80/1-17

Dr 633
UCRL-52170

PRELIMINARY DESIGN CALCULATIONS FOR A FIELD-REVERSED MIRROR REACTOR

W. C. Condit
G. A. Carlson
R. S. Devoto
J. N. Doggett
W. S. Neef
J. D. Hanson

MASTER

November 5, 1976

Prepared for U.S. Energy Research & Development
Administration under contract No. W-7405-Eng-48

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research & Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights.

NOTICE

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Energy Research & Development Administration to the exclusion of others that may be suitable.

Printed in the United States of America

Available from

National Technical Information Service

U.S. Department of Commerce

5285 Port Royal Road

Springfield, VA 22161

Price: Printed Copy \$: Microfiche \$3.00

Page Range	Domestic Price	Page Range	Domestic Price
001-025	\$ 3.50	326-350	10.00
026-050	4.00	351-375	10.50
051-075	4.50	376-400	10.75
076-100	5.00	401-425	11.00
101-125	5.50	426-450	11.75
126-150	6.00	451-475	12.00
151-175	6.75	476-500	12.50
176-200	7.50	501-525	12.75
201-225	7.75	526-550	13.00
226-250	8.00	551-575	13.50
251-275	9.00	576-600	13.75
276-300	9.25	601-up	"
301-325	9.75		

* Add \$2.50 for each additional 100 page increment from 601 to 1,000 pages;
add \$4.50 for each additional 100 page increment over 1,000 pages.

LAWRENCE LIVERMORE LABORATORY
University of California, Livermore, California 94550

UCRL-52170

PRELIMINARY DESIGN CALCULATIONS FOR A FIELD-REVERSED MIRROR REACTOR

W. C. Condit, G. A. Carlson, R. S. Devoto, J. N. Doggett,
W. S. Neef, and J. D. Hanson

MS. date: November 5, 1976

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights.

Contents

Abstract	1
Introduction	1
The Field-Reversed Mirror	2
Physics Assumptions	4
Computation of Plasma Parameters	6
Reactor Parameters	10
Cost Estimates	16
Conclusions	18
References	20

PRELIMINARY DESIGN CALCULATIONS FOR A FIELD-REVERSED MIRROR REACTOR

Abstract

The field-reversed-mirror (FRM) is a plasma-confinement scheme that uses the diamagnetic current of the deuterium-tritium (D-T) fuel plasma to convert the open-field pattern of a conventional mirror machine into a closed-field pattern that should have considerably improved plasma-confinement properties. If realized, this improvement in confinement will considerably improve the energy gain of the mirror reactor and will reduce the cost of producing energy. The FRM device is described and compared with earlier field-reversed

configurations. The overall energy and particle-balance relations are given and used as bases for preliminary engineering design calculations of the major components of an FRM reactor. We find that the engineering design leads to simple structures (e.g., circular coils, cylindrical vacuum tanks) for which it is easy to make reasonably accurate cost estimates. A cost optimization has been performed; the optimum D-T reactor is found to have a fusion output of 23 MW/cell from a 5-cm radius, 15-cm long plasma in a field of roughly 10 T.

Introduction

In the past year, a group at Lawrence Livermore Laboratory (LLL) has been studying possibilities for enhancing the power gain factor, Q , to be expected from a mirror fusion reactor. An overall view of this work is contained in an earlier report.¹ The field-reversed mirror (FRM) appears to be one of the new configurations most likely to lead to a high- Q fusion reactor with both modest cost and modest system sizes. We have begun a conceptual design process to see what such a system

might look like, what magnetic fields and plasma parameters would be needed, and what the optimum system size for lowest-cost net electric power generation might be. The purpose of this work was to find out, in a preliminary way, whether the FRM idea would lead to a viable fusion reactor design; in other words, whether it was worth working on. We were pleased to find that the FRM reactor looks extremely attractive based on the physics we know today. The present report documents our

current knowledge and the reasons for our continuing interest.

A preliminary magnetic fusion energy reactor design always involves a number of physics assumptions. The present work is no exception, particularly in the areas of particle loss rates and hydromagnetic stability, where theoretical work is preliminary. In these areas, we have been as optimistic as current knowledge

allows. We have assumed that observed stable configurations can be scaled to higher temperatures and B fields, and we have assumed that energy and particle losses occur at classical ion random walk rates (intermediate between the strictly classical Spitzer diffusion rate² and the extremely pessimistic diffusion rates such as arise from Bohm diffusion).

