LIMITATIONS OF HEAVY ION SYNCHROTRON ACCELERATION FOR INERTIAL FUSION

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SUMMARY

The potential benefits from heavy ion inertial fusion motivate the rapid development of a program to test the principle. To define the program, accelerator parameters which have not hitherto been commonly considered must be studied interactively with basic questions of space charge limitations and charge exchange. Beam lifetime and power output efficiency may ultimately lead to a linear accelerator as the choice for an ignition device. For proof of principle, however, at power levels way beyond present inertial fusion experience, synchrotrons may have applicability at lower cost. The power and energy which can be delivered by the accelerating system to the reaction chamber are limited by space charge defocussing and intra beam charge exchange scattering, both of which are beam density dependent. These put constraints on linac injector energy, synchrotron aperture, synchrotron magnetic rigidity, acceleration time, ion species and charge to mass ratio.

The accelerator system considered is classical. A linear accelerator injects into a synchrotron which accelerates the ion beam to the full energy delivered to the target. The maximum energy deliverable by a synchrotron is treated in section I. The targetting parameters and the energy gained through synchrotron acceleration completely determine the synchrotron aperture. These are discussed in sections II and III. The ion range in material is treated in section IV. The problem of intrabeam scattering is considered in section V. Finally, in section VI is a discussion of examples to meet specified goals.

I. Space Charge Limited Energy Deliverable by a Synchrotron

The space charge defocusing is computed for a beam confined by the focusing channel to an annular volume of cross sectional radius r . We assume for simplicity that the charge distribution within this radius is uniform. The equation of motion for a particle a distance r from the center is

$$\ddot{r} = \left[\frac{2Ze^2n}{pc^2\gamma^3} - \frac{1}{\partial_g^2} \right] r \quad (1)$$

The second term in the bracket is the unperturbed local betatron oscillation function which describes the weak beam focusing. The first term describes the modifying effect of the beam self-interaction. The ion charge is Ze , the linear particle density is n , the ion momentum is p , the particle velocity in units of the velocity of light is β , the Lorentz contraction factor is γ . The distortion of the betatron oscillation function caused by the self-interaction of the beam can be expressed as a change in the number of betatron wavelengths per revolution,

$$\Delta Q = \frac{1}{2\pi} Z^2 e^2 N_{st} \frac{1}{\gamma^3} \frac{1}{\partial_g^2} \frac{1}{\beta} B_F \quad (2)$$

Here N_{st} is the total number of stored particles, m is the ion mass, ϵ the transverse beam emittance (assumed to be the same horizontally and vertically) and B_F is the longitudinal bunching factor.

*Work performed under the auspices of the U.S. Energy Research and Development Administration.

In order to avoid resonant oscillations of the beam, the change in the number of betatron wavelengths should be limited to $\Delta Q \leq 0.25$. ΔQ is largest at injection so it is at injection that the number of stored particles will be limited because of space charge defocusing.

The emittance at injection must be found if the limitation to the number of stored particles is to be computed. The emittance is determined by the beam size at the pellet target. The emittance at a target of radius a is

$$\epsilon_{tgt} \leq a \frac{R_q}{f} \quad (3)$$

f is the focal length of the last focusing element and is comparable to the target chamber radius. R_q is the lens half aperture. We choose $a = 0.1$ cm, $R_q = 39$ cm and $f = 10$ m. The emittance at injection is computed assuming that the beam size is damped adiabatically in the synchrotron.

$$\epsilon_{inj} \leq a \frac{R_q}{f} \frac{(\beta^-)_{tgt}}{(\beta^-)_{inj}} \quad (4)$$

The subscripts tgt and inj refer to quantities at the target and at injection. The number of stored particles becomes

$$N_{st} \leq 2\pi \frac{1}{\epsilon_p} \Delta Q B_F \frac{a R_q}{f} \frac{\beta^-}{2\pi} (\beta^-)_{tgt} (\beta^-)_{inj} \quad (5)$$

ϵ_p is the classical proton radius. This is a complete parameterization of the maximum number of stored particles in terms of the targetting energy, the injection energy, the atomic number, the ion charge, the dimensions of the final transport system, the reaction chamber radius and pellet size.

