Sh. 724

COO/2597-2

MASTER

STUDIES ON METHODS OF REDUCING HEAT LOSSES
FROM FLAT PLATE SOLAR COLLECTORS

Progress Report, August 1, 1974—January 31, 1976

K. G. T. Hollands
G. D. Raithby
T. E. Unny

June 1976

Work Performed Under Contract No. E(11-1)-2597

Waterloo University
Ontario, Canada

ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
Division of Solar Energy

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

This report has been reproduced directly from the best available copy.

Available from the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22161

Price: Paper Copy \$6.00 (domestic)
\$8.50 (foreign)
Microfiche \$3.00 (domestic)
\$4.50 (foreign)

STUDIES ON METHODS OF REDUCING
HEAT LOSSES FROM FLAT PLATE
SOLAR COLLECTORS

Progress Report
for the Period

August 1, 1974 to January 31, 1976

K.G.T. Hollands
G.D. Raithby
T.E. Unny

University of Waterloo
Waterloo, Ontario, Canada
N2L 3G1

Date Published June, 1976

PREPARED FOR THE UNITED STATES ENERGY RESEARCH
AND DEVELOPMENT ADMINISTRATION
DIVISION OF SOLAR ENERGY
UNDER CONTRACT NO. E(11-1)-2597

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *EB*

ABSTRACT

Improvements to flat plate solar collectors for heating and cooling of buildings have been evaluated through three parallel studies. The first study has demonstrated suppression of free convection by a honeycomb at all angles of inclination. Reductions by a factor of 5 in the free convective heat loss are reported, and equations and charts permitting the rational design of honeycombs for convection suppression in solar collectors are presented. The second study has resulted in a precise, easily programmable set of equations for the free convective heat loss through the air gaps in flat plate solar collectors, for any angle of tilt. The third study has theoretically and experimentally demonstrated that the solar transmittance of a flat transparent cover for a solar collector can be very substantially increased by V-corrugating. Since V-corrugating should also decrease the free convective and radiant losses, substantial improvements in collector efficiency are predicted.

CONTENTS

	Page
Abstract	i
Contents	ii
List of Figures	iv
1. Summary	v
2. Introduction	1
3. Free Convection Across Inclined Air Layers Constrained by a Honeycomb	5
4. Free Convection Across Inclined Air Layers Not Con- strained by a Honeycomb	15
5. Solar Transmittance Of a V-Corrugated Transparent Sheet	21
6. Conclusions	25
7. References	26
8. List of Contributing Personnel	28
9. Patents Applied For	29
10. Nomenclature	30
11. List of Papers Based on Studies Described in This Report	32
12. Appendices	33
Appendix A: "Natural Convection Across Inclined Honey- comb Panels", R.L.D. Cane, K.G.T. Hollands, G.D. Raithby, T.E. Unny.	34
Appendix B: "Numerical Study of Free Convective Heat Transfer Across Rectangular Cells". S. Rajaram, T.E. Unny, K.G.T. Hollands, G.D. Raithby.	61

CONTENTS(cont'd)

	Page
Appendix C: "Radiant Transmittance of V-Corrugated Trans- parent Sheets, With application To solar Col- lectors", B.E. Sibbitt, K.G.T. Hollands.	103

LIST OF FIGURES

	Page
3-1 Experimental Nusselt numbers as of function of Rayleigh number, for honeycombs of various aspect ratios.	9
3-2 Plot of $C(\theta)$ vs. θ . $C(\theta)$ is to be used in equation (3-6)	12
4-1 Plot of the Free convective heat transfer coefficient across an air gap as a function of the gap spacing, L .	18
5-1 Predicted Transmittance of sheets of Lexan, Mylar and glass, all V-corrugated at an opening angle of 30° .	22

1. SUMMARY

This report describes the results of three studies, all of which are related to methods for improving the efficiency of flat plate solar collectors. The first study relates to the suppression of free convective heat transfer in an inclined air layer by means of insertion of a honeycomb. This study has demonstrated experimentally that considerable suppression is possible in the inclined position, a reduction of 80% being observed for a honeycomb of aspect ratio of 5. Equations and charts permitting rational design of honeycombs for convection suppression in solar collectors are given.

The second study relates to the free convection in an inclined air layer not containing a honeycomb, such as occurs in a conventional flat plate collector. This experimental and theoretical study has resulted in a set of definitive relations predicting the free convective heat loss as a function of the relevant variables. Some recommendations for the best spacing between the absorber plate and the transparent cover, and between adjacent transparent covers, are given.

The third study relates to the use of a V-corrugated transparent sheet, rather than a flat sheet, as an inner cover in a solar collector. This analytical and experimental study has demonstrated that the solar transmittance of the V-corrugated sheet will generally be higher than the corresponding flat sheet, so that increased collector performance can be expected.

2. INTRODUCTION

Methods must be found for reducing heat losses from the conventional flat plate solar collector, if it is to perform efficiently in solar heating and cooling systems for buildings. In particular some additional mechanism or mechanisms must be incorporated into the design in order to substantially decrease losses from the collector top. These mechanisms must be economic, durable and practical. More important, they must not substantially decrease the solar radiation input to the collector plate.

The development of new such mechanisms, and the study of critical problems associated with mechanisms already suggested, was the broad objective of the research described in this report.

This broad objective was pursued through three parallel studies:

(i) a study of suppression of free convective heat transfer across inclined air layers by the insertion of a honeycomb;

(ii) a definitive study on free convective heat transfer through inclined air layers without a honeycomb;

(iii) a study of a new device for minimizing top losses, - the corrugated inner cover.

The specific objectives of these individual studies were as follows:

Study 1: Natural convection heat losses in tilted honeycomb collectors.

That honeycombs suppress free convective heat transfer in the horizontal position had already been demonstrated; moreover the conditions under which suppression is achieved in this situation had been largely defined. However, very little was known about these phenomena for the inclined position. The phenomena were therefore examined, both theoretically and experimentally. The experimental study, carried out on the University of Waterloo Natural Convection Apparatus, was to determine if virtual suppression of convective heat transfer is possible in the inclined condition, and if so, to try to define the conditions under which virtual suppression takes place. The theoretical study was to examine the base flow in the honeycomb cell, so as to point the way to the proper interpretation of the experimental results.

Study 2: Natural convection in inclined air layers.

The objective of this study was to make a new set of measurements on this important problem, using the University of Waterloo Natural Convection Apparatus; and to interpret the results, using recently-acquired theoretical knowledge of the fluid flow in fluid layers, so as to produce a set of reliable, accurate and simple equations for predicting free convective heat loss across parallel plates in solar collectors.

Study 3: The V-corrugated inner cover.

The V-corrugated inner-cover is structured in such a way so as to redirect reflected solar rays towards the absorbing plate so that they are not lost. The objective of this study was to find the conditions under which the overall transmittance of a V-corrugated inner cover is greater than that of the plane cover, if such conditions exist.

The results of Study 1 are given in Section 3. Briefly, it was found that although complete suppression of free convection heat transfer by a honeycomb is not possible in the inclined air layer, very large reductions in the combined convective and conductive loss across the air layer (of the order of a factor of 5 for a honeycomb of aspect ratio 5) can be achieved provided the honeycomb is properly sized. However, care must be taken in selecting the honeycomb for a specific application since in some cases a honeycomb can actually increase the free convective heat transfer across an inclined air layer. Plots of Nusselt number versus Rayleigh number are presented from which the rational design of honeycomb for convective suppression can proceed. A single correlation equation is given for the Nusselt number, covering a broad range of the relevant variables.

The results of Study 2 are given in Section 4 of this report. Two equations for the free convective heat transfer across an inclined air layer are given, one for the range of θ (the angle of inclination,

measured from the horizontal) from 0 to 60 degrees, the other for θ between 60 and 90 degrees. These equations are recommended for the design and computer modeling of conventional flat plate collectors.

The results of Study 3 are given in Section 5. In this study it was found that V-corrugating a thin sheet of transparent material does in fact improve its solar transmittance. Predicted solar transmittance of V-corrugated sheets of different plastics and glass, suitable for solar collectors, are presented.

3. FREE CONVECTION ACROSS INCLINED AIR LAYERS CONSTRAINED BY A HONEYCOMB

Much of the relevant details to this study are given in References [1] and [2]. A summary of the experimental program is given in a manuscript entitled "Natural Convection Across Inclined Honeycomb Panels" by R.L.D. Cane, K.G.T. Hollands, G.D. Raithby and T.E. Unny. This manuscript is contained in Appendix A of this report. The present section of this report will therefore be devoted to summarizing the results, and to discuss their application to the design of solar collectors.

The basic theory of the honeycomb solar collector is contained in [3]. A review of the state-of-the-art in their design is given in [4], which also contains the relevant bibliography. The honeycomb fills a two-fold purpose: the first is to suppress radiant loss from the plate to the outer glass cover; the second is to suppress the free convection heat transfer across the air gap bounded by the plate and the glass cover. The present study deals primarily with the free convection suppression mechanism. Although considerable studies had dealt with this free convection problem in the horizontal position ($\theta = 0$) very little hard design information existed for the inclined position. In fact there had been some dispute in the literature as to whether convection suppression does in fact take place if the honeycomb is inclined.

The free convective heat transfer across a honeycomb heated on the lower inclined face and cooled on the upper face, can be characterized by a Nusselt number, Nu , which represents the ratio which the combined conductive and free convective heat transfer through the air makes to the corresponding heat transfer when the air is stagnant. Hence, a Nusselt number of unity indicates purely conductive heat transfer and complete suppression of convective motion. The Nusselt number can be shown to be a function of the following dimensionless groups:

$$Nu = Nu(Ra, Pr, A, \theta, C, N, H, \epsilon, \epsilon_{p1}, \epsilon_{p2}) \quad (3-1)$$

All quantities are defined in the nomenclature. The Rayleigh number representing approximately the ratio of buoyant to viscous forces on the air, is the appropriate free convective group. The Prandtl number, Pr , is very nearly constant for air at .71. The group N , as well as ϵ , ϵ_{p1} , ϵ_{p2} , accounts for the effect on the free convection of the radiative (long-wave) heat transfer across the honeycomb. The groups C and H account for the effects of the heat conduction along and across the honeycomb walls.

In this study, experimental measurements were carried out to determine the functional relationship in equation (3-1), for values of the parameters pertinent to candidate honeycomb solar collectors. Thus, the values of N , C , H and ϵ were made approximately equal to

their corresponding values in the plastic honeycomb collectors [5]. For glass honeycombs which are also under development [6], the values of ϵ and C will be higher than those in our experiments, with the result that the actual heat transfer in the collectors will be less than that predicted from the present measurements. Consequently, designs based on these experiments should be conservative when used in design of glass honeycombs. The actual values of N , C , H and ϵ are given in Appendix A.

In the present experiments, the plate emissivities were: $\epsilon_{p1} \cong \epsilon_{p2} \cong .06$. In a solar collector with a glass cover $\epsilon_{p2} \cong .9$, and $\epsilon_{p1} \cong .9$ for a black-painted absorber plate and $\epsilon_{p1} \cong .15$ for a selective surface absorber plate. Consequently, a close match on ϵ_{p1} and ϵ_{p2} was not obtained. However, provided A is relatively large, the effect of ϵ_{p1} , ϵ_{p2} may be expected to be slight, although further experiments should be carried out in the future to confirm this.

The remaining groups in equation (3-1) are Ra , θ and A ; thus for the present experiments:

$$Nu = Nu (Ra, \theta, A) \quad (3-2)$$

The primary purpose of the present experiments was to determine the functional relations (3-2). The range of variables of interest was

$0 < Ra < \sim 10^7$, $0 \leq \theta \leq 90^\circ$, $A = 2, 3, 4$ and 5 . The results are shown in Figure 3-1.

In the horizontal case ($\theta = 0^\circ$) the Nusselt number is unity, indicating complete suppression of free convective heat transfer, until a critical value of Ra is reached after which the Nusselt number rises sharply with Rayleigh number. This critical value of Ra is called the critical Rayleigh number.

For the inclined case the Nusselt number is never exactly unity even at very small Rayleigh number, indicating that as previously noted by Charters and Peterson there is never a complete suppression of convective motion. However, the Nusselt number increases rather slowly with Ra until it reaches approximately 1.2 after which it rises rather steeply with Ra . This Ra at the point of departure (or departure Ra) occurs at approximately the same Ra as the critical Rayleigh number for the horizontal case.

Also shown in Figure 3-1, by the solid ($\theta < 90^\circ$) and dashed ($\theta = 90^\circ$) lines is the Nusselt number which would be observed if there were no honeycomb to suppress free convection. Clearly at Rayleigh numbers around the corresponding critical Rayleigh number at $\theta = 0^\circ$ (denoted by Ra_{co}) considerable suppression of free convective heat transfer is observed. For example for $A = 5$, $\theta = 45^\circ$ and $Ra = 8 \times 10^5$ the Nusselt number with a honeycomb is less than one-fifth of what

Figure 3-1

Experimental Nusselt numbers as of function of Rayleigh number,
 for honeycombs of various aspect ratios.

would occur if there were no honeycomb present; the free convective component of the Nusselt number, $Nu - 1$, is one twenty-fifth of the corresponding no honeycomb value.

It should be noted that in some instances the honeycomb can actually increase the free convective heat transfer across the air layer. For example at $A = 2$, $\theta = 45^\circ$ and $Ra = 7 \times 10^5$ the Nusselt number with a honeycomb is about 25% larger than without a honeycomb. This underlines the fact that if the honeycomb is not properly sized for the application of suppressing free convection, an actual augmentation of free convection can take place.

The measurements in Figure 3-1 are for square-celled honeycombs. However, measurements were also carried out in this study on a hexagonal honeycomb. The results were found to be the same, to within about 20%, provided the comparison is made at the same hydraulic diameter. Thus, the information in Figure 3-1 should be applicable to square or hexagonal honeycombs. However, their application to other, nonsymmetric cell shapes, such as rectangles, is not recommended.

