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CALCULATION OF MONTHLY AVERAGE INSOLATION ON TILTED SURFACES

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ABSTRACT

Several simplified design procedures for solar energy systems require monthly average meteorological data. Monthly average daily totals of the solar radiation incident on a horizontal surface are available. However, radiation data on tilted surfaces, required by the design procedures, are generally not available.

A simple method of estimating the average daily radiation for each calendar month on surfaces facing directly towards the equator has been presented by Liu and Jordan.[1] This method is verified with experimental measurements and extended to allow calculation of monthly average radiation on surfaces of a wide range of orientations. The results are presented in analytic and in tabular form for latitudes ranging from 25° to 55°, surface azimuth angles of ±45°, and for collector slopes within the range of practical interest.

INTRODUCTION

Estimates of the monthly average solar radiation incident on surfaces of various orientations are required for solar energy design procedures, heating load calculations, and other applications. Monthly averages of the daily solar radiation incident upon a horizontal surface are available for many locations. However, radiation data on tilted surfaces are generally not available.

A simple method of estimating the average daily radiation for each calendar month on surfaces facing directly towards the equator has been developed by Liu and Jordan. Their method is described here and compared with the work of Page[2] and with additional experimental measurements. The method is then extended so that it is applicable for surfaces oriented east or west of south.

ESTIMATION OF AVERAGE DAILY RADIATION ON SURFACES FACING DIRECTLY TOWARDS THE EQUATOR

The average daily radiation on a horizontal surface, $\overline{H}$, for each calendar month can be expressed by defining $\overline{K}_T$, the fraction of the mean

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daily extraterrestrial radiation, $\overline{H}_0$.

$$\overline{K}_T = \overline{H}/\overline{H}_0 \tag{1}$$

$$\overline{H}_0 = \frac{1}{(m_2-m_1)} \sum_{n=m_1}^{m_2} (H_0)_n \tag{2}$$

where

m_1 and m_2	are respectively the days of the year at the start and end of the month
$(H_0)_n$	is the extraterrestrial radiation on a horizontal surface on day n of the year which is given by

$$(H_0)_n = \frac{24}{\pi} I_{sc}[1+0.033 \cos(\frac{360n}{365})][\cos\phi\cos\delta\sin\omega_s + \omega_s 2\pi/360 \sin\phi\sin\delta] \quad ...(3)$$

where

I_{sc}	is the solar constant
n	is the day of the year given for each month in Table 1
ϕ	is the latitude
δ	is the solar declination which can be approximately expressed

$$\delta = 23.45° \sin[360(284+n)/365] \tag{4}$$

ω_s	is the sunset hour angle

$$\omega_s = -\tan\phi \tan\delta \tag{5}$$

$\overline{H}_0$ can be conveniently estimated from equation (3) by selecting for each month, the day of the year for which the daily extraterrestrial radiation is nearly the same as the monthly mean value. Using the 16th day of each month can lead to small errors in $\overline{H}_0$, particularly for June and December. Recommended days for each month are given in Table 1. $\overline{H}_0$ is tabulated for each month as a function of latitude in Table 2. The value of the solar constant used in the construction of Table 2 is 4871 kJ $hr^{-1}m^{-2}$ (Thekaekara and Drummond)[3], which is approximately 3% lower than the value used by Liu and Jordan[1,4] and Page.

The average daily radiation on a tilted surface, $\overline{H}_T$, can be expressed

$$\overline{H}_T = \overline{R}\ \overline{H} = \overline{R}\ \overline{K}_T \overline{H}_0 \tag{6}$$

where $\overline{R}$ is defined to be the ratio of the daily average radiation on a tilted surface to that on a horizontal surface for each month. $\overline{R}$ can be estimated by individually considering the beam, diffuse, and reflected components of the radiation incidence on the tilted surface. Assuming diffuse and reflected radiation to be isotropic, Liu and Jordan[1] have proposed that $\overline{R}$ can be expressed

$$\overline{R} = (1-\overline{H}_d/\overline{H})\overline{R}_b + \overline{H}_d/\overline{H}(1+\cos s)/2 + \rho(1-\cos s)/2 \tag{7}$$

where

$\overline{H}_d$ is the monthly average daily diffuse radiation

$\overline{R}_b$ is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month

s is the tilt of the surface from horizontal

ρ is the ground reflectance. Liu and Jordan[4] suggest that ρ varies from 0.2 to 0.7 depending upon the extent of snow cover.