The Field-Reversed Mirror

The idea of a closed-field-line geometry using a poloidal field, with field lines encircling an azimuthal current, was proposed early in the controlled thermonuclear research program for the toroidal Z-pinch and the Astron thermonuclear reactor.^{3,4} The idea is shown schematically in Fig. 1. In the Astron scheme, the azimuthal current was carried predominantly by high-energy (beam) ions — those with enough energy to encircle the axis of the device in roughly circular orbits. These beam ions were embedded in a low-energy background plasma. In the course of the Astron theoretical work, it was noted that increasing the pressure of the background would cause a significant fraction of the current to be contributed by the low-energy (plasma) particles via their diamagnetic effect.⁵ The FRM lies

at the ultimate limit of this process: a field-reversed configuration, localized by a weak magnetic mirror, in which the closed field lines are

Fig. 1. Schematic of field-reversed mirror reactor.

provided entirely by the plasma currents. Of course, as the relative importance of the beam particles decreases, there is a continuous transition from the Astron to the FRM case. Reactor designs in the Astron limit were studied extensively in the early 1970's.⁶ The designs seemed very attractive, but the reactors would be of large unit output (about 5 to 6 GW_e). To enable comparisons, in this paper we shall present reactor parameter sets in the opposite limit; i.e., no distinct beam contributions to the magnetic field will be considered.

The quantitative distinction between an Astron geometry and an FRM geometry is best made on the number of gyroradii included in the minor radius of the plasma. The geometry and variables are shown in Fig. 2.

Fig. 2. Mean ion and electron energies (E_i , E_e), magnetic field (B_0), and Q versus neutral-beam energy for $a/\rho_i = 5$, $R_p/a = 2$, and $L/R_p = 3$ (with elongated cross section).

If the gyroradius ρ_i (based on the plasma mean energy and vacuum field) is much less than the minor radius a , one has the FRM case. If there is a class of beam particles with gyroradius $\rho_b \geq a$, one has the Astron case. In the Astron limit, stable E-layers have recently been observed. It is clear from examining theoretical models of field-reversed equilibria that if ρ_i/a is too small, the configuration is adequately described by the hydromagnetic equations and is in general unstable.⁸ Recent experimental work by a group at the Kurchatov Institute⁹ has shown that field reversal in elongated plasma toroids with greatly diffused current sheaths is apparently magneto-hydromagnetically (MHD) stable; i.e., the toroids survive for 100 or more sound transit times across the major radius. Similar experiments have been done at Garching,¹⁰ and further work is planned at Los Alamos Scientific Laboratory (LASL).¹¹ In a recently developed theoretical framework for analyzing the stability of field-reversed layers,⁸ the stability criterion is cast in the form of an energy principle with five terms, three of which are familiar from MHD stability theory (zero gyroradius). These three terms on balance indicate instability; the effect of the new terms, which can be seen to have

a stabilizing influence in certain limits, is under study. Analytical examples of layers that are predicted by this theory to be stable have not, as of this writing, been found, possibly either due to failure to search out the relevant parameter space for the new terms or from omissions in the theory (which does not consider line-tying of the open field-lines). Therefore, we rely on the experimental results. In the Russian work, the ratio $S = a/\rho_1$

was about 5 for stable plasmas (see Ref. 1, Appendix C, for an analysis of this point). In the Garching work, the ratio was approaching 10. To study the consequences of varying the plasma size, we have assumed $S = 5$ to 10 for our reactor studies. The Russian workers made no attempt to sustain their field-reversed configuration with neutral-beam injection; however, for purposes of evaluation, we shall assume that this can be done.

Physics Assumption

Within the broad framework of a multi-orbit, beam-sustained FRM configuration, there are many possible variations. To have a specific starting point for design calculations, we make the following additional physics assumptions:

- Tangential injection of ions is needed for startup.
- Radial injection of ions is needed for sustaining operation.
- An elongated toroidal shape is most stable.
- A diffuse profile is required (cubic or parabolic).
- D-T fuel ions also carry the azimuthal current.
- D-T is introduced only by monoenergetic beam injection.

- Alpha deposition is unimportant throughout the bulk of the plasma.
- Plasma is lost on time scales related to ion-ion collisions.