For the range of variables: $30 \leq \theta \leq 90$ and $A \geq 4$, which covers the main range of interest for solar collectors for the heating and cooling of buildings a single correlation equation has been found for all the relevant data of Figure 3-1 to within $7\frac{1}{2}\%$. The correlation equation is:

$$Nu = 1 + .89 \cos(\theta - 60^\circ) \left(\frac{Ra}{2420A^4} \right)^{2.88 - 1.64 \sin \theta}, \quad \frac{Ra}{A^4} < 6000 \quad (3-3)$$

Although based on data for $A \geq 4$, equation (3-3) is approximately valid for $A = 3$ as well, particularly near the departure Rayleigh number. The application of this equation to the design of honeycomb collectors will now be discussed.

Although design for complete suppression of free convection by the honeycomb is desirable, in the inclined position this is not realistic since there exists some free convective heat transfer no matter how small the Rayleigh number. For engineering purposes it is preferable to design for a Nusselt number of about 1.2, since this gives maximum convection suppression, and involves only a slight convective penalty. Substituting $Nu = 1.2$ in (3-3) and solving for A there results:

$$A = f(\theta) \left(\frac{Ra}{2420} \right)^{\frac{1}{4}} \quad (3-4)$$

where:

$$f(\theta) = \left[4.45 \cos(\theta - 60^\circ) \right]^{\frac{1}{11.52 - 6.56 \sin \theta}}, \quad 90^\circ \leq \theta \leq 30^\circ \quad (3-5a)$$

This expression is also valid for $\theta = 0^\circ$ if $f(\theta)$ is taken as 1.072:

Figure 3-2

Showing plot of $C(\theta)$ for use in equation (3-6).

$$f(\theta) = 1.072$$

$$\theta = 0^\circ \quad (3-5b)$$

Linear interpolation between (3-5a) and (3-5b) is recommended. Figure 3-2 shows the recommended plot of $C(\theta) = 1.03 f(\theta)$ vs θ .

For air at atmospheric pressure and moderate temperatures ($200F \leq T_m \leq 500F$), equation (3-4) can be expressed dimensionally as:

$$A = C(\theta) (1 + 2x)^{\frac{1}{2}} \cdot x \cdot \Delta T^{\frac{1}{4}} L^{3/4} \quad (3-6)$$

where $x = 100/T_m$. Here T_m is the mean air temperature, $(T_1 + T_2)/2$, expressed in degree K, ΔT is the temperature difference across the honeycomb, $T_1 - T_2$, expressed in degree K and L is depth of the honeycomb expressed in cm. This equation is based on an expression for $\frac{\beta}{\lambda \nu}$ derived in [7]. For example, suppose it is required to design a hexagonal honeycomb to suppress free convection currents in an air layer 5 cm (~ 2 inches) deep, in a solar collector for which the design absorber plate temperature is 375K ($\sim 220F$) and the glass cover temperature is 310K ($\sim 100F$). The inclination of the collector is to be 45 degree from the horizontal. Then from Figure 3-2 $C(\theta) = 1.274$; also $T_m = 342.5$, $x = .292$, $\Delta T = 65$ and $L = 5$. Therefore, the required aspect ratio is:

$$A = 1.236 x (1 + 2 x .292)^{\frac{1}{2}} x .292 x 65^{\frac{1}{4}} x 5^{3/4} \quad (3-7)$$

$$A = 4.44$$

Since $A = L/D$ the required width across the cell is $D = L/A = 5/4.44 = 1.12$ cm.

In addition to the experimental study an analytical study was also undertaken in this program. The details of this study are given in a manuscript entitled "Numerical Study of Free Convective Heat Transfer Across Rectangular Cells" by S. Rajaram, T.E. Unny, K.G.T. Hollands and G.D. Raithby contained in Appendix B of this report. This study concentrated on the honeycomb cell inclined at $\theta = 90^\circ$ so that the heating and cooling are from vertical side faces. The general mass, momentum and energy conservation equations were solved using a numerical scheme. The studies were in the first instance fully three-dimensional. However, it was found that the effect of the third dimension was small so that two-dimensional solutions were sought for the latter part of the study. A comparison between the experimental heat transfer rates and numerical predictions was found to be quite satisfactory. The analytical computations were based purely on base-flow and did not allow for instabilities. The agreement between theory and experiment is therefore taken as being indicative that at this inclination, the flow is stable, and the heat transfer is just that due to base-flow. This has been useful knowledge in understanding the nature of the convection. The fact that a honeycomb inserted in an air layer can actually increase the heat transfer, if the aspect ratio is not properly sized, is born out by the theory.

4. FREE CONVECTION ACROSS INCLINED AIR LAYERS

NOT CONSTRAINED BY A HONEYCOMB

Measurements on this important problem were carried out on the University of Waterloo Natural Convection Apparatus. One set of measurements was carried out covering the range of angles, θ , from zero degrees (horizontal air layer) to 75 degrees. The results of these measurements and their correlation into a single correlation equation is reported fully in Reference [8]. (Since this reference is readily available its contents will not be repeated in this report.) The correlation equation is:

$$\begin{aligned} \text{Nu} = 1 + 1.44 \left[1 - \frac{1708}{\text{Ra} \cos \theta} \right]^{\bullet} \left(1 - \frac{1708 (\sin 1.8\theta)^{1.6}}{\text{Ra} \cos \theta} \right) \\ + \left[\left(\frac{\text{Ra} \cos \theta}{5830} \right)^{1/3} - 1 \right]^{\bullet}, \quad 0^{\circ} \leq \theta \leq 75^{\circ}, \quad (4-1) \end{aligned}$$

where square brackets like so: $[]^{\bullet}$, indicate that if the quantity inside the brackets is negative, the quantity is to be taken as zero, i.e.:

$$[x]^{\bullet} = (|x| + x)/2$$

where x is any quantity.

Measurements were also made in the range $75^\circ \leq \theta \leq 90^\circ$.

However, because of the nature of the Waterloo Natural Convection Apparatus (in particular its nonzero aspect ratio and the fact that the measurements are made only over the central part of the slot) it was felt that these measurements were not representative of the free convective heat transfer across the very small aspect ratio slots typical of air layers in nonhoneycomb solar collectors. (Here aspect ratio, A , is defined as the ratio of the distance across the layer L , to the length along the layer H , as measured along either the heated or cooled bounding surface, in the upslope direction.) As an alternative, a theoretical approach was used, based on the method developed in Reference [9]. The prediction of this theory was then compared with the considerable experimental data which already exists on this problem for the vertical layer ($\theta = 90^\circ$). The agreement was found to be very good. On this basis a recommended correlation equation was obtained. The full details of this aspect of the study is given in Reference [10]. The recommended relation for air is:

$$Nu = \left[1, .288 \left[\frac{\sin \theta Ra}{A} \right]^{\frac{1}{4}}, .039 (\sin \theta Ra)^{1/3} \right]_{MAX}, 75^\circ \leq \theta \leq 90^\circ, \quad (4-2)$$

where the subscript MAX indicates that, at a given Ra , the maximum of the three quantities separated by commas should be taken.

Some explanation should be given as to why the aspect ratio dependence is given in equation (4-2) and not in (4-1). It is felt that for inclinations within 60 degrees of horizontal, the effect of

aspect ratio should be very slight provided $A < .1$; this should cover the full range of interest for solar collectors, (except for honeycomb collectors which are treated separately). However, in the case of near vertical ($70^\circ \leq \theta \leq 90^\circ$) collectors, the aspect ratio can have a significant effect if $A > .01$. Since some flat plate collectors may conceivably have A as large as 0.08 (e.g. $L = 2$ inch, $H = 24$ inch), the effect of A must be included.

Figure 4-1 shows a dimensional plot of Equation 4-1, which underlines the effect of the gap spacing, L , and angle of inclination, θ , on the free convective heat transfer coefficient h across the air gap. The quantities ϕ_1 , ϕ_2 and ϕ_3 are functions of the temperatures T_1 and T_2 of the bounding lower and upper surfaces respectively:

$$\phi_1 = \frac{464}{(\bar{T}_m + 200)^{1/3} \bar{T}_m^{1/2}} \quad (4-3)$$

$$\phi_2 = \left(\frac{T_1 - T_2}{50} \right)^{1/3} \quad (4-4)$$

$$\phi_3 = \frac{1428 (\bar{T}_m + 200)^{2/3}}{\bar{T}_m^2} \quad (4-5)$$

where

$$\bar{T}_m = (T_1 + T_2)/2 \quad (4-6)$$

Figure 4-1

Plot of the free convective heat transfer coefficient across an air gap as a function of the gap spacing, L .

For a flat plate collector with one glass cover, T_1 is the plate temperature, T_2 is the glass cover temperature. The functions ϕ_1 , ϕ_2 and ϕ_3 have been so defined that for $T_1 = 325\text{K}$, $T_2 = 275\text{K}$, $\phi_1 = \phi_2 = \phi_3 = 1$, corresponding very approximately to a single glass cover collector used for heating a building in northern U.S. in the winter.

On the basis of Figure 4-1 consideration can be given to recommendations for the optimum spacing between the collector plate and a glass cover, or between two glass covers in a flat plate collector. Since the free convective losses are proportional to the convective coefficient, h , it is desirable to choose L so as to minimize h . The minimum in h occurs when L is infinity, - a clearly impractical option. The decision of the best spacing must therefore take account of factors other than purely free convective heat loss, - for example the shading of the plate by the sides of the collector, and the cost penalties associated with very deep collectors, including material, packaging and installation costs. A spacing of 5 cm as recommended in Reference [4] appears to be a good compromise option. Another option is to design for the local minimum in h shown by the curves - i.e. those occurring at $L \approx .7 \text{ cm}$ for $\theta = 0^\circ$ and $L \approx 1 \text{ cm}$ for $\theta = 70^\circ$, for $T_1 = 325\text{K}$, $T_2 = 275\text{K}$. This gives a free convective loss only slightly greater than that for $L \approx 5 \text{ cm}$ and also gives a very compact design. The location of this local minimum is given to a good approximation by:

$$L_{\min} = \frac{.71}{\cos \theta \phi_2 \phi_3} \quad \text{cm}$$

In our opinion the decision for the best spacing should be made on the basis of a systems study taking into account all of the relevant cost penalties and shading factors.

5. SOLAR TRANSMITTANCE OF A V-CORRUGATED TRANSPARENT SHEET

The full description of this study is given in detail in Reference [11]. A manuscript entitled "Radiant Transmittance of V-Corrugated Transparent Sheets with Application to Solar Collectors" by B.E. Sibbitt and K.G.T. Hollands summarizes the study and is included in Appendix B of this report. The findings are briefly summarized in this Section.

The analytical and experimental studies demonstrates that the transmittance of a flat transparent sheet can be very substantially increased by V-corrugating. The analytical studies were based on a Monte Carlo radiant analysis, allowing for Fresnel reflections at interfaces, polarization effects and some forward scattering, as well as absorption of radiation in the sheet. The experimental studies were carried out on a teflon sheet V-corrugated with an opening angle of 30° . They demonstrated the principal of increase in transmittance over the flat sheet and were in good agreement with the theory.

On the basis of these results predictions were made of the increase in transmittance expected for a number of candidate sheet materials which may be suitable for solar collectors. Figure 5-1 shows the predictions for three such sheet materials, all corrugated at an opening angle of 30° . The sheets are: .003 inch thick Lexan, .003

Figure 5-1
 Predicted Transmittance of sheets of Lexan, Mylar and glass,
 all V-corrugated at an opening angle of 30°

inch thick Mylar, and .063 inch thick glass. Very substantial increases in transmittance compared to the corresponding flat sheet are expected. In these plots, the angle β is the angle of incidence of the solar radiation to the plane of the sheet: $\theta = 0$ corresponds to the incident rays running across the grooves, $\theta = 90^\circ$ corresponds to incident radiation running along the grooves. In practice the grooves would probably be made to run in the East-West direction so that the $\theta = 90^\circ$ situation would be approached.

The implication of these results to the design of flat-plate collectors should be clear. Insertion of a V-corrugated inner cover between the absorber plate and an outer glass cover will result in a higher transmittance than if a flat sheet had been used. In addition to this it should give better radiant shielding than the flat sheet because it will establish a gradient in temperature along the sheet in the direction normal to the plate. Moreover, it should give better free convection suppression than the flat sheet, since (if the angle of opening is sufficiently small), stagnant air will be trapped in the V's. Consequently, very considerable gains in efficiency should be expected on using V-corrugated inner covers rather than flat inner covers. Further studies on this concept are definitely recommended.

If the V-corrugated cover is made a few inches deep (from peak-to-peak) it becomes in principle very similar to the honeycomb concept. It should however, be much easier to manufacture than a

honeycomb. It can for example be directly extruded; for plastics it can be set-up as a folding operation. Techniques for folding are already developed by the paper industry.

6. CONCLUSIONS

1. Effective convection suppression is possible by a honeycomb at all angles of inclination from zero to ninety degrees. Reduction by a factor of 5 in the combined convection and gaseous conduction heat losses have been achieved at forty-five degree tilt. Sufficient information in the form of equations and graphs, are given in this report, to permit the rational design of honeycombs suitable for solar collectors.
2. The free convective heat loss across the air gaps in flat plate nonhoneycomb solar collectors can be predicted with confidence by equations (4-1) and (4-2). The optimum gap spacing for a collector, decided so as to minimize free convection heat loss, must be decided on the basis of a total systems study. However, a gap size of approximately 5 cms appears to be good option; so does a gap size of approximately $(.7/\cos \theta)$ cm where θ is the angle of tilt measured from the horizontal.
3. The transmittance of a flat transparent sheet can be increased very substantially by V-corrugating it. Such a sheet, placed between the outer glass cover and the absorber plate in a flat-plate solar collector should give very substantial gains in efficiency, and extend the operating temperature range.

