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THREE-BODY CALCULATIONS AT LOS ALAMOS

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1. Introduction

Approximately seven years ago the Los Alamos-Iowa collaboration[1] began its program of investigating the trinucleon systems by solving the Faddeev equations in configuration space. This work was motivated by four goals: (1) By working in configuration space, where intuition is greatest, investigate graphically those trinucleon properties which are determined by specific features of wave functions; (2) Produce benchmark calculations against which new techniques and numerical methods can be measured; (3) Investigate the effect of the Coulomb interaction between the two protons in ^3He and in the p-d system; (4) Systematically investigate the various trinucleon observables. Configuration space is particularly well-suited for investigating the Coulomb problem. The singularity and discontinuity problems associated with the Coulomb (momentum space) t-matrix are transformed into boundary condition problems in configuration space. One simply adds the Coulomb potential to the strong interaction. In order to produce accurate numerical solutions powerful techniques were adopted which have not frequently been used in nuclear physics. These spline methods together with collocation techniques combine the power of Gaussian quadrature procedures with the flexibility and strength of finite element approaches to solving partial differential equations. Further details are given by G. L. Payne in his contribution to this workshop[2]. The union of these methods allows one to calculate wavefunctions at the same qualitative level of accuracy as the eigenvectors. Observables can therefore be calculated with considerable confidence.

2. The Faddeev Approach

2.1 Overview

The Faddeev approach[3] to solving the Schrödinger equation involves rather different formal procedures than other methods, such as the hyperspherical expansions or the Rayleigh-Ritz variational technique. The latter methods expand the wavefunction in some type of basis and solve directly for the expansion parameters of that basis. The Faddeev method, on the other hand, expands the potential as a (nonterminating) series, each term of which acts only in a single nucleon-nucleon partial wave (e.g., 1S_0). Each "interacting pair" of particles in a given partial wave, when coupled to partial waves of the remaining (noninteracting) "spectator", leads to one or two three-nucleon partial waves, or "channels". The infinite series of potential terms must be truncated in performing a numerical calculation, but the angular momentum barrier suppresses the contribution of higher partial waves at low energies and in the bound state.

2.2. Two-body Force Results

Including all nucleon-nucleon partial waves with total angular momentum $j \leq 4$ (34 channels) typically produces a converged answer at the level of 10 keV in the bound state, with the $j=4$ waves producing only 20 keV out of 58 MeV for the RSC potential[4]. This result, first achieved recently by the Los Alamos-Iowa group, improves the previous standards of 18 channels ($j \leq 2$) and 5 channels (positive parity, $j \leq 1$). Adding 29 channels to the latter case for the RSC potential enhances the binding by 320 keV. Commonly used potential models, such as Reid Soft Core (RSC)[5], Argonne V_{14} (AV14)[6], and Super Soft Core (C) (SSCC)[7], produce binding energies of [7.35, 7.67, 7.53] MeV, respectively. These are underbound by approximately 1 MeV. However, it has been known for several decades[8] that if the tensor force is weakened, the tendency is to increase the binding. The reason is that the deuteron (with one $T=0$ pair) is even more sensitive to the tensor force than the triton (with $\frac{3}{2}$ $T=0$ pairs), where typically more than half the potential energy derives from that force. Weakening the tensor force requires an enhanced central force in order that the deuteron maintain the same binding energy. This drives up the triton binding energy because of the bigger deuteron lever arm with respect to the tensor force. Recent calculations[9-10] with unpublished versions of the Bonn potential[11], whose deuteron D-state probability is less than 5 percent, give 8.0-8.3 MeV binding. A major difficulty in pinning down the tensor force is the poor quality of some of the nucleon-nucleon data below 100 MeV, which allows considerable flexibility in the nucleon-nucleon forces.

Current experiments at Karlsruhe[12] and planned experiments at SIN are expected to help remedy this problem[13].