The requirement of tangential injection for startup is evident from computer calculations of the startup phase (Ref. 1, Appendix E).^{*} We presently envision using a high-voltage ion diode for startup.¹² It seems

^{*} Tangential injection here means that the neutral beam is aimed tangentially to the major radius of the plasma. Since the neutral atom may be ionized anywhere along its transit chord through the plasma, we do not in general have radial velocity = 0 at the ionization point.

clear that once a reversed state is established, it can be maintained by keeping the plasma pressure gradient and current profile steady; i.e., if $\vec{\nabla}P = \vec{j}_1 \times \vec{B}$, then $\vec{j}_1 = (\vec{B} \times \vec{\nabla}P)/B^2$ is automatically correct to maintain the required B configuration. More sophisticated methods of maintaining a current have been proposed by Ohkawa.¹³ We are currently planning to modify the computer code, SUPERLAYER, used for calculating startup, to study the sustained operation question; until clear evidence is available on this point, we shall assume radial injection (although our evaluation of tangential injection shows that it does not lead to large changes in reactor design — see Ref. 1, Appendix D).

The assumption of a plasma shaped like an elongated toroid with a diffuse profile is, again, based on the Kurchatov work. Where specific numbers are needed, we shall take the overall length of the plasma as six times the minor radius; that is, $L = 6a$. The longer the plasma, the larger the power output, and the computer calculations in Ref. 1 indicate that our present assumption is conservative.

In regard to the density profile

of the plasma, we have evaluated both cubic and parabolic profiles and find only modest differences in reactor parameters. In the interest of simplicity, we have assumed for our initial study that D-T fuel ions carry the azimuthal current and that a third ion species is not needed. In the same spirit, we have assumed that all D and T ions are injected at a single energy; i.e., there is no cold refueling such as has been suggested for some other magnetic fusion energy reactors. Since we shall show later in this paper that the FRM reactor looks attractive even with these restrictions, we feel that this procedure is justified. If we took advantage of the possibility either of partial cold refueling or of using high-energy protons to carry part of the azimuthal current, our analysis might be more encouraging. Also for simplification, we have neglected alpha deposition while noting that alphas have large gyro-orbits and that they will probably deposit most of their energy in the cold plasma on the open field lines at the edge of the high-density region.

Plasma losses will be discussed at length in the next section.

Computation of Plasma Parameters

To evaluate the FRM, it is necessary to compute the expected plasma parameters. To do so, we have formed a simple model with a number of free parameters that can be adjusted to fit either experimental measurements or other, more detailed, theoretical calculations. We consider only conditions in the toroidal reversed-field layer because the surrounding plasma is assumed to be confined only by weak mirrors; ions diffusing into this region will be quickly lost. Basic to the model are one-point (zero-dimensional) equations for the central ion number density n_i and the mean ion and electron energies $\bar{E}_i$ and $\bar{E}_e$. We assume an isothermal plasma and only one average ion of mass $A_i = 2.5$ amu and allow for the expected density gradient by an imposed density profile of either cubic or parabolic shape, as appropriate. We take particle and energy losses to be diffusive in nature and neglect the α -particles. Except for very high-energy beams, the α gyroradius (at 3.5 MeV) will exceed the plasma dimensions; the α -particles will thus be lost without energy deposition in the plasma.

With these assumptions, the appropriate zero-dimensional equations

are

$$\frac{\partial n_i}{\partial t} = 0 = \dot{n}_0 - \frac{n_i}{\tau_p} \quad (1)$$

$$\begin{aligned} \frac{\partial (n_i \bar{E}_i)}{\partial t} &= 0 \\ &= \dot{Q}_{ie} - \frac{n_i \bar{E}_i}{\tau_{Ee}} - \frac{5}{3} \frac{n_i \bar{E}_e}{\tau_p} - \dot{Q}_s - \dot{Q}_B \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial (n_i \bar{E}_i)}{\partial t} &= \dot{n}_0 E_{inj} - \frac{n_i \bar{E}_i}{\tau_{Ei}} \\ &- \frac{5}{3} \frac{n_i \bar{E}_e}{\tau_p} - \dot{Q}_{ie} \end{aligned} \quad (3)$$

$$n_e = n_i \quad (4)$$

In these equations, $\dot{n}_0$ is the rate of increase of central density due to beam absorption; E_{inj} is the beam energy (assumed the same for D and T); τ_p , τ_{Ee} , and τ_{Ei} are the particle, electron energy, and ion energy-confinement times, respectively; and $\dot{Q}_{ie}$ is the ion-electron energy-transfer rate per unit volume for the assumed Maxwellian distribution function of ions and electrons given in W/m³ by¹⁴

$$\dot{Q}_{ie} = \frac{n_i (\bar{E}_i - \bar{E}_e)}{\tau_E}, \quad (5)$$

where

$$\tau_E = \left(\frac{5.396 \times 10^{18} A_1 \bar{E}_e^{3/2}}{n_e^2 k n_i} \right) \times \left(1 + \frac{\bar{E}_i}{183 A_1 \bar{E}_e} \right)^{3/2}, \quad (6)$$

with $\bar{E}$ in keV and n in m^{-3} . For these calculations, n_i has been taken as the typical value of 20. The synchrotron and bremsstrahlung loss terms $\dot{Q}_s$ and $\dot{Q}_b$ are computed from standard expressions.^{15,16} For the synchrotron losses, the wall reflectivity has been taken as 0.8.

