PARAMETERS

- $\{l_i\}$ length of all limb segments
- $\{M_i\}, M_{\text{body}}$ mass distribution
- $\{I_i\}$ rotational inertia of limb segments
- $\{Z_i\}$ characteristic impedance of joint actuators
- $\{\tau_i^m\}$ maximum torque available at each joint
- $\{\theta_{m_i}\}$ maximum angular excursion at each joint
- $\{\beta_i\}$ specific power required for power-handling components

MISSION DESIGN PARAMETERS

- T_p mission duration under maximum power
- T_L mission duration when loitering
- R maximum range of vehicle { one-way mission
two-way mission
- X_H maximum cross-sectional area viewed horizontally
- X_V maximum vehicle cross-sectional area viewed vertically
- Q_p waste heat under maximum power
- Q_L waste heat when loitering

SYSTEM DESIGN PARAMETERS

- N_L number of legs
- N_J number of joints on each leg
- Configuration of vehicle and functions of joints

produce a decent first-cut starting point, and should limit the motion design problem to one of reasonable size.

One could have doubts that an apt estimate can be made for optimal leg count and joint function, because these parameters can range from 0 to ∞ without restriction. At this point, biology provides a helpful hint, which may be unreliable but seems intuitively valid. Biological limbed systems may or may not be absolutely optimal in any sense, but they surely cannot be significantly sub-optimal, or evolution would have eliminated them by now.

For the sort of applications considered here, nature has never chosen a rigid-segment solution with over 500 degrees-of-freedom (total number of controllable motions, abbreviated D.O.F.). If one rules out snakes and fish as reasonable approaches to this problem, one never finds over 100 D.O.F.'s per limb. Ten seems to be the upper limit on the number of limbs on creatures that have any functional or environmental resemblance to the WLRNVs under consideration.

So, if nature is any guide, the choices that are available for a rational first cut in RNV design are quite limited in number. Obviously, 1 leg will hardly do for operation on water, since compared to several legs a monopod design has fairly poor control and maneuverability. In addition, it is not endowed with the virtue of expendable limbs mentioned previously.

The same sort of objections could well be raised to bipeds, and to WLVs optimized for hopping. Bipedal balance and accurate hopping seem to be particularly uncontrollable motions for a water surface-operating vehicle. All the drag losses involved in plunging feet in and out of the water do not contribute to desired vehicle motion, and are thus unnecessary losses of energy/fuel. Thus, the heavy plunging that would accompany hopping,

bounding, or bipedal motion seems to be comparatively wasteful.

However, there are other evident advantages to a bipedal design. It may be marginally unstable, but it is vastly more maneuverable than multi-leg designs. Consider the problem of turning a spider around. In contrast, a human being can do an about-face to any direction he pleases with two quick foot motions. Not only can the biped turn faster, but he can start and stop with great rapidity simply by allowing his body to "fall" backwards or forwards over his feet, employing gravity to rapidly change his momentum. In contrast, multi-leg designs usually cannot get their center of mass beyond the support base of their feet, and are thereby restricted to starting and stopping under their own power.

These sort of advantages epitomize a general trade-off that accompanies the number-of-legs decision. More legs offers the possibility of greater stability, and a greater number of expendable locomotive units. Fewer legs will exhibit lower stability, but will generally provide greater maneuverability.

Other problems begin to arise with increasing leg count, and also with increasing numbers of joint-per-leg. These have to do with control complexity and control versatility. As the number of D.O.F.'s increase, the size of the state-space of functions that must be stored for control grows exponentially. As such, every additional leg opens up vast new gait and motion-type possibilities, but also calls for exponentially increasing control complexity, unless the potential state-space is severely restricted by fiat.

This facet of the choice of leg count is counter-balanced at the low end by the complexity of control for the balancing problem. Monopeds, bipeds, and tripeds are all by definition unstable at some time during their

motion. Control for such vehicles will be complicated by balancing problems that will not occur for higher leg-count vehicles.

The last restriction on leg count, indeed on leg configuration, has to do with symmetry. Nature seems to avoid odd numbers of limbs. Discounting the non-rigid-limbed star fish, there are no 3-, 5-, 7-, 9-, or 11-limbed creatures. This avoidance of odd leg count may not be optimal, but it does seem sensible (ignoring other issues) from a control complexity viewpoint to practice symmetry about the vehicle-transverse plane. If this symmetry is maintained, left and right are identical, turning capability is symmetric, and a single control system will suffice for motion in either direction.

This alone does not rule out odd numbers of legs, since the odd leg could be exactly on the plane of symmetry. However, the running geometry of leg extension dictates that this control leg must flex to a shorter length during this motion if the body is going to follow any course that is not a circular arc about this leg. This flexure can either be accomplished by bending at the knees or by extending sideways. If the latter is chosen, then bilateral symmetry is violated; the former alternative is clearly structurally disadvantageous, as can be easily seen by attempting to run with knees bent and noting the usually high tensions developed.

So, even though odd leg counts could conceivably prove optimal for some applications, it seems wise for the moment to concentrate the consideration on even numbers of legs. Perhaps if there is sufficient interest for a later study, the actual comparison of reasonable odd legged vehicles can be made, but for the moment it seems reasonable to ignore the possibility.

Thus the leg count choice could be rationalized as choosing between 2, 4, 6, 8, 10 and 12. Bipedal motion may be simplest, but it will have the plunging inefficiency and the balancing problems already described, and

thus does not appear as a wise choice for near-term WLRNVs. At the high end, 12 legs hardly provides much more stability than 10, and probably requires large increases in control complexity. For 8, 10 and 12 leg counts there is evidently a problem of the legs being able to collide even for a small range of motion. As such either the body must be large so that substantial leg excursions do not cause collisions, or small excursions must be used which do not make full advantage of the legs.

Thus the 8, 10 and 12 legged vehicles will either have to be comparatively enlarged (or just lengthened) to give the legs space which only can employ to full advantage. Long vehicles with in in-line legs seem to lack sideways maneuverability (e.g., consider the lobster or crab). As such, the most reasonable many-legged vehicles will tend towards radial symmetry somewhat like the octopus, and their added complexity will mean that they only become optimal once the capability has been achieved to operate well after substantial leg-loss.

Thus one might envision three different sorts of vehicles. The first sort is a 4-or perhaps 6-legged vehicle which is stable, somewhat maneuverable, and simpler in control for near-term achievability. The next sort has a leg count on the order of 10, deployed in something like radial symmetry, and fully exploits the versatility of many gaits and the intrinsic system hardness associated with many expendable limbs. The last sort of vehicle is the far-term biped, whose extreme maneuverability might be required for missions performed under very heavy enemy counteraction. The increased vehicular complexity and balancing complexity, respectively, of these last two types require that they be developed only after the first sort has been pioneered.

Considering any of these sorts of vehicles, there is a common framework of terminology that should be defined. The limbs must move to effect motion. In general walking, running, scuttling, crawling, and even hopping are cyclic

motions. The cyclic motion of each limb while the vehicle is in such a motion is called a 'limb cycle.' In general, all limbs will undergo similar cycles during a vehicle motion (during "regularly realizable" gaits).

The entire vehicle moves as a pattern of limb cycles. There will be a continuous spectrum of possible variations of the position histories of the joints during each limb cycle, so that rates and body-wobble, velocity and foot thrust, etc. can all be continuously varied as versions of the same motion.

There are also other patterns of motion that can't be described as continuous variations from a given pattern. These form fundamentally separate classes of motion which are termed gaits. A trot is a different gait than a canter or gallop for the horse. Likewise one could conceive of running, jogging, walking, skipping, and hopping as the distinct gaits of humans. It should be clear that variations in the limb cycles of walking can achieve a variety of walking speeds, and exaggerate non-essential motions as in the duck walk, the extreme hip motion once practiced by females, bow-legged walking, etc. However, there is no continuous variation that will turn a walking gait (in which the heel strikes first and the body can stop stably) into a running gait (in which the toe strikes first and the body position is statically unstable, and would fall over forward, if stopped).

Gaits in general should be classified as a change in the pattern of the motion rather than in the limb cycles. For a four-legged WLW, one could envision gaits that kept 3, 2, 1 or no feet on the ground at the same time. There are different orderings of the limb cycles (see Figure 23) that make for different gaits as well. In general, the number of conceivable gaits goes up very rapidly with the number of legs.

Howsoever, one must realize that, although one could vary the limb cycles within a given gait to achieve a wide range of motions, a given gait will only be a useful vehicle motion across a narrower range. A horse does not

FIGURE 17

GAITS FOR THE QUADRUPEL

There are 5040 distinct gaits for quadrupeds, discounting simultaneous kickoffs or impacts of feet. Of these, none are strictly stable.

I. One foot off ground: "creeping" gaits 3 weakly stable

Foot numbering

stable gaits are:

Cyclic

"Crawl"

II. Two feet off ground at once: "trotting" gaits none stable

Only weakly stable gaits are those that keep diagonal pairs of feet on ground at same time

III. Three feet off ground at once: "galloping" gaits none stable

IV. Four feet off ground: "running" two feet on ground at once

V. Four feet off ground for most of cycle: "hounding" one foot on ground at a time

have the power to walk at 40 mph, nor do his legs have the strength to run at 2 mph. Even if a gait is mechanically possible, often there is another that will involve a more efficient use of power, or of vehicle structural strength margins, for the same desired maneuver. Again the horse can achieve the same speed with a slow canter as with a fast trot, but the one chosen is usually the canter, for efficiency reasons unless the horse is fatigued (only low power available) in which case the trot is preferable, for power consumption reasons.

Thus the vehicle motion problem consists of determining what gaits are mechanically available for the desired motion, which among these will be more optimal (with respect to some cost criteria) and which limb cycles should be used to realize the desired motion from this gait. In general, the specification of the limb cycle is the most complex, since it involves generating the control signal for each joint at each time during the cycle. The choice of gaits is a matter of defining a complex set of boundaries in the mathematical motion-characterizing-parameter space which surround the region(s) where a known gait is better than all alternate ones.

Control is then simply a matter of the quantitative characterization of the motion that is desired, and the look-up of the resulting parametric point in this hyperspace map of boundaries. This informs the RNV of the best gait for its purpose, and then limb cycles for that gait with the given characterization (speed = X, wobble = Y, turning radius = Z etc.) can be looked up or calculated to give the best time history for each joint's control that will accomplish the desired (parametrically characterized) motion.

Unfortunately, it is difficult to specify beforehand which gait will do best for a given motion. Since different gaits are not continuously connected by smooth variations in the limb cycles, a given numerical optimization procedure cannot find the single optimum

over a number of gaits. Each gait must be optimized to perform the desired motion, and then the performance of all gaits can be compared to find the best one.

The vehicle motion design problem can thus be reduced to specifying the number of legs and type of joints, determining all possible gaits and writing codes to simulate/optimize them, finding the mechanical limits to performance for each gait, and comparing the performance of the optimal from different gaits performing the same maneuver in order to establish the optimal boundary map just described.

This problem is far from easy, because the "efficiency" one would like to optimize is a function of physical parameters (e.g. work, drag, heat-losses etc.). Thus the best motion will be the one whose physical attributes are the best. A study of vehicle physics must therefore be made in order to write optimization codes. These codes will be slow and bulky due to the complexity of vehicle physics, and, as such, generation of a single optimum limb cycle (given a gait and a parametric description of the desired motion) may be relatively expensive in computer time. Fortunately, this computing may be done off-board and not in real time, as will be discussed subsequently.

It will thus require considerable effort before enough optimal solutions for a pair of distinct gaits are completed to draw a single threshold/boundary which tells a given vehicle which gait will be better under the mission circumstances anticipated for it. More generally, the WLRNV will doubtless have to settle for a nominal choice of gait and a semi-optimal limb cycle to realize that gait. It will be much easier to get an interpolation between a few known optimal solutions (termed a "semi-optimal" solution subsequently) than to get a dense enough net of real optimal solutions in the motion-parametric hyperspace to achieve true optima under all vehicle operating circumstances.

TYPES OF JOINTS

Before an attempt is made at evaluating vehicle physics, a mathematical characterization suitable for a physical model must be made to form such a model. One need simply isolate all of those vehicle elements that effect vehicle physics and ignore the rest. For motion physics, it is enough to simply consider a vehicle as a set of rigid segments of known masses and angular inertias. These segments are acted on by forces, take on kinetic and potential energy, etc., and in general are governed in their motion by the familiar laws of classical mechanics.

These rigid bodies that represent the limb segments and the vehicle body cannot be treated as simply as rigid bodies in inertial space. Unfortunately the pieces, each of which are subject to Newton's $F=MA$ or, equivalently, to Lagrange's equations $\frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i$ are also hung together by joints that couple the forces and energy of the diverse parts, i.e. have non-holonomic constraints on their motions. In fact, all of these parts stay attached to each other at the joints that connect them.

There are two types of joints that are feasible with modern mechanical hardware: those that swivel about an axis, and those that slide. Unfortunately the sliding variety have a number of comparative deficiencies. In general, slide bearings are a more impoverished technology than rotational bearings, and in general their performance is worse because the loads that the bearing must withstand while moving fall on fewer rollers/balls in a linear bearing than an axial one. Not only is the present technology inferior, but the nominally free motion of a bearing actually exhibits a low drag — which helps dissipate power — and linear bearings are more lossy than rotating bearings. Thus sliding assemblies of a given load-bearing capability will be lossier, as well as heavier, than their rotating equivalent.

Even more important in a limbed vehicle, sliding action is much less useful in providing geometrical freedom to a limb (see Figure 18). Along the same lines, a rotating axle that is under control provides a fulcrum which can selectively transform rotating kinetic energy to/from pure translational energy by clamping or moving freely. Clearly if one's ankle froze while walking, fairly drastic motion changes could be achieved, compared to those possible if the ankle was replaced by a sliding leg. Thus for an equivalent complexity of implementing one of the two types of joints, a rotating joint buys greater range of motion transformation than any realistic sliding joint (assuming that a sliding joint would not allow a limb segment to shrink its length to zero).

Thus, until there is any reasonable evidence to the contrary, it can be concluded that sliding joints are comparatively useless. To simplify the physical model, this analysis shall simply ignore sliding joints (although they are not difficult to include). Of the remaining rotating joints one can now classify two types of rotating joints: "twisters" and "elbows."

An "elbow" is exactly like the elbow one can find on human arms. If one defines a line that runs from shoulder to elbow, then the elbow is characterized by a single degree of freedom of rotation about an axis perpendicular to that line. On the other hand there is obviously another degree of freedom between the human fibula and tibia, the ability to twist the upper arm illustrated simply by the switch from forehand to back-hand. This degree of freedom is a "twister," defined by the axis of rotation being parallel to that line between shoulder and elbow.

Before continuing to make these types of joints into a geometrical model for the constraints that operate between limb segments, it would be well to detach the reader from anthropomorphic imagery. In the abstract definition of "elbow" and "twister," only the direction of the axis of rotation was specified with respect to the

FIGURE 18

TYPES OF JOINTS

Joints provide those degrees-of-freedom over which the Mechapod has direct control

I. ELBOW

Can be mono or biflexural monoflex (as in human elbow):

Mathematical expression:

Rotation about

$$\begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_{N+1} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_N$$

II. TWISTER

Much like human degree-of-freedom (D.O.F.) between shoulder + elbow

Mathematical representation is rotation about

$$\begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_{N+1} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_N$$

$$\begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_{N+1} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \parallel \\ \perp \\ T \end{bmatrix}_N$$

III. SLIDER

Just like telescoping segment

Relatively useless joint for locomotion

IV. SHOULDER

Just like human shoulder

Isomorphic to 2 elbows at same location

- V. All other joints for rigid-segment mechapods can be modeled as series of elbows + twisters concentrated at one point. In general one twister angle α and two elbow angles θ θ are sufficient to represent arbitrary orientation joints, since $\{\theta, \alpha, \theta\}$ are the "Euler Angles"

preceeding limb segment, and nothing has been said about the extent of possible rotation. The human elbow flexes from straight through about 135° , but there is no reason why an automata cannot rotate its "elbow" through 360° . The same goes for twistors, and it is easy to conjure the picture of robot arm using a screwdriver in a continuous rotation.

However, even though such continuous rotations are possible, in the case of the "elbow" it is mechanically disadvantageous and geometrically unnecessary to build continuously rotating structures. Thus a human-type elbow/knee will suffice for real designs, although attainable flexure may be bidirectional (see Figure 18). In the case of the twister, 360° degree rotation may be actually useful, but full, continual rotation will occasion the use of electrical slip rings, as well as making fluidic and pneumatic power transmission beyond this joint quite difficult.

It also should be remarked that other sorts of joints are easily conceivable. The first that comes to mind is a "shoulder," which can easily be seen as a pair of elbows at the same location, with perpendicular axes of rotation. Other possibilities (non-perpendicular joint axes being the foremost) turn out to be mathematical superpositions of elbows and twistors, once the rotations are expressed as vector transformations (rotator matrices).

Given this universal characterization, one can then express the full geometry of the vehicle by specifying an angle at each individual elbow or twister. This gives a full description of the configuration of the vehicle at a given time and, with data that locates any point on the body and specifies rotation pitch, yaw, and roll of that point with respect to some inertial coordinate system, the inertial positions of each and every position of the vehicle are known at a given time. Taking derivatives will give velocities and accelerations, integrating the squared velocity of all of the mass will

FIGURE 19

ROTATOR MATRICES

(Sometimes known as direction-cosine matrices)

Consider an initial coordinate system in 3 spaces:

which must be rotated (by motion at a joint) into another system:

This can be done by breaking up the rotation into 3 steps: rotation about $\hat{e}_1$, then about $\hat{e}_2$, then about $\hat{e}_3$

At each step, the unit vector being rotated about acts as an axis, and does not undergo a transformation, while the other two get rotated:

$$\hat{e}_{A'} = \cos\theta \hat{e}_A + \sin\theta \hat{e}_B$$

$$\hat{e}_{B'} = -\sin\theta \hat{e}_A + \cos\theta \hat{e}_B$$

or, in matrix notation

$$\begin{bmatrix} \hat{e}_{A'} \\ \hat{e}_{B'} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \hat{e}_A \\ \hat{e}_B \end{bmatrix}$$

Then, we simply choose the fixed axis

to write $\begin{bmatrix} \hat{e}_{1''} \\ \hat{e}_{2''} \\ \hat{e}_{3''} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$ and $\begin{bmatrix} \hat{e}_{1'''} \\ \hat{e}_{2'''} \\ \hat{e}_{3'''} \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \hat{e}_{1''} \\ \hat{e}_{2''} \\ \hat{e}_{3''} \end{bmatrix}$ and

$$\begin{bmatrix} 1' \\ 2' \\ 3' \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1''' \\ 2''' \\ 3''' \end{bmatrix}$$

call these 3 x 3 matrices

$$R_1(\theta_1), R_2(\theta_2), R_3(\theta_3)$$

where the subscript denotes which vector is unchanged

then $\begin{bmatrix} 1' \\ 2' \\ 3' \end{bmatrix} = \underbrace{\begin{bmatrix} R_3(\theta_3) & R_2(\theta_2) & R_1(\theta_1) \end{bmatrix}}_{G(\theta_1\theta_2\theta_3)} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ expresses the desired notation

Note that the overall transformation matrix G is unique, but the three rotator matrices that comprise it are not unique unless the order of rotation operations is specified:

$$G = R_1(\theta_1) R_2(\theta_2) R_3(\theta_3) = R_2(\theta'_1) R_1(\theta'_2) R_3(\theta'_3)$$

The three non-unique angles that specify a rotation $\{\theta_1\theta_2\theta_3\}$ are known as the Euler Angles

give kinetic energy, and the mass-weighted summation of positions above a reference plane gives potential energy. In short, the specification of a joint angle for every joint and a trio of position values for 1 point on the body, plus a trio of pitch/yaw/roll angles (as functions of time), is sufficient specification for the physical model.

VEHICLE PHYSICS

In general, the information desired from a physical model of the vehicle can be summarized as: upon specifying the control given at each joint, how does each part of the vehicle move and what forces and powers does this motion require. For the physical model, there is a fundamental difference between the model of the joint control as a force input or as a geometrical control. In general, the mechanical transducers available to actuate the joints provide force, but the force applied may be position-dependent, velocity-dependent, power-limited, etc.; available actuators deliver forces which will be complex functions of the positions, velocities, accelerations, etc. of the joints motions, as well as of the controlling signal.

At this point, one therefore is well-advised to decouple the problem by asserting some simplified form of the physical control of a joint, and then to enforce that form by appropriate feedback after the problem solution has been found. Indeed, the physical model might most aptly assert that the limited vehicle joints are excited by a pure time-dependent source of torque, then evaluate the subsequent physics to find the resulting joint position/velocity/acceleration parameters, and finally input these into the known dependences of the control to find the control input that would have generated that force.

This system may be employed to get a relatively "pure" physical model by asserting non-dependent inputs at the joints. However in an attempt to vary the control and see if the resulting motion is more desired, one is forced to vary the relatively simplistic input to the physical model and accept the variation in the control that falls out of the result. The cost of this straightforward physical model is thus a difficulty in directly varying the control to see the effects. Small variations in all input parameters must be made to obtain an

invertable matrix of control sensitivities that allows a desired infinitesimal change in the control signal to be inputted to the model, but differential-variation shifting will not be of use for large variations that may be required in the control.

An evident question arises: Why should one care what the control signal is, if it can be supplied to generate any motion that is decided upon? The answer is that the control may be a more meaningful parameter in terms of understanding the effect of a variation than is the physical input. In the dual case, where the input is assumed to be a position, velocity or (angular) acceleration at each joint, the physical input begins to take on some meaning as it depicts the geometry of the moving vehicle.

In point of fact, however, an iterative approach to vehicle motion design using the physical model never needs to be performed. Instead, numerical optimization routines may be employed to determine local maxima of a specified vehicle performance evaluation function over a discretized time sequence of values for the control inputs. Thus such a procedure would be employed to determine optimal motions for a given vehicle and a given gait. Such a gait would be imposed in the initial estimates of the control input histories, and since other gaits can't be achieved by a continuous variation of limb cycle motion histories, the final optimum that is found must be the optimum for the initial gait.

In this way, one never has need to precisely evaluate what the control inputs imply in terms of the motion the vehicle will execute. Instead the best solutions in terms of a maximized performance evaluation function will fall out of the optimization procedure, allowing the force and position of the optimum control solution to be inputted into the mechanical function of the joint actuator. All that is needed is a simple physical model for the system with a single input per joint.

The most frequent interpretation of the mechanics

problem is that applied forces cause geometrical motion. In this context, force is the input from which motion is calculated. It appears, however, that for the limbed vehicle control problem both Newtonian and Lagrangian formulations are easier to solve if one considers the geometry as the specified input during motion and uses the physical model to calculate the forces and torques that would produce such motion. In particular, it is clear that there is just one controllable input for each joint, and only the joints are directly controllable to effect motion.

If one specifies an angle at each joint which is free to move, there is a torque associated with that specification which causes the joint to be in that position. Thus the joint angles for each elbow and twister in the model of the vehicle are the variable inputs to the model which the optimizer alters to find optimal motions for each joint. The physical model determines the dual variable, the torque τ at each joint required to motivate that D.O.F., and the product $P = \tau \frac{d\theta}{dt}$ is the instantaneous power that the actuator at that joint is contributing to limb motion.

There are also six geometric variables in the problem that are not to be varied by the optimizer but must be found from the physical model. The most evident choice of the definition for these variables are the X,Y,Z location of the body, and the pitch, yaw, and roll of the body: $\theta_x, \theta_y, \theta_z$. More specifically, a geometrical point within the body known as the body-center must be chosen as the reference for the X,Y,Z coordinates that describe absolute body location in inertial space. Three orthogonal axes must also be defined with respect to the body in order to define orientation. Also, since angular rotations do not commute, one must also specify the order in which pitch, yaw, and roll rotations are performed to be able to unambiguously specify body orientation.

These six variables totally describe the physical

FIGURE 20

CLASSES OF MODELS

The vehicle is a collection of rigid bodies (ignore elasticity to get lumped parameter model)

I. CHOICE OF PHYSICAL MECHANICS

- NEWTONIAN $\sum_j \vec{f}_{ij} = M_i \ddot{\vec{x}}_i$ (familiar as $F = MA$)
requires action-reaction force pairs (redundant force variables)
- LAGRANGIAN $L = T - V$ $T = \text{total kinetic energy of vehicle}$
 $V = \text{total potential energy of vehicle}$
$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \leftarrow \text{generalized force}$$

uses generalized coordinates (q_i) $\Rightarrow$ easy to constrain

II. SEQUENCE OF CALCULATIONS FOR JOINT MOTION

- PRE-CONSTRAINED incorporates the fact that the rigid bodies that comprise the vehicle are constrained to rotate about axes (joints) fixed with respect to those bodies directly into coordinates used by physical model.
- POST-CONSTRAINED allows one to use a simple model for the rigid bodies in metric 3-space, and constraint for joint rotation afterwards.

III. CHOICE OF KNOWNs AND UNKNOWNs

- FORCE INPUT interprets mechanics as "force causes motion"; use ideal force (torque) sources to drive the model and calculates motion as the desired output.
- POSITION INPUT interprets mechanics as "motion requires force"; use ideal position (angle) sources to drive the model and calculate the forces required for such motions.

NOTE: This choice does not apply to the six coordinates in the model (body position and orientation in inertial space) which are not under control by the vehicle and therefore can't be considered as inputs.

position of the body. These plus the joint angles specify the position of every part of the vehicle. However, only the joints are controllable, and these six variables cannot be specified or estimated to achieve the optimum motion; they are attained only as a result of limb motion. Just as a man cannot directly control the physical position of his navel or the axis of his torso without motion on the part of his joints, the WLV cannot independently control body center position and body orientation, except through the control of joint angles acting through the physics of body motion.

The way these six variables fall out of the physical model, which (as already described) has N_{joints} (the number of joints) of arbitrary controls and N_{torques} as unknowns, is through the constraints that are placed on the vehicle's feet (limb terminators). Obviously the feet have position and force characteristics of their own. In the case of pontoon feet, there is a drag force that is a function of velocity relative to the water and immersion depth (among other parameters). Unfortunately, the trick of turning this possibly complex function into a position source (as was done for joint actuation) can't be used here.

The immersion depth, the relative velocity to the fluid, the angle of attack, etc. of a pontoon foot are not parameters that can be freely chosen. In the previously discussed framework, only the joints are under direct control, and as with the six variables of body geometry, all of the above (immersion depth, etc.) are already specified. The optimizer is therefore not free to vary these parameters independent of the joint angles.

It has been proposed that the six variables that were just discussed could be exchanged for foot-oriented variables in order to simplify the evaluation of foot forces, which are critical physical inputs that are not directly under control. This approach isn't of great utility, since the six missing variables that must be added to the joint-angle control inputs to specify all of

FIGURE 21

LIMB SEGMENT AND JOINT REPRESENTATIONS

Define a coordinate system of unit vectors for each limb-segment fixed with respect to that segment in the parallel, perpendicular, and transverse directions.

Define a rotation point at each end of the segment through which the axis of the joint at the end must pass. Then the unit parallel vector $\hat{e}_{\parallel}$ is parallel to the line connecting those two points $\overline{AB}$, and points toward the adjacent segment closest to the foot.

For the physical model, each limb segment is represented by the quantities:

$$l_i = \text{length of segment } \overline{AB}$$

$$M_i = \text{mass of limb segment}$$

$$I_i = 3 \times 3 \text{ rotational moment tensor defined in } \hat{e}_{\parallel}, \hat{e}_{\perp}, \hat{e}_{\tau}$$

The limb segment will be modelled as a rigid body.

The limb segment is terminated at each end by a (logical) joint which is represented by a rotator matrix. There are two sets of these joints:

coordinate transformation expressed by rotator matrix $R_T(\theta)$

coordinate transformation expressed by rotator matrix $R_{\tau}(\alpha)$

Note that more complex joints can be easily expressed by the use of virtual limb segments with $l_i = 0$, $M_i = 0$, $I_i = 0$.