$\overline{R}_b$ is a function of the transmittance of the atmosphere (except during times of equinox) which depends upon the atmospheric cloudiness, water vapor and particulate concentration. However, Liu and Jordan suggest that $\overline{R}_b$ can be estimated to be the ratio of extraterrestrial radiation on the tilted surface to that on a horizontal surface for the month. For surfaces facing directly towards the equator,

$$\overline{R}_b = \frac{\cos(\phi-s)\cos\delta\sin\omega_s' + \pi/180\ \omega_s'\sin(\phi-s)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \pi/180\ \omega_s\sin\phi\sin\delta} \tag{8}$$

where

ω is the hour angle which is 15°x(hours from solar noon), afternoons, positive, mornings negative

ω_s' is the sunset hour angle for the tilted surface which is given by

$$\omega_s' = \min\left[\omega_s,\ \arccos[\ -\tan(\phi - s)\ \tan\delta]\right] \qquad (9)$$

Page has calculated values of $\overline{R}_b$ for five surface orientations of several latitudes by integrating the direct radiation on the tilted and horizontal surface calculated at hourly intervals for a standard direct radiation curve. Values of $\overline{R}_b$ calculated from equation (8) are in reasonably good agreement with the values tabulated by Page as seen in Table 3. Page's values are slightly more conservative, i.e., closer to unity.

Since measurements of $\overline{H}_d$, the monthly average daily diffuse radiation are rarely available, $\overline{H}_d$ must be estimated from measurements of the average daily total radiation. A number of investigators have found that the diffuse radiation fraction, $\overline{H}_d/\overline{H}$, is a function of $\overline{K}_T$. Shown in Fig. 1 are the relationships reported by Liu and Jordan and Page which can be expressed

$$\frac{\overline{H}_d}{\overline{H}} = \begin{cases} 1.390 - 4.027\overline{K}_T + 5.531\overline{K}^2_T - 3.108\overline{K}^3_T & \text{(Liu \& Jordan)} \quad (10a) \\ 1.00 - 1.13\overline{K}_T & \text{(Page)} \quad (10b) \end{cases}$$

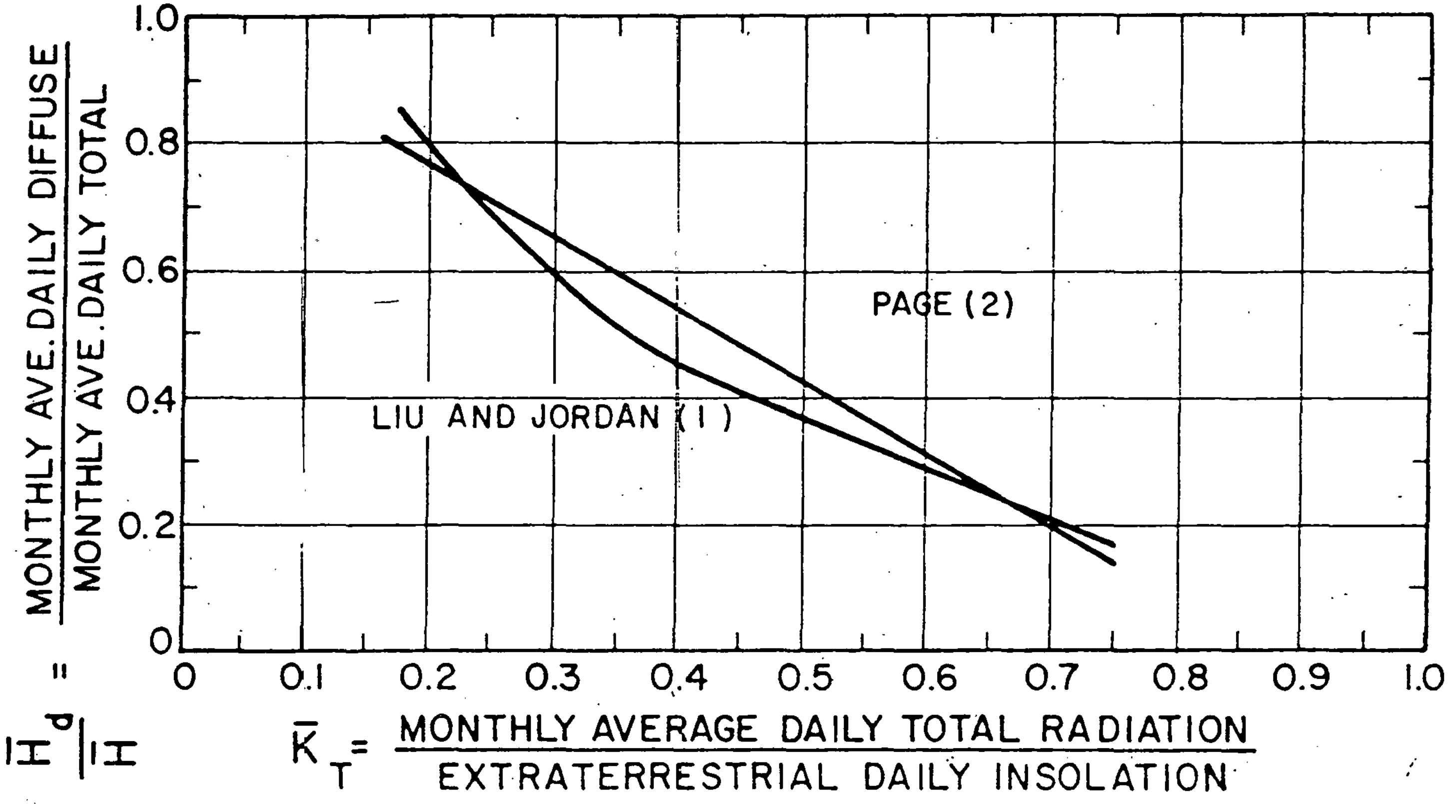
Page's relationship, which was derived from experimental measurements at 10 stations, tends to agree more closely with the additional measurements reported by Choudhury[5], Stanhill[6], and Norris[7]. The discrepancy is apparently due, at least in part, to the fact that a shade ring correction factor was applied to all reported diffuse radiation measurements except those taken at Blue Hill, Massachusetts which Liu and Jordan used to derive their relationship. Page's relationship probably results in a more accurate estimate of the diffuse radiation fraction; however, values of $\overline{R}$ estimated from Eq. 7 with $\rho = 0.2$ tend to agree more closely with experimental measurements when the Liu and Jordan relationship is used, as shown in the next section.