The magnetic field is related to the plasma pressure via the total β . We take β as given, either from experiment or from an equilibrium code, and compute B_0 in Tesla along with the solution to the other equations from

$$B_0 = \left[\frac{2.68 \times 10^{-22}}{\beta} (n_e \bar{E}_e + n_i \bar{E}_i) \right]^{1/2}. \quad (7)$$

For the present calculations, we have assumed $\beta = 1.5$, a value used in previous field-reversed reactor studies.⁶

For given confinement times, a very large family of solutions to Eqs. (1)-(10) depending on both $\dot{n}_0$ and E_{inj} is possible. However, as discussed earlier, diffuse density

profiles are expected. Therefore, for most of our studies we choose the characteristic beam-attenuation length of the plasma to yield a cubic profile. The fraction of neutral beam that penetrates a distance x into the plasma is given by

$$f(x) = \exp \left[- \int_0^x \langle \sigma_T \rangle n_i dz \right], \quad (8)$$

where $\langle \sigma_T \rangle$ is the sum of the cross sections for ionization and charge transfer¹⁷ averaged over the ion and electron Maxwellian velocity distributions. For radial beam injection, the distance x is measured along the plasma radius. For isothermal plasmas with density profiles of the form $n = n_0 \{1 - (r/a)^j\}$, where r is the radial distance from the minor axis, Eq. (8) becomes

$$f(x) = \exp \left[- \frac{j}{j+1} n_i \langle \sigma_T \rangle x \right] \quad (9)$$

Results for the mirror fusion engineering research facility (FERF) design¹⁸ suggest that a cubic profile ($j = 3$) can be expected for radial injection if $f(a) \approx 1.75$. It is clear that we can obtain this value by adjusting $\dot{n}_0$ to yield the desired $f(2a)$ through its dependence on n_i . Note that $\langle \sigma_T \rangle$ depends also on $\bar{E}_i$, E_{inj} , and $\bar{E}_e$, albeit only very weakly on the latter.

To complete the description, we need expressions for the particle- and energy-confinement times. The classical result of $\tau_p, \tau_{Ee} \approx a^2 \tau_{ei} / \rho_e^2$, where ρ_e is the electron gyroradius and τ_{ei} is the electron-ion scattering time, has seldom been observed in experiments. We have therefore made the more conservative assumption of

$$\begin{aligned}\tau_p &= a^2 \tau_{ii} f(j) / \rho_i^2, \\ \tau_{Ee} &= \tau_{Ei} = a^2 \tau_{ii} / \rho_i^2,\end{aligned}\quad (10)$$

where ρ_i is the ion gyroradius, and τ_{ii} is given by¹⁹

$$\tau_{ii} = 3.58 \times 10^{-14} \frac{A_i^{1/2} \bar{E}_i^{3/2}}{n_i \ln \Lambda}, \quad (11)$$

and where $f(j)$ is a factor that accounts for the effect of the cubic density profile. In Eq. (11), the mean ion number density for the plasma, $\bar{n}_i$, is used.

We have also considered Bohm diffusion as a loss mechanism. This amounts to estimating the diffusion coefficient as $D = \rho_e^2 \omega_{ce}^2 / 8$.²⁰ At the high temperatures and modest magnetic fields and densities we are considering for the FRM reactor, Bohm diffusion is 8 to 12 orders of magnitude faster than classical diffusion and, since $\rho_e^2 = \rho_i^2 (m_e / m_i)$, it is well over 5 orders faster than the ion loss rates we used in the energy

balance calculations. It is clear that any loss rate approaching the Bohm rate will be fatal to the FRM. Also, since the size of the plasma appears to be limited to a small number of gyroradii, one cannot avoid the problem by going to a plasma with a large minor diameter.

Equations (1) through (11) have been combined in a computer code that enables all quantities to be evaluated as a function of the neutral-beam energy E_{inj} . Calculations have been made for a torus of circular cross section and for a torus with a highly elongated cross section. The latter would yield higher values of Q and appears possible both from experiments⁹ and theory (Ref. 1, Appendix E). Similarly, a torus with an aspect ratio near unity is indicated; we have taken the major radius $R = 2a$. Results for this geometry are plotted in Fig. 2 for $S = a / \rho_i = 5$ and a length/radius ratio $L/a = 6$. We note quite favorable Q values (for D-T and a fusion energy of 17.6 MeV) of 4 to 6 for beam energies of 150 to 300 keV. At the higher beam energies, the required magnetic field is quite high; however, it appears that for a small penalty in Q we can lower the beam energy and thus the required B_0 to values attainable with inexpensive superconductors.

