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MHD STABILITY PROPERTIES OF BEAN-SHADED TOKAMAKS

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ABSTRACT

A study of the MHD stability properties of bean-shaped tokamak plasmas is presented. For ballooning modes, while increased indentation gives larger β stable configurations, the existence and accessibility of the second stable region is sensitive to the pressure and safety factor profiles. The second stable region appears at lower β values for large aspect ratio and moderately high q -values. Finite-Larmor-radius (FLR) kinetic effects can significantly improve the stability properties. For low q ($\lesssim 1$) operation, long wavelength ($n \sim 2,3$) internal pressure driven modes occur at modest β_p values and accessibility to higher β operation is unlikely. Indentation modifies the nature of the usually vertical axisymmetric instability, but the mode can be passively stabilized by placing highly conducting plates near to the tips of the plasma bean. At constant q , indentation has a stabilizing effect on tearing modes.

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1. INTRODUCTION

Several large tokamak experiments [1,2] have recently reached β -values near to the minimum required for tokamak fusion reactors. While these encouraging results are consistent with theoretical MHD stability analyses, they are close to the limits expected for pressure driven modes. Indeed, the comparison of theory and experiment for the β -saturation mechanisms associated with resistive ballooning modes [3] and with the fishbone-internal kink mode [4] indicates that such MHD models should be considered seriously. The deterioration in confinement with increasing β adds incentive to the search for configurations with improved stability properties, since these would reduce the risks associated with the design of ignition experiments, improve the flexibility of design tradeoff studies for future reactors, and may even permit the consideration of advanced fuel fusion reactors.

Detailed numerical parameter and β -optimization studies have shown that, for modestly elongated plasmas, operation at very small aspect ratio ($R/a \sim 2.5$) and moderately low q offers one possible approach to higher stable β tokamak configurations [5,6,7]. At the present time, this is the conventional approach to increasing β , and experiments to take advantage of this scaling will come into operation in the near future [8]. Small aspect ratio tokamaks, however, present numerous technical difficulties as power reactors, and alternative approaches, valid at larger aspect ratio, are considered attractive. This issue is one of the motivations for this work.

There are at least two basic effects, strong magnetic shear and magnetic well stabilization, which can be exploited to provide enhanced stability. Increased shear can be realized by poloidal cross-sectional shaping and is already partly responsible for the improved β values of Dae-shaped tokamak configurations over those with circular or elliptical cross sections. That further improvement can be obtained by enhancing the weighting of field lines

in good curvature regions by indenting the inboard small major radius side of a tokamak, has been suggested in earlier studies [6,7,9,10,11]. However, a detailed analysis of the consequences of this bean shaping on ballooning and other global MHD instabilities has not, until recently, been undertaken. The second effect, which is associated with the large outwards equilibrium Shafranov shift which occurs at large β , has also attracted considerable attention [12,13]. While a localized analysis clearly supports the concept of a second stable ballooning regime at large β , detailed numerical calculations [14,15] have shown that it is difficult to gain access to such regimes on all the plasma cross-sectional magnetic surfaces simultaneously, except at very large aspect ratios or at high q . Hence, it appeared that such high- β regions would be difficult to achieve practically.

That a combination of these two effects could provide for a ballooning mode stable path to very high β values in medium aspect ratio, slightly elongated, tokamak plasmas, was reported recently by Chance et al. [16]. Studies of the internal kink mode [17] in bean-shaped plasmas, indicating the possibility of complete stability, further increased interest in these configurations. As a result, the Princeton PDX experiment is currently being modified to enable a study of some of these advantages. As part of the effort associated with this reconfiguration, and also because of the possible implications for future devices, parameter studies of some of the more important MHD modes in bean-shaped plasmas have been carried out, and are the subject of this work.

In the next section we review briefly some of the standard techniques used in this work, describe the equilibrium configurations, and discuss some of the limitations of the present study. The results of a localized ballooning analysis are presented in Sec. 3. We concentrate primarily on the

effects of the safety factor profile and the scaling with aspect ratio, but also included are studies of variation and optimization of the pressure profile, the effect of elongation and of the stabilizing influence of finite Larmor radius effects. In Sec. 4 we consider several long wavelength MHD modes. The axisymmetric (vertical) instability is particularly interesting because it occurs with no β threshold and must be stabilized in order to create and sustain a bean-shaped plasma. Since the $n = 1$ internal kink mode is stabilized by indentation, it is also of interest to study other low n internal modes for $q_0 < 1$ as possible candidates for "fishbone" activity. A study of the $n = 1$ tearing mode in low β , bean-shaped plasmas is included since it illustrates that strong indentation can reduce other instability driving mechanisms, in addition to those associated with the plasma pressure. A discussion of the main results and some implications for the Princeton Beta Experiment (PBX) device are given in Sec. 5.

2. EQUILIBRIUM AND STABILITY METHODOLOGY

Our present understanding of β -limiting processes in tokamaks is based heavily on ideal, single fluid, MHD equilibrium, and stability analyses. The simple MHD model has the virtue of being well understood, both qualitatively and quantitatively, and it is generally accepted that only configurations which have favorable properties in this model are likely to be viable candidates for serious consideration as future reactors. Because of the complexity of the equations involved, detailed analysis can only be carried out numerically. The calculations presented here are similar to those carried out in support of the design of most present day tokamaks and for several future reactor designs.

Carrying out an MHD analysis requires the calculation of suitable

equilibrium solutions of the Grad-Shafranov equation, in which the plasma is modelled as an infinitely conducting fluid surrounded by a vacuum region and supported by an external field [18]. This is called a free-boundary equilibrium. For practical application, it is necessary to specify the external field in considerable detail in order to determine the shape of the plasma surface. Alternatively, one can prescribe a reasonable shape for the plasma surface, solve the fixed-boundary equilibrium problem with an infinitely conducting shell on this surface, and leave the necessary external equilibrium field as a separate calculation. This is the approach most convenient for theoretical parameter studies and is the one followed for the majority of calculations presented here. We define the plasma surface by [16],

$$\begin{aligned} X(\theta) &= \hat{X} + \rho \cos \theta, \\ Z(\theta) &= E \rho \sin \theta, \end{aligned} \quad (1)$$

where $\rho = 1 + B \cos \theta$, $t = C \sin \theta$, $0 \leq \theta \leq 2\pi$. An illustration is given in Fig. 1, with $B = 0.6$, $C = 97^\circ$, and $E = 0.895$. To quantify the effects of indentation $l \equiv d/2a$, we vary $\hat{X}$, E , and C in Eq. (1) so that the elongation (b/a) and the aspect ratio (R/a) remain constant.