Thus a vehicle may be represented by more logical joints than its actual complement of controllable physical joints.

By convention limb segments are indexed with increasing integers going down the limb, away from the body, toward the feet.

BODY COORDINATE REPRESENTATION FOR USE IN PRE-CONSTRAINED MODELS

Break vehicle up into logical limb segments for vector representation. Define a body center (B.C.) at some point fixed inside the vehicle body, and connect rotation points outward down each leg therefrom:

Using the logical limb segments to represent these vectors, the positions of a point in any limb can be found as the sum of limb segment vectors ℓ_i , using two types of rotator matrices

$$R_{\text{elbow}} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{\text{twist}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

denoting position vectors in inertial space by X

$$\vec{X}_{\text{joint } j \text{ limb } k} = \vec{X}_{\text{shoulder of limb } k} + \sum_{i=1}^j [\ell_i \ 0 \ 0] \prod_{i=1}^j R_{ki} \begin{bmatrix} \tau_{11} \\ \tau_{\perp} \\ \tau_t \end{bmatrix}_{\text{shoulder } k}$$

Note that one must use the product of rotator matrices because

$$\begin{bmatrix} \tau_{11} \\ \tau_{\perp} \\ \tau_t \end{bmatrix}_{\text{joint } j \text{ limb } k} = R_{k,j-1} \begin{bmatrix} \tau_{11} \\ \tau_{\perp} \\ \tau_t \end{bmatrix}_{\text{joint } j-1 \text{ limb } k} = R_{k,j-1} R_{k,j-2} \begin{bmatrix} \tau_{11} \\ \tau_{\perp} \\ \tau_t \end{bmatrix}_{\text{joint } j-2 \text{ limb } k} = \text{etc.}$$

In order to get shoulder locations and orientations, define six body-center coordinates (B.C.C.'s): Position $(X_0, Y_0, Z_0) = \vec{X}_0$ and three Euler angles with unique ordering PITCH, TILT, ROLL called $\theta_Y, \theta_Z, \theta_X$. Body orientation matrix = $R_2(\theta_Y) R_3(\theta_Z) R_1(\theta_X)$.

Each shoulder location can then be defined by a matrix in the $\tau_{11}, \tau_{\perp}, \tau_t$ coordinate system fixed in the body a_k (pre-specified by vehicle geometry).

$$\vec{X}_{k,j} = \vec{X}_0 + \sum_{i=1}^j [\ell_i \ 0 \ 0] \left(\prod_{i=1}^j R_{k,i}(\theta_{n,t}) \right) R_2(\theta_Y) R_3(\theta_Z) R_1(\theta_X) [a_k] \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$$

Any point within a limb segment can be found by replacing $[\ell_j \ 0 \ 0]$ with that point's vector coordinate in the $\tau_{11}, \tau_{\perp}, \tau_t$ system in the last term of this sum. Note that the position of any point in the body is defined by the G.B.C.C. variables $(X_0 Y_0 Z_0 \theta_X \theta_Y \theta_Z)$ and all of the logical joint angles θ_{k_i} on that limb between the point and B.C.

the geometry would only suffice to characterize the position and orientation of one foot. Once the position of one foot is known, the joint angles give the position of all of the body - but unless that foot is the only foot in the water at the time, all of the other immersed feet have positions and orientations that are extremely complex functions of the joint-angle inputs. Only (possibly) bipeds will therefore tend to show a decreased-complexity benefit when using a foot-centered coordinate system.

For the sort of vehicles this document considers, the fact remains that the geometrical motion of the feet in the water are very complex functions of the joint angles and six body coordinates. These complex functions can be calculated, and only then can they be inserted into the physical model for foot forces to generate the foot forces associated with the vehicle motion that is occurring.

Thus, the physical model is an extremely complex coupled set of equations, even when the location of every position of vehicle mass is known as a function of time. Six body coordinate variables must be found from complex equations which basically determine the body motion that results from the inertia, gravity, joint, and foot forces acting on a body in a known configuration. Thus the six body coordinates are coupled back into their differential equations by the instantaneous vehicle geometry. The joint torques help determine the six body coordinates, which in turn govern the vehicle configuration, which specifies gravitational forces, etc., which in turn contribute changes to the joint torques through the implied constraints that keep the limb segments from separating (as they usually would if fully arbitrary joint and gravitational forces were applied to neighboring segments).

As such, the best one can say about the vehicle physics model is that it will be a complex coupled set of differential equations, discussed in detail in the

next section. Since there are six unknown body-coordinates, N_{joint} unknown torques, and $6 N_{\text{feet}}$ unknown foot forces, there will be a large number (over 30) such equations. In either Newtonian or Lagrangian mechanics, each of these equations has a large number of terms.

In general, one can think of this as an extremely complicated problem with N_{joint} inputs and many unknowns. The number of unknowns can be much larger than the number of known inputs to the problem, and still the solutions will show an independent dimensionality greater than the number of inputs. This is because the equations of physics represent an indeterminate amount of information (e. g. conservation of momentum can determine three arbitrarily variable components of motion due to arbitrarily many independent forces). Thus there is no way that one can assert that all but N_{joint} of the unknowns are determined by those N_{joint} unknowns. Also, there is no way to reduce the number of equations by asserting that the number of true independent unknowns is necessarily equal to the number of variables.

If one can tackle and solve these equations for a reasonable problem (as has not been done through the present), then a true understanding of the forces and energies involved in a given vehicle motion can be investigated. Once torques and body coordinates are known, one has position and force data for every part of the physical model. These can be used to calculate the required control signals for actuator inputs, and can also be used with actuator models to calculate actual power losses in the actuators. In general the actuators will waste power (even when they're not moving) at a rate that is a function of the torques and angular rates they effect on each joint.

Data from the physical model is thus a sufficient and necessary statistic for the computation of wasted power. One can hardly expect to be able to optimize vehicle motion when one can't calculate power wastage, which is first among the performance parameters to be

minimized. The optimization problem therefore cannot be tackled without first tackling the physical model. It also turns out that these optimization procedures add at least an order of magnitude above the number of computation required to solve the mechanics problem to solve the optimization problem. Since the physical mechanics (physical models) are so complex, special attention must be afforded to getting reasonably quick solutions to the physical motion problem, lest the generation of optimal solutions require prohibitive amounts of computer power or time.

It might seem strange that the general physics problem for a limbed system has not been solved previously. In point of fact, a substantial number of limbed vehicles have been built over the last two decades, and it would seem rather unlikely that no one has tackled the problem. Indeed there are such investigations, but these have been based on approximations that are quite unacceptable in the context of the present problem.

First, it appears that this document may contain the first consideration of limbed motion on water. On solid land, feet are constrained to provide thrust by the action of static and sliding friction. As such the least lossy, obviously optimal solutions to motion generation will all keep feet which are in contact with the ground absolutely fixed with respect to the ground by the action of static friction. This type of foot behavior is more easily included in the physics problem as a geometric constraint (zero horizontal foot motion) than as a motion-dependent force. The no-motion constraint is particularly easy to add to a variational dynamics (Lagrangian mechanics) model without any great increase in complexity.

On the other hand, feet-with-drag moving buoyantly over water cannot be represented easily as motion constraints, but are actually easier to introduce into Newtonian models than the more abstract geometric constraints. The actual finite sensitivity of drag to motion of the pontoon actually proves numerically helpful

to the computational accuracy/stability of vehicle control, since Coulomb friction exhibits a singularity at zero sliding velocity.

There is also a fundamental approximation that has been made in almost all previous analyses: inertia forces have been ignored. This is a very desirable assumption, since it effectively eliminates the dynamic complexity of the problem while leaving a reasonable physical model. Ignoring inertial forces removes the cross-coupled coriolis and centripetal acceleration contributions to the motion physics, the only remaining forces being gravitational or control-imposed at the joints. As such there is no time-dependence to the resulting model (neither control inputs or gravitational forces being affected by the rate at which time passes).

This may appear as a poor abstraction on which to base physical conclusions - that velocities and accelerations do not effectively enter into the problem. However, the results are asymptotically correct as the rates of vehicle motion are slowed toward zero. Indeed, they have proved to be quite adequate for the slow hydraulic and pneumatic limbs that have been built to date, and are being implemented in large numbers by industrial robotics technology today.

Unfortunately, the WLRNV's *raison d'etre* is not quasi-statically moving objects around, or holding them, or accurate, slow manipulation of weighty loads. The missions of WLRNVs demand speed and agility. A running velocity of 10 meters-per-second (~ 18 knots) requires angular speeds of at least 10 radians/sec (95 RPM) for vehicles with 1 meter limbs. This implies that there will be centripetal accelerations of on the order of 10 g at the feet. In such a regime, one can hardly afford to neglect dynamic inertial forces (such as the aforementioned centripetal forces) in comparison to gravitational forces.

Thus the approximation that holds well for sluggish industrial robots and slow rough terrain limbed locomotion vehicles is simply a poor one for WLRNVs. Even more than

the possibility of severely violating this assumption, the WLRNV purposefully tries to overwhelm static contributions with dynamic ones. For the avoidance of counter-fire, the WLRNV should attempt as irregular a motion as is achievable. Indeed, running with only 1 or 2 (i.e., with almost any gait of a four-legged WLV) leg in the water at a given time is statically unstable, and the WLRNV depends on inertia to keep it from falling over before it can put another foot down, just as humans do.

So, there does exist a reasonable corpus of previous limbed RNV control work; it is simply unsuitable for the WLRNV physics model, however. The suggestions made by the no-foot-motion and no-inertial-forces approximations may nonetheless be helpful in fast limbed vehicle design, and in attempting to make intelligent distributions of mass and power throughout the vehicle. However, the LLL WLRNV Project has been forced to tackle the relatively difficult, novel, and unsolved problem of creating an extremely general model of such vehicles, whose only assumption is that of the rigidity (lack of significant flexure) of the vehicle structure.

The general solution to this problem is not immediately required by the Project, since such a solution is merely the capstone of the motion design problem, which can be addressed for near term needs by using nominal motion solutions arrived at by study of advanced animal motion. The vehicle design, the evaluation of systems and components and of operational feasibility does not depend on realizing optimised motion; any limb cycles and gaits that create effective motion will do for a first approximation. A tight design and firm upper bounds on vehicle performance, however, can only be attained if the optimal motion design problem is fully solved, and this can occur only if the physical model can be evaluated in a sufficiently simple form to make optimization computations practical.

LAGRANGIAN AND NEWTONIAN DYNAMICS

The task of the physical model for the WLRNV is to relate the motion of the joints, the torque at the joints, the motion of the body, and the motion of the feet. These are the essential variables that must interrelate within the model: powers, forces, and stresses can all be derived simply if these variables are specified as functions of time. One of the early fundamental alternatives discussed in the last section was which of these qualities could be assumed to be known inputs, since independent actuation of N_{joint} joints assured that at least N_{joint} of the physical variables were under direct, arbitrary control.

The next fundamental alternative was the choice of geometric representation for the vehicle that would result in a form for the physical model that was easy to evaluate by computer. One such representation has already been presented: the angle-dependent representation that characterizes vehicle geometry as a stick-figure of fixed-length lines with known twist or elbow angles where those lines meet.

Another conceivable representation assigns a separate, six position-and-orientation set of variables to each limb segment and to the body and feet. This representation obviously has many more geometrical variables, compared to the previous representation (which averaged close to one variable per limb-segment in specifying sufficient position data for each part in the vehicle). It can be shown easily by contradiction that none of the variables in the joint-angle representation can be eliminated. (For if it could be, then that joint would not be free to rotate under control, and the two line segments it connected could be replaced by a straight line, eliminating that variable from the representation.)

Thus the joint-angle representation is a geometric characterization (including the 6 body coordinates) that uses the fewest variables. How then does the six-variable-per-rigid-segment representation avoid including extra

degrees of geometric freedom the vehicle does not possess? The answer is that there must be constraint equations which insure that the "knee-bone-is-connected-to-the-thigh-bone," that the limb segments cannot float arbitrarily in space with respect to one another but are constrained to meet and revolve about one (or 2 for a shoulder) axis only at each joint.

So one can set forth a pair of extreme alternatives for the second option in the form of the physical model: either one can go with minimum-variable-representations like the joint-angle, body-coordinate representation or use separate-segment-coordinate representations which must also include constraint equations. There is an entire spectrum between these two, characterized by use of some joint-angles and some separate-coordinate representations for different parts of the vehicle. By necessity such mixed models must also include some constraint equations.

Perhaps it might not be evident why one should be at all interested in a separate-coordinate model. The reason revolves around the fact that the joint angles must make their effect felt through the products of rotator matrices on coordinate systems that are different in each limb system. It is therefore very complex to express velocities and accelerations of limb segments, and coordinate-system-rotations must be continually performed, in order to use the joint angles as a representation of vehicle geometry.

The last choice that can be made among the possible physical models that will analyze vehicle physics is the choice between the different forms of classical mechanics: Newtonian and Lagrangian. Newtonian mechanics is comprised of coupled differential equations of the familiar $F=ma$ type. It deals with time derivatives of the motion and with forces. Lagrangian mechanics gives equally valid results, but deals in variational calculus-formulated equations $\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q_i$ which are differential equations in terms of generalized coordinates and Lagrangian functions. This form deals with energies and work, taking derivatives with respect to (perhaps physically non-existent) coordinate

VELOCITY AND ACCELERATION IN BODY COORDINATES

j = logical joint index varies from 1 at should to $N_{\text{joint}} = N_J$

K = limb index, arbitrarily assigned, varies from 1 to $N_{\text{leg}} = N_L$

i, ℓ, M used as dummy indices

$\ell_{K,j}$, $M_{K,j}$, $I_{K,j}$, and a_K are fixed by vehicle design and do not vary for vehicle physics models.

$X_0 = [X_0 \ Y_0 \ Z_0]^T$, θ_X , θ_Y , θ_Z , and all θ_{Kj} are geometrical unknowns that specify body configuration at any point in time.

R_{Kj} is the rotator matrix of logical joint (K,j) , and assumes that the model keeps track of which axis that joint rotates about

$$\begin{aligned}
 R_{Kj} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos & \sin \\ 0 & -\sin & \cos \end{bmatrix} \text{ or } \begin{bmatrix} \cos & 0 & \sin \\ 0 & 1 & 0 \\ -\sin & 0 & \cos \end{bmatrix} \text{ or } \begin{bmatrix} \cos & \sin & 0 \\ -\sin & \cos & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 \dot{R}_{Kj} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin & \cos \\ 0 & -\cos & -\sin \end{bmatrix} \text{ or } \begin{bmatrix} -\sin & 0 & \cos \\ 0 & 0 & 0 \\ -\cos & 0 & -\sin \end{bmatrix} \text{ or } \begin{bmatrix} -\sin & \cos & 0 \\ -\cos & -\sin & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
 \ddot{R}_{Kj} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\cos & -\sin \\ 0 & \sin & -\cos \end{bmatrix} \text{ or } \begin{bmatrix} -\cos & 0 & -\sin \\ 0 & 0 & 0 \\ \sin & 0 & -\cos \end{bmatrix} \text{ or } \begin{bmatrix} -\cos & -\sin & 0 \\ \sin & -\cos & 0 \\ 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

$\dot{R}_{Kj}$ = derivative of matrix with respect to its angle.

Also, for simplicity let $R_{K,0} = R_2(\theta_Y)$, $R_{K,-1} = R_3(\theta_Z)$, $R_{K,-2} = R_1(\theta_X)$.

Let b_{Kj} = fixed row vector for a given calculation, usually $[\ell_{Kj} \ 0 \ 0]$; then

$$\vec{x}_{Kj}(\vec{b}_0) = \vec{x}_0 + \sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} \vec{a}_K, G_{Kj} \prod_{i=-2}^j R_{Ki}$$

where $b_{Kj} = [\ell_{Ki} \ 0 \ 0]$ for all but the last term in the sum, when it equals $\vec{b}_0$. Letting

$\vec{V}_0 = \frac{d}{dt} \vec{x}_0$, $\vec{A}_0 = \frac{d}{dt} \vec{V}_0$, we can compute velocities at any point:

$$\vec{V}_{Kj}(\vec{b}_0) = \vec{V}_0 + \sum_{i=-2}^j \vec{b}_{Kj} \left(\sum_{M=-2}^j \frac{\partial G_{Kj}}{\partial \theta_{KM}} \dot{\theta}_{KM} \right) \vec{a}_K, \text{ where we}$$

define $\theta_{K,0} = \theta_Y$, $\theta_{K,-1} = \theta_Z$, $\theta_{K,-2} = \theta_X$ for simplicity,

$$\text{but } \frac{\partial G_{Kj}}{\partial \theta_{KM}} = \begin{bmatrix} j \\ \prod \\ i=M+1 \end{bmatrix} R_{Ki} \cdot \dot{R}_{KM} \begin{bmatrix} M-1 \\ \prod \\ i=-2 \end{bmatrix} R_{Ki}, \text{ or } = 0 \text{ if } M > j.$$

VELOCITY AND ACCELERATION IN BODY COORDINATES (CONTINUED)

But since rotator matrices have the property $R^{-1} = R^T \rightarrow G^{-1} = G^T$,

$$\frac{\partial G_{Kj}}{\partial G_{KM}} = G_{Kj} G_{KM}^{-T} \dot{R}_{KM} G_{KM} = G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM}$$

$$\vec{V}_{Kj}(\vec{b}_0) = \vec{V}_0 + \sum_{M=-2}^j \dot{\theta}_{KM} \left[\sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM} \vec{a}_K \right]$$

$$\vec{A}_{Kj}(\vec{b}_0) = \vec{A}_0 + \sum_{M=-2}^j \ddot{\theta}_{KM} \left[\sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM} \vec{a}_K \right] +$$

$$\sum_{M=-2}^j \sum_{\substack{\ell=-2 \\ \ell=M}}^j \dot{\theta}_{KM} \dot{\theta}_K \left[\sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_M^T \dot{R}_{KM} G_{KM} G_K^T \dot{R}_K G_K \vec{a}_K \right]$$

for $\ell < M$

$$\left[\sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_K^T \dot{R}_K G_K G_{KM}^T \dot{R}_{KM} G_{KM} \vec{a}_K \right]$$

for $\ell > M$

parameters to derive a "least action."

In almost every complex dynamics problem, the usual approach is to use the Lagrangian form. If constraints are embedded in the choice of the generalized coordinates, and the Lagrangian computed using this "preconstrained" representation, there generally turns out to be fewer equations and less unknowns in the Lagrangian form. Thus the Lagrangian form incorporates constraints within the formulation without the redundant coordinates (or dually, forces) that accomplish those constraints that appear in the Newtonian form.

The non-technical reader might be prompted to say that there should only be one sort of physics that describes vehicle motion. This is true, for if one chooses the generalized coordinates in a Lagrangian formulation to be inertial coordinates, and treats the constraints subsequently, the resulting equations are identical to those of the Newtonian formulation. However, when it comes to incorporating constraints, the Lagrangian form eliminates all of the equal-and-opposite forces of the Newtonian form.

Indeed, for the present problem, the Newtonian formulation has many redundant forces that can't be cancelled. Each limb segment is affected by six independent forces and six independent torques, and is capable of motion in six independent inertial coordinates $(X, Y, Z, \theta_x, \theta_y, \theta_z)$ before constraints are added. In comparison there are only two variables, a joint angle and its accompanying torque for each limb segment in the Lagrangian formulation, because the constraints are implicit in the formulation. In addition, constraints are hard to impose on the Newtonian form since a constraint $F(X_1, X_2, \dots, X_n) = 0$ must be inverted into $X_1 = g(X_2, X_3, \dots, X_n)$ and brute force-substituted into the equations of the Newtonian form, often creating a complex implicit equation.

Besides containing more variables and having a variety of difficulties implementing the many geometric constraints, the Newtonian formulation is plagued by a functional complexity that usually means that it is more difficult to

express and substitute within the many Newtonian equations without human error. Thus the Newtonian form seems to be the distinctly poorer choice, and at best, after a vast amount of error-prone mathematical manipulation, gives the same result as the Lagrangian form.

The choice is not quite as clear as one would think, however. Indeed, this document may well stand alone in its advocacy of the Newtonian form as the superior arrangement of the physical model for computer evaluation. To begin with, one of the dominant physical effects for WLRNV physics is the drag action of the feet. This drag is not easily expressed as anything but a velocity-dependent force. Since in general Lagrangian forms can only deal with non-potential energy forces by expressing them as work performed, the Lagrangian form cannot deal directly with the known models for the hydrodynamic drag on the feet, which in turn is so critical to WLRNV motion.

This drawback of the Lagrangian form turns out to be correctable by the addition of extraneous degrees of freedom. By assuming that the ankles are capable of degrees of freedom they never really use (virtual displacements), the Lagrangian techniques can find the force associated with that degree of freedom at a given time. If then that D.O.F. is never exercised (the virtual displacement equals zero) the real vehicle is being modeled and the force associated with that D.O.F. has been found. Thus, once the model calculates foot motion, the drag-model for the foot can be evaluated to obtain a force, which can become a known input to the Lagrangian model. Thus, with a meagre penalty, the Lagrangian form can deal readily with a force-output model for the drag feet, and still appears superior. As for other, intermediate choices, one would not want to proceed with any separate-coordinate-representation geometrical models in the Lagrangian formulation (including partial, compromise choices), since they only produce Newtonian-type equations, being basically inertial-type coordinates.

The first choice is not much more arbitrary for the

Lagrangian form, since using the joint torques as the N_{joint} known parameters (specified by the control) gives differential equations with terms containing $\ddot{\theta}_i, \dot{\theta}_i \dot{\theta}_j$, and τ . One must therefore use these equations to solve for $\ddot{\theta}$, which involves a matrix inversion (better than solving an implicit function by far, but still requiring manipulation of a $N_{\text{joint}} \times N_{\text{joint}}$ matrix), and then time-integrate twice to get values to plug into the equations for the next time step. (A computer can only solve differential equations at a finite number of time steps.)

In contrast, if the N_{joint} joint angles are the controlled inputs, their derivatives can be taken and the resulting values plugged into the complex functions of the Lagrangian equations. It turns out (note formulation) that the torques are isolated on the right hand side of these equations, so that straight computation (without differential equation solutions) gives the joint torques without matrix inversion. Of course it should be remembered that six differential equations for $X, Y, Z, \theta_x, \theta_y$, and θ_z must be solved, including a 6×6 matrix inversion to get the body coordinates, but the torque input choice has these same problem also.

One may therefore state that the best of Lagrangian models uses a joint-angle representation of geometry and N_{joint} θ inputs as its known variables. Of course extra D.O.F.'s must be added (and the foot-force model included), to integrate the effect of the feet into the otherwise energy-conservative model. It turns out that, even with the marked superiority of the Lagrangian formulation, the complexity of velocities using the joint-angle representation proves a major challenge for practical implementation of this model. Using the joint angles and body coordinates to describe vehicle geometry, the position of the masses in the limbs are found as sums of products of rotator matrices, each matrix being determined by one joint angle. This form may be tractable, but taking a time derivative to obtain velocity gives chain rule contributions for each of the matrices in a string of each product. Thus velocity ends

up containing a sum of N_{joint}^2 products.

The Lagrangian formulation unfortunately requires kinetic energies, which involve squaring these velocities and mass-integrating the squared velocity over a limb. Squaring velocity gives sums of N_{joint}^3 products. Summing $1/2 M V^2$ for each segment in a limb bring this up to N_{joint} matrix products to get the kinetic energy. The $\frac{\partial}{\partial \dot{q}}$ does not change the number of terms, but unfortunately there are $N_{\text{joint}} \dot{\theta}_i \dot{\theta}_j$'s in the evaluation of T , the kinetic energy, so that time derivatives require $\dot{\theta}_i \frac{\partial f(\theta)}{\partial \dot{\theta}_i}$, which bring the count of matrix products in $\frac{\partial}{\partial \dot{q}}(\frac{\partial L}{\partial \dot{q}})$ up to N_{joint}^5 .

This may not appear unacceptable, even after noticing there is also an N_{joint}^5 sum of matrix products in $\frac{\partial L}{\partial q}$ also, but one must remember that the N_{joint} is not simply the number of joints in a limb (which could be below 5) but includes the three body angles θ_x, θ_y , and θ_z . As such, $N_{\text{joint}} > 7$ if the vehicle is to move at all. Thus each Lagrangian equation includes at least 2×10^5 matrix products or a million floating products. Recalling that there are at least $4 \times (N_{\text{joint}} - 3) + 6$ equations, each with different matrix products in the Lagrangian formulation, execution of one time step of this formulation on a very fast (10^7 multiplications-per-second) computer would take 2 seconds.

The previous calculation is also quite optimistic. Not only was the time for arithmetic additions ignored, but the angular $I\omega^2$ kinetic energy, the foot terms, the gravitational potential energy, and a variety of other terms were ignored. An alternate solution that saves greatly on computation (by saving these matrix products instead of recalculating them) would require storage of at least N_j^2 3×3 matrices and, at 10^6 memory recalls per second (again a substantial requirement), one limb motion-calculating time step would require only .5 seconds.

A reasonable observer would quickly say: If the joint angle representation is causing such problems, then consider something more redundant and less complex. In the separate-coordinates-per-segment representation, the complicated kinetic energy terms are just $1/2 M_{\text{segment}}$

EULER-LAGRANGE DYNAMICS

Using Pre-Constraint via Body Coordinates

Assume model is constrained so that all positions can be expressed via body coordinates, thereby restricting motion to feasible logical joint rotations

Lagrangian $L = T - V = \text{Kinetic} - \text{Potential energy}$. Let $\vec{b}_{0_{Kj}}$ be the center of mass of the (K,j) logical limb segment. The translational K.E. $= \sum 1/2 M V^2$. Let

$$H_{KMj} = \sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM} a_K$$

$$\vec{V}_{Kj}(\vec{b}_{0_{Kj}}) = \vec{V}_0 + \sum_{M=-2}^j \dot{\theta}_{KM} H_{KMj} \quad \vec{V} \cdot \vec{V} = \vec{V}_0 \cdot \vec{V}_0 + \sum_{M=-2}^j \dot{\theta}_{KM} \vec{V}_0 H_{KMj}$$

The translational K.E. for whole vehicle =

$$+ \sum_{M=-2}^j \sum_{\substack{\ell=-2 \\ \ell < M}}^j \dot{\theta}_{KM} \dot{\theta}_{K\ell} H_{KMj} H_{K\ell j}$$

$$2T_X = \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} M_{Kj} \sum_{M=-2}^j \sum_{\ell=-2}^j \dot{\theta}_{KM} \dot{\theta}_{K\ell} H_{KMj} H_{K\ell j} +$$

$$\vec{V}_0 \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} M_{Kj} \sum_{M=-2}^j \dot{\theta}_{KM} H_{KMj} + M_{veh} \vec{V}_0 \cdot \vec{V}_0$$

Each limb segment is also a rotating coordinate frame

$$\vec{\omega}_{Kj} = \frac{d}{dt} \begin{bmatrix} \hat{t}_1 \\ \hat{t}_2 \\ \hat{t}_3 \end{bmatrix}_{Kj} = \frac{d}{dt} G_{Kj} \begin{bmatrix} \hat{t}_X \\ \hat{t}_Y \\ \hat{t}_Z \end{bmatrix} = \sum_{i=-2}^j \dot{\theta}_{Ki} G_{Kj} G_{Ki}^T \dot{R}_{Ki} G_{Ki} \begin{bmatrix} \hat{t}_X \\ \hat{t}_Y \\ \hat{t}_Z \end{bmatrix} =$$

$$\sum_{i=-2}^j D_{Kj} \begin{bmatrix} \hat{t}_X \\ \hat{t}_Y \\ \hat{t}_Z \end{bmatrix} \dot{\theta}_{Ki}$$

The rotational kinetic energy has the form $1/2 I \omega^2 = 1/2 \omega^T I \omega = 1/2 \text{tr} [\omega_{Kj}^T I_{Kj} \omega_{Kj}]$ so that

EULER-LAGRANGE DYNAMICS (CONTINUED)

$$2T_R = \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} \text{tr} [D_{Kj}^T I_{Kj} D_{Kj}], \text{ and we can write}$$

$$T = \left[\frac{1}{2} M_{veh} \dot{\vec{V}}_0 \cdot \dot{\vec{V}}_0 + \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} \left[M_{Kj} \left(\dot{V}_0 \sum_{M=-2}^j \dot{\theta}_{KM} H_{KMj} + \sum_{M=-2}^j \sum_{\ell=2}^j \dot{\theta}_{KM} \dot{\theta}_{K\ell} H_{KMj} H_{K\ell j} \right) + \sum_{M=-2}^j \sum_{\ell=2}^j \dot{\theta}_{KM} \dot{\theta}_{K\ell} \text{tr}[D_{Kij}^T I_{Kj} D_{Kij}] \right] \right]$$

$$V = \text{Potential Energy (gravitational only)} = \sum Mgh \quad \text{or}$$

$$V = \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} M_{Kj} [0 \ 0 \ 1] \vec{x}_{Kj} (\vec{b}_{0Kj}) = M_{veh} Z_0 + \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} M_{Kj} \sum_{i=-2}^j b_{Kj} G_{Kj} \phi_K^2$$

Then one formulates the Lagrangian mechanics with generalized coordinated $X_0, Y_0, Z_0, \theta_X, \theta_Y, \theta_Z, \theta_{Kj}$ + virtual displacements. N_V per limb of these generalized forces τ_{Kj} corresponding to the variables

Using the system of equations:

$$\underbrace{\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\theta}_{Kj}} \right) - \frac{\partial L}{\partial \theta_{Kj}} = \tau_{Kj}}_{N_J \cdot N_L + \text{virtual } N_V \text{ equations}} \quad \underbrace{\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{x}_{X,Y,Z}} \right) - \frac{\partial L}{\partial x_{X,Y,Z}} = 0}_{\substack{\text{or} \\ X_0, Y_0, Z_0 \quad X_0, Y_0, Z_0 \\ \text{6 equations}}}$$

- In the torque-input formulation, one assumes the τ_{Kj} are specified and solves $N_L(N_J + N_V) + 6$ nasty non-linear coupled O.D.E.s.
- In the angle-input formulation, one assumes all $\theta_{Kj}(t)$ are specified, solves for $N_L(N_J + N_V)$ variables the θ_{Kj} while solving the 6 O.D.E.s for $\vec{A}_0$ and $\ddot{\theta}_{X,Y,Z}$. Since L depends on the 6 B.C.C.s which can't be prespecified, the 6 B.C.C.s must be found stepwise from the O.D.E.s before the τ_{Kj} can be solved for, but the τ_{Kj} s do not affect the solution of the O.D.E.s.