Shown in Fig. 3 are similar results for the same parameters as Fig. 2, but with $S = 10$. Q is considerably higher for this geometry, a not surprising result since both τ_p and Q scale as S^2 . The actual Q is not quite this much higher because the longer τ_p results in more energy transfer to the electrons, and $\bar{E}_1$ is therefore reduced. The B field is also lower because the restriction on the beam annulation [c.f. Eq. (9)] is met by a lower n_1 .

Results for the same geometry as Fig. 2, but for a parabolic density profile, are plotted in Fig. 4. Q values are comparable to those of Fig. 2, but higher B fields are required, reflecting the higher central number density.

Fig. 3. Mean energies, magnetic field, and Q versus neutral-beam energy for $a/p_1 = 10$ (other quantities as in Fig. 2).

Fig. 4. Mean energies, magnetic field, and Q versus neutral-beam energy for parabolic profile (other quantities as in Fig. 2).

For comparison, in Fig. 5 we show results with $S = 5$ for a circular cross section and cubic profile. The lower Q value is due to the fact that losses occur over the entire surface; in the elongated cross section, losses out the ends were neglected. Other parameters are approximately the same as in Fig. 2.

Although much higher Q is obtainable with $S = 10$, we have chosen to base our design on $S = 5$. Our choice is based principally on the successful Russian experiments⁹ that operated near this value. The choice of beam energy and other engineering parameters such as cell length is dictated by economic considerations, the subject of the next section.

Fig. 5. Mean energies, magnetic field, and Q versus neutral-beam energy for circular cross section (other quantities as in Fig. 2).

Reactor Parameters

In this section, we shall first describe the overall configuration of an FRM power reactor. Then we shall describe the details of the parametric analysis that we are using to optimize the design for minimum power cost. Because the parametric analysis is still in an early stage of development, the results reported here should be considered preliminary.

We shall assume that an FRM reactor consists of a number of plasma cells (10) on a common axis (see Fig. 6), held apart by weak magnetic mirrors in the nearly uniform field of a long superconducting solenoid. The magnetic mirrors are produced by water-cooled copper coils placed between the plasma and the blanket. To form a magnetic well, copper Ioffe-bar conductors are also placed at this location.

The neutron-moderating blanket and the shield for the superconducting

coil are cylindrical shells. We are considering both a high- and low-energy multiplication blanket. The high-multiplication blanket contains beryllium and achieves an average M (blanket thermal energy per fusion neutron/14.1 MeV) of 1.7 through $n,2n$ reactions. The low-multiplication blanket achieves an M of 1.2 due to exothermic neutron capture. Tritium is bred in solid, lithium-containing compounds. The blanket is cooled by helium: hot and cold manifolds pass axially through the reactor, and individual axial segments of the blanket are cooled by parallel flow circuits. The exit helium is used to generate steam for a steam power cycle that has an efficiency of 40%.

As we shall show, the optimum magnetic field strength for the superconducting solenoid appears to be about 10 T, thus permitting the use of niobium-titanium superconductor.

Fig. 6. Layout of multicell field-reversed mirror reactor.

Typical section at beam ϵ

(b)

Fig. 6. (Continued).

The superconducting solenoid is restrained by a simple hoop structure. Outside the coil and coil structure is a cylindrical vacuum vessel.

Neutral-beam injectors are arranged along the length of the reactor, aimed radially inward. As we shall show, injection energies of ~ 200 keV are desirable; thus, to obtain high injection efficiency ($\sim 80\%$), negative-ion neutral-beam injectors must be used.

At each end of the reactor, a single-stage direct converter converts into electricity about 50% of the energy of the plasma that is leaked out the ends. The remaining energy is removed by helium coolant and added to the steam power cycle. The direct converter chambers contain the primary vacuum pumping system for the reactor, consisting of helium-cooled condensation panels operated at 4 K.

The entire reactor, including the injectors and direct converters, is assumed to be oriented vertically and enclosed in a reinforced-concrete containment building. Depending on the desired electrical output of the power plant, more than one reactor may be placed in the same building.