The energy delivered to the target is expressed in terms of the number of particles available and the kinetic energy, T , of each particle.

$$W = N_{st} T_{tgt} \quad (6)$$

The upper limit to the energy deliverable to the target in terms of the accelerator parameters is

$$W \leq \sqrt{T} \Delta Q \frac{1}{m_p} \frac{1}{\beta^-} \frac{1}{r_p} \frac{a R_q}{f} \left(\frac{Z}{A} \right)^{3/2} \left(\frac{e}{c} \text{HR} \right)^{1/2} eV_{inj} \quad (7)$$

$$\cdot \left(\frac{\sqrt{2} \gamma_{inj} \sqrt{1 - \gamma_{inj}}}{1 + \gamma_{tgt}} \right)$$

or

$$W \leq 0.680 \frac{a R_q}{f} \left(\frac{Z}{A} \right)^{3/2} \left(\frac{e}{c} \text{HR} \right)^{1/2} eV_{inj} \quad (8)$$

$$\cdot \left(\frac{\sqrt{2} \gamma_{inj} \sqrt{1 - \gamma_{inj}}}{1 + \gamma_{tgt}} \right)$$

We adopt the notation of synchrotron magnetic rigidity (e/c HR), which is the ion momentum divided by ion charge and of linac potential energy (eV), which is the ion energy divided by ion charge. Units of ion energy,

ion momentum and distance are 30V, 30V/c and cm. The maximum deliverable energy is in units of megajoules. The relativistic factors in the brackets are close to unity for all examples to be considered.

The solid curves of Figures 1, 2 and 3 are computed from Equation (3) for several upper limits of deliverable energy. Their practical range of application depends upon the synchrotron aperture.

II. Synchrotron Aperture

The beam size is largest at the time ions are injected into the synchrotron.

$$\epsilon_s = \epsilon_{inj} \quad (9)$$

The emittance at injection is related to the target parameters by equation (4). The maximum value of the synchrotron beta function is assumed to be 25 meters. Approximating Equation (4) by its nonrelativistic form and with the target parameters as before,

$$\epsilon_{inj} = 2.7 \sqrt{\frac{T_{inj}}{r_{tgt}}} = 2.7 \sqrt{\frac{V}{r_{tgt}}} \quad (10)$$

We arbitrarily decide to keep the half aperture of the beam less than 7.1 centimeters which implies that the ratio between the injection and target energies is

$$\alpha = \frac{T_{inj}}{T_{tgt}} = 48 \quad (11)$$

III. Kinematic Constraints

The kinematics of the heavy ion motion completely determines the size of the synchrotron,

$$T_{tgt} = \alpha T_{inj} \quad (12)$$

$$1 < \alpha < 48 \quad (13)$$

The energy momentum relation is

$$\alpha T_{inj} = \frac{p_{tgt}^2}{2m_p A} \left[\frac{2}{1 + \gamma_{tgt}} \right] \quad (14)$$

The injection voltage is expressible in terms of the magnetic rigidity of the synchrotron at its maximum energy,

$$\alpha V_{inj} = \frac{1}{\alpha} \frac{1}{2m_p} \frac{Z}{A} \left(\frac{e}{c} HR \right)_{tgt}^2 \left[\frac{2}{1 + \gamma_{tgt}} \right] \quad (15)$$

This kinematic relation is shown in figures 1, 2 and 3 as the dashed curves. It is shown as the maximum synchrotron rigidity versus injector voltage for four ratios of the initial and the final kinematic energies. The curve labeled $\alpha = 1$ is the situation in which there is no acceleration in the synchrotron and is the case which applies to an accumulator. The other extreme is the case $\alpha = 48$, for which the kinetic energy is increased by a factor of 48 through the synchrotron acceleration. A factor of 48 appears to be a large but reasonable one to achieve in some cases.

Superimposed on the same curves are the maximum deliverable energy which was computed through Equation (8).

IV. Particle Range

The ion beam is to deliver its entire energy to the pellet and must therefore come to rest in the pellet. If this is to be done through ionization energy loss, the energy of a single ion is limited. The ion range should be less than two grams per square centimeter. If the ion is uranium, the ion energy should not exceed 100 GeV. If a lighter ion like iodine were used, the maximum energy would be 30 GeV. The limits due to ion range are shown in figures 1, 2 and 3 for uranium.

V. Intra Beam Scattering

The intra beam ion collisions sometimes result in ion-ion charge transfer. After such a collision, the ion is lost from the synchrotron. The rate of ion collision is

$$\frac{dN}{dt} = \frac{1}{\tau} N_{st} \gamma \quad (16)$$

N_{st} is the total number of particles stored, ρ is the ion density, γ is the relative ion velocity, τ is the ion exchange cross section. The relative velocity between two ions with momenta $p + \Delta p_1$ and $p + \Delta p_2$ is

$$\frac{1}{4} \frac{\gamma^2}{c^2} = \frac{1}{4} \gamma^2 \left[1 + \left| \frac{p_1}{p} - \frac{p_2}{p} \right| \right] \sin^2 \frac{\pi}{2} + \frac{\pi^2}{4} \left(\frac{p_1}{p} - \frac{p_2}{p} \right)^2 \quad (19)$$