To specify an equilibrium, two surface functions must be given. We take

$$P(\psi) = P_0 \left[1 - \left(\frac{\psi}{\Delta\psi} \right)^\alpha \right]^\beta, \quad (2)$$

with $\Delta\psi$ the poloidal flux within the plasma. Most of the calculations reported here have $\alpha = \beta = 2$, resulting in flat profiles near the magnetic

axis ($\psi = 0$) and plasma edge ($\psi = \Delta\psi$). For the second surface function, we specify the safety factor profile,

$$q\left(\frac{\psi}{\Delta\psi}\right) = \sum_{i=0}^3 q_i \left(\frac{\psi}{\Delta\psi}\right)^i, \quad (3)$$

with the coefficients, q_i , chosen to give prescribed values for q and its first derivative at the magnetic axis and at the plasma surface. Characterizing each equilibrium is an average β value which we define as $\langle\beta\rangle \equiv 2\mu_0/pdV/B^2dv$.

When the equilibrium problem is posed as above, the toroidal plasma current is determined self-consistently from the pressure balance equation and cannot be separately adjusted. In Fig. 2, we illustrate some of the features of the current profiles associated with the choice $q(0) = 1.03$, $q(1) = 4.2$, $q'(0) = 0.84$, $q'(1) = 9.0$. Comparison of Fig. 2(a), and Fig. 2(b) indicates the variation with indentation at low $\langle\beta\rangle$ ($\sim 0.1\%$), while Fig. 2(b) and Fig. 2(c) ($\langle\beta\rangle \sim 10\%$) illustrate the effect of varying β at similar indentation. Since the current profile cannot be measured directly in tokamak experiments, one can really only speculate about whether or not these profiles are realistic. At low β , indentation primarily modifies the current profile near the plasma surface, for q -conserved sequences such as these, and the bulk current profile is relatively unaffected. At very high β values, the current profiles become peaked near the plasma edge and additional understanding of plasma evolution is required before one can judge whether such configurations are achievable without auxiliary (non-Ohmic) current drive schemes. It has also been suggested [19] that the proximity of the separatrix surface in strongly indented plasmas may act to keep the safety factor profile monotonic near the plasma edge, avoiding the anomalous current penetration usually

attributed to field line reconnection associated with resistive double tearing modes. If this should be the case, the Ohmic currents may penetrate on the plasma skin time and current profiles, as in Fig 2(c), might arise naturally.

The MHD stability of the equilibria generated was studied with a variety of numerical codes. For localized ideal and resistive interchange modes, the quantities D_I , D_R were evaluated on each of the equilibrium flux surfaces [20]. Short wavelength modes were studied by solving the infinite- n ballooning equation [21] (in the incompressible limit), and a critical- n value was obtained by applying the quantization condition to the WKB solution [22]. This analysis was carried out on every third flux surface, and the equilibrium was considered ballooning stable only if it was stable on each of the surfaces tested. For long wavelength modes ($n = 0 - 3$), the PEST 2 code [23] was used to determine the marginal stability boundaries, and PEST 1 [24] was run to illustrate the nature of the eigenfunction. The resistive PEST [25] code was used to study tearing modes.

During the course of the parameter studies reported in the next sections, more than a thousand equilibria were generated and several thousand stability calculations were carried out. Since it was obviously not feasible to monitor the numerical accuracy of each case, convergence studies were performed on representative cases, and the various numerical parameters in these codes were then set to values adequate to resolve those situations. Convergence studies of the equilibria and ballooning results were performed at modest indentation, with $\langle \beta \rangle \sim 15\%$, and the results were compared for several different coordinate systems. Good agreement between the critical- n estimates from the ballooning code and PEST 2 helped to establish an adequate number of expansion functions for the latter. For most of this work, equilibria were then generated on a 51×121 , (ψ, θ) equal arc length mesh; the equilibria were mapped to a 97×128

PEST coordinate mesh for the ballooning study and for $n = 0$ modes, and to a 200×128 mesh for the $n = 2, 3$ internal modes. The finite- n PEST studies used 100 radial finite elements and 30 Fourier modes. The resistive PEST calculations used up to 400 radial elements and 20 Fourier modes.

3. BALLOONING MODES

The existence of a high- β , second region of stability to ballooning modes has been predicted by various theoretical analyses. With modest aspect ratio and typical q -profiles ($1 \leq q \leq 2 - 4$), Chance *et al.* [16], showed not only that indentation lowers the value of β corresponding to the second stable region, but also that there exists a critical indentation beyond which no internal ballooning modes exist. The calculations of this section were carried out to determine how sensitive the existence of the second stable region and the critical indentation for accessibility are to a variation of the various equilibrium parameters. For most of the work the results will be compared to the case studied previously [16], which we shall see was, in many senses, quite optimal. With $q(0) = 1.03$, $q(1) = 4.2$, $q'(0) = 0.84$, $q'(1) = 9.0$, $\alpha = 2$, $\beta = 2$, $R/a = 4$, $b/a = 1.396$, typical profiles are shown in Fig. 3. The marginal stability diagram for infinite- n ballooning modes as a function of indentation is reproduced as the dashed curve in Fig. 4.

3.1 Aspect ratio scaling

The effect of varying the aspect ratio of the rectangular box containing the bean-shaped plasma is illustrated in Figs. 4 and 5. In Fig. 4, marginal stability curves are given for two larger aspect ratio devices, in addition to the standard case. At small indentations, these results are consistent with previously established results for circular plasmas, showing that the lower

critical- β value decreases with aspect ratio. However, we see that the second stable region also sets in at smaller values of β as the aspect ratio is increased. This is particularly noticeable at zero indentation where, for these profiles, only the case for which the aspect ratio equals 10 actually enters the second stable region. The dramatic decrease in the upper critical β , from about 15% at $i = 0$ to approximately 5% at $i = 0.1$, provides an excellent illustration of the effect of even small bean shaping.

In Fig. 5 we have plotted the marginal β values, as a function of R/a at a fixed indentation ($i = 0.304$), close to the critical indentation for complete stability. We see again that higher stable β values in the first region are obtained with low aspect ratio. As R/a is decreased, the lower critical $\langle\beta\rangle$ scales approximately as $(R/a)^{-3/2}$, but in light of the definition of $\langle\beta\rangle$, it is not straightforward to compare this in detail with previous scaling laws for circular or dee-shaped configurations [6]. In the limit of small aspect ratio, however, the second stable region becomes increasingly remote. At $R/a \sim 2$, for example, equilibria with $\langle\beta\rangle \sim 80\%$ are still unstable. As the aspect ratio is increased, the region of instability rapidly shrinks, making it appear more likely that operation above the ballooning limit should be possible. However, at this fixed value of indentation, the instability region does not vanish, at least out to $R/a \sim 20$, which is as far as the present study has been carried. In fact, as seen from Fig. 4, at larger aspect ratio the critical indentation for complete stability at all β is increased, making it necessary to deform the plasma shape more to create an accessible path to the second stable region. Schemes which take advantage of the additional stabilization associated with the injection of high energy particles to pass directly through the unstable ballooning region are also likely to benefit from operating at larger aspect ratio [26].

3.2 Safety factor scaling

A number of calculations have been performed to determine the effect of $q(\psi)$ on the standard accessibility curve of Fig. 4. This is important since varying the q -profile within this model can strongly affect the resulting current profile.