2.3. Three-Nucleon Forces

In addition to forces which depend only on the coordinates of two nucleons, there are other forces, three-nucleon forces, which depend in a nontrivial way on the simultaneous coordinates of the three nucleons[14]. Such forces are expected to be much weaker overall than the two-body potentials; order of magnitude estimates give 1 MeV potential energy (out of a total of 50 MeV). The archetype of three-body forces (TBF) in atomic, nuclear, and classical physics is the mutual "distortion" force caused by the change of shape of composite objects which interact with each other. The classical example is the earth's tidal force on a satellite, caused by the moon's attraction on the oceans. The atomic example is the long-range Axilrod-Teller force[15] from three mutually distorted atoms. The prototype long-range nuclear TBF arises from Δ -excitation of a nucleon by a pion emitted from a second nucleon, which pion is absorbed by a third nucleon after the virtual Δ decays. Many other mesonic processes also contribute. A major technical problem with these forces is that certain components depend on the way the nucleons in the nucleus are taken off-shell, which demands consistency between the two- and three-nucleon force models[16].

2.4. Three-Nucleon Force Results

Although it is trivial in principle to incorporate a TBF in bound state calculations, it is somewhat more complicated in practice, because the partial-wave series converges more slowly. Nevertheless, a number of calculations have been carried out at Los Alamos and elsewhere. Four TBF models have been used: Tucson-Melbourne (TM)[17], Brazilian (BR)[18], Urbana-Argonne (UA)[19], and Hajduk-Sauer (HS)[20]. The latter model has Δ -components in the wavefunction and only a single set of calculations exists. The TBF effect in this model is implicit; the net increase in binding is 0.3 MeV. Calculations with the other models[21] produce much more binding, in general, but have a disturbing sensitivity to the assumed short-range behavior, usually mediated by a pion-nucleon form factor. The range of this phenomenological cutoff is poorly known. The "standard" version increases the binding by roughly 1.5 MeV for the TM and BR models, and is insensitive to the two-body potential model used. However, if one assumes other plausible cutoffs[21] much more or much less binding accrual is found.

3. Observables

3.1 Scaling

Many combinations of two- and three-body force models have been solved at Los Alamos and yield a wide range of binding energies, E_B , most of which are not close to the physical triton energy. In those cases are the values or various calculated physical observables to be taken seriously? An example is the rms radius, which is very sensitive to the asymptotic portion of the wavefunction $[-\exp(-\kappa\rho)/\rho^{5/2}]$, where $\kappa = E_B^{1/2}$. If one naively assumes that this asymptotic form holds everywhere, the radius should scale as $E_B^{-1/2}$. This is indicated in Fig. 1, where the individual points result from calculations at Los Alamos[22] of the (point nucleon) rms trinucleon charge radii, plotted versus the corresponding binding energies for diverse models including a Coulomb force in ${}^3\text{He}$. The small spread of these "theoretical data" indicates scaling behavior with the binding energy, and the experimental data from a recent Saclay analysis[23] are in good agreement with the simple fits.

Fig. 1. Calculated trinucleon rms radii together with data.

The isoscalar, or mass, combination of radii does indeed vary as $E_B^{-1/2}$, while that combination sensitive to the ${}^3\text{He}$ - ${}^3\text{H}$ density difference scales more nearly as E_B^{-1} . The latter density component is largely determined by the overlap of S- and S'-states, and the latter state decreases rapidly with increased binding.

A second example of scaling is the ${}^3\text{He}$ Coulomb energy, E_C , calculated in perturbation theory with a dipole nucleon form factor. The scaling plot is depicted in Fig. 2. At the physical triton binding energy one finds $E_C = 652 \pm 2 \text{ keV}$, which is slightly higher than the value produced by the hy-

perspherical approximation, which works well[22], used in conjunction with experimental charge form factor data. This difference can be traced to the apparent inability of the impulse approximation to reproduce all features of the trinucleon charge form factors. Both results are significantly lower than the observed 764 keV binding energy difference. Clearly, in these two examples of scaling any individual calculated point is virtually useless. Only the set of calculations provides substantial insight.

Fig. 2. Calculated trinucleon Coulomb energies.

3.2 Charge densities and form factors.

Interest in the trinucleon form factors, $F_{ch}(q^2)$, has been augmented by the recent tritium experiment at Saclay[23] and the nearly completed one at Bates[24]. The data are plotted in Figs. 3 and 4 versus squared momentum transfer, together with impulse approximation calculations from Los Alamos for 34 channels[25], the RSC two-nucleon potential model, and with various (or no) TBF models. The data in the region of the diffraction minima are in obvious disagreement with the calculations. All of the TBF results are similar, and correspond roughly to the same binding energy.

