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Spatial Coherence of Synchrotron Radiation

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Theory and measurement of spatial coherence of synchrotron radiation beams are briefly reviewed. Emphasis is given to simple relationships between electron beam characteristics and far field properties of the light beam.

Introduction

Synchrotron Radiation (SR)^{1,2,3} has been widely used since the 80's as a tool for many applications of UV, soft X rays and hard X rays in condensed matter physics, chemistry and biology. The evolution of SR sources towards higher brightness has led to the design of low-emittance electron storage rings (emittance is the product of beam size and divergence), and the development of special source magnetic structures, as undulators. This means that more and more photons are available on a narrow bandwidth and on a small collimated beam; in other words there is the possibility of getting a high power in a coherent beam. In most applications, a monochromator is used, and the temporal coherence of the light is given by the monochromator bandwidth. With smaller and smaller sources, even without the use of collimators, the spatial coherence of the light has become appreciable, first in the UV and soft X ray range, and then also with hard X rays. This has made possible new or improved experiments in interferometry, microscopy, holography, correlation spectroscopy, etc.^{4,5,6,7,8,9,10}. In view of these recent possibilities and applications, it is useful to review some basic concepts about spatial coherence of SR, and its measurement and applications. In particular we show how the spatial coherence properties of the radiation in the far field can be calculated with simple operations from the single-electron amplitude and the electron beam angular and position spreads. The gaussian approximation will be studied in detail for a discussion of the properties of the far field mutual coherence and the estimate of the coherence widths, and the comparison with the VanCittert-Zernike limit.

I. SPATIAL COHERENCE (SC)

First let us remind some concepts and define some symbols about SC of a quasi-monochromatic field in general. If we have paraxial propagation of a random electromagnetic field $f(\mathbf{p})$ (where $\mathbf{p} = (x, y)$ is a point in the transverse plane) along a direction z , the field $f(\mathbf{p})$ at $z = 0$ propagates in the Fresnel approximation

$$f_z(\mathbf{p}) = \frac{1}{(\lambda z)^2} \int f_0(\mathbf{p}_0) e^{-\frac{ik}{z}(\mathbf{p}-\mathbf{p}_0)^2} d^2\mathbf{p}_0, \quad (1)$$

and in the Far Field (FF) (Fraunhofer region), where most observations are done, we have

$$\tilde{f}(\mathbf{k}) = F_{\mathbf{p} \rightarrow \mathbf{k}} f(\mathbf{p}) = \frac{1}{\sqrt{2\pi}} \int f(\mathbf{p}) e^{i\mathbf{k} \cdot \mathbf{p}} d^2\mathbf{p} \quad (2)$$

where we have dropped the index 0 and indicated with $F_{\mathbf{p} \rightarrow \mathbf{k}}$ the Fourier transform operator from $\mathbf{p}$ to $\mathbf{k}$ domain. Here we use as a variable the transverse component of the wavevector $\mathbf{k} = (k_x, k_y)$, the observation angle is then

$$\theta = \mathbf{k} \frac{\lambda}{2\pi}, \quad (3)$$

According to eq. (2), angles are expressed in terms of reciprocal space coordinates, as is natural in diffraction optics.

“Second-order” statistical properties of the field are described by the “mutual intensity” (m.i.) (see¹¹), i.e. the ensemble average of the products of fields at two points.

It is convenient to express the m.i. as a function of the average and difference coordinates: $\mathbf{p}_1 = \mathbf{p} - \Delta\mathbf{p}/2$, $\mathbf{p}_2 = \mathbf{p} + \Delta\mathbf{p}/2$ and, in reciprocal space, $\mathbf{k}_1 = \mathbf{k} - \Delta\mathbf{k}/2$, $\mathbf{k}_2 = \mathbf{k} + \Delta\mathbf{k}/2$

$$Mf(\mathbf{p}, \Delta\mathbf{p}) = \langle f^*(\mathbf{p} - \Delta\mathbf{p}/2) f(\mathbf{p} + \Delta\mathbf{p}/2) \rangle \quad (4)$$

For simplicity we will also use these symbols:

$$If(\mathbf{p}) = Mf(\mathbf{p}, 0)$$

the intensity and

$$Cf(\Delta\mathbf{p}) = \int Mf(\mathbf{p}, \Delta\mathbf{p}) d^2\mathbf{p}$$

the (integrated) autocorrelation. The degree of (spatial) coherence is defined as

$$\mu f(\mathbf{p}, \Delta\mathbf{p}) \equiv Mf(\mathbf{p}, \Delta\mathbf{p}) / \sqrt{I(\mathbf{p} - \Delta\mathbf{p}/2) I(\mathbf{p} + \Delta\mathbf{p}/2)} \quad (5)$$

The Fresnel propagation of the m.i. can be expressed as:

$$Mf_z(\bar{\mathbf{p}}, \Delta\bar{\mathbf{p}}) = \frac{1}{(\lambda z)^2} \int Mf_0(\mathbf{p}, \Delta\mathbf{p}) e^{i\frac{k}{z}(\bar{\mathbf{p}} - \mathbf{p})(\Delta\bar{\mathbf{p}} - \Delta\mathbf{p})} d^2\mathbf{p} d^2\Delta\mathbf{p} \quad (6)$$

and in the FF

$$\left\langle \tilde{f}(\mathbf{k} - \frac{\Delta\mathbf{k}}{2}) \tilde{f}(\mathbf{k} + \frac{\Delta\mathbf{k}}{2}) \right\rangle = \int Mf(\mathbf{p}, \frac{\Delta\mathbf{p}}{2}) e^{i\mathbf{p} \cdot \Delta\mathbf{k} + \mathbf{k} \cdot \Delta\mathbf{p}} d^2\mathbf{p} d^2\Delta\mathbf{p} \quad (7)$$

or, with our simplified notation,

$$M\tilde{f}(k, \Delta k) = F_{\mathbf{p} \rightarrow \Delta\mathbf{k}} F_{\Delta\mathbf{p} \rightarrow \mathbf{k}} Mf(\mathbf{p}, \Delta\mathbf{p}).$$

From this, two useful reciprocity relations connecting source and FF intensity/coherence properties can be derived^{12,13}:

$$FCf(\mathbf{k}) = IFf(\mathbf{k}) \quad (8)$$

$$FI f(\Delta\mathbf{k}) = CFf(\Delta\mathbf{k}) \quad (9)$$

and reciprocal ones interchanging source and FF. Properties of non-stationary random functions can also be described by the Wigner function (WF) (which is a photon number distribution in phase space, if divided by $\hbar\omega$)^{14,15,16}:

$$\begin{aligned} Wf(\mathbf{p}, \mathbf{k}) &= \int \langle f(\mathbf{p} - \Delta\mathbf{p}/2) f^*(\mathbf{p} + \Delta\mathbf{p}/2) \rangle e^{i\Delta\mathbf{p} \cdot \mathbf{k}} d^2\Delta\mathbf{p} \\ &= \int \left\langle \tilde{f}(\mathbf{k} - \Delta\mathbf{k}/2) \tilde{f}^*(\mathbf{k} + \Delta\mathbf{k}/2) \right\rangle e^{i\Delta\mathbf{k} \cdot \mathbf{p}} d^2\Delta\mathbf{k} \end{aligned} \quad (10)$$

In fact from the definition we see that Fourier-transforming the WF with respect to $\mathbf{k}$ one gets the m.i. of $f(\mathbf{x})$, while transforming with respect to $\mathbf{x}$ gives the m.i. of $\tilde{f}(k)$. We also remind that the intensity at the object plane $If = \int Wf d\mathbf{k}$ and in the far field $I\tilde{f} = \int Wf d\mathbf{x}$.

An equivalent description, with essentially the same characteristics, could be obtained with the Ambiguity function¹⁸

$$Af(\Delta\mathbf{p}, \Delta\mathbf{k}) = \int \langle f(\mathbf{p} - \Delta\mathbf{p}/2) f^*(\mathbf{p} + \Delta\mathbf{p}/2) \rangle e^{i\mathbf{p} \cdot \Delta\mathbf{k}} d^2\mathbf{p} \quad (11)$$

Both Wigner and Ambiguity functions are real (almost always positive) functions and can be considered as a phase space energy density: notice that this phase space area is dimensionless. Wf propagates in the same way of the “radiance” (or “brightness”) of geometrical optics:

$$Wf_z(\mathbf{p}, \mathbf{k}) = Wf_0(\mathbf{p} - \frac{\mathbf{k}}{k}z, \mathbf{k}), \quad (12)$$

and the same for $Af_z(\Delta\mathbf{p}, \Delta\mathbf{k})$.

A. Gaussian approximation

A gaussian model (also called a gaussian Schell model¹⁹) of a partially coherent field has a radiance which has a 4-D gaussian distribution in phase space. From now on let us for simplicity consider one transverse dimension, say x (and k_x will be called k for short): we have then $[Wf \text{ or } Af]$ of the form (using our previous symbols):

$$Wf(x, k) = N_1 \exp\left(-\frac{1}{2} \frac{x^2}{\sigma_I^2}\right) \exp\left(-\frac{1}{2} \frac{k^2}{s_I^2}\right) \quad (13)$$

the MI is then

$$Mf(x, \Delta x) = N_2 \exp\left(-\frac{1}{2} \frac{x^2}{\sigma_I^2}\right) \exp\left(-\frac{1}{2} \frac{\Delta x^2}{\sigma_M^2}\right) \quad (14)$$

where

$$\sigma_M = \frac{1}{s_I} \quad (15)$$

(in agreement with eq. 8), if we define σ_M as the width of $Cf(\Delta x)$. Here we have indicated with N the normalisation constants)

This MI clearly satisfies separability between x and Δx (Walther's condition¹⁶). When $\sigma_M \ll \sigma_I$ we have the “quasi-homogeneous” approximation ($Mf(x, \Delta x) = If(x)\mu f(\Delta x)$), and the factor function of x has the meaning of the intensity¹⁷.

We easily see that this model satisfies the Schell condition (that's why it is also called “gaussian Schell” model) that the degree of coherence depends only on the separation between two points Δx): eq. 14 can be written:

$$Mf(x, \Delta x) = N \exp\left\{-\frac{1}{2} \frac{(x - \Delta x/2)^2}{2\sigma_I^2}\right\} \exp\left\{-\frac{1}{2} \frac{(x + \Delta x/2)^2}{2\sigma_I^2}\right\} \exp\left\{-\frac{1}{2} \frac{\Delta x^2}{\sigma_\mu^2}\right\} \quad (16)$$

where

$$\frac{1}{\sigma_\mu^2} = \frac{1}{\sigma_M^2} - \frac{1}{4\sigma_I^2} \quad (17)$$

In particular, we see that for a perfectly coherent gaussian beam, $\sigma_M = 2\sigma_I$. The Schell and Walther conditions are satisfied simultaneously only for a plane wave and gaussian wave: writing the two conditions,

$$\left[If\left(x + \frac{\Delta x}{2}\right) If\left(x - \frac{\Delta x}{2}\right)\right]^{\frac{1}{2}} \mu f(\Delta x) = Mf(x, \Delta x) = If(x) mf(\Delta x) \quad (18)$$

If we apply the logarithm and call $h(\Delta x) = \log[\mu f(\Delta x)/mf(\Delta x)]$, $L(x) = \log[If(x)]$ Eq. 18 becomes:

$$2L(x) - L\left(x + \frac{\Delta x}{2}\right) - L\left(x - \frac{\Delta x}{2}\right) = 2h(\Delta x)$$

By Taylor expanding L we see that in order for the left term to be dependent only on Δx , terms higher than 2 must be 0, i.e. a Gaussian, exponential or flat intensity only.

II. MUTUAL INTENSITY OF SYNCHROTRON RADIATION

A characteristic of SR is that it is the random superposition of a large number of rather collimated elementary waves emitted by each electron of the beam^{14,15,20}. Let us call $\tilde{a}(\mathbf{k})$ the well-known far-field amplitude (or square root of the intensity) emitted by a single electron. It can be seen as the FT of the amplitude at the source $a(\mathbf{p})$, which of course is not a Dirac delta because of the diffraction corresponding to the limited angular aperture (and this is a limit to the possibility of localizing an electron by observing or imaging the emitted SR). The electron beam is characterized by a transverse spatial distribution $g(\mathbf{p})$ and an angular distribution $\gamma(\mathbf{k})$, which are to a good approximation both gaussian. The ratio of the beam size and angular aperture is called the beta function and it is known from the machine physics. Usually the source is in a place where position and angular distribution are uncorrelated; otherwise it is possible to define an effective source position at the “waist” point where the two distributions are uncorrelated. The “waist” points may be different for the vertical and the horizontal distributions.

We will consider for simplicity one transverse coordinate, say x and k . The superposition of all elementary contributions can be best described in phase space, where the Wigner function (or Ambiguity function) can be obtained by a convolution of the two distributions (electron, and single-electron light)^{14,15}.

$$Wf(x, k) = g(x)\gamma(k) ** Wa(x, k) \quad (19)$$

where $**$ indicated the convolution with respect to both variables. The source and FF mutual intensities are then²¹:

$$Mf(x, \Delta x) = \tilde{\gamma}(\Delta x)[g(x) * Ma(x, \Delta x)] \quad (20)$$

and

$$M\tilde{f}(k, \Delta k) = \tilde{g}(\Delta k)[\gamma(k) * M\tilde{a}(k, \Delta k)] \quad (21)$$

In particular, for the FF intensity:

$$If(k) = \gamma(k) * I\tilde{a}(k) \quad (22)$$

In order to give estimates of sizes and correlation distances of SR, it is useful to use a gaussian approximation for the SR distributions $a(x)$ and $\tilde{a}(k)$. Actually they are not gaussians, but this approximation is rather good for two reasons: $g(x)$ and $\gamma(k)$ being gaussians, the convolution is close to a gaussian, except on the tails (as $a(x)$ has long tails), and the part that is used is just the central one.

With this gaussian approximation, the source and FF are characterised by 6 gaussian widths.

Let us call σ_I the characteristic width of the intensity (so that $I(x) = \exp(-x^2/2\sigma^2)$) at the source, and s_I the FF intensity width. The M.I. of the source is given by eq. 14

As we have seen (eq. 14), the degree of coherence $\mu f(x)$ has a width which is related to the other two widths by:

$$\frac{1}{\sigma_\mu^2} = \frac{1}{\sigma_M^2} - \frac{1}{4\sigma_I^2} \quad (23)$$

And analogously, if we use s for the FF widths:

$$\frac{1}{s_\mu^2} = \frac{1}{s_M^2} - \frac{1}{4s_I^2} \quad (24)$$

On the other hand, if we apply the reciprocity relations (Eq. 8, 9) to the gaussian case, we have:

$$s_M = \frac{1}{\sigma_I}, \text{ and } s_I = \frac{1}{\sigma_M} \quad (25)$$

This is illustrated in fig 1

FIG. 1: Illustration of the source - far field reciprocity relations (eq. 25), note that in the gaussian case the ratio between beam width and coherence width is the same in the near field and far field (as well as in all sections in between²⁰)

We want now to correlate these widths with the electron and SR characteristics. We approximate the single-electron FF amplitude with

$$\tilde{a}(k) = \exp\left(-\frac{k^2}{4\rho^2}\right) \quad (26)$$

In this way we have defined ρ as the gaussian width of the FF intensity. The angular width ρ is of the order of the relativistic factor of the electrons (multiplied by $2\pi/\lambda$, in our reciprocal space units)^{9,22,23}

If we also apply to the gaussian case eqs. 19, we get

$$s_I^2 = s_e^2 + \rho^2, \text{ and } \sigma_I^2 = \sigma_e^2 + 1/4\rho^2 \quad (27)$$

Putting together these relations, we can eventually determine the intensity and coherence properties of the FF as a function of electron beam (and single-electron radiation) data:

$$s_I = (s_e^2 + \rho^2)^{1/2}, \quad s_M = (\sigma_e^2 + 1/4\rho^2)^{-1/2} \\ s_\mu = \left(\sigma_e^2 + \frac{1}{4\rho^2} - \frac{1}{4(s_e^2 + \rho^2)}\right)^{-1/2} \quad (28)$$

In the perfectly coherent limit ($s_e \ll 1/\rho$ and $s_e \ll \rho$) we have $s_\mu = \infty$, $s_I = \rho$ and $s_M = \rho$. The quasi-homogeneous case is when $s_e \gg \rho$ and $\sigma_e \gg 1/\rho$: in this case

$$s_\mu = (\sigma_e^2 + 1/4\rho^2)^{-1/2} \simeq 1/\sigma_e \quad (29)$$

This result coincides with the VanCittert-Zernike theorem, valid in the limit of a completely incoherent source. In general, however (for a rather coherent beam, that is a beam produced by an electron beam with small σ_e and s_e), the VanCittert-Zernike theorem needs a correction²¹.

It may also be of interest to know the resolution for imaging the source on the basis of FF intensity and coherence measurements. In principle, we can get both σ_e and s_e by measuring s_I and s_M or s_μ : from the previous equations we see that from eq 27 we get

$$s_e^2 = s_I^2 - \rho^2, \\ \sigma_e^2 = 1/s_M^2 - 1/4\rho^2. \quad (30)$$

However in practice the low precision of correlation measurement with the unfavorable propagation of errors, makes the method usable only if $4\rho^2/s_M^2 - 1$ and $s_I^2/\rho^2 - 1$ are not much smaller than one, i.e. the beam is not much smaller than the diffraction limit).

In these remarks we have considered always a quasi-monochromatic component of the field; in other words we imagine the light to be filtered before by a monochromator. It may be worthwhile to mention that SR, and in particular the radiation from undulators, is not “cross-spectrally pure” as defined by Mandel²⁴, as the spectrum depends on angle, and then the spatial coherence and spectral characteristics cannot be separated, a subject that has not yet been analysed in the literature.

III. EFFECTS OF QUALITY OF OPTICAL ELEMENTS.

In recent machines where spatial coherence becomes appreciable over a fraction of the photon beam width, or in other words is very well collimated (near the diffraction limit), the effect of imperfection of optical elements, as mirrors^{25,26} or Berillium windows²⁷ strongly influences the beam quality. For mirrors, if the rms slope error is ς , this must be compared with $\theta_{coh} = \lambda s_\mu/2\pi$: in order to have small distortions we should have $\varsigma \ll \theta_{coh}$. For windows, a uniform illumination will become non-uniform, with a contrast

$$C = 2\pi h/\lambda(n-1)$$

Some authors have called this degradation of beam quality a “reduction of coherence”^{27,28}. Actually this is not precise²⁹, as the speckle-like field produced by a random deflection from a rough surface (or refraction from a rough window) is still capable of producing interference fringes in a Young experiment if the original wave was spatially

coherent. In fact, the optical path (as a function of x,y) is fixed in time, it is a single realization of a random function, in other words a deterministic function (although not known in detail). We have to distinguish averages in time from averages over an ensemble of optical elements with similar statistical properties. The measure of correlation distance is given by σ_μ or s_μ , not by σ_M or s_M , as the latter ones maybe short, for example, in a perfectly coherent light with strong and rapid spatial variations of intensity.

In other words, coherent light stays coherent, even after passing through a random media. The photons in a coherent volume in phase space never mix with others as a consequence of the Liouville theorem. However when we perform a measurement, we normally measure projections (intensity) or slices (interferometry) in phase space. In the case of a Young's slits experiment for example, the two slits act as slices in the phase space, the beams diffracted from the slits have lost directionality, and different volumes in phase space are therefore mixed. In a intensity interferometry experiment, we integrate the phase space distribution over the angles³⁰.

FIG. 2: propagation of the wigner function: (top) a gauss-shell beam propagates in free space, and a coherent volume is selected by two slits. (bottom) the same beam after passing through a random phase object

IV. MEASUREMENTS

The first soft x-ray interferometric measurements with synchrotron radiation were performed by Polack et al³¹ using two mirrors with an angle between them of 2.25 arcmin at 6⁰ grazing angle. Coherence measurements using Young slits have been performed by many groups in the soft X-ray range^{32,33,34,35}. Takayama used a young-slit experiment to characterize the emittance of the electron beam³⁶.

In the hard x-ray the first interferometric measurement of the beam coherence was performed using two mirrors at grazing incidence acting as slits^{37,38,39} (Fig. 3). Normal slits have also been applied⁴⁰.

Other measurements of coherence have been performed by diffracting x-rays from a wire^{41,42}, using Talbot effect⁴³, a mask of coded apertures called a uniformly redundant array (URA)⁴⁴. Other techniques include using nuclear resonance from a rotating disk and measuring the spatial coherence in the time domain (the rotating disk acts as a 'prism' of increasing angle)⁴⁵, and intensity interferometry⁴⁶. The latter has been used to measure the spatial as well as longitudinal coherence⁴⁷ and characterize the 3 dimensional x-ray pulse widths. Variation of the visibility of a speckle pattern can also be used as an indication of the coherence width⁴⁸.

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FIG. 3: experimental setup used to perform hard x-ray interferometric characterization of the coherence. By moving D or changing the angle of incidence, or the height h of one mirror one can study the vertical coherence, while by tilting one mirror it is possible to study the horizontal coherence³⁸.

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- ¹ see for ex. *Handbook on Synchrotron Radiation*, E. E. Koch, Ed., Vol.1-4, North Holland, Amsterdam (vol.1 in 1983).
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