EVALUATION OF EULER-LAGRANGE MECHANICS

Virtual displacements must be added to the model to determine force-related phenomena. They are entered as extra segments with non-physical (logical) motions, zero mass, and inertia (sometimes zero length), whose angles (generalized) coordinates are frozen:

6 virtual segments (massless) allow 3 arbitrary foot forces and 3 arbitrary foot torques to enter the model.

Bending moments:

To instrument structure forces
not otherwise available in model

2 zero length segments with
angles frozen at $\alpha_1 = \alpha_2 = 0$

BEST CASE COMPUTATION

Ignore all terms in L except

$$L_B = \sum_{K=1}^{N_L} \sum_{j=1}^{N_J} \frac{M_{Kj}}{2} \sum_{M=-2}^j \sum_{\ell=-2}^j \dot{\theta}_{KM} \dot{\theta}_{K\ell} H_{KMj} H_{K\ell j}$$

$$H_{KMj} = \sum_{i=2}^j \vec{b}_{Kj} G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM} a_K$$

Ignore all but $N_L(N_J + N_V)$ equations, ignore all terms except

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\partial L_B}{\partial \dot{\theta}_{Kj}} \right) &= \frac{\partial}{\partial t} \sum_K \sum_j \frac{M_{Kj}}{2} \sum_M \sum_{\ell} \dot{\theta}_{KM} \dot{\theta}_{K\ell} H_{KMj} H_{K\ell j} ; \quad \text{ignore } \ddot{\theta} \text{ term} \\ &= \sum_K^{N_L} \sum_j^{N_J} \frac{M_{Kj}}{2} \sum_{M=-2}^j \sum_{\ell=-2}^j \dot{\theta}_{KM} \dot{\theta}_{Ki} 2 \sum_{i=-2}^j \frac{\partial H_{KMj}}{\partial \dot{\theta}_{Ki}} H_{K\ell j} \end{aligned}$$

$$\frac{\partial H_{KMj}}{\partial \dot{\theta}_{Ki}} = \sum_{i=-2}^j \vec{b}_{Kj} G_{Kj} G_{KM}^T \dot{R}_{KM} G_{KM} G_{Ki}^T \dot{R}_{Ki} G_{Ki} a_K \quad \text{for } i < M; \text{ reverse } i \text{ and } M$$

for $i > M$

EVALUATION OF EULER-LAGRANGE MECHANICS (CONTINUED)

As such 8 matrix multiplications in $\frac{\partial H_{KMj}}{\partial \theta_{Ki}} H_{Klj}$;

ignoring scalar multiplications and additions, a four-legged RNV requires

$$\frac{\partial}{\partial t} \left(\frac{\partial L_B}{\partial \dot{\theta}_{Kj}} \right) 4 \sum_{j=-2}^{NJ} \sum_{M=-2}^j \sum_{l=-2}^j \sum_{i=-2}^j (8 \text{ matrix mpy's}) = 8 (NJ + 3)^4 \text{ matrix multiplies.}$$

Entire system solution requires the $\frac{\partial L_B}{\partial \dot{\theta}_{Kj}}$ terms (which are just as complex) as well in each of $(N_j \cdot N_L + 6)$ equations, requiring approximately $4;16 (NJ + 3)^3$ matrix multiplies, if $N_j = 4$ (no virtual motions, very simple leg). This is about 1 million matrix floating multiplies per second (i.e. the best available technology) and noting that Rs are only 2×2 matrices (effectively) while Gs are non-sparse 3×3 , one time step takes at least 2 seconds.

Modelling vehicle phenomena with reasonable timesteps of $1/2$ the characteristic times of the vehicle implies that this model would at best compute 200X real time; we have ignored all of the scalar operations and most of the terms in

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0 !$$

At 300 points per limb cycle and 100 gradient steps to find an optimum (assuming differential approximation hold valid for a variational optimization), it would take 1 day of computing on a very fast processor to find optimal motion.

$(\dot{\vec{X}}_j \cdot \dot{\vec{X}}_j) + 1/2 [I]_{3 \times 3} (\text{segment}) \dot{\vec{\omega}}_j \cdot \dot{\vec{\omega}}_j$. The derivatives of the Lagrangian formulation cause no problems, but one quickly sees that what results is a pure Newtonian form!

Indeed, the Newtonian form turns out to be manifestly easier, so the least complex choice of Lagrangian formulation would put all limb segments, feet, and body into separate-coordinate representations yielding a poorly arranged Newtonian form. Then one is faced with the question of how best to evaluate a Newtonian form, especially since it is so difficult to impose the required constraints into the Newtonian formulation.

Indeed there turns out to be one alternative for the first two choices described that turns out to be straightforward in Newtonian form. Instead of using the standard interpretation that forces cause motion, one can interpret mechanics such that known motion requires certain calculable forces. Thus, if one specifies the motion with the available N_{joint} control inputs, the forces that produce that motion can be calculated.

Now the usefulness of the joint-angle representation comes to light, because not only do N_{joint} specified angles in this model provide a full description of the motion of all masses given in the six body coordinates, but the representation implicitly contains the constraint equations. If the vehicle performs a motion that can be specified by just the joint angles and the six body coordinates, then the limb segments stay attached to one another and only move as rotations about the specified axis at the joints. Thus any motion that can be performed by an arbitrary history for the N_{joint} angles will be a properly constrained motion, and the constraint equations have effectively already been substituted into the motion before the forces are calculated.

One can easily envision that time-histories for all of the N_{joint} θ_j 's may be specified for a limb cycle (perhaps as a Fourier series). These (smooth) histories can be operated on analytically to give exact values of angular velocity and angular acceleration at each joint. These

NEWTONIAN DYNAMICS

Given all C.D.O.F. position histories $\theta(t)$, the rotator matrices can be used to find position, velocity, and acceleration down each limb given the 6 B.C.C.'s

$$\vec{x}_{Kj}(\vec{b}_0) = \vec{x}_{Kj-1}(\text{end point}) + \vec{b}_0 G_{Kj} \vec{a}_K ; \quad \vec{b}_0 = \text{limb coord. of endpoint or center of mass}$$

$$G_{Kj} \vec{a}_K = R_{Kj} (G_{Kj-1} \vec{a}_K)$$

$$\vec{v}_{Kj}(\vec{b}_0) = \vec{v}_{Kj-1}(\text{endpoint}) + \vec{b}_0 \dot{G}_{Kj} \vec{a}_K$$

$$\dot{G}_{Kj} \vec{a}_K = \dot{R}_{Kj} (G_{Kj-1} \vec{a}_K) + R_{Kj} (\dot{G}_{Kj-1} \vec{a}_K)$$

$$\vec{A}_{Kj}(\vec{b}_0) = \vec{A}_{Kj-1}(\text{endpoint}) + \vec{b}_0 \ddot{G}_{Kj} \vec{a}_K$$

$$\ddot{G}_{Kj} \vec{a}_K = \ddot{R}_{Kj} (G_{Kj-1} \vec{a}_K) + R_{Kj} (\ddot{G}_{Kj-1} \vec{a}_K) + 2 \dot{R}_{Kj} (\dot{G}_{Kj-1} \vec{a}_K)$$

So, starting with $\vec{x}_0, \vec{v}_0, \vec{A}_0, \theta_X, \theta_Y, \theta_Z, \dot{\theta}_X, \dot{\theta}_Y, \dot{\theta}_Z, \ddot{\theta}_X, \ddot{\theta}_Y, \ddot{\theta}_Z$, we can calculate recursively all required $\vec{x}, \vec{v}, \vec{A}$ down each limb by successively calculating and storing the vectors $\vec{x}_{Kj}, \vec{v}_{Kj}, \vec{A}_{Kj}, \dot{G}_{Kj}, \ddot{G}_{Kj}$ in each segment.

The a model of the drag forces on the foot can give $\vec{f}_{\text{foot}}, \vec{\tau}_{\text{foot}}$ using the $\vec{x}_{\text{foot}}$ and $\vec{v}_{\text{foot}}$ from the above calculations. The foot forces and torques (= 0 foot out of water) are the propagated back up the limbs by Newton's laws:

$$\vec{f}_{Kj}^{\text{upper}} = \vec{f}_{Kj}^{\text{lower}} + M_{Kj} \vec{A}_{Kj}(\text{center of mass } \vec{b}_0)$$

$$\vec{\tau}_{Kj}^{\text{upper}} = \vec{\tau}_{Kj}^{\text{lower}} + I_{Kj} (\ddot{G}_{Kj} \vec{a}_K) + \vec{f}_{Kj}^L B_L + \vec{f}_{Kj}^U B_U$$

B_L and B_U are the matrices of moment arms or end forces about mass centers

NEWTONIAN DYNAMICS (CONTINUED)

Equilibrium is asserted at every joint:

$$\vec{f}_{KJ-1}^{\text{lower}} = R_{Kj} \vec{f}_{Kj}^{\text{upper}} \quad \vec{\tau}_{KJ-1}^{\text{lower}} = R_{Kj} \vec{\tau}_{Kj}^{\text{upper}}$$

With these equations the forces and torques are propagated back up the limbs to find

$$\vec{A}_0 = \frac{1}{M_{\text{body}}} \sum_{L=1}^{N_L} \vec{f}_{\text{shoulders}}; \quad \vec{\theta}_{X,Y,Z} = I_{\text{body}}^{-1} \left[\sum_{\text{shoulders}} \tau + f B \right]$$

Then $\vec{A}_0$ and $\vec{\theta}_{X,Y,Z}$ are numerically integrated to give new $x_0, v_0, \vec{\theta}_{X,Y,Z}$, $\vec{\theta}_{X,Y,Z}$ to begin a new time step.

Computation

Downward propagation requires $6 \cdot 2 \times 2$ matrix products, 3 vector products per limb segment, or $N_L \cdot N_J$ 27 multiplies for downward propagation.

Foot force model: 1 3×3 matrix product (to resolve orientation of surface) plus table lookup and perhaps 10 more multiplies = $N_L \cdot 37$ multiplies.

Upward propagation requires 2 vector multiplies, 1 3×3 matrix multiply, and 2 2×2 matrix multiplies, or $N_L \cdot N_J \cdot 41$ multiplies for upward propagation.

Body motion requires N_L vector multiplies and 1 3×3 matrix multiply, plus about 60 multiplications for fourth-order numerical integration. Total is $N_L(68N_J + 38) + 80$ multiplies per time step. For quadruped with $N_J = 4$ this gives a 1.5 ms timestep on a 10^7 floating-multiply/second machine ($N_J = 8$ —most complex model required—takes only 2.4 ms). This is almost a real time simulation, and would derive an optimal motion in about 2 minutes.

values may then inserted into the derivative versions of the geometrical equations that give positions for mass in the limbs. Even for accelerations, only $N_{\text{joint}} \cdot N_{\text{LEG}} \cdot 9$ products of 3×3 matrices are necessary to get the acceleration and angular acceleration for all masses in the vehicle.

Once acceleration is known, force and torque are simply found from the coupled $F = ma$ type of equations, along with the equal-and-opposite-reaction equations (see formulation). These equations are not differential equations and will have no numerical stability problems for computer implementation. In fact, the majority of these equations are linear, except for those that find foot force from foot motion by the physical model for foot drag. Unfortunately the six body coordinates are still unknown, but $\ddot{X}_0, \ddot{Y}_0, \ddot{Z}_0, \ddot{\theta}_x, \ddot{\theta}_y, \ddot{\theta}_z$ can be found without matrix inversion, once the forces that the limbs exert on the body have been calculated from the simultaneous force equations.

Thus one can envision a compact cyclic procedure for implementing one time-step of this Newtonian formulation. The time histories of motion for each joint-angle, being specified throughout the cycle (angle, angular velocity, and angular acceleration), are sampled for this time step from a stored list of the histories of these derivatives. The values are used in a differential-geometric formulation that takes nine N_{joint} matrix multiplications and stores $N_{\text{joint}} \cdot 9 \cdot 3$ vectors (see formulation). This gives the position, velocity, and acceleration histories for all masses in the body.

Starting then at the feet, position and velocity vectors determine the drag forces according to the physical foot model. This gives forces and torques at the extreme end of the limbs. Then $F = ma$ and $\tau = I\dot{\omega}$ are used to sequentially find forces and torques ascending the limbs toward the body. (Note that there was no need to simultaneously solve the coupled force equations.) Eventually one arrives at the forces at all of the

uppermost ends of the limbs, giving net force and torque on the body. Using $\Sigma \vec{F} = m\ddot{\vec{x}}$ and $\Sigma \vec{\tau} = I\ddot{\vec{\omega}}$ gives the second derivatives of the six body coordinates. Highly accurate numerical integration techniques for just such complex-function differential equations as the Body $\ddot{\theta} = F(\theta\text{'s and Body } X, Y, Z \text{ and Body } \theta_x, \theta_y, \theta_z)$ exist (of which the predictor-corrector is probably most effective here), and may be used with $\dot{X}$ and $\dot{\omega}$ to give accurate values for the six body coordinates to be used in the calculations of the next time step.

Not only does this cyclic evaluation advance one time step very rapidly, but it appears that this sort of routine is especially well suited to being separated for simultaneous, cooperative solution by a number of computers. (The reasons why this should be considered will become amply evident in Part II.) It should be noticed that, unlike the Lagrangian formulation, in all of the equations in this Newtonian formulation, the complexity resides in the geometrical calculations of accelerations from θ inputs.

Observing the nature of these calculations, one notices that angles from another leg never enter the geometrical calculation for a given leg. In fact, the physics in each leg is decoupled until forces are summed from all legs to contribute to $(\ddot{X}, \ddot{\theta}_b)$. Thus each leg can be processed independently by a different computer with no need to interact except with a central computer that calculates the time-step changes in the six body coordinates.

In fact, exploiting the multi-computer concept to fullest advantage in order to achieve minimum cycle time, one could notice that the determination of the motion of each limb segment or foot is separate beyond the first $\theta_x, \theta_y, \theta_z$ D.O.F.'s. Thus one processor per variable could be employed, if they could all communicate to receive new data each time step, and to determine their neighbors' force environments, to achieve the desired flow of the force calculations.

Several things should, however, be noticed about this streamlined method. The actuation and computations for this model will require extreme dynamic range on the parameters. When a pontoon-foot first descends onto the water surface, relatively large shock-like pulses will propagate up the attached limb across a few time steps. Thus many of the forces and torques in a limb could stay at low values for most of a limb cycle, only to peak at large values during small fractions of the cycle.

Obviously one of the features of an optimal solution will be to keep these force spikes minimized to conserve structure weight, but that does not mean the physical model will never face high peak loadings, and the wide sensor accuracy range and possible numerical instability problems associated therewith. This aspect will possibly mean that this otherwise ideal control method cannot be readily adapted to walking motion on land, where foot force undergoes true discontinuities at contact time or zero velocity and may cause severe numerical shocks in the physical model.

OPTIMIZING VEHICLE MOTION

Now that a scheme to practically calculate vehicle physics is presumably in hand, it is appropriate to elaborate on the uses of that calculation, in particular the nature and function of computer programs that make use of the physical-model program to achieve optimal vehicle motion. There are essentially only N_{joints} variables if the vehicle physics problem is under control by the vehicle. The actual variable being controlled may not be the joint angle, but could either be an angle, angular velocity, or torque variable that is a function of the other two variables and a control signal.

So long as it is possible to find the angle and torque at each joint as a function of time that is in some sense productive of an "optimal" vehicle motion, it hardly matters exactly what sort of physical activation (torque or geometric) is being provided at that joint, since the necessary control can be calculated implicitly, given the required torque, angle, etc. and the properties of the actuator. It should be noted that this procedure appears satisfactory, but indeed the whole optimization procedure may fail if in fact the implicit equation that gives the control from torque, velocity, etc. is singular, or nearly so.

Singular points should not disturb the optimality of a solution that just fills in the control-history with a continuous value over that singular point, so long as no unmonitored variable in the model (e.g. yaw ankle force) becomes momentarily unbounded. However, for a large class of problems, if there is a singular point in the actuator function, the optimal solution will dwell for a period (known as a "singular arc") at that insoluble point in the actuator model where a control can't be determined.

One should be able to see that, unlike a true singularity (where no control exists that will accomplish the desired actuation), non-unique points where more than

one value of the control will accomplish the optimal motion are no problem, since one can arbitrarily pick the most convenient of these and still attain the optimal motion. Usually non-unique controls will occur far more frequently than truly singular controls. However, it will often happen that more control or excessive rates of control change will be demanded which the physical actuator cannot deliver.

Bounded controls constitute one of the least well-behaved domains of optimization theory. In fact, for the present treatment, a practical definition of optimality will be adopted to avoid these problems of physical capacity-of-actuator bounds. The optimum will be defined as that solution which minimizes a cost function. Included in this cost function will be a non-linear cost on those parameters that are so limited. Since the only real purpose of this component of cost is to restrict the range of a control variable to within actuator limits, and has nothing to do with real optimality of the motion, this parameter will be arbitrary. The designer is then free to choose this parameter so that the optimal motion value never exceeds its limit during the optimal limb cycle.

The solution that results from such a procedure will be somewhat arbitrary, and is unlikely to be as truly optimal as the real solution to a bounded-control optimization problem. But not only will this choice suffice to deliver the sort of smooth, no wasted motion, no wasted energy, no unnecessary structure-loading motion we want, but indeed could actually be closer to a practical optimum than the bounded-control solution. It must be remembered that hard limits are as much an engineering artifice as was the arbitrary penalty function that effectively eliminates actuator overloads.

In fact, if one is sufficiently sophisticated, one can actually determine a lifetime-expectation for the actuator in terms of the hardness of control. Often some valid cost that has a true bearing on vehicle effectiveness

can be derived from the smooth lifetime-projection vs. control-strength curve that is more meaningful than the hard-limit region-of-operation model that results in the bounded control problem. In any event, bounded control optima will look very much like cost-function-optimized results, if the cost function used for the limited variable is very steeply non-linear near these limits. If this cost were actually made into a step rise at the hard limit the two solutions would be the same.

Having concluded that it is indeed possible to arbitrarily vary N_{joint} angles-histories over the limb cycle and get an achievable control that does provide these angle histories, one moves to the question of how to vary those angles to get optimal motion. This problem then is no longer concerned with actual controls, but exists on a more abstract plane that is particularly well adapted to the physical model and its computer evaluation. Keeping in mind that a cost term must be added for each control parameter that has operative limits, the control question becomes: What is defined as optimal?

In order to make use of the engineering and mathematical wisdom of past work in optimization theory, one defines an optimal motion as one that minimizes some sort of quantitative cost function. This function can include not only values of various physical parameters during the limb cycle, but integrals of these parameters over all or part of the cycle. There are then techniques of variational calculus that allow one to extremize (i.e. find local maxima and minima of) this cost function.

What sort of variable might be desirable to include in a cost function? One obviously wants to minimize useless motion such as needlessly shaking the limbs and overly rigorous plunging of the pontoons on contact with the water. Thus the cost to be minimized should include the mechanical power that had to be furnished by the joints to obtain the desired vehicle motion, and also the power that was lost during the limb cycle to drag, actuator and bearing losses.

COST FUNCTIONS

Power Utilization

One-shot motion: $J_p = \int_{t_0}^{t_f} \underbrace{\left[\sum_{\text{all joints}} \tau_{\text{joint}} \dot{\theta}_{\text{joint}} \right]}_{\text{locomotive losses include drag losses}} dt + \underbrace{\int_{t_0}^{t_f} \left[\sum_{\text{all joints}} \frac{1}{2} f(\theta_{\text{joint}}, \dot{\theta}_{\text{joint}}) \tau_{\text{joint}}^2 \right] dt}_{\text{actuator losses}}$

Cyclic motion: $J_p = \oint \left[\sum_{\text{all } j} \tau_j \dot{\theta}_j + \sum_{\text{all } j} \frac{1}{2} f(\theta_j, \dot{\theta}_j) \tau_j^2 \right] dt$

To keep torque below $\tau_{\max}$, use abrupt function

$$J_{\text{limit}} = \int_{t_0}^{t_f} \sum_j g\left(\frac{\tau_j}{\tau_{j\max}}\right) dt \text{ for one shot or } \oint \sum_j g\left(\frac{\tau_j}{\tau_{j\max}}\right) dt \text{ for cyclic motion}$$

Can use hard limits $g(Y) = K_{\text{STEP}}(Y-1)$, K very large,
or extremely soft limits $g(Y) = K Y$, K moderate;
conventional control uses $g(Y) = \frac{1}{2} Y^2 = \frac{1}{2} \frac{\tau^2}{\tau_{\max}^2}$.

A more effective $g(Y) = KY^4$, where K is moderate.

Every stress in the structure can be expressed as a weighted sum of all forces and torques in the Newtonian Model:

$$\sigma = F(\text{all forces and torques in vehicle}) = \sum_{\text{all } F} K_i F_i + \sum_{\text{all } \tau} K_i \tau_i$$

This function can be simplified to have only a few significant contributions

$$\begin{aligned} \text{one shot } J_{\text{limit}} &= \int_{t_0}^{t_f} \left[\sum_{\text{all worst case points}} g\left(\frac{\sigma(\text{point, all forces})}{\sigma_{\max}}\right) \right] dt = \\ &= \int_{t_0}^{t_f} \left[\sum_{\text{all worst case points } x_j} g\left(\frac{\sum K(x_j) F_j}{\sigma_{\max}}\right) \right] dt \end{aligned}$$

$$\text{cyclic motion: } J_{\text{limit}} = \oint \left[\sum_{x_j} g\left(\frac{1}{\sigma_{\max}} \sum_{\text{all forces in model}} K(x_j) F_j\right) \right] dt$$

Stability J = energy required to tip vehicle over in easiest direction, averaged over one-shot or cyclic period.

These power terms can only be expressed as integrals of power terms such as $\tau_{\text{joint}} \cdot \omega_{\text{joint}}$ over the limb cycle. If vehicle mission range is the only concern, then one power loss is as serious as any other, and both mechanical power input for running and real power losses should add in cost. On the other hand, if vehicle infrared signature is particularly critical for a given design, then power loss in those devices that will heat up due to these losses should carry a larger weighting factor compared to the cost of other power drains.

Another quantity that we might like to maximize is vehicle stability, which can be defined as the amount of energy it will take to knock the vehicle over. Unstable vehicles don't take any such energy; they fall over without provocation. However any vehicle supported by several feet can be overturned if sufficient energy is provided to elevate and move its center of mass beyond the polygon described by the feet, at which point the vehicle falls over.

Overturning a vehicle could be done with a variety of forces in a number of directions, but one can be sure that it can't be done without sufficient energy to move the center of mass out from the support-polygon of weight-bearing feet. Thus, at any instant, one could calculate the energy increment required to overturn the vehicle. One could envision finding the minimum pushover-energy at the most unstable instant in the cycle, and subtracting this worst-case pushover energy from the cost function.

Unfortunately the functional that expresses the minimum of energy over time and pushover direction is particularly analytically intractable. Instead, a reasonable alternative would be to maximize the pushover energy, averaged over all directions and moments of time. Since this energy should be a smooth function of direction, the average energy will be qualitatively similar to minimum energy in directional variations. Time-averaging corresponds to a probabilistic design, because the expectation of the time before enemy action overturns the vehicle is

just a function of the average pushover energy and the average ordinance impact energy, since the time of impact is quite arbitrary. In contrast, the minimum pushover-energy over time corresponds to a worst-case design philosophy with respect to vehicle overturn, and unrealistically credits the opposition with being able to attack the vehicle at its least stable moment.

The cost function thus resembles the integrated power usage over the limb cycle covering a unit distance minus a weighting factor times cycle-averaged vehicle overturn energy. It should be recalled that the basic motion design problem is totally cyclic, since the majority of vehicle mission time and hardware capability center on the ability of the WLRNV to go places. Thus the foremost motion design will be the one that results in highest performance "running." Continuous running is a cyclic problem and, since all vehicle phenomena repeat each cycle, the optimum might just as aptly be defined over a single cycle.

Of course there are a number of motions to be optimized, but the prime concern is with high velocity vehicle, cruise. This being the case, there is one other sort of term to include in the cost which operates slightly differently from the energy-type terms: structure loads. Obviously too 'jerky' a motion might be overtaxing vehicle components structurally, without exceeding available actuator output, while still being optimal in the previous cost functions minimization.

So it would seem wise to add structure-loading terms to the cost function. However, there is qualitatively little point to trying to reduce the structure loads in a motion that is already sufficiently below yield strengths and fatigue limits already. Likewise, a direct attempt to penalize for structural loads could well lend to a sluggish and unnecessarily delicate motion that would waste power and be needlessly unstable.

An obvious solution is to employ the same scheme as was used for bounded controls: put a coefficient in

front of the structure-loading terms in the cost, and vary this arbitrary parameter to keep the loads in the resulting "optimal" motion within design limits. Unfortunately this still somewhat corrupts the optimum, since there is still a benefit attached to increasing reduction of structure loads well below the yield and fatigue limits. Thus the motion that would be calculated with such a cost still emphasizes unnecessarily low loads.