In Fig. 6 we have repeated the calculations of the standard case with flatter q -profiles, generated using Eq. (3) with $q(1)$ equal to 2.7 and 3.2, but keeping $\Delta q'(1)/q(1)$ fixed. This is a model which, in a crude sense, tends to keep the separatrix surface at a fixed distance from the plasma edge. At small indentations, it can be seen that a reduction of the edge q has the effect of removing the second stable region at high β . This appears to be a continuous process in which, at $q(1) \sim 3.2$, even the concept of a critical indentation vanishes. For smaller $q(1)$, while there is a considerable improvement in β for large indentations, no second stable region appears. Nevertheless, it is a characteristic of these high- β , bean-shaped equilibria ($\beta \gtrsim 10\%$) that the WKB critical- n values are quite large ($n_c \gtrsim 100$), for $i \gtrsim 0.2$, and these modes may be stabilized by kinetic effects [27]. For $i \lesssim 0.2$ the boundary of the first stable region scales as $\langle \beta \rangle q(1) \sim \text{constant}$.

The trend shown in Fig. 6 suggests that increasing the edge q value above 4.2 would reduce the critical β for the onset of the second stable region at small indentations and this is shown in Figs. 7 and 8. Here are presented the results of a detailed study for the case $q(1) = 5.2$, which was carried out over a much wider range of β values to gain an understanding of the connection between changes in the equilibrium and the ballooning stability properties as β is increased. Figure 7 gives the stability boundary for this configuration, while in Fig. 8 we indicate the regions of the plasma cross section which are

unstable to ballooning modes for various values of the indentation.

In discussing these diagrams we concentrate first on the ballooning stability properties of those surfaces contained inside surface # 90 of Fig. 8. This information is shown by the solid lines on both diagrams. From Fig. 8 it can be seen that, at small indentation, a broad range of surfaces become unstable at low β , centered around the region where p' is a maximum (near surface 60). This corresponds to the first critical β boundary for $\langle\beta\rangle \approx 2\%$ on Fig. 7. As β increases to the second β boundary near 15%, the last surfaces to stabilize are nearer to the magnetic axis where the global shear is relatively weak. For configurations with $i < 0.4$ this stable region persists until $\langle\beta\rangle \sim 35\%$, when ballooning instability again occurs, this time for a range of surfaces closer to the plasma edge. This corresponds to the upper stability boundary near $\langle\beta\rangle \sim 40\%$, $i \sim 0.2$. Again this range spreads out, involving surfaces nearer to the plasma centre which had, at low β , already entered the second stable region. This effect might not have been expected from previous analytic work based on a local perturbation of equilibrium quantities [29], and reflects the significant modifications to the shape of the surfaces as the magnetic axis shifts at very large β .

Superimposed on Fig. 7 and 8 (dashed curves) are the modifications to this interpretation if we include the ballooning analysis of those equilibrium surfaces between surface # 90 and the plasma edge (#97). Then we find that, at large indentation ($i > 0.3$), equilibria which are stable over the inner 90% of the plasma cross section become ballooning unstable near the plasma edge and there is no second stable region, although there is a strong improvement in the first stability boundary near $i = 0.4$. For $i < 0.3$ a second stability region appears; for $0.25 < i < 0.3$ this ends abruptly when ballooning instability occurs near the plasma edge. For $i < 0.25$ the surface instability

does not occur and the stability boundary moves up close to 40%. Thus, if we include the stability properties of surfaces very close to the plasma surface, we would conclude that the stable regions are confined to lie below the first stability boundary (extended by the dashed curve to $i \sim 0.5$), and inside the region bounded by the solid line near $i = 0.1$ and the dashed curve near $i = 0.3$. We are unable to reach a firm conclusion about this situation because of uncertainties in the accuracy of the numerical equilibrium calculations near the plasma edge. Since only two numerical equilibrium zones are available for representation of the last 10% of the plasma cross section (the equilibrium code uses $\phi^{1/2}$ as a coordinate), the interpolation on the equilibrium mesh may not be sufficiently accurate there, although it is much more accurate inside surface #9. Considerable effort would be required to improve this resolution. Nevertheless, we have included a discussion of this issue because it illustrates the pitfalls associated with numerical calculations of this type which have often lead to difficulties in comparing studies by different authors [29]. The systematic trend which is seen in these results near the plasma surface is not indicative of large numerical errors and it is likely to persist. If it does, we do not believe the impact on the overall stability properties is significant — the critical n numbers are large, the associated growth rates are small, and small modifications in the profiles near to the edge would most likely remove the offending unstable surfaces, moving the stability boundary back to the solid curve of Fig. 7.

When the solid curve of Fig. 7 is considered together with Fig. 6, we obtain a consistent picture of the effect of increasing $q(1)$. For small $q(1)$ improvements in the maximum stable β values occur at large values of the indentation. As $q(1)$ is increased, the stability boundary is bent upwards and in towards smaller indentations, producing an increasingly large second stable

region. Our calculations show that there is always a ceiling to this second stable region, which moves up as $q(1)$ is raised. [This was not observed previously for the standard ($q = 4.2$) case, because the calculations were not carried to sufficiently high β .] While this feature clearly favors large $q(1)$, the value of the critical indentation for accessibility to this second region becomes larger — and hence more difficult to achieve practically.

In addition to the strong dependence on the value of q at the plasma boundary, it is also to be expected that the stability properties should be sensitive to q at the magnetic axis. To illustrate this, we doubled the value of $q(0)$, holding $q(1)$ fixed at 4.2, thus reducing the global shear and increasing the connection length. As seen in Fig. 9, for the case $R/a = 10$ already given in Fig. 4, this has the effect of significantly reducing the critical indentation, which now occurs near $i = 0.07$. Nevertheless, while the second stable region exists even for D-shaped plasmas ($i = 0$) at this aspect ratio, some indentation is helpful in finding ballooning stable paths from low- β configurations.

In addition to separately adjusting the q -profile near the magnetic axis or the plasma surface as done above, it is possible to study the effect of q by scaling the toroidal field at constant $\beta_p \equiv 4\pi p dV / (\mu_0 I_\phi^2 R)$. For low- β equilibria, or if the adjustment is small, the principal effect is a uniform scaling of $q(\phi)$, but both q and q' can be significantly altered at higher β . Figure 10 illustrates the results obtained by taking each of the set of equilibria used to generate the standard case of Fig. 4 and examining the ballooning stability after scaling $q(0)$ to values different from 1.03. As already seen in Fig. 9, increasing $q(0)$ reduces the instability region, making accessibility easier. When $q(0)$ is reduced below 1, it becomes increasingly difficult to stabilize internal, pressure-driven modes. As we shall see in Sec.

4, long wavelength, $n \sim 2,3$ modes with broad eigenfunctions appear in this region of parameter space.

