THE COST OF TRITIUM BRED
IN A CRITICAL FISSION REACTOR

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ABSTRACT

Elementary equations related to the "six-factor" formula for criticality are used to derive an expression for the cost of tritium, bred in a fission reactor, in terms of the cost of fissile material. Reasonable parameters give a value about two-thirds of that usually quoted (80 times the price of fissile) for fusion-fission reactor self-bred tritium. However, the handling costs and hazards associated with large-scale LWR tritium production are not assessed.

THE COST OF TRITIUM BRED IN A CRITICAL FISSION REACTOR

I. Introduction

An equation for the cost of tritium bred in a fission reactor can be derived from consideration of criticality. Elementary formulas (which are reasonably accurate when empirical constants are used) keep the derivation simple and qualitatively obvious.

The six-factor formula for the multiplication constant of a fission reactor is $k = \eta \epsilon p f P_f P_{th}$. (See any introductory reactor theory textbook, e.g. J. Duderstadt and L. Hamilton, Nuclear Reactor Analysis.) The non-leakage probability, $P = P_f P_{th}$, depends on the shape and size of the reactor. The resonance escape probability, p , depends on the fertile material and the degree of fuel "lumping". The fast fission factor, ϵ , depends on the reactor spectrum and the fertile material. The remaining two factors are neutrons per absorption, η , and the thermal utilization, f . These can be rewritten as $\eta f = \nu F / (1 + \alpha)$, where the neutrons per fission, ν , and the capture-to-fission ratio, α , are properties of the fissile material and the spectrum. F is the probability that a thermal neutron will be absorbed by the fissile material:

letting $h = \frac{\Sigma_{abs}^{fissile}}{\Sigma_{abs}^{fuel}}$, $F = hf$ and $\eta = \nu v / (1 + \alpha)$.

Only the enrichment affects h ; F depends on the enrichment and the degree of poisoning.

The addition of lithium to a fission reactor lattice to breed tritium will decrease k by decreasing F ; however, the reactor must be maintained critical so a compensating change is required. P_f and P_{th} are not candidates for the compensation since increasing the size would increase the power; ν , α , and ϵ are not candidates since they are intrinsic properties of the reactor type. The two factors which are available to restore k are the resonance escape probability p and the fissile utilization F .

II. Compensation with the resonance escape probability.

Compensation with p is easy to analyze and will be dealt with first. Let Σ_a^f = absorption in fissile material, Σ_a^{all} = absorption in all materials (present initially), Σ_a^L = absorption in lithium (producing tritium). Then

$$F = \Sigma_a^f / \Sigma_a^{all} ; F' = \Sigma_a^f / (\Sigma_a^{all} + \Sigma_a^L) ; \text{ so } F/F' = 1 + \Sigma_a^L / \Sigma_a^{all} .$$

Now require $p' = p F/F'$ so that $p'F' = pF$ and $k'=k$. This causes a decrease in the fissile conversion ratio C (the number of fissile atoms bred per fissile absorption). Based on the same physical model used to derive the six-factor formula,

$$C = \left[\nu \epsilon P_f (1-p) + \nu \epsilon P_f p P_{th}^f (1-h) \right] / \nu \epsilon P_f p P_{th}^{fh} \text{ or}$$

$$C = (1-p)/(pFP_{th}) + (1-h)/h. \text{ Since } h \text{ is not changed and } (pF) \text{ is}$$

held constant, $(C-C') = (p'-p)/(pFP_{th}) = p(F/F'-1)/(pFP_{th}) =$
 $(\Sigma_a^L / \Sigma_a^{all})/(FP_{th}) = (\Sigma_a^L / \Sigma_a^f)/P_{th} \approx \Sigma_a^L / \Sigma_a^f$ (since thermal non-
leakage $P_{th} \approx 1.$) .

Thus the effect of increasing $\hat{p}$ is to decrease the reactor internal fissile conversion ratio. This either will require that extra makeup fuel be purchased or else will reduce the spent fuel credit by an equal value. The value of this fuel (plus assorted handling charges which will be ignored) is assigned as the cost of the bred tritium. Since ΔC is the loss per fissile absorption, a factor $(\Sigma_a^f / \Sigma_a^L)$ is needed to normalize to the tritium breeding rate; furthermore a factor of $(239/3)$ is required to adjust the values per atom (or per mole) derived above to specific (per unit mass) values. Thus letting H^T = specific cost of tritium and H^f = specific price of fissile material; $H^T = H^f * (239/3) * (\Sigma_a^f / \Sigma_a^L) * (\Sigma_a^L / \Sigma_a^f) / P_{th}$ or $H^T \approx H^f * (239/3)$. This reflects a simple mole-for-mole tradeoff between bred tritium and bred fissile.