7. REFERENCES

1. Cane, R.L.D., "Free Convection in Inclined Air Layers Constrained by a Square-Celled Honeycomb", M.A.Sc. Thesis, Department of Mechanical Engineering, University of Waterloo, Waterloo, Ontario Canada, (1976).
2. Rajaram, S., "Numerical Study of Free Convective Heat Transfer Across Rectangular Cells", M.A.Sc. Thesis, Department of Mechanical Engineering, University of Waterloo, Waterloo, Canada, (1976).
3. Hollands, K.G.T., "Honeycomb Devices in Flat Plate Solar Collectors", Solar Energy, 9 p 159, (1965).
4. Buchberg, H., Catton, I. and Edwards, D.K., "Natural Convection in Enclosed Spaces: A Review of Application to Solar Energy Collection"; Journal of Heat Transfer Vol. 98, No. 2, pp 182-188, (1976).
5. Marshall, K.N., Wedel, R.K., Damman, R.E., "Development of Plastic Honeycomb Flat-Plate Solar Collectors", Lockheed Missiles and Space Company, Inc., Report No. LMSC/D462879, (1976).
6. Buchberg, H., Edwards, D.K., Mackenzie, J.D., "Transparent Glass Honeycomb Structures for Energy Loss Control", University of California, Los Angeles Report No. UCLA-ENG-7625, (March 1976).

7. Hollands, K.G.T., "Natural Convection in a Horizontal Layer of Air With Internal Constraints", Ph.D. Thesis, McGill University, Department of Chemical Engineering, (1967).
8. Hollands, K.G.T., Unny, T.E., Raithby, G.D., Konicek, L., "Free Convection Across Inclined Air Layers", Journal of Heat Transfer, Vol. 98, No. 2, pp 189-193, (1976).
9. Raithby, G.D. and Hollands, K.G.T., "A General Method of Obtaining Approximate Solutions to Laminar and Turbulent Free Convection Problems", Advances in Heat Transfer, Vol. 11, pp 265-315, Academic Press, New York, (1975).
10. Raithby, G.D., Hollands, K.G.T., Unny, T.E., "Free Convection Heat Transfer Across Fluid Layers of Large Aspect Ratios; ASME Paper No. 76-HT-45, presented at the 16th National Heat Transfer Conference, St. Louis, August 8-11, (1976).
11. Sibbitt, B.E., "The Solar Transmittance of a V-corrugated Diathermous Sheet", M.A.Sc. Thesis, University of Waterloo, Department of Mechanical Engineering, Waterloo, Ontario Canada, (1976).

8. LIST OF CONTRIBUTING PERSONNEL

Principal Investigator:

K.G.T. Hollands, Associate Professor, Department of Mechanical Engineering.

Faculty Associates:

T.E. Unny, Professor, Department of Civil Engineering.

G.D. Raithby, Associate Professor, Department of Mechanical Engineering.

Graduate Students:

R.L.D. Cane, Research Assistant, Department of Mechanical Engineering.

S. Rajaram, Research Assistant, Department of Mechanical Engineering.

B.E. Sibbitt, Research Assistant, Department of Mechanical Engineering.

J.F. Orgill, Research Assistant, Department of Mechanical Engineering.

Dean Smart, Research Assistant, Department of Mechanical Engineering.

Samy El Cherbiny, Research Assistant, Department of Mechanical Engineering.

Consultant:

L. Konicek, CSICC Fellow, Division of Building Research, National Research Council of Canada, Ottawa, Ontario, Canada.

Lab Technician:

M. Kaptein, Thermal Engineering Lab., Department of Mechanical Engineering.

9. PATENTS APPLIED FOR

1. "Corrugated Cover Plate for Flat Plate Collector"
K.G. Terry Hollands, Bruce Sibbitt
Files, U.S. Patent Office, April 26, 1976,
assigned Serial No. 680, 605.

10. NOMENCLATURE

Note: This nomenclature is for the body of this report only and does not apply to the Appendices.

- A aspect ratio of honeycomb cell, or of an inclined air layer;
 $A = L/D$.
- C wall admittance group; $C = k_f L / k_w \delta$.
- D hydraulic diameter of honeycomb cell, [m]; also for an air layer of small aspect ratio, the length of layer, measured in the upslope direction, along the heated bounding surface.
- g acceleration due to gravity [m/s].
- H honeycomb cell coupling group, $H = k_f \delta / k_w L$.
- k, k_f thermal conductivity of air, [W/mK].
- k_w thermal conductivity of honeycomb wall, [W/mK].
- L height of honeycomb panel, or distance across air layer measured normal to heated bounding surface [m]. (except in eqn. 3-6 where [cm])
- N radiation group; $N = 4\sigma T_m^3 L / K_f$.
- Nu Nusselt number; $Nu = qL / (k_f \Delta T)$.
- Pr Prandtl number of air; $Pr = \nu / \lambda$.
- q free convective component of heat transfer across honeycomb panel or air layer, [W/m²].
- Ra Rayleigh number; $Ra = g\beta \Delta T L^3 / (\nu \lambda)$.
- ΔT temperature difference across honeycomb or fluid layer;
 $\Delta T = T_1 - T_2$, [K].
- T_1, T_2 temperature of lower and upper bounding faces of honeycomb or air layer respectively, [K].
- T_m mean absolute temperature of air layer; $T_m = (T_1 + T_2) / 2$, [K].

- t thickness of honeycomb wall, $[m]$.
- β coefficient of volumetric expansion of air, K^{-1} , except in Section 5 where it represents the angle of incidence of solar radiation measured from the normal to the plane of a V-corrugated sheet.
- δ half-thickness of honeycomb wall, $\delta = t/2$.
- ϵ total emissivity of single film of honeycomb wall, at T_m .
- $\epsilon_{p1}, \epsilon_{p2}$ total emissivity of plates bounding honeycomb on top and bottom respectively
- λ thermal diffusivity of air, $[m^2/s]$.
- ν kinematic viscosity of air, $[m^2/s]$.
- θ angle of inclination of heated bounding surface to honeycomb panel or air layer; also azimuthal angle of incidence of solar radiation on V-corrugated sheet, measured from along the normal to the grooves. $[deg]$.
- σ Stefan Boltzman constant $[wm^2k^4]$.
- ϕ_1, ϕ_2, ϕ_2 functions of T_1 and T_2 defined by equations (4-3) to (4-5).

11. LIST OF PAPERS BASED ON STUDIES

DESCRIBED IN THIS REPORT

1. Hollands, K.G.T., Unny, T.E., Raithby, G.D. and Konicek, L., "Free Convection Across Inclined Air Layers", Journal of Heat Transfer, Vol. 98, No. 2, pp 189-193, (1976). (Presented originally at 15th National Heat Transfer Conference, San Francisco, August, (1975).
2. Raithby, G.D., Hollands, K.G.T., Unny, T.E., "Free Convection Heat Transfer Across Fluid Layers of Large Aspect Ratios", ASME Paper No. 76-HT-45, to be presented at the 16th National Heat Transfer Conference, St. Louis, August 8-11, (1976).
3. Sibbitt, B.E., Hollands, K.G.T., "Radiant Transmittance of V-Corrugated Transparent Sheets, With Applications to Solar Collectors", to be presented at the Winter Annual Meeting, ASME, New York, December, (1976).
4. Cane, R.L.D., Hollands, K.G.T., Raithby, G.D. and Unny, T.E., "Convection Suppression in Inclined Honeycombs", presented at the International Solar Energy Society Meeting in Los Angeles, July 28,-August 1, (1975).
5. Cane, R.L.D., Hollands, K.G.T., Raithby, G.D. and Unny, T.E., "National Convection Across Inclined Honeycomb Panels", submitted to the Journal of Heating Transfer.

APPENDIX

APPENDIX A

NATURAL CONVECTION ACROSS INCLINED
HONEYCOMB PANELS

R.L.D. Cane
Graduate Assistant

K.G.T. Hollands
Associate Professor

G.D. Raithby
Associate Professor

Department of Mechanical Engineering

T.E. Unny
Professor
Department of Civil Engineering

University of Waterloo
Waterloo
Canada

May, 1976

APPENDIX

ABSTRACT

Experimentally-obtained Nusselt number-Rayleigh number plots are presented for free convective heat transfer across honeycomb panels inclined with respect to the horizontal at angles of 0, 30, 45, 60 and 90 deg and heated from below. Aspect ratios of honeycomb cells of 2, 3, 4 and 5 are included; the fluid contained in the cells is air. Substantial suppression of free convection is observed, when compared with an air layer of the same depth but not containing a honeycomb. Heat transfer associated with the base flow is found to be of only moderate importance at angles near horizontal, but of considerable consequence at angles near vertical. The honeycomb walls used were partly transparent to thermal radiation, and a strong radiative coupling is observed.

APPENDIX

NOMENCLATURE

A	aspect ratio of honeycomb cell; $A = L/D$.
C	wall admittance group: $C = k_f L / (k_w \delta)$.
C*	revised wall admittance group, which includes radiative effect, defined by equation (4).
D	honeycomb cell width, see Figure 1, for square cell; distance across hexagonal flats for hexagonal honeycomb [m].
g	acceleration of gravity $[m/s^2]$.
H	cell coupling group, $H = k_f \delta / k_w L$.
k_f	thermal conductivity of fluid, $[W/mK]$.
k_w	thermal conductivity of honeycomb wall, $[W/mK]$.
L	height of honeycomb panel, see Figure 1, [m].
N	radiation group; $N = 4\sigma T_m^3 L / k_f$.
Nu	Nusselt number; $Nu = qL / (k_f \Delta T)$.
Pr	Prandtl number of fluid; $Pr = \nu / \lambda$.
q	free convective component of heat transfer across honeycomb panel, $[W/m^2]$.
Ra	Rayleigh number; $Ra = g\beta \Delta T L^3 / (\nu \lambda)$.
Ra_D	modified Rayleigh number; $Ra_D = g\beta \Delta T D^4 / L \nu \lambda$.
ΔT	temperature difference across honeycomb; $\Delta T = T_1 - T_2$, [K].
T_1, T_2	temperature of lower and upper bounding faces of honeycomb, respectively, [K].
T_m	mean absolute temperature of honeycomb walls; $T_m = (T_1 + T_2) / 2$, [K].
t	thickness of honeycomb wall (see Figure 1), [m].
α	total absorptivity of single film of honeycomb wall to radiation from a black body at T_m .
β	coefficient of volumetric expansion of air, $[K^{-1}]$.

APPENDIX

δ	half-thickness of honeycomb wall, $\delta = t/2$.
ϵ	total emissivity of single film of honeycomb wall, at T_m .
ϵ_p	total emissivity of plates bounding honeycomb on top and bottom.
λ	thermal diffusivity of fluid, $[m^2/s]$.
ν	kinematic viscosity of fluid $[m^2/s]$.
θ	angle of inclination of honeycomb face from horizontal, [deg].
ρ	total reflectivity of single film of honeycomb wall to radiation from a black body at T_m .
σ	Stefan-Boltzmann constant $[W/m^2 K^4]$.
τ	total transmittance of single film of honeycomb wall to radiation from a black body at T_m .

APPENDIX

INTRODUCTION

Although considerable attention has been given to free convective heat transfer across honeycomb panels in the horizontal position, [1-7], very little information exists for predicting free convective heat transfer across inclined panels. Notwithstanding the studies which have touched on the inclined case, [8-11], there is insufficient published information at the present time to permit the rational design of honeycombs for particular applications. The situation has been recently reviewed in [7].

An important proposed use of honeycombs is for the purposes of suppressing the free convective heat transfer which would otherwise occur in an inclined layer of fluid, heated from below. Such a layer occurs in the conventional flat plate solar collector. The insertion of a honeycomb into this fluid layer is known, in the case of a horizontal layer, to completely suppress convection currents for all Rayleigh number, Ra , less than a critical value, denoted by Ra_c . In the case of an inclined layer, it is known that a finite convective motion, called the base flow, exists for any Ra greater than zero. This base flow, which is driven by the component of gravity along the heated bounding surface of the honeycomb panel, is expected to make only a modest contribution to the heat transfer, at least until very high Rayleigh numbers are reached. However, the actual magnitude of the heat transfer associated with the base flow, and the Rayleigh number at which it becomes significant, are at the present time largely unknown. Also largely unknown is the critical Rayleigh number at which this base flow becomes unstable. This latter instability is expected to be due to the "top-heavy" nature of the fluid layer and hence is expected to be driven by

the component of gravity normal to the heated bounding surface.

The present paper presents an extensive set of measurements on this problem, for a square-celled, diathermous honeycomb. Experimental plots of Nusselt versus Rayleigh number are presented for angles of inclination, θ , measured from horizontal ranging from zero to ninety degrees, and for aspect ratios ranging from 2 to 5. The fluid used is air and in all cases (except at $\theta = 90^\circ$), heating is from the lower face of the honeycomb.

DESCRIPTION OF EXPERIMENT

All experiments were performed on the University of Waterloo Natural Convection Apparatus which is fully described elsewhere [3], [12], [13]. The procedure and data reduction was also the same as described in these papers. The combined heat transfer coefficient for radiation and honeycomb-wall conduction was assumed to be dependent only on the honeycomb configuration (i.e. independent of pressure and inclination). Its value was determined by measurement in the horizontal position at subcritical pressure. Full details of complete experiment are given in [14].

Sketched in Figure 1, the honeycombs were constructed from .038 mm (.0015 inch) thick polyethylene film, by heat-sealing pre-slit strips at periodic join-lines. The width of the join-lines was kept small (approximately .25 mm), thereby ensuring nearly square cell corners. Table 1 gives the actual dimensions of the honeycombs used.

As in [3], polyethylene was chosen for the honeycomb material because, in thin films, it has a very high transmittance to long-wave ($3\mu\text{m}$ to $30\mu\text{m}$) thermal radiation. This high transmittance, along with the

APPENDIX

low emissivity, of the polished copper plates which form the bounding surfaces of the honeycomb in the test apparatus, was originally expected to ensure a de-coupling of the thermal radiation mode of heat transfer across the honeycomb, from the free convective mode. However, as will be discussed later in the paper, a strong radiative coupling was still observed. Hence a full specification of the radiant parameters is necessary.

