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**RADIAL ZONING OF LARGE LMFBRs, PART II:
AN ANALYTICAL APPROACH**

by

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U.S. GOVERNMENT PRINTING OFFICE

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August 1975

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ABSTRACT

The optimum nuclear radial power peaking factor as a function of radial zoning is investigated for large carbide LMFBRs of the 5000 MWt class. Four different fuel volume fractions are considered. The two radial zones are characterized by different plutonium enrichments. The dependence of the optimum power peaking factor on the volume and enrichment ratios of outer to inner core is investigated for static conditions and for burn cycles corresponding to first core loading. A simple phenomenological model is used to determine the functional dependence of the optimum enrichment split on the ratio of inner to outer core volume.

RADIAL ZONING OF LARGE LMFBRs, PART II: AN ANALYTICAL APPROACH

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1. INTRODUCTION

Minimization of the nuclear power peaking factor by means of radial zoning is considered for large LMFBRs of the 5000 MWt class. For these reactors, the optimum radial power peaking factor exhibits a broad minimum as a function of the ratio of outer core to total core volume, but is extremely sensitive to the ratio of outer to inner core enrichments. In this analysis, the dependence of the optimum enrichment split on the volume ratio of the outer to inner core is investigated for static conditions and for the burn cycle corresponding to first-core loading. A finite-cylinder uncontrolled core, three feet in height and approximately 200 cm radius was chosen to simulate the actual reactor core. The two radial zones of the core are characterized by different plutonium enrichments. Carbide fuel was used in the study.

The plan of the report is as follows: in Section II, the results of the static zoning analysis based on DARCID² neutronic calculations are discussed. A pellet density of 95% T.D. was assumed for the carbide fuel and fuel volume fractions 0.30, 0.35, 0.40, and 0.45 were considered. A family of curves for the optimum enrichment split ϵ_2/ϵ_1 vs. the ratio of the outer to total core volume V_2/V are among the graphs included.

In Section III, we explore a simple analytical approach based on an idealized model to determine the optimum ϵ_2/ϵ_1 as a function of V_1/V_2 . From this analysis we extract a phenomenological relation containing two parameters which depend on the material properties of the core.

In Section IV, these parameters are determined by a least squares fit for the model core and a comparison of the results with the data obtained in Section II is made.

Section V includes a discussion of the optimization results for first core burnups. Calculations were based on the 1-D version of the REBUS-2 code. In Section VI, it is shown that the analytical approach of Section III is appropriate for the determination of the midcycle ratio $\bar{\epsilon}_2/\bar{\epsilon}_1$, where $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ are the average inner and outer core enrichments. The extrapolation to the beginning-of-cycle optimum enrichment split is discussed.

Section VII contains a summary and discussion of the results.

11. STATIC ZONING OF CARBIDE CORE FOR FOUR FUEL VOLUME FRACTIONS

The assumptions for the model carbide reactor are summarized in Table I. The lack of an axial blanket is a reflection of the one-dimensional calculations. The core is assumed to be uncontrolled and unpoisoned and its fuel composition corresponds to beginning-of-cycle conditions. Ten percent of the power is taken to come from radial and axial blankets, so that the core is characterized by a power of 4500 MWt. Four cases are considered, corresponding to fuel volume fraction (FVF) = 0.45, 0.40, 0.35, and 0.30 respectively. The atom density used in the neutronic calculation (based on DARC1D²) corresponds to a pellet density of 95% T.D. The plutonium enrichment is defined as ϵ = mass of fissile plutonium isotopes/mass of the heavy metal.

For a fixed ratio of the outer core volume to total core volume, V_2/V , the ratio of outer to inner core enrichment, ϵ_2/ϵ_1 , was varied to determine the minimum radial power peaking factor. The resulting curves, examples of which are given in Fig. 1 (see also Reference 1) take the forms of "vees"; the first leg of each "vee" corresponds to the peak power occurring at the center of the core, while the second leg corresponds to peaking in the outer core. Fig. 2 displays the broad minima near $V_2/V = 0.3$ in the radial peak/average power ratio as a function of V_2/V for the four fuel volume fractions considered. In the range $V_2/V \simeq 0.2$ to 0.4 the optimum power peaking factor varies by less than 2%. It is a decreasing function of the fuel volume fraction.

Plots of the optimum ϵ_2/ϵ_1 versus V_2/V for the four cases appear in Fig. 3. The dashed lines, which closely overlap the curves except in the region $V_2/V \lesssim 0.25$, correspond to the predicted optimum ϵ_2/ϵ_1 from the analysis described in Sections III and IV.

III. FUNCTIONAL DEPENDENCE OF OPTIMIZED ENRICHMENT SPLIT ϵ_2/ϵ_1 ON THE RATIO OF INNER CORE TO OUTER CORE VOLUME V_1/V_2 (STATIC CONDITIONS)

As shown in Section II of this report, for a given choice of V_2/V , the radial power peaking factor is a sensitive function of the enrichment split ϵ_2/ϵ_1 . Since the optimized power peaking factor has a broad minimum as a function of V_2/V , the practical problem is reduced to the determination of the optimum enrichment split as a function of V_2/V .