Now we shall describe the step-by-step procedure used in the parametric analysis to design a particular reactor. The primary independent

physics parameter is the injection energy. The physics model described earlier then yields the required magnetic field strength, the size of the plasma layer, the plasma energy and density, the fusion power, and the plasma Q (fusion power divided by injected neutral-beam power). Some of these results are shown in Fig. 2. The fusion power per cell is shown in Fig. 7.

For each value of injection energy, the engineering design begins with the coupled problem of the internal dimensions (radius and length) of the reactor cell and the design of the copper coils. First, we specify a trial length L for the

Fig. 7. Power cost and fusion power versus injection energy for optimum cell lengths.

reactor cell. Then, after adding the neutron flux contribution from each plasma layer, we set the first-wall radius R_w so that the peak first-wall neutron wall loading Γ_N is a preselected value. For the geometries discussed in this paper we have set $\Gamma_N = 4 \text{ MW/m}^2$, a figure based on typical results from the Yin-Yang mirror reactor study.²¹ For simplicity, we assume that the mirror coils, Ioffe bars, and blanket first wall are all at the first-wall radius. Because the copper coils are always small in cross section, this is a good approximation. There is a lower limit placed on the first-wall radius: it is not allowed to be less than the radial extent of the plasma toroid plus two alpha particle gyroradii.

At this point in the calculation, we check to verify that the cell length L satisfies the criterion $L > 1.2 R_w$. At less than this spacing, the axial mirror ratio (coil to center) of a pair of coils is less than unity. The optimum distance from an economic point of view is always somewhat greater than this minimum spacing because, as this spacing is approached, very large mirror currents are required to maintain the mirror ratio greater than unity (see Fig. 8).

Fig. 8. Projected cost of electrical power versus cell length for $E_{inj} = 300 \text{ keV}$.

With the mirror coils defined in length and radius, the axial mirror ratio may be varied by changing the current in the mirror coil. (Recall that a large, uniform, solenoidal field is established by the superconducting coil.) Just how high the axial mirror ratio should be is not obvious. Because the adjacent layers must be restrained by the mirrors from coalescing due to their mutual attraction, the specific numerical procedure we use is to require a small axial mirror ratio of at least 1.0001. This ratio is defined to include the fields from all coils and all plasma layers except the one in the cell under consideration; i.e., it is the vacuum

mirror ratio seen by a given plasma layer. For the current in the Ioffe bars, we require at the major radius of the plasma a small radial well with a ratio of 1.0001.

An approximate formula of Taylor²² is used to calculate the ratio for this well; more accurate coil calculations may show the required well to be somewhat different. Although these choices of mirror ratios are somewhat arbitrary, the resultant mirror coils and Ioffe bars contribute a negligible amount to the overall reactor cost, and higher currents could be specified.

Now we must determine the cross-sectional area of the copper mirror coils and Ioffe bars. An economic trade-off exists: coils with large cross sections minimize the resistive power loss in the coils but maximize the neutron attenuation in the coils. We therefore perform an optimization to minimize the sum of the resistive loss in the coils and the potential for electrical power production which is lost because of neutron attenuation in the copper. The latter quantity is approximated by multiplying the neutron heating of the copper by M and by the thermal efficiency of the steam power cycle. Within this optimization is another optimization; i.e., for a given total resistive loss in the coils, the loss is appor-

tioned between the mirror coils and Ioffe bars so as to minimize neutron interception. This completes the design of the reactor cell and copper coils for a single trial cell length.

In general, larger trial cell lengths result in smaller first-wall radii (because of the criterion on wall loading by neutrons) and smaller copper coil losses. Of course, the power per unit length of solenoid also decreases. The calculation of all costs determines the optimum cell length for the injection energy being considered. Because the first-wall radius is a weak function of cell length, and the copper coil losses tend to be quite small as soon as the condition $L = 2 R_w$ is exceeded by a small margin, we have found that small cell lengths, rather close to the minimum allowed, give the lowest cost per unit output.

The remainder of the reactor is designed from the first wall outward. The thickness of the cylindrical shell blanket is set at 1 m. The design thickness for the superconducting coil shield involves an economic trade-off: thicker shields reduce the neutron heating of the coil and thus the cost of cryogenic refrigeration, but they increase the size and cost of the coil, the coil structure, the vacuum vessel, and the containment

building. We calculate an optimum shield thickness for each geometry. However, because of the exponential attenuation of neutrons in the shield, we have found that the optimum shield thickness is always in the narrow range of 70 to 80 cm. The superconducting solenoid is placed outside the shield. The thickness of the coil-restraining structure (a single, stainless-steel, cylindrical shell) is calculated based on a 60,000-psi

design stress at 4K. The main vacuum tank surrounding the coil structure is assumed to be aluminum, and is designed to ASME code as a long cylindrical vessel. (This neglects the stiffening effect of the ends and any webbing, and thus overestimates the weight.) The containment building is sized to hold the reactor, complete with its injectors and direct converters.

