**An Optimization Model for Air
Quality Analysis in Energy Facility Siting**

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ANALYSIS IN ENERGY FACILITY SITING

W. R. Emanuel, B. D. Murphy,¹ D. D. Huff,
C. L. Begovich,¹ and J. F. Hurt²

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ABSTRACT

EMANUEL, W. R., B. D. MURPHY, D. D. HUFF, C. L. BEGOVICH,
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The siting of energy facilities on a regional scale is discussed with particular attention to environmental planning criteria. A multiple objective optimization model is proposed as a framework for the analysis of siting problems. Each planning criterion (e.g., air quality, water quality, or power demand) is treated as an objective function to be minimized or maximized subject to constraints in this optimization procedure. The formulation of the objective functions is illustrated by the development of a siting model for the minimization of human exposure to air pollutants. This air quality siting model takes the form of a linear programming problem. A graphical analysis of this type of problem, which provides insight into the nature of the siting model, is given. The air quality siting model is applied to an illustrative siting example for the Tennessee Valley area.

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INTRODUCTION

The necessity of a regional approach to the planning of energy systems which addresses larger geographical areas than specific sites is generally recognized (Tinney and Enns, 1974). There are two fundamental reasons for this requirement. The first is the regional nature of the planning variables used in energy system design such as the distance to load centers or the exposure of population to air pollutants. The second is the synergistic nature of the impacts of energy systems. An example is the increase in the impact on aquatic biota in river reaches where thermal plumes combine. This characteristic leads to a cumulative impact of a power system which is greater than the sum of the impacts of each facility. These two characteristics of energy systems and their associated impacts require a total system approach to analysis and assessment.

Increasing awareness of the social and environmental impacts of energy facilities has prompted the consideration of a broad spectrum of planning criteria in the siting of facilities. Limited availability of resources and legislative standards have placed many additional constraints on the choice of power system configurations. With this increase in complexity decision makers are using systems analysis techniques in the solution of energy system planning problems. The desire to choose a "best" solution with respect to all planning criteria from a set of possibilities has prompted the application of optimization techniques to the planning of energy systems. The most general problem is to determine a power system

configuration which satisfies all constraints and minimizes a set of objective functions representing various planning criteria.

A fundamental concept in the analysis of energy systems is the characterization of the system with respect to facility location, size, and type (i.e., nuclear or coal power plant). This analysis will be referred to as the energy facility siting problem in this report. The general trends which arise from the analysis are usually the most important result. The recurring question of whether to employ a few large facilities or a larger number of small facilities to meet demand is exemplary of the type of question to be addressed here. Without a great deal of data for calibration and verification of models used in the siting analysis, an exact selection of sites and assignment of activity levels will generally not be possible.

Numerous planning criteria or objectives must be considered when analyzing energy systems (e.g., cost, environmental impact, demand). Frequently, these objectives are conflicting in that a decision which improves the system with respect to one objective will create a less desirable situation with respect to another. With past emphasis on engineering and cost criteria this problem of conflicting objectives has generally been handled by assigning weighting factors to each objective function. The weighting factors reflect the relative worth of the objectives. Often the weighting functions are determined by assigning econometric values to objectives, such as a dollar cost for the abatement of pollutants.

In the first section of this report, an alternate approach to the optimization of siting patterns will be discussed. This approach constitutes a general framework for the energy facility siting problem. This general approach description will be followed by the formulation of an optimization model for the air quality aspects of the siting problem. Results of a case study are presented in the final section of the report.

ENVIRONMENTAL OBJECTIVES IN ENERGY FACILITY SITING

Increasing concern for the environment has led to efforts to incorporate quantifiable environmental planning criteria in systematic planning procedures for siting energy facilities. Examples of such criteria are air and water quality. The goal is to formulate an approach which incorporates environmental considerations in a way compatible with techniques used for engineering and economic planning.

When the siting of energy facilities is considered on a regional scale (e.g., state or larger), the number of planning criteria and constraints is generally large. As previously indicated, this is because of the interaction between facilities. Optimization techniques are therefore desirable in selecting preferred energy system configurations which satisfy all constraints. However, if insufficient representation of a planning criterion occurs in the optimization model, the resulting energy system configuration is usually not suitable. In this section a multiple objective optimization framework for energy facility siting is described. This approach allows for incorporation of each planning criterion as an objective function to be minimized (or maximized) by optimization procedures. In this way all objectives receive adequate treatment. The use of multiple objective optimization models for energy facility siting is first described. This is followed by a clear statement of the multiple objective optimization problem in mathematical terms. One approach for solution of these problems is then briefly described.

Multiple Objective Optimization

The siting of energy facilities involves a large number of planning objectives. Examples include the distance to resources, the distance to load centers, air quality, cost, and water quality. These and many other criteria are considered either quantitatively or qualitatively to determine the spatial arrangement of facilities. Because of the complexity of the siting problem (especially with respect to the number of planning objectives), decision makers generally desire quantitative information on as many variables as possible. In addition analytical techniques are used whenever suitable to assist in making the siting decision. One of the most commonly used types of analysis is optimization. Values are assigned to a set of decision variables (e.g., size and location of power plants) either to minimize or maximize one or more objective functions subject to a set of constraints. The equations and data used in this optimization approach are generally referred to as an optimization model. The problem is construct an optimization model for energy facility siting which adequately incorporates environmental objectives.

A classical approach to the multiple objective problem is to combine the objectives to form one criterion. Consider a set of n objectives functions:

$$\{f_1(\underline{x}), f_2(\underline{x}), \dots, f_n(\underline{x})\} \quad (2.1)$$

where $\underline{x}$ is a vector or set of decision variables (i.e. choice of facility size, location and type). Each $f_i(\underline{x})$ corresponds to a particular planning objective such as air quality or cost and is a function of the decision variables (i.e., choice of facility size, location and type). The classical approach is to form a new single objective function by combining the $f_i(\underline{x})$ to give

$$h(\underline{x}) = \theta_1 f_1(\underline{x}) + \theta_2 f_2(\underline{x}) + \dots + \theta_n f_n(\underline{x}) \quad (2.2)$$

where the θ_i are parameters used to form a linear combination of the n objective functions. The parameters θ_i are a measure of the relative worth or importance of the objective functions and are often called weighting coefficients. A common method of choosing the θ_i is to determine the econometric value of objective i . An example is the use of abatement cost as a measure of value or cost associated with pollution. This choice of θ_i results in a $h(\underline{x})$ measured in econometric units. When planning emphasis is on minimizing cost and maximizing profit, this is a desirable arrangement. The problem is reduced to a single objective amenable to standard optimization techniques (linear programming, nonlinear programming). Pantell (1976) provides further motivation for this approach.

Two major difficulties are associated with using the parametric approach described by (2.2). It is generally difficult to determine adequate values for the weighting factors. If an approach such as the econometric one is used, the resulting $h(\underline{x})$ is generally biased to emphasize economic considerations. This bias may be more

reflective of the analyst's feelings than those of the decision makers' or their constituents. Furthermore, it is often impossible to determine adequate values for the true costs or worth of many objectives (e.g., water quality). This task, which can only be accomplished with information from decision makers, is often assigned to the analyst who is forced to base his calculations on limited data (e.g., estimates of abatement costs). The use of constant coefficients as weighting factors gives rise to the second major difficulty. The relative worth or value of objective functions is generally dependent on their level of attainment. For example the importance of an air quality objective function relative to a cost objective is much greater for air pollutant concentrations which are at a level threatening public health. Progress has been made in overcoming this difficulty by the utility function approach (Pantell, 1976). A further expansion of utility functions is presented here. The approach is called the surrogate worth trade off method and was first proposed by Haimes and Hall (1974) for application to water resource systems.

With this motivation the multiple objective optimization problem is now presented in mathematical terms. As presented here, the optimization problem is designed for energy facility siting studies. Consider a vector of decision variables

$$\underline{x} = [x_1, x_2, \dots, x_m]^T \quad (2.3)$$

where x_i is the activity level (e.g., generating capacity) of the facility assigned to site i and T indicates the transpose operation. The multiple objective optimization problem is to find values of the decision variables $\underline{x}$ which minimize a set of objective functions subject to site suitability constraints:

$$\underset{\underline{x}}{\text{minimize}} \underline{f}(\underline{x}) = [f_1(\underline{x}), f_2(\underline{x}), \dots, f_n(\underline{x})]^T$$

such that

$$\begin{aligned} \underline{g}(\underline{x}) &\leq \underline{0} \\ \underline{x} &\geq \underline{0} \end{aligned} \quad (2.4)$$

The set of site suitability constraints is expressed as a p-vector of functions $\underline{g}(\underline{x})$. This set of constraints is mainly composed of upper limits on site activity levels of the form

$$g_i(\underline{x}) = x_i - x_{i \max} \leq 0. \quad (2.5)$$

Load constraints also appear in the vector $\underline{g}(\underline{x})$. In general any optimization model for energy facility siting can be put in the form of (2.4) by suitable transformations. These transformations are discussed for the linear case in the next section.

The most significant characteristic of the multiple objective optimization model (2.4) is that in general there is no optimal solution in the sense that there does not exist a set of decision

variable values which simultaneously minimize all objective functions while satisfying the constraint set. This condition leads to the necessity for a trade off or compromise between objectives. This is the central issue of making decisions. The parametric approach of combining objective functions with weighting factors as specified by (2.2) does not provide an adequate treatment of these trade offs.