COMPARISON WITH EXPERIMENTAL RESULTS

Long-term measurements of the radiation incident on both tilted and horizontal surfaces are scarce. Measurements of the radiation incident on a horizontal and a south-facing vertical surface in Blue Hill, Massachusetts (latitude 42.2°N) for the years 1952 to 1956 have been presented by Liu and Jordan. In Table 4 experimental values of $\overline{R}$ are compared with values estimated from Eq. 7 with $\rho = 0.2$ using the diffuse

Figure 1

RELATIONSHIP OF $\frac{\bar{H}_d}{\bar{H}}$ TO $\bar{K}_T$



radiation fraction relationships of Liu and Jordan (Eq. 10a) and Page (Eq. 10b). In Table 5, similarly calculated values of $\overline{R}$ are compared with experimental values for a 38° north-facing surface at Highett, Victoria, Australia (latitude 37.9°S) for the years 1966 through 1968[8]. Based on this experimental data, it appears that Liu and Jordan's relationship for the diffuse radiation fraction (Eq. 10a) results in more accurate values of $\overline{R}$ than does Page's (Eq. 10b). It is possible that the "underestimated" diffuse radiation fraction arising from Eq. 10a tends to cancel errors caused by the conservative assumptions of isotropic diffuse radiation and a ground reflectance of 0.2.

ESTIMATION OF AVERAGE DAILY RADIATION ON SURFACES ORIENTED EAST OR WEST OF SOUTH

Liu and Jordan's method of calculating $\overline{R}_b$ can be extended so that it is applicable for surfaces which are not oriented directly towards the equator by integrating the rate of extraterrestrial radiation on the surface for the period during which the sun is both above the horizon and in front of the surface and then dividing this result by $\overline{H}_0$. In this case

$$\begin{aligned}\overline{R}_b = \Big\{ & [\cos s \sin \delta \sin \phi]\, \pi/180[\omega_{ss} - \omega_{sr}] \\ & -[\sin \delta \cos \phi \sin s \cos\gamma]\, \pi/180[\omega_{ss} - \omega_{sr}] \\ & +[\cos \phi \cos \delta \cos s][\sin \omega_{ss} - \sin\omega_{sr}] \\ & +[\cos \delta \cos \gamma \sin \phi \sin s][\sin \omega_{ss} - \sin\omega_{sr}] \\ & -[\cos \delta \sin s \sin \gamma][\cos \omega_{ss} - \cos\omega_{sr}]\Big\} \\ & \Big/ \Big\{2\, [\cos \phi \cos \delta \sin \omega_s + \pi/180\, \omega_s \sin \phi \sin \delta\Big\} \quad (11)\end{aligned}$$

where

γ is the surface azimuth angle, i.e. the deviation of the normal to the surface from the local meridan, the zero point being due south, east negative, and west positive