A better solution would be to adopt some realistic non-linear load dependent cost, such that the 'cost' of structure loads is essentially zero until those loads get close to yield/fatigue limits, the passing of which would damage the vehicle. Because some function of structure loads would have to be used in any event so that negative loads (tensions vs. compressions) were not beneficial without limit, this scheme is only slightly more computationally expensive. The first scheme proposed simply included the absolute value of the structure loads in the cost. An attractive non-linear alternative would be to take the fourth power of the loads into the cost (or some similarly stiff function), a function that is small for a substantial region around zero and increases rapidly near the extremes of its effective range.

This alternative will evidently suffice to keep structure loads in a reasonable range. One might well ask, in fact, why no such scheme was proposed for the bounded control variables. In fact, this is exactly the best way to handle the control bounds if the real cost (in terms of degraded vehicle component life, etc.) is known for controls beyond the bounds. However, often the function is not known, and the use of this sort of functional cost is perhaps needlessly cumbersome.

Also, it should be noted that there will be little point in even worrying about structure loads for some time, due to the order of the vehicle design procedure. Once vehicle physical parameters have been defined (e.g. limb lengths, mass distribution), the optimization routines are run to find the optimal motions for all

gaits and for the finite-duration motions (zig-zag, cornering, etc.) just described. Then, using the physics for these optimal motions, the highest required actuator outputs and structure loads that the vehicle must produce or withstand are found (assuming it is controlled to produce only optimal motions). Thus the capabilities of the structure and actuators are found after solution of the optimization problem, and one can optimize without any bounded variables.

Things will not be so pleasant indefinitely, however. Often a vehicle is designed and deployed, and new mission requirements will arise that must utilize a fixed vehicle design. These mission requirements will imply different cost functions when new optimized motions are defined. If the vehicle was designed to withstand the worst loading in one set of motions, and a mission profile update attempts to add new motion 'alphabets', it could be expected that some of the new motions might occasion overloads in the structure and actuators. In each cases, the bounded-variable techniques just outlined must be used.

The exact use of the cost function (whatever physical variables may be involved) should now be discussed. The reader, contemplating this sort of problem which involves finding "optimal" motion once a vehicle has been specified, will generally think in terms of getting the best performance out of a fixed design. For the primary cyclic-cruising problem of greatest concern, the vehicle performance is expressed simply in terms of attainable cruise speed.

Indeed it would seem that a reasonable performance parameter that should be maximized is specific velocity, V_{sp} , the distance traveled by the vehicle, given a fixed amount of energy (fuel) to expend, or its differential form, the velocity that can be maintained, given a fixed amount of power. These two forms are easily related and, since one is just the integral of the other, the optimized motion should be the same. However, the two interpretations have been pointed out because they relate to different

types of component limitations in the power subsystem.

In the cyclic problem, this parameter is simply the distance traveled per cycle over the energy consumed per cycle. As such, it does not exactly correspond to an available power limit, since there could be required-power peaks during the limb cycles so that the resulting motion did not make full use of the available power during most of the cycle. In any event, the maximization of this single parameter, which seems to be exactly the expression one desires for WLRNV cruise performance, turns out to have integrals in both denominator and integrator.

Classical optimization theory treats functions-of-integrals (such as ratios) by a clever trick: one expresses the entire integral as a new state variable in the problem, governed by the differential equation which is the inverse of the integral equation that defines the new variable. Then the cost function is just a function of state and control variables, without any integrals. Using this trick is computationally somewhat costly for the present problem, since it introduces new and complex differential equations, whose complexity comes from the fact that the power being integrated is a function of every geometric and torque parameter solved for by the physical model.

There is, however, a very attractive alternative to maximizing specific velocity: minimize the power-cost integral while constraining the cruise velocity to be fixed. This can be done for a whole range of interesting cruise velocities. The specific velocity ratio can then be computed, and its maximum can be found by one-parameter gradient methods that should require about a dozen computer solutions of the "optimal" solution problem. Not only is the optimizer simplified greatly by this one-integral cost, with the resulting calculations not taking much more computer time than maximizing V_{sp} , but it has useful additional benefits.

At times it may be desirable to cruise at less than the maximum-performance velocity, as in rendezvous

maneuvers, and surveillance sweeps. At other times, it could obviously be very useful to cruise at much higher speeds, even at the cost of great inefficiency, to evade pursuit or to get out of the line of enemy fire.

This latter problem involves the possibility of constraining power to be fixed, and solving the optimization problem to find the fastest cruise speed of which the vehicle is capable. This problem turns out to be much more computationally complex (it is not true that maximum velocity for fixed power will be the same curve as minimum power for fixed velocity), because power is a more complex (integral of sums of products) variable than is velocity and is more difficult to constrain.

Also, it turns out that maximum velocity is not determined by readily attainable amounts of available power, but rather by the available speed of the control system, so this problem requires no near-term solution.

A constant-velocity constraint is just one of the constraint equations that must be included in the minimum-power optimization problem. The problem as it has been sketched is cyclic: there are cyclic constraints that the vehicle must be in exactly the same geometry at the end of the cycle as at the beginning, except for horizontal translations. If motion is desired in the X direction, the Y, Z, θ_x , θ_y , θ_z as well as all of the joint angles must be the same at the end of the cycle as at the beginning. These are the so-called cyclic constraints.

The constant velocity constraint is not as simple as specifying that $\dot{X} - X$ has a constant value, where t_f and t_0 are cycle end and begin times, respectively. This is due to the fact that one of the variables to be optimized is the cycle duration t_f . Unlike all of the other parameters, this one has a single value, and is not a function to be defined over an interval of continuous time. In order to make the velocity constraint effective, one expresses it as an integral constraint over the cycle, where the cycle-average of $\dot{x}$ is equal to the velocity:

$$\frac{1}{t_f} \int_0^{t_f} (\dot{x} - V_{\text{cruise}}) = 0 \rightarrow \int_0^{t_f} (\dot{x} - V_c) = 0.$$

This produces an integral constraint which does not explicitly depend on the value of t_f . Such a constraint can very easily enter the problem by adjoining it to the cost with a non-time-dependent Lagrange multiplier. The constraint then enters directly into the form for the desired solution, and the constraint equation itself provides the extra information needed to determine the extra variable; the Lagrange multiplier.

Once the velocity constraint is adjoined, the problem looks exactly like the well-known Boltza problem, where the cost is a pure integral over the cycle. The six body coordinates, which are considered to be states, are governed by differential equations that determine their second derivatives, given the N_{joint} control variables (θ_{joint}) and their first and second derivatives. Unfortunately these differential equations are a bit too complicated to write simply in closed form.

However, it is easy to see that neither the integrand under the cost integral (the Lagrangian L is a scalar) or the functions (F) that determine X , Y , Z , θ_x , θ_y , θ_z are explicit functions of time. In fact, both L and F are totally computable if $\vec{X}$, $\vec{\theta}$ and θ_{joint} , θ_j , and $\dot{\theta}_j$ are specified. Thus we can partition the variables in this problem into the state variables $\vec{X}$, $\dot{\vec{X}}$, $\vec{\theta}$, $\dot{\vec{\theta}}$, and N_{joint} θ_j 's and $\dot{\theta}_j$'s, while there are N_{joint} arbitrarily variable inputs $\ddot{\theta}_j$.

In this representation, all of the states can be expressed as differential equations involving the other states and the inputs: i.e.

$$\ddot{\vec{X}} = F(\vec{X}, \vec{\theta}, \dot{\vec{X}}, \dot{\vec{\theta}}, \theta_j, \dot{\theta}_j, \ddot{\theta}_j), \quad \ddot{Y} = F, \quad \ddot{Z} = F, \quad (\text{etc.})$$

As such there is a vector function that gives the derivatives of $(2 N_{\text{joint}} + R = M)$ states depending on those states plus the inputs: $\text{state}_i = F_i$ (all state + all inputs). These functions are also adjoined inside the cost integral with M time-dependent Lagrange multipliers λ , in order to get the Hamiltonian $H = L + \lambda^t F$.

Since it turns out that the Hamiltonian is not an explicit function of time, ($\frac{\partial H}{\partial t} = 0$), the optimal solution

will be governed by the so-called first integral:
 $H = \text{constant}$. This is a fortunate situation, since otherwise one must resort to the so-called Euler-Lagrange equations $\dot{\lambda}_i^t = \frac{\partial H}{\partial x_i}$, which double the number of coupled differential equations. Matters become even more agreeable when the condition that asserts optimality of t_f is applied. Since there are no non-integrated components of the cost, the necessary condition for optimal t_f , known as "transversality" is $H(t_f) = 0$.

Thus the entire problem of finding an optimum requires just that $H(t) = 0$ or $L + \lambda^t f = 0$ throughout the cycle. Since L and F are computable throughout the cycle, the equation $L + \lambda^t f = 0$ gives one condition on the M λ 's which is in the nature of an orthogonality to F . This equation, however, is true for all points in time during the cycle and, even though there could conceivably be M different solutions to $L + \lambda^t f = 0$, only one will be "continuous" with the one from the last time step. Since F and L will be continuous in time, the smooth value for λ is the one to be chosen.

VEHICLE DESIGN OPTMIZATION

The previous section dealt with a method to determine the "optimal" control that gave an "optimal" motion for a given vehicle. The real problem of designing a WLRNV must include the motion design solution just discussed as a sub-design. However, the motion design turns out to be coupled into the rest of the design problem in a complicated fashion.

The optimal motion calculations make great use of the physical model, and as such are dependent on the physical parameters of limb length, joint configuration, mass distribution, and foot-drag characteristics. These parameters are fixed for a given vehicle design, but their values are determined in part by a knowledge of the optimal motion during the design process. The optimal motion specifies all of the required torques and forces, and is thereby used to find the required structure and actuator strengths. Both of these terms depend on the results of optimized-motion evaluations, but they in turn govern the mass of actuator and structure required. As such the mass distribution of the vehicle depends on the results of the optimization, and the optimization depends on the mass distribution.

Such design problems are usually tractible mathematically, at least with the aid of a computer, as the solution of simultaneous non-linear (and perhaps implicit) parametric equations problems. For the WLRNV, however, matters becomes much more difficult, since the physical model is extraordinarily complex, and the closed form solution for any optimal motion will only be more so. A good design will be one that makes intelligent use of its mass distribution, its joint and limb geometries etc. and, as such, an adequate vehicle design that is specified before the desired motion is known simply will include unnecessary strength, mass and size.

Despite the fact that the optimal motion can be

written in closed form, only with great difficulty can a 'best' design still be found by a super-optimization. First, it must be decided which motions are the ones around which a vehicle should be designed. As a good first approximation, one could say that the "best" design is the one which operates with the least power at a specified velocity. This is not the same problem that was discussed in the last section, since the best design will not be able to run faster than this velocity, because that would require greater structural/actuator strength than is needed for the specified running. This would require more mass than was used for the specified running, and could not be an optimal design due to this extraneous mass. Therefore, the best design cannot run faster than the specified velocity.

The problem is a coupled one, where part of the equations are not in closed form. As in all problems of this type, the solution must be found iteratively. One must start by estimating the physical parameters of vehicle geometry and mass distribution, then insert these into the motion optimizer and see how much capability the estimate has included that is not required. Conversely, the estimated parameters might result in a vehicle which would not be capable of performing the mission, more structure mass or actuator torque being required for even the optimal motion than the estimate included.

With a measure of how much in error the estimate was, one can correct the estimate in such a way that the next one will be closer to having no under- or over-capabilities. Procedures for such iterative design exist, among which gradient algorithms and second-order algorithms are the most popular. It should be noted, however, that there are a very large number of variable parameters, including every actuator strength, every limb mass, foot size and shape, power available (i.e. power plant mass), gear ratios in the actuator outputs, and lengths of every joint-to-joint distance (i.e., P

variables in all).

As such, even gradient algorithms involve taking P variations of the function to be minimized (e.g. the over-capability, K), in order to form a $P \times P$ matrix, which is inverted to calculate the next estimate. The second-order algorithms attempt to fit the second-order terms in a Taylor series approximation as well, and thus require P^2 variations. The variations are those of a function, K , which is too complex for analytic derivations to be calculated effectively, so that $\frac{\partial u}{\partial x_i}$ must be found by making a slight change Δ in one parameter, going through the extensive calculations to compute the K that results, and approximating $\frac{\partial K}{\partial x_i}$ as $\frac{K(x_i) - K(x_i - \Delta)}{\Delta}$.

Not only is this inaccurate for large variations, but it requires $P + 1$ solutions of the whole model (including optimization and assignment of foot-size, structure weight, etc. to the results) in order to compute the lowest K . There is no way to perform design optimizations without the partial derivatives, which in turn requires these extensive computations (which are also inaccurate). Such inaccuracy implies that the algorithm must make small changes from estimate to estimate in order to keep close to the path toward the optimum.

Thus it will take a substantial number of estimates for the algorithm to converge in a stable manner. It also should be noted that the gradient algorithm, the only one that can be used without either having analytic expressions for K or computing higher partial derivatives of K , also converges rather poorly. Thus one might expect perhaps 30 iterations of the entire algorithm might be required before a reasonable estimate of a vehicle parameter evolved to a best design choice.

Considering the fact that a four-legged vehicle could well have over 50 crucial design parameters (16 motors, 8 limb tubes with strengths in bending, tension, and buckling, 16 gear ratios, etc.), the number of passes through the design procedure would be well over 1000.

This is rather unfortunate, since the procedure includes the motion optimizer, which alone takes on the order of hours of computer time to reach a solution. Thus, in order to get a best overall vehicle design, a fairly powerful computer must be left to the task for well over a month.

This problem would definitely be worth the expense of a solution for any WLRNV, because adding performance capability to any one component that the rest of the vehicle cannot exploit would be a waste. Actuators strong enough to break the limbs they move, power plants and conditioners that are too massive due to unused capability to handle uncalled-for loads, limb lengths that make the desired gait inefficient — none of these should be allowed, especially in a design that might be mass-produced. Avoiding the mass production of unnecessary capability in high-technology components is clearly worth even months of computer time.

If such a solution is ever arrived at, it will be a notable event in engineering design history. Few design problems in the realm of physical machinery are both as analytic and as complex as the detailed design of a WLRNV. More often designs incorporate non-analytic factors that depend on indeterminate reliability, or acceptability to human users. Such problems do exist in WLRNV design, but they exist at the military-vehicle specification level and do not affect the parametric physical design.

On the other hand, an obvious question that the designer must answer is: Can one avoid solving this problem, aren't there other ways? It would be well to avoid the problem not only due to computer costs involved in solving it, but because a multi-month time lag for a change of specification means idle personnel and major penalties incurred. Indeed, the whole problem does simplify considerably if one eliminates the optimal motion determination by using a nominal motion.

But what means does one use to devise a good nominal

motion? Choosing a poor one is equivalent to designing human muscle and bone to execute St. Vitus' dance: there will be a great deal of wasted capability, even when the human is walking at maximum velocity or power efficiency. Obviously some sort of design will result from a biomorphic extrapolation for the nominal motion, which would perhaps suffice for research and one-of-a-kind vehicles. But in order to make such nominalism work respectably, there must be some intuitive understanding of what optimal motions look like.

The large computational endeavor involved in just the motion optimization means that such understanding will be sparse in the near term. However, once there is a background of previous solutions, there is an attractive alternative to human nominalization, namely its computerized equivalent: extrapolation/interpolation to the intuitive solution that is also based on past data.

In other words, in the far term, when a substantial number of good nominal solutions have been found, extrapolation can be done in the design parameter space to attain a good substitute for the full, multi-month design optimization. One could either extrapolate simply the optimal motion results, removing the $P + 1$ optimizer passes required for each estimate in the gradient algorithm, or one could extrapolate the design parameters themselves. This latter solution may be quicker, but the former will result in a better design, since the inaccuracies are in the motion specification, where they cannot result in fundamental incapacities.

The ability to form a tight parametric design for the physical aspects of the WLRNV is thus seen to be possible but costly and, as experience with WLRNV design grows, reasonable alternatives that permit rapid solutions to the motion problem will be worthwhile even for one-of-a-kind WLRNVs. The ability to perform such a nominal design, however, is not the same as the ability to achieve the most desirable vehicle.

The concept of the most desirable vehicle must be defined in some context, if it is to be investigated. The military might be most prone to reference desirability to the performance of a particular WLRNV mission or class of missions. Evaluating desirability for a single archetypal mission is a benchmark-performance viewpoint, while evaluations over a class must involve some weighted averaging of performance that represents a probabilistic viewpoint.

Either way, one might like to attempt to formulate a mathematical problem which gives some solution for the most desirable design for a given mission/class of missions (m/com). Unfortunately there is one major complication that makes this rather impractical, if not impossible. This mission, if it is any sort of realistic mission at all, is fraught with random variables and overall performance is governed by their effects. Waves are the easiest example of physical inputs to the vehicle which affect the motion that the vehicle must execute to travel, and therefore their amplitude, phase, frequency, etc. are all random variables that effect mission performance.

There are other random variables which contribute to the problem, but the important fact is that the resulting m/com performance will itself be a function of many such random variables. In order to construct relevant probabilistic models for mission outcome, one would need to know the distribution of such performance randomness. To define a most desirable vehicle design, the expected value of that random variable is all that is required.

Unfortunately the expected performance is not found analytically by any direct route from the expected value of wave parameters, wind parameters, patterns of counteraction, etc.; one must compute probabilities first. Because of the complexity of vehicle motion (which can't be arranged in a closed form), the effect of wave geometry on walking does not become any express-

FIGURE 32

HIERARCHY OF OPTIMIZATIONS

ion that can be used to compute the probabilities. One is therefore generally unable to compute the expected performance, and can only evaluate single elements of performance for a given instance of wave geometry.

However, probabilities do relate to the real world, and expectations are the asymptotic results of averaging large numbers of instances (according to the weak law of large numbers that applies to the random variables in this problem). Indeed, if one is but willing to choose several wave geometries for the entire mission at random, according to a reasonable probability distribution for wave geometry, then insert each geometry into a physical model to get a performance, and average the performances, the resulting average will be close to the expectation.

This sort of procedure can be used to create a measure of m/com performance and to define a so-called "stochastic optimum." Unfortunately, having no a priori idea of how many samples will be needed to result in a given accuracy of the calculated expectation/average (which will in general be a function of the structure of the random process that is involved), there will be at least 10 values to be calculated for averaging in order to get a value that is at all precise. This means that the motion of a vehicle design over the sample wave geometry must be evaluated at least 10 times for each value of K.

Now, if one wants to find a stochastic optimum for the most optimal design (i.e., maximize the expectation of K), one is faced with the same sort of problem as was solved the physical optimal design, except that each calculation of K requires tenfold more motion calculations. Each motion calculation involves an optimization to get the best motion for that design over those waves. In short, one would expect at least 0.3 hours of computation on a moderately powerful computer if the averages were to converge adequately with 10 samples. This appears to be a very substantial penalty to pay for a

computerized solution.

Instead, it would seem more reasonable simply to build and examine the performance of the vehicles than to wait for such computer solutions. Optimally designed vehicles should not perform too poorly, so long as the cost function used to define optimal motion was closely related to good performance on the mission, e.g. a vehicle designed where specific velocity was maximized without wasted capability would hardly be a good choice for a decoy RNV whose mission was to move rapidly and erratically in limited areas.

However, one can expect that as knowledge of WLRNVs grows, one will be able to apply extrapolation techniques to the stochastic optimization problem. As vehicle capability grows, longer, more complex missions can be modelled by existing means, using older extrapolations to get optimal designs that take the whole mission (instead of just one sort of optimizable motion) into account. Eventually, one may expect to see a body of applications knowledge that predicts true maximum-desirability designs directly from a description of the mission. In this way there will eventually be a return for the complexity penalty incurred by the WLRNV concept: many vehicle design D.O.F.'s mean that one can be very well adapted to a particular mission.

In the very long term, it is quite evident that the design of these vehicles is a multi-variate problem (over the many gaits and maneuvers that comprise a mission) that may exceed detailed human comprehension. Thus the design procedure will increasingly leave the human arena, to be invested in larger computer design systems. The eventual goal of such a program is to be able to take a definition of a mission and turn out a choice of all vehicle physical parameters that does the best job of performing that mission in a probabilistic environment.

POWER PLANT

After one decides on the physical parameters of a vehicle, in either an optimal or a nominal way, there is still a large amount of complex design work necessary to create a vehicle with those parameters. Among the foremost of these parameters to be thereby determined are those that involve power and energy, two critical quantities for an autonomous vehicle. The aim of this section will not be to design the power subsystem (given a choice of relevant parameters), but to discuss the relative importance of the parameters for the design of power equipment.

Obviously, if each joint's actuator has a specified size and mechanical impedance (e.g. gear ratio), one has enough information to set limits on the amount of power it can handle. Thus the specification of physical parameters is enough to give an upper limit for P_{joint} . It should be recalled that actuators can be thought of as power transformers, and as such neither store energy nor exhibit limits to changing power flux through them. Thus the function of an actuator is largely characterized by the power it transduces at any time, and this value has an upper limit.

There are only a specific number of places that the onboard power/energy can ultimately go, the most productive being mechanical work done by the actuators (as this helps move the vehicle). There are also power losses in these same actuators, losses in foot-drag, losses in the power conditioning/generating system, power losses in the electronics, and the power expenditures for communications.

Some approximations could be made for first-cut design. The electronics that processes signals and issues control commands is relatively low-power for high-speed operation: one might expect that the power consumption of the electronics will be much smaller than that of the mechanical system. To be parametric, one could define a quantity $\underline{P}$ that is the maximum amount

FIGURE 33

POWER SUBSYSTEM LAYOUT

* Indicates optional components whose inclusion might prove useful

of power available onboard the RNV at any time. Much of this power may not be generated at any given time, so that it need not be wasted. The fractional power consumption of the control electronics $F_{p\_computer}$ is usually minimal, so that the above assumption amounts to saying $F_{p\_computer} \ll 1$.

A similar approximation could be made for the sensory/communications gear, although most portions of such equipment probably need not operate continuously. The rest of the loss terms, and the actuator work cannot either be neglected or assumed to be negligably small, since the operation of the vehicle is characterized by the presence of fluctuating power and drag (synergy).

Thus one could envision the power-use tree for the vehicle (Figure 38), where each branch can draw up to $F_{p\_branch} \times \underline{P}$ at any time. At most a total power $\underline{P}$ is available to this tree, but at any given time only $\underline{P}^*$ is being generated and consumed. Thus there is some use factor U_p which represents that fraction of the available power $\underline{P}$ which is being used on the average. Any vehicle design where $U_p < 0.5$ might be considered a substantially sub-optimal design, because it included bulky power equipment that was substantially under-utilized.

In order to get a better design, $\underline{P}$ should be no larger than the maximum value of $\underline{P}^*$, the instantaneous power drain, for otherwise the power system can generate more power than it can ever use. Likewise $\underline{P}^*$ should never fall too far below $\underline{P}$ for prolonged periods, or there will be substantial underutilized capability. One might ask, however, how one can control the power consumption $\underline{P}^*$, since it would appear to be totally specified by the motion design, not the vehicle design. Of course, one could exert a special effort to design motions of the class having $\underline{P}^* = \text{constant} = \underline{P}$, but such motions would not in general result in efficient travel.

There are two alternatives within the power system. The first is the obvious: let the power generators

supplying the varying loads be throttled or pulsed in such a way that power generated equals power consumed, and accept the unnecessary hardware weight and volume involved in not generating $\underline{P}$ continually. The other solution is to level loads with secondary storage. If the main powerplant generates $\underline{P}^*$ continuously, it is never under-used and will have no performance or durability degradation due to throttling, etc. Then this power $\underline{P}^*$ is applied to buffers (batteries, flywheels, etc.) which store power when $\underline{P} > \underline{P}^*$ and release power when $\underline{P}^* > \underline{P}$.

The power buffer concept allows continuous operation of the prime generators, and allows $U_p \approx 1$, but may still not be the optimal alternative, since the power buffer (secondary energy storage) has a certain weight and volume which must be less than that of the extraneous power plant otherwise required, if this concept is to prove better than the $U_p < 1$ alternative described above.

One could also conceive of power buffers at a variety of levels in the power tree. Underutilized transformers, underutilized actuators, even underutilized cabling could be upgraded with load-leveling power storage capabilities. This upgrading makes sense only if the weight and volume of the storage elements is less than the average unused weight and volume in the underutilized device.

This line of reasoning brings out the major motivation of design in the power subsystem: weight reduction. There are two general qualities a design for a WLRNV could optimize: performance and reliability. Since it turns out that the control system is likely to be substantially less reliable than the other subsystems, only vehicle performance is an apt figure-of-merit for the power subsystem. Assuming that the power subsystem must provide the power required for motion, control, and communication, the only way a power subsystem design can improve performance beyond meeting power requirements is

GENERALIZED VARIABLES IN THE POWER SUBSYSTEM

$\underline{P}$ = Maximum power that the subsystem could generate, that the vehicle could use, or that power subsystem components could channel without burnout.

F_{pi} = Fraction of $\underline{P}$ that the i^{th} component could use, dissipate, or transduce, either in worst-case or before burnout.

always ≤ 1

$\underline{P}^*(t)$ = Instantaneous power drain of entire vehicle.

$\bar{P}_i$ = Average power used/transduced by i^{th} component.

$\bar{P}_{tot}$ = Average power used by entire vehicle.

$U_{pi} = \frac{\text{Average power capacity of component } i}{\text{Maximum power capacity of component } i} \leq 1$

$U_p = \frac{\text{Average power use of vehicle}}{\underline{P}} \leq 1$

The use factors U_{pi} are measures of unnecessary power handling capability in i^{th} component.

$\beta_i = \frac{\text{Maximum power capabilities of } i^{th} \text{ component}}{\text{Mass of } i^{th} \text{ component}}$

$\alpha_i = \frac{\text{Total energy storage capability of } i^{th} \text{ component}}{\text{Mass of } i^{th} \text{ component}}$

Specific powers and energies (β_i and α_i) are qualitatively fixed by choice of component technologies

$T = \text{Storage times desired from power buffers}$
 $= \frac{\alpha_{\text{buffer}}}{\beta_{\text{users}} \prod P_{\text{users}}} \cdot \frac{M_{f \text{ buffer}}}{M_{f \text{ users}}}$

$M_{fi} = \text{Mass fraction of component } i = \frac{\text{mass component } i}{\text{total vehicle mass}} \leq 1$

to minimize its mass.

Minimizing power system mass leaves more mass for other motion-enhancing capabilities (e.g. more versatile gearboxes, extra vehicle hardening, less lossy bearings) which can use the extra available mass to improve vehicle performance, or the entire vehicle's mass can be reduced to directly enhance performance. The alternative of buffering power to reduce total system mass has already been described. A number of similar alternatives exist in the regimes of power generation, transformation, and consumption.

Eventually one is forced to compare and evaluate competing component technologies to specify the power subsystem. The most convenient parameter that can be used to compare components is the specific power, β , equal to the maximum power that can be input/output to or from the component, divided by its mass. Often, it turns out that a technology can be characterized by a fairly constant β , as for example internal combustion engines for automobiles typically produce 1 hp/lb. In other cases (such as fluidic pumps), β is a strongly varying function of some design choice (such as operating pressure).

Thus, one strives to employ the highest β technologies in the power system to increase vehicle performance. Other power system components do not transform power from one form to another, but store it. The parameter β is almost meaningless for such devices, since the rate at which energy can be extracted (which will determine $\beta = \frac{\partial \text{energy}}{\partial \tau} \cdot \frac{1}{\text{mass}}$) is governed by different phenomena than those that store the energy. Thus one can define the energy storage parameter α , equal to energy stored over mass of storage medium, that characterizes the performance of a storage scheme. It should be noted that each storage technology will be characterized by separate α and β parameters, i.e. capacitors store energy very inefficiently (low α), but show high β due to the high rate at which that energy can be extracted.

One can then consider a design parameter for each storage device, the storage time T . This time gives the power-handling capacity of a storage element by $\beta = \frac{\alpha}{T}$. Unlike α and β , however, storage time T is not only a property of the technology being used, but may also be chosen by the designer from within a fairly broad range available within the technology, e.g. the mass of a full gas tank is a widely variable parameter, roughly bounded between 1 oz. and 100 tons, for gas-tank energy storage technology.

