MASTER

INVESTIGATION OF ACCURACY AND COMPUTATION TIME OF A HIERARCHY OF GROWTH RATE DEFINITIONS

BY
P.J. MAUDLIN, R.C. BORG
AND K.O. OTT

PURDUE UNIVERSITY
SCHOOL OF NUCLEAR ENGINEERING
WEST LAFAYETTE, INDIANA 47907

NOTICE
This report was prepared as an account of work
sponsored by the United States Government. Neither
the United States nor the United States Energy
Research and Development Administration, nor any of
their employees, nor any of their contractors,
subcontractors, or their employees, makes any
warranty, express or implied, or assumes any legal
liability or responsibility for the accuracy, completeness,
or usefulness of any information, apparatus, product or
process disclosed, or represents that its use would not
infringe privately owned rights.

July 1977

Prepared for

THE U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
UNDER CONTRACT NO. EY-76-S-02-4116.M000

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

EP

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed or represents that its use would not infringe privately owned rights.

ABSTRACT

A numerical illustration of the hierarchy of four logically different procedures for the calculation of the asymptotic growth of fast breeder fuel is presented. Each hierarchy level is analyzed in terms of accuracy and computational effort. Using the first procedure as reference, the fourth procedure, which incorporates the isotopic breeding worths, $\vec{w}^*$, requires a minimum amount of effort with a negligible decrease in accuracy.

1. INTRODUCTION

Determination of the breeding capability of a plutonium-uranium fast breeder reactor (FBR) requires the consideration of the equilibrium operation of a closed fuel cycle.^{1, 2, 3, 4} Consistent and convenient measures of the growth of fast breeder fuel have been derived by Ott and Borg³ from the mathematical description of the closed cycle equilibrium operation. A hierarchy of four logically different procedures for the calculation of the asymptotic growth of breeder fuel have been identified by Ref. 3. The aim of this report is to illustrate this hierarchy numerically, including an analysis of the differences in accuracy and use of computer resources between procedures.

II. GROWTH RATE CALCULATION PROCEDURES

The four computational procedures for the determination of the asymptotic growth rate of fuel, denoted by γ^∞ , based on reuse of plutonium in identical reactor systems, operated in the closed fuel cycle mode are reviewed here³:

P.1. The first procedure consists of the straight forward use of the mathematical formulation of the basic doubling time concept as a measure of the asymptotic growth of fuel in a system of reactors, expanding at a rate that is determined by the surplus breeding of fuel:

$$\gamma^\infty = \frac{\frac{d\hat{N}_i(t)}{dt}}{\hat{N}_i(t)} = \frac{\sum_i w_i \frac{d\hat{N}_i(t)}{dt}}{\sum_i w_i \hat{N}_i(t)} = \frac{\ell_{n2}}{t_D} \quad (1)$$

One calculates the accumulating number of all fuel isotopes $[\hat{N}_i(t)]$ for a park of reactor systems and determines the asymptotic growth rate, e.g. as slope in a semilogarithmic plot (comp. Fig. 1). The asymptotic growth rate is the same for each isotope as well as for any combination of isotopes; i.e., the weight factors in Eq. (1) are arbitrary.

P.2. The second procedure is deduced from the first one; it employs only quantities from a single reactor system rather than from the reactor park.

One solves the space dependent equilibrium fuel cycle problem, forms the production and loss rates of all plutonium isotopes (P_i^∞ and L_i^∞), finds the equilibrium number of fuel isotopes in the core, blankets and out-of-pile cycles, N_{Si}^∞ , and calculates the equilibrium growth rate as:

$$\gamma^{\infty} = \frac{\sum_1 w_1^{CM} (P_1^{\infty} - L_1^{\infty})}{\sum_1 w_1^{CM} N_{S1}^{\infty}} = \frac{\ln 2}{t_D} \quad (2)$$

Here, the w_1 have formal influence upon γ^{∞} , but have been shown to be practically arbitrary with very little impact on the results⁵, if Eq. (2) is actually applied for an equilibrium state. If actual delays in the plutonium reuse are considered, the plutonium compositions in numerator and denominator of Eq. (2) are different. This difference needs to be evaluated by using the substitutional critical mass worths (comp. the discussion of this subject in Ref. 7).