ω_{sr} and ω_{ss} are the sunrise and sunset hour angles on the tilted surface, given by

$$\omega_{sr} = -\min\left[\omega_s, \arccos[(AB+\sqrt{A^2-B^2+1})/(A^2+1)]\right] \quad \text{if } \gamma < 0 \quad (12)$$
$$\omega_{ss} = \min\left[\omega_s, \arccos[(AB-\sqrt{A^2-B^2+1})/(A^2+1)]\right]$$

$$\omega_{sr} = -\min\left[\omega_s, \arccos[(AB-\sqrt{A^2-B^2+1})/(A^2+1)]\right] \quad \text{if } \gamma > 0 \quad (13)$$
$$\omega_{ss} = \min\left[\omega_s, \arccos[(AB+\sqrt{A^2-B^2+1})/(A^2+1)]\right]$$

$$A = \cos\phi/[\sin\gamma \tan s] + \sin\phi/\tan\gamma \quad (14)$$

$$B = \tan\delta\left[\cos\phi/\tan\gamma - \sin\phi/[\sin\gamma \tan s]\right] \quad (15)$$

An example demonstrating this method of estimating radiation on tilted surfaces follows.

EXAMPLE

Estimate the monthly averages of daily radiation incident on a surface tilted 43° from horizontal facing due south in Madison, Wisconsin (43° N latitude) and compare them with those incident if the surface were oriented 15° west of south.

Daily averages value of $\bar{H}$, the radiation incident on a horizontal surface can be found in reference (9). The mean daily extraterrestrial radiation, $\bar{H}_0$, for each month can be determined from equation (3) (using the days of the year in Table 1) or from Table 2 with interpolation. The ratio $\bar{H}/\bar{H}_0$ determines $\bar{K}_T$ for each month which can be used to calculate $\bar{H}_d/\bar{H}$ from equation (10a) **or (10b)**. (Equation (10a) is used in this example.) $\bar{R}$ is calculated from values of $\bar{R}_b$ for each month for both the south (equation (8)) and the 15° west of south (equation (11)) surfaces. The average daily radiation on each of the surfaces is the product of $\bar{R}\bar{H}$ for each month. These results are displayed in Table 6.

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TABLE 1

Recommended Average Day for Each Month

MONTH	DAY OF THE YEAR	DATE
January	17	Jan. 17
February	47	Feb. 16
March	75	Mar. 16
April	105	Apr. 15
May	135	May 15
June	162	June 11
July	198	July 17
August	228	Aug. 14
September	258	Sept. 15
October	288	Oct. 15
November	318	Nov. 14
December	344	Dec. 10

TABLE 2

MONTHLY AVERAGE DAILY EXTRATERRESTRIAL RADIATION KJ/m^2

LATITUDE	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
25	23902.	28115.	32848.	37111.	39356.	40046.	39606.	37832.	34238.	29413.	24909.	22669.
30	21034.	25679.	31141.	36436.	39569.	40706.	40071.	37534.	32917.	27213.	22161.	19714.
35	18069.	23072.	29200.	35497.	39530.	41129.	40292.	36976.	31348.	24820.	19296.	16687.
40	15043.	20319.	27040.	34303.	39247.	41328.	40281.	36166.	29542.	22255.	16344.	13626.
45	11998.	17448.	24577.	32869.	38737.	41322.	40055.	35118.	27515.	19541.	13344.	10579.
50	8987.	14490.	22131.	31209.	38025.	41147.	39644.	33851.	25283.	16705.	10342.	7605.
55	6082.	11486.	19423.	29345.	37152.	40863.	39100.	32391.	22863.	13778.	7396.	4791.

TABLE 3

COMPARISON OF VALUES OF $\overline{R}_b$ FROM PAGE[2] AND EQUATION (8)

	ϕ = 30°				ϕ = 40°			
	ϕ-s = 0		Vertical		ϕ-s = 0		Vertical	
	Page	Equation (8)	Page	Equation (8)	Page	Equation (8)	Page	Equation (8)
Jan	1.61	1.66	1.49	1.59	2.15	2.26	2.11	2.32
Feb	1.40	1.43	1.06	1.13	1.72	1.79	1.50	1.59
Mar	1.18	1.20	0.64	0.67	1.35	1.38	0.93	0.96
Apr	0.99	1.00	0.29	0.30	1.07	1.06	0.48	0.48
May	0.89	0.87	0.13	0.11	0.90	0.88	0.27	0.25
June	0.84	0.87	0.06	0.05	0.84	0.80	0.19	0.17
July	0.85	0.84	0.09	0.08	0.85	0.83	0.22	0.21
Aug	0.94	0.94	0.21	0.21	0.98	0.98	0.37	0.37
Sept	1.09	1.12	0.45	0.50	1.20	1.24	0.69	0.74
Oct	1.30	1.35	0.88	0.97	1.57	1.64	1.24	1.36
Nov	1.53	1.60	1.33	1.46	1.98	2.12	1.86	2.10
Dec	1.67	1.74	1.61	1.74	2.30	2.42	2.36	2.58