Assuming the parameter T has been specified, an energy storage device has an overall β that characterizes it. This β comes from the power that can be input/output divided by the mass of the storage medium plus the mass of the storage/retrieval power conditioning hardware. Thus $\beta_{\text{overall}} = \left(\frac{1}{\beta_{\text{stor}}} + \frac{1}{\beta_{\text{xfrm}}} \right)^{-1}$, because the power flux into/out of the storage medium is the same as that handled by the storage/retrieval power conditioners, while the mass adds. (For example, a flywheel energy storage system has a certain mass of rotating matter devoted to storing energy, and a mass of motor/generator devoted to getting power into/out of the flywheel.)

In this way, we see that for any two components that handle the same power, their β 's add in parallel. This notion is useful, because for radically different β 's their parallel combination is essentially the lower of the two β 's. Thus if a fluidic pump/motor puts power through a gearbox into the actuation of a joint, and $\beta_{\text{motor}} \gg \beta_{\text{Gearbox}}$, then there is hardly much point to attempting to reduce motor weight, since it will be an insignificant part of the overall actuator weight.

One could attempt to extend this concept to the entire power system design, but its use would be marginal because one transformer could serve 3 actuators, etc.; the power fluxes through various vehicle components are not the same. However, one could analytically extend the concept by employing the fractional powers Fp_i to

FIGURE 35
POWER SUBSYSTEM CHARACTERIZATION
By Manipulation of Generalized Variables

PARALLEL COMBINATION of β s

If two components are employed in series then one delivers all of its power to the other. If one has a larger maximum power handling capability than the other, than the larger is capable of burning out the smaller. Thus the larger capability cannot be used and both mass and cost are being squandered on wasted capability. Wise design therefore matches maximum power on components in series, and since mass adds:

$$M_{\text{series components}} = M_{\text{each component}}$$

P_{max} each component is the same

$$\text{Therefore } \beta_{\text{series components}} = \left[\sum \frac{M_i}{P_{\text{Max}}} \right]^{-1} = \beta_1 || \beta_2 || \dots \beta_{\text{Last}}$$

Specific powers of series components combine in parallel.

FANOUT OF POWER TREE

If, at any level, power distribution branches out to a number of users/transducers, one can define functional powers

$$f_{pi} = \text{maximum fraction of power trunk } i \text{ drawn through branch } j$$

$$\text{Then } P_{\text{branch}} = f_p \cdot P_{\text{trunk}} \rightarrow \beta_{\text{trunks branches}} = \left[\sum \frac{f_{pi}}{\beta_i} \right]^{-1}$$

gives β for all branches combined (also works for fan-in of branches).

COMBINATION OF IDENTICAL BRANCHES

If identical branches form parallel power paths branching into or out of a given power trunk, then an each worst-case prediction is $\beta_{\text{overall}} = \frac{\beta_{\text{branch}}}{\# \text{branches}}$, which could be found by similarizing the f_{pi} in the previous formulation. However, often the demand of identical branches is mutually exclusive in time. If this is the case $\beta_{\text{overall}} = \frac{\beta_{\text{branch}}}{U_{p \text{ branch}, \# \text{branches}}}$ allows us to deal with the multi-user non-constant drain phenomena represented by the actuators (locomotive end users)

refer all power use to $\underline{P}$, the total available power.

Using this method one could say that $\frac{\beta_i}{F_{p_i}}$ combine in parallel, and when all the fractional parts are computed for $\sum_i F_{p_i} = 1$, one arrives at a total specific power β_{tot} .

However, one must remember that there could be a variety of power sources and power transformers, as well as power consumers. Thus there would be a trio of β_{tot} 's, the β_{gen}^t , β_{xfm}^t , and $\beta_{consump}^t$. Each of these must be computed as parallel combinations of the $\frac{\beta_i}{F_{p_i}}$ of each device handling a portion of $\underline{P}$ in that stage. Then, since every watt generated must be conditioned transformed and then consumed, one defines $\beta_{power} = \frac{\underline{P}}{\text{Mass of power subsystem}}$, or $\beta_{gen}^t // \beta_{xfm}^t // \beta_{consump}^t$.

Of course, by the time one reaches this point, one may have overextended an analytic approach into a problem where it is of too general nature to be of great use; it is easier to sum up mass and then divide, using the true value of $\underline{P}$. However, this analytic approach has a certain non-dimensionality to it, as well as conferring some intuitive benefit. The characterization of the power subsystem in terms of β removes all issues of the size/scale of the referenced vehicle. It emphasizes the fact that any technology with β much lower than the other β 's in the system will dominate the weight and determine the performance of the vehicle much more strongly than the higher β components.

Unfortunately, the problem can be characterized with these β 's, but not designed with them. The individual component technologies have β 's that are functions of size/scale, so that the β design techniques are indirectly influenced by scale. As such the β representation of the vehicle is of little use for exact design, but serves as a good descriptor of where the mass requirement is, and can be used as a coarse design technique. If one has a choice of technology, or operating point for a technology (e.g. T , pressure for fluidics, frequency for A.C. electric power, etc.), one

would do best to choose those whose β 's are similar to all the others in the power system. Since increasing β generally means expensive performance increases and lower component reliability, it doesn't pay to increase $\beta_{\text{technology}}$ much beyond the other β 's in the system, since it will be marginally less effective in decreasing sub-system mass. Conversely, decreasing β_i below that of the other components might be economical, but the mass of the sub-systems using that β_i would be likely to dominate the power system mass.

One may be forced in some instances to let one weight contribution dominate the others. This eventuality becomes evident when one considers the possibilities for power sources. There are two basic alternatives for the power source: onboard and offboard. Offboard power will have to be transmitted to the vehicle, and power cables essentially destroy the advantages of the WLRNV. The remaining options for offboard power are electromagnetic wave power transmission to the vehicle. Such options are not available in the near term, though when available, are likely to be lighter than the equivalent onboard equipment (and would moreover allow indefinite operation, though not arbitrary range).

On the other hand, onboard power must come from some form of energy storage. This storage could be chemical (as from a gas tank or battery), mechanical (flywheel, spring, etc.), or electromagnetic, but whatever form it takes, there is weight required. This weight will be comparatively large, because the effective $\beta = \frac{\alpha}{T}$ must be low for missions of large T , since all vehicle power must come from this storage medium (primary storage). The length of the mission will therefore increase proportionally with α_{primary} , and can generally be increased by decreasing the β that is allowed for primary storage.

Thus, for an onboard primary power source, one could increase the duration of the mission (equal to T/U_p)

by decreasing β of the primary storage. This will be a reasonable procedure, so long as α is high enough to give $\beta_{\text{prime}} = \frac{\alpha}{T} \geq [\text{all other } \beta\text{'s in the power system}]$. Beyond this point, one increases mission duration by letting β_{prime} dominate the other β 's, resulting a design that becomes increasingly an all-fuel vehicle. However, by allowing designs where the fuel tank becomes a dominant fraction of the vehicle to achieve a greater range/mission time, one degrades the hardness of the WLRNV by an effectively larger fuel tank cross section against counter-fire.

This is only one example of conceptual design in terms of β 's, studying the way they combine to determine the mass distributions within the power subsystem; it also turns out that β 's are critical for overall vehicle performance. It has also clear that α of the primary storage determines mission duration for onboard power source vehicles.

The object of the design of this subsystem is to supply the power for all necessary vehicle functions with as little weight expenditure as possible. This turns out to be exactly the same problem as maximizing the available power for motion (if one ignores non-actuator power drains and structure mass), given a constant vehicle mass, when one considers actuators as power transformers and therefore includes them in the power subsystem weight. Thus, the solution of the power distribution problem just sketched maximizes performances and, even though careful attention must be paid to weight calculations, the β approach gives a good intuitive picture of the primary issues and motivations in the power subsystem design.

VEHICLE MECHANICS

As previously mentioned, a WLRNV contains four distinguishable sub-systems: power, locomotion, structure, and control. There is some overlap, since the joint actuators both provide locomotion and consume power. Likewise some of the structure is particularly applied toward locomotive loads, while the rest is needed to support and mount non-structural components. Of the four, the locomotive subsystem is hardest to encapsulate or separate from the other three. The locomotive system is also the most critical for WLRNV motion, as the other three can be considered as supportive of the locomotive functions.

The power subsystem has already been discussed briefly, and although it is essential to vehicle operation, this sub-system seems to be entirely molded to suit the power demands of control and locomotion. As such, if locomotion design is complete, and the control system requires a steady known power supply, the power subsystem can be designed simply by selecting the technologies of its parts over a specified range of storage times T . The design of the power subsystem is hardly trivial, and is coupled to the other sub-systems designs via component masses, but nonetheless poses no fundamental design questions.

Likewise, the design of the structure of the vehicle involves few novel problems, since if one knows the loads that gravity and locomotive forces will impose on the structure, one can design the structure in a straightforward fashion. However, unlike the other three sub-systems, the vehicle structure will be the only subsystem that is too complex to practically admit a full analytic design. A structural design involves so many configuration options that it is doubtful whether all extraneous material in the structure could be eliminated with analytical design tools.

On the other hand, structure is particularly

amenable to empirical design. If an attempt to lighten the mounting for a bearing results in an unanticipated stress concentration which causes fatigue failure at some future time, it is doubtful that such a problem would be noticed, no less if it could be solved analytically. Instead the design is lightened until it breaks in operation over an unacceptably short life. General shapes for parts of the structure can be found from intuition, solved for exact shape under simplified loading models, and implemented. For the complex, time-varying loading occurring in a WLRNV, it is doubtful that optimized structure shapes and compositions can be found analytically, but they can be effectively sought empirically as vehicle experience grows, and with a reasonably optimal results within a few vehicle generations.

In contrast, a fairly good design can be achieved analytically the first time for the control and power subsystems. Of these two, the control system and its problems will be novel, so that unexpected problems may be encountered herein. However, once one is working at all, it will not require years of experience to achieve a good control subsystem design.

In contrast to the other three subsystems, the locomotive subsystem turns out to incorporate both novelty and non-analyticity, so that experience will be necessary to get a good design, as well as to get any design working. As such, locomotion appears to be the toughest sub-system design problem in the entire vehicle's design. However, it is the least critical to WLRNV operation, although it directly impacts WLRNV performance.

A design that does a poor job of locomotion is likely to still be capable of motion, even though range, maneuverability, and other vehicle performance parameters may be poor in consequence. On the other hand, a poor power design is likely to be either qualitatively incapable or destructive of its own parts. A poor

FIGURE 36

LIMB CYCLE PROPULSION

Motion for only one leg of 4 is shown for clarity

NOTE that appreciable forces are exerted on the body only during thrust phase

structural design either fails catastrophically or is effectively incapable of motion. A poor control design could well be considered hopeless. In contrast, any locomotion is still locomotion.

As such, a WLRNV program can afford to wait for the requisite locomotion experience to accumulate with the passage of time, and still be able to produce effective vehicles. This line of reasoning justifies WLRNV research and deployment, since one can confidently expect that vehicle performance for any given mission will improve dramatically as knowhow grows. However, it still does not pay to leave a wide variety of locomotive questions uninvestigated, as reasonable early designs are needed if the WLRNV concept is to be attractive enough to encourage further effort.

Consider how a WLRNV must locomote. Obviously thrashing the legs around does little good, as the vehicle must in some sense push coherently against the underlying water. Thus there must be a thrusting portion of the limb cycle that produces a net force on the vehicle in the direction of desired motion. This thrust must not only push the vehicle forward; it should also push the vehicle upward to make up for effects of gravity when the vehicle is transiently undersupported, i.e. when its legs are out of the water.

Just as obviously, the feet must be taken out of the water at some time during the limb cycle, to enjoy the benefit of high foot drag, limbed propulsion. It is of course possible for such a vehicle to move while keeping all feet in the water all the time, much the same way one can move in a chair with rollers without using foot propulsion, by shifting weight back and forth and making use of non-linear drag of the rollers. This mode of locomotion is markedly inferior and, though sea snakes execute it relatively impressively, it simply makes little sense for a high speed, long range WLRNV.

Thus good limbed locomotion over water should have a limb cycle with a push phase, when the foot moves

backward with respect to the body and the water, generating thrust, and a recovery phase when the foot is removed from the water and moves rapidly forward with respect to the body to begin the next thrusting cycle. All limbs on the body must execute such cycles in order to achieve the desired sort of motion. It is conceivable that different feet can move through different cycles at different frequencies. However, in order to achieve smooth forward motion, it intuitively seems superior to move all legs at the same frequency at roughly constant phase shifts from each other, i.e. to execute cyclic phased limb motion, as essentially all animals do.

(The relative phases of the various limb cycles are a good way to characterize the gaits. If one can solve the general motion design problem, optimal limb motion may vary from leg to leg and gait to gait. But, for a four-legged vehicle, there is evidently the slowest gait with only 1 foot off the water at a time, the next with two feet off the water, etc. Several such gaits are possible as variations of the sequencing pattern of foot motions, such as both front feet off the water then both back feet, or diagonal feet off the water simultaneously, or left/right off the water simultaneously. Envisioning all of the possible gaits eventually includes the various bounding and galloping modes that involve the vehicle's feet all leaving the water entirely for substantial fractions of the motion cycle.)

POWERED LOCOMOTION

The previous description allows us to separate the locomotive problem into several positions. The first questions that must be resolved have to do with the thrust portions of the limb cycle. The second range of questions have to do with support of the weight of the vehicle while moving. The third set have to do with propelling the leg throughout the limb cycle. Lastly, one should consider questions of inertia and balance. Already we are in a position to specify answers to these problems adequate to achieve a locomotable vehicle, but careful consideration must be given if we wish to make locomotive predictions and ascertain required developments.

In this section, we will primarily treat the straightforward approach to achieving limbed locomotion. All of the previously described problems involve issues of power. In this section we will simply assume that controllable actuator power is available at whatever joints we wish to move. This regime of locomotion will be referred to as "powered" because of these assumptions.

In the next section we will consider ways to improve locomotive efficiency without simply applying more power from the vehicle power plant. This will involve a number of proposed locomotive enhancement techniques. Since these techniques have not been experimentally evaluated, it is difficult to scope their potential analytically, and they should be considered simply as promising suggestions which could result in sizable improvements above the predictions laid down in this section.

When a foot is immersed in water (or in contact with solid land), it may perform two functions: support and propulsion. However, the control of the actuators gives total arbitrary selection of the actual supportive and propulsive forces that foot is providing at any moment. Indeed, the foot could be servo-controlled to

FIGURE 37

AMPHIBIOUS FOOT DESIGN

exert absolutely no force on the vehicle at all: The mere fact that a foot is in the water is no indication of the particular contribution it is making to either support or locomotion.

Thus, the so-called "push phase" of the limb cycle actually has an internal history. At first contact, control is striving to insert the foot into the water to desired depth and attitude with as little plunging loss as possible. Once inserted, all joints in the leg cooperate to achieve a requisite propulsive force. During the middle of the push phase the knee(s) are being bent to keep body motion relatively smooth, and as such the supportive forces are most desirable in this interval when the bent knee affords best control of the vertical foot forces due to buoyancy. As the limb elongates toward the end of the push phase, all joints are moving again, delivering thrust. Finally, there is the foot de-insertion, in which the object is to minimize deplunging losses, followed by a final kickoff that serves to help reverse leg motion for the return cycle.

The vehicle, when operating on water, tends to equilibrate the buoyant forces of its immersed feet with the configuration of its vehicle weight, thereby determining the mean foot immersion depth. However, the large number of controllable degrees-of-freedom (joints) in the vehicle allow a redistribution of body weight support over the limbs without changing the position or attitude of the feet. Thus, by using the redundant control of many joints, it is possible to specify the immersion depth of any foot during its cycle, as well as foot motion, between wide limits.

The immersion depth and foot attitude can be used to modulate foot drag. Because drag is used to provide thrust forces via motion relative to the water, foot immersion depth is a fundamental thrust control parameter. It is, therefore, extremely useful to attempt to increase the bandwidth of this controllable variable,

so that we can maintain close drag control even for very rapid limb cycles. Thus a major foot design parameter will be the vehicle equilibration time constant, which expresses the latency time over which vehicle weight and foot buoyancy equilibrate.

With this motivation, it is appropriate to briefly discuss foot design, to serve as a lead-in to the familiar problems of locomotive subsystem design. Firstly, feet must provide buoyancy. One might envision a non-buoyant design, but slapping the water to support the vehicle is hardly a quiet way to move, even if the energy spent to support the vehicle in such a fashion were unimportant. Buoyant feet also afford essentially instantaneous conversion to a loitering configuration (something not available with inflatable alternatives).

Requisite buoyant forces basically constrain foot volume. If we consider a nominal military WLRNV with 2 meter limbs weighing on the order of 250 pounds, we see that the foot size for extremely buoyant operation is quite modest: If one desires total vehicle support by 2 of the 4 legs, each foot need only immerse about 2 cubic feet of hollow space at its nominal immersion depth. This means that a 2 meter limb WLRNV requires a foot slightly larger than a large watermelon.

Next come the questions of equilibration times, for if the requisite 2 cubic feet must be configured as 4 foot diameter disks, foot design and use will be clumsy. Buoyant equilibration times are just a function of the foot cross section at the water surface. The effective buoyant spring constant of the foot is just pA , or (assuming that the feet are about 1 foot in depth) the spring constant of the nominal vehicle is about 250 lbs/foot for 2 of its feet in the water. (One could as well design for 3.2 or 1.7 feet in the water, if such corresponded to the average duty cycle of required body support.) To obtain a damping coefficient leading to determination of an equilibration time, we estimate a reasonable value of 23 lb/ft/sec, this corresponding to

FIGURE 38

FOOT DESIGN

a linearized model for the aerodynamic drag on a bobbing vehicle, and is equivalent to representing the drag of the body with a 4 foot diameter plate. Thus a very conservative calculation gives equilibration time constant of 0.1 of a second for feet of reasonably non-planar geometry, implying that the equilibration time could be reduced into the range of dozens of milliseconds without absurd foot geometries.

Finally, the foot design must provide drag for thrust generation. In order to investigate the design of drag fins, it is first necessary to specify both the required thrust per foot and the maximum allowable foot-vs-water velocity. The foot is much like an unpropelled hull, and must be heavily immersed to be net buoyant. Thus, it may be expected to encounter an abrupt non-linearity in hull drag at around 30 knots speed relative to the water. One would definitely like to operate significantly below this point so that laminar flow near the foot resulted, thereby minimizing foot force noise contribution to the vehicle control problem.

When one attempts to achieve laminar flow in water past a foot of this size, however, the resulting velocities that keep foot motion below Reynolds numbers of the order of 10^4 are on the order of 10 cm/sec. This turns out to be unacceptably small, given the requirement of reasonably sized drag fins. If one wishes to maintain full maneuverability at as low as 3 knots foot-to-water speed, then the requisite foot force must be available at foot-to-water velocities of about 0.5 meters/second, which may or may not cause turbulent flow. "

One can then use this velocity specification and the momentum drag approximation (stagnated water pressure across 1 side of the fin) to find the fin area for any required force. In this (reasonable) approximation, the drag force is proportional to the foot-fin area.

This gives an equivalent pressure which, at the above velocity, is about 2 lbs/ft^2 . Thus, a watermelon-sized foot with 1 ft^2 of drag fins on its sides and

bottom would generate about 10 lbs of momentum drag at 3 knots speed relative to the water. At 30 knots relative speed this could increase to 1,000 lbs force, and (with three feet in the water) a 250-lb. vehicle could jerk about at 12g!

The above approximation simply made use of momentum drag due to flow separation behind the fin. With properly surfaced and perforated fins one could expect to do much better, exploiting viscous drag. With growing experience in foot hydrodynamic design, one could then expect to boost the maximum foot drag from the acceptable, reasonable magnitudes discussed above to much higher levels, by enhancing both viscous and hydrodynamic drag terms. Eventually one could expect to achieve drags that would give sufficiently solid footing to sustain 10 g accelerations at as low as 1 knot foot-to-water velocities, with reasonable fin areas.

We now consider how the vehicle feet are to be employed. Still concerned with the thrusting phase, we can set up a simple force balance between drag-limited foot force and drag-absorbing body aerodynamics. With the actuators providing the power, if we consider the foot thrusting temporarily as steady-state (i.e. consider only time averages), we can use drag models to find the velocity ratio of a backsliding run. If we express drag forces in generalized drag-coefficient form $F = C_D V^2$ and equate forces for a steady state, we get $V_{\text{body}} =$

$$V_{\text{foot-to-water}} \sqrt{C_{D\text{foot}} / C_{D\text{body}}}$$

We could continue to elaborate this model by evaluating viscous and momentum terms in the complicated drag coefficients. The final result would, however, be dubious, due to the lengthy train of assumptions. It is sufficiently enlightening to evaluate the square root only semi-quantitatively. How fast would a streamlined WLRNV body with 2-meter legs have travel to experience 10g worth of deceleration due to aerodynamic drag? Obviously, well over 100 knots, since a car (which presents far more area to the wind) does not decelerate

at over 1g (21 mph/sec) when shifted into neutral at 120 mph.

This, then, is the origin of the above-referenced promise of great speeds for WLRNVs: Floating feet can move backwards through water near the turbulent limit around 30 knots and provide enough time-averaged force to propel forward the vehicle to which they are attached at over 100 knots. An immediate question then is: "Do we have the power to invest in the thrust phase to achieve this level of acceleration performance?" Continuing to use the nominal value of 10g at 30 knots of available foot force for a 250-lb. vehicle, we calculate the power to maintain such acceleration to be about 200 kw. This means that the entire vehicle must perform with a β_t of almost 2,000 watts/kg.

Unfortunately, while present power and motor technologies could achieve this performance, β_t must take into account considerable excess weight in early vehicles, so that overall vehicle acceleration may performance could be expected to be as much as an order of magnitude behind the performance levels implicit in power components in the near term. In the far term, however, one could expect to have the experience necessary to operate at the specific power limits of the actuator/prime mover technologies, as well as to develop specialty light-power technologies, and thereby achieve 100 knot speed and 10g acceleration levels.

The power-drain analyses may be continued somewhat further, to develop a simple performance model. If we write the drag forces in terms of a drag coefficient $F_D = C_D V^2$, and refer all drag terms to the vehicle cruise velocity V_o with a scale factor $v = \frac{V}{V_o}$, then we can write $V_o = 3 \sqrt{\frac{P}{C_D}} v = 3 \frac{v^{3/2} P}{C_D}$ as the attainable cruise velocity of the vehicle. This can be put into the general functional form $V = 3 \sqrt{\phi}$ where ϕ is a variable that will be called performance. It turns out that almost all power drain models can be expressed in this form. Since all power loss terms must be made up by the

same power source, the power drains add, while the properties of the vehicle used in the model stay fixed. Thus, the $P = \sum KV_o^3$ form can be rearranged to show that performances (like the series β 's previously) combine in parallel!

This is a convenient fact, because whenever one must deal with cubic models such as this drag model (or the cyclic loss model below), one can evaluate overall performance by combining ϕ 's in parallel from each model. This indicates relative importances, because substantially smaller terms have vanishingly small effects on ϕ_{tot} . However, one cannot simply design a vehicle way, since there are also velocity models that are linear or quadratic with power.

While considering power models, one also finds those that are linear in velocity. The simplest of these is running uphill (upwave). Since the gravity load on the vehicle is $mg \sin \theta$, where θ is the angle of uphill slope, we find $P_{req'd} = M_{tot} g \sin \theta V_o$. This gives $V_o = \frac{\beta_t \phi}{g \sin \theta}$, where β_t is the overall specific power of the vehicle and ϕ is the fraction of P invested in doing uphill work. Note that V_o goes up linearly with the performance parameter β_t , implying that we should be striving to keep β_t as close as possible to the β_i of the components ($\beta_t \lesssim \beta_i$ for any $i > 1$, due to parallel combination), as well as to develop higher β technologies for components.

One can also use the same analysis made in the problem of hauling fixed loads. If the load exerts a drag force D , and we normalize D to the vehicle weight W by $D = kW$, we find the form $V_o = \frac{\beta_t \phi}{kg}$ which also gives a load-hauling limitation for the vehicle $k \lesssim \phi \frac{\beta_t}{V_{og}}$. This model may seem simple, but turns out to be useful since we can say that $\beta_t/g \approx 3$ knots is very readily attainable (requiring β_{tot} of only 100 watt/kg) and $\beta_t/g \approx 15$ knots approaches the limits of some present components, while $\beta_t/g = 75$ knots would require special adapta-

tions of technology, as well as substantial engineering, to attain.

The last important linear-velocity model is quite a curious one. It originates whenever one assumes that there is aerodynamic lift as well as drag. Indeed, the prudent designer should consider deliberately introducing lift in higher speed WLRNVs, so that some of the support burden is taken off the feet and the limb actuators during high-speed running. The feet should be kept out of the water as much as possible, especially during high speed running, to avoid plunging/deplunging losses, while actuators having fixed mechanical impedance must be geared-up at high velocities, so that less torque will be available to support bodyweight loads in the limbs; thus, significant lift would be substantially advantageous.

Assuming that one seeks to keep a fraction $q < 1$ of the vehicle's weight off the feet by lift, the lifting force is $q g M_{\text{tot}}$, while the power drain to keep the vehicle cruising in steady state is $\underline{P} = V_0 D$. Using the lift-to-drag ratio L/D (which is a fixed property of an aerodynamic body) we find $V_0 = \frac{\phi \beta t}{qg} \left(\frac{L}{D}\right)$. The ratio $\left(\frac{L}{D}\right)$ can be made as high as 10, while an ordinary body design would (unintentionally) have a $\left(\frac{L}{D}\right)$ of order unity.

The last type of problem requiring some modelling in the locomotive subsystem is that of driving the limb around the limb cycle. After the push phase, the foot is withdrawn from the water (or solid surface) and must be cycled around to a point further forward than the beginning of the last cycle. This requires the rearward velocity to be halted and reversed, a higher velocity minimum-drag acceleration forward, and then a reversal to rearward velocity again before the insertion point.

This requires power at every stage, because the acceleration needs power to increase kinetic energy, and the decelerations must dissipate power because almost no present actuator technology is readily capable of returning power to the power subsystem. Even the use of

generators to perform decelerations would still waste some of the kinetic energy being removed by cyclic decelerations.

