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**A Bayesian Zero-Failure (BAZE) Reliability
Demonstration Testing Procedure and Its
Application to a Rankine Dynamic Radioisotope
Power Conversion System**

by

H. F. Martz, Jr.
R. A. Waller

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NOTATION

a	- prior shape parameter
b	- prior scale parameter
BAZE	- <u>B</u> ayesian <u>Z</u> ero Failure
BIPS	- Bratton Isotope Power System
δ_0	- specified cycle failure-rate (failures/cycle)
ERDA	- Energy Research and Development Administration
$f(\lambda)$	- prior probability density function of the failure-rate λ
$f(\lambda 0 \text{ failures})$	- posterior probability density function of the failure-rate λ conditional on 0 observed failures in a test of nt unit-hours
FMEA	- Failure Mode and Effects Analysis
γ	- posterior assurance
$(1-\gamma)$	- posterior risk
$I(a, x)$	- incomplete gamma function
	- $\int_0^x y^{a-1} e^{-y} dy$
k	- discrimination ratio (λ_1/λ_0)
KIPS	- Kilowatt Isotope Power System
λ	- failure-rate
λ_0	- specified failure-rate
λ_1	- criterion failure-rate ($k\lambda_0$)
λ^*	- generic failure-rate
m	- required component duty cycle
MFR	- Median Failure Rate
MMD	- Mean Mission Duration
n	- number of test units
n_0	- required number of test units
N	- number of test cycles
$(nN)_0$	- required unit-cycles of testing
nt	- unit-hours of testing

$(nt)_0$	- required unit-hours of testing
$P(\cdot)$	- Probability
$P(x \text{ failures} \lambda)$	- conditional probability of x failures in a test of nt unit-hours with an underlying failure-rate of λ for each item
$P(0 \text{ failures})$	- unconditional probability of passing the BAZE test
$P(0 \text{ failures} \lambda_* \leq \lambda \leq \lambda^*)$	- conditional probability of passing the BAZE test
$P(x \text{ failures})$	- unconditional probability of x failures in a test of nt unit-hours
$P(\lambda_* \leq \lambda \leq \lambda^* x \text{ failures})$	- conditional probability that the failure-rate is contained in the interval $[\lambda_*, \lambda^*]$ given x failures in a test of nt unit-hours
PCS	- Power Conversion System
POC	- Posterior Operating Characteristic
RTG	- Radioisotope Thermoelectric Generator
SFR	- Specified Failure Rate
SNS	- Space Nuclear Systems
γ_y	- $100(y)$ th percentile of the standard gamma distribution
t_0	- required test duration
t	- test duration
X	- failure-time random variable

A BAYESIAN ZERO-FAILURE (BAZE) RELIABILITY
DEMONSTRATION TESTING PROCEDURE AND ITS APPLICATION
TO A RANKINE DYNAMIC RADIOISOTOPE POWER CONVERSION SYSTEM

by

H. F. Martz, Jr. and R. A. Waller

ABSTRACT

A Bayesian Zero-Failure (BAZE) reliability demonstration testing procedure is developed. The procedure may be used to verify component failure rates associated with both real-time dependent and cycle-dependent chance failure mechanisms. A constant failure-rate model with a gamma prior distribution is assumed. The procedure is used to obtain sample test plans for components of a proposed Rankine power conversion system.

I. INTRODUCTION AND PURPOSE

Most government and military contracts for hardware development include a numerical reliability requirement in the specifications. For example, consider the Space Nuclear division, ERDA, proposal solicitation No. SNS-RFP-75-2-CAO, November 4, 1974, regarding the Kilowatt Isotope Power System (KIPS) Ground Demonstration Unit. This is a Phase I project within the overall proposed KIPS program. The system is to meet a lifetime requirement of 7 years and a mean mission duration (MMD) of 5 years. To satisfy that specification, the system must have a failure-rate λ of no more than 11.64×10^{-6} failures/hr. In addition, most contracts require quantitative assurance that the minimum acceptable reliability requirement, e.g., the system failure-rate above, has been

achieved. For example, the document SNS-2 entitled "Reliability Engineering Program Requirements Publication," Space Nuclear Systems, ERDA, states in Section 5.2 that "the contractor shall provide the test requirements needed to reasonably assure SNS of demonstration of the minimum acceptable reliability." A typical reliability demonstration statement is that a system failure-rate requirement of X failures/hr. be demonstrated with Y% confidence.