The nonexistence of an optimal solution leads to the definition of a noninferior solution. A noninferior solution is one in which no improvement can be obtained in any of the objectives without causing a simultaneous degradation of at least one of the other objectives (Haimes et al., 1975). Any solution which does not satisfy this condition is referred to as inferior. Clearly only noninferior solutions are of interest, so that the problem becomes one of selecting preferred solutions from the set of noninferior solutions. Some sort of additional criteria are required for selection of preferred solutions.

The Surrogate Worth Trade Off Method

The selection of preferred solutions is based on indifference on the part of the decision maker. The indifference band is defined to be the subset of the noninferior set where the improvement of one objective function is equivalent in the mind of the decision maker to the necessary degradation of the others (Haimes et al., 1975). Since no other solution can be demonstrated superior, solutions belonging to the indifference band will be considered preferred solutions. The surrogate worth trade off method is based on the

determination of a surrogate worth function which has a value of zero when the decision maker is indifferent concerning the trade off between objectives.

A trade off rate function will now be defined and developed. Associated with the multiple objective optimization problem (2.4) is the single objective optimization problem:

$$\underset{\underline{x}}{\text{minimize}} f_1(\underline{x})$$

subject to

$$\begin{aligned} f_j(\underline{x}) &\leq \varepsilon_j & , & \quad j = 2, 3, \dots, n \\ g_k(\underline{x}) &\leq 0 & , & \quad k = 1, 2, \dots, \mu \\ \underline{x} &\geq 0 \\ \underline{\varepsilon} &\geq 0 \end{aligned} \tag{2.6}$$

where $\underline{\varepsilon} = [\varepsilon_2, \varepsilon_3, \dots, \varepsilon_n]^T$ is a vector of constraints on all objective functions except the first. In formulation of (2.6), frequently referred to as the ε -constraint problem, maximum allowable levels have been specified for all but one objective function. Trade off rate functions naturally arise in the solution of (2.6).

Since (2.6) is a single objective problem an optimal solution in the usual sense exists. The generalized Lagrangian for (2.6) is

formed by adjoining the various constraints to the objective function:

$$L = f_1(\underline{x}) + \sum_{k=1}^p \mu_k g_k(\underline{x}) + \sum_{j=2}^n \lambda_j (f_j(\underline{x}) - \varepsilon_j) \quad (2.7)$$

where $\underline{\mu} = [\mu_1, \mu_2, \dots, \mu_p]^T$ and $\underline{\lambda} = [\lambda_2, \lambda_3, \dots, \lambda_n]^T$ are vectors of Lagrange multipliers. Necessary conditions for an optimal solution of (2.6) are the Kuhn-Tucker conditions (Kuhn and Tucker, 1950):

$$\frac{\partial L}{\partial \underline{x}} = \frac{\partial f_1(\underline{x})}{\partial \underline{x}} + \sum_{k=1}^p \mu_k \frac{\partial g_k(\underline{x})}{\partial \underline{x}} + \sum_{j=2}^n \lambda_j \left(\frac{\partial f_j(\underline{x})}{\partial \underline{x}} \right) = \underline{0} \quad (2.8)$$

$$\mu_k g_k(\underline{x}) = 0 \quad , \quad k = 1, 2, \dots, p \quad (2.9)$$

$$\lambda_j (f_j(\underline{x}) - \varepsilon_j) = 0 \quad , \quad j = 2, 3, \dots, n \quad (2.10)$$

$$\mu_k \geq 0 \quad , \quad k = 1, 2, \dots, p \quad (2.11)$$

$$\lambda_j \geq 0 \quad , \quad j = 2, 3, \dots, n \quad (2.12)$$

$$x_i \geq 0 \quad , \quad i = 1, 2, \dots, m \quad (2.13)$$

Equations (2.8) through (2.12) are solved simultaneously for an optimal solution, $\underline{x}^*$, $\underline{\lambda}^*$, and $\underline{\mu}^*$.

Now consider the variation of the Lagrangian (2.7) with respect to the ϵ -constraint on the j th objective function. By partial differentiation

$$\frac{\partial L}{\partial \epsilon_j} = -\lambda_j \quad (2.14)$$

so that the Lagrange multiplier associated with the j th ϵ -constraint is a measure of the sensitivity of the Lagrangian to changes in the constraint value. Since at optimality the Lagrangian is equal to the optimal value of f_1 , λ_j is also a measure of sensitivity of L with respect to $f_1(\underline{x}^*)$. In a similar manner it can be demonstrated that λ_{ij} , the Lagrange multiplier adjoining the j th ϵ -constraint to the i th objective function unconstrained, is the sensitivity of the optimal value of the i th objective function to the ϵ -constraints on the remaining $n-1$ objective functions. This derivation leads to the definition of the trade off rate function

$$T_{ij} = \frac{\partial L_i^*}{\partial \epsilon_j} = -\lambda_{ij}^* \quad (2.15)$$

where L_i^* is the optimal value of the Lagrangian when the i th objective function is left unconstrained, and λ_{ij}^* is the optimal value of the Lagrange multiplier used to adjoin the j th ϵ -constraint to the i th objective function. The value of T_{ij} can be calculated from (2.15) for several constraint values ϵ_j . A polynomial approximation to T_{ij} as a function of ϵ_j can be

determined by least squares techniques. The trade off rate functions provide very useful information to decision makers. These functions express the marginal worth of the i th objective function due to a unit change in the j th objective function.

The trade off rate functions given by (2.15) are used as a basis for forming a surrogate worth function W_{ij} between each pair of objective functions. The decision maker is provided the value of the trade off rate function (2.15) and corresponding values of objectives functions. He expresses his feelings about the improvement of objective i and the corresponding degradation of objective j in the following manner:

$$W_{ij} = \begin{cases} > 0 & \text{when } T_{ij} \text{ marginal units of } f_i(\underline{x}) \text{ are preferred} \\ & \text{over one marginal unit of } f_j(\underline{x}) \\ \\ = 0 & \text{when } T_{ij} \text{ marginal units of } f_i(\underline{x}) \text{ are equiva-} \\ & \text{lent to one marginal unit of } f_j(\underline{x}) \\ \\ < 0 & \text{when } T_{ij} \text{ marginal units of } f_i(\underline{x}) \text{ are not pre-} \\ & \text{ferred over one marginal unit of } f_j(\underline{x}) \end{cases} \quad (2.16)$$

The W_{ij} are monotonic functions of the trade off rate functions. The values of decision variables corresponding to $W_{ij} = 0$ are considered a preferred solution to the multiple objective optimization problem (2.4). A band of indifference will be defined by the union of the indifference bands for the various W_{ij} . This indifference

band corresponds to values of the trade off rate function which by (2.15) specifies the Lagrange multipliers. The Kuhn-Tucker conditions are invoked to compute the decision variable values for the preferred solution.

The multiple objective optimization decision framework is illustrated in Fig. 1. As indicated, the conflicting objectives are compromised through the use of surrogate worth functions developed through interaction with decision makers. The decision makers are provided with trade off rate functions as a measure of the effect of choices.

Fig. 1. Schematic representation of multiple objective optimization framework for energy facility siting.

AIR QUALITY OPTIMIZATION MODEL FORMULATION

The multiple objective optimization approach to planning described in the previous section, generally requires the formulation of an optimization model for each objective function to be considered. This is a difficult task for most environmental planning criteria. Many of the problems which arise in the development of optimization models for environmental variables are discussed by Pantell (1976). The successful application of optimization techniques to planning problems involving environmental variables depends on further study of the difficulties which arise in adequately formulating an optimization model. This section describes such a model in which the objective function is the minimum exposure of human population or vegetation to air-dispersed pollutants. Computer codes for the implementation of this model are briefly documented in the appendix.

Problem Statement

In the specified region under consideration, it is assumed that a set of candidate or feasible sites for the development of energy facilities has been selected. These are selected in various ways including the use of automated screening techniques. A set of site suitability factors is overlaid on the computer as it exists in space for the region under consideration. Water availability, proximity to load centers, transportation availability, seismic risk, population distribution, fuel availability, and distribution of wildlife habitats are some of the factors which can be considered

in this screening process. The candidate sites used in case studies with the optimization model proposed here were selected by this computer technique using software and data developed by Voelker (1976). Initial emphasis has been on coal combustion facilities and the site suitability factors associated with them. The set of candidate sites is designated as:

$$S = \{s_1, s_2, \dots, s_{NS}\} \quad (3.1)$$

where NS is the number of candidate sites.

Associated with each candidate site are constraints on the size and type of facility which can be constructed. Many of the factors listed above are important in determining the maximum size and suitable type of a facility at each site. Perhaps the most significant environmental criterion constraining size of energy facilities is the availability of water. The amount of water required will depend on the particular technology employed. The choice of facility sizes and cooling technology using an optimization model has been discussed by Marks and Borenstein (1970). In the present model a simple constraint on the generating capacity of the coal combustion plant is used:

$$\begin{aligned} P_1 &\leq P_{1 \text{ max}} \\ P_2 &\leq P_{2 \text{ max}} \\ &\vdots \\ P_{NS} &\leq P_{NS \text{ max}} \end{aligned} \quad (3.2)$$

where P_1 is the generating capacity at plant 1, and $P_{1 \max}$ is the maximum capacity which can be installed at site 1. The magnitude of the constraints can be varied to analyze the effect of various cooling technologies on the solution of the optimization model. The availability of water is incorporated in the determination of $P_{1 \max}$ through a flow frequency analysis based on USGS data (Riggs, 1972). While many additional factors may effect the value of $P_{1 \max}$, initial emphasis is placed on water availability because it is the determining factor in many cases.