Not only does the cyclic motion represent a power drain, but it also has maximum power requirements that are connected to locomotion with readily predicted drag coefficients. In particular, one can take the first Fourier series term for all motions in the limb cycle, and take a cycle-averaged approach to finding total angular moments. Consider the first harmonic at any given joint $\theta = \theta_{\max} \sin Kt$, where the period of the limb cycle is $\frac{2\pi}{K}$. This can be differentiated to give:

$$\begin{aligned}\omega &= K \theta_{\max} \cos Kt \\ \dot{\omega} &= -K^2 \theta_{\max} \sin Kt \\ \tau &= -K^2 I \theta_{\max} \sin Kt\end{aligned}$$

Power use is then $\omega\tau = -K^3 I \theta_{\max}^2 \frac{1}{2} \sin 2Kt$ on an instantaneous basis, which can be integrated over half the cycle to give the acceleration losses, or its absolute value taken and integrated over the entire cycle for a total loss model. Before this is done, it is simplest to assert the constraint of foot velocity to eliminate K . If one asserts that $V_o = \delta \ell \omega_{\max}$, where ℓ is the length of the limb (or limb segment) in question and δ is a factor that relates the cycle-averaged contribution of that joint to motion (which must be averaged, since the leg is not of constant length during the cycle due to knee flexure). This gives $K = \frac{V_o}{\ell \delta \theta_{\max}}$ to substitute into the expression for maximum cyclic power

$$P_m = \frac{1}{2} K^3 I \theta_{\max}^2 = \frac{1}{2} \frac{V_o^3 I}{\ell^3 \delta^3 \theta_{\max}}$$

and for cycle-integrated power,

$$\bar{P} = \frac{\phi V_o^3 I}{\ell^3 \delta^3 \theta_{\max}}$$

But now the use of a normalizing constant μ can give the angular moment I in terms of limb mass and limb

$$\text{length } \mu = \frac{I}{M_{\text{leg}} \ell^2}$$

PARAMETRIC MODELS OF LIMBED LOCOMOTION

I. The Lifting Run

Assume fraction q of vehicle mass supported by lift rather than foot impact

$$L = qw = q M_{veh} g$$

Power is expended propelling vehicle against aerodynamic drag

$$\underline{P} = DV = \left(\frac{D}{L}\right) q M_{veh} g V$$

$$\text{Assuming } \beta_{prop} = \frac{P_{lost\ to\ drag}}{M_{veh}} = P_D \beta_T \text{ where } \beta_T = \frac{P_{avail}}{M_{veh}}$$

$$\text{and } P_D \lesssim 1 \text{ gives } V = \frac{P_d \beta_T}{qg} \left(\frac{L}{D}\right)$$

II. The Uphill Run

Same model as above except $D = W \sin\theta$

$$\underline{P}_{loss} = DV = M_g \sin\theta V \rightarrow V = \frac{P_d \beta_T}{g \sin\theta}$$

$$\text{III. } \underline{\text{Hauling loads}} - \text{load force } L = K W_{veh} = \text{Drag by same formalism } V = \frac{P_d \beta_T}{K g}$$

$$\text{IV. } \underline{\text{Aerodynamic Drag}} \quad F_{Drag} = C_D V^2 \quad \underline{P} = C_D V^3 \text{ gives}$$

$$V = 3\sqrt{\frac{\underline{P}}{C_D}}; \text{ hard to reduce this form without a good model for drag coefficient}$$

C_D .

$$\text{V. } \underline{\text{Large cubic performance model}} \quad V = 3\sqrt{\phi}$$

where ϕ = performance scale factor ≤ 1

$$\delta = \text{velocity ratio of foot to } \omega_{motor_{max}} \quad \ell_{leg} \approx 0.9$$

$$\theta_M = \text{maximum angular swing of prime joint}$$

$$\ell = \text{leg length}$$

$$\beta_T = \text{specific power of vehicle} = \frac{P_{avail\ onboard\ max.}}{M_{veh}}$$

$$M = \text{angular moment ratio} = \frac{I_{leg}}{M_{leg} \ell^2} \approx 0.2$$

$$\bar{P} = \frac{\phi V_o^3 M_{leg} \mu}{\ell \delta^3 \Theta_m}$$

In this form we see that only the shoulder motion will contribute a significantly sized $\bar{P}$ to locomotive power drain, so that we can increase the cyclic-loss factor $\phi = \frac{\pi}{2}$ for all power lost by about 40% to obtain

$$\bar{P}_{veh} = \frac{\phi V_o^3 M_{4legs} \mu}{\ell \delta^3 \Theta_m},$$

and if we define $\tau = \frac{M_{body}}{M_{tot}}$ we can then write V_o in terms of β_{tot} :

$$V_o = \sqrt[3]{\Phi_{cyclic}} = \sqrt[3]{\frac{\pi}{2} \frac{\phi \delta^3 \Theta_m \ell \beta_t}{\mu (1-\psi)}}$$

There are several features that should be noted about this form. Performance Φ is proportional to β_t and to limb length. Running speed increases with larger angular excursion on the joints, as one would expect from watching humans run. Also, it is very important to get μ as close to zero and ψ as close to one as possible, for then Φ increases at essentially no extra cost in locomotor complexity.

We can evaluate this form for any number of good estimates for the parameters. It is easy to worst-case, since all of these functional parameters have definite limits: $\psi \leq 1$, $\mu \leq 1$, $\phi \leq 2$, $\delta \leq 1$, $\Theta_m \leq \pi$. The results one would expect are shown in Figure 4S, and they are undeniably encouraging.

This is the second type of cubic model to be formulated, and is capable of being combined in parallel with any of the drag-produced cubic models derived earlier. The validity of this model is certainly no greater than the previous ones, for although the algebra was more complex, the first harmonic and cyclic-averaging (of δ and I) assumptions were necessary. As such, one would expect that only the performance factor ϕ need be modified to correct these type of models, since the fundamental dependency was correct.

PERFORMANCE FOR VARIOUS ASSUMPTIONS

Linear Models

$$V = P_d \frac{\beta_T}{K}$$

P_d = scale factor   optimistic $P_d = 1$

conservative $P_d = 0.3$

β_T = vehicle technology parameter. Current actuator technologies have β_s of over $3000 \frac{\text{watts}}{\text{kg}}$, current prime mover technologies over $5000 \frac{\text{watts}}{\text{kg}}$ assuming all $f_{pi} = 1$, $v_{pi} = 1$ to be conservative, and that power components are designed to comprise 10% of vehicle weight (again, conservative), and

$$(\text{conservative}) \beta_T = 300 \frac{\text{watts}}{\text{kg}}, \quad (\text{pessimistic}) \beta_T = 100 \frac{\text{watts}}{\text{kg}}$$

For lifting run at $(\frac{L}{D}) = 2$ (for controllability), $q = 9$, corresponding to running uphill at 27° or hauling 45% of the vehicle's weight as payload implies $\rightarrow K = 2.222$

$$V(\text{opt. } P_d, \text{ cons. } \beta_T) = 68 \text{ mtr/sec} = 152 \text{ miles/hr} = 132 \text{ knots}$$

$$V(\text{cons. } P_d, \text{ cons. } \beta_T) = 20 \text{ mtr/sec} = 46 \text{ miles/hr} = 40 \text{ knots}$$

$$V(\text{cons. } P_d, \text{ pess. } \beta_T) = 6.8 \text{ mtr/sec} = 15.2 \text{ miles/hr} = 13.2 \text{ knots}$$

Cubic Models

$$V = \phi^{1/3} \quad \phi = \frac{\pi \phi \delta^3 \theta_M^2 \beta_T}{2\mu(1-\psi)}$$

ϕ = performance fudge factor   optimistic = 1

conservative = .1

δ = dimensionless velocity (should be ≤ 0.7 for running, 0.9 for nominal motion)

θ_M = maximum angular excursion of limb during stride ($\frac{\pi}{2} < \theta_M < \pi$ running)

μ = moment ratio (should be between 0.2 and 0.5)

$\psi = \frac{M_{\text{body}}}{M_{\text{tot}}}$ (at worst case, $\psi = 0$ but usually $\psi \approx 0.5$ for a 2 mtr limb length)

Then

$$V(\text{worst case}) = 3.2 \text{ mtr/sec} = 7.15 \text{ miles/hr} = 6.2 \text{ knots}$$

$$V(\text{worst case, but cons. } \beta_T) = 4.6 \text{ mtr/sec} = 10.3 \text{ miles/hr} = 9.0 \text{ knots}$$

$$V(\text{cons., with good motion}) = 7.6 \text{ mtr/sec} = 16.9 \text{ miles/hr} = 14.7 \text{ knots}$$

$$V(\text{cons., good motion, good design}) = 12 \text{ mtr/sec} = 26.8 \text{ miles/hr} = 23 \text{ knots}$$

$$V(\text{optimistic}) = 26 \text{ mtr/sec} = 58 \text{ miles/hr} = 50 \text{ knots}$$

It should be recalled that β_T , the vehicle power technology was conservative throughout. There is little doubt that with some knowhow β_T could be increased by an order-of-magnitude, $\phi \approx 1$ achieved, and $\mu = 1.5$, $\psi = 0.7$ with some special design. Thus, we could predict for deployable vehicles,

$$V(\text{deployable}) = 78 \text{ mtr/sec} = 17.5 \text{ miles/hr} = 152 \text{ knots}$$

From the simplicity of the various assumptions made, one would not expect estimates of Φ to be in error by more than a factor of 2, and one would expect the results from these forms to be correct to within $\pm 40\%$ (RMS). These simple locomotion models also usefully account for the functional dependence of performance on limb length and technology level (through β_{tot}), as well as characterizing a number of useful sub-design parameters, such as δ and μ . These parametric dependences will probably be highly useful during the development phase of WLRNVs, since they represent the state-of-the-art for more advanced capability prediction and size scaling evaluations that will be necessary to specify the next generation of vehicle design.

In any event, the results arising from the reasonable choices made for parameters in this section indicate the promise of the WLRNV concept. Indeed, if presently predicted performance were an order-of-magnitude poorer the WLRNV would still be of very considerable interest to the Navy because of the promise that has been amply demonstrated in essentially all the technological endeavors of the 20th century: that at least one and perhaps two orders-of-magnitude improvement in overall performance may usually be brought to a new technology, simply by adapting extant ones to meet the special requirements of a new frontier.

LOCOMOTION ENHANCERS

A relatively small population of limbed locomotive vehicles have been built before by the technological community. Every such one has been slow and clumsy, compared to the performances asserted as attainable for WLRNVs. The essential reason for such relatively low performance appears to be that such vehicles were not designed for speed and acceleration, but rather for load-carrying and rough terrain traversing. A much larger number of limbed locomotors have been built by the commercial world as toys. Walking dolls, walking toy dogs, etc. have been made in vast number with almost no technological development and with only 1 actuator!

Both the WLRNV and the rough-terrain-LVs have definite uses for multiple actuators, which are mostly required by the extremely variable geometry of the surface to be traversed and the desired versatility of motion. There are, however, significant lessons to be learned from toys. If one allows oneself the toymakers' artifices (strings and cables, linkages, and springs), the previous analysis appears somewhat crude. It asserted that limbed motion required continual power expenditure, with all torque in a joint being supplied by the actuator attached to that joint. However, the vastly more complex/versatile biological locomotors use shared torque between degrees of freedom, compliance (springiness), and cabled crosscoupling between joints.

In short, the actuator-per-joint philosophy may simplify control slightly, but it probably is not the most power-efficient way to achieve limbed locomotion. Instead of having to use power buffers and load levelling to minimize peaking loads in locomotive power histories, one could conceive of using cables and linkages to get each actuator to contribute to several joints in such a way as to level such loads. The simulation/optimization procedures sketched in the foregoing will give

optimal θ and τ histories for the joints, but there is no reason why N torques need be supplied by N actuators (required for independent joint control) in a 1:1 mapping.

Thus, once limbed locomotion is well-understood, it may well be that linkages and cabled coupling will be seem to be beneficial to the power-levelling and control problems. Of course cables are always of interest since they provide a way to put the actuators closer-to/in the body, thereby minimizing μ and $(1-\Psi)$. This cannot exactly be considered enhancement, however, since the location of the actuator with respect to its joint was never specified in the above analyses. Cable-driven systems are definitely advantageous (though difficult) and, even if they are appropriate for use in the near term, they will eventually allow attainment of the electromechanical analogs of the all-cable legs of birds (i.e. very high Ψ).

Considerably more interesting than permuting the locations and connection of the actuators with linkages and cables is the entire panoply of useful techniques falling in the category of impedance shifting. Non-linear linkages and cabling systems can provide varying mechanical transformance ratios which may help greatly to better match the actuators to their loads during motion.

Power limits on a component come in two varieties, hard and soft. The hard limit is the maximum power that the actuator can draw without failing. The soft limit is associated with the maximum power it can drive into the mechanical load. In the unenhanced system, the actuator must be specified by its soft power limit. If one allows impedance shifting, one could advance to performance levels associated with the hard limit with the same actuator. This would provide significantly more power at those points in motion cycles where the actuator and its locomotive loads are significantly mismatched.

Non-linear transmissions (linkages and cables) would provide time-dependent impedance matching (with a clever design) during locomotion. Even more useful, however, would be the ability to match impedances where they are substantially (orders-of-magnitude) mismatched. This latter situation occurs when the required actuation is either radically slower or faster than the actuator. The solution to this is the gearbox, for if the actuator could be geared up or down, it could be made to better match the load and deliver correspondingly more power.

Therefore gearboxes will be some of the most critical motion performance enhancers, since they allow impedance matching over a wide speed range, and thereby permit the control system to vary maximum torque or speed, and thus use more of the hard limit maximum power capabilities. Besides these direct advantages, they can be used as modulatable brakes (since power losses are much higher in low gear), saving power that would otherwise have to be dissipated in the actuators (to cause braking) during deceleration.

Gearboxes are just multiple-speed transmissions, and as such imply the use of clutches. So, postulating gearboxes, one also naturally assumes a corresponding clutch technology. This affords two very important enhancements: braking and free-wheeling. If one clutches a joint to the nearby limb structure, that joint is frozen, allowing no-power holding at any joint. If this clutch is held closed without power then vehicle loitering requires absolutely no actuator power drain. As long as a joint is frozen, no power is being used there, so that braked intervals at various joints during motion would save energy.

Free wheeling promises even greater relative power savings. Once a joint is moving in the proper direction, it may be declutched and requires no further power for some time. Since the gearbox and actuator are then disconnected, there is essentially no loss mechanism at that joint. If the proper modification were made to the

cost function in the limb-cycle-optimizer to allow for the absence of losses when the joint was frozen or free-wheeling, the optimal motion solutions would contain frozen and free-wheeling intervals to take advantage of these power and energy-conserving enhancers.

Another of the toymakers' gambits that would be useful is the spring, which may also be considered as impedance shifting in the complex Z plane. More intuitive than complex impedance, one may simply consider the meaning and usefulness of reactive loads. Instead of wasting power they store it. Mass is then the conjugate form of mechanical storage, and between springiness and mass one gets the phenomena known as resonance. The addition of springs to limbed locomotors would thus allow direct energy storage (at the end-user, the joint) and exploitation of resonance phenomena.

At first glance resonance may only seem an inconvenience. It makes control more complex (but not much, in the concept to be presented), and it is not versatile because of its fixed frequency. But applied to the most fundamental use — generation of the acceleration and deceleration torques of the limb cycle — one sees the possibility of greatly increasing the cyclic ϕ . The analogy that intuitively motivates spring utilization is manifested in human locomotion: Walking 100 yards takes more than an order-of-magnitude less total energy than does running 100 yards, and almost none of that difference is due to aerodynamic drag. When "walking" one is taking advantage of gravity loading and the pendulum mode of the leg to drive the limb cycle. By modulating θ_m , the angular length of stride, velocity is controlled while the gravity-inertia resonance provides the cyclic driving torques at no power cost. When a human runs, however, his leg muscles must provide those rapidly time-varying acceleration and deceleration torques at ever higher, non-resonant frequencies. The simplest form of WLARNV pays this price too, in the cubic performance requirements of limb cycle power with vehicle velocity.

Now consider using a spring to produce a higher resonant frequency than the limb pendulum mode. The pendulum mode provides 'free' cyclic torques, with a maximum cruise velocity near 3 knots for a 1 meter limb, increasing as $\sqrt{l}$ beyond this. If one used a spring, this maximum could be extended to a higher speed regime, thereby allowing ϕ in the cyclic ϕ to approach zero (there will always be some V^3 terms, due to cyclic losses).

However, to move considerably slower than this spring resonance and still have enough θ_m to move the limb foot well clear of the water, one would have to declutch the spring from the joint. If one adds a clutch between spring and joint to retain power-efficient low-speed operation, then one can obtain a superior application. The clutch-declutch cycle on a spring can then be used to reflect the motion of a joint: at a given angular velocity ω , declutch the gearbox/actuator and clutch in the spring; the spring compresses until all available kinetic energy goes into spring potential energy, then reverses the sign of ω and starts to return the stored energy to kinetic form; when stored spring energy hits zero again, declutch the spring and the joint is moving at $-\omega$.

In this way, appropriate springs and clutches could be used to totally eliminate acceleration/deceleration limb cycle losses in the joint actuators: just reflect at the hip to get reversal from forward slew to set up for foot insertion into the water, and reflect after foot kickoff to get into forward slew. The net energy costs of the cyclic limb motion would be drastically reduced, and the period of the cycle is now totally controllable once again, simply by varying the length of the declutched intervals. In this way the limb cycle becomes a type of tuned non-linear oscillator, and the actuators need only make up the intrinsic losses of this oscillator and provide the push-phase power.

It may also prove useful to put a freeze clutch on

the spring after the declutching clutch, so that the spring could retain stored energy indefinitely. The spring, unlike the actuator, delivers its energy to its load as fast as the load can accept it, having no practical power limits. The actuator could then be used to 'charge' the spring for 'ultra-push' quick starts. Equally importantly, the springs could also be used to absorb all of the kinetic energy of the joints, then freeze and declutch, resulting in a ultra-fast stop that conserves the energy of motion to reapply for a quick restart, simply by reapplying the torques of the wound springs.

Thus, with enhancement, we may go considerably beyond the continuous power requirements of the simplest type of limbed locomotion into an almost energy-conservative regime, where only necessary propulsion losses require make-up power. We may match impedances to extract the highest possible performance from a given actuator. In short, we may enhance performance by at least an order of magnitude in subsequent generations of WLRNVs. By so boosting the cyclic Φ , the dimensionless vehicle performance the Φ_{tot} then determines the vehicle's applications potentials. But with time, the growing WLRNV knowhow that creates ultra-high-drag feet, streamlines vehicles and reduces component dissipation will also boost Φ , so that the semi-quantitative performance predictions of the last chapter will become quite conservative, even without advances in the several component technologies, (which, however, are also certain to occur, whether WLRNVs are developed aggressively or not).

THE STRUCTURAL PROBLEM

The role of the structural subsystem is quite self-evident. The whole WLRNV must hold together and perform with all its limbs and body intact under the most severe locomotive stresses. If there is some extreme locomotion or even accident which must be endured in order to complete a mission, then the structural elements of the vehicle must withstand it. On the other hand, the structure must be just as light as possible, since structure weight is useless for increasing agility, speed or mission effectiveness of a WLRNV.

The major question is how little structure a WLRNV can have, while withstanding the most severe structure loads. The structure must be considered as a system component whose design is dictated by the forcing loads of other components, including the limb shells, the body frame, the body shell, the joint mechanisms, bearings, axles, and mounting for other locomotive, power, or control components. The worst-case loads are expressed in terms of the worst-case forces expected to be withstood by the structure.

Such forces (structure loads) can all be referred to one characteristic parameter of the vehicle, its weight. This normalization, expressing structure loads as $\alpha = \frac{F_{\text{load}}}{g M_{\text{tot}}}$ gives a dimensionless parameter representing loading which can be referred to designs in the locomotive framework via the M_{tot} (total vehicle mass).

It might appear inappropriate to express the problem in terms of α 's, except that one can readily normalize α . A typical example is the limb-bending moment, which is interesting because it is obviously one of the most likely modes of structural failure. The limb may be thought of as a loaded beam (bent at the elbow) through which axial, bending, and twisting loads are applied by external loads at the feet and self-constrained loads providing the torque at the joints. Due to the actuator-

produced torques in the joints, the structure has internally loaded points as well as end loads.

There are only three failure modes in the simplest (worse-case) approximation, a piece of tubing with an elbow bending in it. This piece of tubing is taken to be of constant diameter, to simplify the analysis. (We admit this simplification because this constant diameter representation can only be stronger than a tapered diameter limb tube, drilled for lighter weight.) This worst case tube can be readily analysed for its three basic failure modes: tensile axial, bending, and twisting. There are two distinct modes of twisting failure, one in each tube, and two modes of bending failure (perpendicular or transverse) referred to either limb tube.

Elastic phenomena are linear in their responses to structure loads. Thus we may determine the structural loadings in terms of stresses which are everywhere the sum of six sets of forces. This can be simplified even further, since the stress axes and the failure modes in one tube are collinear: one may define uncoupled failure phenomena in each limb segment separately, and merely match boundary conditions at the elbow to get the single-beam result.

In fact, one may proceed even further, simply by noting that we may detect failure by calculating stresses from the loading forces. First in the entire bent tube then in two segments, and then in six modes per each pipe, one asserts that one may decompose the failure event into mutually exclusive subsets whose phenomenology is upheld by stress continuity. Thus, we have mutually independent boundaries for acceptable loads in the 10 possible failure modes. We next note that if these boundaries are independent, we can safely worst-case each of them, and know that for every possible loading we then have a worst-case. This worst case is then defined over all possible sum geometries, and we have defined the failure set of a guaranteed-conservative

FIGURE 41

LOADED TUBE MODEL

Consider the limb as a bent tube loaded by forces and moments at each end. Elbow is considered to be "rigid" and the motor torque there τ_e is actually embedded in the 12 end loads by equilibrium. This goes as well for any other controllable D.O.F.'s in limb.

This is equivalent to
is equivalent to

Model each limb segment as a tube for bending, twisting, and axial loadings

Cross section

Idealize as cylindrical tubular beam; total length L

$$\text{Bending Moment } I_{11} = \frac{\pi}{16} R^3 t$$

$$\text{Twisting Moment } I_T = 2\pi R^2 t$$

$$\text{Cross Section} = 2\pi R t$$

Pick t/r as a dimensionless thickness parameter;

$$\text{then } \sigma_{\text{bending}} = \frac{\tau_{\text{across axis}} R}{I_{11}}$$

$$\sigma_{\text{twisting}} = \frac{\tau_{\text{about axis}}}{I_T}$$

$$\sigma_{\text{axial}} = \frac{F_{\text{axial}}}{\text{cross-section}}$$

If σ is the maximum allowable stress, taking into account the possibility of multi-axial loading, the stress in the beam should stay less than σ for bending, twisting, and axial loading. Since all of these loads are worst when $\theta = 0$ (the limb is straight), the design should match stress at $\theta = 0$ and maximum allowable loading to σ in order to achieve minimal weight structures:

$$\sigma = \frac{F_{\text{shear}} \cdot \ell \cdot 16}{\pi R^3 (t/r)} = \frac{F_{\text{shear}} \ell/2 \cdot 1/2}{\pi R^3 (t/r)} = \frac{F_{\text{axial}}}{2\pi R t} = \frac{F_{\text{axial}} 1/2}{\pi R^2 (t/R)}$$

model as the union of 10 disjoint failure sets.

This is an extremely straightforward way of getting an absolute upper bound on the structure weight required to give a specified limb strength. Indeed, if the union of disjoint worst-case failure modes is empty for all specified loads, then there exists a tubular model that can endure to those loads because they are the worst-geometry loads of the tube model. Furthermore if there is a tubular model that can endure such loadings, then there must be a lighter optimally-designed structure that can to likewise.

The first of these modes to consider is bending failure. In any of its four possible geometries (i.e., perpendicular or transverse failure in either segment), the worst case geometry is when the elbow is relaxed so that the limb extends to its maximum length ℓ . In this mode, either transverse structure loads or an elbow torque could cause failure. Failure in bending is totally due to excessive moment. Thus the strength of the straight (worse-case) tube model must withstand the sum of the elbow moment (on the transverse axis only) plus the moments of the end transversal forces.

These forces are entirely determined by the possible acceleration of the vehicle, α , the maximum vehicle acceleration divided by g , which is about the same magnitude as the structure load divided by W . Then for 10 g body movement $\alpha = 10$, and for 1/2 g operation (acceleration to about 0 to 60 mph speed in six seconds) $\alpha = 1/2$. Defining the long tube to be a cylindrical beam with radius R and thickness t , one may select a ratio of acceptable $\frac{R}{t}$, and then find an effective bending moment I_{II} ,

$$I_{II} = \left(\frac{\pi}{16} \frac{t}{R}\right) R^4.$$

With an applied bending moment and a maximum loading of αW applied at the tube ends (this being the worst possible locomotive load), all that is needed is a largest acceptable stress for the required set of operations. Calling this maximum allowable stress σ , we recall $\alpha = \frac{\alpha W \ell R}{I_{II}}$ is the maximum stress encountered in bending.

To resist this type of loading from any possible direction, the structure must be a tube of continuous strength, depending on which end could be clamped. Thus the worst case requires a tube of constant radius and thickness, which must weigh $M_{\text{limb}} = \rho 2\pi R \ell \tau$. This allows us to form the useful equation $M_{\text{tot}} = \frac{\sigma I_{\text{II}}}{\alpha g \ell R}$, giving

$$\frac{M_{\text{limb}}}{M_{\text{tot}}} = \mathcal{Z} = \alpha g \ell^2 \frac{R^2 \tau}{I_{\text{II}}} 2\pi \left(\frac{\rho}{\sigma}\right) = 16 \alpha g \ell \left(\frac{\ell}{R}\right) \lambda \left(\frac{\rho}{\sigma}\right)$$

This last parameter $\left(\frac{\rho}{\sigma}\right)$ is the single parameter that characterizes our materials technology. It has a value for machinable, high strength aluminum alloys of $4 \times 10^{-6} \frac{\text{sec}^2}{\text{M}^2}$, for the best steel near 6.5×10^{-6} , for beryllium (the best of all metals) of $3 \times 10^{-6} \frac{\text{sec}^2}{\text{M}^2}$, while fiber composites could do more than an order-of-magnitude better in the far term. The parameter λ is somewhat greater than 1, and is a factor used to include the mass of mounting hardware that should also scale in weight proportionally to the bending moment.

The $\left(\frac{\ell}{R}\right)$ parameter is known as a slenderness ratio S , and the limb weight fraction goes up directly in proportion to it. This explains why the slender-limbed-type insect designs are not possible in the larger animals, since slenderness must be traded for size as biological designs are implemented with a low, fixed $\left(\frac{\sigma}{\rho}\right)$.

We see that structure weight fractions can be expressed totally in terms of accelerations (α), materials technology $\left(\frac{\sigma}{\rho}\right)$, slenderness, length, and mounting state of the art (λ). This imposes the problem quite nicely, and indicates where we can trade off design variables against each other while maintaining a given structure weight. It automatically signals when we are going to have to sacrifice for structure weight, and what types of vehicles are entirely too heavy.

The next set of loads of concern are axial loads. The geometrical worst case for these is still the single straight tube geometry, with a pure tensile or compressive

FORCING AND FAILURE IN TUBE MODEL

Possible Failure Modes

- I. Bending: 1) $\uparrow_{\perp}$ failure in upper segment
 2) $\uparrow_T$ failure in upper segment
 3) $\uparrow_{\perp}$ failure in lower segment
 4) $\uparrow_T$ failure in lower segment
- II. Twisting: 1) Shear failure in upper segment
 2) Shear failure in lower segment
- III. Axial: 1) Tensile failure in upper segment
 2) Buckling failure in upper segment
 3) Tensile failure in lower segment
 4) Buckling failure in lower segment

Also, bending failure can occur by sub-area buckling in which the compressed side of the tube crumples. All buckling failure modes can be eliminated by increasing t/r and decreasing effective density of material (i.e. drilling holes in thicker tubing, spacing concentric thinner tubes, radial honeycombing, etc.) Note this fix does not work beyond the $F = N\pi^2 EI_{rod}/\ell^2$ limit for a solid rod; this limit is not of interest for modern structural technology (metals and composites), but is the one which forces biological systems (using bone and chitin) to endoskeletal (rods) rather than the lighter exoskeletal (shells) design.

Twisting failure and bending failure have the same functional form, but bending occurs at 64 lower loads $\rightarrow$ twisting failure mode is inactive. Once buckling is eliminated, axial failure has the same form as bending failure, but with $1/32 \times 1/S$ the stress for the same load. For any limb where S (the slenderness ratio) $\times \ell/R$ is > 1 , axial modes are also inactive.

$$\pi R^2 \sigma = F_{load} \cdot \ell \cdot 16 = \alpha_{load} g M_{tot} \cdot \ell \cdot 16$$

$$M_{limb} = \rho 2\pi R t \ell = \lambda M_{tot}$$

$$\lambda \pi R^2 \sigma = \alpha g \rho 2\pi R t \ell \cdot \ell \cdot 16 \quad \text{or}$$

$$\lambda = \ell \left(\frac{\ell}{R} \right) 16 \alpha g \left(\frac{\rho}{\sigma} \right) \lambda \quad ; \quad \left(\frac{\sigma}{\rho} \right) \text{ is technological parameter called specific-strength that characterizes the material.}$$

λ is a weight scale factor that allows for excess weight proportional to loading, and could also be used to include bending and twisting stresses contributions toward multi-axial failure.

load. At first, it would seem that the concern should be buckling under compression. However, it turns out that the choice of $(\frac{\tau}{R})$ is totally arbitrary if one is willing to fabricate the metal shell in unusual geometries. If one fabricates the tube of radial fins of the thickness τ at a radius R , the weight is not affected because strength is conserved by using the thinner continuous tube as a continuum approximation to a discrete structure. Thus one trades density for $(\frac{\tau}{R})$ at constant R , and one can (arbitrarily) make buckling occur at any axial load.