The spectral bi-normal radiative transmittance of the polyethylene film was measured on a spectro-photometer and, assuming no scattering and an index of refraction for polyethylene of 1.52, the total normal transmittance τ , reflectance ρ , and absorptance, α , for a black-body source at $T_m = 305K$ (550R) (the average temperature on the experiments) were calculated. The result was $\tau = .819$, $\rho = .067$, and $\alpha = .114$. The total normal emissivity at 305K is consequently $\epsilon = .114$ and the corresponding hemispheric value was calculated to be .130. Most of the absorption occurred in two bands, one at $6.9 \mu m$, the other at $14.2 \mu m$. The copper plate emissivity was $\epsilon_p = 0.065$.

Since the temperature difference across the honeycomb was kept small ($\approx 5K$), it is permissible to assume that the radiative interaction can be linearized. Assuming this, and assuming also that a grey radiant analysis applies, the Nusselt number can be shown to be a function of the following dimensionless groups:

$$Nu = Nu(Ra, Pr, \theta, A, C, H, N, \rho, \epsilon, \epsilon_p) \quad (1)$$

(In fact, ρ need not be listed, as will be demonstrated later.)

Certain of the variables were kept constant for all experimental runs, Thus, since all experiments were with air at moderate temperature,

APPENDIX

$Pr = .71$; also $\rho = .067$, $\epsilon = .130$, $\epsilon_p = .065$, as noted above. Other variables were kept "quasi-constant". Thus, except for one honeycomb, C was kept at $166 \pm 25\%$; H was kept at $6 \times 10^{-5} \pm 25\%$; and N was kept at $15.6 \pm 25\%$. Since it was anticipated that rather large variations in these quantities was required in order to have a significant effect on Nu , these variables were considered constant. Consequently, for the present experiment:

$$Nu = Nu(Ra, \theta, A) \quad (2)$$

The primary purpose of the experiment was to examine the effect of Ra , θ and A , and the range of values of these variables was consequently made large: the ranges were: for A , $A = 2, 3, 4$ and 5 ; for θ , $\theta = 0, 30, 45, 60$ and 90 degrees; and for Ra , $\sim 10^3 < Ra < \sim 10^6$. In all, six honeycombs were tested; their various dimensions and pertinent dimensionless groups are given in Table 1.

An additional, very different honeycomb was also tested. This honeycomb will be described later in the paper.

RESULTS

The measured Nusselt number-Rayleigh number plots are shown in Figure 2. Also shown on these graphs, for comparison, are the Nusselt number curves which would apply for free convective heat transfer across slots of very small aspect ratio, but containing no honeycomb [13], [15]. Thus the difference between the data and the curves is an indication of the effectiveness of the honeycomb in suppressing free convection.

The data for $\theta = 0^\circ$ are exactly as have been observed earlier [1], [3] - i.e., a Nusselt number of unity, indicating complete convective

APPENDIX

suppression, until a critical Rayleigh number is reached, after which there is a dramatic rise in Nu with Ra . The inclined data show the same general trend, but with an initial gradual rise in Nusselt number, making it difficult to define a critical Rayleigh number for the inclined case. This region of initial gradual rise is almost certainly associated with the base flow which, as discussed earlier, occurs for any $Ra > 0$ (provided $\theta \neq 0$) and is driven by the component of gravity along the bounding surface (i.e., $g \sin \theta$). The more dramatic rise in Nusselt No., noted at values of Ra slightly greater than the corresponding Ra_c for $\theta = 0$, is most probably due to the "top-heavy" instability, driven by the component of gravity normal to the bounding surfaces, $g \cos \theta$. At $\theta = 90^\circ$ there appears to be no instability of this kind and it is therefore felt that the observed heat transfer is due entirely to base flow. This is supported by the close agreement noted in [16] between the present data and the predictions of a numerical solution for the base flow.

It is interesting to note that the results in Figure 2 show that at some Rayleigh number, and at low aspect ratios, the effect of the honeycomb can be to increase the heat transfer over that which would occur with no honeycomb present. At least in the inclined and near vertical cases, this result is felt to be due to a breaking up of the boundary layers which would otherwise form along the heated and cooled faces, by the honeycomb walls. Such an effect was also found in [16], for $\theta = 0$, based on a numerical solution of the governing equations.

The same data are plotted in slightly different form in Figure 3, where a modified Rayleigh number, Ra_D , defined by:

APPENDIX

$$Ra_D = \frac{g\beta\Delta TD^4}{\nu\alpha L} = Ra/A^4$$

has been used. This group has been put forward previously by Edwards [17] for large aspect ratio stability studies at $\theta = 0$ and by Hart [18] for large aspect ratio base flow studies at $\theta = 90$ degrees. Consequently, for large aspect ratios this group alone was anticipated to characterize the heat transfer, for a fixed angle θ , at least for small and moderate Rayleigh number, and Figure 3 shows that this is in fact the case, provided $A \geq 4$ and $\theta \neq 0$. This useful result implies that, provided $A \geq 4$ and $\theta \neq 0$:

$$Nu(Ra, \theta, A) = Nu(Ra(A/5)^4, \theta, 5) \quad (3)$$

thereby permitting extrapolation of the results of Figure 2 to aspect ratios higher than 5, and also interpolation between $A = 4$ and $A = 5$.

As a check on dynamic similarity, and to test the hypothesis that, over the ranges of C , H and N covered in the experiments, the effect of these variables is negligible, Figure 4 was plotted. This figure shows a comparison of the measured Nusselt numbers for two different honeycombs (Honeycombs 1 and 2, Table 1), both having the same values of A (namely 3), but of different dimension and, consequently, slightly different values of C , H and N . The close agreement should indicate the insensitivity to these variables. Close agreement was also found when Honeycombs #3 and 4, which are both of aspect ratio 2, were compared.

CRITICAL RAYLEIGH NUMBER AT $\theta = 0$

The critical Rayleigh numbers exhibited in Figure 2 for $\theta = 0$

APPENDIX

can be compared with the various theories which apply to this stability problem. Since there is no theory which both (a) accounts for radiation and (b) is applicable to square cells, we compare first with the predictions of Catton [6] which is applicable to isolated square cells, but which ignores radiation. The comparison is shown in Figure 5. Since the actual values of C for the honeycomb is $C = 145 \pm 40\%$, the measured critical Rayleigh numbers are seen to be about 60% higher than predicted by this theory. The honeycombs appear to behave as if they have a constant value for $C = 10$. The measured critical Rayleigh numbers of [3] are also shown in Figure 5. These honeycombs were fabricated from .004 inch thick polyethylene film, using a pressure sensitive tape. Allowing for the presence of the tape, the values of C for these honeycombs are calculated also to be of order 100. In [3] it was argued by one of the authors that the difference of these measured values of Ra_c from theory was due to convective interaction between cells, a quantity which should be represented by the group H . Continuation of this argument would indicate that the present data should have higher critical Rayleigh numbers than those of [3]. Clearly they do not. Consequently, it is now felt that as indicated in [7], the radiation interaction, represented by the group N , is the more likely explanation of the discrepancy.

Although considerable work on the radiation effect on the critical Rayleigh number has been done by Edwards and Sun [5], [8], [19], no theory is yet available which applies directly to the present problem. All theories assume opaque side-walls and black end walls, whereas in the present instance the side-walls are partly transparent and the end walls are

APPENDIX

of low emittance. Moreover, none of the theories are for square cells. The difficulty of the non-opaque side-walls can be overcome by the following artifice.

Consider a hypothetical isolated cell having side-walls of one half thickness of those of the experiment; let the side-walls be immediately bounded on the outside of the cell with a perfect reflector of thermal radiation which reflects in a perfectly specular manner. A moment's consideration shows that, due to the symmetry of the honeycomb about any side-wall, this hypothetical cell behaves radiantly (and thermally) in a manner identical to the actual honeycomb cell. Therefore the honeycomb cell can be considered to be opaque, with the same emissivity as that of the actual film, but with specular rather than diffusely reflecting side-walls. (Note that this also implies that ρ can be dropped from the list of variables in (1), since the reflectance and transmittance are lumped together and the effective reflectance is $1-\epsilon$.) Edwards' and Sun's theories are for diffuse rather than specular side-walls; in applying their theory to the present problem, one would expect, therefore, differences of the order of those applying to diffuse vis-a-vis specular radiant analysis. (It is recognized that in long passages such as a honeycomb cell, quite substantial differences of this type are found.)

We choose the predictions of [5] which, in fact, apply to an infinite cylinder; we therefore assume that a square cell of the same hydraulic diameter behaves in the same manner. This theory predicts that the effect of radiation on stability can be accounted for by using an effective value of C , namely C^* , given by:

APPENDIX

$$\frac{1}{C^*} = \frac{1}{C} + \frac{\epsilon}{4-\epsilon} \cdot \frac{N}{A^2} \quad (4)$$

Values of C^* are tabulated for each honeycomb in Table 1. The average value of C^* of 18 is less than a factor of two different from the required 10 to give good agreement. In view of the various assumptions, this agreement must be considered to be quite good. It should be noted that a specular analysis would be expected to produce a stronger radiative effect and this is what is required to fit the experiments. More refined theories [8] which allow for finite height of cylinder have also been examined; however, they show no better agreement than that just discussed.

Values of C^* for the honeycombs of [3] shown in Figure 5, were calculated in a similar manner, allowing for the presence of the pressure sensitive tape, which was assumed to be opaque, by taking area weighted averages of the tape and film emissivities. Values for C^* of the order of 3 were calculated, putting them in excellent agreement with Catton's theory as can be seen in Figure 5.

It should be noted that the strong radiative coupling just discussed for $\theta = 0$ is probably equally applicable to the inclined results of Figure 2 as well. However, there is insufficient experimental and theoretical information available at the present time to permit quantification of the magnitude of its effect.

HEXAGONAL HONEYCOMB

Honeycombs considered for flat plate solar collectors are likely to be different from those of the present study in that: (i) they may not be square-celled, and (ii) the wall material will likely be

APPENDIX

thicker and have higher emissivity. To examine the effect of these two changes, another honeycomb was studied on the apparatus. Details of this honeycomb are as follows: cell shape, - hexagonal; wall material, - polycarbonate (Lexan, General Electric Reg. T.M.); average wall thickness = .01 cm; distance across flats, - $D = .95$ cm; aspect ratio = $L/D = 5$; wall emissivity = .39; $C = 94$, $H = 10.7 \times 10^{-5}$, $N = 11.9$, $C^* = 16.1$. Figure 6 shows the measured Nusselt numbers for this honeycomb. Also plotted for comparison are the results for a square-celled polyethylene honeycomb of the same aspect ratio (honeycomb #5). The broad behaviour of the Nusselt number is the same for both honeycombs; interestingly, the hexagonal honeycomb permits less heat transfer at $\theta = 0$ and more for $\theta > 0$. Disagreement is largest at Rayleigh numbers well past critical; it does not usually exceed 20%. It should be noted that the two honeycombs are compared for the same hydraulic diameter.

CONCLUSIONS

1. Free convective heat transfer can be effectively suppressed by insertion of a thin walled honeycomb, even in the inclined position.
2. Figure 2, along with the extrapolation (and interpolation) formula given by equation (3) can be used for designing thin walled, partly-transparent square-cell or quasi-square-cell honeycombs suitable for use in inclined solar collectors. Discrepancies in Nusselt number of the order of 20% may be expected, since it is not likely that all dimensionless groups will be exactly matched, nor will the cell shape be completely matched. Further research is required to establish completely the effect of these parameters.

APPENDIX

3. Even though a honeycomb material may be largely transparent to thermal radiation, a coupling of the radiative and free convective modes may still be evident. Stability theories accounting for radiant coupling, but based upon opaque side-walls can be applied with reasonable success to partly transparent side-walls; however, for better results, the theories should be based upon specularly reflecting side-walls, rather than diffuse.

APPENDIX

ACKNOWLEDGMENTS

This work was supported by the Energy Research and Development Administration (ERDA) of the U.S. Government, under contract number E(11-1)-2597.

The National Research Council of Canada provided the original funds for building up the apparatus, for which thanks are also given.

The authors are also indebted to Professor D.K. Edwards for a useful discussion involving the radiation effects, and to Professors Edwards and Catton of UCLA for providing the theoretical part of Figure 5 for tracing.

We wish also to thank Mr. K.N. Marshal of Lockheed Palo Alto Research Laboratories, for providing the hexagonal honeycomb for testing, and Mr. Dean Smart and Mr. Samy El-Cherbiny for carrying out the tests on this honeycomb.

APPENDIX

TABLE 1

Properties of Six Polyethylene Honeycombs Tested

<u>Honeycomb #</u>	<u>L inch (cm)</u>	<u>D inch (cm)</u>	<u>A —</u>	<u>C —</u>	<u>Hx10⁵</u>	<u>N —</u>	<u>C*</u>
1	3.000 (7.62)	1.000 (2.54)	3	200	1.25	18.99	13.1
2	1.875 (4.76)	0.625 (1.59)	3	125	2.00	11.87	19.0
3	2.500 (6.35)	0.625 (1.59)	4	167	1.50	15.82	25.3
4	3.125 (7.94)	0.625 (1.59)	5	208	1.200	19.78	31.7
5	2.000 (5.08)	1.000 (2.54)	2	133	1.900	12.66	8.7
6	1.250 (3.18)	0.625 (1.59)	2	83	3.000	7.91	12.7

APPENDIX

REFERENCES

1. Catton, I., and Edwards, D.K., "Effect of Side-walls on Natural Convection between Horizontal Plates Heated from Below", J. Heat Transfer, Vol. 89, No. 4, Nov. 1967, pp. 295-299.
2. Sun, W.M., and Edwards, D.K., "Natural Convection in Cells of Finite Conducting Walls Heated from Below", Heat Transfer 1970, Proceedings of the 4th International Heat Transfer Conference, Versailles, France, Sept. 1970, Paper NC 2.3, Elsevier Publ. Co., Amsterdam.
3. Hollands, K.G.T., "Natural Convection in Horizontal Thin-Walled Honeycomb Panels", J. Heat Transfer, Vol. 95, pp. 439-444, (1973).
4. Kent, A.C., and Bowyer, J.M., "Thermal Convection in Air of Small Cells Heated from Below", Ind. Eng. Chem. Fundam. Vol. 11, No. 3, pp. 319-323, (1972).
5. Edwards, D.K., and Sun, W.M., "Effect of Wall Radiation on Thermal Instability in a Vertical Cylinder", Int. J. Heat Mass Transfer, Vol. 14, pp. 15-18, (1971).
6. Catton, Ivan, "Effect of Wall Conduction on the Stability of a Fluid in a Rectangular Region Heated from Below", J. Heat Transfer, Vol. 94, pp. 446-452, (1972).
7. Buchberg, H., Catton, Ivan, and Edwards, D.K., "Natural Convection in Enclosed Air Spaces: A Review of Application to Solar Energy Collection", ASME Paper 74-WA/HT-12; to appear in Journal of Heat Transfer, 1976.
8. Sun, W.M., "Effect of Arbitrary Wall Conduction and Radiation on Free Convection in a Cylinder", Ph.D. Thesis, University of California, Los Angeles, (1970).
9. Charters, W.W.S., and Peterson, L.F., "Free Convection Suppression using Honeycomb Cellular Materials", Solar Energy, Vol. 13, pp. 353-361, (1972).
10. Chun, K.R., and Crandall, W.E., "Effect of Mylar Honeycomb Layer on Solar Collector Performance", ASME Paper No. 74-WA/HT-11, Presented at Winter Annual Meeting, N.Y., 1974.
11. Edwards, D.K., Arnold, J.N., Catton, I., "End Clearance Effects on Rectangular-Honeycomb Solar Collectors", Paper Presented at the meeting, ISES 75, Los Angeles, Calif., July 28-Aug. 1, 1975.
12. Hollands, K.G.T., and Konicek, L., "Experimental Study of the Stability of Differentially Heated Inclined Air Layers", Int. J. Heat Mass Transfer, Vol. 16, pp. 1467-1476, (1973).

APPENDIX

13. Hollands, K.G.T., Unny, T.E., Raithby, G.D., and Konicek, L., "Free Convective Heat Transfer Across Inclined Air Layers", ASME Paper No. 75-HT-S5; also J. Heat Transfer, 1976 (in press).
14. Cane, R.L.D., "Free Convection in Inclined Air Layers Constrained by a Square-Celled Honeycomb", M.A.Sc. Thesis, Department of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada, 1976.
15. Raithby, G.D., Hollands, K.G.T., and Unny, T.E., "Free Convection Heat Transfer Across Fluid Layers of Large Aspect Ratios", to be presented at the National Heat Transfer Conference, ASME-AIChE, St. Louis, Aug. 8-11, 1976.
16. Rajaram, S., "Numerical Study of Free Convective Heat Transfer Across Rectangular Cells", M.A.Sc. Thesis, Department of Mech.Eng., University of Waterloo, Waterloo, Canada, (1976).
17. Heitz, W.L., and Westwater, J.W., "Critical Rayleigh Numbers for Natural Convection of Water Confined in Square Cells with L/D from .5 to 8", J. Heat Transfer, Vol. 93, pp. 188-195 (1971). (Discussion by D.K. Edwards.)
18. Hart, J.E., "Stability of Thin Non-rotating Hadley Circulations", J. Atmos. Sci., Vol. 29, No. 4, pp. 687-697, (1972).
19. Edwards, D.K., and Sun, W.M., "Prediction of the Onset of Natural Convection in Rectangular Honeycomb Structures", Paper 7/62, 1970 International Solar Energy Society Conference, published by the Australian and New Zealand Section of the Society, 191 Royal Parade, Parkville, Victoria 3052.

APPENDIX

LIST OF FIGURES

- Figure 1: Figure showing dimensions and method of fabricating square-celled honeycomb.
- Figure 2: Experimental results for various aspect ratios and angles of inclination in the form of Nusselt number-Rayleigh number plots.
- Figure 3: Experimental results plotted in terms of Ra_D rather than Ra .
- Figure 4: A check on dimensional similarity and the importance of variations in the groups C , H and N . These two honeycombs have the same aspect ratio A , but are of different actual dimensions (see Table 1).
- Figure 5: Comparison of the critical Rayleigh numbers at $\theta = 0$, predicted by Catton [6], and those measured in the present study and in [3].
- Figure 6: Comparison of two honeycombs, one square-celled, the other hexagonal, compared at the same hydraulic diameter ($A = 5$). The hexagonal-celled honeycomb also has a higher wall emissivity.

APPENDIX

FIGURE 1

APPENDIX

FIGURE 2

FIGURE 3

APPENDIX

FIGURE 4

APPENDIX

FIGURE 5

APPENDIX

FIGURE 6

APPENDIX

APPENDIX B

NUMERICAL STUDY OF FREE

CONVECTIVE HEAT TRANSFER ACROSS

RECTANGULAR CELLS

By

S. Rajaram (1)

T. E. Unny (2)

K. G. T. Hollands (3)

G. D. Raithby (4)

- (1) Graduate Student, Dept. of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada
- (2) Professor, Dept. of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada
- (3) Associate Professor, Dept. of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada
- (4) Associate Professor, Dept. of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada

APPENDIX

ABSTRACT

This paper deals with a numerical study of natural convection induced in a rectangular cavity (square in cross-section), whose square, vertical faces are maintained at different fixed temperatures. The Boussinesq equations of fluid flow were solved numerically in three dimensions using a computationally stable, implicit finite difference scheme. Following a method suggested by Mallinson and De Vahl Davis, the equations in the steady state, which are a set of coupled, non-linear elliptic partial differential equations, have been converted to parabolic form by the introduction of false transient terms and then solved using a marching procedure through time, with varying time steps for different equations. The equations have been formulated in terms of vorticity and stream-function. A Prandtl number of 0.72 (air) was assumed and the problem has been solved for perfectly conducting and adiabatic boundary conditions for side walls. Aspect ratios from 0.2 to 5 and Rayleigh numbers up to 1.5×10^6 have been considered. An interesting feature of the results is that for a given Rayleigh number, there is a value of the aspect ratio (≈ 1) at which the Nusselt number is maximum.

APPENDIX

INTRODUCTION

Thermal convection in a rectangular region is of interest from the point of view of both fundamental fluid mechanics, and engineering applications. One important example of the latter is the evaluation of free convective heat transfer in honeycomb solar collectors. Of particular interest to the present paper is the special case where the vertical end faces are maintained at different fixed temperatures (Figure 1), so that the heating is from the sides rather than from below.

Although considerable study (for example [1] - [3] has been given to this problem for the case of small aspect ratio, A , ($A = L/D$, Figure 1), very little, if any, attention has been given to the other extreme case, where $A > 1$. An important exception is the recent analytical, Galerkin-method, study of Catton et al [4], which included, within its wide range of parameters, the case of $A = 5$. As might be expected, their predicted Nusselt numbers for the aspect ratio was considerably less than that for large aspect ratio, for the same Rayleigh number, indicating substantial convection suppression by the side-walls. Their study was two-dimensional and assumed infinite Prandtl number.

A number of studies have attacked the problem using finite difference techniques, although none have considered A as great as 5, and all have been two-dimensional. Moreover the Rayleigh number range investigated has been very moderate. Included in these studies are the important early studies by Elder [5], Wilkes and Churchill [15], Gershuni et al [6], and De Vahl Davis [7]. The first fully 3-dimensional

APPENDIX

numerical solution to the cavity problem (but not that specific cavity shown in Figure 1) appears to be that of Aziz and Hellums [8]. Their maximum Rayleigh number was 3500.

An interesting general approach applicable to both 2 and 3 dimensional problems has recently been outlined by Mallinson and De Vahl Davis [9]. Although they give no detailed results, they describe a method for solving a set of coupled elliptical equations, the essential feature of which is the conversion of all equations into parabolic form by the addition of false transients, and marching through distorted time. The method is a variation on the scheme proposed independently by Douglas [10] and Brian [11].

The present paper applies this scheme (i.e. that described in [9]) to the square-cross-section cell problem shown in Figure 1. The problem formulation, and, in the first instance, solution, is fully three-dimensional. However, it was found that for the square-cross-section case, the effect of the third dimension (i.e. y - dimension) was small, and hence subsequent solutions were all made two-dimensional. Results cover an aspect ratio range of from .2 to 5 and Rayleigh numbers ranging up to 10^6 and greater. The Prandtl number was kept at .72, which is that of interest in solar collectors. Referring to the D x L faces in Figure 1 as side walls, solutions are given for two classes of boundary conditions on the side walls, adiabatic and perfectly conducting.

APPENDIX

FORMULATION OF THE PROBLEM

Governing Equations

Making the Boussinesq approximation, the equations of state are:

$$\rho = \rho_0 [1 - \beta(T - T_0)]$$

$$\kappa = \text{constant}$$

$$\nu = \text{constant}$$

$$\beta = \text{constant}$$

where ρ , β , ν and κ denote respectively the mass density, volumetric expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid. Further T is the temperature of the fluid at the physical point under consideration and T_0 denotes temperature at a specified state.

The equations of motion and energy can be written as

$$\frac{D\bar{V}}{Dt} = -\nabla p / \rho_0 - \beta(T - T_0) \bar{g} + \nu \nabla^2 \bar{V} \quad (1)$$

$$\nabla \cdot \bar{V} = 0 \quad (2)$$

$$\frac{DT}{Dt} = \kappa \nabla^2 T \quad (3)$$

where $\bar{V}$ is the velocity vector, p is the pressure given by

$p = (P + \rho_0 g h)$ where P is the static pressure and h is a vertical coordinate measured upwards. Further $\bar{g}$ is the gravitational vector.

APPENDIX

The variables are non-dimensionalized using suitable scale factors. They are L , (a dimension of the cavity), L^2/κ , κ/L , $\rho_0^2 \kappa^2/L^2$ for length, time, velocity and pressure, respectively and $\theta = (T - T_0)/(T_1 - T_0)$. Here, L is considered as the cross dimension of the layer (Fig. 1).

Equations (1) to (3) now become

$$\frac{\partial \bar{V}}{\partial t} = -(\nabla \times \bar{V}) \times \bar{V} - \nabla P - Ra Pr \theta \bar{g} + Pr \nabla^2 \bar{V} \quad (4)$$

$$\frac{\partial \theta}{\partial t} = -\nabla \cdot (\bar{V} \theta) + \nabla^2 \theta \quad (5)$$

where $P = (p + \rho_0 (\bar{V} \cdot \bar{V}))$

Further, $Ra = \beta g (T_1 - T_0) L^3 / \kappa \nu$ is the Rayleigh number and $Pr = \nu / \kappa$ is the Prandtl number.

A scalar stream-function ψ as such does not exist for three dimensional flows. But for a solenoidal vector field which obeys the three dimensional continuity equation, $\nabla \cdot \bar{V} = 0$, a so-called vector potential $\bar{\psi} = \psi_1 i + \psi_2 j + \psi_3 k$ does exist, such that velocity components are obtainable from the curl of $\bar{\psi}$, and given by

$$\bar{V} = \nabla \times \bar{\psi} \quad (6)$$

The arbitrariness present in the definition of $\bar{\psi}$ is removed by requiring that $\bar{\psi}$ be solenoidal, that is,

APPENDIX

$$\nabla \cdot \bar{\psi} = 0 \quad (7)$$

Vorticity, τ , is then obtained as

$$\bar{\tau} = \nabla \times \bar{V} = \nabla \times \nabla \times \bar{\psi} = -\nabla^2 \bar{\psi} \quad (8)$$

Equation (4) may now be rewritten as

$$\frac{\partial \bar{\tau}}{\partial t} = -\nabla \times (\bar{\tau} \times \bar{V}) - RaPr (\nabla \times \bar{\theta g}) + Pr \nabla^2 \bar{\tau} \quad (9)$$

$$0 = \nabla^2 \bar{\psi} + \bar{\tau} \quad (10)$$

$$\bar{V} = \nabla \times \bar{\psi} \quad (11)$$

Equations (9) to (11) and equation (5) are to be solved for the velocity and temperature fields.

The method of false transients as proposed in [9] makes two simple changes to the above set of equations. A fictitious transient term is inserted on the left hand side of equation (10) and the resulting time derivatives as well as the time derivatives in the momentum equations are given modified coefficients.

The equation set then becomes

$$\frac{Pr}{\alpha_\tau} \frac{\partial \bar{\tau}}{\partial t} = -\nabla \times (\bar{\tau} \times \bar{V}) - Ra Pr (\nabla \times \bar{\theta g}) + Pr \nabla^2 \bar{\tau} \quad (12)$$

APPENDIX

$$\frac{1}{\alpha_{\psi}} \frac{\partial \bar{\psi}}{\partial \tau} = \nabla^2 \psi + \bar{\tau} \quad (13)$$

$$\frac{\partial \theta}{\partial \tau} = -\nabla \cdot (\bar{V} \theta) + \nabla^2 \theta \quad (14)$$

If a steady state exists, it will be attained as a final time independent solution for $\bar{\tau}$, $\bar{V}$, $\bar{\psi}$, and θ . The rate at which this will be attained depends upon the values of α_{τ} and α_{ψ} . The effect of these time constants is to distort the artificial time steps. According to the procedure adopted in solving equations (12), (13), (14) and (11) progressively through time, increasingly higher values of α 's are used to each equation until α is made equal to unity in the final equation (14), this being the last equation that is solved in the marching procedure.

Boundary Conditions

Non-dimensionalizing the lengths using L as a length scale, the limits for x are 0 and 1, the limits for y are 0 and D/L and the limits for z are 0 and D/L .

The specification of boundary conditions for the vector potential has been fully discussed by Hirasaki and Hellums [12]. For a plane impermeable surface, the vector is normal to the surface and its gradient is zero.

