department of electrical engineering

DEFORMATION OF LIQUID DROPLETS AND GAS BUBBLES IN ELECTRIC FIELDS

Research Report #1

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Target Research and Fabrication Group
Laser Fusion Program
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DEFORMATION OF LIQUID DROPLETS AND
GAS BUBBLES IN ELECTRIC FIELDS

I. Introduction

When liquid droplets or gas bubbles in liquid dielectrics are subjected to electric fields, ellipsoidal elongation in the direction of the electric field occurs. The direction of this elongation is independent of the relative dielectric constants or electrical conductivities of the two media; the major axis of the ellipsoidally deformed bubble or droplet is always parallel to the applied electric field. See Figure 1.

Garton and Krasucki [1] solved the theoretical problem of the deformation of droplets and bubbles by assuming that the deformed spheres become prolate spheroids. Their solution was not exact, however they summarized calculations which provided justification of their assumption of ellipsoidal deformation to a high degree of accuracy. They derived relationships between deformation and applied electric field which are implicit in terms of the ratio γ ,

$$\gamma = \frac{a}{b} \quad (1)$$

where a is the semi-major axis and b is the semi-minor axis. Though inconvenient to use, these expressions may be employed to calculate γ as a function of the electric field, for a given set of droplet or bubble parameters. Their theory compared favorably to experiment. Garton and Krasucki also derived expressions for the stability of droplets and bubbles to electric field induced elongation. They found that only for permittivity ratios in excess of about 20, would droplets or bubbles exhibit a threshold of instability. Because the case of a perfectly conducting droplet is equivalent to the limiting case of $\epsilon_2/\epsilon_1 \rightarrow \infty$, an

Figure 1. Elongated Bubble in Dielectric
Liquid or Liquid Droplet in
Uniform Applied Electric Field

Instability threshold is exhibited by perfectly conducting droplets in an electric field.

Of interest in the present study is the behavior of dielectric droplets and gas bubbles in electrostatic levitation, positioning, and control devices for laser-fusion target applications. Various methods of target control and production using electric field forces are presently being investigated at Colorado State University under contract from the Laser-Fusion Target Production Group at Lawrence Livermore Laboratory. Among these are (i) charged droplet dynamic levitation in quadrupolar fields, (ii) polarized (uncharged) droplet dynamic levitation in hexapolar fields, and (iii) static levitation of gas bubbles in non-uniform electric fields. Because of the exacting tolerance requirements for the sphericity of laser fusion targets, there is concern over the electric field induced elongation effect which may exist in some or all of these schemes. These proposed tolerances suggest upper limits on the axis ratio γ [2]:

$$\gamma - 1 < 10^{-3} \quad (2)$$

which may severely limit the maximum allowable electric field in certain electrostatic levitation schemes. Calculations of the deformation of bubbles in cryogenic hydrogen were undertaken by Bliss [3], but only for large deformations up to $\gamma \sim 2$. In this report, these calculations are extended down to very small deformations, that is:

$$10^{-4} < \gamma - 1 < 10^{-1} \quad (3)$$

for droplets of cryogenic liquid hydrogen (H_2) and bubbles in liquid hydrogen at its triple point. Also, the effect of pressure on the

deformation has been explored. Further, approximate explicit analytic expressions for $\gamma(E)$ have been obtained for the limit of small deformations.

II. Theory and Calculations

A. Theory of Garton and Krasucki

Garton and Krasucki [1] used an expression for the balance of surface forces at the droplet/bubble interface to obtain a mathematical expression for the deformed surface. The resulting equation was compared to that of an ellipsoid of revolution and found to be very close. Using the known depolarization and geometrical factors of an ellipsoid, they arrived at expressions which relate the ratio $\gamma = a/b$ to the imposed uniform electric field E . Two important cases were considered: incompressible liquid dielectric droplets, and compressible gas bubbles in dielectric liquids. Garton and Krasucki's results are in cgs units [1], and so for convenience, the equivalent equations in mks units are summarized below.

Incompressible Dielectric Droplet:

$$E_{OZ} = \left\{ \frac{\sigma}{\epsilon_1 R_0} \right\}^{1/2} \left\{ \frac{\epsilon_1}{\epsilon_1 - \epsilon_2} - L_z \right\} \left\{ 2\gamma^3 A \right\}^{1/2}, \quad (4)$$

where

$$L_z = \frac{1}{3\gamma^2} \left[1 + \frac{3}{5}(1-\gamma^{-2}) + \frac{3}{7}(1-\gamma^{-2})^2 + \dots \right], \quad (5)$$

$$A = 2\gamma - 1 - \gamma^{-2}. \quad (6)$$

All quantities are defined in the Nomenclature section. Note that the depolarization factor L_z is approximated by the first few terms of a Taylor Series expansion based on Stratton's expression [4]. The deformation γ depends only on E_{OZ} , the applied uniform electric field, and not on pressure. This is due to the incompressible nature of the droplet.