In this section we adopt an idealized model in order to understand the general features of such a relation between ϵ_2/ϵ_1 and V_1/V_2 . This model, illustrated in Fig. 4, is based on a number of assumptions which are described below. The validity of these assumptions for the neutronic calculations described in Section II is discussed.

We shall assume that for a given choice of the radius of the inner core R_1 , the plutonium enrichments ϵ_2 and ϵ_1 are such that the power densities at the center of the core ($r=0$) and at the inner boundary of the outer zone ($r=R_1$) are equal. This corresponds to the (static) minimum radial power peaking factor for the given volumes V_1 and V_2 (R_2 and R are held fixed throughout). Further, for V_2/V in the optimum range, $V_2/V \approx 0.2$ to 0.5, we assume

- a) the radial flux in the inner core is an extremely slowly decreasing function of the radius r ,
- b) the radial flux in the outer core is also slowly decreasing with increasing r , but its curvature is much greater than the corresponding curvature of the inner core flux,
- c) the flux is zero at the radial boundary of the reactor, $r=R$,

- d) in all three regions, the flux is separable as a function of energy and space, i.e.,

$$\phi(E, r, z) = C(E)\phi(r)Z(z).$$

The neutronics calculations described in Section II were based on the ARC System One-Dimensional Diffusion Theory Capability, DARC1D,² and eight energy groups were used in the multigroup analysis.⁴ In Fig. 5, we have plotted the first seven flux groups for a particular case of the carbide model reactor corresponding to FVF = 0.4, $V_2/V = 0.4$, and an optimum enrichment split $\epsilon_2/\epsilon_1 = 1.2055$. The assumptions above can be considered as fairly accurate for the inner core, less so for the outer core and the regions of the blanket and reflector. Therefore, we look for the idealized model to give the overall behavior of ϵ_2/ϵ_1 , with corrections to be introduced later.

The assumption (d) implies that the radial solutions in the three regions can be expressed by

$$\phi_1(r) = AJ_0(\beta_1 r), \quad 0 \leq r \leq R_1; \quad (3.1)$$

$$\phi_2(r) = CJ_0(\beta_2 r) + DY_0(\beta_2 r), \quad R_1 \leq r \leq R_2; \quad (3.2)$$

$$\phi_3(r) = EJ_0(\beta_3 r) + FY_0(\beta_3 r), \quad R_2 \leq r \leq R; \quad (3.3)$$

while the axial solution in all three regions is

$$Z(z) = A' \cos(\alpha z), \quad 0 \leq |z| \leq H/2. \quad (3.4)$$

where α satisfies $\tan(\alpha \frac{H}{2}) = \frac{1}{d}$, and H is height of reactor, d = the extrapolation distance. The radial buckling β_i and axial buckling α satisfy

$$B_i^2 = \alpha^2 + \beta_i^2, \quad i = 1, 2, 3. \quad (3.5)$$

where B_i^2 is the buckling in region i of the reactor. In terms of the average diffusion coefficient $\bar{D}$ and average macroscopic cross sections, the one-energy buckling is

$$B_i^2 = \frac{(\bar{\Sigma}_f)_i - (\bar{\Sigma}_a)_i}{\bar{D}_i} \quad (3.6)$$

where

$\bar{\Sigma}_f$ = average macroscopic fission cross section

$\bar{\Sigma}_a$ = average macroscopic absorption cross section of fuel and moderators (coolant, structure).

From Eqs. (3.5) and (3.6) we know that the radial buckling is a linear function of enrichment. We shall write

$$B_i^2 = f\epsilon_i - g, \quad i=1,2 \quad (3.7)$$

and to the first approximation, the parameters f and g are taken as independent of inner and outer core properties. Thus we have

$$\frac{\epsilon_2}{\epsilon_1} = \frac{\beta_2^2 + g}{\beta_1^2 + g} \quad (3.8)$$

Applying the boundary conditions on the fluxes and the normal currents at $r=R_1, R_2$, and R , and assuming the diffusion coefficients of the inner and outer core are approximately equal, we obtain the following condition for solution:

$$\begin{array}{ccccccccc} J_0(\beta_1 R_1) & -J_0(\beta_2 R_1) & -Y_0(\beta_2 R_1) & 0 & 0 & & & & \\ J'_0(\beta_1 R_1) & -J'_0(\beta_2 R_1) & -Y'_0(\beta_2 R_1) & 0 & 0 & & & & \\ 0 & J_0(\beta_2 R_2) & Y_0(\beta_2 R_2) & -J_0(\beta_3 R_2) & -Y_0(\beta_3 R_2) & = 0 & (3.9) & & \\ 0 & J'_0(\beta_2 R_2) & Y'_0(\beta_2 R_2) & -\frac{\bar{D}_1}{\bar{D}_2} J'_0(\beta_3 R_2) & -\frac{\bar{D}_1}{\bar{D}_2} Y'_0(\beta_3 R_2) & & & & \\ 0 & 0 & 0 & J_0(\beta_3 R) & Y_0(\beta_3 R) & & & & \end{array}$$