For the surface $x = 0$, for example

APPENDIX

$$\frac{\partial \psi_1}{\partial x} = 0, \quad \psi_2 = \psi_3 = 0$$

At non-slip surfaces, the tangential derivatives of the velocity components are zero. Thus at $x = 0, 1$ the vorticity components become

$$\tau_1 = 0, \quad \tau_2 = -\partial w / \partial x, \quad \tau_3 = \partial v / \partial x$$

The boundary condition on θ are simple. Two types of boundary conditions were used - (a) perfectly conducting side walls and (b) adiabatic side walls. The planes $x = 0$ and $x = 1$ are maintained at different uniform temperatures.

The complete set of boundary conditions on $\bar{v}$, $\bar{\tau}$, $\bar{\psi}$ and θ are shown in Table 1.

APPENDIX

Table 1

Boundary Conditions At The Walls

	$x = 0$ and $x = 1$	$y = 0, y = D/L$	$z = 0, z = D/L$
u :	0	0	0
v :	0	0	0
w :	0	0	0
ψ_1	$\partial\psi_1/\partial x = 0$	$\psi_1 = 0$	$\psi_1 = 0$
ψ_2	$\psi_2 = 0$	$\partial\psi_2/\partial y = 0$	$\psi_2 = 0$
ψ_3	$\psi_3 = 0$	$\psi_3 = 0$	$\partial\psi_3/\partial z = 0$
τ_1	$\tau_1 = 0$	$\tau_1 = \partial w/\partial y$	$\tau_1 = -\partial v/\partial z$
τ_2	$\tau_2 = -\partial w/\partial x$	$\tau_2 = 0$	$\tau_2 = \partial u/\partial z$
τ_3	$\tau_3 = \partial v/\partial x$	$\tau_3 = -\partial u/\partial y$	$\tau_3 = 0$
θ	at $x = 0, \theta = 1$ at $x = 1, \theta = 0$		
For adiabatic side walls		$\frac{\partial\theta}{\partial y} = 0$	$\frac{\partial\theta}{\partial z} = 0$
For perfectly conducting side walls		$\theta = (1 - x)$	$\theta = 1 - x$

APPENDIX

For the special case of two-dimensional flow, the boundary conditions are

$$u = v = 0 \quad \text{at } x = 0, 1$$

$$u = v = 0 \quad \text{at } z = 0, D/L$$

$$\psi_2 = 0 \quad \text{at } x = 0, 1$$

$$\psi_2 = 0 \quad \text{at } z = 0, D/L$$

$$\tau_2 = - \partial w / \partial x \quad \text{at } x = 0, 1$$

$$\tau_2 = \partial u / \partial z \quad \text{at } z = 0, D/L$$

$$\theta = 1 \quad \text{at } x = 0$$

$$\theta = 0 \quad \text{at } x = 1$$

$$\partial \theta / \partial z = 0 \quad \text{at } z = 0, D/L \text{ for adiabatic side walls}$$

$$\theta = (1-x) \quad \text{at } z = 0, D/L \text{ for conducting side walls}$$

APPENDIX

SOLUTION PROCEDURE

Solving of Equations

The field equations (12) to (14) are solved numerically using an alternating direction implicit method. The method adopted is the one suggested by Mallinson and De Vahl Davis [9]. This method is a variation of the scheme proposed independently by Douglas [10] and Brian [11]. It is to be noted that the field equations are written in a conservative form that guarantees conservation of vorticity in the finite difference formulation of the problem.

A uniform mesh is employed throughout and central differences have been used.

To start the numerical procedure, initial assumed distributions of $\bar{\psi}$ and θ are given as inputs. From these distributions values of $\bar{\tau}$ and $\bar{V}$ are computed. The main iterative loop starts with the advancement through one time step of $\bar{\psi}$ using equation (13). This is followed by calculation of new velocity fields from equations (11) and the determination of vorticity boundary conditions. A new θ field is found from equation (14) and finally new values of $\bar{\tau}_1$, $\bar{\tau}_2$, and $\bar{\tau}_3$ are obtained from equation (12). The cycle is continued until convergence is reached, that is, until steady state has been obtained.

The method requires the solution of many small independent systems of simultaneous equations. The method was chosen for its efficiency in terms of the number of operations required per time step.

APPENDIX

Calculations by Mallinson and De Vahl Davis [9] revealed that both in two and three dimensions, with α_τ and $\alpha_\psi = 1$, the upper limit for time step to avoid numerical instability was given by $\Delta t = 0.8(\Delta h)^2$, where Δh is the grid size. Reasoning that the oscillations in the solution were driven by the source term in the vorticity equation, they reduced α_τ to less than 1. By using time steps for the vorticity equation between 0.1 and $0.15(\Delta h)^2$, the time step for the stream function and energy equations were increased to between 4 to 10 times $(\Delta h)^2$. In other words, they found that the effective time step for the stream-function and energy equations could be increased beyond the permissible limit of $0.8(\Delta h)^2$ by the same factor by which it could be reduced in the vorticity equation.

In the present case, a number of tests were conducted with the time step in the vorticity equation equal to $0.5(\Delta h)^2$ and the time steps in the stream-function and energy equations in the range of 4 to 10 times $(\Delta h)^2$. The solution started to diverge rapidly as the time step in the energy and stream-function equation was increased. Therefore, it was necessary to determine the appropriate time steps by trial and error.

The range of values in our scheme were the following:

$$\alpha_\tau - 0.005 \text{ to } 0.05$$

$$\alpha_\psi - 0.50 \text{ to } 1.00$$

APPENDIX

Time step for vorticity equation: $0.05(\Delta h)^2 - 0.1(\Delta h)^2$

Time step for stream-function equation: $0.5(\Delta h)^2 - 2.5(\Delta h)^2$

Time step for energy equation: $1(\Delta h)^2 - 2.5(\Delta h)^2$

where (Δh) is the grid size.

The criterion for convergence is that the absolute value of the ratio of the difference between new and old value to the old value be $\leq \epsilon$, for all variables, where $\epsilon = 10^{-5}$.

That is

$$\left| \frac{\text{New value} - \text{old value}}{\text{old value}} \right| \leq \epsilon$$

This condition must be satisfied by $\bar{\psi}$, $\bar{\tau}$, θ and $\bar{V}$. Convergence is declared to have been reached if this condition is satisfied for two successive iterations. This requirement was intended to prevent premature termination of the iterations at the turning point of a slow oscillatory solution.

Due to storage limitations on the computer the grid size was limited to 0.1 in all cases. The vorticity stream-function technique contains ten variables. However, these can be reduced to eight by computing the velocities when required and by solving for the three components of vorticity consecutively. However, this will result in an increase in solution time.

The numerical solution for the set of equations was obtained for various Rayleigh numbers up to 1.5×10^6 . The solution for

APPENDIX

lower Rayleigh numbers was used as the starting values for the solution at the next higher Rayleigh number.

Heat-Transfer Calculation

The parameter of practical importance is the Nusselt number, Nu . The Nusselt number is a dimensionless quantity describing the heat transfer. It is convenient to determine the local Nusselt number at a point by considering the temperature gradient on a vertical wall since the heat transfer through a bounding wall is purely by conduction. For any point in the interior, both conduction and convection have to be taken into account.

The local Nusselt number at a point on the hot wall is given by

$$Nu_z = - \left(\frac{\partial \theta}{\partial x} \right)_{x=0}$$

The average Nusselt number is defined as

$$Nu = -A \int_0^{1/A} \left(\frac{\partial \theta}{\partial x} \right)_{x=0} dz$$

$\left(\frac{\partial \theta}{\partial x} \right)_{x=0}$ is calculated using three points, a point on the wall and two interior points as

$$\left. \frac{d\theta}{dx} \right|_j = (-\theta_{j+2} + 4\theta_{j+1} - 3\theta_j)/2(\Delta x)$$

At low Ra , when $(\partial \theta / \partial x)_{x=0}$ tends to 1 for all z , Nu tends to 1 indicating heat is transferred across the layer purely by conduction.

The average Nu is evaluated by numerical integration.

APPENDIX

RESULTS AND DISCUSSION

One of the interesting outcomes of the calculations has been a determination of effect of the vertical side walls on the fluid motion. Table 2, below, lists the absolute value of the maximum stream-function components in the three directions at various Ra. The results are for a cell of aspect ratio unity and with conducting side walls.

TABLE 2

Ra	ψ_x	ψ_y	ψ_z
1000	0.00401	2.476	0.00436
5000	0.00404	4.042	0.00502
10000	0.00512	5.708	0.00615
15000	0.00763	6.821	0.00836
20000	0.00925	7.983	0.00974
25000	0.0112	8.836	0.0107
30000	0.0124	9.761	0.109
35000	0.0129	10.303	0.0109
40000	0.0135	10.734	0.0120
60000	0.0141	11.387	0.0123
80000	0.0146	12.917	0.0126

An inspection of the table indicates that the strongest component is ψ_y and that, in comparison, ψ_x and ψ_z are of a negligible magnitude. Since the stream-function is a measure of the total quantity of fluid in motion, this result clearly indicates that the motion is almost two dimensional, even for high Ra. Consequently, only 2 dimensional solutions were subsequently obtained. Results given here after

APPENDIX

are 2-dimensional.

In figure 2 are shown the temperature profiles along the mid-height of the plane $y = 1/2$ for an aspect ratio of unity. The temperature profile has been shown for various Ra and for both conducting and insulating side walls.

At low Ra (1000) the temperature profile is virtually linear. The flow of heat across the cell from the hot wall to the cold wall under these conditions is due almost entirely to conduction. However, as Ra increases, convection becomes increasingly significant. The temperature profiles now show a progressive departure from linearity. The horizontal temperature gradient in the central portion becomes smaller, showing that the amount of heat flowing across the cell from the hot wall to the cold wall directly by conduction is diminishing. This is the transition regime.

At very high Ra , the horizontal gradient of temperature in the central portion becomes negative and there are steep gradients of temperature at the hot and cold walls. The flow now in the boundary layer regime, is so strong that hot fluid has been carried over to the cold side of the cavity and vice versa. Conduction in the central portion of the cavity is actually opposite to the overall direction of heat flow. There is very little convection in this region, the fluid motion being essentially vertical. The curves show larger deviation from conduction when the side walls are insulating.

APPENDIX

The heat transfer results are presented in figures (3) to (12) as plots of average Nu vs Ra for each aspect ratio and also for conducting and insulating side walls.

The vertical square cavity has been the subject of some investigations. Several comparisons with the present work are possible.

For the conducting side wall, comparison with the results of De Vahl Davis [7], is satisfactory (fig. 3) though the results of Gershuni et al [6] show large deviations. It may be noted that the above authors considered a $Pr = 1$ while in the present work $Pr = 0.72$. For the insulating side walls good agreement is found with the results of Elder [5] and De Vahl Davis [7]. (fig. 4).

Since this work was motivated by a need to reduce natural convection losses in solar collectors, the question most relevant is that of the effectiveness of the rectangular cavity in reducing convective losses. An inspection of the heat transfer results clearly indicates that it depends on the aspect ratio.

The results show that for a given Ra , there is an aspect ratio giving maximum heat transfer. This aspect ratio is approximately unity. As the aspect ratio is increased beyond unity there is a sharp reduction in heat transfer. The Nu decreases rapidly with increasing aspect ratios. However, as the aspect ratio is decreased below unity, there is only a slight reduction in heat transfer with decreasing aspect ratios. This effect is demonstrated

APPENDIX

in figures (13) and (14).

There is a practical implication of this result. If honeycombs are provided in flat plate collectors of vertical or near vertical orientation they should be such that the aspect ratio of these rectangular cells be made larger than unity for effectiveness in reducing heat transfer. Otherwise the honeycomb can actually increase the heat transfer. With a very high aspect ratio a large number of cells would be required and hence in any practical situation the proper aspect ratio has to be selected through consideration of an economical cost function.

Comparison with the experimental work of Cane et al [13], [14] is possible for aspect ratios 2,3,4 and 5. This has revealed an interesting feature. In figures 5,6,7 and 8 the heat transfer results obtained by the numerical solution has been compared with those obtained by experiments [13] for aspect ratios mentioned above. There seem to be discrepancies between these results. The maximum deviation is of the order of 40%. The discrepancy could perhaps be explained in terms of the differences in boundary conditions between the experiment and theory. The boundary conditions for the numerical model was either perfectly conducting side walls or perfectly adiabatic side walls. However, a different boundary condition exists in the experiment. This is what is called a "repetitive" boundary condition. In such a case the gradient of temperature normal to the side wall at a point on the wall is equal

APPENDIX

to the gradient of the temperature normal to the wall at a corresponding point on the opposite wall. This condition exists due to the hot fluid moving up the hot wall and the cold fluid moving down on the cold wall. This has the effect of a counter-flow heat exchanger. This arrangement has a tendency to reduce transfer of heat and might explain the lower heat transfer rate obtained in experiments.

Another interesting feature is revealed by comparison with experiments. The equations describing the fluid motion in the numerical model are that of the base flow. The heat transfer results obtained numerically therefore suggest large heat transfer rates associated with the base flow and that the experimentally obtained heat transfer results are also due to the base flow. The comparison therefore suggests that there is no instability in the base flow filling the honeycomb in the range of Rayleigh numbers investigated.

APPENDIX

SUMMARY AND CONCLUSIONS

A stable implicit finite difference scheme and the method of false transients has been used to obtain solutions to equations of motion and energy in three dimensions governing the flow of air in rectangular cells caused by a temperature gradient normal to the body force. Several aspect ratios have been investigated for both perfectly conducting and adiabatic walls.

The motivation for this work arose from a desire to suppress natural convection heat losses from honeycomb solar collectors. Results have clearly indicated that this depends upon the aspect ratio. For a given Rayleigh number there appears to be a maximum heat transfer rate at an aspect ratio near unity.

Another significant feature is that in the case of the vertical geometry, the base flow contributes significantly towards heat transfer and there does not appear to be instabilities associated with the base flow up to high Rayleigh numbers.

Finally, it has been found that for the case of a square cross-section cell, the problem can be treated essentially as a 2-dimensional one.