An additional constraint enters the optimization model due to the demand for increased generating capability. In this model a total increase in capacity for the region is assumed such that

$$P_1 + P_2 + \dots + P_{NS} \geq P_D \quad (3.3)$$

where P_D is the increase in demand. Greater resolution could be obtained by disaggregating this demand through a load center analysis. The optimization problem is to assign generating capacities to the candidate sites to minimize exposure indexes for air pollutants and satisfy the constraints (3.2) and (3.3). The problem is not well posed if

$$P_{1 \max} + P_{2 \max} + \dots + P_{NS \max} \leq P_D \quad (3.4)$$

since the maximum capacity would be required at each site.

Objective Function Calculation

Formulation of the objective function for the air quality optimization model is difficult. Each candidate site is the source of air pollutants. Initial emphasis is on the sulfur pollutants, SO_2 and sulfate. These pollutants are likely to be of main importance with the expansion of the coal power system (Shriner et al., 1977). An air dispersion model must be used to estimate ground level concentration of pollutants in the region of interest due to these sources. Because of interest in regional analysis, a long range model based on wind trajectory analyses has been selected.

A wind trajectory model developed by the National Oceanic and Atmospheric Administration (Heffter et al., 1975) is used for the analysis of long range pollutant transport (up to 2000 km). This long range air transport and diffusion model uses tropospheric wind data which are available for about 100 stations throughout the U.S. Roughly one quarter of these stations report upper air wind observations every six hours and the remainder report every twelve hours. Wind trajectories are averaged by the model throughout the mixed layer, and thus an average tropospheric wind vector is obtained for each station at each reporting time. Pollutant advection is determined by constructing wind trajectories from the point of origin of a pollutant. Trajectories are composed of three hour segments which are obtained by backward or forward extrapolation of the nearest observations in time. Observations within 300 nautical miles of the beginning of a trajectory segment are averaged to obtain the

wind vector for the new segment. Distance to the point of observation is used to weight each observation in the form of an inverse square and directional weighting is such that upwind and downwind observations (as defined by the previous trajectory segment) are given more weight.

The model imposes a time dependent Gaussian diffusion on the trajectories and thereby determines the lateral and vertical mixing of the pollutant. The vertical diffusion is assumed to be expressible by a simple diffusion coefficient, U_z , which in the case of the studies mentioned here had a value of $5 \text{ m}^2/\text{s}$. The lateral diffusion is treated by a relationship where T_y (lateral standard deviation of Gaussian plume) is simply proportional to time. This relationship for T_y is obtained by a fit to a great variety of conditions. Data on diffusion more pertinent to this kind of work are in short supply and all calculations on long range diffusion of power plant effluents should be appraised with that fact clearly in mind. The calculations reported here, however, are climatological in nature. They refer to a period of one month (November) during which worst case meteorological conditions are expected and are therefore not so much dependent on the diffusion of individual trajectories as they are on the overall pattern of air mass movements.

The region under consideration is divided into a set of cells by overlaying a square grid system. A cell which is, 0.25° on a side is convenient for this type of analysis. Cell size may be varied depending on resolution of the dispersion model chosen. The cell system is schematically described in Fig. 2. Candidate sites are

specified to a spatial resolution of a 0.125° cell (i.e., the standard 7.5' quadrangle). A typical pattern of candidate sites is also shown in Fig. 2. The trajectory model is used to estimate the ground level concentration of pollutants in each cell due to a unit source at each candidate site. These calculations provide a matrix of transfer coefficients,

$$\underline{D} = [d_{ij}] \quad (3.5)$$

where $i = 1, 2, \dots, NS$, and $j = 1, 2, \dots, NC$. NS is the number of candidate sites and NC is the number of cells. This matrix serves as a parameterization of the air dispersion characteristics of the region. The ground level concentrations can be averaged for any desired time period. An important choice used in the preliminary analysis is a typical winter month. It should be noted that neither short range nor episodic considerations have been incorporated in the transfer matrix $\underline{D}$. Initial emphasis is on the regional problem; however, these finer resolution points can be incorporated in this framework.

To calculate ground level concentrations in each cell due to non-unit sources at each candidate site, a source model for each pollutant is required. Generally, these must relate the pollutant source strength to generating capacity. A typical example is given by the Environmental Protection Agency (1973) for SO_2 resulting from coal combustion:

$$E_i = 0.38 \beta_i \eta_i P_i \quad (3.6)$$

where E_i is the SO_2 source strength at site i , measured in lbs, β_i is the fractional concentration of sulfur in the coal at site i , η_i is the fractional combustion efficiency at site i and P_i is the generating capacity at site i measured in megawatts. Clearly this model must be modified to reflect the use of control procedures such as scrubbers. The actual source model used should be designed to simulate emissions from planned facilities. In any event, a model which is linear in plant capacity is desired. This requirement allows the use of linear optimization schemes. Using a source strength model such as (3.6) and the matrix of transfer coefficients (3.5), the average ground level concentration of pollutants in each cell due to sources at all candidate sites is calculated:

$$R_j = \sum_{i=1}^{NS} d_{ij} E_i \quad (3.7)$$

The optimization model is then constructed to minimize an exposure index. With initial emphasis on sulfur pollutants, two exposure indexes have been considered. The first is the exposure of population to the pollutants and is calculated as the product of ground level concentration of pollutants in a cell and the population of the cell. The exposure indexes for all cells are summed to give a regional index:

$$J_P = \sum_{j=1}^{NC} R_j G_j \quad (3.8)$$

where G_j is the population in cell j . By substituting (3.7)

$$J_P = \sum_{j=1}^{NC} \sum_{i=1}^{NS} d_{ij} E_i G_j \quad (3.9)$$

The second exposure index is for vegetation and is similarly given as

$$J_V = \sum_{j=1}^{NC} \sum_{i=1}^{NS} d_{ij} E_i V_j \quad (3.10)$$

where V_j is the number of acres in cell j in a particular type of vegetation. The exposure indexes (3.9) and (3.10) are used as objective functions in the optimization model.

The use of an exposure index rather than an actual measure of the effects of air pollutants as an objective function is a subtle point. From a purely practical viewpoint, the actual effects are not sufficiently characterized to allow for adequate quantification in this type of objective function. The exposure index is a viable alternative. The principal objection to the use of an exposure index is that the pollutant effects are generally of a threshold nature. There are no significant effects of the pollutant until a threshold concentration is reached beyond which the effect may be very pronounced. This concept has been fundamental in establishing legislative standards for environmental quality. This threshold approach seems invalid for optimization models used in environmental planning. The optimization model described here is used to site facilities added to an existing energy system to meet an increase in demand at a particular time. If the objective function is

insensitive to variations in decision variables below a critical threshold (as postulated for many air pollutant effects), the optimum solution is not adapted for planning at these levels. If the increase in system capacity results in environmental quality just below the threshold, future planners have no tolerance for increases.

The optimization model for air quality can now be stated as:

determine P_i for $i=1,2,\dots,NS$ to

$$\underset{P_i}{\text{minimize}} J = \sum_{j=1}^{NC} \sum_{i=1}^{NS} d_{ij} E_i G_j$$

$$E_i = \gamma_i P_i$$

$$P_1 \leq P_1 \text{ max}$$

$$P_2 \leq P_2 \text{ max}$$

$$\vdots$$

$$P_{NS} \leq P_{NS} \text{ max}$$

$$P_1 + P_2 + \dots + P_{NS} \geq P_D$$

(3.11)

The coefficient γ_i is used to designate any desired source coefficient. This type of optimization model is a linear programming problem. Numerous routines are available for computer solution. A brief description of the linear programming problem and an illustrative example will follow.

Linear Programming

Linear programming is a technique for solving a specific class of constrained optimization problems. In this type of mathematical

programming problem, the objective function (the quantity to be optimized) is a linear function of the decision variables (those variables which can be manipulated to affect the optimum). In addition, any constraints on the values of the decision variables must be linear functions of those variables. Specifically, the linear programming problem can be stated as:

Find the values of the decision variables $\underline{x}$;

$\underline{x} = [x_1, x_2, \dots, x_n]^T$; which maximize the linear function

$$f(\underline{x}) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \quad (3.12)$$

subject to the constraints

$$x_j \geq 0, \quad j = 1, 2, \dots, N \quad (3.13)$$

and

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1N}x_N &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2N}x_N &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mN}x_N &\leq b_m \end{aligned} \quad (3.14)$$

where the a_{ij} , b_i and c_j are problem specific constants.

The problem can be conveniently written in matrix notation as:

$$\underset{\underline{x}}{\text{maximize}} \{f(\underline{x}) = \underline{C}^T \underline{x}\} \quad (3.15)$$

subject to

$$\underline{x} \geq \underline{0} \quad (3.16)$$

$$\underline{A} \underline{x} \leq \underline{b} \quad (3.17)$$

where $\underline{C}$ is a n -vector of objective function coefficients; $\underline{A}$ is a $m \times n$ matrix of coefficients for the constraint equations; $\underline{B}$ is a m -vector of right-hand-side constants for the constraint equations; and $\underline{x}$ is a n -vector of decision variables. The linear programming technique can be used to solve the power plant siting problem posed in previous sections of this report.

In a general setting, the linear programming problem may take on a form somewhat different than that of equations (3.15), (3.16) and (3.17). Other problem formulations can generally be converted to this canonical form by the use of elementary transformations (Taha, 1971). Three of these transformations are sufficient for the analysis reported here.