3.3 Pressure profile effects

In the previous section we have seen that the stability properties of bean-shaped plasmas can be quite sensitive to the safety factor profile. In all of those studies a fixed pressure profile, Eq. (2) with $\alpha = \beta = 2$, was employed. Since this profile has small values of p' near the magnetic axis (where the global shear is small), one might expect that it would be favorable with respect to localized ballooning modes. An indication of the advantage of flat pressure profiles has been seen previously in the study of the stabilization of the internal kink mode [17], and, as in that work, we might expect here a strong dependence on the pressure profile.

In Fig. 11 we illustrate that this is indeed the case with two separate calculations. First, we set $\alpha = 1$, $\beta = 2$ [so that $p'(0) \neq 0$], and repeat the stability calculations for the standard case. The results are given by the dashed curve. At modest indentation the first stability boundary is essentially identical to the standard case, but there is no dramatic improvement at large indentation, and no evidence of any second stable region, even up to $\langle R \rangle \sim 20\%$ with $i = 0.35$. Moreover, in that region, the WKB critical- n value is small ($\sim 3,4$), and it is therefore unlikely that kinetic effects would provide any significant stabilization.

While this result is pessimistic, the fact that the standard case does possess a second stable region suggests that some optimization should be possible, and it is of interest to determine what shape the pressure profile must adopt to accomplish this. An iterative procedure to carry this out can be developed from the ballooning mode equation. Consider a low- β equilibrium at fixed indentation which is in the first stable region to ballooning modes. Since

the pressure gradient appears in the infinite- n ballooning equation parametrically, it is reasonable to assume that it could be increased locally on any specific magnetic surface before that particular surface becomes marginally stable. If these adjustments are carried out, the total average β would be increased. Of course, it is necessary to take into account the fact that these modifications in the pressure profile, in turn, lead to a new toroidal equilibrium, which must be computed in order to make the optimization self-consistent. If, at any stage in the iteration between the equilibrium calculation and the solution of the ballooning equation an instability appears, we reduce p' on unstable surfaces to the marginally stable values. Following this procedure for fixed indentation and safety factor profile, either the sequence of equilibria generated will extend to arbitrarily high β , or the process will converge to a pressure profile which is marginally stable to a ballooning analysis on each magnetic surface. This profile then produces the optimal β .

The results of carrying out such a procedure for five values of i , are also shown in Fig. 11. We see that, with modest indentation, significant improvement in the first region of stability can be achieved over both of the fixed pressure profiles discussed earlier. In Fig. 12, optimized pressure profiles are shown for several values of indentation. By comparison with the flat pressure profiles of the standard case, we see that as the cross section is indented the gains have been made by increasing the pressure gradient near to the plasma surface, where the effective indentation of the magnetic surfaces and hence weighting of the good curvature region is larger. In addition to the improvement at small indentation, these calculations demonstrate a smaller critical indentation which must be provided in order to reach the second stable region. Indeed calculations at $i = 0.2$ with optimized pressure profiles remove the instability region of the

standard case. However, we have not yet devised a successful strategy for minimizing the second stable region marginal stability boundary at small indentations. This aspect will be the subject of further studies.

3.4 Effect of plasma elongation

In previous studies it has been established that increasing the ellipticity of the plasma surface can improve the stability to internal pressure-driven modes [5]. These results apply to the critical β corresponding to the stability boundary of the first stable region, and it is of interest to extend this study to configurations exhibiting the second stable region.

The results of such a study are given in Fig. 13. Here we have considered three sequences of equilibria with different values of elongation, b/a (see Fig. 1), but with identical pressure ($\alpha = \beta = 2$) and safety factor profiles $q(1) = 2.7$ (as in Fig. 6), $i = 0.25$, and $R/a = 3$. For each case we have plotted the WKB critical- n value versus average β . As β is increased, ballooning modes first appear with $n_c = \infty$, corresponding to the stability boundary of the first stable region. At low β these results illustrate that elongation increases the possible stable β values. Above the marginal β value, increasing the pressure lowers the value of n for which instability occurs, with longer wavelength modes becoming unstable. If the equilibrium configuration is to display a second stable region, n_c must eventually become infinite again as β is raised. For the sequences studies here, only the case $b/a = 1$ has this property, entering a second stable region at $\langle\beta\rangle \approx 10\%$. At higher values of β the configuration becomes unstable again near the plasma surface, a feature similar to that discussed with respect to Fig. 7. The other configurations, with elongated boundaries, do not display a second stable region, although the results for the standard case ($b/a = 1.4$) indicate a strong improvement in stability for $\langle\beta\rangle > 15\%$, and it could be

expected that FLR effects would stabilize these modes. However, from the results for the $b/a = 1.7$ case, it can be clearly seen that vertically elongating bean-shaped plasmas does not enable easier access to the second stable region. On the other hand, these results suggest that reducing b/a below 1 should lead to additional reduction in the β value corresponding to the second stable region, and, therefore, that some indentation could allow accessibility to the second stable region exhibited by tokamaks with horizontally elongated cross sections [30].

3.5 Finite ion Larmor radius stabilization

It has been established that for large values of n , kinetic modifications to the MHD analysis of internal ballooning modes can be very important [27]. In particular, the stabilizing finite gyroradius contribution from ion diamagnetic drifts acts directly against the interchange driving mechanism and becomes significant when $k_{\perp}^2 \rho_i^2$ is comparable to L_p/L_k . Here, ρ_i is the ion gyroradius and L_p and L_k are the respective scale lengths for the pressure gradient and field-line curvature. Since the worst ideal MHD cases exist for the smallest value of L_p , substantial modifications can result even for small values of $k_{\perp} \rho_i$. Previous calculations have shown that when both the radially nonlocal (finite- n) corrections and the finite gyroradius effects are simultaneously included, the resultant estimates of critical β can be significantly more optimistic for specific equilibria [27]. Hence, an analysis of this type for bean-shaped tokamak equilibria is of considerable interest.

We have applied the procedure described in Ref. [27] to representative bean-shaped tokamak equilibria, and found that the kinetic effects can lead to significantly more optimistic β -criteria. This is illustrated in Fig. 14 where the kinetically modified stability boundaries are shown as solid

curves. For typical parameters, the actual values of $k_{\perp} \rho_i$ in these calculations remain well below unity.

In this analysis attention was focused on kinetic modifications due to the ion diamagnetic drift term, since this factor alone directly influences the interchange driving mechanism of ideal MHD ballooning modes. However, other kinetic effects, such as wave particle resonances and trapped particle dynamics, can also be important. In particular, collisional and collisionless dissipative processes can destabilize both the drift and shear Alfvén waves leading to residual growth rates on the drift wave time scale. Here again the dominant kinetic modes or microinstabilities are strongly influenced by curvature effects. Since bean-shaped tokamak equilibria generally exhibit more favorable curvature properties, it is likely that microinstabilities will also be favorably influenced. These effects will be investigated in future studies of bean-shaped configurations.