The additional restriction on the solution is obtained from our optimization condition, namely

$$P_1(r=0) = P_2(r=R_1), \quad (3.10)$$

where $P_1(r)$ and $P_2(r)$ are the power densities in the inner and outer cores, respectively. It is straightforward to find that in the one-energy group approximation, Eq. (3.10) leads to the relation

$$\frac{c_2}{c_1} (\text{optimum}) = J_0'(\beta_1 R_1) . \quad (3.11)$$

With the aid of Eqs. (3.8), (3.9), and (3.11), and the assumptions (a) through (d), we may express the radial buckling B_1^2 as an explicit function of R_1^2 (R_2 and R are held fixed throughout). Assumptions (a) and (b) may be translated to read

$$\begin{aligned} \text{(a)} \quad \beta_1^2 R_1^2 &\ll 1, \\ \text{(b)} \quad \beta_2^2 R_2^2 &\ll 1, \end{aligned} \quad (3.12)$$

$$\beta_2^2 \gg \beta_1^2,$$

and for small arguments the Bessel functions and their derivatives can be approximated by

$$\begin{aligned} J_0(\beta r) &\simeq 1 - \left(\frac{\beta r}{2}\right)^2 \\ J_0'(\beta r) &\simeq -\frac{\beta^2 r}{2} \\ Y_0(\beta r) &\simeq \frac{2}{\pi} [\ln \beta r - 0.11593] \\ Y_0'(\beta r) &\simeq \frac{2}{\pi \beta r} \end{aligned} \quad (3.13)$$

from Eq. (3.8) and the second part of assumption (b), we have

$$\epsilon_1 \approx \epsilon_1^* \left(\frac{gR_2^2}{g} \right) \quad (3.14)$$

Finally, keeping only the leading terms in the expansion of the determinant (3.9), we obtain the expression (valid in the approximate range $V_1/V \approx 0.2$ to 0.5)

$$\epsilon_1 \approx \frac{gC}{g} \left[\frac{1}{R_2^2} + \frac{1}{R_2^2 - R_1^2} \right] \quad (3.15)$$

where

$$C = R_2 \frac{\bar{D}_3}{D_2} \left[\frac{J_0(\epsilon_3 R_1) Y_0'(\epsilon_3 R_1) - Y_0(\epsilon_3 R_1) J_0'(\epsilon_3 R_2)}{J_0(\epsilon_3 R_1) Y_0(\epsilon_3 R_2) - Y_0(\epsilon_3 R_1) J_0(\epsilon_3 R_2)} \right] \quad (3.16)$$

and C is a positive number. We have taken ϵ_3 to be independent of R_1 in the first approximation. From (3.15) and the first-order approximation of $J_0(\epsilon_3 R_1)$, it follows that

$$\frac{\epsilon_2}{\epsilon_1} \approx 1 + \frac{C}{g} + \frac{C}{g} \frac{R_1^2}{R_2^2 - R_1^2} = 1 + \frac{C}{g} + \frac{C}{g} \frac{V_1}{V_2} \quad (3.17)$$

Eqs. (3.7) and (3.15) imply the following: as we vary V_1 , holding $V = V_1 + V_2$ fixed, the value of ϵ_1 corresponding to optimum ϵ_2/ϵ_1 , (i.e., the optimized power peaking) is a symmetrical function of V_1 and V_2 , i.e.,

$$\epsilon_1 \propto \frac{1}{V_1 V_2} + \text{constant}. \quad (3.18)$$

Then ϵ_1 has a minimum at $V_1/V \approx V_2/V = \frac{1}{2}$. A comparison of this prediction with the data obtained from the neutronics calculations described in Section II indicates that this is a good approximation. This is demonstrated in Fig. 6, where the optimum inner core enrichment ϵ_1 has been plotted as a function of V_2/V for the four fuel volume fractions 0.45, 0.40, 0.35, and 0.30.

Second-order corrections to p_1^2 in the one-energy group approximation could be included by keeping higher-order terms in the expansions of the Bessel functions. However, such corrections are not very meaningful without including the corrections coming from the relaxation of the other simplifying assumptions. Instead of embarking on a more detailed analysis, we note that the general features of our results are retained if we assume

$$\frac{\epsilon_2}{\epsilon_1} \text{ (static opt)} = \exp a \exp \left(b \frac{V_1}{V_2} \right), \quad (3.19)$$

with $a \ll 1$, $b \ll 1$. The parameters remain to be determined. We regard Eq. (3.19) as a phenomenological relation, and investigate its application to the model carbide core in the next section.

IV. LEAST SQUARES DETERMINATION OF PARAMETERS a AND b IN $\epsilon_2/\epsilon_1 \text{ (static opt)} = \exp a \exp \left(b \frac{V_1}{V_2} \right)$.

The form of Eq. (3.19) lends itself readily to a simple least squares determination of the parameters a and b . Since the relation is expected to be accurate only in the optimum range of V_2/V , and because the exponential function is smoothly varying, the fit was simplified by limiting it to three data points at $V_2/V = 0.3, 0.4, 0.5$. The corresponding values of optimum ϵ_2/ϵ_1 were obtained from the neutronic calculations based on DARC1D and described in Section II.