The angle between the ion trajectories is π . The density is

$$\rho = \frac{N_{st}}{2\pi R} \frac{1}{B_f} \quad (20)$$

The beam oscillation amplitude and angle are approximated by

$$\epsilon = \sqrt{\epsilon_s}, \quad \sigma = \sqrt{\epsilon_s} \quad (21)$$

In the limit $\alpha/p \ll 6 \ll 1$, the charge exchange rate then becomes

$$\frac{dN}{dt} = \frac{1}{2\pi R} \frac{1}{B_f} N_{st} \gamma \frac{Z}{A} \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\epsilon_s}} \quad (22)$$

Substituting

$$R = mcZ\gamma/Z_e H$$

and combining equation (22) and equation (6)

$$\frac{1}{N} \frac{dN}{dt} = \frac{1}{2\pi^2} \frac{1}{B_f} \frac{e}{m_p c} \frac{1}{A} H \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\epsilon_s}} \quad (23)$$

With the synchrotron parameters as before,

$$\frac{1}{N} \frac{dN}{dt} \leq 37 \frac{W}{I} \frac{1}{\sqrt{2}} \frac{1}{Z} \frac{1}{10^{-12}} H_{(Tesla)} \quad (24)$$

The expression applies at the peak synchrotron energy where the collision rate is close to its maximum value. Note that the ion interchange rate is half the particle loss rate since two ions are involved per collision.

The charge exchange cross sections can be no greater than $\sim 10^{-12} \text{ cm}^2$, the geometric cross section. Estimates have been made⁴ for the charge interchange cross sections and they may be as small as 10^{-12} cm^2 . Precise calculations are difficult. The examples which follow will illustrate that good measurements are needed.

VI. Examples and Discussion

The cost of constructing a synchrotron can be significantly less than that of a linear accelerator particularly if the energy gain in the synchrotron is large. We therefore examine the extent to which synchrotron acceleration can be used at two different power levels: first for a power plant and second for a test of principle.

Stored energy between 3 and 10 megajoules is required to detonate a pellet used in a power plant.⁵ A synchrotron capable of delivering 10 megajoules per pulse could operate well below the space charge limit with a final energy up to 48 times the linac energy. For example, a 2 GeV linac injecting into a synchrotron which accelerates a singly charged uranium ion beam to

traction only, consider injection from a linac of 0.3 to 10 volts into a synchrotron or a fixed field power and energy experiments in the 10 to 10¹⁰ eV range relevant to pellets, reactors and accelerators. The linac voltage would still be expanded in parallel. (The synchrotron might ultimately be used as an experimental accelerator ring.) Whether this optimally advances the program or is a diversion is still not clearly defined.

An example of an accelerator designed for a test of principle would be one which delivered 100 kilojoules. In that case, something like a uranium ion in a charge state 10 might be used. (See Fig. 2.) A linac injector of 200 MeV would inject into a synchrotron of 23 GeV/c magnetic rigidity. The ion charge interchange rate $1/N \, dN/dt = 0.7\%$ per second. To arrive at this collision rate, $H_{ave} = 0.5 \, T$ and $\tau = 10^{-25} \, cm^2$ as before. With such a device, a thermonuclear burn could be initiated at a substantially lower capital cost than a laser beam fusion trigger of comparable deliverable energy. The cycle time of a synchrotron is seconds compared to laser cycle times of hours.

The usefulness of synchrotron acceleration for a fusion device depends heavily upon finding an ion with a charge interchange cross section much smaller than the geometric cross section. This is probably not the case for singly charged uranium but a closed shell ion is likely to be satisfactory. It is important that cross section measurements be available when an accelerator design is chosen. Note that the $z = 1$ case, the accumulator ring, does not have the charge interchange lifetime problem.

Synchrotrons are possibilities for ion fusion development. R.F. linacs and accumulators may offer advantages for a fully developed energy generation complex, considering R.F. power requirements, overall energy efficiency, etc. As an example, for illus-

Fig. 1. Maximum synchrotron magnetic rigidity vs injector voltage for uranium $Z = +1$. (—) Space-charge limit of energy deliverable to target. (----) Kinematic relation between magnetic rigidity and injector voltage.

Fig. 2. Maximum synchrotron magnetic field, vs. injector voltage for uranium $Z = +10$.

Fig. 3. Maximum synchrotron magnetic rigidity vs. injector voltage for uranium $Z = +100$ or iodine $Z = +53$ (approximate to within relativistic corrections).

References

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