APPENDIX

ACKNOWLEDGEMENTS

This work was supported by the Energy Research and Development Agency of the United States, under contract number E(11-1) 2597. The support of this agency is gratefully acknowledged. The authors are also thankful to Mr. R.L.D. Cane for discussions on the comparison with experiment.

APPENDIX

REFERENCES

- [1] Batchelor, G. K., "Heat Transfer by Free Convection Across A Closed Cavity Between Vertical Boundaries at Different Temperatures", Quart. J. Appl. Math., 12 (1954), 209-233.
- [2] Eckert, E. R. G. and Carlson, W. O., "Natural Convection In An Air Layer Enclosed Between Two Vertical Plates With Different Temperatures", Int. J. Heat Mass Transfer, 2 (1961) 106-120.
- [3] Elder, J. W., "Laminar Free Convection In A Vertical Slot", J. Fluid Mech., 23 (1965) 77-98.
- [4] Catton, Ivan, P.S. Ayyaswamy and Clever, R.M. "Natural Convection Flow in A Finite Rectangular Slot Arbitrarily Oriented With Respect To The Gravity Vector." Int. J. Heat Mass Transfer, 17 (1974) 173-184.
- [5] Elder, J. W., "Numerical Experiments With Free Convection In A Vertical Slot", J. Fluid Mech. 24-(1966) 823-843.
- [6] Gershuni, G. Z., Zhukhovitskii, E. M., and Tarunin, E. L. "Numerical Investigation of Convective Motion In A Closed Cavity", Mekhanika Zhidkosti I Gaza, Vol. 1, No. 5 (1966) 56-62.
- [7] De Vahl Davis, "Laminar Natural Convection In An Enclosed Rectangular Cavity", Int. J. Heat Mass Transfer, 11 (1968) 1675-1692.

APPENDIX

- [8] Aziz, K., and Hellums, J. D., "Numerical Solution of Three-Dimensional Equations of Motion For Laminar Natural Convection", Physics Fluids 10 (1967), 314-324.
- [9] Mallinson, G. D. and De Vahl Davis, G., "The Method of False Transients For the Solution of Coupled Elliptic Equations", Journal of Computational Physics 12, (1973) 435-461.
- [10] Douglas, J., "Alternating Direction Methods for Three Space Variables", Numerische Mathematik, 4 (1962) 41-63.
- [11] Brian, P. L. T., "A Finite Difference Method of High-Order Accuracy for the Solution of Three-Dimensional Transient Heat Conduction Problems", A.I.Ch.E., J. 7 (1961) 367-370.
- [12] Hirasaki, G. J., and Hellums, J. D., "A General Formulation of the Boundary Conditions on the Vector Potential In Three Dimensional Hydrodynamics", Quart. Appl. Math. 16 (1968) 331-342.
- [13] Cane, R. L. D., Hollands, K.G.T., Raithby, G.D. and Unny, T.E., "Convection Suppression in Inclined Honeycombs", Presented at the International Solar Energy Society Meeting in Los Angeles, July 28, - August 1, 1975.
- [14] Hollands, K.G.T. "Studies On Method Of Reducing Heat Losses From Flat Plate Collectors." University of Waterloo Research Institute, Annual Progress Report on ERDA contract E(11-1) 2597.

APPENDIX

LIST OF FIGURES

- FIG. 1. Geometry of the Rectangular Region.
- FIG. 2. Temperature Distribution at mid height of cavity for $A = 1$
- FIG. 3. Plot of average Nusselt number vs Rayleigh number for $A = 1$. Conducting Side Walls.
- FIG. 4. Plot of average Nusselt number vs Rayleigh number for $A = 1$. Adiabatic side walls.
- FIG. 5. Plot of average Nusselt number vs Rayleigh number for $A = 2$.
- FIG. 6. Plot of average Nusselt number vs Rayleigh number for $A = 3$.
- FIG. 7. Plot of average Nusselt number vs Rayleigh number for $A = 4$.
- FIG. 8. Plot of average Nusselt number vs Rayleigh number for $A = 5$.
- FIG. 9. Plot of average Nusselt number vs Rayleigh number for $A = 1/2$.
- FIG. 10. Plot of average Nusselt number vs Rayleigh number for $A = 1/3$.
- FIG. 11. Plot of average Nusselt number vs Rayleigh number for $A = 1/4$.
- FIG. 12. Plot of average Nusselt number vs Rayleigh number for $A = 1/5$.
- FIG. 13. Plot of average Nusselt number vs Aspect ratio for a conducting side walls.
- FIG. 14. Plot of average Nusselt Number vs Aspect ratio for an adiabatic side walls.

APPENDIX

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

APPENDIX

Fig. 10

Fig. II

Fig. 12

Fig. 13

Fig. 14

APPENDIX

C

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK.

APPENDIX C

RADIANT TRANSMITTANCE OF V-CORRUGATED
TRANSPARENT SHEETS
with Application to Solar Collectors

B.E. SIBBITT

Research Engineer
National Research Council
Ottawa, Canada

K.G.T. HOLLANDS

Associate Professor
Thermal Engineering Group
Department of Mechanical Engineering
University of Waterloo
Waterloo, Canada

May 1976

APPENDIX

ABSTRACT

This paper demonstrates both theoretically and experimentally that the radiant transmittance of a flat specular transparent sheet can be increased if it is V-corrugated. This principle can be made use of in a flat plate solar collector, by placing such a V-corrugated sheet between the absorbing plate and the outer cover. The theoretical predictions of the radiant (solar) transmittance of a V-corrugated sheet are based on a Monte Carlo type analysis, with allowance for Fresnel reflections, polarization effects, and forward scattering. The experiments are performed on an FEP Teflon sheet, V-corrugated with an opening angle of 30 degrees, and are found to be in good agreement with the theory. The predicted solar transmittance of V-corrugated sheets, fabricated from several plastics, suitable for solar collectors, are presented.

APPENDIX

LIST OF SYMBOLS

H	height of VCTS, Figure 2, [m]
I_p, I_s	intensity of p-polarized and s-polarized radiation, respectively, [W/m ² st]
I_u, I_v	third and fourth Stokes coefficients, respectively [W/m ² st]
K	absorption coefficient [m ⁻¹]
K_λ	spectral absorption coefficient [m ⁻¹]
L	thickness of sheet (Figure 2), [m]
n	index of refraction of sheet material
P	pitch of VCTS, (Figure 1), [m]
r_p, r_s	interfacial reflectivity between air and a dielectric of index of refraction n , for p-polarized and s-polarized radiation, respectively [equation (1.c)]
R	random number
$ R $	reflectivity matrix
S	scattering coefficient
x, y, z	co-ordinates, see Figure 3, [m]
α^*	absorptance of thin transparent sheet
β	angle of incidence of ray from normal to plane of VCTS, Figure 2
θ	circumferential angle of incidence of ray on VCTS, Figure 2
λ	wavelength, [μ m]
ρ^*	reflectance of thin flat transparent sheet
ρ_p^*, ρ_s^*	reflectance of thin flat transparent sheet to p and s-polarized components, respectively [equation (1.b)]
τ^*	reflectance of thin flat transparent sheet

APPENDIX

τ_p^*, τ_s^*	reflectance of thin flat transparent sheet to p and s-polarized components, respectively [equation (1.b)]
$\tau(\beta, \theta), \tau(\beta, \theta, x)$	directional-hemispheric transmittance of VCTS; $\tau(\beta, \theta, x)$ is the local value at x ; $\tau(\beta, \theta)$ is the value averaged with respect to x
ϕ	angle of opening of VCTS, Figure 2
χ	see equation (1.d)
ψ	angle of incidence on flat transparent sheet, measured from normal to sheet

APPENDIX

INTRODUCTION

Low temperature solar energy-heat converters that are more efficient than the conventional flat plate solar collector are required for building heating and cooling systems. Several candidate advanced converters are therefore under study and development. This paper relates to the evaluation of one suggestion for an advanced solar converter - namely, the V-corrugated transparent sheet (VCTS) collector [1].

The principle of the VCTS collector is very similar to that of the transparent honeycomb collector. In this latter collector the honeycomb effectively reduces the collector loss coefficient, by suppressing free convection and radiant transfer from the absorber plate to the outer cover. Equivalent heat loss reduction could be obtained by a sufficient number of flat transparent sheets (inner covers) placed between, and parallel to, the absorber plate and the outer cover (Fig. 1(a)). However, since for the honeycomb, incident solar rays reflected off the honeycomb walls are reflected forward toward the absorber plate (Fig. 1(b)), whereas for the set of inner covers, rays reflected off the various covers are reflected back away from the absorber plate (Fig. 1(a)), the solar transmittance of the honeycomb is considerably greater than the equivalent set of inner covers. Consequently, the honeycomb collector is more efficient. In brief, the honeycomb forward-reflects (or forward-scatters) whereas inner covers back reflect (or back-scatter).

This capacity for forward-reflection can be designed into geometrical configurations of transparent sheets other than the honeycomb shape. One of these is the V-corrugated transparent sheet, shown in Figure 1(c). The VCTS consists of a thin transparent specular sheet

APPENDIX

(probably plastic) formed (by, say, extrusion or vacuum molding) into a V-corrugated shape as sketched. The profile of the sheet is assumed to be invariant in the direction normal to the plane of the drawing. That (at least for near-normal incidence, and at least for the angle of opening shown) the reflections are forward, is indicated in the sketch. Provided the opening angle, ϕ , (Figure (2)), is sufficiently small, the VCTS should suppress the free convection between the absorber plate and the outer cover. Moreover, provided the plastic is thin and opaque to thermal radiation, radiant loss suppression should also be affected (otherwise the absorber plate may be given a selective surface). Although full experimental and theoretical studies are required to establish it, the VCTS shows promise of improving efficiencies almost as much as the honeycomb collector does. It has the added advantage of being much easier to fabricate.

The present paper deals with one aspect of the VCTS collector performance, namely its radiant (solar) transmittance. As discussed, the VCTS is expected to have a higher transmittance than the corresponding flat transparent sheet, since the first reflected ray is forward-reflected. However, a number of factors, such as increased absorption and scattering, and increased reflectance at interfaces (due to higher angles of incidence), could negate this expectation. In order to check its validity, a general theory for the transmittance of the VCTS, which takes into account absorption in the film material, Fresnel reflection at interfaces and scattering, was required. Moreover, since it is known that in forward-reflecting, specular geometries, polarization effects can be important [2], the theory should also account for polarization. This paper presents such a theory,

APPENDIX

and also describes an experiment designed to check the theory.

THEORY

The transparent sheet which comprises the VCTS is assumed to be a smooth-surfaced thin sheet of a dielectric material. Although allowance will be made for radiant absorption in the sheet, the material extinction coefficient is assumed to be not so large as to affect reflectivity (i.e., $k \ll n$). The index of refraction, n , and the absorption coefficient of the sheet material, K , are both assumed to be independent of wavelength and in this sense the analysis is grey. However, for most materials of practical interest, the absorption is weak, so that the optically thin limit is approached. In this instance, use of the incident mean absorption coefficient for K should give exact results. The geometrical variables and co-ordinate system are defined in Figure 2. The angle of opening, ϕ , and sheet thickness, L , are assumed to be specified. Emission of radiation by the sheet is not of interest to the present problem; nor is the history of the radiation which is absorbed. Consequently, for the purposes of analysis, the sheet itself can be considered to be uniformly at zero absolute temperature. The top face ($z = 0$ plane) of the VCTS is assumed to be uniformly irradiated with unpolarized radiation of some known (solar) spectral distribution and unidirectional along directions β and θ . The $z = -H$ plane is black at zero absolute temperature. The material between the V's is a transparent gas ($n = 1$). Required is the directional transmittance of the VCTS, $\tau(\beta, \theta)$ defined as the ratio of the irradiation on the $z = -H$ plane to the irradiation on the $z = 0$ plane.

APPENDIX

In the first instance, an analysis based on the method of images for specular surfaces was considered. However, with both reflections and transmittances occurring, an algorithm covering all possible combinations of ray paths becomes so complex as to be virtually impossible to set up. Moreover such methods are not generally applicable to problems where an accounting for angular reflectance dependence and polarization effects is required. Consequently a Monte Carlo approach was adopted. Although Monte Carlo techniques have been applied to a host of radiant problems, to our knowledge, this is the first time it has been applied to a problem including polarization effects and multiple interactions with a diathermous sheet. Consequently a number of the procedures adopted may be of general interest.

One of these is the method of handling the radiant interaction of a ray with a single flat sheet. It is well known that some parts of an incident ray suffer multiple internal reflections: these photons reflect back and forth with some absorption taking place on each pass across the sheet. A strictly Monte Carlo approach would involve making a probabilistic decision, at each interface, as to whether or not the ray is reflected and, on each pass, to determine whether or not it is absorbed. However, the average photon interaction with the flat sheet can be predicted analytically, using classical methods. Thus the transmittance τ^* , reflectance ρ^* , and absorptance α^* for radiation incident with angle of incident ϕ and with intensity I_p for the polarization component normal to the incident plane, and I_s for the component parallel to that plane, is given by the set of relations:

APPENDIX

The method of handling polarization effects is also of interest, since, to our knowledge, polarization has not previously been incorporated into a Monte Carlo analysis. Extensive use was made of the methods and equations developed by Edwards and Bevans [3], and Edwards and Tobin [2]. The photon bundle is characterized at any point by an intensity vector $|I| = |I_p, I_s, I_u, I_v|$, where I_p and I_s are the components in two reference perpendicular directions, and I_u and I_v are the third and fourth Stokes coefficients. When the bundle is incident locally on a single flat sheet, a transformation matrix given in [3] must first be applied to its intensity vector so as to make one of the reference directions perpendicular to the plane of incidence for the sheet. Then the decision is made, as outlined previously, as to whether the bundle is reflected, transmitted or absorbed. If it is reflected, its intensity vector is modified as it must be multiplied by the reflectivity matrix $|R|$ which is also given in [3]. In the case of a dielectric, as is assumed here, the expression for $|R|$ reduces to:

$$|R| = \begin{vmatrix} r_p & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & \sqrt{r_p r_s} & 0 \\ 0 & 0 & 0 & \sqrt{r_p r_s} \end{vmatrix}$$

(Here it has been assumed that those photons which suffered internal reflections have their polarization altered in the same way as those which reflect from the first interface.) It can be shown that since all surfaces are dielectrics, I_v is unaltered from its original value of

APPENDIX

zero, so that, in fact, only three element intensity vectors need be carried.