However, it is pointless to augment buckling strength beyond the point at which compressive creep sets in. This is essentially the same point as the tensile yield stress limit, so that the tensile and compressive failure modes are considered as one. Then the load is αW , which must be less than $\sigma 2\pi R\tau$ to resist failure. This tube in length ℓ weights $\rho 2\pi R\tau\ell$, so that

$$\frac{V_{\text{limb}}}{V_{\text{tot}}} = \mathcal{J} = \frac{2\pi R\ell\tau\rho}{2\pi R\tau} \alpha g = \alpha g\ell \left(\frac{\rho}{\sigma}\right)\lambda$$

Comparing this to the bending form, we see that, for any $S > 1$ limb design, the bending failure modes are always more critical than axial tensile failure modes.

The required (dimensionless) vertical accelerations, though > 1 to support the vehicle weight, will usually be smaller than the peak horizontal accelerations required of the vehicle. This being the case, we can just add the slight contribution of the superimposed bending loads by approximating this as a 20% increase in λ in the comparable form for bending.

The last failure mode is twisting, and the associated stresses are worst for the straight tube geometry. However, in this geometry there is no reasonably large loading condition. The only condition in which there are relative large loads applied to this mode is for the elbow bent as a 90° angle. In this mode, there is

approximately $\ell/2$ length about to be yielded, and a force of αW acting on an $\frac{L}{2}$ length arm.

St. Venant twisting approximations can be used on the twisting-loaded segment. Unlike bending and axial loading, the critical stresses in twisting are shear stress, the maximum sustainable stresses of which are usually $1/2$ of the previous σ , which was a tensile stress. In the shear-flow approximation $\frac{\alpha WL}{2} = \sigma_s 2\pi R^2 \tau$, and again the limb weighs $\rho 2\pi R \ell \tau$, so that

$$\lambda = \frac{\rho 2\pi R \ell \tau \alpha_s \ell}{2\sigma_s 2\pi R^2 \tau} = \frac{1}{2} \lambda \alpha g \ell s \left(\frac{\rho}{\sigma_s}\right)$$

This has exactly the same form as the bending form, and should be loaded by the same worst case accelerations, while the $\left(\frac{\rho}{\sigma_s}\right)$ has doubled due to shear failure. Thus twisting loads are very nearly $\frac{1}{16}$ of the size of bending loads, so that the use of $\lambda=2$ allows a 20% margin for extra axial stress, 30% margin for twisting requirements, and another 50% for the possibility of worst-case multi-axial loading superposing to bring us closer to the failure surface in $\sigma_x \sigma_y \sigma_z$ space.

Thus we have a single calculation that gives an upper bound on limb mass fraction (λ), thereby telling the designer exactly what can and can't be done. This accounts, with appropriate conservatism in the factor λ , for all axles, bearings in tubes, and mountings therefor. The only unaccounted for component is the body shell. However, this part is a product of a hardness design, since a non-military vehicle would function quite satisfactorily with a thin fiberglass container for a body shell. (The body skeleton and mountings inside are included in the overall λ_{tot} .) Since the shell is the result of a hardness-oriented design, only the classified ordnance literature would give a good idea of currently attainable technologies and requirements in body-shell hardness. For the purposes of this document, it is sufficient to treat the body-shell as a separate component with a non-structural purpose whose weight fraction

z_{hard} is specified by the military user who supplies the technology. The user simply specifies how many pounds are required to protect a vehicle of the desired mass, and the design can proceed with this mass fraction z_{shell} defined (which should be $\leq 30\%$, except for very special missions).

The previous mass fraction model is useful for design, since it bounds alternatives. However, it does not define performance points, but just sets limits on the parametric region in which those points can be. It is possible to evaluate the performance associated with a structurally limited operating point. This model uses the vehicular torque requirements to give performance limits, rather than specifying maximum acceleration.

There are two sorts of required motive torques, static and dynamic. The thrust phase torques can be considered to have been covered in the α discussion, since this describes the g-equivalent model of power drag-limited running, as well as acceleration. Static torques are those required to support the vehicle against gravity. These as well fit best into the model in which $\alpha = 1$, as did the push-phase torques. Dynamic torques, however, are those the structure must withstand due to the kinematics of the actuators and springs.

For this analysis we employ the first-harmonic approximation to get a qualitative grip on structure-limited velocity. Using $\tau_{\text{max}} = K^2 I_{\text{II}} \Theta_m$ as before, and eliminating K^2 by $V_o = \delta \ell \kappa \Theta_m$ gives $\tau_{\text{max}} = \frac{I \Theta_m V^2}{\delta^2 \ell^2 \Theta_m^2}$. This can be substituted into the bending limit (acknowledging a λ_{tot} for the entire structure) to give an overall maximum velocity that the structure can withstand.

$$\sigma = \frac{\tau_m R \lambda_{\text{tot}}}{I} \quad \text{gives} \quad \tau_m = \frac{\sigma I}{\lambda_{\text{tot}} R};$$

$$V^2 = \frac{\tau_m \Theta_m}{I_{\text{mb}}} \delta^2 \ell^2 = \frac{\sigma I \Theta_m \delta^2 \ell^2}{\lambda_{\text{tot}} R \mu M_{\text{tot}}};$$

$$\text{but } M_{\text{tot}} = M_{\text{limb}} = \rho 2\pi R \ell \tau \text{ and } I_{\text{II}} = 16 \left(\frac{\tau}{R}\right) R,$$

$$\text{so that } V^2 = \frac{\sigma}{\rho} \frac{S}{\mu} \frac{g^2}{\theta_m} \text{ or } V = \frac{S(\sigma/\rho)\theta_m\delta^2}{\mu \lambda_t}$$

This is the quadratic model referred to earlier, and arises from the quadratic dependence of velocity on torque that one expects in rotating systems. Its basic result is that slender limbs row faster, even as they take up more fractional vehicle weight, and it demonstrates the importance of advancing know how in reducing λ and μ . It also tells us that those vehicles adapted particularly for speed will be forced to employ long spiderlike legs (as in the horse or the giraffe) in order to stretch the vehicle performance available from materials technology.

Lastly, this can be combined with both the static requirements from the bending model and the cubic-velocity-drag performance formulation to give a "nothing wasted" design. This design philosophy appears to produce an excellent first cut design, once the motion requirements have been specified. It is based on the assertion that no mass should be wasted on a vehicle capability that cannot be used.

If the quadratic velocity V is higher than the cubic V_0 , then the structure will withstand loads beyond the limits of the actuators to produce. In this way one would have too much structure mass, since some could be thrown away and the vehicle would still never break. If, on the other hand, V_0 is larger than V , one cannot operate the actuators at full power without breaking the structure, and thus actuator mass is being wasted, since a lighter actuator that delivers less peak torque would do the job as well. Thus the match point $V = V_0$ gives the "nothing wasted" design.

PARAMETRIC VEHICLE STRUCTURE MODELS

I. Static Limit $\beta_T = \frac{\text{limb mass}}{\text{veh. mass}} = 16\alpha \left(\frac{\ell}{R}\right) \ell \left(\frac{\rho}{\sigma}\right) g N_{\text{leg}} \lambda$

where $\alpha = \frac{\text{max. thrust}}{\text{vehicle weight}}$; $\left(\frac{\ell}{R}\right) = \text{limb slenderness ratio (from 3 to 30)}$

$\ell = \text{limb length}$

$\left(\frac{\rho}{\sigma}\right) = (\text{specific strength})^{-1} = \left(\frac{\text{yield strength}}{\text{density}}\right)^{-1}$

$N_{\text{leg}} = \text{number of legs on vehicle}$

$\lambda = \text{structure use scale factor} \geq 1$

II. Dynamic Limit

From cyclic constraints in recycle phase, assume temporal frequency

$\theta = \theta_M \sin Kt$; balancing torque requirements gives $\tau = \frac{I \theta_M V^2}{\delta^2 \ell^2 \theta_M^2}$ or

$V = \ell \sqrt{\delta^2 \frac{\tau_{\text{max}} \theta_M}{I}} = \sqrt{\frac{S \bar{\sigma} \theta_M}{M \lambda}} \delta^2$, where

$\bar{\sigma} = \frac{\sigma_{\text{yield}}}{\rho} = \text{specific strength}$

$S = \text{limb slenderness ratio} \frac{\ell}{R}$

$\theta_M = \text{max. angular excursion of prime joint}$

$M = \text{angular moment ratio} \frac{I}{M_{\text{leg}} \ell^2}$

$\lambda = \text{structure use scale factor}$

III. Power Balance (Maximum velocity allowed by structure)

$V < V_0$ means too much structure mass

$V > V_0$ means there is enough propulsion power to break the structure

$V = V_0$ gives $\frac{\bar{\sigma} S \theta_M \delta^2}{M \lambda} = \ell \frac{\pi}{2} \frac{\lambda \delta S \beta_T}{\eta \bar{\sigma}}$; $V_0 = V = \sqrt[3]{\frac{\pi \delta^3 \theta_M \ell \beta}{2 M (1-\psi) \phi}}$

Required structure technology: $\sigma = S \lambda \sqrt[3]{\frac{M}{\theta}} \left(\frac{\beta_T \ell}{\eta}\right)^{2/3}$

Matched design $V_0 = \sqrt[3]{\frac{\gamma g \ell^3 \beta_T^2 \lambda^2 S^3}{\nu^2 \sigma^3 (1-\psi)}} = \frac{\beta \ell}{\eta} \left(\frac{\lambda S}{\bar{\sigma}}\right)^{2/3} \sqrt[3]{g \ell} \sqrt{\frac{\gamma}{1-\psi}}$

DESIGN AND PERFORMANCE

Structurally feasible limits within which to build a quadruped

$$\beta = 16 \lambda g(\frac{\rho}{\sigma}) \alpha S, \text{ at all } < .25$$

For technology, $(\frac{\sigma}{\rho})$ is best for aluminum alloys (i.e., 7000 series) among the common engineering materials (beryllium is better but inadvisable for large-scale production)

$$\sigma = 98 \text{ ksi} = 6.7 \times 10^8 \frac{\text{Nt}}{\text{M}^2}; \rho \approx 2.75 \frac{\text{Kg}}{\text{cc}} = 2.75 \times 10^3 \frac{\text{Kg}}{\text{M}^3}$$

$$g(\frac{\rho}{\sigma}) = 4 \times 10^{-5} \text{ M}^{-1} \text{ for off-the-shelf, commonly-used technology}$$

$$= 10^{-4} \text{ M}^{-1} \text{ (if one wishes to be pessimistic)}$$

for $\ell = 2 \text{ mtr}$, the limit $= 0.25$ gives $\alpha S = 7.8$, assuming essentially no knowhow ($\lambda = 10$) and pessimism about materials. For the vehicle to support its own weight, $\alpha > 1/2$ implies $S < 15$. Allowing existing structures and materials technology, the maximum $\alpha S \approx 80$ should be easily achievable (without exotic technology), giving 10 g accelerations at $S = 8$.

Parametric Design

Assume a mission design target of $V = 50 \text{ knots}$ with 10 g acceleration capabilities. V is 26 mtr/sec, so that $V\alpha g$, the requisite motive power, require β_{tot} of only 2500 watts/Kg, which is quite compatible with a 50 knot goal on deployment timescales.

From the structural model,
$$V^2 = \frac{S(\frac{\sigma}{\rho})\theta_M \delta^2}{M\lambda}$$

The 50 knot goal required $M \approx 0.2$, $\theta_M \approx \pi$, $\delta \approx 0.9$, $\beta \approx .08$.

Thus mission definition gives $\frac{S}{\lambda} = 2.2 \times 10^{-4}$; since $\lambda < 100$ and $S > 1$, this means that velocity is essentially never structure-limited!

$$V > \sqrt{\frac{S(\frac{\sigma}{\rho})\theta_M \delta^2}{M\lambda}}$$

$$= .08 = 16 \lambda (4 \times 10^{-5}) \cdot 10 \cdot 2.5 \text{ implies}$$

$$\lambda S = 6.25 \text{ for a 2 meter limb}$$

Thus a high-g vehicle requires stubby ($\geq 13 \text{ inch}$ diameter) legs and the best structures knowhow ($\lambda \rightarrow 1$). A low (~ 1) g vehicle could have $S \rightarrow 30$ and still require only reasonable structure knowhow ($\lambda = 2$).

If the performance technology were available, could perhaps be allowed to double, but this does not help greatly. Only if one is willing to engineer with the fiber composite materials (another order of magnitude in $\frac{\sigma}{\rho}$) will the 10 g regime be accessible with slender legs ($S > 30$), although 3 g operation could perhaps be realized from a metals technology.

THE CONTROL PROBLEM

Finally, we consider the control subsystem for the WLRNV. Even before substantial on-board 'intelligence' is specified, it is obvious that high speed WLRNV control is one of the most challenging control problems encountered to date. However, as difficult as these relatively novel control problems may be, they are definitely soluble. Eventually such control subsystem will be as routinely created as those presently found in airplanes, though they are as complex as those found in current spacecraft systems.

The problem is one of both bulk and novelty: bulk because of the dozens of sensory signals that must be received and analyzed at bandwidths well above 100 Hz, and novelty due to the requirement for flexible control of vehicular motion under the random conditions of wind and sea. Indeed, control is both the vital element and the crucial unknown of WLRNV technology; everything else appears relatively straightforward.

Since these considerations indicate that control is the crux of the RNV scheme, this problem must be evaluated very thoroughly before WLRNV can be declared to be a viable vehicular technology. However, not only have very powerful and general techniques been developed in the control area, but the LLL Smart Vehicles project has successfully investigated the conceptual feasibility of a moderately direct scheme for implementing the required control for WLRNV systems. There can be no doubt that the WLRNV control problem can be solved with extant control technology, that which has created microradian pointing-and-tracking controllers for large mass and moment slewing on millisecond timescales, and which routinely steers spacecraft across interplanetary distances with sub-part-per-billion error. Hardware implementation of such solutions will be initially bulky since the relevant timescales are small and the number of

INSTRUMENTATION LIST

PROPRIOCEPTIVE

Motor current and voltage measured to give torque and speed information.
 $2 \cdot N_{\text{CDOF}}^*$ signals

ANGULAR

Rotary encoders on joints, digitally enhanced to give θ and $\dot{\theta}$ information.
 $2 \cdot N_{\text{CDOF}}$ signals

ACCELEROMETERS

Read $\vec{a} + \vec{g}$, $\vec{\omega}$ in limb segment coordinate system; 6 accelerometers per limb segment required
 $6 \cdot N_{\text{LS}}^*$ signals

STRAIN GAGES

Determine structural loads F, M, in half-bridge pairs: require at least $6 \cdot N_{\text{LS}}$ signals

THERMISTORS

Monitor motors and transformers for over-power operation: at least
 N_{CDOF} signals

PRESSURE SENSORS

Resolve buoyant force distribution on feet: require at least $6 \cdot N_{\text{FOOT}}^*$ signals

VIBRATION (ACCELEROMETER) SENSORS

Detect foot vibration due to turbulence in water: probably $3 \cdot N_{\text{FOOT}}^*$ signals

POWER SYSTEM MONITORING

Measure and sample all supply levels to ascertain correct power flow

ATTITUDE SENSORS

Instrument body orientation: 3 signals

GYROSCOPIC PLATFORM

Body orientation, velocity, and position

OCULAR

Scanning laser rangefinder
 Stereo solid-state scanning array vision
 Time-of-flight narrow-band narrow-pulse scanning array

TACTILE

Whiskers - strain sensing to detect prospective contact
 Surface compression - tactile map for docking

- * N_{CDOF} = Number of controllable degrees from freedom
- * N_{LS} = Number of physical limb segments
- * N_{FOOT} = Number of feet

processes being controlled is large, but relatively little precision of control is required, and the vehicle mechanics are certainly capable of effecting the specified control, as noted previously. Moreover, the LLL Project asserts capability of performing WLRNV control at the present time. Basically, the solution consists of decomposing the problem into naturally separated regimes which can be re-integrated ever more thoroughly as WLRNV operating experience is gained. Since early generations of research technology in WLRNV control can be generated in periods of the order of a year at 500 K\$ cost levels, such experience will be gained both quickly and inexpensively; a few years of WLRNV control research will result in demonstration of sophisticated control capability.

Separating the problem into several blocks to solve it is allowed by the decoupling implicit in four assumptions in the design, the most basic assumption of which is that we are able to exactly specify the motion of the vehicle at any time, as long as the proposed motion does not exceed structure or locomotive limits. A second is that we are able to combine and use stored motion programs (gaits) to exactly specify such motions. The third assumption is that any desired course of the vehicle can be executed by merging together intervals of gaits drawn from a stored 'alphabet' of motions. The fourth is that quasi-optimal courses that adjust the desired mission histories (stored before leaving the WLRNV's mother ship) can be calculated from local ocean surface 'maps' by an on-board decision computer.

Thus, we have separated the overall problem into a hierarchical one. At the bottom level is the layer of hardware which obtains exact compliance to low level motion control decisions from the physical vehicle. Immediately above this is a series of spooling devices with considerable memory, whose task it is to

recall and combine stored limb cycles and gaits. Above this is a course decision computer that executes mission plans by selecting which of and how long the stored intervals of limb cycles and gaits will be applied. To the side of the main control hierarchy, service processors adapt sensory inputs into forms most useful to the control procedure just described. The most complex of these, probably consisting of several computers, is the vehicle eye and its processing gear which delivers real time ocean surface maps to the decision processor.

This scheme runs the gamut from dedicated hardware to several complete digital processors in the mini-computer range. By separating the problem into a hierarchy of problems, it becomes solvable in parts. These parts will eventually have to be interlinked by a variety of communication protocols, but so long as each such linkage functions at the required throughput bandwidth, the total problem is solved. This solution, perhaps implementable in as few as 20,000 1976-level integrated circuits, can be miniaturized by the aerospace thick-film hybridization techniques into sub-breadbox volume of under 100 lbs. weight with current technology. Since the solid state technology expects to be gaining three orders-of-magnitude in circuit density over the next decade, the entire Smart WLRNV digital control system can be fit into less than 1 liter volumes by deployment dates in the late 1980s.

Stated simply, the required control can be implemented acceptably with this year's technology, and by the time the entire WLRNV technology has matured to the mass deployment point, such control implementation will be trivial. Moreover, as noted above, failure to start WLRNV technology on such a learning curve in the near future involves the pointless sacrifice of an aspect of future military superiority implied by continuing technological asymmetry in semiconductor technology.

The primary innovation of the Livermore approach is found in the lowest layer of vehicle control. Every previous limbed vehicle effort has aimed for much lower speed of operation, and has performed calculation of vehicle motion physics for control. These calculations have been severely simplified by removing the non-linear acceleration terms (e.g., the coriolis and centripetal terms in the acceleration transformation for rotating coordinate frames).

- The LLL genera of WLRNV, on the other hand, is specifically aimed at high speed operation, in which the inertia forces are generally much higher than either the gravity or actuator damping forces. Thus, ignoring the inertia terms is quite unacceptable, and the required vehicle physics calculations become truly formidable, as noted above. In contrast, the LLL Project's control scheme will essentially employ high gain feedback and the vehicle itself to solve these vehicle physics problems in real time, without data-processing hardware. Each vehicle joint is arranged as a very high gain ($\approx 300X$) servo-loop, which has the non-linearities of motion physics as an intrinsic feature of its feedback loop. As such, they can be treated quite satisfactorily by considering the non-linear terms to be species of noise with characteristic correlation times comparable to those of the mechanical components of the vehicle.

In essence, the non-linear physics forces only change on 10^2 millisecond timescales. If one positions a servo-loop in the control system of a vehicle limb with a response time in the 1 millisecond range or faster, the limb loads associated with the non-linear forces appear as slowly drifting errors to the loop, and are cancelled out by its error-suppressing response. Thus, by using high bandwidth servo systems, we can force each joint to be a position control system with essentially no error. The linear errors between commanded

and actual angles are quite predictable by linear system theory, and treating the non-linear forces as error sources with a correlated statistical distribution allows one to design a system that rejects such slow mechanical (e.g., non-linear physics) contributions by as much as 80 dB (for a sophisticated compensation design).

The only difficulty with this promising approach is that implementation may pose some problems. For full implementation of a 1 kHz bandwidth servo-loop, one must, of course, have actuators with close to that much bandwidth. This requirement effectively removes from contention all actuator technologies except the D.C. motor technology. But to obtain any reasonable torque from this technology one must gear it down heavily to get into the desired impedance range for coupling power into limb locomotion. Gear trains have ratio irregularities, slip-stick binding, hysteresis, and worst of all, backlash. A high gain servo system has almost no chance of being guaranteed-stable with these sorts of response imperfections in the loop.

However, the motor itself behaves almost linearly, with a small amount of ω^2 drag in the bearings. Better yet, the motor is a linear electro-mechanical device, and as such its speed can be deduced from its instantaneous electrical parameters (voltage and current), via linear differential equations. This simplifies the task of the servo-system designer, for by using measured electrical variables he can design a guaranteed-stable velocity feedback loop. This sort of loop also has the benefit of 1 less integration, permitting the use of so-called integral feedback to square the rejection ratio of relatively slow non-linear noise!

Thus, one implements a high gain velocity servo-loop containing only the motor, thereby eliminating the non-linear problems of impedance-shifting. This loop is then included in an outer loop for position control,

which maintains the high bandwidth with much lower gain. The outer loop includes the gear train and the rotary encoder at the joint, and can readily be designed to remain stable and give absolutely accurate position control (due to encoder information). Both inner and outer loops can be implemented as sampled data systems, allowing the control feedback processing for a number of joints to share the same digital hardware.

Thus we have sketched the lowest level of the control system that effectively forces the physical vehicle to do exactly as it is commanded by the higher levels of the system. This lowest level of control may therefore be thought of as the analog of the cerebellum in mammalian limbed locomotion control.

It must be recalled, however, that if the motions demanded of this system require torques that cannot be actuator-generated or structurally withstood, then the actual joint angle cannot track the commanded angle, and a monitoring system will therefore be necessary to check for acceptably low servo error and alert higher control levels when error exceeds its allowable range. This even can only happen when there is an accident or emergency, since the next higher control level is selecting among stored limb cycle trajectories and feeding sequential position instructions to the joints. These trajectories should be the result of motion physics optimizations, and would not normally require excessive torques. Since the offline simulation did include all non-linear inertial terms, the stored trajectories include all of the motion torques required.

This spooling control level can be split into two pieces, the trajectory spooler which delivers data streams to the joints, and the motion generator which orders and synchronizes motion. With the inclusion of "hold it" commands of the appropriate durations, the

motion generator is capable of specifying and synchronizing vehicle trajectory segments according to the force information sensed in the limbs. This allows compensation for the quasi-random location of the water surface, by allowing the limb motion to continue or halt prematurely, so that the water surface is properly positioned (relative to limb feet) to provide adequate buoyant and thrust forces.

Thus, at the motion generator level, short motion intervals are chosen and synchronized according to the instantaneous limb load situation to adapt vehicle motion to the changing water surface. Besides being able to speed up or slow down the motion trajectories, the motion generator can also pick deeper or shallower foot immersion paths to allow for water level fluctuations during a limb cycle. It must therefore include a fading operator that allows it to shift from deeper to shallower limb cycles without requiring abrupt changes in limb velocity and excessive joint torques. By use of such capabilities, the motion generator can use stored optimal limb cycles in the trajectory spooler to produce motions of variable velocity, turning radius, and upwave slope motions, employing the speed-up/slow-down operator. Synchronization and decisions at this level are all performed by digital calculations and tabular look-up. These multi-dimensional tables give both decision thresholds and speed-up values. As such, the motion generator is also more of a special purpose lookup machine than a computer, although it does some digital processing of force and speed information. It may be thought of as akin to the mammalian motor cortex.

At the level above the motion generator sits the motion-specifying decision processor, which is the first true computer in this control system. Since decisions on course routing must be made on the scale of seconds rather than milliseconds, the lower control

levels have reduced the computer decision load to the point that a fast minicomputer can adjust the vehicle course by specifying the motions. Motions are specified to execute a course, which is actually calculated from the time-varying wave height map generated by the ocular processors. This processor is the mammalian cerebrum analog.

The eyes of the WLRNV will most probably be scanning laser range-finders, since range data on the ocean surface is most vital. This can be postprocessed by already demonstrated techniques to yield a height map of the scanned points. Since the wave height distribution is mostly a low spatial wave number phenomenon, a relatively sparse scan that represents from 10^3 to 10^4 points on the ocean should be interpolatable to a reasonable model of the ocean surface in front of the vehicle. The surface-mapping optical range-finder must include threshold detectors on its detector to ascertain if range points came from froth or a singularity, thereby allowing the surface reconstruction in the ocular processor to discard that point as being informative about general wave shape, while tagging it as representing uncertain footing. The decision processor can then use such data to avoid getting its feet caught in wavecrests. Froth/singularity detection can also be used as a routine check for safe footing during the recycle time of each limb. Thus, some of the scan points will be devoted to confirming reliable footing around the nominal foot touchdown area. With electro-optic beam scanning, it is straightforward to direct a scanning range-finder at over 10^4 points in the time it takes to divert a foot from uncertain footing to a clear area.

The data processing required to locate good footing once the nominal footing area is determined to be unsound is a significant problem. To solve it will require a third computer which uses all available scan data to model the geometry and velocity of the local

ocean surface. Thus, the vehicle's eye would be kept in use at all times, but only a fraction of the time would be devoted to detecting froth/singularity patterns in the nearby water. Since the vehicle and the patterns maintain constant relative velocities for periods of the order of several seconds, much of the data acquisition capability of the eyes may be devoted to crest modelling in a throw-out-data-as-you-go fashion, thereby requiring relatively very little computation to model slow-moving wavecrest systems in a Kalmanesque fashion.

Once this "filter" has locked onto the position of a new wavecrest, it can track the wave system along with the other systems being modelled. By being in possession of the time-varying position of all wavecrest systems, a very simple gradient procedure can be implemented to optimize the safety of limb footing about the nominalized touchdown position. Thus, the WLRNV derives all the local benefits of vision while using only two minicomputer-scale processors executing algorithms less complicated than those already developed and demonstrated for similar problems (e.g., block counting and stacking involving hidden surface recognition). Both processors operate by getting incremental information from each scan and throwing the data points away for each scan after they have been filtered into the vehicle's data base of the properties of the surrounding ocean. The first processor establishes and maintains a surface map, and the second one a crest pattern map, but neither need provide more than 1 mapping per second. Since the rate of data influx is far below video rates, we can therefore use conventional processors to obtain video-level performance while avoiding the data bulk and rate problems of video processing (by using appropriate filtering techniques).

The eye, its dedicated postprocessor, and two digital processor-evaluators comprise a major arm of the

control service hardware. The other arm may or may not need to be a computer, and is the emergency situations processor, which is used to constantly monitor all sensory signals for an emergency condition. This is a demanding operation involving up to megasamples per second of sensory data, though much of this may be pre-processed to reduce bandwidth requirements. If an emergency status is detected, associated sensors must be checked to ascertain if the sensor associated with that status has faulted, rather than a true emergency having occurred. If this is not the case, this processor must call up a decoding tree with that sensor's address and the current mode of vehicle operation to diagnose the emergency. Such diagnoses may then be passed to the decision processor via interrupt, so that it may correct the emergency situation without having to resort to excessive analysis. With proper design, an emergency response system may be perfected to deal effectively with failures in less than 1 limb cycle, so that any local problem need not propagate.