Letting

$$y_i \equiv \ln (\epsilon_2/\epsilon_1)_i, \quad i=1,2,3 \quad (4.1)$$

$$x_i \equiv (V_1/V_2)_i, \quad (4.2)$$

and

$$\chi^2 = \sum_{i=1}^3 [y_i - (a + bx_i)]^2, \quad (4.3)$$

we obtain solutions to the equations

$$\frac{\partial \chi^2}{\partial a} = 0, \quad \frac{\partial \chi^2}{\partial b} = 0. \quad (4.4)$$

they are given by

$$a = \left[\sum_{i=1}^3 y_i - \frac{\sum_{i=1}^3 x_i \sum_{i=1}^3 y_i}{\sum_{i=1}^3 x_i} \right] / \left[\sum_{i=1}^3 x_i - \left(\frac{\sum_{i=1}^3 x_i}{3} \right)^2 \right], \quad (4.5)$$

and

$$b = \left[3 \sum_{i=1}^3 x_i y_i - \frac{\sum_{i=1}^3 x_i \sum_{i=1}^3 y_i}{3} \right] / \left[3 \sum_{i=1}^3 x_i^2 - \left(\sum_{i=1}^3 x_i \right)^2 \right] \quad (4.6)$$

Table II contains the solutions for a and b for the model carbide reactor described in Section II (see Table I) and for the four fuel volume fractions considered. In Fig. 3, the solutions to Eq. (3.19) based on these values of a and b have been plotted (dashed lines) along with the solutions obtained by means of the DARCID neutronic calculations described in Section II (solid lines). The agreement is extremely good in the range $V_2/V = 0.25$ to 0.6.

The parameters a and b have been plotted in Fig. 7 as functions of the product of the pellet density and the fuel volume fraction, $\rho D \cdot FVF$. The variable $\rho D \cdot FVF$ was chosen because the neutronics calculations are based on this product. Therefore, a and b may be determined for other pellet densities by means of this graph or by interpolation in Table II.

V. OPTIMUM ZONING FOR 300 DAY BURN CYCLE: FIRST CORE LOADING, UNCONTROLLED CORE

In general, the power distribution in the large LMFBR core changes shape as burnup proceeds. Addition or subtraction of control poison can be done in such a way as to minimize this change and thus to minimize the change in the nuclear power peaking factor. The static, or beginning-of-cycle, zoning optimization would be appropriate in this case. For the uncontrolled core, optimization of the power peaking factor must be defined differently.

For the range of fuel volume fractions under consideration, the conversion ratio of the inner core is always greater than unity. It is also greater than the outer core conversion ratio. As a result, since the total power remains constant, the power density at the core center increases with burnup, while it decreases in the outer core due to power buildup in the inner core and in the radial blanket. Thus, for example, the peak power may occur in the outer core at the beginning of cycle but may shift to the core center by the end of cycle. For the uncontrolled core, the optimum zoning corresponds to this situation, with the additional condition that the magnitude of the peak/average power ratio of the core be the same at beginning and end of cycle.

A 300 day burn cycle corresponding to first-core loading was considered for the model carbide reactor of Table I. k_{eff} was taken to be unity at the beginning of cycle for the fuel volume fractions 0.45, 0.40, and 0.35, where the reactor gains reactivity during burnup, and at the end of cycle for FVF = 0.30, where the reactor loses reactivity. For fixed outer to total core volume ratio, V_2/V , the beginning-of-cycle enrichment split, ϵ_2/ϵ_1 (BOC), was determined from REBUS-2⁽³⁾ calculations to meet the conditions for optimizing the radial power peaking factor. The resulting optimum power peaking factors are plotted as functions of V_2/V in Fig. 8. As expected, these factors

are higher than in the static zoning. However, the dependence on the fuel volume fraction is reversed: the power peaking factor increases with increasing FVF in burnup optimization. Also, the broad minima of the power peaking factors are shifted to larger values of V_2/V compared to the static case. The corresponding optimized ϵ_2/ϵ_1 (BOC) are plotted as a function of V_2/V in Fig. 9.

In Figs. 10-13, the dependence of the radial peak/average power on the number of days of burnup is shown. In each case, the burn cycle has been optimized, i.e. the optimum radial power peaking factor is given by the BOC or EOC value of the peak/average power. During the first part of the cycle, the peak/average power decreases as the peak power, occurring in the outer core, decreases. At the time when the peak power occurs simultaneously at core center and in the outer core, the curve reaches a minimum. The increasing power density of the inner core is then responsible for the rising curve to the end of the burn cycle. In Fig. 14, the peak fission density of the inner and outer core, respectively, is plotted as a function of burnup days for a representative case, FVF = 0.40. The intersections of each pair of curves corresponds to the minima in the peak/average power in Fig. 12 (not to be confused with the optimum nuclear power peaking factor for the burn cycle).

The behavior of the power distribution during burnup showed great sensitivity to small changes in the input ϵ_2/ϵ_1 (BOC). This sensitivity is illustrated in Figs. 15-17. In Fig. 15, where the peak/average power vs. burnup days is plotted for FVF = 0.45 and $V_2/V = 0.30$, a 3% change in ϵ_2/ϵ_1 (BOC), for instance, results in a 9% change in the optimum power peaking factor. Moreover, the whole character of the burn cycle is changed, with the peak/average power in the core steadily decreasing from beginning to end of cycle. In Fig. 16, where the peak/average power is given as a function of the fuel volume fraction, this sensitivity is shown to increase with increasing fuel volume fraction: a 1% change in ϵ_2/ϵ_1 (BOC) causes 2% and 3% changes in the peaking factor for

$FVF = 0.30$ and $FVF = 0.45$, respectively. Finally, Fig. 17 is included to demonstrate the sensitivity of the end-of-cycle fission density at core center to small variations in ϵ_2/ϵ_1 (BOC). Here we see that in the case $FVF = 0.45$, $V_2/V = 0.5$, a 1% change in ϵ_2/ϵ_1 (BOC) can result in an 8% variation in the end-of-cycle fission density at core center.