1. The minimization of a function is equivalent to maximizing the negative of the function:

$$\underset{\underline{x}}{\text{minimize}} f(\underline{x}) \equiv \underset{\underline{x}}{\text{maximize}} -f(\underline{x}) \quad (3.18)$$

2. The direction of an inequality may be changed by multiplying both sides of the inequality by -1:

$$\underline{A} \underline{x} \leq \underline{b} \equiv -\underline{A} \underline{x} \geq -\underline{b} . \quad (3.19)$$

3. An equality constraint may be replaced by two inequality constraints of opposite direction:

$$\underline{A} \underline{x} = \underline{b} \equiv \underline{A} \underline{x} < \underline{b} \text{ and } \underline{A} \underline{x} > \underline{b} . \quad (3.20)$$

To illustrate the linear programming technique, a problem involving two decision variables will be discussed. Consider the minimization of a population exposure to air pollutant index which is given by

$$J = c_1 P_1 + c_2 P_2 \quad (3.21)$$

where P_i is the activity level of the i th facility which is a source of air pollution. As an example it will be assumed that facility 1 exposes three times as many people as facility 2 so that

$$J = 3P_1 + P_2 . \quad (3.22)$$

Further assume that

$$\begin{aligned} P_1 &\leq 2000 \\ P_2 &\leq 3000 \end{aligned} . \quad (3.23)$$

These are constraints on the size of the facilities. An equality constraint is placed on the problem if the total capabilities of the two facilities must meet a specified demand:

$$P_1 + P_2 = 4000 \quad (3.24)$$

It should be noted that the problem would be ill-posed if the demand were greater than the sum of the constraints on facility size.

The solution to this problem can be determined by examining Fig. 3. The solution must lie along the line representing the demand equality constraint and within the shaded region due to the inequality constraints. Values of the decision variables satisfying these requirements are referred to as feasible solutions to the linear programming problem. A series of parallel lines is generated by the objective function (3.22) as various values of the decision variables are considered. The objective function line which intersects the demand equality constraint line within the shaded region and corresponds to a minimum value is the desired solution. It can be shown that the critical intersection will be at a vertex of the area bounded by the inequality constraints and the equality constraints (Spivey, 1966). In Fig. 3 the solution must correspond either to point (1):

$$P_1 = 1000$$

$$P_2 = 3000$$

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Fig. 3. Graphical solution for a linear programming example.

or to point (2):

$$P_1 = 2000$$

$$P_2 = 2000 \quad .$$

These critical points can be substituted in the objective function (3.22) to determine which is the optimal solution. Point (1) corresponds to an optimum with an objective function value of 6000.

An important point is demonstrated by this example problem. At optimality, one of the decision variables will assume a value which lies on an inequality constraint. In other words, at least one inequality constraint will be binding. In the above example P_2 was equal to 3000 at optimality. The constraint on P_2 was therefore binding on the optimal solution. If the P_2 inequality constraint represents a legislated environmental safety regulation, a value of P_2 which lies on this limit is probably undesirable. This characteristic of the linear programming approach can generally be avoided by more complicated nonlinear formulations.

The two variable example problem presented here demonstrates the nature of the linear programming approach to optimization. However, there are usually a large number of decision variables involved in a given problem. For cases of more than two decision variables, a numerical technique implemented on a digital computer is usually required to determine the optimal solution. The simplex method developed by Dantzig (1963) and various modifications are the most commonly used approaches to the numerical solution. Several general

purpose programs have also been developed for the computer solution of the linear programming problem (e.g., MPS/360). Since a great deal of data manipulation was required to calculate the objective function coefficients in the power plant siting problem discussed here, a FORTRAN subroutine for linear programming was chosen (Schneeberger et al., 1967).

AIR QUALITY OPTIMIZATION: An Example

The optimization model for air quality described in the previous section was designed for application in regional planning situations. In this section a brief example is presented. The main purpose is to further illustrate the type of problem which can be addressed, rather than to carry out a detailed case study. Also discussed is the integration of large regional data bases which are used in calculating objective function coefficients. The region for the example is first described, followed by an explanation of the optimization model calibration for the region. Some sample results are then presented.

For this example an area in the eastern United States is considered. Specifically the area between 81° and 89° west longitude and 31° and 34° north latitude was selected. The area contains the drainage basin of the Tennessee River and parts of the Ohio and Cumberland Rivers. The region has a large coal resource and coal combustion power plants form a large part of the existing power system. An expansion in the nation's coal power system would likely manifest itself quickly in this area which has a long history in the use of coal.

An actual region was chosen for this example to ensure as much realism as possible; however, the analysis provided here is in no way complete. By using real terrain and actual data, some of the problems encountered in applying the optimization approach are highlighted. The development of methodologies which are not applicable in real world situations is also avoided to some extent.

The results of this example should not be taken as indicative of the results derived from a complete regional analysis with respect to air quality.

A sketch map of the region under consideration is shown in Fig.

4. Five candidate sites for coal combustion facilities were selected using an automated site screening technique (Voelker, 1976). Variables including distance to load centers, distance to coal resources, available transportation, water availability and distance to population are overlaid for each county in the region. Counties with a high rating in a systematic scoring system are considered good choices for locating energy facilities. The five counties chosen by this procedure are Colbert, Morgan, and Jackson in Alabama and Rhea and Hawkins in Tennessee. Identification of potential sites in these five counties was further refined through a topographic map study aimed mainly at water availability. A 0.25 cell was chosen in each county which seemed to be acceptable for construction of a power plant. These cells represent the specification of candidate sites.

Associated with each candidate site is an upper limit on the size of facility which can be constructed. As indicated in the previous section, the upper limits on plant size are constraints in the linear programming model. The limiting factor on site suitability for coal combustion facilities is frequently water availability. The analysis of water availability is difficult since competing users and flow regulation must be considered. The counties selected here provide adequate water resources for a large

Fig. 4. Sketch map for the region used in the example study.

power plant (perhaps 1500 MW) independent of the availability of that water. It will be assumed for this example analysis that a maximum capacity of 1500 MW can be constructed at each site. A planned increase in generating capacity for coal fired power plants in the region of 5000 MW is assumed.

Using the trajectory dispersion model briefly described in the previous section, ground level concentrations for SO_2 were calculated in 0.25° cells due to a source at each candidate site. The EPA source model given in equation (3.6) was used. The distribution of sulfur pollutant at ground level due to sources of unit strength at all candidate sites is shown in Fig. 5. Clearly any long range air dispersion model could be used to perform these calculations. Initially a period of one month in the winter season has been chosen for analysis.

The distribution of population in the region was obtained from 1970 census data (U.S. Bureau of the Census 1970). The census data have been disaggregated to 0.25° cells using a technique similar to that described by Haaland and Heath (1973). This format for census data lends itself to use with the optimization model where air dispersion characteristics are parameterized using a cell system. The population distribution for the region is shown in Fig. 6.

As indicated in the previous section, it is sometimes desirable to use the exposure of various classes of land use to air pollutants as an objective function to be minimized in the optimization. For this example a county land use data base was employed (USDA, 1967). Twenty classes of land use are available in this data base. The

Fig. 5. Ground level concentration distribution for SO_2 pollutant from candidate sites.

Fig. 6. Population distribution for example region.

data are specified by number of acres in each county in each land use category. Since it is generally unreasonable to disaggregate county level data of this type to a finer spatial resolution, the ground level concentrations of SO_2 for each 0.25° cell were averaged for the county. A weighted average using the inverse of distance from the county centroid to the center of the 0.25° cell in question was used. The distribution of the sum of several agricultural land-use categories is shown in Fig. 7.

The data sets described here are sufficient to calibrate the optimization model for air quality. The objective function becomes

$$J_P = 1290 P_1 + 1616 P_2 + 1492 P_3 + 741 P_4 + 631 P_5 \quad (4.1)$$

while the constraint set is given by

$$\begin{aligned} P_i &\leq 1500, \quad i=1,2,\dots,5 \\ \sum_{i=1}^5 P_i &\geq 5000 \end{aligned} \quad (4.2)$$

The solution for this example is given in Table 1.

Examination of Table 1 and Figs. 5, 6 and 7 yields several interesting interpretations of the case study results. The candidate site in Morgan county is not used, and only a small facility is sited in Jackson county. Since these three candidate sites are clustered around the Huntsville demand center in northern Alabama, a simple inference is that planning for minimum exposure of population to air pollutants drives the facility siting pattern away

Fig. 7. Distribution of agricultural land use in the example region.

Table 1. Results of the five site example. A demand of 5000 MW was assumed.

SITE NUMBER	MAXIMUM PLANT SIZE	COMPUTED PLANT SIZE
1	1500 MW	1500 MW
2	1500 MW	0
3	1500 MW	500 MW
4	1500 MW	1500 MW
5	1500 MW	1500 MW

from this type of clustering. In Figure 5 it is clear that the trajectories of pollutants from sites 1, 2, 3 and 4 overlap and combine resulting in high pollutant concentrations. These areas of high concentration are seen to correspond to high population density areas in Fig. 7. Note that site 5 is reasonably isolated and is assigned a high activity level. If a small plant at site 3 as indicated in Table 1 is not feasible, an alternate fuel resulting in different pollutants or the use of improved abatement procedures may be a reasonable alternative.

This example serves to illustrate the nature of the optimization model for air quality as described in detail in the previous section. In an actual application many more constraints would be imposed on the model. However, the solution will lie on binding constraints thus eventually eliminating the necessity of considering the whole set. Even this simple example serves to illustrate the necessity of an optimization approach. With sufficient field data collection for validation of the model, this model provides a useful planning tool. The main application is in conjunction with other optimization models forming the multiple objective framework described in the previous section.