Experimental measurements reported by Edwards et al [4] indicate that scattering may occur in thin plastic sheets in the short-wave (solar) part of the spectrum and that it is predominately forward scattering. Consequently, forward scattering was incorporated into the Monte Carlo algorithm, as represented by a quantity S . When the decision is made that the photon bundle is to be transmitted, a random number, $R(0 \leq R \leq 1)$, is generated; if $0 \leq R < 1-S$, the bundle is transmitted directly as discussed earlier. If $1-S \leq R \leq 1$, the bundle is assumed to be scattered. The direction of the scattered bundle is assumed to differ by not more than 90 degrees from the incident bundle, but not to back scatter (i.e., scatter in the same direction as a reflected bundle). The actual direction is decided by generating two random numbers to fix the cone angle and the circumferential angle, but restricting the range of the circumferential angle so as not to include back-scattering.

The Monte Carlo algorithm, which is incorporated into a computer program, proceeds in the following way. First the input parameters, β , θ , n , (Kt) , ϕ and S are specified. Since the value of $\tau(\beta, \theta, x)$ averaged over x from 0 to P is required, the photon bundle release point co-ordinate x is taken as one of a predetermined set of quadrature points, x_i , based on sixth order Gaussian quadrature; the weighted average of the values of $\tau(\beta, \theta, x_i)$ will later give the required average value, $\tau(\beta, \theta)$.

The bundle is assigned an intensity vector $|\frac{1}{2}, \frac{1}{2}, 0, 0|$,

APPENDIX

released from x_1 in direction β, θ , and traced until it strikes the VCTS. Here the local angle of incidence, ψ , is calculated, and the decision as to whether transmission, reflection or absorption takes place at this sheet is determined, using the method described previously. If not absorbed, the bundle continues to be traced until it either strikes a new sheet of the VCTS, in which case the process is repeated; or until it strikes the $z = 0$ plane, in which case it has been reflected by the VCTS; or until it strikes the $z = -H$ plane, in which case it has been transmitted by the VCTS. Registers of bundles transmitted, reflected and absorbed by the VCTS are maintained and if any of these events occurs, the corresponding register is increased by one, a new photon bundle is released from the $z = 0$ plane, and the process is repeated. After a predetermined number of bundles, N , have been released, the VCTS transmittance, $\tau(\beta, \theta)$, is then estimated as the number in the transmittance register, divided by N . The geometrical problems of determining the intersection points and angles were handled in the same way as described by Bannerot and Howell [5] for a similar geometry. Full details are given in [6].

As a compromise between computer cost and accuracy, the number of quantum bundles sent was fixed at $N = 1200$ (200 per quadrature point x_1). This number gives an uncertainty in $\tau(\beta, \theta)$ based upon 95% confidence levels of [7]:

$$\Delta\tau = .056 \sqrt{\tau(\beta, \theta) (1 - \tau(\beta, \theta))}$$

Values of $\tau(\beta, \theta)$ close to unity are expected. Thus, for $\tau(\beta, \theta) = .99$, $\Delta\tau = .0056$; for $\tau(\beta, \theta) = .95$, $\Delta\tau = .013$.

APPENDIX

EXPERIMENT

Simply stated, the method used in the experiment was to place the VCTS between the source and a radiometer, take a radiometer reading, remove the VCTS, take another radiometer reading; the transmittance is then the ratio of these two readings. The apparatus is sketched in Figure 3. The source, not shown in the figure, was a 1000 watt, 3200 K filament lamp with a quartz envelope; it was located 3.5 meters from the test VCTS and subtended a solid angle at the VCTS of about 10^{-4} steradians. The apparatus is also designed to operate with the sun as a source; a reference radiometer is therefore provided to ensure that the sun's radiation has not altered over the period of a reading. Since the VCTS is known to scatter radiation, a radiometer with a flat angular (as well as spectral) response was required; an Eppley Black and White Model 8-48 Radiometer was used. To ensure that the radiometer receives all scattered radiation, the VCTS would have to be infinite in size. The actual size of the VCTS was 41 cm square; the radiometer was 4 cm below the lower plane of the VCTS so that, for $\beta = 0$, all radiation scattered by less than 84 degrees from its original path is accounted for. At large angles of β , this problem becomes more critical; consequently, highly reflecting specular mirrors were placed around the outer edge of the VCTS, thereby creating indefinitely extending mirror images of the VCTS. The box (Figure 3) containing the radiometer and supporting the VCTS was painted on the inside with optical black paint, as was a cardboard sheet which shaded the radiometer's otherwise white body. This box was located on two gimbals to allow proper setting of the angles β and θ : first, the reference line for the source was obtained by adjusting the lower gimbal so as to align the sighting device; then the upper gimbal

APPENDIX

were adjusted to give the β and θ settings.

The VCTS was constructed by weaving .025 mm thick FEP teflon film over a pre-strung weft consisting of very thin (diameter $< .14$ mm) nylon thread which was wound on a stiff frame. The opening angle ϕ was made 30° and the pitch $P = 1.25$ cm, giving a height $H = 2.4$ cm. The measured transmittance of the threads themselves (without the teflon) was .9945. From this figure a slight correction factor, to be applied to later readings, was obtained.

The radiometer's detection area of about 7 cm^2 was not sufficient to average the x dependence of $\tau(\beta, \theta, x)$; it was therefore necessary to take a number of readings at different values of the x co-ordinate location of the centre of the radiometer, covering a range of $2P$. The fourteen Chebyshev quadrature points over the range 0 to $2P$ were used.

In order to compare the experimental measurements with the theoretical predictions, the value of the incident mean absorption coefficient, K , was required. The spectral coefficient, K_λ , was determined from normal-hemispheric spectral transmittance measurements on flat sheet supplied by the film manufacturer. The data was for 5 thicknesses of teflon sheet ranging from .127 mm to 24 mm, but extended only up to .75 μm . From this data and the index of refraction of teflon, $n = 1.344$, the quantity K_λ was calculated using equations (1). It was found to be independent of sheet thickness and given closely by $K_\lambda = .43\lambda^{-2.28} \text{ cm}^{-1}$ where λ is in μm . Binormal spectral transmittance data was available for $\lambda > .75 \mu\text{m}$ and indicated no absorption bands out to 2.7 μm . The incident mean extinction coefficient is given by:

APPENDIX

$$K = \frac{\int_0^{\infty} K_{\lambda} I_{\lambda i} d\lambda}{\int_0^{\infty} I_{\lambda i} d\lambda}$$

The incident radiation was assumed to be the same as that for a black body at 3200K except for a lower wavelength cut-off at $\lambda = .28 \mu\text{m}$, and an upper wavelength cut-off at $\lambda = 2.8 \mu\text{m}$, both of which were due to the transmission properties of the glass dome on the radiometer. The integrated value of K was found to be $K = .56 \text{ cm}^{-1}$ assuming that the relation $K_{\lambda} = .43\lambda^{-2.28} \text{ cm}^{-1}$ is valid for $\lambda > .75 \mu\text{m}$, and $K = .93 \text{ cm}^{-1}$ assuming K_{λ} remains constant for $\lambda > .75 \mu\text{m}$, at its value at $\lambda = .75 \mu\text{m}$. The average of these, $K = .745 \text{ cm}^{-1}$, was chosen. The corresponding values for the sun at air mass 2 as the source were 1.32, 1.45 and 1.39 cm^{-1} , respectively.

As a preliminary check on the apparatus and method, the transmittance of a flat teflon sheet, ($\phi = 180^\circ$), was measured and compared to the predictions of equations (1). Agreement was found to be excellent. Full details are given in [6].

COMPARISON OF THEORY AND EXPERIMENT

Figure 4 shows the comparison between theory and experiment for the experimental teflon VCTS, in the form of a plot of $\tau(\beta, \theta)$ vs. β with $\theta = 0$ (incident radiation along the V's) and $\theta = 90$ degrees (across the V's). At $\beta = 0$ the average measured transmittance is .985; the predicted value is .98; the value for the flat sheet is .96. Thus, the principle of increase in transmittance due to V-corrugating appears to be verified. However, because the index of refraction is very small, the gain in transmittance is very slight. (Greater gains

APPENDIX

are predicted for other materials as will be discussed shortly.) A reduction in transmittance as β is increased is both predicted and observed, although significant decrease is not noted until relatively large values of β are reached. This reduction is not nearly so pronounced for $\theta = 90$ degrees as for $\theta = 0$, indicating that for a solar collector facing south, the V's should be run in the east-west direction. Insufficient information was available to determine the scattering factor S for the teflon sheet used; the predictions shown in Figure 4 are for $S = 0$ and $S = .02$. Slightly better agreement, on average, is found for $S = .02$. The most probable reason for the discrepancy between theory and experiment is the fact that the sides of the V's are not precisely flat. This meant that no two V's were exactly alike so that integration over only two pitches (using the 16 quadrature points) was not a sufficiently broad sample. It also meant that the rays in the actual experiment did not maintain their θ direction. At high values of β the discrepancy is probably due to the finite size of the experimental VCTS, the effect of this becoming more pronounced at higher angles.

Considering these differences, the agreement shown in Figure 4 is quite satisfactory. It is therefore of interest to predict the transmittance of other VCTS's.

THEORETICAL PREDICTIONS

Figure 5 shows the predictions for the transmittance of sheets having values of n and KL typical for those of the plastics MYLAR and LEXAN, and for common glass assuming the sun as the source.

APPENDIX

(Properties for the plastics were taken from [8]; those for the glass from [9].) All are for opening angles $\phi = 30^\circ$, and for $S = 0$. It is obvious from this figure that for $\theta = 0$, transmittance enhancement (up to 10%) occurs in all cases, and for $\theta = 90$ degrees, it is always observed until relatively large values of β are reached. The higher the index of refraction, n , the greater is the increase. To examine the effect of changing ϕ , Figure 6 shows the predicted transmittance of teflon sheet V-corrugated at 30 degrees and 60 degrees. Transmittance enhancement, almost as large as that for 30 degrees, is observed for $\phi = 60$ degrees.

CONCLUSIONS

(1) The transmittance of a flat transparent sheet can be increased by V-corrugating it, at least for angles of incidence within 45 degrees of normal.

(2) This principle can be used in flat plate solar collectors where such a V-corrugated sheet can have the added advantage of suppressing convection and radiant losses more effectively than a flat sheet.

(3) A Monte Carlo analysis for predicting the transmittance of such a sheet is in satisfactory agreement with experiments.

APPENDIX

REFERENCES

1. Hollands, K.G.T., "Studies on Reducing Heat Losses from Flat Plate Collectors", Proceedings of the Workshop on Solar Collectors for Heating and Cooling of Buildings, S.L. Sargent, Editor. Published by National Science Foundation and University of Maryland, May, 1975.
2. Edwards, D.K., and Tobin, R.D., "Effect of Polarization on Radiant Heat Transfer Through Long Passages", Journal of Heat Transfer, Trans. ASME, Series C, Vol. 89, May, 1967, p. 132.
3. Edwards, D.K., and Bevans, J.T., "Effect of Polarization on Spacecraft Radiation Heat Transfer", AIAA Journal, Vol. 3, 1965, p. 1323.
4. Edwards, D.K., and Roddick, R.D., "Basic Studies on the Use and Control of Solar Energy, Final Progress Report", NSF G 9505, Dept. Of Engineering, U.C.L.A., 1961, p. C-12.
5. Howell, J.R., and Bannerot, R.B., "The Evaluation of Surface Geometry Modifications to Improve the Performance of Solar Energy Collectors", Report: NSF/RANN/SE/GE-41003/TR/74/1, University of Houston, Dec., 1974.
6. Sibbitt, B.E., "The Solar Transmittance of a V-Corrugated Diathermous Sheet", M.A.Sc. Thesis, Dept. of Mech.Eng., University of Waterloo, Waterloo, Canada, 1976.
7. Siegel, R., and Howell, J.R., Thermal Radiation Heat Transfer, McGraw-Hill, New York, 1972.
8. Marshall, K.N., Wedel, R.K., and Bell, G.A., "Development of a Honeycomb Covered Flat Plate Solar Collector", 2nd Quarterly Progress Report, LMSC-D409620, Lockheed Palo Alto Research Laboratory, Palo Alto, California, March, 1975.
9. Threlkeld, J.L., Thermal Environmental Engineering, Prentice-Hall, Englewood Cliffs, 1970, p. 325.

APPENDIX

ACKNOWLEDGMENTS

This research was supported by the Energy Research and Development Agency of the U.S. Government, under Contract No. E(11-1)-2597. The authors are also indebted to Professors J.R. Howell and R. Bannerot for their suggestion that the Monte Carlo technique be applied to this problem, and for some useful discussions on its use.

APPENDIX

CAPTIONS FOR FIGURES

1. Paths of reflected rays when solar radiation is incident upon a solar collector containing (a) a number of inner transparent covers; (b) a transparent honeycomb; (c) a V-corrugated transparent sheet. Only first reflections are shown, for clarity.
2. Sketch of VCTS showing dimensions and co-ordinate system.
3. Sketch of apparatus used for measuring transmittance of VCTS.
4. Comparison of experimental and theoretical values of $\tau(\beta, \theta)$.
5. Predicted solar $\tau(\beta, \theta)$ for two plastic and one glass VCTS; all are based on $S = 0$, $\phi = 30^\circ$.
6. Predicted solar $\tau(\beta, \theta)$ for a teflon VCTS at $\phi = 30^\circ$ and $\phi = 60^\circ$.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6