P.3. The third procedure is devised to avoid the large computational expense of the solution of the space dependent equilibrium fuel cycle problem. Although this procedure (the continuous integrated model^{8,9,10} procedure) is derived as rigorous condensation of the fuel cycle problem, it requires practically only an approximate solution of the space dependent problem. From the latter solution, one calculates the coefficients for the "continuous fuel cycle model." This latter model assumes for a closed equilibrium fuel cycle the form of an eigenvalue problem

$$\underline{C}^{\infty \rightarrow \infty} = \gamma_C^{\infty \rightarrow \infty} \underline{C}^{\infty \rightarrow \infty} \quad (3a)$$

Multiplication with arbitrary weight factors gives the following expression for γ_C^{∞} :

$$\gamma_C^{\infty} = \frac{(\vec{w}, \underline{C}^{\infty \rightarrow \infty})}{(\vec{w}, \underline{n}^{\infty})} \quad (3b)$$

The solution of Eq. (3a) yields the asymptotic growth rate of the core fuel (γ_C^∞) and the equilibrium composition of the core plutonium, $\vec{n}^\infty$. γ_C^∞ gives the core doubling time t_{CD} .

From the core-fuel growth rate, γ_C^∞ , one can calculate the reactor-fuel growth rate, γ^∞ , by multiplying with the ratio of core to system fuel. Note, the physical doubling time (t_D) always includes the "doubling" of the blanket and out-of-pile plutonium.

P.4. The fourth procedure is devised for an application to a variety of different reactors which may be considered, e.g., during the conceptual design phase. It exploits the fact that the weight factors in the first three procedures are practically arbitrary. The w_i may therefore be determined such that they provide computational advantages, e.g., by making it unnecessary to find the correct equilibrium fuel composition.

One can postulate Eq. (3b) to yield the correct result for any (allowed) fuel composition, $\vec{n}$, i.e.:

$$\gamma_C^\infty = \frac{(\vec{w}^*, \vec{C}^\infty \vec{n})}{(\vec{w}^*, \vec{n})} \quad (4a)$$

The weight factors which make Eq. (4a) independent of $\vec{n}$ are the solution of the following adjoint fuel cycle problem:

$$\vec{C}^\infty \vec{w}^* = \gamma_C^\infty \vec{w}^* \quad (4b)$$

which has the same eigenvalue as Eq. (3a). The w_i^* are called "isotopic breeding worth" values. Typical values for a 1000 MWe LMFBR are:

$$(w_9^*, w_0^*, w_1^*, w_2^*) = (1.0, 0.52, 1.10, 0.26) \quad (4c)$$

The proposed fourth procedure consists of the application of the isotopic breeding worth values [e.g., from Eq. (4c)] to find the doubling time for reactors of the same general type as the reference reactor. One calculates the reaction rates (P_1 and L_1) and the core-plutonium inventory (N_1) from a static multigroup diffusion theory solution and forms:

$$\gamma_C^\infty = \frac{\sum_i w_i^* (P_1 - L_1)}{\sum_i w_i^* N_1} = \frac{\ell_{11} \lambda}{t_{CD}} \quad (5)$$

The advantages of the growth rate formulas, Eqs. (4a) and (5), as compared to currently applied formulas are: the results of Eq. (4a) are independent, or the results of Eq. (5) are nearly independent of the fuel composition; both definitions are stationary about the reference reactor.