III. Compensation with the fissile enrichment.

The simplest method of compensating for a poison is to increase the density of the fissile material. The effects of this include changes in: R^f , the fission rate; R^F , the fissile breeding rate; and R^L , the lithium absorption rate. Several useful ratios of these will be evaluated in terms of the cross sections given previously plus Σ_a^n , the absorption in nonfissile

materials; plus N^L , N_O^f and N^f , the lithium density and the initial and adjusted fissile densities.

Initially $F = N_O^f \sigma_a^f / (N_O^f \sigma_a^f + \Sigma_a^n)$, which must equal the final $F = N^f \sigma_a^f / (N^f \sigma_a^f + N^L \sigma_a^L + \Sigma_a^n)$. Rewriting each of these in the same manner shows $N_O^f \sigma_a^f = \Sigma_a^n F / (1-F)$ while $N^f \sigma_a^f = (\Sigma_a^n + N^L \sigma_a^L) F / (1-F)$. Define $g \equiv (1-F)/F$ and subtract to get $N^f \sigma_a^f - N_O^f \sigma_a^f = N^L \sigma_a^L / g$ so that the change in fissile density required to offset N^L is

$$\Delta N^f = (N^L \sigma_a^L) / (g \sigma_a^f) = (N_O^f / g) (N^L \sigma_a^L) / (N_O^f \sigma_a^f) = N_O^f \lambda / g,$$

where $\lambda \equiv (N^L \sigma_a^L) / (N_O^f \sigma_a^f)$.

A cost is incurred due to this increased enrichment, but it must be allocated to the tritium actually produced. This is done by adjusting for the relative tritium-to-fission rate: $(\Delta N^f / R^L) = (R^f / R^L) * (\Delta N^f / R^f) = (R^f / R^L) (N_O^f / R^f) (\lambda / g)$. Note that (N_O^f / R^f) is inversely proportional to the initial specific power density: $P = (R^f * \text{Fission Energy}) / (N_O^f * \text{At. Wt.} / \text{Avogadro's No.})$ or $P = (R^f * E_f) / (N_O^f * M^f / N^A)$.

The factor (λ / g) , defined in terms of parameters which might not be available, can be eliminated as follows. Consider the equation $(R^f / R^L) \equiv (N^f \sigma_f^f) / (N^L \sigma_a^L) = [N_O^f (1 + \lambda / g) \sigma_f^f] / (N^L \sigma_a^L) = [N_O^f (1 + \lambda / g) * (\sigma_a^f / (1 + \alpha))] / (N^L \sigma_a^L) = (1 + \lambda / g) / [\lambda (1 + \alpha)]$, where α is the capture-to-fission ratio, $(1 + \alpha) = \Sigma_{\text{abs}}^f / \Sigma_{\text{fis}}^f$. Rearranging, $(1 + \lambda / g) * (R^L / R^f) = g(1 + \alpha) \lambda / g = g(1 + \alpha)(1 + \lambda / g) - g(1 + \alpha)$, which is

solved for $(\lambda/g) = (R^L/R^f)/g(1+\alpha)-(R^L/R^f)$. Therefore

$$\Delta N^f/R^L = (R^f/R^L) \left\{ (R^L/R^f) / \left[g(1+\alpha)-(R^L/R^f) \right] \right\} \left[(N^A E_f)/(PM^f) \right] = \left[(N^A E_f)/(PM^f) \right] / \left[g(1+\alpha)-(R^L/R^f) \right] .$$

Assuming that tritium is to be bred, in some number of fission reactors, to be used as makeup fuel for DT burning fusion reactors with an average net tritium breeding ratio T less than T_0 (the required value ≈ 1), the parameter R^L/R^f can be converted to an expression relating the reactor performance characteristics. Given S = ratio of fission reactor thermal power to fusion reactor thermal power, and Q = fusion energy multiplication:

$$R^L/R^f = (T_0 - T) \left[(1/Q)(1/E_{fus}) \right] / \left[S/E_{fis} \right] = 11.38 (T_0 - T)/(SQ) .$$