This extreme sensitivity, characteristic of the large fuel volume fractions of the large LMFBRs, makes it very important to determine the optimum ϵ_2/ϵ_1 (BOC) as accurately as possible. Although some performance characteristics such as the reactor breeding ratio remain relatively insensitive to changes in the enrichment ratio,¹ others such as fluence, burnup swing $\Delta k/k$, will be more strongly affected. Therefore, it is worthwhile to investigate the possibility of extending the static zoning analysis to include the first-core burn cycle. This analysis is described in the next section.

VI. ANALYTICAL APPROACH TO FIRST-CORE BURN CYCLE OPTIMIZATION

We recall that the condition for optimum static zoning was that the power density peak simultaneously in the inner and outer core. This provided the constraint $P_1(r=0) = P_2(r=R_1)$ given in Eq. (3.10), which, in addition to the boundary conditions on the flux enabled us to derive a relation for ϵ_2/ϵ_1 (static optimum). We might at first expect, then, that we could apply a similar analysis to the core valid at a given time during the burn cycle when similar conditions exist, namely when the peak/average power is a minimum (see, e.g., Figs. 12 and 14). This is not a valid assumption, however, because the plutonium enrichment, uniform in each core, low at the beginning of cycle, becomes a function of radius during burnup. For example, at midcycle, the inner core enrichment increases with radius, in general, from core center to the outer zone in the optimized

case. This is because the beginning-of-cycle power distribution peaks in the inner core at its outer boundary; see, for example, Fig. 18. On the other hand, the midcycle enrichment of the outer core decreases with increasing radius.

Nevertheless, we shall be able to apply the analysis of Section III if we use the volume-averaged enrichments of the inner core $\bar{\epsilon}_1$ and of the outer core $\bar{\epsilon}_2$. Imagining a core fueled with uniform enrichments $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ in the inner and outer cores, respectively, we define the corresponding fluxes $\phi_1(\bar{\epsilon}_1, \vec{r}, t)$ and $\phi_2(\bar{\epsilon}_2, \vec{r}, t)$ and power densities $P_1(\bar{\epsilon}_1, \vec{r}, t)$ and $P_2(\bar{\epsilon}_2, \vec{r}, t)$. For these quantities, we shall assume that at some time t' during the burn cycle, the conditions for static optimization are met, namely,

$$P_1(\bar{\epsilon}_1, r=0, t') = P_2(\bar{\epsilon}_2, r=R, t') \quad (6.1)$$

and that conditions (a) - (d) of Sect. III hold. Then, carrying out the same analysis as in Sect. III, we expect

$$\frac{\bar{\epsilon}_2}{\bar{\epsilon}_1} (t=t') = \exp a' \exp (b' \frac{V_1}{V_2}) \quad (6.2)$$

where t' is the time at which (6.1) holds, and a' and b' are parameters analogous to those determined in Sect. III.

If Eq. (6.2) is to be useful, the time to reach "optimization conditions", namely t' , should be independent of V_1/V_2 and also of FVF. The simplest assumption is that t' corresponds to midcycle, in our case, 150 days. We found, in fact, that $\bar{\epsilon}_2/\bar{\epsilon}_1$ (midcycle) corresponding to the optimized burn cycle is in remarkably close agreement with ϵ_2/ϵ_1 (static optimum), in strong support of

the assumptions above. This is demonstrated in Table III. Comparisons of $\bar{\epsilon}_2/\bar{\epsilon}_1$ (midcycle) to values calculated from

$$\bar{\epsilon}_2/\bar{\epsilon}_1 \text{ (midcycle)} = \exp a \exp b \frac{V_1}{V_2} \quad (6.3)$$

are given in Fig. 19, where a and b were taken from Table II.

Thus we conclude that $a' \sim a$ and $b' \sim b$. We recall that for static zoning in the unpoisoned core, a and b were approximately linear functions of $PD \times FVF$, i.e. the product of the pellet density and the fuel volume fraction. The identification $a' \sim a$ and $b' \sim b$ can be made if we consider a and b to be functions of the product of the "fuel" pellet density and "fuel" volume fraction, where "fuel" denotes the fuel plus the accumulated fission products (in the simulated reactor burnup of REBUS-2,⁽³⁾ these quantities remain almost constant during burnup).

It is important to note that the actual power distribution at midcycle will not correspond to $P_1(r=0) = P_2(r=R_1)$. This is illustrated schematically in Fig. 20, and also in Fig. 18. However, the effect of the small radial variation in ϵ_1 and ϵ_2 at midcycle should have only a second-order effect on the flux shape, since, e.g., $J_0(\beta_1 r) \sim 1 - \frac{\beta_1^2}{4} r^2$, and the buckling $\beta_1^2 = \epsilon_1 + \text{const.}$ This is born out by comparison of Figs. 5 and 21. In the former, the flux groups are plotted for a case of static optimization. The latter shows the midcycle flux groups for an optimized burn cycle.