Future dynamic radioisotope power conversion systems will likely require stringent reliability specifications because the missions are of lengthy duration. In addition, such systems are necessarily more complex than present static radioisotope thermoelectric generator (RTG) systems because of higher power output requirements. In order to quantitatively demonstrate system conformance to such high reliability requirements, pertinent resource-effective and statistically sound test plans must be developed. An early consideration of such plans has numerous benefits. First, the early availability of these plans will permit adequate planning of scarce resources such as test facilities, test budget, personnel, etc. Second, the early availability of such plans will assist in determining the number of components to be produced for testing purposes. Third, and perhaps the most important benefit of early demonstration test planning is that the required test duration is determined early in the program. This is a valuable aid in developing an efficient program plan which best utilizes available resources. Another relevant consideration when testing high-reliability systems, such as the KIPS and Bratton Isotope Power System (BIPS), is as follows: For high-reliability hardware, traditional (classical) test plans usually are not as resource-effective as Bayesian test plans. For illustration, suppose specifications require verification that a component has a failure-rate no greater than 3.5×10^{-6} failures/hr. with at least 50% confidence. Assuming a constant failure-rate model and classical test plans, 198,029 unit-hours of testing with no failures is required. On the other hand, a corresponding Bayesian test plan requires only 117,829 unit-hours of testing with no failures. However, it is remarked

that the Bayesian plan requires the use of so-called "prior information" (see Section II). Thus, in this example, the use of Bayesian plans yields a 68% test resource improvement over the classical plans. This illustrates one attractive and practical aspect of Bayesian test plans, particularly to the contractor.

The purpose of this report is to develop a general Bayesian Zero-Failure (BAZE) reliability demonstration testing procedure which can be used to demonstrate component/subsystem/system reliability conformance to stated requirements. The procedure presented is a general one and its use is not restricted to power conversion systems. However, it is used here to develop test plans for verifying the reliability of components of a proposed Rankine ground demonstration power conversion system (PCS). This system is being developed by Sundstrand Energy Systems, the prime contractor for the KIPS program.

A general discussion and introduction to both classical and Bayesian methods is presented in Section II. The BAZE procedure is developed in Section III. Section IV describes the procedure used to fit prior distributions to components of a proposed Rankine PCS. Section V presents a selection of sample reliability demonstration test plans for component verification of the Rankine PCS. Finally, some additional comments regarding cycle-dependent chance failure mechanisms and component level testing with a system's requirement are presented in Section VI. A summary and some conclusions are also given in Section VI.

II. RELIABILITY DEMONSTRATION TESTING

Over the past two decades numerous classical reliability demonstration testing methods have been devised for various failure-time distributions. Classical demonstration test plans for components having an exponential failure-time distribution (constant failure-rate) may be fixed time (Type I Censoring), fixed number of failures (Type II Censoring), or sequential tests. In addition, such tests may be conducted either with or without the replacement of failed items when failures occur during the test. One example

of a classical procedure is MIL-STD-781C which gives various test procedures derived under the assumption of a constant failure-rate. These tests are based on various levels of producer's and consumer's risk, discrimination ratios, and the confidence level of the test. In these tests, a simple statistical hypothesis of the form

$H: \lambda = \lambda_0$ (specified failure-rate)

$A: \lambda = \lambda_1$ (maximum acceptable failure-rate)

is tested, where λ is the (unknown) failure-rate of the device and $\lambda_1 > \lambda_0$. The producer's risk is the probability that, if H is true, A will be accepted, while the consumer's risk is the probability that, if A is true, H will be accepted. The discrimination ratio is defined as the ratio of λ_1 to λ_0 . The text by Mann, Schafer, and Singpurwalla (1974) gives an excellent discussion of the basis upon which these and other classical test plans have been developed.

The reliability demonstration testing environment in industry is generally hindered by several major constraints. Among the most serious are those due to limited test budget and limited time for conducting the demonstration test. The use of Bayesian methods has recently received a great deal of attention as a means of adhering to such constraints. Basically, the Bayesian idea is to combine two sources of information by use of the well-known Bayes Theorem [see Waller and Martz (1975)]. The resultant test plans are resource-effective, without sacrificing quality protection. Frequently, during the course of a contract, information regarding component performance is acquired by such means as reliability predictions, design, development, and production tests, and the observation of similar products in field use. Such information is referred to as "prior information," since it represents the sum total of engineering judgement and belief concerning component performance prior to conducting the demonstration test and observing the test results. The second source of information is the quantitative test results themselves. Both sources of information are combined via Bayes Theorem to produce a statement such as the

probability that the failure-rate does not exceed the required value is $X\%$. Bayesian demonstration test plans frequently permit reaching a decision regarding component reliability with fewer test resources than required by classical test plans, as illustrated in Section I. If the prior information supports an adequately reliable component, then less testing will usually be required compared to the classical case. If the opposite is the case, then more testing may be required. An introduction to the use of Bayesian methods in reliability is given by Waller and Martz (1975), (1976).

One of the earliest references to Bayesian reliability demonstration plans is that of Bonis (1966). Since that time numerous Bayesian schemes have been developed. Easterling (1970) presented a somewhat modified Bayesian demonstration procedure. Schafer and Singpurwalla (1970) developed a sequential Bayes procedure for obtaining required test plans. Recently, Schafer (1969), (1971), and (1973) has considered three types of Bayesian plans: (1) Bayesian fixed time tests, (2) mixed Bayesian/classical, and (3) Bayesian sequential tests. Following along these same lines, