Antiproton Tune Measurements

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ABSTRACT

A system to measure the tunes of a single antiproton (or proton) bunch was built and has been commissioned. The system achieved high sensitivity with a novel closed-orbit suppression system. The use of high bandwidth directional pickups and kickers in conjunction with precise timing gates enabled the measurement of the tune of a single bunch.

HARDWARE DESCRIPTION

A functional schematic of a tune measuring system pickup is shown in Fig. 1. There were two identical systems: one for vertical and one for horizontal. The circuit was designed and built by Dave McGinnis. The beam position signal was obtained from the difference of the signals from the two pickup electrodes (top and bottom for vertical, left and right for horizontal). The signal was then amplified, sampled, and held. The pickup and electronics were constructed to have a high-bandwidth (about 100 MHz) and, consequently, the circuit had a high time resolution (better than a single bucket). The design strategy was very different from that of the resonant Schottky detector at A17.¹ The noise level obtained with the sample-and-hold system was not as good as the resonant detector, but the time resolution was much better. With the sample-and-hold system it was easy to measure the tune of a single bunch of protons or antiprotons

The interesting components of the signal are at the betatron sidebands, but the largest component is often the signal that arises from the closed orbit error. A low frequency component of the signal was derived from the output of the sample-and-hold circuit and applied to a low-pass filter. The output of the low pass filter was used to control the attenuation on one of the pickup electrodes. The attenuator was varied mechanically with a stepping motor. By properly choosing the attenuation the closed orbit error can be largely eliminated from the difference signal (Δ). The proper attenuation was obtained by a feedback loop in which a non-zero low frequency signal at the sample-and-hold (S/H) actuated the stepping motor via the stepping motor controller and thereby reduced the low frequency signal at the S/H. This feedback loop is sometimes referred to as an "auto-zero" circuit.

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Fig. 1. Block diagram of the tune system pickup.

The auto-zero circuit can be implemented in other ways. The use of a mechanical attenuator controlled by a stepping motor is in some ways awkward. However, the variable mechanical attenuator has overwhelming advantages that make it an extremely attractive choice: the ability to withstand high voltages, excellent linearity over a wide dynamic range, and a low noise figure. These characteristics are important, since the common-mode signal on the pickup can be 100 V while the betatron signal can be as small as 10's of μV . The major disadvantage of the stepping motor-mechanical attenuator scheme is its slow response time, which is unimportant for the slowly ramping Tevatron.

The pickup system does not have sufficient sensitivity to measure reliably the antiproton tune without exciting the beam. The existing damper kicker system was used to stimulate the beam at the betatron frequency. A signal near the betatron frequency (at 27 kHz, for example) was gated for 20 nsec period when the beam was present in the kicker. The gated signal was mixed (up-converted) with the 53 MHz rf signal. The resulting signal was applied to the damper driver amplifier and passed to the two strip-line electrodes in the damper kicker via a 180° hybrid(Δ).

Fig. 2. Block diagram of the tune system kicker.

The gate allows one to selectively excite bunches. The 53 MHz up-conversion removes the dc component of the signal, and reduces the length of the transient response from the amplifier. Since the kicker system excites individual bunches, it is possible to use the resonant Schottky pickup at A17 as well as the F0 pickup described above to measure antiproton tunes.

SYSTEM CONSIDERATIONS

There were two separate systems: one for horizontal motion and one for vertical motion. Each channel had a pickup and kicker. The pickup sample and hold timing and

the kicker gate timing are derived from the antiproton rf and the pbar marker so the changes in timing are tracked when the antiprotons are clogged. The system timing can also be modified to measure proton tunes by interchanging upstream and downstream ends of the pickup and kicker and by using the proton rf and proton marker.

The proton signal is rejected in several ways. Both the pickup and the kicker are directional. The pickup and kicker each have a proton signal rejection better than 20 dB. This directionality may be sufficient to separate proton and antiproton signals for any values of cogging.

However, additional isolation between proton and antiproton tunes is provided by the use of precise timing gates. If the proton beam arrives either 2 buckets before or after the antiproton beam, the signal suppression is about 30 dB. A one bucket separation still provides significant rejection. The positions of the electrodes relative to the center of F0 (a collision point) is given below. The table is based on mechanical drawings, not direct beam measurements.

Table I. Positions of the pickup and kicker electrodes

Electrode	Distance from F0 in units of rf buckets (+=downstream proton direction)
Horizontal Pickup	38.3
Horizontal Kicker	5.5
Vertical Pickup	-3.8
Vertical Kicker	-4.2

The kicker was excited with a network analyzer and the beam response was measured on the pickup, resulting in the measurement of the tune spectrum. It is possible to measure a response at any multiple of the revolution frequency plus or minus the betatron frequency. Since the sample rate is once per revolution, the bandwidth is limited to half the revolution frequency. This bandwidth contains only one betatron line. Measurements at one betatron sideband can therefore be used to predict the result of all other measurements. The theoretically infinite number of sidebands present are combined by the action of the sample-and-hold circuit. These bands would be distinct if a linear circuit (not a sample-and-hold) circuit were used. In particular, it is not possible to compare the widths of the upper and lower betatron sidebands to determine the chromaticity with this system. The "lower" sideband (*i.e.*, the measurement of the tune band at 20 kHz) actually contains components of both the upper and lower sidebands.

THEORY OF TRANSVERSE SCHOTTKY SIGNALS

The Schottky power density can be written as (see Appendix I)

$$P_T(f) = (ef_0)^2 \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \int_0^{A_{L\max}^2} \rho(A_L^2, A_\beta^2) J_k^2 \left[\frac{\phi_{\max}}{h} \left(n + \zeta \frac{\xi}{\eta} \right) \right] \frac{dA_L^2}{f_0 \left(\frac{dQ}{dA_\beta^2} \right)},$$

In all of the measurements (except one), the externally driven response - not the Schottky signal - was measured. We assume that the "real part" of the driven response is identical to the Schottky signal, except that the spectrum is simply proportional to the particle density -

not the density times the betatron amplitude. However, the driven response is measured with an unknown phase shift. Theoretically, we expect the driven response to be completely imaginary when the frequency of the drive is outside the band of particle oscillation frequencies. We use this expectation to apply an arbitrary phase shift to the measured data so that the corrected data have approximately zero real part for the portion of the spectrum that lies above and below the resonant response. While the data are generally consistent with the theoretical expectation, the procedure used is an "eyeball" fit, and the real part is not identically zero outside the particle distribution.

Several Schottky spectra have been calculated (see Eqn. 1) for the Tevatron to give the reader some idea of what is expected. The spectrum shown in Fig. 3 consists of a central line at the beam tune and several sidebands spaced by the synchrotron frequency. The envelope of the spectrum is determined by the value of the chromaticity. Figs. 3, 4, and 5 compare different chromaticities. The figures are drawn with a noise floor which is 60 dB below the peak; actual measurements seldom, if ever, have such a good signal to noise ratio and fewer synchrotron lines would be seen in actual measurements.

Fig. 3. A spectrum calculated for the Tevatron Schottky band $n+Q=500+20.575$ when chromaticity is 0.

Calculated Tevatron Schottky Spectrum

Fig. 4. A spectrum calculated for the Tevatron Schottky band $n+Q=500+20.575$ when chromaticity is 1.

Calculated Tevatron Schottky Spectrum

Fig. 5. A spectrum calculated for the Tevatron Schottky band $n+Q=500+20.575$ when chromaticity is -1.

The line widths of the spectra come mainly from the amplitude dependence of the tune. Fig. 6 is an example of a spectrum with a smaller amplitude dependence of the tune and a larger synchrotron frequency spread (fuller buckets). The measured Tevatron spectra appear to be more like Figs. 3-5 than like Fig. 6.

Calculated Tevatron Schottky Spectrum

Fig. 6. A calculated spectrum where the synchrotron frequency spread is much larger than the amplitude dependence of the tune.

EXAMPLES OF TUNE MEASUREMENTS

Although the system was not designed to see Schottky signals, we observed on one occasion a beam signal with no excitation applied. At the time of this measurement we had a single proton bunch at 150 GeV. The spectrum obtained is shown in Fig. 7. Both the horizontal and vertical spectra were quite noisy, and there were peaks that were not near the known betatron frequencies. However, we are confident that the lines identified in the spectra below are truly the tune lines because of the signature of the 80 Hz sideband modulation by the synchrotron frequency. Other, higher amplitude, lines (not shown in Fig. 7) were visible in regions removed from the tune lines. The sources of the noise lines was not determined, but it is known that timing jitter in the sample-and-hold circuit timing causes significant amplitude fluctuations. It was possible to minimize this effect by changing the relative delays of the two strip-line electrodes in the pickup so that the input to the sample and hold was relatively flat ($dV/dt=0$) at the instant when the sample was taken. This fine tuning was crucial in obtaining Fig. 7, but unnecessary when external excitation was applied to the beam.

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Fig 7. Schottky signal (i.e., no excitation from the kicker) of a beam at 150 GeV.

Fig. 8 shows the driven response of the same beam at a later time. The spectrum is considerably less noisy (and the tune is slightly different).

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Fig 8. Driven response of a beam at 150 GeV.

Fig. 9 shows the effect of long range interactions in creating a tune spread. Both spectra in Fig. 9 were taken at the end of a normal 6x6 store with the antiprotons cogged by +21 buckets. The upper trace (which has been shifted upwards for clarity) was taken with the protons in the machine and the lower trace was taken after the protons had been removed. It is clear that the beam-beam effect increases the width of the individual lines—indicating a tune spread that is somewhat larger than the synchrotron frequency spread. The markers show the frequencies of the horizontal and vertical tunes after the protons are removed. The most prominent peak on the upper trace is the shifted and broadened vertical tune line.

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TRACE A: F1 D1*K1

Cogged +21 buckets

Fig 9. Tune spectra of an antiproton beam cogged +21 buckets from the collision point. The upper trace was taken with the proton beam present (bunch intensities near 100×10^9) and the lower trace was taken with the proton beam absent. The upper trace was translated upward by 1/2 division for clarity.

On another occasion, a tune spectrum was obtained while the beams of a normal 6x6 store were colliding. These spectra were taken with the resonant pickup at A17 and are shown in Figs. 10 and 11. The measurements are sensitive to the upper and lower betatron sidebands near 21.4 MHz (between harmonic numbers 448 to 449). Figs. 14 and 15 were taken under identical conditions except that the protons were removed from the Tevatron with the scrapers. These spectra allow an estimate of the tune spread of the beam in collisions.

The bottom trace in Figs. 10 and 11 is the “real part” of the response. It is found by multiplying the measured data by a phase that yields zero real part outside the beam distribution. The real part is not exactly zero outside the beam distribution so the “best” phase adjustment has some arbitrariness. Figs. 12 and 13 show the same data, but with a more extreme estimate of the “real part” that was obtained with an alternate choice of phase.

Figs. 12 and 13 also have markers showing the interpretation given to the various peaks. Comparison of the upper edge of the tune spectra (which should correspond to the small amplitude part of the beam) in Fig. 10 and the unshifted tune in Fig. 14 yields an estimate for the horizontal tune spread. A more sophisticated analysis would take into account the effect of chromaticity on the tune spread when the beams are in collisions. Similarly Figs. 11 and 15 allow an estimate of the vertical tune spread.

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TRACE A: F1 D1*K1

TRACE B: F1 D1*K1

Fig 10. Upper horizontal antiproton sideband with the beams in collisions.

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TRACE A: F1 D1*K1

TRACE B: F1 D1*K1

Fig 11. Upper vertical antiproton sideband with the beams in collisions.

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TRACE A: F1 D1*K1

TRACE B: F1 D1*K1

Fig 12. Upper horizontal antiproton sideband with the beams in collisions. The data is the same as Fig. 10, except that a different phase was used to obtain the real part of the response.

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Fig 13. Upper vertical antiproton sideband with the beams in collisions. The data is the same as Fig. 11, except that a different phase was used to obtain the real part of the response.

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Fig 14. Upper horizontal antiproton sideband taken under the same conditions as Fig. 10, except that all the protons have been removed from the Tevatron.

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Fig 15. Upper vertical antiproton sideband taken under the same conditions as Fig. 11, except that all the protons have been removed from the Tevatron.

A measure of the system's ability to isolate a specific bunch is shown in Figs. 16 and 17. In Fig. 16, the normal response (upper curve) is compared to the response with the kicker pulse intentionally mistimed by setting it one bucket early. Fig. 17 is similar, but the timing is set one bucket late. The difference in the signal level with the single bucket timing error exceeds 13 dB in either case. It is somewhat surprising that the largest pulse response occurs when the signal is delayed: it makes one suspect that the labels on Fig. 16 and 17 should be reversed. However, the main point of Fig. 16 and 17 was to check that the resonance associated with the proton tune (labeled by the marker on Fig. 16) was primarily caused by the excitation of the antiproton beam. The reduction in signal level in Figs. 16 and 17 when the kicker is mistimed is consistent with this interpretation.

Fig. 16. Horizontal tune with the beam in a normal 6x6 store. The lower trace was taken with the kicker timing intentionally set early by one bucket.

Fig. 17. Horizontal tune with the beam in a normal 6x6 store. The lower trace was taken with the kicker timing intentionally set late by one bucket.

The tune spectra do not always look like the theoretical spectra. Often it is not completely clear which line is the "central" tune line. In order to identify the central line the synchrotron frequency can be changed by lowering the rf voltage. Fig. 18 shows an example of the identification of the central line (center of screen) when the synchrotron frequency was changed from 37.5 to 34.0 Hz.

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Fig. 18. Two tune spectra showing the change in synchrotron frequency when the rf amplitude was changed from 1.10 to 0.88 MV.

MYSTERIOUS FEATURES

There are some features of the measurements that are not well understood. Some of these are:

1. The effect of coupling between horizontal and vertical tunes is not well understood. In general, the two tunes generate an interference pattern that depends of the frequency difference between the tunes and the synchrotron frequency.
2. The synchrotron lines are not generally symmetric around the central line. Some of the effect can be explained by the convolution of the synchrotron frequency spread and the amplitude dependence of the tune (see Figs. 4-6), but the effect is rather small (except for contrived distributions). One speculation is that the asymmetry is caused because of positive feedback arising from collective effects.
3. The coupling between antiproton and proton beams (when they are in collision) is not understood other than to note that the effects should be similar to the horizontal and vertical coupling.

CONCLUSIONS

A single bunch tune measuring system has been tested. The system has a high sensitivity and a high rejection for signals from the other bunches.

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Appendix I

Formulas for Schottky Spectra

A1. REVIEW OF RF FORMULAS FOR A STATIONARY BUCKET

The synchrotron frequency for a infinitesimally small amplitude oscillations is

$$f_s(0) = f_0 \sqrt{\frac{\eta h e V}{2\pi \beta^2 E_0}}$$

where

$$\begin{aligned} f_0 &= \text{revolution frequency} \\ V &= \text{rf Voltage} \\ h &= \text{rf harmonic number} \\ \beta &= v/c \\ E_0/e &= \text{beam energy in eV} \\ \eta &= 1/\gamma_t^2 - 1/\gamma^2 \end{aligned}$$

The maximum energy deviation ($E_{\max}$) is related to the maximum phase deviation ($\phi_{\max}$) according to

$$\Delta E_{\max} = E_b \sin \frac{\phi_{\max}}{2}$$

where E_b is the bucket height and is given by

$$E_b = \sqrt{\frac{2\beta^2 E_0 e V}{\pi \eta h}}$$

The synchrotron frequency for particles with maximum phase deviation $\phi_{\max}$ is given by

$$f_s(\phi_{\max}) = f_s(0) \left[\frac{\pi}{2K\left(\sin^2 \frac{\phi_{\max}}{2}\right)} \right]$$

where K denotes the complete elliptic integral of the first kind.