III. NUMERICAL RESULTS

III.A. Reactor Depletion Analysis Description

The 1000 MWe modified Clinch River Breeder Reactor (CRBR) design is used as basis for the numerical comparisons presented here. A fuel-reloading cycle period of 304 days (which corresponds to a fuel burnup of 130 MWd/Kg) and a three assembly type, batch scatter loading scheme comprise the in-pile fuel management program for both core and blankets. An external fuel cycle (EFC) with a fuel residing time of an additional 304 days is also considered; thus, the out-of-pile time and material both average one quarter of the total. A 2% EFC loss and ^{241}Pu decay are also taken into account. The initial core plutonium composition was assumed to be equilibrium Light Water Reactor discharge⁴, and the initial blanket plutonium is an approximate FBR loading equilibrium value.

Operation of the CRBR design is modeled using the program 2DS¹¹, treating the energy dependence of the neutrons with six energy groups. Two-dimensional (r - z) cylindrical geometry is used breaking the core into two enrichment zones and axial and radial blankets each into four zones. Surplus fuel discharged from the reactor system is assumed to become immediately part of the external fuel cycle, where it stays for 304 days. The average CDC 6500 CPU time consumed per fuel-reloading cycle is approximately 700 seconds.

The low values for the doubling times obtained here is the result of an error in the uranium volume fraction of the axial blanket. This error does not affect the relative differences between the growth rates (or doubling times) to which this paper is addressed.

III.B. First Procedure: Accumulating Park Fuel

A detailed depletion calculation of the accumulation of plutonium isotopes in an expanding park of identical FBR's was performed. The "identical" reactor systems were modeled as described in Sec. III.A. Figure 1 illustrates the results of this calculation on a semi-log graph. The abscissa represents the total atoms residing in all locations (cores, blankets and EFC's) that exist in the park at time "t". The initial number of park reactors is one.

At small times the park plutonium isotopes, $\hat{N}_i(t)$, are in transition and thus all curves have different logarithmic slopes. However, at large times the park isotopes reach dynamic equilibrium, and each increases with a common constant logarithmic slope or asymptotic fuel growth rate (γ^∞). The asymptotic slope of these curves, given by Eq. (1), has a value of 0.07605 yr^{-1} . The CPU time consumed in this calculation and thus associate with the first procedure increases roughly as

$$T_{p1}(\text{sec}) \approx \text{constant } e^{\gamma^\infty t} . \quad (6)$$

Eq. (6) represents a substantial computing effort.

Despite the discontinuous nature of the batch refueling scheme, the park curves of Fig. 1 are surprisingly smooth. This somewhat peculiar feature is better understood after study of Figs. 2 and 3. Figure 2 shows the park growth of a single fuel batch. At any point in time, each reactor system contains a total of four fuel batches; three in-pile and one residing in the EFC. The isotopic park growth of one of these four batches is observed in Fig. 2 over the time range of 40 fuel-reloading cycles.

Each curve shown in Fig. 2 displays two different time dependencies:

- D1. A primary, overall exponential increase due to the increasing number of reactor systems contained in the park; the initial park condition is 1 reactor whereas after 40 cycles the park has expanded to include almost 12 reactors.
- D2. A secondary, cyclic time dependence of $4\Delta t$ in duration ($\Delta t = 304$ days) due to the batch cycle of $3\Delta t$ resident exposure time in-pile and $1\Delta t$ of residence in out-of-pile. The regular "dips" in these curves reflect the EFC loss and decay during the out-of-pile residence time.

Figure 3, which is a composite of Figs. 1 and 2, shows the single batch curves (multiplied by a factor of 4) plotted over the smooth system curves of Fig. 1. Obviously, the growth of fuel in a park of reactors is described by very smooth curves, because they result from the addition of the four fuel batch curves. Each batch cycle period of $4\Delta t$ is shifted by $1\Delta t$, $2\Delta t$, and $3\Delta t$ from the other three, and thus the sum of these four offset curves tends to smear out the EFC "kinks" and yields smooth curves. This summation has practically eliminated the secondary time dependence defined above by D2.