Table 4

Comparison of Experimental[a] and Estimated Values of $\overline{R}$ For a Vertical Surface Facing South at Blue Hill, Mass Lat. 42° 13' N

Month	K_T	$\overline{R}$ Experimental[a] (1952-1956)	R Estimated From Eqs. 7 and 10a	R Estimated From Eqs. 7 and 10b
Jan.	0.411	1.80	1.72	1.55
Feb.	0.445	1.38	1.31	1.22
Mar.	0.445	0.93	0.91	0.87
Apr.	0.440	0.61	0.62	0.62
May	0.481	0.44	0.47	0.48
June	0.524	0.39	0.41	0.42
July	0.528	0.42	0.42	0.44
Aug.	0.485	0.54	0.55	0.55
Sept.	0.485	0.79	0.79	0.77
Oct.	0.466	1.23	1.18	1.11
Nov.	0.421	1.60	1.61	1.46
Dec.	0.422	1.94	1.91	1.72

[a] Source: Liu and Jordan (1962)[1]

Table 5

Comparison of Experimental and Estimated Values of $\overline{R}$ For A 38° Surface Facing North at Melbourne, Australia (Lat. 37.9° S)

Month	K_T	$\overline{R}$ Experimental[b] (1966-1968)	$\overline{R}$ Estimated from Eqs. 7 and 10a	$\overline{R}$ Estimated from Eqs. 7 and 10b
Jan.	0.46	0.85	0.88	0.89
Feb.	0.46	0.94	0.96	0.96
Mar.	0.41	1.10	1.09	1.06
Apr.	0.40	1.29	1.27	1.21
May	0.34	1.37	1.41	1.33
June	0.34	1.50	1.54	1.44
July	0.37	1.50	1.55	1.42
Aug.	0.39	1.34	1.34	1.28
Sept.	0.38	1.15	1.14	1.10
Oct.	0.39	0.98	0.99	0.98
Nov.	0.41	0.88	0.90	0.90
Dec.	0.42	0.84	0.86	0.87

TABLE 6

CALCULATION OF DAILY AVERAGE RADIATION ON A 43° SURFACE IN MADISON

Month	$\overline{H}$	$\overline{H}_0$	$\overline{K}_T$	$\overline{H}_d/\overline{H}$	$\overline{R}_b$ $\gamma=0°$	$\overline{R}_b$ $\gamma=15°$	$\overline{R}$ $\gamma=0$	$\overline{R}$ $\gamma=15°$	$\overline{H}_T$ $\gamma=0$	$\overline{H}_T$ $\gamma=15°$
	KJ m^2 day^{-1}	KJ m^2 day^{-1}							KJ-m^2-day^{-1}	KJ m^2 DAY^{-1}
JAN	6412	13226	0.485	0.384	2.53	2.47	1.92	1.88	12300	12100
FEB	9224	18612	0.496	0.374	1.95	1.90	1.57	1.54	14500	14200
MAR	13992	25762	0.543	0.337	1.46	1.44	1.28	1.27	18000	17800
APR	16527	33429	0.494	0.376	1.09	1.09	1.03	1.03	17100	17100
MAY	19821	39011	0.508	0.365	0.88	0.89	0.90	0.91	17900	18000
JUNE	23073	41348	0.558	0.325	0.80	0.81	0.85	0.86	19600	19700
JULY	23241	40136	0.579	0.310	0.84	0.85	0.87	0.88	20300	20400
AUG	19762	35552	0.556	0.327	0.99	1.00	0.98	0.98	19400	19400
SEPT	16397	28499	0.575	0.313	1.30	1.30	1.19	1.18	19500	19400
OCT	11277	20684	0.545	0.335	1.77	1.73	1.49	1.47	16800	16500
NOV	6311	14472	0.435	0.428	2.36	2.30	1.75	1.71	11000	10800
DEC	5632	11785	0.473	0.390	2.75	2.68	2.04	2.00	11500	11300