A2. LONGITUDINAL SCHOTTKY SPECTRA

The beam current of a single particle is

$$i(t) = e \sum_{n=-\infty}^{\infty} \delta[t + \Delta t \cos(\omega_s t + \phi_{s0}) - nT]$$

where we have approximated the synchrotron motion by simple harmonic motion. This approximation will be good if $\phi_{\max} = \omega_0 \Delta t$ is small. A useful identity is

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) = \frac{\omega_0}{2\pi} \sum_{n=-\infty}^{\infty} e^{in\omega_0 t},$$

where

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T}.$$

Applying this identity to the beam current, the sum over delta functions can be converted to a sum over revolution harmonics

$$i(t) = ef_0 \sum_{n=-\infty}^{\infty} e^{in\omega_0 t} e^{in\omega_0 \Delta t \cos(\omega_s t + \phi_{s0})}$$

The exponent with the cosine function can be expanded in terms of Bessel functions by using the identity:

$$e^{ikz \cos \theta} = \sum_{k=-\infty}^{\infty} i^k J_k(z) \cos(k\theta)$$

The result is that the current of a single particle is given by:

$$i(t) = ef_0 \sum_{n=-\infty}^{\infty} e^{in\omega_0 t} \sum_{k=-\infty}^{\infty} i^k J_k(n\omega_0 \Delta t) \cos[k(\omega_s t + \phi_{s0})]$$

The current from many particles add incoherently because of the random phases and the spectral density is given by

$$P_L(f) = (ef_0)^2 \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \int_0^{A_L^2 \max} \rho_L(A_L^2) [J_k(n\phi_{\max})]^2 dA_L^2$$

where $\rho_L(A_L^2)$ is the particle density in longitudinal phase space, $A_L = \sin(\phi_{\max}/2)$, and the frequency satisfies the condition $f = f_0 + kf_s(A_L^2)$.

A3. TRANSVERSE SCHOTTKY SPECTRA

The betatron frequency is given by

$$\frac{d\phi_b}{dt} = \left[Q + \xi \frac{\Delta p}{p} \sin(\omega_s t + \phi_{s0}) \right] \omega_0$$

and the betatron phase is

$$\phi_b = \omega_0 Q t - \frac{\xi \omega_0}{\omega_s} \frac{\Delta p}{p} \cos(\omega_s t + \phi_{s0}) + \phi_{b0}$$

The dipole moment (amplitude times current) is

$$d(t) = e A_\beta \cos \left[\omega_0 Q t - \frac{\xi \omega_0}{\omega_s} \frac{\Delta p}{p} \cos(\omega_s t + \phi_{s0}) + \phi_{b0} \right] \\ \times \sum_{n=-\infty}^{\infty} \delta \left[t + \Delta t \cos(\omega_s t + \phi_{s0}) - nT \right]$$

As was done for the longitudinal case, the delta-function can be expanded into Fourier components:

$$d(t) = \frac{e A_\beta f_0}{2} e^{i \left[\omega_0 Q t - \frac{\xi \omega_0}{\omega_s} \frac{\Delta p}{p} \cos(\omega_s t + \phi_{s0}) + \phi_{b0} \right]} \\ \times \sum_{n=-\infty}^{\infty} e^{i n \omega_0 t} e^{i n \omega_0 \Delta t \cos(\omega_s t + \phi_{s0})}$$

and the frequency modulation can be expanded in terms of Bessel functions.