In addition to increased enrichment, there will be a reduction in the fissile conversion ratio. As before, $C = (1-p)/pF + (1-h)/h$, but now with p held constant and F restored,

$$C_0 - C = (1-h_0)/h_0 - (1-h)/h = 1/h_0 - 1/h . \text{ Using the definition of } h,$$

$$C_0 - C = 1/h_0 - (\Sigma_{ao}^{fuel} + \sigma_a^f \Delta N^f) / (\Sigma_{ao}^f + \sigma_a^f \Delta N^f) ,$$

and after substituting for ΔN^f and dividing by Σ_{ao}^f ,

$$C_0 - C = 1/h_0 - (g/h_0 + \Sigma_a^L / \Sigma_{ao}^f) / (g + \Sigma_a^L / \Sigma_{ao}^f) .$$

Combining terms and multiplying through by the cross section ratio,

$$\begin{aligned} C_0 - C &= (1/h_0 - 1) / (1 + g \Sigma_{ao}^f / \Sigma_a^L) = (1/h_0 - 1) / (1 + g \Sigma_a^f / \Sigma_a^L - g \sigma_a^f \Delta N / \Sigma_a) \\ &= (1/h_0 - 1) / (g \Sigma_a^f / \Sigma_a^L) . \end{aligned}$$

Since the conversion ratio is in terms of breeding per fissile atom absorbed, finding the amount of lost fissile per triton bred requires multiplying by the ratio of the fissile absorption rate to lithium absorption rate, yielding:

$$(C_O - C) R_a^f / R_a^L = (C_O - C) \Sigma_a^f / \Sigma_a^L = (1/h_O - 1)/g = (1 - h_O)/(h_O g) = (f_O - F)/(1 - F) .$$

The specific cost of tritium (e.g. \$/g) can now be calculated in terms of the specific cost of fissile material, several reactor parameters, the relative breeding rate, and the capital charge rate (I) on the increased inventory:

$$H^T = (239/3) * [H^f (C_O - C) + I * H^f (\Delta N^f / R^L)] .$$

$$\text{Thus } (H^T / H^f) (3/239) =$$

$$(f_O - F)/(1 - F) + (I/P) \left[N^A E_f / M^f \right] / \left[g(1 + \alpha) - (T_O - T) E_{fis} / (SQ E_{fus}) \right] =$$

$$(f_O - F)/(1 - F) + (I/P) \left[0.02558 \left(\frac{MW}{kg} \right) / \left(\frac{\%}{y} \right) \right] / \left[g(1 + \alpha) - 11.38 (T_O - T) / (SQ) \right] .$$

The reactor parameters can be replaced by "typical" values for an LWR;

$$v = 2.87 \quad \alpha = .354 \quad \eta = 1.65 \quad f_O = .710$$

$$p = 0.87 \quad F = .553 \quad g = .810 \quad h = .778$$

$$P_f = .97 \quad P_{th} = .99 \quad \epsilon = 1.02 \quad P = 1.454 :$$

$$\text{so } (H^T / H^f) (3/239) = 0.352 + 0.01759 I / \left[(1.096 - 11.38(T_O - T) / (SQ)) \right] .$$

Taking now $T_O = 1$, $T = 0$, $Q = 8$, and $S = 5$, representing the symbiosis of LWR's with a maximum fissile fuel producing hybrid, and $I = 15\%$, $H^T / H^f = (239/3) * (0.352 + .264/.812) = 0.677 * (239/3)$. This is about two-thirds the cost which was calculated for preserving

k by adjusting the resonance escape probability. However, the various reactor performance factors could change this greatly, as Figures 1 and 2 show.

IV. Tritium breeding in a hybrid reactor.

Perhaps surprisingly, the same equations for the cost of breeding tritium apply to fusion-fission hybrid reactors. Consider first the minimum-power hybrids, containing fertile material. Source and secondary neutrons which escape capture in the fertile material will thermalize and be captured by lithium to produce tritium. (Neglecting the parasitic absorption which would be minimized in this type of reactor by high lithium-6 concentrations.) Thus variation of a parameter analogous to the resonance escape probability will effect a mole-for-mole tradeoff on the breeding of fissile material and tritium. In this case $(H^T/H^F)(3/239) = 1.0$.