Cost Estimates

We have not yet designed the components of the reactor in much detail; however, they are in most cases simple (circular coils, cylindrical tanks, etc.), and we feel that adequate preliminary cost estimates can be made. We have calculated direct capital costs using 1975-76 prices except where noted and have not included indirect cost factors for construction facilities, engineering, interest during construction, contingency, etc. Table 1 gives the basis for our preliminary cost estimates. For a number of components, the estimates were borrowed from other reactor studies conducted at LLL and are not based directly on the concurrently developed FRM design shown in Fig. 6. In future

work, we shall be able to improve our cost estimates for these components. Although all of the estimates are subject to change, and additional cost categories may be added, we believe our present results to be reasonably accurate.

The $\$/kW_e$ cost of the reactor is calculated by dividing the total direct capital cost by the net electric power of the plant. Figure 7 shows this cost as a function of injection energy for cases of optimum cell length. Curves are shown for two values of blanket energy multiplication, $M = 1.2$ and 1.7 . We see that the minimum costs occur for injection energies near 250 keV. The minimum cost is $\$950/kW_e$ for $M = 1.2$ and $\$730/kW_e$ for $M = 1.7$.

Table 1. Basis for cost estimate.

Item	Basis for estimate
Superconductor	Present prices reduced by 3.5 factor for mass production. ^a
Winding	1/2 of Yin-Yang winding cost per A·m. ^b
Structure	Stainless steel at \$11/kg
Power supply	Proportional to coil energy, normalized to FERF results. ^c
Refrigeration system	Proportional to coil neutron heating, normalized Moir <i>et al.</i> results. ^b
Blanket (with beryllium)	\$280 thousand/m ³ . ^b
Blanket heat-transfer loop	\$37/kW handled. ^b
Shield	80% stainless steel at \$11/kg.
Vacuum tank	Aluminum at \$22/kg.
Vacuum pumping system	1/2 of vacuum tank cost.
Neutral-beam injectors	\$200/kW injected.
Diagnostics and control	\$100 thousand/cell.
Tritium handling systems	Normalized to Moir <i>et al.</i> results. ^b
Direct converters	\$100/kW of charged particles. ^b
Direct converter heat-transfer loop	\$35/kW handled. ^b
Main containment building	Proportional to volume, normalized to Moir <i>et al.</i> results. ^b
Thermal conversion system	\$28/kW handled. ^b
Turbine building	\$5/kW handled. ^b

^aFrom Ref. 23.^bFrom Ref. 21.^cFrom Ref. 18.

Also shown on Fig. 6 are the fusion power and the net electric power per reactor cell.

Figure 5 shows that the cost variation is quite small over a range of injection energy from 200 to 300 keV. At lower energies, the cost increases primarily because of lower Q's and the resulting high recirculating power fractions. At higher energies, the cost increases because the more slowly increasing Q no longer compensates for the higher magnetic field strengths that are required. For the $M = 1.7$ case, the magnet and magnet structure account for 16% of the total cost at the optimum energy, but 38% of the total at 400 keV.

Table 2 gives reactor parameters for a near-optimum design with $M = 1.7$. We chose the slightly off-optimum case with 200-keV injection energy because the required superconductor field strength is lower, making possible the use of niobium-titanium superconductor. The optimum design occurs for 250-keV injection. The power output (23 MW/cell) and B field (11 T) are then slightly higher than for the 200-keV-injection example in Table 2, and the plasma dimensions are slightly smaller. The power cost for this high-field case is only about 2% lower than the cost in Table 2.

Conclusions

The reactor costs shown in Fig. 7 are extremely attractive. This might be expected *a priori* for a system with a high β and a simple magnetic-field configuration.

We are making a detailed engineering design for a reactor with the parameters given in Table 2 and are

working to reduce the number of physics extrapolations and assumptions needed. We feel confident that, if questions such as the start-up and maintenance of an FRM configuration can be answered with reasonable technology, the end product will be a very useful form of MFE reactor.

Table 2. Typical FRM reactor design.