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APPENDIX

DOCUMENTATION AND SOURCE PROGRAM LISTING FOR COMPUTER
IMPLEMENTATION OF THE OPTIMIZATION MODEL FOR AIR QUALITY ANALYSIS

The optimization model for air quality analysis in the siting of energy facilities as presented in this report is a linear programming model. Numerous packages are available for the computer solution of such problems. The computer program described in this appendix is constructed around a routine for Talako's symmetric method (Talako, 1963). The FORTRAN subroutines listed here are due to Schneeberger, Burrus and Rust (1967) with modifications by Jane Cope of the Computer Sciences Division, Oak Ridge National Laboratory. As indicated in the development of the model, the greatest computing burden is the calculation of objective function coefficients. These calculations require the manipulation of large data bases. Because of this data handling requirement, a FORTRAN program designed around a subroutine for linear programming is desirable.

The main program and subprograms listed here are in ANS FORTRAN with the following major deviations:

- (1) Due to the limited word length of IBM computers, all floating point variables are specified double precision using the IBM statement, `IMPLICIT REAL*8`. These statements must be modified in each routine for compilers not offering the `IMPLICIT` statement. The double precision specification should be removed for machines of reasonable word length.
- (2) The apostrophe has been used as a delimiter for literal data.

The program has been run on the IBM 360/91 with the FORTRAN-H compiler and on the DEC system 10 with the FORTRAN-10 compiler. This program was developed as a research tool. Major improvements may be required for some applications.

Data Format

Print switches: IPRT, INPRT, MPRT, IGRT one card 4I5.

These switches control the level of printer output. If a switch is 0, print is not provided. If a switch is 1, print is provided.

IPRT--trajectory coefficients

INPRT--population data

MPRT--constraint matrix

IGRT--population grid (census data averaged to 0.25 cell).

Number of candidate sites: NPL one card I5

Number of cells: NCL one card I5

Number of constraints: NCON one card I5

Trajectory coefficients: 4(I5, I5, E10.0) as many cards as necessary. The first field is the cell number containing the source. The second field is the cell receiving the pollutant. The third field specifies the ground level concentration for a unit source. A blank in the source field indicates the end of data for a particular source.

Population data: (I5, 10X, 3I10) one card for each cell. The first field is the cell number. The second field is population for

the cell. The remaining fields specify the coordinates of the southwest corner of the cell.

Control card for population data output: one card 2I5.

These parameters control the frequency of output under the IGRT option. The first field is vertical frequency the second horizontal frequency.

Sulfur content of coal: (10E6.1) on as many cards as necessary.

One field for each site.

Efficiency specification: one card 2E10.3

The first field is the combustion efficiency for coal. The second field is the source term coefficient for the air pollutant of interest.

Maximum generating capacity: (8E10.5) as many cards as necessary.

One field for each site. These are the site suitability constraints.

Required load: one card E10.5.

This is the equality constraint on the total capacity of all plants.

Program Structure

The main program is used mainly as a driver for the various sub-routines. Some routine input and output functions are performed.

AIROBJ -- Calculation of objective function coefficients based on pollutant concentrations and air dispersion characteristics.

CONSTR -- Inequality constraints are put in the correct format.

- PMAXC -- Site suitability constraints and regional load specifications are inputted and checked for consistency.
- SUCONT -- Source model for air pollutants is factored in objective function coefficients.
- TALAKO -- Solution of linear programming problem.

The structure of the program is given in Figure 8.

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Fig. 8. General structure of the computer code implementing the air quality optimization model.

```

C*****
C      OPTIMAL POWER PLANT SITING--AIR QUALITY ANALYSIS
C      W. R. EMANUEL  ENVIRONMENTAL SCIENCES DIVISION
C      OAK RIDGE NATIONAL LABORATORY
C      P. O. BOX X   OAK RIDGE, TENNESSEE 37830
C
C      THIS PROGRAM AND SUBROUTINES PROVIDE A LINEAR
C      PROGRAMMING MODEL FOR POWER PLANT SITING.
C      THE PERFORMANCE INDEX IS THE MINIMUM EXPOSURE
C      OF EITHER POPULATION OR LAND USE CATEGORIES TO
C      AIR DISPERSED POLLUTANTS.
C
C      J. F. HURT, UNIVERSITY OF KENTUCKY IS RESPONSIBLE
C      FOR ORIGINAL DEVELOPMENT OF THE CODE.
C
C      SUBROUTINE TALAKO WAS PROVIDED BY B. RUST AND
C      J. COPE COMPUTER SCIENCES DIVISION ORNL.
C*****
C      VARIABLES OF MAJOR INTEREST
C      A--MATRIX OF CONSTRAINT COEFFICIENTS
C      B--VECTOR OF CONSTRAINT RHS ELEMENTS
C      C--VECTOR OF OBJECTIVE FUNCTION COEFFICIENTS
C      P--VECTOR OF DECISION VARIABLES(POWER PLANT ACTIVITIES)
C      S--VECTOR OF SENSITIVITIES
C
C      STORAGE HAS BEEN PROVIDED FOR 20 DECISION VARIABLES AND 20
C      INEQUALITY CONSTRAINTS.
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      DIMENSION A(20,20),B(20),C(20),P(20),S(20)
C      DIMENSION IID(20),JID(20)
C      COMMON/OPTION/IPRT,INPRT,IGRT
C
C      SPECIFY I/O DEVICES.
C
C      IN=5
C      IOUT=6
C
C      READ SWITCHES INDICATING WHETHER OR NOT TO PRINT, RESPECTIVELY.
C      TRAJECTORY COEFFICIENTS (IPRT), POPULATION DATA (INPRT), CONSTRAINT
C      MATRIX (MPRT), OR THE POPULATION GRID(IGRT). THE FORMAT IS 4I5.
C      1 = PRINT , 0= DO NOT PRINT.
C      READ(IN,50)IPRT,INPRT,MPRT,IGRT
C      50 FORMAT(5I5)
C
C      INPUT THE NUMBER OF SITES
C      READ(IN,10) NPL
C      10 FORMAT(I5)
C      IF(NPL.LE.10) GO TO 28
C      WRITE(IOUT,29)
C      29 FORMAT(1H , 'NUMBER OF SITES IS .GT. 10')
C      STOP
C      28 CONTINUE
C

```



```

C INPUT THE NUMBER OF CELLS
  READ(IN,11) NCL
  11 FORMAT(I5)
  IF(NCL.LE.550) GO TO 30
  WRITE(IOUT,31)
  31 FORMAT(1H , 'NUMBER OF CELLS IS .GT. 550')
  STOP
  30 CONTINUE

C
C PRINT GENERAL TITLES
  WRITE(IOUT,12) NPL,NCL
  12 FORMAT(//,1H , 'OPTIMAL SITING FOR AIR QUALITY',/,1H , I5,2X,
    1 'SITES',10X,I5,2X, 'CELLS')
  READ(IN,11) NCON

C
C DEVELOP OBJECTIVE FUNCTION
  CALL AIROBJ(NPL,NCL,C)
  WRITE(IOUT,13)
  13 FORMAT(//,1H , 'OBJECTIVE FUNCTION COEFFICIENTS',/,1H ,
    1 'SITE NUMBER',3X, 'COEFFICIENT')
  DO 14 J=1,NPL
  14 WRITE(IOUT,15) J,C(J)
  15 FORMAT(1H , I5,10X,E14.5)

C
C DEVELOP CONSTRAINT EQUATIONS
  CALL PMAYC(NPI)
  CALL CONSTR(NPL,A,B)
  IF(MPRT.NE.1) GO TO 55
  WRITE(IOUT,32)
  32 FORMAT(//,1H , 'CONSTRAINT MATRIX ....')
  DO 16 J=1,NCON
  WRITE(IOUT,17) (A(J,K),K=1,NPL)
  17 FORMAT(1H ,10E10.3)
  16 CONTINUE
  55 CONTINUE
  WRITE(IOUT,33)
  33 FORMAT(//,1H , 'RIGHT HAND SIDE -- MAXIMUM CAPACITY AT SITES',
    */,1H , 'SITE',11X, 'CONSTRAINT VALUE')
  DO 18 J=1,NCON
  18 WRITE(IOUT,19) J,B(J)
  19 FORMAT(1H , I5,10X,E14.5)

C
C LINEAR PROGRAMMING
  ITT=0
  MR=20
  CALL TALAKO(A,B,C,VAL,MR,NCON,NPL,ITT,110,JID,KSOL)

C TEST FOR VALID SOLUTION
  IF(KSOL.EQ.0) GO TO 20
  IF(KSOL.EQ.-1) GO TO 21
  IF(KSOL.EQ.1) GO TO 22
  WRITE(IOUT,23) KSOL
  23 FORMAT(1H , 'VALUE OF KSOL IS INVALID' , I5)
  STOP
  22 WRITE(IOUT,24)
  24 FORMAT(1H , 'MULTIPLE SOLUTIONS ARE POSSIBLE')
  GO TO 20
  21 WRITE(IOUT,25)
  25 FORMAT(1H , 'PROBLEM IS NONFEASIBLE OR UNBCUNDED')

```

```

      STOP
20  CONTINUE
C
C OUTPUT THE DECISION VARIABLES
  WRITE(IOUT,34)
34  FORMAT(//,1H , 'PLANT ACTIVITY ALLOCATION',
  */1H , 'SITE',11X, 'ACTIVITY')
  DO 26 JJ=1,NPL
    J=JJ
    P(J)=XS(J,A,B,C,MR,NCON,NPL,IID,JID)
26  WRITE(IOUT,27) J,P(J)
27  FORMAT(1H ,15,10X,E14.7)
C
C OUTPUT THE DUAL VARIABLES AS SENSITIVITIES
  WRITE(IOUT,82)
82  FORMAT(//,1H , 'SENS. OF OBJECTIVE FUNCTION TO SITE CONSTRAINTS',
  */1H , 'SITE',11X, 'SENSITIVITY')
  DO 80 J=1,NPL
    JJ=J
    S(JJ)=US(JJ,A,B,C,MR,NCON,NPL,IID,JID)
80  WRITE(IOUT,81) JJ,S(J)
81  FORMAT(1H ,15,10X,E14.7)
  STOP
  END