In order to estimate $\epsilon_2/\epsilon_1(\text{BOC})$ for the optimized burn cycle, we consider the time rate of change of the enrichments ϵ_1 and ϵ_2 . Since these quantities change slowly and approximately linearly with burnup (see, e.g., Fig. 22⁵), to a very good approximation, we can write

$$\bar{\epsilon}_i(t) = \epsilon_i(\text{BOC}) [1 + \alpha_i t], \quad i=1,2 \quad (6.4)$$

where α_i is a constant which can be determined from any burnup calculation with a burnstep of length, say, t_1 :

$$\alpha_i = \left[\frac{\bar{\epsilon}_i(t_1)}{\epsilon_i(\text{BOC})} - 1 \right] / t_1, \quad i=1,2 \quad (6.5)$$

It follows then that the optimized beginning-of-cycle enrichment split can be approximated by

$$\frac{\epsilon_2}{\epsilon_1} (\text{BOC optimum}) \approx \frac{1 + \frac{1}{2} \alpha_1 T}{1 + \frac{1}{2} \alpha_2 T} \times \exp a \exp \left(b \frac{V_1}{V_2} \right), \quad (6.6)$$

where T = length of the burn cycle. The factor $[1 + \frac{1}{2} \alpha_1 T] / [1 + \frac{1}{2} \alpha_2 T]$ in (6.6) is relatively insensitive to small variations in $\epsilon_2/\epsilon_1(\text{BOC})$. For example, for $\text{FVF} = 0.45$ and $V_2/V = 0.30$, the two inputs $\epsilon_2/\epsilon_1(\text{BOC}) = 1.353$ and 1.421 yielded values of this factor of 1.0534 and 1.0545, respectively. Thus, a good estimate of the optimized $\epsilon_2/\epsilon_1(\text{BOC})$ could be obtained from one burnup calculation and Eq. (6.5) provided the first guess of $\epsilon_2/\epsilon_1(\text{BOC})$ is reasonably good.

It should be emphasized that the methods and equations of this section are intended only as a way of obtaining a fairly good estimate of the optimum enrichment split $\epsilon_2/\epsilon_1(\text{BOC})$. Because of the great sensitivity of some burnup characteristics, it may be desirable to use it as a first guess input subject to more refinement.

VII. SUMMARY AND DISCUSSION

The investigation of radial zoning in a cylindrical model of an uncontrolled carbide reactor in the 5000 MWt class was carried out. The zoning parameters were the ratio of the outer core volume to total core volume, denoted by V_2/V , and the ratio of the outer to inner core fissile enrichment, ϵ_2/ϵ_1 .

Both static zoning and first-core burn cycle zoning were investigated. In the former, the optimum power peaking factor decreased with increasing fuel volume fraction; the reverse trend occurs for burn-cycle zoning. Burn-cycle zoning also resulted in an increase in the optimum peaking factor and a shift to larger optimum values of V_2/V over the static zoning.

Since the optimum radial power peaking factor exhibits a broad minimum as a function of the ratio of the outer core to total core volume, the emphasis of the analysis was on determining the behavior of the optimum value of ϵ_2/ϵ_1 for a given ratio V_2/V . For fixed V_2/V , the power peaking factor is extremely sensitive to the ratio of outer to inner core enrichments.

An analytical approach was adopted and applied first to the case of static zoning. Applying a one-group approximation to an idealized model of the core led to a simple phenomenological formula for optimum ϵ_2/ϵ_1 in terms of the volume split V_1/V_2 , given by Eq. (3.19). Two parameters are involved which are approximately linear functions of the fuel volume fraction.

Subsequently it was shown that the expression also holds for the midcycle ratio of the volume-averaged enrichments, $\bar{\epsilon}_2/\bar{\epsilon}_1$. Since $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ change very slowly with burnup, extrapolation to the beginning-of-cycle optimum enrichment split ϵ_2/ϵ_1 can be estimated by means of Eq. (6.6), where the parameters α_i , $i = 1, 2$, are obtained from burnup calculations.

One of the most significant results of the investigation is the sensitivity of the power distribution in the reactor during burnup to small changes in the ratio ϵ_2/ϵ_1 . The consequences of this behavior, which becomes more pronounced as the fuel volume fraction increases is currently under investigation, but it is worthwhile to point out here that it is important to take it into account when performing any kind of burnup calculations.

Finally, a remark is in order concerning the emphasis of the report on determining the minimum power peaking as a function of the enrichment split ϵ_2/ϵ_1 for a given value of V_1/V_2 . It is possible to derive an analytical expression for the minimum radial power peaking factor in terms of V_1/V_2 (or V_2/V) using the idealized model of Section III. The phenomenological parameters involved could be obtained for the model core by methods similar to those employed in Section IV. The usefulness of such an expression, however, is limited by two factors: (1) the volumes of the outer and inner core vary discontinuously for actual reactors, and (2) the optimum radial power peaking factors are relatively insensitive to changes in V_2/V in the optimum range $V_2/V \approx 0.3$ to 0.4 .