Parameter	Value
Beam energy	200 keV
Plasma minor radius	0.013 m
Plasma major radius	0.062 m
Plasma length	0.19 m
Size parameter (S)	5
Vacuum field	8.2 T
Ion energy	77 keV
Electron energy	29 keV
Peak density	3.6×10^{21} ions/m ³
Total energy	205 kJ/cell
Fusion power	18.5 MW/cell
Plasma pressure ratio (β)	1.5
Blanket thickness	1 m
Blanket multiplication factor	1.7
Shield thickness	0.8 m
Superconducting solenoid radius	2.4 m
Wall radius	0.61 m
Cell length	1.1 m
Injected current	17 A/cell
Injected power	3.5 MW/cell
Plasma profile	Cubic
Mirror coil current	46 kA
Ioffe bar current	10 kA
Mirror coil cross section	13 cm ²
Ioffe bar cross section	14 cm ²
Resistive power	0.28 MW/cell
Neutron heating	0.38 MW/cell
Net power	10.2 MW _e /cell
Fusion energy gain (Q)	5.3
Total cost	\$745/kW _e

References

1. W. C. Condit, T. K. Fowler, and R. F. Post, *Status Report on Mirror Alternatives*, Lawrence Livermore Laboratory, Rept. UCLRL-52008 (1976).
2. L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience, NY, 1962), p. 42.
3. J. L. Tuck, "Controlled Thermonuclear Research at Los Alamos," in *Proc. Second U. N. Intern. Conf. Peaceful Uses Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 32, p. 3.
4. N. C. Christofilos, "Astron Thermonuclear Reactor," in *Proc. Second U. N. Intern. Conf. Peaceful Uses Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958) Vol. 32, p. 279.
5. D. V. Anderson, *et al.*, "Theoretical Study of E-Layer and Plasma Behavior in Astron," in *Proc. Fourth Intern. Conf. Plasma Physics, Madison, WI, 1971* (IAEA, Vienna, 1971), vol. I, p. 137.
6. N. C. Christofilos, "Astron Plasma Parameters Confined in the Closed Magnetic Well of a Proton E-Layer," in *Proc. Energy '70, Intersociety Energy Conversion Engineering Conf., Las Vegas, 1970* (ANS, 1970), p. 1-36.
7. J. J. Bzura, *et al.*, *Phys. Rev. Lett.* **29**, 256 (1972).
8. R. N. Sudan and M. N. Rosenbluth, *Phys. Rev. Lett.* **36**, 972 (1976).
9. A. G. Eskov, *et al.*, "Plasma Confinement in a Pulsed System with a Compact Toroidal Configuration," in *Proc. Seventh European Conf. Controlled Fusion and Plasma Physics, Lausanne, 1975* (CRPP, 1975), p. 55; see also Ref. 1, p. 20.
10. A. Eberhagen and W. Grossman, *Z. Phys.* **248**, 130 (1971).
11. K. Thomas, Los Alamos Scientific Laboratory, private communication (1976).
12. D. S. Prono, J. W. Shearer, and R. J. Briggs, "Intense, Pulsed Ion-Diode Sources and Their Application to Mirror Machines," in *Proc. Intern. Topical Conf. Electron Beam Research and Technology, Albuquerque, 1975* (Sandia Laboratories, Rept. SAND75-5122, 1976) vol. I, p. 575.
13. T. Ohkawa, *Fuel. Fusion* **10**, 185 (1970).
14. T. F. Morse *Phys. Fluids* **6**, 1420 (1963).
15. D. Rose and M. Clark, *Plasmas and Controlled Fusion* (Wiley, NY, 1961), p. 233.
16. B. A. Trubnikov, *Zh. Eksp. Teor. Fiz. Pis'ma Red. [JETP Lett.]* **16**, 25 (1972).

17. A. C. Riviere, *Nucl. Fusion* 11, 363 (1971).
18. T. H. Batzer et al., *Conceptual Design of a Mirror Reactor for a Fusion Engineering Research Facility (FERF)*, Lawrence Livermore Laboratory, Rept. UCRL-51617 (1974).
19. S. I. Braginskii, "Transport Processes in a Plasma," in *Reviews of Modern Physics*, M. A. Leontovich, Ed. (Consultants Bureau, NY, 1965), p. 205.
20. L. Spitzer, *Phys. Fluids* 3, 659 (1960).
21. R. W. Moir et al., "Mirror Reactor Studies," in *Proc. Sixth Intern. Conf. Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, 1976* (IAEA, Geneva, in press); detailed Lawrence Livermore Laboratory report in preparation.
22. J. B. Taylor, "Plasma Confinement in Magnetic Wells," in *Plasma Physics* (IAEA, Vienna, 1965), p. 449.
23. J. R. Powell, "Design and Economics of Large DC Fusion Magnets," in *Proc. Applied Superconductivity Conf., Annapolis, 1972* (IEEE, 1972), p. 346.