There is little doubt that the processing required in such a hierarchal WLRNV control system is tractable even at present, and will become exponentially easier with time, as the associated technologies advance. The fundamental practicality question at the present is then one of memory. To store 100 types of limb cycles for 20-30 vehicle joints on a 1 millisecond time grid requires perhaps 10^6 words of storage for the trajectory generators alone!

This size scale is encountered throughout the remainder of the WLRNV control problem. The motion generator might easily use 10^6 words of tables to decide what must be done on the basis of reduced force data and instructions from the motion-specifying decision processor. The decision processor could definitely use $\sim 10^6$ words for program storage (as could the emergency processor) to decode all eventualities

CONTROL DATA-PROCESSING NEEDS

Sensory Data (~12 Bit Words)

Proprioceptive: 24-32 actuators with 4 sensors each at 1 KHz max. sample rates, or 128,000 words/sec, max.

Limb sensing: 8 limbs with 30 signals @ 200 Hz max. samples rates, or 48,000 words/sec, max.

Other non-ocular: approximately 200 signals max. at 200 Hz max. sample rates, or 40,000 words/sec, max.

Ocular: approximately 10^2 point scan 50 times per second gives X,Y,Z values accurate to >1 word (assume 2), or 30,000 words/sec, max.

Total influx: $\leq$ 250,000 12 bit words/sec, or 3 megabaud data input rates.

Motor Data

Signals to 24-32 actuators at 1 KHz rates

Torque control @ 12 bits

Control of $\leq$ 10 clutches @ 1 bit

Total outflux: $\leq$ 0.75 megabaud.

Course Control

Ocean surface moves with characteristic times of $\geq$ 0.1 second, for all surface phenomena larger than 1 foot.

For course decisions, one is only interested in the nearest 100 feet of surface in a path cone of about $\pm 45^\circ$.

Area $\approx$ 8,000 ft² of 8,000 data points on the 1 ft characteristic grid of surface phenomena.

Total required information flux to describe this surface is then about 80,000 12 bit words per second, maximum.

Since the surface shapes are heavily correlated in space and time, and the vehicle is only interested in those parts of the area that look promising, 30,000 12 bits words/sec is probably quite adequate and 80,000 12 bit words/sec is definitely more than sufficient to map/utilize the changing ocean surface as a traction medium for WLRNV traversal.

and digest the requirements of a complex mission. In short, although control could definitely be done with about 20,000 integrated circuits (2,000 per processor and 500 per dedicated processor using 1976 circuits), it would have to be accompanied by at least 5,000 16K memory chips to provide the several millions of words of requisite data and program storage.

In any case, we may be confident that complex vehicular control can be done in under 30,000 1976-level integrated circuits. In specially heat-sunk hermetic packages, each such circuit occupies about .005 cubic inches, and the entire assembly could fit in a package 6" x 6" x 6", employing nothing but present circuitry and packaging technologies. As time passes, more and more of the required WLRNV processing functions will be ever more highly integrated, and specialty packaging technology will be developed for assemblies requiring chip counts of 10^4 - 10^5 , as inevitable consequences of advancing semiconductor technology. These developments will combine, in addition to the continuing factor-of-2 annual gain in device density which the semiconductor industry has maintained for the two past decades to make WLRNV control a most straightforward subsystem to implement on mass deployment timescales.

STAGED SOLUTION OF THE WLRNV PROBLEM

The entire problem of the WLRNV concept can be summed up as: "Novel vehicular technologies synthesized from available component technologies still face substantial problems in the integration of components and subsystems to the vehicular level." WLRNV development will encounter problems never seen in any previous class of vehicle. It is impossible to predict a priori what all of these problems and their solutions will be, even though preliminary modelling has produced the general result that they can be built and successfully operated.

We have already shown what can be done with 1976 components and technological capabilities. However, there is a qualitative difference between a collection of parts that can handle the required loads/powers and a functioning WLRNV. Creating a jet fighter or a formula racecar (vehicles of comparably challenging technology) out of whole catalogs of parts is called "knowhow," which, in its essence, is having sufficient mastery of the relevant technologies to successfully design around the unexpected problems of implementing that vehicular scheme. How to put it together, how to pick the right parts for the job, and how to get it working are the basic questions of the WLRNV concept, and will remain such until at least one such vehicle is working well.

Consider the scope of the problem. We want a vehicle with a very large number of precisely moving parts, we assert the feasibility of a complicated and never-before-done mode of locomotion (walking on water), and we require the control to be completely computerized for a RNV scheme; the final vehicle is comparably complicated as a spacecraft. This does not mean that it must be anywhere near as expensive as space hardware, since required reliability at all points in the development cycle can be reduced drastically and sufficiently many WLRNVs will be produced to descend a learning curve much

further, relative to spacecraft. (Even with the relatively few spacecraft manufactured NASA, observed an 80% learning curve descent rate).

Assuming that one makes enough WLRNVs (more than 10^4 and perhaps more than 10^5), production can be streamlined and optimized, and mass production and quality control techniques perfected; the 'fleet price' for even this complex a vehicle could thereby drop well below $\$10^5$ in the distinctly foreseeable future (e.g., below the cost of a well-instrumented and structurally advanced private plane) and eventually below the $\$10^4$ level in the more distant future. But complexity, more than putting a lower limit on the pricetag, raises major feasibility issues. How can a technology as novel and complex as that of the WLRNV be developed without paying costs on the National program scale and spending 1-2 decades of time, as did NASA?

The important difference between the WLRNVs and spacecraft is that even the most crude and least reliable WLRNVs will work! This fact allows all of the problems in the development stage of these vehicles to be decoupled, and for parallel development to occur. For instance, since a crude, overweight structure can still walk (if its feet have enough buoyancy), those who are optimizing and improving power plant technologies for WLRNVs need not worry about the structure problems when developing and testing their ideas; those working on clutch-spring motion enhancers can operate independently of those doing improved hydrodynamic foot design; the people who improve and specialize actuators can ignore the problems of those who improve control and enhance control reliability. In short, at any instant the best state-of-the-art in each of the many vehicle subsystems can be crudely integrated to get a state-of-the-art vehicle that will locomote for subsystem testing, as well as for overall vehicle evaluation. Such a situation contrasts sharply

with the problems encountered in space technology, where valid tests could be performed only after each subsystem design took all the others into account, and a multi-million dollar testbed (booster, satellite, etc.) was built and flown repeatedly. Building and testing a new generation of development-phase WLRNVs should cost on the order of 300-500 K\$, giving a vehicle development program at least three orders-of-magnitude cheaper than any comparable one in the space program. In other words, although they are complex, the WLRNVs are of relatively small size and use reasonably mundane components (as compared to plasma guns, klystrons, ultra-refractory materials, etc.); each generation of these machines in a development program will be relatively inexpensive.

Though the development of WLRNV hardware may not be costly, there is an irreducible minimum of qualified engineering personnel required, who contribute the bulk of the cost of a new vehicle generation. Research-level WLRNVs may cost very little to design, simply because they are not exhaustively engineered. Military vehicles mission-qualified WLRNVs, in contrast, are of a different class, since their design must insure, for instance, that no structural component fails from stress corrosion, no bearing receives excessive wear in service, no single electronic failure cripples the vehicle, etc. This sort of exhaustive engineering, which attempts to eliminate every foreseeable failure mode, takes on the order of thousands of man-years of design effort, while the research-stage vehicles (one which could afford to be somewhat less than totally reliable) are likely to require as few as 10 man-years. In other words, a highly complicated mechanical and electronic object like a WLRNV is relatively inexpensive to demonstrate and test in the lab, while comparatively costly to design for mil-spec level deployment. A decade of developmental research is thus likely to be a very

small fraction of the cost of a half decade of immediately pre-deployment design. Since developmental research also serves to define the boundaries of available performance, it will be possible to examine how potentially useful the WLRNV concept is to the Navy at each stage in such research.

A substantial in-depth research effort is in order not only to resolve currently unsolved problems, and gain the experience and knowhow to circumvent subsequent design problems, but also to provide a series of performance and potential applications benchmarks to Navy sponsors that indicate whether the then-current level of WLRNV technology is worth the relative heavy costs of militarized design and deployment. This staged philosophy of vehicular research is also unquestionably important at the beginning of an WLRNV project. It is possible to perceive this document purely as a semi-quantitative discussion of blue-sky possibilities (though this is hardly justified in view of the fact that every component needed for a moderately high performance WLRNV can even now be obtained off-the-shelf.) Nonetheless, the gap between a paper proposal and a real WLRNV undergoing performance tests is a qualitative one. Legitimate Navy interest in WLRNVs will grow only as the hardware achievability is demonstrated to ever greater extents over time. Feasibility-demonstration is thus necessary to establish the distinction between a realistic concept that can be practically implemented in hardware, and an interesting blue-sky idea that happens to call upon only extant component technologies.

Fortunately, the decoupling discussed earlier with respect to vehicle research and design functions very well in the earliest phases of research, and perhaps provides the only reliable way that the Navy can ascertain that its sponsorship of such work is worthwhile. The earliest WLRNVs can operate satisfactorily

FIGURE 47

FIRST WLRNV DESIGNS

- Provide experimental testbed for design equations.
- Provide experience for established parametric values.
- Provide a test for motion—histories derived from simulation.
- Act as a modifiable “skeleton”, as more and more servipede capabilities are added.
- Provide, at every stage, the most easily feasible demonstration of the capabilities of the LLL Limbed Locomotion Project

- An upward-compatible family of machines, each built as a logical extension of the last:

Mod 0	Version 1	Mechanically viable limb
	Version 2	Servoed + Driven limb
Mod 1	Version 1	First vehicle, no on-board electronics or sensors
	Version 2	Adds proprioceptive sensors
	Version 3	Add metric space control
	Version 4	Fixed gaits available
Mod 2	Version 1	Power plant on-board
	Version 2	Power electronics on-board
	Version 3	Digital buffers + servosystem on-board
Mod 3	Version 1	On-board limb-control computer
	Version 2	Everything but decisions done on-board
Mod 4		Eye subsystem integrated
Mod 5		Weight reduction effort in structures gear boxes electronic packaging

as basic feasibility demonstration systems without motion enhancers, without eyes, and without emergency processing and monitoring. At the very lowest level they can operate (though probably not in a militarily useful fashion) without decision processors, without most of their sensors, and even without an on-board power plant. This is the very essence of the proposed "staged solution" approach to WLRNVs, that feasibility and capabilities can be demonstrated long before essentially any of the more detailed, time-consuming and costly sub-investigations are required.

Such an approach gives a staged sequence of proof by WLRNV developers that a given vehicle performance level can be accomplished, with a corresponding sequence of approvals by project sponsors. The sponsor's confidence in the project can be validated by capability demonstrations at the completion of each performance level, with an exposure at each level which is only commensurate with the performance gain to be realized. The first such level consists of a collection of driven limbs associated with a vehicle body, a construction with all limb power and control cabled into it, a rudimentary vehicle that demonstrates only that purposeful walking on water can be accomplished. Addition of an integrated, on-board power plant will then allow meaningful speed and motion-efficiency trials. After this level of performance is demonstrated, more sophisticated control can be developed and integrated on-board to demonstrate vehicle versatility and maneuverability.

By this performance level, the basic feasibility of WLRNVs should have been sufficiently established to justify the substantial costs of electronic packaging of the vehicle control system by aerospace methods. The next developmental stage will then be able to demonstrate rudimentary but completely on-board vehicle control, allowing the WLRNV to operate completely

stand-alone for the first time. This level of feasibility demonstration of the RNV concept should afford ample justification to fund the mechanical design effort required to add motion enhancers and optimized feet to the vehicle, and to start shedding excess structure weight, so as to further improve overall vehicle performance. At the end of this stage, feasibility testing should provide valid inputs for relatively detailed predictions of range and durability for various types of missions. Continuing development would then permit a reasonable determination to be made as to when and if pre-deployment militarized design work should be commenced.

This document has outlined the WLRNV concept and sketched its potential performance based on existing technology, and has also suggested a reasonable path on which to proceed the present juncture to a military WLRNV. With the proposed staged development philosophy, no expenses are incurred for research and development than cannot be justified on the basis of results from the immediately preceding stage, and subsequent demonstrations of ever-increasing concept feasibility/viability/performance are only funded when results of the previous stage meet previously determined benchmarks. The stages themselves allow a flexible Project timescale, because they are completed when the specified new layer of feasibility/capability is demonstrated. Phases may obviously be re-sequenced so as to maximally exploit particularly rapid progress in technologies especially pertinent to one phase but not others.

Even so, the LLL Project expects mass-production and deployment of Smart, Water-traversing Limbed Remotely Navigated Vehicles to be feasible in the late 1980s in a surprise-free scenario, if adequately funded development aimed at this goal is commenced in the near future.

SUMMARY

In this document we have described what Smart Water-traversing Limbed Remotely Navigated Vehicles (SWLRNVs) are, what some of their uses may be, how they move, and scoped out their expected locomotive performance levels. The rationale for such systems, the required technologies, the functional components, and the problems that will be involved in the hardware implementation of these vehicles were also discussed. The first major segment of the paper served to project how useful and important the WLRNVs could be to Navy missions. The conclusion of that segment can be summarized: too potentially valuable in naval warfare to ignore the concept, or to delay its advanced development and evaluation.

The second major section (following a brief description of the major sub-systems of a WLRNV) contained a discussion of the motions that such a vehicle would have to execute in order to perform the interesting missions sketched in the first segment. This discussion included the substantial problem of vehicle physics simulation, and outlined how one would use such simulation plus optimization codes to calculate the "best" motions for the vehicle to follow in order to achieve efficient locomotion. Some discussion was also included of how one could then use such motion-simulating programs to get a vehicle design optimized by computer to match its locomotion and mission-specific goals. All of these formulations were intended to be performed by computer offline before a vehicle was constructed.

The third segment began with a brief description of the separable problems of WLRNV technology and continued to discuss the power sub-system that supplies other vehicle components with the wherewithall to execute active functions. This sub-system was viewed as taking a definitely supportive role, and was probed to achieve

a model in which it could be determined if this sub-system would impose limits on vehicle performance, using current power generation and handling technologies. The results were encouraging, in that they showed that current technologies provided power-to-weight ratios that were adequate to keep the power sub-system components at a reasonable fraction of total vehicle weight (i.e., not dominating the vehicle weight budget), while still providing the power levels that would be required by the best of the present locomotion technologies.

The next segment completed this line of power-oriented inquiry by developing models for the locomotive functions of the WLRNV. Using worst-case limits whenever possible, a number of simple models were formed for vehicle performance. WLRNVs were found to operate at respectably high velocities when operating with currently available power systems technologies (speed estimates for various WLRNV development levels ranging from 6 knots to 132 knots were derived, postulating only available technologies). All of these models were vehicle velocity-oriented, because range models would have required some forecast of power loss rates during motion; such knowledge that will not be available until after successful detailed vehicle simulation has been accomplished.

Accelerations were not at issue in the locomotive section either, because in purely locomotive models the available peak vehicular acceleration is selectable as a variable in the mechanical design. In the subsequent section, the structural issues of the WLRNV were discussed, structure requirements were modelled, and a number of calculations presented. Some of the first of these brought out that the price of high acceleration capabilities is paid in structure weight, which may be necessary but is never beneficial per se. This section also identified some structural limits on velocity, which did not so much limit the range of achievable velocities as they selected the limb lengths

that were necessary to achieve that velocity without breaking. Between the structure models and the locomotive models, it was possible to define a nominal design procedure yielding the physical specifications of a vehicle from the requisite motion and technological parameters.

Two technological parameters were shown to control performance: the specific power and the specific strength. Present technology (which is certainly not specialized to meet WLRNV needs), is capable of delivering 30 knots of vehicle speed and 3 g's of acceleration without optimism in the parameters that express the state-of-the-art in vehicle engineering. There is little doubt that an effort to adapt existing devices to WLRNV needs could produce at least an order of magnitude improvement in available specific power, and that the employment of the new fiber-composite materials technology could deliver an order of magnitude improvement in specific strength. This implies that we could reasonably expect 100+ knot vehicle speeds at over 10 g peak vehicle accelerations in such vehicles deployed a decade hence, along with improved reliability.

The next segment was essentially the only one that included projections at all beyond existing technology. Its topic was the control and guidance of this collection of limbs to get motion on a highly random and time-varying surface (such as the ocean). This is admittedly a problem which has not been addressed to a substantial extent previously. However, it is not significantly more difficult than the other complex control and instrumentation problems that have been solved since the Second World War. A scheme was presented that used only current control-theoretic building blocks and control/signal-processing techniques to implement the required control sub-system.

In short, no discoveries will be necessary to build a control sub-system, but rather a pioneering effort to integrate a number of previously uncombined

techniques. Considerable effort will have to be spent on a demanding electronics miniaturization effort to realize the full potential of this class of naval vehicle, since the weight specifications for the required multi-computer control system are stringent. Like other phases of the WLRNV hardware problem, this miniaturization technology exists and has been pioneered in aerospace applications.

The last segment concluded the considerations of this document with the inquiring: If one wanted a WLRNV technology, how would one go about establishing one? The problem thus posed turned out to be a rather pleasant one in at least its early portions, since passing through each of the early generations of WLRNVs would be relatively inexpensive ($\leq \$10^6$ per generation) moreover, the various problems involved could be separated and sequenced to generate a staged program wherein the sponsor would be able to evaluate at each stage the progress that had been made, and the prospects for the next stage. This situation was shown to extend down to the beginning of the effort, wherein feasibility could be validated at increasingly more interesting levels, without ever requiring the sponsor to make a relatively large, high risk commitment to the WLRNV scheme.

Thus, what initially appears to be a quite advanced scheme may be reasonably commenced as soon as basic technological feasibility can be shown to exist. That showing has been possible only in the last few years, due to the recent advanced in solid-state technology that makes control of these vehicles possible. Since a good part of the attractiveness of the WLRNV stems from the fact that America could establish a substantial lead in what might prove to be a very militarily effective naval platform, it would seem reasonable to begin the inexpensive, basic feasibility validation effort at the present time. Such a step would generate the real possibility of deploying usefully

large numbers of personnel-conserving, highly economical,
militarily important WLRNVs supporting our naval forces
late in this century.

FIGURE 48

**SMART WATER-TRAVERSING LIMBED REMOTELY NAVIGATED VEHICLES
ARE INTERESTING BECAUSE**

- They are the only viable small vehicle usable in high sea states.
- They are small and work on the ocean surface, they are much harder to detect than any other conventional craft.
- They have no human pilot, are capable of high-g evasion, and will continue to operate after direct hits that would have crippled a human crew.
- They have the prospect of providing surface platforms possessing unprecedented speed and maneuverability.
- Unlike manned information-gathering craft, they impose almost no penalty for missions in excess of 10 hours (no need to rotate shifts of crewmen, no food/lavatory requirements, etc.) and, in their "loitering mode", waterbugs could perhaps perform their missions for days to weeks.
- They are cheap enough to use for one-way missions.
- They are mass-producible.
- They are inherently reliable — almost impossible to sink and, in the event of in-use failure, the vehicle will not be destroyed.
- They maximally exploit continuing technological asymmetries between the U.S. and its potential opponents.
- They are economically highly cost-effective for a wide spectrum of Navy missions.

BIBLIOGRAPHY

Anonymous, "The Walking Truck," Machine Design 41, 32-34 (1969).

J. R. Ashley, A. Pugh, "Logical Design of Control Systems for Sequential Mechanisms," Int'l Journal of Production Research 6, No. 4, 291-302 (1968).

R. A. Baldwin, "Application of Digital Fly-By-Wire to Fighter/Attack Aircraft," Presented at 17th Mtg of the Guidance & Control Panel of AGARD, Geilo, Norway, 9/24-26/73, Publ. for NATO by Techn. Editing & Repro. Ltd, England (1974).

M. Bekker, "Theory of Land Locomotion," University of Michigan Press, Ann Arbor (1962).

R. Bolles, R. Paul, "The Use of Sensory Feedback in a Programmable Assembly System," Stanford Artificial Intelligence Lab. Memo AIM-220; Computer Science Department Report No. CS-396 (1973).

B. Bresler, F. R. Berry, "Energy and Power in the Leg During Normal Level Walking," Institute of Engineering Research Report Series II, Issue 15, University of California at Berkeley (1951).

C. K. Chow, D. H. Jacobson, "Studies of Human Locomotion via Optimal Programming," Harvard University Division of Engineering and Applied Physics Report No. 617 (1970).

J. E. Cotes, F. Meade, "The Energy Expenditure and Mechanical Energy Demand in Walking," Ergonomics 3, No. 2, 97-119 (1960).

G. Y. Eastman, "The Heat Pipe," Scientific American, 38-46, May 1968.

R. E. Fikes, P. E. Hart, N. J. Nilsson, "Learning and Executing Generalized Robot Plans," Artificial Intelligence 3, 251-288 (1972).

- P. M. Fitts, "The Information Capacity of the Human Motor System in Controlling the Amplitude of Movement," *Journal of Experimental Psychology* 47, No. 6, 381-391 (1954).
- A. A. Frank, "An Approach to the Dynamic Analysis and Synthesis of Biped Locomotion Machines," *Med. & Biol. Engineering* 8, 465-476 (1970).
- A. A. Frank, "Automatic Control Systems for Legged Locomotion Machines," University of Southern California USCEE Report No. 273 (1968).
- A. A. Frank, R. B. McGhee, "Some Considerations Relating to the Design of Autopilots for Legged Vehicles," *Journal of Terramechanics* 6, No. 1, 23-35 (1969).
- A. Freedy, I. C. Hull, et al., "A Computer-Based Learning System for Remote Manipulator Control," *IEEE Transactions SMC-1*, No. 4, 356-363 (1971).
- C. Gans, "How Snakes Move," *Scientific American*, 82-96, June 1970.
- K. C. Garner, "Evaluation of Human Operator Coupled Dynamic Systems," *Ergonomics* 10, No. 2, 125-138 (1967).
- J. Gray, How Animals Move, Cambridge University Press, London (1959).
- M. Hildebrand, "Symmetrical Gaits of Horses," *Science* 150, 701-708 (1965).
- J. P. Hong, L. Liporace, "A Terrain and Object Model Derived From Range Data and Useful to a Robot," *Proc. 6th Annual Hawaii Int'l Conf. on Systems Sciences*, Publ. by Western Periodicals, No. Hwd, CA, 469 (1973).
- W. W. Hooker, "A Set of r Dynamical Attitude Equations for an Arbitrary n -Body Satellite Having r Rotational Degrees of Freedom," *AIAA Journal*, 8, No. 7, 1205-07 (1970).
- A. K. Jain, "A Study of Regularly Realizable Gait Matrices," Master's Thesis, Ohio State University at Columbus (1970).

E. G. Johnsen, "Humanoids: The New Remote Systems Technology," Nuclear News 12, 37-48, January 1969.

E. G. Johnsen, W. R. Corliss, "Teleoperators and Human Augmentation," NASA SP-5047 (An AEC-NASA Technology Survey) (1967).

W. L. Kilmer, W. S. McCulloch, J. Blum, "A Model of the Vertebrate Central Command System," International Journal of Man-Machine Studies 1, 279-309 (1969).

P. D. Lawrence, W. C. Lin, "Statistical Decision Making in the Real-Time Control of an Arm Aid for the Disabled," IEEE Transactions on Systems, Man, and Cybernetics SMC-2, No. 1, 35-42 (1970).

A. Liegeois, M. Renaud, "Mathematical Model of Articulated Mechanical Systems in Relation to Automatic Control of Their Movements," C.R. Acad. Sc. Paris, t. 278, No. 18, Series B, 799-801, publ. by Acad. of Sciences, Paris (1974).

M. E. Kahn, B. Roth, "The Near-Minimum-Time Control of Open-Loop Articulated Kinematic Chains," Transactions of the ASME, 3, 164-172 (1971).

K. Matsushima, H. Katagiri, "A Fundamental Study on the Simple Eyes of Industrial Robots Applying the Correlation Technique," Bulletin Tokyo Inst. of Technology (Japan), No. 107, 99-112 (1971).

R. B. McGhee, "Abstract Quadruped Locomotion Automata," University of Southern California USCEE Report No. 130 (1965).

R. B. McGhee, "Finite State Control of Quadruped Locomotion," Simulation 9, 135-140 (1967).

R. B. McGhee, "Improved Techniques for the Control of Remote Devices," Final Scientific Report Grant No. AFOSR-70-1901, USAF Off. of Scientific Research (1971).

R. B. McGhee, "A Mathematical Theory for Legged Locomotion Systems," Proc. of the 1970 Midwest Symposium on Circuit Theory, 1.3.1-1.3.14.

R. B. McGhee, A. K. Jain, "Some Properties of Regularly Realizable Gait Matrices," Mathematical Biosciences 13, Nos. 1 & 2, (1972).

R. S. Mosher, "Exploring the Potential of a Quadruped," SAE Paper No. 690191, 836-843 (1969).

D. E. Okhotsimskiy, et al., "Algorithms for Controlling a Walking Apparatus Capable of Overcoming Obstacles," National Technical Information Service JPRS 62018 (1974).

D. Y. Okhotsimskiy, A. K. Platonov, G. K. Borovin, I. I. Karpov, "Digital Computer Simulation of the Motion of a Stepping Vehicle," Engineering Cybernetics 10 (3) 410-419 (1972).

D. Y. Okhotsimskiy, A. K. Platonov, et al., "Computer Simulation of the Movement of a Walking Machine," Institute of Applied Mathematics, USSR Academy of Sciences, Moscow (1971), National Technical Information Services JPRS 57018 .

D. Y. Okhotsimskiy, A. K. Platonov, "Control Algorithms for a Stepping Vehicle Capable of Overcoming Obstacles," Rept'd at 3rd International Conf. on Artificial Intelligence, Stanford, CA (1973).

R. Tomovic, R. B. McGhee, "A Finite State Approach to the Synthesis of Bioengineering Control Systems," IEEE Trans. on Human Factors in Electronics, HFE-7, No. 2, 65-69 (1966).

R. Tomovic, "Robots for Exploration of Hostile Environments," Proc. 4th IFAC. Belograd, Yugoslavia (1971).

M. Vukobratovic, D. Hristic, V. Ciric, M. Zecevic, "Analysis of Energy Demand Distribution Within Anthropomorphic Systems," Journal of Dynamic Systems, Measurement & Control Transactions of the ASME 95, No. 4, 402-443 (1973) .

M. Vukobratovic, J. Stepanenko, "Mathematic Models of General Anthropomorphic Systems," Mathematical Biosciences 17, 191-242 (1973).

M. Vukobratovic, J. Stepanenko, "On the Stability of Anthropomorphic Systems," Mathematical Biosciences 15, 1-37 (1972).

R. Tomovic, R. Bellman, "A Systems Approach to Muscle Control," Mathematical Biosciences 8, 265-277 (1970).

M. Vukobratovic, D. Hristic, Z. Stojiljkovic, "Development of Active Anthropomorphic Exoskeletons," Medical & Biological Engineering 12, No. 1, 66 (1974) .

M. Vukobratovic, A. A. Frank, D. Juricic, "On the Stability of Biped Locomotion," IEEE Trans. on Bio-Medical Engineering BME-17, No. 1 25-36 (1970).

V. A. Tucker, "The Energetics of Bird Flight," Scientific American, 70-78, May 1969.

D. E. Whitney, W. J. Book, P. M. Lynch, "Design and Control Considerations for Industrial and Space Manipulators," Proc. 1974 Joint Automatic Control Conf., Austin, Texas, June 18-23, 1974, 591-598, Publ. by Amer. Inst. of Chemical Engineers, NY (1974) .

D. M. Wilson, "Insect Walking," Annual Review of Entomology 11, 103-133 (1966).

D. E. Whitney, "Resolved Motion Rate Control of Manipulators and Human Prostheses," IEEE Trans on Man-Machine Systems MMS-10, 2, 47-53 (1969).

"Comparison of Materials," Materials Engineering 2-75, 18-58 (1975).

"First Annual Report for the Development of Multi-moded Remote Manipulator System," C-3790, The Charles Stark Draper Lab., Div. of MIT, Cambridge, Massachusetts (1972).

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