Compressible Dielectric Bubbles:

$$K_{0z} = \left\{ \frac{\epsilon_1}{\epsilon_1 - \epsilon_2} \right\}^{\frac{1}{2}} \left\{ \frac{\epsilon_1}{\epsilon_1 - \epsilon_2} - 1 \right\} \left\{ \frac{8A^2 B}{3\gamma(\epsilon_1 - \epsilon_2)} \right\}^{\frac{1}{2}} \left\{ \cosh \theta \right\}^{\frac{1}{2}} \quad (7)^*$$

where A and L_z are previously defined and

$$B = 2\epsilon_1 \gamma^3 - \epsilon_2(1 + \gamma^2), \quad (8)$$

$$\theta = \frac{1}{3} \cosh^{-1} \left[\frac{Pr_0}{\sigma} \left\{ \frac{27\gamma^5(\epsilon_1 - \epsilon_2)^3}{2B^3} \right\}^{\frac{1}{2}} \right] \quad (9)^*$$

Usually, it is assumed that the bubble permittivity $\epsilon_2 = \epsilon_0 \approx 8.854 \cdot 10^{-12} \text{ F/m}$, the permittivity of free space. The quantity r_0 is an equivalent radius of the compressible bubble under zero external pressure:

$$r_0 = \left\{ \frac{R_0^3 P}{2\sigma} + R_0^2 \right\}^{\frac{1}{2}}. \quad (10)$$

where R_0 is the actual undeformed bubble radius, P is the external hydrostatic pressure, and σ is the interfacial surface tension. Eq. (7) reflects a dependence of γ on the external pressure P which is found to be significant only for deformations in excess of $\gamma \sim 1.01$.

The results of Carton and Krasucki [1] converted to mks units and summarized above may be supplemented by the relationships for two very important special cases, those of a perfectly conducting incompressible droplet or bubble. The equations may be obtained by letting the ratio of permittivities go to infinity, that is

$$\frac{\epsilon_2}{\epsilon_1} \rightarrow \infty \quad (11)$$

* replace cosh by cos throughout if the argument of $\cosh^{-1}$ in Eq. (9) is less than one.

Incompressible Conducting Droplet:

$$E_{O_z} = \left\{ \frac{\sigma}{\epsilon_1 R_0} \right\}^{\frac{1}{2}} \left\{ L_z \right\} \left\{ 2\gamma^{\frac{1}{3}} A \right\}^{\frac{1}{2}} \quad (12)$$

where L_z and A are defined by Eq. (5) and (6).

Compressible Conducting Bubble:

$$E_{O_z} = \left\{ \frac{\sigma}{\epsilon_1 r_0} \right\}^{\frac{1}{2}} \left\{ L_z \right\} \left\{ \frac{8A^2(1+\gamma^2)}{3\gamma} \right\} \left\{ \cosh \Theta \right\}^{\frac{1}{2}} \quad (13)^*$$

Hardest to envision, the compressible conducting bubble case may apply in a situation where an elastic electrically conducting skin or surface layer forms on a gas bubble due to impurities in the dielectric or chemical reaction at the surface.

B. Small Deformation Limit

Calculations using Eq. (4) and (7) have been performed using a programmable desk-top calculator and the log-log plotted curves of γ versus E_{O_z} , shown in §II.C, indicate that for $1.0 < \gamma < 1.1$,

$$\gamma \approx 1 + \beta D_0 E_{O_z}^2, \quad (14)$$

where $D_0 = 2R_0$ is the droplet diameter and β is a function of the fluid properties including surface tension and dielectric constant. This result prompted an analysis of Eq. (4) and (7) to obtain Taylor series expressions for γ as a function of E_{O_z} . The results are summarized below.

Incompressible Dielectric Bubbles:

$$\gamma \approx 1 + \frac{\epsilon_1 D_0}{16\sigma} \left[\frac{\epsilon_1}{\epsilon_1 - \epsilon_2} - \frac{1}{3} \right]^{-2} E_{O_z}^2 \quad (15)$$

Compressible Dielectric Bubbles:

$$\gamma \approx 1 + \frac{\sqrt{3} \epsilon_1 r_0}{16\sigma} \left[\frac{\epsilon_1}{\epsilon_1 - \epsilon_2} - \frac{1}{3} \right]^{-2} \cosh \theta_0 E_{0z}^2 \quad (16)$$

$$\theta_0 = \frac{1}{3} \cosh^{-1} \left[\frac{3\sqrt{3} P r_0}{4\sigma} \right] \quad (17)$$

For the "high pressure" limit of $P \gg \frac{\sigma}{2R_0}$, Eq. (16) reduces to Eq. (15).