For a hybrid in which fissile enrichment is used to produce more than the minimum amount of power the situation is very similar to that of the critical fission reactor. The extra power is produced by using the source and secondary neutrons which escape the fertile region to drive a sub-critical fission lattice. If lithium is added to this type of blanket the power output will fall unless: a) the leakage from the fertile region is increased, to raise the effective lattice source; or b) the multiplication factor is restored. The former course will lead, as before, to the mole-for-mole tradeoff. The restoration of k, however, will give very much the same cost-of-tritium equations as were derived for the critical fission reactor, since that derivation was independent of the magnitude of k.

The difference will be in the interpretation of the factor R^L/R^f , which is now the ratio of tritium production to fission just in the hybrid. Scaling to the fusion reaction rate; R^L equals T , the tritium breeding ratio. R^f divided by the fusion rate can be written in terms of the energy multiplication:

$(R^f + s)E_{fis} = (Q-1)E_{fus}$, where s gives the number of fertile fast fissions induced per fusion neutron.

Therefore $R^L/R^f = T / [(Q-1)(E_{fus}/E_{fis}) - s] = 11.38T / (Q-1-11.38s)$,

and for the hybrid, $(H^T/H^f) * (3/239) =$

$(f_o - F) / (1 - F) + (I/P)(0.02558) / [g(1 + \alpha) - 11.38T / (Q-1-11.38s)]$.

There is a much greater range of possible values of f and F for a hybrid blanket, due to the relaxation of the criticality requirement. Note, however, that the term $(f_o - F) / (1 - F)$ will always be less than one since $F \leq f_o < 1$. In fact, for a hybrid blanket composed of a fertile layer backed by a lithium and fissile region, $h = 1$ and $f = F$, so the first term is zero.

The second term will be small if Q is sufficiently large. For $F = 0.75$ and $\alpha = 0.50$ (appropriate to a blanket of lithium - beryllium - plutonium - fluoride salt) $g(1 + \alpha) = 0.50$. Assuming $s = 0.4$ and $Q = 19$ (corresponding to a blanket $M = 23.5$) it appears that 0.59 is an upper limit for T . This implies that in such a hybrid it would not be possible to preserve a constant k merely by increasing the enrichment while T was raised from 0 to 1; the required R^L/R^f appears to be too large.

used this example corresponded to a blanket in which T was already 1.09. In such a case it is more appropriate to evaluate the cost of incremental tritium, normalizing R^L to the increment. This yields the same equation but with ΔT in place of T. Assuming $s = 0.4$, $Q = 19$, $P = 1.55$, and $I = 15$, the cost of incremental tritium in this blanket would be $(H^T/H^f)(3/239) = 0.2475/(0.50 - .84\Delta T)$, which would be 0.55 for raising T from 1.09 to 1.15.

The full effect of requiring $T = 1$ from a blanket which initially produces no tritium cannot be assessed from these equations. This is due to the fact that the hybrid does not require a critical blanket. Therefore the constraint that $k = \text{constant}$ which was the basis of the whole fission reactor analysis is not applicable.

V. Discussion.

The conclusion to be drawn from all these calculations is that purely economic analyses are not a sufficient basis for deciding whether tritium for a hybrid should be self-bred or purchased. Without a clear and substantial economic advantage, the practical problem of actually producing tritium in an LWR without degrading its power-producing capacity becomes important; especially if a symbiotic hybrid is justified to prolong the lifetime of current LWR technology.

In addition there are the safety problems. Batch processing of tritium could increase the inventory of volatile radionuclides in an LWR by an order of magnitude, a clearly undesirable condition. Furthermore, the bred tritium would have to be shipped from the

producer to the consumer, introducing a risk of transportation accidents.

Unfortunately, these issues are very difficult to quantify. Since the formulas developed above represent only the direct costs of LWR-produced tritium without the intangibles, and since they do not show a significant economic advantage, it would seem best to assume that the tritium fuel for fusion reactors should be internally produced.

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Fig. 1. The main part of this figure is a contour plot of H , the molar price of tritium divided by the molar price of fissile material. $H = (H^T/H^f) \cdot (3/239)$ as defined in the text. The auxiliary plots are for convenience in relating the parameters of H to the contour region. Fixed values are assumed for $\alpha = 0.354$ and $f = 0.71$ and $F = 0.55$.

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Fig. 2. Additional contour plate of H as in Figure 1, but in $F = 0.50$ and $F = 0.60$.