References and Footnotes

1. M. J. King and W. P. Sarthoil, "Radial Zoning of Large LWRs, Part I." ANL-AFP-19.
2. L. C. Just, R. Henryson, II, A. S. Kennedy, S. D. Spack, B. J. Toppel, and P. M. Walker, "The System Aspects and Interface Data Sets of the Argonne Reactor Computation (ARC) System," ANL-7711 (1971).
3. A. P. Olson, "A User's Manual for the Reactor Burnup System, REBUS-2," FRA-TM-62 (1974).
4. Upper limits of the neutron energies for groups 1 through 8 were 10 MeV, 1.287 MeV, 0.369 MeV, 98.0 keV, 24.2 keV, 8.05 keV, 1.84 keV, and 0.214 keV, respectively. (The contribution of the eighth energy group is generally 2 or 3 orders of magnitude smaller than the leading contributors, and have not been included on the graphs of Figs. 5 and 21).
5. In Figs. 6 and 22 the enrichment ϵ is defined as the fissile enrichment, i.e., $\epsilon \equiv \text{mass of fissile isotopes/mass of heavy metal}$.
6. The peak/avg power can be expressed as the product of the radial peak/avg power and the axial power peaking factor. The latter was calculated with the assumption of a cosine power distribution in the axial direction, and for the model reactor under consideration, is given by 1.2307.

Table I. Model of Carbide-Fueled Reactor

Power in core	4500 MWt
Fuel	Mixed Uranium-Plutonium Carbide
Coolant	Sodium
Structural Material	Stainless Steel SS316
Radius of Core	200 cm
Outer Radius of Radial Blanket	220 cm
Outer Radius of Radial Reflector	230 cm
Height of Core	91.44 cm

Isotope Fractions:

^{235}U	0.002
^{238}U	0.998
^{239}Pu	0.680
^{240}Pu	0.194
^{241}Pu	0.102
^{242}Pu	0.024

Pellet Density of Fuel 95% T.D.

k_{eff} 1.0000

	Case 1	Case 2	Case 3	Case 4
Fuel volume fraction	0.30	0.35	0.40	0.45
Structural volume fraction	0.15	0.15	0.18	0.20
Coolant volume fraction	0.55	0.50	0.42	0.35

Table II. Least Squares Solutions for Parameters a and b

in ϵ_2/ϵ_1 (static opt) $\approx \exp a \exp (b V_1/V_2)$
for the Model Carbide Core of Table I^(a)

Pellet Density of Carbide Fuel (PD)	Fuel Volume Fraction (FVF)	PD*FVF (T.D.)	a	b
.95 T.D.	0.30	0.2850	0.03949	0.1279
	0.35	0.3325	0.02953	0.1205
	0.40	0.3800	0.02028	0.1114
	0.45	0.4275	0.01373	0.1031

(a) Least squares fits were based on values of ϵ_2/ϵ_1 (opt) corresponding to $V_2/V = 0.3, 0.4$, and 0.5 and obtained by means of neutronics calculations described in Section II.

Table III. Comparison of Midcycle Enrichment Splits for Optimized First-Core Burn Cycles and Static-Zoning Optimized Enrichment Splits

Fuel Volume Fraction	$V_2/V^{(a)}$	Midcycle ^(b)	
		$\bar{\epsilon}_2/\bar{\epsilon}_1$	Static ϵ_2/ϵ_1 (opt)
(±0.005)			
0.45	0.5	1.126	1.124
	0.4	1.182	1.182
	0.3	1.287	1.290
0.40	0.5	1.143	1.141
	0.4	1.205	1.206
	0.3	1.317	1.324
0.35	0.5	1.164	1.162
	0.4	1.231	1.234
	0.3	1.353	1.364
0.30	0.5	1.180	1.182
	0.4	1.254	1.262
	0.3	1.381	1.402

(a) Outer core to total core volume ratio.

(b) $\bar{\epsilon}_2/\bar{\epsilon}_1$ is the ratio of the outer-core average enrichment to inner-core average enrichment at 150 days.

Fig. 1. Radial power peaking factor as a function of enrichment split for static (beginning-of-cycle) conditions. First and second legs of the "vees" correspond to the power peak occurring in the inner and outer core, respectively.

Fig. 2. Optimum radial power peaking factor as a function of outer core volume for static (beginning-of-cycle) conditions and fuel volume fractions 0.30, 0.35, 0.40, 0.45.

Fig. 3. Optimized beginning-of-cycle enrichment split ϵ_2/ϵ_1 as a function of outer core volume for static (beginning-of-cycle) conditions. The dashed lines correspond to solutions of $\epsilon_2/\epsilon_1 = \exp(a) \exp(b \frac{V_1}{V_2})$, where parameters a and b are determined in Section IV.

Fig. 4. Schematic illustration of radial flux distribution in the cylindrical model reactor described in Section III.

Fig. 5. Radial flux distribution for seven energy groups⁴ for the case fuel volume fraction = 0.4, outer-to-total core volume = 0.4, and optimum enrichment split = 1.2055. The static (beginning-of-cycle) power peaking factor has been minimized for the given ratio of outer to total core volume.

Fig. 6. The fissile enrichment⁵ of the inner core, ϵ_1 , vs. the ratio of inner core to total core volume, $V_1/V = 1 - V_2/V$, for four fuel volume fractions 0.30, 0.35, 0.40, 0.45. The enrichment split ϵ_2/ϵ_1 was optimized as a function of V_1/V_2 .

Fig. 7. The parameters a and b , determined in Section IV by a least squares fit to the equation ϵ_2/ϵ_1 (static optimum) $\exp a \exp (b V_1/V_2)$, versus the product of the fuel volume fraction (FVF) and the pellet density (PD) of the fuel (PD is the fraction of the theoretical density).

Fig. 8. Optimum radial power peaking factor as a function of outer core volume for a burn cycle of 300 days corresponding to first-core loading. Four fuel volume fractions are considered, 0.30, 0.35, 0.40, and 0.45.

Fig. 9. Optimized beginning-of-cycle enrichment split ϵ_2/ϵ_1 as a function of outer core volume for 300 day burnup corresponding to first-core loading. (The peak/avg power is the same at beginning and end of cycle.) Four fuel volume fractions are considered, 0.30, 0.35, 0.40, and 0.45.

Fig. 10. The radial peak/avg power as a function of burnup days in a 300 day burn cycle corresponding to first-core loading and fuel volume fraction 0.30. Three ratios of outer core to total core volume, $V_2/V = 0.3, 0.4, 0.5$, are considered.

Fig. 11. The radial peak/avg power as a function of burnup days in a 300 day burn cycle corresponding to first-core loading and fuel volume fraction 0.35. Three ratios of outer core to total core volume, $V_2/V = 0.3, 0.4, 0.5$, are considered.

Fig. 12. The radial peak/avg power as a function of burnup days in a 300 day burn cycle corresponding to first-core loading and fuel volume fraction 0.40. Three ratios of outer core to total core volume, $V_2/V = 0.3, 0.4, 0.5$, are considered.

Fig. 13. The radial peak/avg power as a function of burnup days in a 300 day burn cycle corresponding to first-core loading and fuel volume fraction 0.45. Three ratios of outer core to total core volume, $V_2/V = 0.3, 0.4, 0.5$, are considered.

Fig. 14. Peak fission densities in inner and outer cores plotted as a function of burnup days in a 300 day burn cycle corresponding to first-core loading and fuel volume fraction 0.40. Three ratios of the outer to total core volume, $V_2/V = 0.3, 0.4,$ and 0.5 were considered. The intersection of each pair of curves corresponds to the minimum in the peak/avg power displayed in Fig. 12.

Fig. 15. Peak/avg power (radial $\times$ axial)⁰ as a function of burnup days in a 300 day burn cycle (first-core loading) for fuel volume fraction = 0.45, and outer core to total core volume $V_2/V = 0.30$. The beginning-of-cycle enrichment split $\epsilon_2/\epsilon_1(\text{BOC}) = 1.353$ corresponds to optimization of the power peaking factor, while $\epsilon_2/\epsilon_1(\text{BOC}) = 1.3665$, 1.3935, and 1.4206 represent 1%, 3%, and 5% changes in this input, respectively.

Fig. 16. Peak/avg power (radial x axial) vs. fuel volume fraction for a 300 day burn cycle and outer core to total core volume $V_2/V = 0.30$. The lower curve shows the change in this factor if the beginning-of-cycle enrichment split is varied by 1% from the optimum.

Fig. 17. The peak fission density of the inner core is plotted as a function of burnup days in a 300 day burn cycle (first-core loading). The beginning-of-cycle enrichment split ϵ_2/ϵ_1 (BOC) = 1.168 corresponds to the optimum power peaking factor for the given volume split, $V_2/V = 0.5$, and fuel volume fraction = 0.45.

Fig. 18. The power density vs. radius of the core at beginning-of-cycle (BOC), midcycle, and end-of-cycle (EOC). The beginning-of-cycle enrichment split ϵ_2/ϵ_1 (BOC) = 1.389 corresponds to the optimum power peaking factor for the 300 day burn cycle for the given volume split $V_2/V = 0.30$ and fuel volume fraction = 0.40.

Fig. 19. The midcycle (150 days) ratio of the average outer core enrichment to average inner core enrichment, $\bar{\epsilon}_2/\bar{\epsilon}_1$, as a function of the volume split V_2/V for four fuel volume fractions. The dashed curves are solutions $\bar{\epsilon}_2/\bar{\epsilon}_1$ (midcycle) = $\exp a \exp (b V_1/V_2)$, with a and b taken from Table 11. ; were obtained from graphical solutions based on KEBUS-2 calculations and correspond to burnup optimization of the power peaking factor.

Fig. 20. Schematic plot of midcycle power distribution for optimized beginning-of-cycle enrichment split (burn-cycle optimization). The solid line corresponds to actual power distribution, while the dashed line corresponds to a power distribution which would result from uniform enrichments $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ in the inner and outer cores, respectively.

Fig. 21. Midcycle radial flux distribution for seven energy groups for the case of fuel volume fraction = 0.40, volume split $V_2/V = 0.3$, and optimum burn-cycle beginning-of-cycle enrichment split ϵ_2/ϵ_1 (BOC = 1.389).

Fig. 22. Fissile enrichments⁵ of inner and outer core plotted as a function of burnup for the volume split $V_2/V = 0.3$ and fuel volume fractions 0.30, 0.35, 0.40, and 0.45.