III.C. Second Procedure: A Single System Calculation

A detailed depletion analysis of a single reactor system (described in Sec. III.A.) was performed and the results are illustrated by the solid lines (teeth) shown in Fig. 4. The abscissa represents the total number of plutonium atoms contained in the system (i.e., in all residing locations: core, blankets and EFC). The ^{239}Pu and ^{240}Pu isotopes quickly reach static equilibrium while more time is necessary for the higher isotopes ^{241}Pu and ^{242}Pu . The cycle-averaged production and loss rates and the average residing plutonium, evaluated

at equilibrium reactor operation, produce a value for γ^∞ , via Eq. (2), of 0.07588 yr^{-1} -- a few tenths of one percent less than the logarithmic slope of Fig. 1. Each fuel-reloading cycle (i.e., each tooth shown in Fig. 4) consumes approximately 700 CPU seconds; thus, the computer time required to generate the equilibrium information for the second procedure can be described by

$$\tau_{p2} \text{ (sec)} \approx 700 \cdot n^{eq} \text{ sec} \quad , \quad (7)$$

where n^{eq} is the number of cycles needed to attain asymptotic reactor fuel. Equation (7) represents a lesser but still sizable computing effort when compared to the first procedure.

III.D. Third Procedure: The Continuous Integrated Fuel Cycle Model

An idealized continuous integrated model^{8,9,10} calculation of the average system behavior was performed. The results are illustrated by the dashed lines shown in Fig. 4. The fuel cycle of the reactor system described in Sec. III.A. is continuously modeled here in an attempt to avoid the large consumption of computer resources associated with the first and second procedures. The goal of the continuous calculation is to split the "teeth" of the second procedure at midcycle (a good approximation of average system behavior).

Transitory deviations of the dashed lines from mid-cycle are observed in Fig. 4, although asymptotic agreement is very good.

The continuous prediction of γ^∞ , found either by forming the eigenvalue fuel cycle problem of Eq. (3) or by using the continuous version of Eq. (2), yields a value of 0.07565 yr^{-1} -- half of one percent low of the slope of Fig. 1.

The CPU time associated with only the continuous model calculation is trivial. However, preparation of the input coefficients required by the model

makes a detailed depletion calculation necessary. This depletion is of the type performed for the second procedure but need only be carried out over three fuel-reloading cycles; an approximate solution for the asymptotic continuous model coefficients is sufficient. Thus, the computer time required for the generation of the equilibrium information needed by the third procedure if the preparatory calculations are fully included, amounts to

$$T_{P3}(\text{sec}) \approx 2100 \text{ sec.} \quad (8)$$

Equation (8) represents a considerable savings in computing resources when compared to the two previous procedures. But, the computation time drops down to several seconds if the preparatory time needs not to be included since the corresponding flux calculations have been performed for other reasons.

III.E. Fourth Procedure: Application of the Isotopic Breeding Worths

The fourth procedure requires only a steady state reaction rate calculation for a typical midcycle configuration with an estimated equilibrium fuel composition. Calculations for a typical midcycle configuration will always be performed for other purposes (power distribution, critical enrichment calculation etc.). The stationary properties of this definition provide then an accurate estimate of γ^∞ , despite the approximate nature of the fuel composition. Use of Eq. (5) and the weight-factor set of Eq. (4c) result in value for γ^∞ of 0.07545 yr^{-1} . This growth rate is calculated using, as typical midcycle information, quantities from the tenth fuel cycle (tenth tooth in Fig. 4). This value is only about 0.8 percent lower

than the P1 results. If the midcycle configuration from the fortieth cycle were to be used in Eq. (5), then the values for γ^m and the relative error would be identical to those for P3.

The CPU time associated with this procedure is nil, since the weights of Eq. (4c) need not be specifically calculated from the "adjoint" fuel cycle problem for any particular case and the midlife configuration is assumed available.