4. LONG WAVELENGTH MHD MODES

In addition to a study of internal ballooning modes, information about the nature of the eigenfunctions of large scale instabilities, and about instabilities when the plasma is surrounded by a vacuum region (external modes), is needed in order to assess the overall stability properties of a tokamak configuration. Previous work on the $n = 1$ mode has shown that the internal kink can be stabilized by indentation, while at finite β and indentation, stability to the external kink depends on wall stabilization [17]. In this section, we consider several additional classes of possible instability, each of which should have some implication for bean-shaped tokamak experiments.

4.1 Axisymmetric ($n = 0$) modes

It is well known that the simple ideal MHD model of an infinitely conducting, noncircular, cross-sectional plasma surrounded by a vacuum region can be unstable to axisymmetric ($\partial_\phi = 0$) motions. If the cross section is vertically elongated, the unstable motion is primarily a rigid vertical shift, while a horizontally flattened shape is unstable to translation in the midplane. Such motions can be passively stabilized on ideal MHD time scales by the presence of nearby conductors, where eddy currents are generated which oppose the motion. It is of interest to determine where to place such conductors most effectively to aid the experiment design. To do this, we have studied the $n = 0$ stability of a typical bean-shaped plasma using various versions of the PEST code, with vacuum boundary conditions approximating different conductor locations. We consider a $\langle\beta\rangle = 3\%$ equilibrium with $i = 0.3$, $q(0) = 0.84$, $q(1) = 3.7$, $R/a = 3.3$, and $b/a = 1.52$. The pressure and poloidal current profiles used are similar to those obtained from Thomson scattering and magnetic flux data in typical PDX shots. The equilibrium was computed as a free-boundary calculation using a specific external coil set (Fig. 25) so that the plasma surface does not conform exactly to Eq. (1).

In Figs. 15 and 16 we illustrate the effectiveness of moving a D-shaped conducting wall (see Fig. 1) closer to the plasma surface. Without a conducting shell (Fig. 15), we can see that the plasma displacement combines a nearly rigid vertical shift with a circular, sliding component which follows the equilibrium flux contours in the indented region of the cross section. As the wall is moved uniformly closer (Fig. 16), it first stabilizes the vertical shift component, so that the instability takes on the form of sliding motion towards the tip of the bean, with smaller growth rate. Since, in this model, the most effective stabilization comes from placing a conductor perpendicular

to the displacement of a constant flux surface, it is clear that it is the proximity of the conducting surface near the tip of the bean which is most important at this stage. The instability becomes marginal when $B = A_w/A = 1 \sim 0.65$.

Further investigation of the stabilizing contribution from the region near the tips of the bean is important for the PBX experiment, since the conducting wall can only be located there, in practice, because the larger major radius region must be kept free to accommodate the planned range of indented plasmas. Such a partial wall was simulated by modifying the definition of the wall location in the PEST code to allow certain sections of the wall to be moved out to a large distance from the plasma. As illustrated in Fig. 17a, the stabilizing effect of discrete conducting plates can then be described by two parameters — the length of the nearby section of the wall and the distance of this section to the plasma surface. Calculations have been performed to determine the dependence on both of these quantities. As an example, Fig. 17b shows the normalized growth rate versus the wall distance, for the section indicated in Fig. 17a. Stability occurs when $B \approx 0.3$, giving a wall separation for the PBX experiment of 9-10 cms.

While an idealized calculation such as this is essential to enable positional control on the extremely fast ideal MHD time scale, it is important to appreciate that, for time scales characteristic of the resistive skin times of the wall or plasma, instability will usually still occur as the stabilizing eddy currents resistively decay. The design of a suitable active feedback system, which takes into account the resistive plasma dynamics, is a critical aspect of any tokamak experiment where non circular, cross-sectional shaping is to be carried out by external poloidal field programming.

4.2 Low n internal modes with $q(0) < 1$

Most tokamaks operate with $q(0) < 1$. While on slow resistive MHD time scales the dynamics of low- β discharges are determined by resistive relaxation inside the $q = 1$ surface, at higher β the stability to pressure driven modes with $q(0) < 1$ may dominate the plasma behavior, especially if long wavelength instabilities are excited. Such a possibility is suggested by the calculations presented in Fig. 10, which showed that the stability properties deteriorated strongly when $q(0)$ was reduced. Since the internal $n = 1$ kink has been shown to be strongly stabilized by indentation, it is natural to consider the behavior of other low n modes at low $q(0)$.

Figures 18 and 20 present the results of a set of calculations for internal $n = 2, 3$ modes carried out in an identical fashion to the internal kink studies of Ref. 15, using the PEST 2 code. For three different values of indentation, each member of a sequence of equilibria with parameters of the standard case of 3.1 were scaled, at constant β_p , to generate sets of equilibria with different $q(0)$. Testing the stability of these permits one to construct marginal stability curves in the β_p vs $q(0)$ plane. Both figures show that at finite β these long wavelength modes set in at modest values of β_p over a range of values of $q(0)$, and present a significant limitation to reaching high- β configurations.

For the $n = 2$ mode of Fig. 18, instability sets in with $\beta = 0$ at the Mercier limit, $q(0) = 0.5$, and the most unstable mode is localized near the magnetic axis and dominated by the $m = 2$ poloidal component. Configurations with $q(0) > 0.5$ are stable at low β_p , but as β_p is increased, a stability threshold appears. Without indentation this threshold is insensitive to $q(0)$ for values ≤ 0.9 , but rises quite rapidly near $q(0) = 1$. For both bean-shaped configurations the threshold increases smoothly as $q(0)$ approaches 1. In that

limit it is interesting to note that the β_p limit decreases with indentation, a result which can also be seen in the $n = \infty$ ballooning limits shown in Fig. 10 with $q(0) = 0.8$ or 0.9 . Figure 19 illustrates the instability eigenfunction of a typical case [$q(0) = 0.7$, $\beta_p = 3.74$, $i = 0.185$]. As $q(0)$ is increased from 0.5 to 1.0 , the modes become nonlocalized in ψ_r with poloidal structure involving many Fourier harmonics, typical of ballooning modes.

For the $n = 3$ mode, shown in Fig. 20, the picture is very similar. It differs from the $n = 2$ case primarily in the tendency for additional localized modes to set in at low β_p near the $q = 2/3$ mode rational surface, and in the fact that the threshold is always lower, as expected from ballooning mode theory. Typical eigenfunctions close to the stability boundary for several different values of $q(0)$ are shown in Fig. 21, illustrating the poloidal and radial structure of the most unstable modes. Figures 21(a) and 21(b) show localized interchange modes which would be well described by the Mercier criterion. As β_p and $q(0)$ are increased, Fig. 21(c) and 21(d) illustrate that the modes become nonlocalized, with poloidal variation involving the coupling of many Fourier harmonics.

In the calculations presented in Figs. 18 and 20, there is no indication of a second stable region for $q(0) < 1$, at least for $\langle \beta \rangle \lesssim 20\%$. If such a region does exist at much larger $\langle \beta \rangle$, the results indicate that it is very unlikely that stable paths exist at low $q(0)$ values. Since the modes have broad eigenfunctions and exhibit a β threshold, they may also be candidates for driving beam injection instabilities, similar to the fishbone modes presently identified with near perpendicular injection in internal kink unstable, PDX configurations.