```

```

      SUBROUTINE AIROBJ(NPL,NCL,C)
C
C THIS SUBROUTINE CALCULATES THE OBJECTIVE FUNCTION COEFFICIENTS
C BASED ON GROUND LEVEL CONCENTRATIONS OF POLLUTANTS FROM THE
C AIR DISPERSION MODEL AND POPULATION DISTRIBUTION TAKEN FROM
C CENSUS DATA.
C
C VARIABLES OF PRINCIPLE INTEREST
C   TC--GROUND LEVEL CONCENTRATIONS OF POLLUTANT
C   NPOP--POPULATION
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TC(6,600),LDAR(4),LRAR(4),TCS(4),NPOP(600),C(1),ID(6)
      DIMENSION ILA(600),ILO(600),IPGRA(20,40)
      COMMON/OPTION/IPRT,INPRT,IGRT
C
C SPECIFY I/O DEVICES
C
      IN=5
      IOUT=6
C
C OBJECTIVE FUNCTION FOR POPULATION EXPOSURE
C
C INPUT TRAJECTORY COEF.
      DO 10 J=1,NPL
      DO 10 K=1,NCL
10    TC(J,K)=0.
      DO 11 J=1,NPL
      DO 13 K=1,150
      READ(IN,14) (LDAR(M),LRAR(M),TCS(M),M=1,4)
14    FORMAT(15,15,E10.0,15,15,E10.0,15,15,E10.0,15,15,E10.0)
      DO 15 KK=1,4
      IF(LDAR(KK).EQ.0) GO TO 11
      ID(J)=LDAR(KK)
      IR=LRAR(KK)
15    TC(J,IR)=TCS(KK)
13    CONTINUE
11    CONTINUE
      IF(IPRT.NE.1)GO TO 20
      WRITE(IOUT,23)
23    FORMAT(//,1H , 'TRAJECTORY DATA .....')
      DO 21 J=1,NCL
21    WRITE(IOUT,22) (TC(K,J),K=1,NPL)
22    FORMAT(1H ,10E10.3)
20    CONTINUE
C
C INPUT POPULATION DATA
      DO 16 J=1,NCL
      READ(IN,17)JCL,K,ILAT,ILONG
17    FORMAT(15,10X,3I10)
      NPOP(JCL)=K
      ILA(JCL)=ILAT
      ILO(JCL)=ILONG
16    CONTINUE
      IF(INPRT.NE.1)GO TO 51
      WRITE(IOUT,52)

```

```

52 FORMAT(//,1H,'POPULATION DATA.....',/,
13X,'SITE',7X,'POP.',10X,'LAT.',10X,'LONG.',
DO 55 MM=1,NCL
55 WRITE(IOUT,53) MM,NPOP(MM),ILA(MM),ILO(MM),
53 FORMAT(1X,15,5X,17,7X,17,7X,17)
51 CONTINUE

C
C FORM 2X2 MATRIX FOR POPULATION.
READ(IN,40) IROW,ICOL
40 FORMAT(2I5)
MK=1
DO 61 II=1,IROW
DO 61 KK=1,ICOL
KK2=ICOL-KK+1
IPGRA(II,KK2)=NPOP(MK)
IF(MK.EQ.NCL) GO TO 65
MK=MK+1
61 CONTINUE
65 IF(IGRT.NE.1) GO TO 36
WRITE(IOUT,77)
77 FORMAT(//,1H,'POPULATION GRID',/)
DO 75 IRT=1,ICOL
WRITE(IOUT,78)
78 FORMAT(' ')
75 WRITE(IOUT,67) (IPGRA(IT,IRT),IT=1,IROW)
67 FORMAT(1H,17I7)
36 CONTINUE

C
C COMPUTE OBJ. FUNC. COEF.
DO 18 J=1,NPL
L=J
CALL SUCONT(NPOP,TC,NPL,NCL,SUM,L)
18 C(J)=-SUM
RETURN
END

```

```

SUBROUTINE CONSTR(NPL,A,B)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION PMAX(10),A(20,20),B(1)
COMMON/PMAXS/PMAX,PLOAD

```

```

C
C INEQUALITY CONSTRAINTS ARE PUT IN THE CORRECT FORMAT
C
C ZERO THE A MATRIX
  DO 10 J=1,NPL
    DO 10 K=1,NPL
      10 A(J,K)=0.0
C MAX. SITE POWER CONSTRAINTS
  DO 11 J=1,NPL
    A(J,J)=1.
    11 B(J)=PMAX(J)
C INCREASED LOAD CONSTRAINT
  L=NPL+1
  DO 12 J=1,NPL
    12 A(L,J)=-1.
    B(L)=-PLOAD
  RETURN
END

```

```

      SUBROUTINE PMAXC(NPL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION PMAX(10)
      COMMON/PMAXS/PMAX,PLOAD
C  MAXIMUM POWER GENERATION AT EACH SITE IS GENERATED IN THIS
C  SUBROUTINE IN THE ARRAY PMAX AND PASSED IN THE COMMON BLOCK PMAXS.
C  A MAXIMUM OF 10 SITES ARE ALLOWED.
C  POWER IS MEASURED IN MEGAWATTS.
C  DEMAND IS INPUTED INTO THE VARIABLE PLOAD.
C
C
C  SPECIFY I/O DEVICES
C
      IN=5
      IOUT=6
C  KIN=0 -- VALUES ARE ENTERED BY ARITHMETIC ASSIGNMENT
C  KIN=1 -- VALUES ARE READ FROM CARDS FORMAT 8E10.5 ON AS MANY CARDS
C          AS NECESSARY.
      KIN=1
      IF(KIN.EQ.0) GO TO 10
      READ(IN,11) (PMAX(J),J=1,NPL)
11  FORMAT(8E10.5)
      READ(IN,15) PLOAD
15  FORMAT(E10.5)
      GO TO 16
10  CONTINUE
C  ARITHMETIC ASSIGNMENT OF MAXIMUMS
      PMAX(1)=1500.
      PMAX(2)=1500.
      PMAX(3)=1500.
      PMAX(4)=1500.
      PMAX(5)=1500.
      PLOAD=5000.
C  CHECK FOR PMAX .GE. PLOAD
16  SUM=0.
      DO 12 J=1,NPL
      SUM=SUM+PMAX(J)
12  CONTINUE
      WRITE(IOUT,25)SUM
25  FORMAT(//,1H , 'THE SUM OF THE MAXIMUM LOADS IS',E14.5,/)
      IF(SUM.GT.PLOAD) GO TO 13
      WRITE(IOUT,14) PLOAD
14  FORMAT(//,1H ,5X, 'PLOAD = ',E10.3,
1  'IS NOT GREATER THAN MAXIMUM ACTIVITY SUM')
      STOP
13  RETURN
      END

```

```

SUBROUTINE SUCONT(NPOP,TC,NPL,NCL,SUM,L)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION NPOP(1),TC(6,600),SUL(10)
IN=5
IOUT=6
IF(L.NE.1)GO TO 6
C INPUT SULFUR CONTENT FOR EACH POWER PLANT BEING OPTIMIZED
  READ(IN,9)(SUL(K),K=1,NPL)
  9 FORMAT(10E6.1)
C INPUT EFFICIENCY VARIABLES
  READ(IN,10)EFF,PCOEF
  10 FORMAT(2E10.3)
  6 CONTINUE
  SUM=0.
C COMPUTE OBJECTIVE FUNCTION COEFFICIENTS
  DO 8 I=1,NCL
    8 SUM=SUM+FLOAT(NPOP(I))*EFF*SUL(L)*TC(L,I)*PCOEF
  RETURN
END

```

```

C      SUBROUTINE TALAKO(A,B,C,VALUE,MR,NR,NC,ITT,IID,JID,KSOL)
C      SOLVES LINEAR PROGRAMMING PROBLEMS BY SYMMETRIC METHOD
C      A MATRIX OF CONSTRAINT COEFFICIENTS
C      B CONSTRAINT VECTOR OF PRIMAL PROBLEM
C      C COST VECTOR OF PRIMAL PROBLEM
C      VALUE OPTIMAL FUNCTIONAL VALUE
C      MR FIRST DIMENSION NO. OF A
C      NR NO. ROWS IN A
C      NC NO. COLUMNS IN A
C      ITT NO. ITERATIONS TO FIND SOLUTION, SET TO ZERO BEFORE FIRST PASS
C      KSOL SOLUTION TAG -- KSOL=0 IF SOLUTION IS VALID AND UNIQUE
C      KSOL=+1 IF OTHER VALID SOLUTIONS MAY ALSO EXIST
C      KSOL=-1 IF PROBLEM IS NONFEASIBLE OR UNBOUNDED
C      IID IDENTIFICATION NOS. FOR ROWS
C      JID IDENTIFICATION NOS. FOR COLUMNS
C      IMPLICIT REAL*8 ( A - H , O - Z )
C      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
C      COMMON/BLOK/DEPS
C      DEPS=1.0-20
C      BIG=1.030
C      KSOL=0
C      IF (ITT) 25,25,50
C
C      INITIALIZE ROW AND COLUMN BOOKKEEPING SYMBOLS
C
C      25 DO 30 I = 1,NR
C      30 IID(I) = - I
C      DO 35 J = 1,NC
C      35 JID(J) = J
C      VALUE = 0.0D0
C      NCMR = NC*MR
C      GO TO 1000
C      50 DO 60 J = 1,NC
C      60 C(J) = -C(J)
C      VALUE = -VALUE
C
C      SCAN FOR POSSIBLE PIVOTS
C
C      IMN=0
C      IPL=0
C      1000 FMAX = 0.0D0
C      IF (ITT.GT.2000) GO TO 9999
C      IMN=0
C      IPL=0
C      2005 FORMAT(1H ,10I5)
C      PRINT 2000, ITT, VALUE
C      2000 FORMAT(1H ,15,2X,D15.6 )
C      PRINT 2001, (B(KJ),KJ=1,NR)
C      PRINT 2001,( C(KJ),KJ=1,NC)
C      2001 FORMAT(1H ,10D13.4)
C      IRO = 0
C      DO 150 J = 1,NC
C      102 IF (C(J)) 150,150,103
C      103 GMIN = BIG
C