The shift in the curves when a TBF is added can be understood in terms of a simple schematic model, if we assume that the primary change results from the binding energy difference. Defining $\lambda = (E_B/E_B^0)^{1/2}$, where E_B and E_B^0 are the binding energies with and without a TBF, the asymptotic wavefunction displayed earlier can be written in the form: $\psi(\lambda\vec{x}, \lambda\vec{y})\lambda^3$, where $\vec{x}$ and $\vec{y}$ are any combination of the two vectors necessary to specify a three-body system. Assuming that this form holds for the entire wavefunction changes the charge density $\rho_{ch}(r)$ to $\lambda^3\rho_{ch}(\lambda r)$, and the charge form factor, $F_{ch}(q)$, to $F_{ch}(q/\lambda)$. That is, the momentum transfer scale "stretches", preserving shapes and values of diffraction maxima, but moving positions of

Fig. 3. Calculated ${}^3\text{He}$ charge form factors together with data.

Fig. 4. Calculated ${}^3\text{H}$ charge form factors together with data.

minima outward. This model, which is only quantitative at quite low momentum transfers, accounts for the shift in the diffraction minima, but not the increase in the height of the diffraction maxima.

The latter effect is outside the province of the stretching model, and results from the strong angular dependence of three-body forces, which can depress the charge density near the origin, in addition to the simple contraction of the density predicted by the stretching model. The ^3He charge density corresponding to the Los Alamos-Iowa point-nucleon charge form factor shown earlier is depicted in Fig. 5, together with the quasi-experimental data[26] obtained by removing the nucleon charge form factors

Fig. 5. Calculated ^3He charge densities and quasi-experimental data.

from the experimental nuclear form factor in an approximate way. The dramatic "hole", which is not reproduced by the calculations, actually is "small" in the sense that only about one percent of the ^3He charge would fill it. The addition of a TBF does depress the central density somewhat, relative to the surface density, but the effect is too small. A similar change is observed in ^3H .

If the impulse approximation appears to fail at large momentum transfers, what mechanism would alleviate the problems? Figures 3 and 4 show that any addition to F_{ch} of a small negative component which vanishes at $q^2 = 0$ and large momentum transfers would tend to solve all of the problems. Meson exchange currents have this shape property, and have occasionally been proposed as a solution. There is no question that they help, if they are introduced in certain approximations. The difficulty is

that the pion-exchange part of the charge operator is a relativistic correction^[27], is model dependent, and is also formalism dependent. The latter difficulty can lead to ambiguous matrix elements unless one also calculates the wave functions for those matrix elements in the same formalism (i.e., including relativistic corrections). This is not possible within the class of common nonrelativistic potential models, and at this stage calculations of exchange currents to ρ_{ch} must be regarded as *ad hoc*.

3.3 N-d scattering lengths.

The nucleon-deuteron system has two spin configurations, doublet and quartet. The latter configuration is primarily sensitive to the deuteron binding energy and is therefore not particularly interesting. The former has long been known to show scaling behavior for the n-d case (i.e., the Phillips line^[28]). Recently, the Los Alamos-Iowa collaboration^[29] showed that the p-d doublet scattering lengths behave similarly (Fig. 6), although the shape of the p-d and n-d curves is clearly different. Complicating the situation is a controversy over the definition of the p-d scattering

Fig. 6. Calculated Nd doublet scattering lengths together with data.

length, and a gross discrepancy between theory and experiment in that case. The theoretical problem has been resolved^[30], but the problem with experiment remains. The n-d datum is in excellent agreement with the fit to the various calculations with and without three-body forces.

4. Conclusions.

The Los Alamos-Iowa group has recently extended the bound state Faddeev calculations to 34 channels, or $j \leq 4$. Most two-nucleon models have

insufficient binding, although commonly used three-nucleon forces can substantially increase this. Regrettably the latter forces are extremely sensitive to the assumed short-range behavior, which makes the binding accrual problematical. Many trinucleon observables display scaling behavior with the corresponding binding energy, and fits to existing calculations may provide the best theoretical estimates. The rms radii are in good agreement with recent experiments, while the charge form factors in impulse approximation are not. The extrapolated n-d doublet scattering length agrees well with experiment, while the p-d case does not.

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