4.3 Effect of indentation on resistive modes

The major emphasis in this work on the aspects of bean shaping has been associated with the enhanced stability to ideal MHD modes. It is useful to consider the effect on resistive MHD modes, and we report here on a study of the $n = 1$ tearing mode carried out using the resistive PEST code [25]. This code carries out the complete asymptotic matching procedure for a finite β , axisymmetric, toroidal MHD equilibrium. It computes the growth rate of low n resistive instabilities by solving the boundary layer dispersion relation obtained by matching numerical solutions of the resistive boundary layer equations [20] around each rational surface to the marginally stable solutions of the ideal MHD equations between these singular regions.

To illustrate the effect of indentation on the kink driving terms we consider two pressureless equilibrium sequences with different values of $q(1)$. In the first sequence we take the safety factor profile of the standard case, ($1.03 < q < 4.2$), so that for $n = 1$ there are three rational surfaces, and we compute the growth rate of the unstable resistive mode. In the second sequence we take a similar profile to the standard case but with $q(0) = 1.1$ and $q(1) = 2.9$, so that there is just one rational surface at $q = 2$. For this case the stability to the $m = 2$ tearing mode is determined by the usual criterion $\Delta'_{22} < 0$, where Δ'_{22} is the ratio of the small to big $m = 2$ poloidal harmonic of the ideal MHD solution across the $q = 2$ surface.

Results for the standard case with a conducting wall on the plasma boundary are shown in Fig. 22. We find that there is only one resistive tearing instability associated with the $q = 2$ surface. This mode is seen to be strongly stabilized by indentation and becomes a damped oscillation for $i > 0.2$. With fixed q profile, this stabilization can be understood in terms of a flattening of the equilibrium current profile at the $q = 2$ surface as i

is increased. While the current gradient usually increases on the outer rational surfaces, the steepening up there is not sufficient to drive $n = 1$ modes unstable. It may destabilize higher n tearing modes near the plasma surface, although these are rarely seen in tokamak devices and are more difficult to destabilize in a cylindrical model. Such modes would lead to small magnetic islands near to the plasma edge, and possibly degrade confinement. Additional work is needed to understand this effect in detail.

When the conducting wall is moved away from the plasma surface, the tearing modes can be strongly destabilized. An example of this is shown in Fig. 23 where we have considered the barely stable case, $i = 0.186$ (arrow) of Fig. 22. With just a small movement of the wall a predominantly $m = 2$ ($n = 1$) tearing mode becomes unstable at the $q = 2$ surface. As the wall is moved out near to 0.2 plasma radii, a second tearing mode becomes unstable. Near to the marginal wall position for this mode, it has the structure of an almost pure $m = 3$ tearing mode, centered at the $q = 3$ surface. As this second mode becomes more unstable, the two modes ($m = 2$ and $m = 3$) mix, each producing islands around both the $q = 2$ and 3 surfaces. The $m = 3$ component grows most strongly as the wall is moved further out to where the ideal MHD external kink mode is destabilized. With $q(1) = 4.2$, this ideal mode is predominantly $m = 5$, and it is clearly the coupling of this, through ellipticity, which causes the $m = 3$ tearing mode growth rate to increase so rapidly.

For the flatter q profile ($1.1 < q < 2.9$) we find stability to the $m = 2$, $n = 1$ tearing mode even at zero indentation (Fig. 24). This is consistent with previous calculations which show that toroidicity can be strongly stabilizing for low shear tokamak equilibria [25]. As the conducting wall is moved away from the plasma surface, the results for an indentation $i = 0.1$ show little variation until the wall is close to the marginal point for the

$n = 1$ (mostly $m = 3$) ideal MHD external kink mode, at which point the $m = 2$ tearing mode is strongly destabilized (Fig. 24, solid dots). We note in Fig. 24 that with this current profile the wall must be very close to stabilize the $n = 1$ external kink. Additional studies are needed to understand the effect of plasma current profile on these external ideal MHD modes.

These results suggest that strong indentation and reasonable profile control may provide for some stability against the nonlinear tearing phenomena which are believed to be responsible for plasma disruptions. Fully 3D MHD nonlinear codes must be employed to study this.

5. DISCUSSION

In this work an extensive parameter survey has been carried out to help evaluate the favorable MHD stability properties of bean-shaped tokamak plasmas. While one must be careful not to extrapolate such results too far from the parameter space considered, it is possible to draw various conclusions, which may be helpful in indicating useful experiments or in suggesting future research areas.

Much of this work has centered around ballooning modes and an understanding of the second stable ballooning region, which offers many attractive possibilities for future tokamak reactor designs. The results presented here support the previous observation that this region is difficult to find in circular and Dee-shaped tokamak configurations, and that indentation is a critical factor.

We have found that the ballooning mode stability properties can be sensitive to equilibrium parameters, especially at high- β values near the second stable region. Variation of the safety factor showed that the second stable region increased as the edge q was raised, although the critical

indentation needed for accessibility increased also. Raising q at the magnetic axis helped to alleviate this problem. Ballooning modes quickly set in as q on-axis was lowered below 1, removing the second stable region. Variation of the pressure profile showed that, while this can eliminate the second stable region, it is also possible to enhance strongly the stability properties, and to reduce the critical indentation needed to reach favorable high- β configurations. Further improvement in the stability properties was also shown to be possible, since near the stability boundaries the ballooning modes have very short wavelengths and kinetic effects were found to exert a significant stabilizing influence.

Variation of the aspect ratio has clarified different options for reaching high- β values; operating in the first stable region at small aspect ratio and low q , or in the second region at larger aspect ratios and higher q . Given a favorable scaling of confinement time with aspect ratio (e.g., Alcator, TFTR) and the possibility of avoiding resistive instabilities through indentation, these results indicate large aspect ratio ($A \sim 6 - 10$) bean-shaped tokamaks operating in the second stable region would be interesting reactor candidates. In addition to providing the local shear which improves the stability properties, the indentation strengthens the poloidal field on the outboard side enhancing the confinement properties.

While a study of internal ballooning modes gives a good indication of the MHD stability properties at high β , it is necessary to supplement this work with consideration of current driven modes and calculations which illustrate the nature of the most unstable perturbations. Adding to the work on external kinks reported before, we examined the $n = 0$ mode, identified the behavior of the unstable motion, and showed that stability could be achieved by placing conducting plates sufficiently close to the tips of the bean-shaped cross

section. We briefly considered the $n = 1$ tearing mode and found that indentation may have a stabilizing effect on some resistive instabilities. Finally, we considered the low- n internal modes with $q(0) < 1$, found that these have moderately low β threshold values, and that they would most likely prevent reaching second stable regions unless $q(0)$ is kept above unity.