```



```

C      A POSITIVE COLUMN INDICATOR HAS BEEN FOUND. SCAN THAT COLUMN
C      FOR POSSIBLE PIVOTS.
C
      IJ = (J-1)*MR
      DO 135 I = 1,NR
      IJ = IJ + 1
      IF (A(IJ)) 135,135,110
C 110 IF (B(I)) 135,116,117
      IF (B(I)) 135,117,117
C 116 IF( ITT.EQ.0 ) GO TO 135
      RATIO = B(I)/A(IJ)
C
C      A CANDIDATE FOR A POSSIBLE PIVOT HAS BEEN FOUND. THE CANDIDATE
C      WITH THE SMALLEST RATIO IS THE POSSIBLE PIVOT FOR THAT COLUMN.
C
      IF (RATIO - GMIN) 120,134,135
C 134 IF( A(IJ)-A(IJ1) ) 135,135,120
      GMIN = RATIO
      IJ1=IJ
C 125 IROW = I
C 135 CONTINUE
      IF (GMIN - BIG) 137,150,150
C
C      IF A POSSIBLE PIVOT HAS BEEN FOUND, COMPUTE THE RATIO, P, OF THE
C      PRODUCT OF THE ROW AND COLUMN INDICATORS TO THE PIVOT ITSELF AND
C      CHECK THIS NEW VALUE OF P AGAINST THE PREVIOUS LARGEST SUCH VALUE.
C
C 137 P = C(J)*GMIN
      IF (P-FMAX) 150,141,140
C 141 IF( IRO.EQ.0 ) GO TO 140
      IF(C(J)-C(JCL))150,150,140
      IF(C(J)-C(JCL))150,150,140
C
C      IF THE NEW P IS LARGER THEN THIS IS THE BEST POSSIBLE PIVOT SO FAR
C
C 140 FMAX = P
      IRO = IROW
      JCL = J
      IPL=IPL+1
C 150 CONTINUE
C
C      NOW SCAN THE ROW INDICATORS FOR NEGATIVE ENTRIES AND THE ROWS
C      CORRESPONDING TO NEG. ENTRIES FOR POSSIBLE PIVOTS.
C
      DO 200 I = 1,NR
C 153 IF (B(I)) 155,200,200
C 155 GMIN = BIG
      IJ = I - MR
      DO 185 J = 1,NC
      IJ = IJ + MR
      IF (A(IJ)) 160,185,185
C 160 IF (C(J)) 167,166,185
      IF (C(J)) 167,167,185
C 166 IF( ITT.EQ.0 ) GO TO 185
C
C      A CANDIDATE FOR POSSIBLE PIVOT HAS BEEN FOUND. THE CANDIDATE
C      WITH SMALLEST RATIO IS THE POSSIBLE PIVOT FOR THAT ROW.
C

```

```

167  RATIO = C(J)/A(IJ)
      IF (RATIO - GMIN) 170,184,185
C    NOTE A(IJ) AND B(I) ARE LESS THAN 0 HERE
184  IF(DABS(A(IJ))-DABS(A(IJ2)))170,185,185
170  GMIN = RATIO
      IJ2=IJ
      JCOL = J
185  CONTINUE
      IF (GMIN - BIG) 187,200,200
C
C    IF A POSSIBLE PIVOT HAS BEEN FOUND, COMPUTE THE RATIO P AND
C    COMPARE WITH THE PREVIOUS LARGEST SUCH RATIO.
C
187  P = -B(I)*GMIN
      IF (P-FMAX) 200,191,190
191  IF( IRO.EQ.0 ) GO TO 190
      IF( DABS( B(I) )-DABS( B(IRO) ) ) 200,200,190
C
C    IF THE NEW P IS LARGER, THIS IS THE BEST POSSIBLE PIVOT SO FAR.
C
190  FMAX = P
      IRO = I
      JCL = JCOL
      IMN=IMN+1
200  CONTINUE
C    PRINT 2002, IRO,JCL,GMIN,RATIO,P,FMAX
2002 FORMAT(1H ,2I5,4E15.4)
C
C    AT THIS POINT EITHER IRO AND JCL ARE THE ROW AND COL. ADDRESSES
C    OF THE BEST POSSIBLE PIVOT OR ELSE THERE WAS NO POSSIBLE PIVOT
C    IN WHICH CASE IRO WILL BE ZERO,
C
      IF(IRO)210,201,210
C
C    IF NO POSSIBLE PIVOT EXISTS AND ONE OF THE ROW INDICATORS IS
C    NEGATIVE OR ONE OF THE COLUMN INDICATORS IS POSITIVE,
C    THEN THE PROBLEM IS NONFEASIBLE OR UNBOUNDED.
C
201  DO 203 I=1,NR
      IF(B(I))202,203,203
202  KSOL=-1
      GO TO 9999
203  CONTINUE
      DO 205 J=1,NC
      IF(C(J))205,205,204
204  KSOL = -1
      GO TO 9999
205  CONTINUE
C
C    IF PROGRAM REACHES THIS POINT, NO POSSIBLE PIVOT EXISTS
C    BECAUSE ALL ROW INDICATORS ARE NON-NEGATIVE AND ALL
C    COLUMN INDICATORS ARE NON-POSITIVE. THE PROBLEM WAS
C    PROBABLY SET UP INCORRECTLY
C
      PRINT 2006
2006 FORMAT(1H0,'NO POSSIBLE PIVOTS. ALL COLUMN INDICATORS',
1 ' ARE NON-POSITIVE AND ALL ROW INDICATORS ARE NON-',
2 'NEGATIVE. IRO = 0. YOU HAVE PROBABLY SCREWED UP',

```

```

3 * IN SETTING UP THE PROBLEM.*/)
KSOL = -1
GO TO 9999

C
C
C IF A PIVOT WAS FOUND THEN THE PROGRAM REACHES THIS POINT AND THE
C ELIMINATION STEP OF THE ALGORITHM IS PERFORMED.
210 IPIV = IRO + MR*(JCL-1)
RPIV = 1.000/A(IPIV)
ITT = ITT + 1
JCMR = IPIV - IRO + 1
LL = 0
C*****CHANGE MADE BY JANE COPE 12/13/76
C MAX=0.00
C *****
DO 230 J = 1,NCMR,MR
LL = LL + 1
IF (J-JCMR) 215,230,215
215 IROJ = J + IRO - 1
A(IROJ) = A(IROJ)*RPIV
CALL DIFF(A(J),A(JCMR),A(IROJ),NR,IRO)
C CB=C(LL)
CC=C(JCL)*A(IROJ)
C(LL) = C(LL) -CC
C*****CHANGE MADE BY JANE COPE 12/13/76
CC=DABS(C(LL))
C MAX=D MAX1(C MAX,CC)
C *****
C DIV = D MAX1(DABS(CB),DABS(CC))
C IF(DIV.EQ.0.000) GO TO 230
C IF((DABS(CB-CC)/DIV).LE.DEPS) C(LL)=0.000
230 CONTINUE
C*****CHANGE MADE BY JANE COPE 12/13/76
IF(C MAX.EQ.0.00)GO TO 241
DO 240 J=1,NC
IF (J-JCMR) 235,240,235
235 CC=DABS(C(J))
IF((CC/C MAX).LT.DEPS)C(J)=0.00
240 CONTINUE
241 CONTINUE
C *****
CALL DIFF(B(1),A(JCMR),B(IRO)*RPIV,NR,IRO)
VV=B(IRO)*C(JCL)*RPIV
C VW=VALUE
VALUE = VALUE - VV
C DIV = D MAX1(DABS(VW),DABS(VV))
C IF(DIV.EQ.0.000) GO TO 231
C IF((DABS(VW-VV)/DIV).LE.DEPS) VALUE = 0.000
231 CONTINUE
B(IRO) = B(IRO)*RPIV
JK = JCMR + NR - 1
DO 260 I = JCMR,JK
260 A(I) = -A(I)*RPIV
C(JCL) = -C(JCL)*RPIV
A(IPIV) = RPIV
ITEMP = IID(IRO)
IID(IRO) = JID(JCL)
JID(JCL) = ITEMP

```

```

C      PRINT 2001,RPIV,C(JCL)
C
C      AT THIS POINT THE ELIMINATION STEP HAS BEEN COMPLETED. CHECK
C      FOR POSITIVE COLUMN INDICATORS AND IF THERE ARE ANY GO BACK AND
C      START ANOTHER ITERATION.
C
C      DO 280 J = 1,NC
C      IF (C(J)) 280,280,1000
280  CONTINUE
C
C      IF THERE ARE NO POS. COL. INDICATORS, CHECK FOR NEG. ROW
C      INDICATORS.
C
C      DO 290 I = 1,NR
C      IF (B(I)) 1000,290,290
290  CONTINUE
C
C      IF THERE ARE NO POS. COL. INDICATORS OR NEG. ROW INDICATORS THEN
C      A SOLUTION HAS BEEN FOUND.
C
C      A ZERO ROW OR COL. INDICATOR MEANS THAT MULTIPLE SOLUTIONS EXIST.
C
300  DO 320 I = 1,NR
C      IF(B(I))320,315,320
315  KSOL = +1
320  CONTINUE
C      DO 340 J = 1,NC
C      IF(C(J))340,335,340
335  KSOL = +1
340  C(J) = -C(J)
C      VALUE = -VALUE
9999 RETURN
C      END