$$d(t) = \frac{e A_\beta f_0}{2} e^{i[(n+Q)\omega_0 t + \phi_{b0}]} \\ \times \sum_{n=-\infty}^{\infty} i^k J_k \left(n \omega_0 \Delta t + \frac{\xi \omega_0}{\omega_s} \frac{\Delta p}{p} \right) \cos[k(\omega_s t + \phi_{s0})]$$

or, equivalently

$$d(t) = \frac{e A_\beta f_0}{2} \sum_{n=-\infty}^{\infty} e^{i[(n+Q)\omega_0 t + \phi_{b0}]} \\ \times \sum_{k=-\infty}^{\infty} i^k J_k \left[\frac{\phi_{\max}}{h} \left(n + \zeta \frac{\xi}{\eta} \right) \right] \cos[k(\omega_s t + \phi_{s0})]$$

where

$$\zeta = \frac{2}{\phi_{\max}} \sin \frac{\phi_{\max}}{2} \frac{2K \left(\sin^2 \frac{\phi_{\max}}{2} \right)}{\pi} \\ \approx 1 \text{ (for small } \phi_{\max} \text{)}$$

The expression can be rewritten as a double sum:

$$d(t) = \frac{eA_{\beta}f_0}{2} \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} e^{i[(n+Q)\omega_0 t + k\omega_s t + \phi_{b0} + \phi_{s0}]} i^k J_k \left[\frac{\phi_{\max}}{h} \left(n + \zeta \frac{\xi}{\eta} \right) \right]$$

Power Spectrum for the Transverse Case

$$P_T(f) = \frac{1}{4} (ef_0)^2 \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \int_0^{A_L^2 \max} \int_0^{A_{\beta}^2 \max} A_{\beta}^2 \rho(A_L^2, A_{\beta}^2) J_k^2 \left[\frac{\phi_{\max}}{h} \left(n + \zeta \frac{\xi}{\eta} \right) \right] \\ \times \delta \left\{ f - f_0 \left[n + Q(A_{\beta}^2) \right] - kf_s(A_L^2) \right\} dA_{\beta}^2 dA_L^2 \\ = \frac{1}{4} (ef_0)^2 \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \int_0^{A_L^2 \max} A_{\beta}^2 \rho(A_L^2, A_{\beta}^2) J_k^2 \left[\frac{\phi_{\max}}{h} \left(n + \zeta \frac{\xi}{\eta} \right) \right] \frac{dA_L^2}{f_0 \left(\frac{dQ}{dA_{\beta}^2} \right)},$$

where $\rho(A_L^2, A_{\beta}^2)$ is the phase space density

$$f - (n + Q_0 + Q' A_{\beta}^2) f_0 - kf_s = 0,$$

and

$$A_{\beta}^2 = \frac{f - (n + Q_0) f_0 - kf_s}{Q' f_0}.$$

Appendix II

This section summarizes some useful identities that arise from the generating function. The generating function is :

$$e^{\frac{z}{2}\left(t - \frac{1}{t}\right)} = \sum_{k=-\infty}^{\infty} J_k(z) t^k$$

Substituting $t = ie^{i\theta}$ one obtains

$$\begin{aligned} e^{iz \cos \theta} &= \sum_{k=-\infty}^{\infty} i^k J_k(z) e^{ik\theta} \\ &= \sum_{k=-\infty}^{\infty} i^k J_k(z) \cos k\theta \end{aligned}$$

Where use has been made of the even or odd properties of the Bessel functions:

$$J_{-n}(z) = (-1)^n J_n(z)$$

Alternatively, substituting $t = e^{i\theta}$ one obtains

$$\begin{aligned} e^{iz \sin \theta} &= \sum_{k=-\infty}^{\infty} J_k(z) e^{ik\theta} \\ &= \sum_{k=-\infty}^{\infty} J_k(z) \begin{cases} \cos k\theta, & k = \text{even} \\ i \sin k\theta, & k = \text{odd} \end{cases} \end{aligned}$$