In this work we have attempted to gain some qualitative understanding of the effect of indentation on various instabilities in bean-shaped tokamaks. The next relevant project would be to optimize the configuration. Since we have identified a sensitivity to a number of equilibrium parameters, this should be an interesting task to undertake. Although this sensitivity complicates any analysis, because it makes it difficult to isolate a single, simple physical mechanism to describe the plasma behavior, it nevertheless suggests that there are a variety of possible procedures to achieve stable high- β configurations - many of which allow accessibility to the second stable region. Several of them, such as plasma shaping and choice of aspect ratio, are within the scope of present day experimental and engineering techniques. Others, which involve control of the pressure and safety factor profiles, are not. However, they may be feasible in future experiments involving, for example, refined RF heating and current drive schemes. At the very least, the studies reported here, which demonstrate the existence of very favorable configurations for a variety of parameter values, should stimulate theoretical work to improve our understanding and the ability to model tokamak behavior. Optimization of the properties illustrated here rely to a large extent on this capability.

With respect to the forthcoming PB4, this work identifies several significant issues. Most importantly, the ability to form indented plasmas relies on controlling the axisymmetric instability. While conducting wall

stabilization can reduce the growth rate from the fast ideal MHD time scale, creating high- β beams will depend on designing active feedback systems which stabilize motions on the resistive times characteristic of confinement experiments. This is particularly important since it is planned to form bean-shaped cross sections by the process of continuous deformation from low- β circular discharges.

A second issue in reaching high- β values involves avoiding "fishbones", which have been identified as a major mechanism for β -saturation in neutral-beam-heated PDX experiments. While sufficient indentation may increase or eliminate the β threshold associated with the internal $n = 1$ kink mode, other low- n ballooning modes studied here may have a similar effect when $q(0)$ drops below 1. Since the calculations presented here did not show any improvement with indentation for these modes, they pose a concern for near perpendicular injection. Achievement of high- β values may then depend strongly on the orientation of the injected neutral beam.

If both these difficulties can be overcome experimentally, additional calculations of the sort reported here will be needed to interpret the consequences of ballooning modes and the effect of indentation. This will clearly be necessary because the sensitivity to various parameters, illustrated by this work, makes it probably impossible to predict whether or not a second stable region can be identified. One obvious uncertainty involves the current profiles associated with the ideal MHD equilibria employed here. These profiles, obtained by specifying $q(\psi)$, can be quite different from what is observed in present day, low- β , almost circular, tokamak discharges. Attempts to find the second stable region with more conventional current profiles have not been successful, and while not entirely conclusive, that suggests that additional profile control would be needed to

achieve all the favorable aspects of indentation. For the PBX experiment, where such control is not available, this may indicate that achieving second stable states is unlikely. However, calculations with equilibrium profiles typical of actual data taken from the PDX experiment [1], show that substantial improvements in β should be possible with indentation. An example of a high- β equilibrium ($\sim 10\%$) with typical parameters for the PBX experiment is shown in Fig. 25. Stability studies have shown that this configuration is stable against the internal kink at least as high as $\langle\beta\rangle \sim 20\%$, and against ballooning modes to $\langle\beta\rangle \sim 9\%$. These values are in marked contrast with $\beta_c = 1.4\%$ (internal kink) and $\beta_c = 2.7\%$ (ballooning) previously reported for PDX circular high- β /low- q discharges.

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REFERENCES

- [1] D. Johnson, et al., Plasma Physics and Controlled Nuclear Fusion Research 1982, Vol. 1, IAEA, Vienna, (1983) 1.
- [2] K. H. Burrell, et al., Nucl. Fusion 23 (1983) 536.
- [3] B. A. Carreras, P. H. Diamond, M. Murakami, J. L. Dunlap, et. al., Phys. Rev. Lett. 50 (1983) 503.
- [4] K. McGuire, R. Goldston, et al., Phys. Rev. Lett. 50 (1983) 891.
- [5] A. M. M. Todd, J. Manickam, M. Okabayashi, M. S. Chance, R. C. Grimm, J. M. Greene, and J. L. Johnson, Nucl. Fusion 19 (1979) 743.
- [6] R. C. Miller and R. W. Moore, Phys. Rev. Lett. 43 (1979) 765; L. C. Bernard and R. W. Moore, Phys. Rev. Lett. 46 (1981) 1286.
- [7] L. C. Bernard, F. J. Helton, R. W. Moore, and T. N. Todd, Nucl. Fusion 23 (1983) 1475.
- [8] J. L. Luxon, N. W. Brooks, et al., Proceedings of the 11th European Physical Society Conference on Controlled Fusion and Plasma Physics, Aachen, (1983), Vol. 7D, Part 1, 99.
- [9] H. P. Furth, J. Killeen, M. N. Rosenbluth, B. Coppi, Plasma Physics and Controlled Nuclear Fusion Research, Vol. 1, IAEA, Vienna, (1966) 103.
- [10] C. Mercier in Lectures in Plasma Physics, EURATOM-CEA CEN EUR 5127e, Luxembourg (1974).
- [11] L. C. Bernard and R. W. Moore, Phys. Rev. Lett. 46 (1981) 1286.
- [12] H. R. Strauss, W. Park, D. A. Monticello, R. B. White, S. C. Jardin, M. S. Chance, A. M. M. Todd, and A. H. Glasser, Nucl. Fusion 20 (1980) 638.
- [13] B. Coppi, A. Ferreira, J. W.-K. Mark, and J. J. Ramos, Nucl. Fusion 29 (1979) 715.
- [14] L. Sugiyama and J. W.-K. Mark, Phys. Lett. 84A (1981) 123.
- [15] D. A. Monticello and S. C. Jardin (to be published).

- [16] M. S. Chance, S. C. Jardin, and T. Stix, *Phys. Rev. Lett.* 51 (1983) 1963.
- [17] J. Manickam, R. C. Grimm, and M. Okabayashi, *Phys. Rev. Lett.* 51 (1983) 1959.
- [18] For example; J. L. Johnson, H. E. Dahled, J. M. Greene, R. C. Grimm, Y. Y. Hsieh, S. C. Jardin, J. Manickam, M. Okabayashi, R. G. Storer, A. M. M. Todd, D. E. Voss, K. E. Weimer, *J. Comput. Phys.* 32 (1979) 212.
- [19] H. P. Furth, private communication.
- [20] A. H. Glasser, J. M. Greene, and J. L. Johnson, *Phys. Fluids* 18 (1975) 875.
- [21] M. S. Chance, et. al., in *Plasma Physics and Controlled Nuclear Fusion Research 1978*, Vol. III, IAEA, Vienna, (1979).
- [22] R. L. Dewar, J. Manickam, R. C. Grimm, and M. S. Chance, *Nucl. Fusion* 21 (1981) 493.
- [23] R. C. Grimm, R. L. Dewar, and J. Manickam, *J. Comput. Phys.* 49 (1983) 94.
- [24] R. C. Grimm, J. M. Greene, and J. L. Johnson, *Math. Comput. Phys.* 9 (1976) 253.
- [25] R. C. Grimm, R. L. Dewar, J. Manickam, S. C. Jardin, A. H. Glasser, and M. S. Chance, in *Plasma Physics and Controlled Nuclear Fusion Research 1982*, Vol. III, IAEA, Vienna, (1983) 35.
- [26] M. N. Rosenbluth, S. T. Tsai, J. W. Van Dam, and M. G. Engquist, *Phys. Rev. Lett.* 51 (1983) 1967.
- [27] W. M. Tang, R. L. Dewar, and J. Manickam, *Nucl. Fusion* 22 (1982) 1079.
- [28] J. M. Greene and M. S. Chance, *Nucl. Fusion* 21 (1981), 453.
- [29] P. N. Vabishchevich, L. M. Degtyarev, and S. Yu. Medvedev, *Sov. J. Plasma Phys.* 9 (1983) 100.
- [30] H. E. Dalhed, R. C. Grimm, and J. L. Johnson, *Nucl. Fusion* 22 (1982) 883.