```

```

SUBROUTINE TRAMAT(A,NR,NC,SIGN,MR,MC)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(1)
C MC ROW DIMENSION OF ORIGINAL MATRIX
C MR COLUMN DIMENSION( FIRST DIMENSION ) OF ORIG. MAT.
C
  IF(NC-NR) 2,1,1
1 DO 10 J=1,NC
  K=J
  IF( J.GT.NR ) K=NR
  DO 10 I=1,K
  LL= (J-1)*MR + I
  KK= (I-1)*MC + J
  T = A(LL)
  A(LL) = SIGN*A(KK)
  A(KK) = SIGN*T
10 CONTINUE
  RETURN
2 DO 20 I=1,NR
  K=I
  IF( I.GT.NC ) K=NC
  LL=(J-1)*MR + I
  DO 20 J=1,K
  KK=(I-1)*MC + J
  T = A(LL)
  A(LL) = SIGN*A(KK)
  A(KK) = SIGN*T
20 CONTINUE
  RETURN
END

```

```

SUBROUTINE DIFF(A,B,C,NR,IRO)
C   TIME SAVING ROUTINE FOR GAUSSIAN ELIMINATION STEP
C   CALLED BY TALAKO FOR EACH COLUMN OF THE MATRIX
C   ALSO CALLED TO TRANSFORM B (PRIMAL CONSTRAINT VECTOR)
IMPLICIT REAL*8 ( A - H , O - Z )
DIMENSION A(1),B(1)
COMMON/BLOK/DEPS
C*****CHANGE MADE BY JANE COPE 12/13/76
AMAX=0.D0
C   *****
DO 50 I = 1,NR
  IF (I-IRO) 20,50,20
20  AA=B(I)*C
C   AB=A(I)
  A(I)=A(I)-AA
C   *****CHANGE MADE BY J COPE 12/13/76
  AA=DABS(A(I))
  AMAX=DMAX1(AMAX,AA)
C*****
C   DIV=DMAX1(DABS(AA),DABS(AB))
C   IF(DIV.EQ.0.D0)GO TO 50
C   IF((DABS(AB-AA)/DIV).LT,DEPS)A(I)=0.D0
50  CONTINUE
C   *****CHANGE MADE BY J COPE 12/13 76
  IF(AMAX.EQ.0.D0)RETURN
  DO 60 I=1,NR
    IF(I.EQ.IRO)GO TO 60
    AA=DABS(A(I))
    IF((AA/AMAX).LT,DEPS)A(I)=0.D0
60  CONTINUE
C   *****
  RETURN
END

```

```

C      REAL FUNCTION XS*8(J,A,B,C,MR,NR,NC,IID,JID)
      UNSCRAMBLES PRIMAL SOLUTION. RETURNS COMPONENT J
      IMPLICIT REAL*8 ( A - H , O - Z )
      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
      XS = 0.0D0
      DO 50 I = 1,NR
        IF (IID(I) - J) 50,20,50
20     XS = B(I)
50    CONTINUE
      RETURN
      END

```

```

      REAL FUNCTION US*8(I,A,B,C,MR,NR,NC,IID,JID)
C      UNSCRAMBLES DUAL SOLUTION, RETURNS COMPONENT I
      IMPLICIT REAL*8 ( A - H , O - Z )
      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
      US = 0.000
      DO 50 J = 1,NC
        IF (JID(J) + 1) 50,20,50
20      US = C(J)
50      CONTINUE
      RETURN
      END

```



```

REAL FUNCTION TRANS*8(I,J,A,B,C,MR,NR,NC,IID,JID)
C  UNSCRAMBLES TALAKO TO FIND TRANSFORMATION MATRIX
C  THEN X = TRANS*BO, WHERE BO = ORIGINAL B VECTOR
C  ALSO U = TRANS(TRANPOSE)*CO, CC = ORIGINAL C VECTOR
IMPLICIT REAL*8 ( A - H , O - Z )
DIMENSION A(1),B(1),C(1),IID(1),JID(1)
DO 20 K = 1,NR
  IF (I - IID(K)) 20,30,20
20 CONTINUE
  GO TO 50
30 DO 40 L = 1,NC
  IF (J + JID(L)) 40,70,40
40 CONTINUE
50 TRANS = 0.0D0
  RETURN
70 KK = K + (L-1)*MR
  TRANS = A(KK)
  RETURN
END

```

```

C      REAL FUNCTION DSL*8(J,A,B,C,MR,NR,NC,IID,JID)
C      FINDS THE DUAL SLACK VECTOR
C      DSL(J) = SUM U(I)*A(I,J) - C(J)
C      I
C      NOTE THAT IF DSL(J).GT.0, X(J) = 0.0
C      IMPLICIT REAL*8 ( A - H , O - Z )
C      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
C      DO 30 K = 1,NC
C      IF (JID(K) - J) 30,60,30
30    CONTINUE
C      DSL = 0.0D0
C      RETURN
60    DSL = C(K)
C      RETURN
C      END

```

```

C
C
C
C
REAL FUNCTION PSL*8(I,A,B,C,MR,NR,NC,IID,JID)
FINDS THE PRIMAL SLACK VECTOR
  PSL(I) = B(I) - SUM A(I,J)*X(J)
  J
  NOTE THAT IF PSL(I).GT.0, U(I) = 0.0
  IMPLICIT REAL*8 ( A - H , O - Z )
  DIMENSION A(1),B(1),C(1),IID(1),JID(1)
  DO 30 K = 1,NR
  IF (IID(K) + I) 30,60,30
30 CONTINUE
  PSL = 0.000
  RETURN
60 PSL = B(K)
  RETURN
  END

```

```

C      REAL FUNCTION PTRAN*8(I,J,A,B,C,MR,NR,NC,IID,JID)
C      ROUTINE FINDS MATRIX WHICH CONVERTS ORIGINAL B VECTOR TO
C      TALAKO RETURNED B, B= PTRAN*BC
C      PTRAN IS NR BY NR
C      NOTE PTRAN CONVERTS ANY VECTOR TO ONE WHICH WOULD HAVE
C      BEEN PRODUCED BY THOSE ITERATIONS IN TALAKO
      IMPLICIT REAL*8 ( A - H , O - Z )
      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
      DO 60 JJ = 1,NC
        IF (JID(JJ)+J) 60,50,60
50      IJ = I + (JJ-1)*MR
        PTRAN = A(IJ)
        GO TO 9999
60      CONTINUE
        PTRAN = 0.0D0
        IF (IID(I)+J) 9999,100,9999
100     PTRAN = 1.0D0
9999    RETURN
      END

```

```

      REAL FUNCTION DTRAN*8(I,J,A,B,C,MR,NR,NC,IID,JID)
C      ROUTINE FINDS MATRIX WHICH CONVERTS ORIGINAL C VECTOR TO
C      TALAKO RETURNED C, C = C*DTRAN
C      DTRAN IS AN NC BY NC MATRIX
C      NOTE DTRAN CONVERTS ANY VECTOR TO ONE WHICH WOULD HAVE
C      BEEN PRODUCED BY THOSE ITERATIONS IN TALAKO
      IMPLICIT REAL*8 ( A - H , O - Z )
      DIMENSION A(1),B(1),C(1),IID(1),JID(1)
      DO 60 II = 1,NR
        IF (IID(II)-I) 60,50,60
50      IJ = II + (J-1)*MR
        DTRAN = A(IJ)
        GO TO 9999
      60 CONTINUE
        DTRAN = 0.0D0
        IF (JID(J)-I) 9999,100,9999
100      DTRAN = -1.0D0
9999 RETURN
      END

```

```

      SUBROUTINE ARRAYD (B,NRR,NCC,LVV,NAME)
C   PROGRAM AUTHORS  J. DUNPHEY OF COMPUTING TECHNOLOGY CENTER AND
C                     L. FELDMAN FORMERLY WITH
C   COMPUTING TECHNOLOGY CENTER, UNION CARBIDE CCRP., NUCLEAR DIV.,
C   OAK RIDGE, TENN.
C
C   CTC ORD PROGRAM NO. 7085.2
      DOUBLE PRECISION COLUMN,B(1)
      DATA COLUMN /8HCOLUMN /
      NR = NRR
      NC = NCC
      LV = LVV
      DO 3 ML = 1,NC,5
      MH = ML + 4
      IF (NC .GE. MH) GO TO 1
      MH = NC
1     MLM1 = (ML-1)*LV
      MHM1 = (MH-1)*LV
      DO 3 LI = 1,NR,50
      NL = LI
      NH = NL + 49
      IF (NR .GE. NH) GO TO 2
      NH = NR
2     PRINT 100,      NAME
100  FORMAT (1H0//1H0,A4)
      PRINT 200,      (COLUMN,J,J=ML,MH)
200  FORMAT (4H0ROW,3X,5(3X,A8,I5,9X))
      PRINT 300
300  FORMAT (1H0)
      DO 3 L=NL,NH
      MML = MLM1 + L
      MMH = MHM1 + L
3     PRINT 400,      L,(B(J),J=MML,MMH,LV)
400  FORMAT (1H ,I3,3X,1P5D25.16)
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      END

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