IV. SUMMARY

The results of the numerical comparison calculated for the various levels or procedures of the hierarchy are illustrated in Table 1 and can be summarized as follows: the first two procedures give about the same asymptotic fuel growth rates, and the corresponding relative errors both with a substantial computational effort of many thousands of seconds on a CDC 6500. Also the third procedure yields the compound doubling time with a negligible error but with a small or even trivial amount of computation time. The fourth procedure, i.e., the application of the breeding worth factors, give excellent growth rate estimates, without any additional computational effort, if the reaction rates for a typical midlife configuration have been determined for other purposes.

REFERENCES

1. B.I. Spinrad, "On the Definition of Breeding," Proc. of the Conf. on the Physics of Breeding, Argonne National Laboratory, October 1959, ANL-6122, 23 (1959).
2. K.O. Ott, "Letter to the Editor of Nuclear Applications and Technology," Nuclear Technology, 9, 451 (1970).
3. K.O. Ott and R.C. Borg, "Derivation of Consistent Measures for the Doubling of Fast Breeder Reactor Fuel," Nuclear Science Engineering, 62, 243 (1977).
4. Y.I. Chang and W.P. Barthold, "The Growth Rate of Large Systems of Breeder Reactors and the Doubling Time Concept," Argonne National Laboratory, FRA-TM-78 (1975).
5. P.J. Maudlin, R.C. Borg and K.O. Ott, "Transitory Fuel Growth Rates for Fast Breeder Reactors," Trans. Am. Nucl. Soc., 26, 235 (1977).
6. A.R. Baker and R.W. Ross, "Comparison of the Value of Plutonium and Uranium Isotopes in Fast Reactors," Proc. of the Conf. on Breeding, Economics and Safety in Large Fast Breeder Reactors, October 7-10, 1963, ANL-6792, 329, Argonne National Laboratory (1963).
7. R.C. Borg and K.O. Ott, "Impact of Delays in Plutonium Use on the Stationary Doubling Time for Fast Breeder Reactors," (to be submitted to N.S.E.).
8. K.O. Ott and A. Jansen, "Long-Time Behavior of Fast Breeders," Proc. Seminar Physics of Fast and Intermediate Reactors, Vol. II pp. 213-228, International Atomic Energy Agency, Vienna (1962).
9. C.W. Terrell, "An Integrated Model for Time Dependent and Equilibrium Fuel Compositions in Fast Breeder Reactors, Ph.D. Thesis, Purdue University (1970).
10. K.O. Ott, C.W. Terrell, R.C. Borg and P.J. Maudlin, "Derivation and Verification of an Integrated Continuous Model for Time Dependent and Equilibrium Fuel Composition in Fast Breeder Reactors," (to be submitted to N.S.E.).
11. W.W. Little, Jr. and R.W. Hardie, "2DB User's Manual," BNWL-831 (1969).

Fig. 1. Detailed calculation of the isotopic growth of FBR fuel in an expanding park of identical reactor systems. Initial park condition is one reactor with an initial fuel condition of LWR plutonium.

Fig. 2. Detailed calculation of the isotopic growth of park fuel contained in one fuel batch-type. Initial fuel condition of the batch is LWR plutonium.

Fig. 3. Composite of Figs. 1 and 2 for comparison purposes.

Fig. 4. Detailed depletion calculation (solid lines or "teeth") for a single reactor system with an initial fuel condition of LWR plutonium. Continuous integrated model calculation (dashed curves) predicting an idealized average reactor behavior.

HIERARCHY	$\gamma^{\infty}(\text{yr}^{-1})$	$t_D(\text{yr})$	$\Delta \gamma^{\infty}(\%)$
P1	0.07605	9.114	0.00
P2	0.07588	9.135	-0.23
P3	0.07565	9.163	-0.53
P4	0.07545	9.186	-0.79

Table 1. Summary of the numerical results illustrating the hierarchy of the four procedures for computation of the asymptotic growth of fuel. Middle column is the compound system doubling time.