In the next section, these approximations are compared to the exact calculated results in the small deformation limit.

C. Calculated Results

Calculations based upon the Eq. (4) and (7) have been performed for droplets of $n\text{-H}_2$ and for gas bubbles in $n\text{-H}_2$ using the triple point liquid property values summarized below:

$$\begin{aligned} \text{relative dielectric constant: } \epsilon_r &= 1.253, \\ \text{surface tension: } \sigma &= .003 \frac{\text{Nt}}{\text{m}}. \end{aligned}$$

Droplet and bubble sizes from $10 \mu\text{m}$ to 1.0mm have been considered. For the compressible bubble case, external pressures over the range from 10^{-1}Pa (10^{-6}atm.) to 10^6Pa (10atm.) have been considered. Figures 2 and 3 show the strikingly accurate square law dependence of $(\gamma - 1)$ upon E_{0z}^2 for small deformations. The assumed pressure for the compressible droplet results in Figure 3 is $P = 10^5 \text{Pa}$ (1atmos.). Numerical determination of the coefficient β of Eq. (14) compares very favorably with the approximate results of Eq. (15) and (16). The differences are smaller than the uncertainties of the fluid parameters.

Figure 2. Deformation of an Incompressible Droplet of Liquid Hydrogen (H_2) at the Triple Point.

Figure 3. Deformation of a Compressible Bubble in Liquid Hydrogen (H_2) at the Triple Point.

	<u>approximate calculated</u>	<u>exact calculated and read from graph</u>
incompressible case:	$\beta = 1.002 \cdot 10^{-11}$ (Eq. 15)	$\beta = 1.005 \cdot 10^{-11}$ (Fig. 2)
compressible case:	$\beta = 1.06 \cdot 10^{-11}$ (Eq. 16)	$\beta = 1.08 \cdot 10^{-11}$ (Fig. 3)

The equations for compressible bubbles, Eq. (7) - (10), indicate a dependence of γ on external pressure P . This pressure dependence is significant only in the limit of large deformations, $\gamma - 1 > 10^{-2}$. Figure 4 shows a plot of γ versus E_{O_2} for two limiting cases of pressure.

$$P > 10^3 \text{ Pa} (10^{-2} \text{ atm}),$$

$$P < 10 \text{ Pa} (10^{-4} \text{ atm}).$$

The curves indicate that in the limit of the small allowable deformations which are typical for laser-fusion targets, external pressure is not a significant parameter.

III. Conclusion

The results of the theoretical calculations for droplets and bubbles of normal liquid hydrogen can be used to determine the maximum electric field allowed for given limits on deformation. If the deformation is small enough, the approximate expressions, Eq. (15) or Eq. (16), may be used to relate the deformation γ to the electric field E_{O_2} . For droplets less than 1.0 mm in diameter, the deformation is found to be essentially independent of external static pressure unless $\gamma - 1 > 10^{-2}$.

The calculations suggest that certain electrostatic orientation schemes may not be useful in the levitation or positioning of droplets or bubbles which are being formed into laser-fusion targets. The magnitude of the "time-average electric field" experienced by the droplets or bubbles in certain levitation structures, including quadrupole and

Figure 4. Effect of Pressure on Deformation of Bubbles in Liquid Hydrogen (H_2) at the Triple Point.

hexapole electrode systems, is difficult to estimate. Because of the very low surface tension of cryogenic hydrogen and its isotopes, elongation of droplets or bubbles of these liquids is difficult to avoid in any electrostatic orientation device.

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NOMENCLATURE

Λ = dimensionless function of γ , see Eq. (6)

a = semi-major axis of ellipsoid

B = function of γ , ϵ_1 , ϵ_2 , see Eq. (8)

D_0 = undeformed droplet or bubble diameter

E_{0z} = uniform applied electric field

L_z = depolarization factor for ellipsoid

R_0 = deformed droplet or bubble radius

r_0 = radius of compressible bubble under zero hydrostatic pressure

β = coefficient of approximate deformation equation

γ = deformation factor, see Eq. (1)

ϵ_1, ϵ_2 = dielectric permittivities

$\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m

ϵ_r = relative dielectric constant

θ, θ_0 = defined quantities, see Eq. (9) and (17)