FIGURE CAPTIONS

- FIG. 1. Parameters used to describe the bean-shaped plasmas studied in this work.
- FIG. 2. Midplane profiles of $J\phi$. Figure (a) and (b) are at low $\langle\beta\rangle \approx 0.1\%$, Fig. 2(c) is at $\langle\beta\rangle \approx 10\%$. Other parameters are as for Fig. 4.
- FIG. 3. Typical pressure, safety factor, and toroidal field ($B_T \alpha g(\phi) \nabla\phi$) profiles used in the stability calculations.
- FIG. 4. Marginal stability boundaries for ballooning modes in three different aspect ratio devices. The "standard" case, $R/a = 4$, is dashed.
- FIG. 5. Marginal stability boundaries for ballooning modes as a function of aspect ratio for fixed indentation, $i = 0.304$, showing the first and second stability regions. Also shown are the boundaries for Dee-shaped plasmas, $i = 0$. Other parameters are as for Fig. 4.
- FIG. 6. Marginal stability boundaries for ballooning modes as $q(1)$ is reduced from the standard case.
- FIG. 7. Marginal stability boundary for ballooning modes with $q(1) = 5.2$. The solid line describes the boundary when ballooning instabilities near the plasma surface are ignored. Accessibility through indentation does not occur when the edge instabilities are included (dashed curve).

- FIG. 8. Showing the regions of the cross section which are ballooning unstable as $\langle\beta\rangle$ is varied for a set of equilibrium sequences with different indentation. The abscissa is proportional to $\psi^{1/2}$.
- FIG. 9. Marginal stability boundary for ballooning modes as $q(0)$ is increased, for $R/a = 10$, bean-shaped plasmas.
- FIG. 10. Ballooning mode stability boundaries showing the variation as $q(0)$ is reduced.
- FIG. 11. Ballooning mode stability boundaries for different pressure profiles; solid line, $\alpha = 2$, dotted line, $\alpha = 1$. Results of profile optimization are also shown.
- FIG. 12. Optimized pressure profiles for a Dee-shaped plasma ($i = 0$) and a modestly indented plasma ($i = 0.185$) compared to the standard profile (dashed). ($\hat{\psi} \equiv 1 - \psi/\Delta\psi$).
- FIG. 13. Critical- n value on the most unstable surface as a function of $\langle\beta\rangle$, for $q(0) = 1.03$, $q(1) = 2.7$, $i = 0.25$, $R/a = 3$ and three different values of the plasma elongation, b/a .
- FIG. 14. Kinetically modified stability boundaries (solid lines) for bean-shaped equilibria with $q(1) = 3.2$ (as in Fig. 6). Solid (open) dots represent equilibria unstable (stable) to ideal MHD ballooning modes. The FLR modified boundaries are shown for two different particle densities.

FIG. 15. Typical $n = 0$, axisymmetric instability for a bean-shaped plasma equilibrium with surrounding vacuum extending to infinity.

FIG. 16. Axisymmetric instability in the presence of a Dee-shaped conducting wall (Fig. 1) at $A_w/A = 1.65$.

FIG. 17(a) Conducting wall used to simulate the use of axisymmetric conducting plates near the bean tips.

FIG. 17(b) Normalized growth rates showing the stabilizing effect of moving conducting walls closer to the plasma surface. Solid dots are for the partial wall of Fig. 17(a), circles for the solid wall of Fig. 1.

FIG. 18. Stability boundary for internal $n = 2$ modes with $q(0) < 1$. Results are given for a Dee-shaped plasma ($i = 0$) and two different values of the indentation.

FIG. 19. Displacement vector of a high β , $n = 2$ internal mode with $q(0) = 0.7$, $\beta_p = 3.74$, and $i = 0.185$. The eigenfunction has finite radial extent.

FIG. 20. Stability boundary for internal $n = 3$ modes for the same equilibrium configurations as Fig. 18.

FIG. 21. Radial structure of unstable eigenvectors from PEST 2, for four different values of $[q(0), \beta_p]$ close to the stability boundary for $i = 0.3$ in Fig. 20. a) $q(0) = 0.31$, $\beta_p = 0.72$, b) $q(0) = 0.6$, $\beta_p = 1.31$, c) $q(0) = 0.68$, $\beta_p = 2.23$, and d) $q(0) = 0.87$, $\beta_p = 3.66$.

- FIG. 22. Effect of indentation on the growth rate of the $n = 1$ tearing mode ($\eta = 10^{-6}$) for bean-shaped plasmas with $q(0) = 1.03$, $q(1) = 4.2$, $R/a = 4$, $b/a = 1.386$. Calculations at values of i indicated by the two arrows give damped, stable modes.
- FIG. 23. Effect of moving the wall away from the plasma surface on Fig. 22, showing the onset of two resistive tearing modes and an ideal kink mode.
- FIG. 24. Effect of indentation (open circles, bottom axis) and wall position, B (dots, top axis), on the $m = 2$, $n = 1$ tearing mode for equilibria with $q(0) = 1.1$, $q(1) = 2.9$.
- FIG. 25. Bean-shaped, flux surfaces for a $\langle \beta \rangle = 10\%$ equilibrium in PBX. Also shown are poloidal field coil locations and external conducting stabilizer plates.

#83T0273

Fig. 1

#83T0186

Fig. 2

Profiles
 $\langle \beta \rangle = 8.75\%$
 $i = 0.304$

Fig. 3

Fig. 4

83T0454

Fig. 5

Fig. 6

#83T0470

Fig. 7

#83T0467

Fig. 8

Fig. 9

Fig. 10

Fig. 11

#83T0466

Fig. 12

Fig. 13

Fig. 14

83P0294

Fig. 15

Fig. 16

#83T0473

Fig. 17

#83T0485

Fig. 18

83T0453

Fig. 19

#83T0469

Fig. 20

Fig. 21

#83T0450

Fig. 22

#83T0486

Fig. 23

Fig. 24

83P0097

Fig. 25

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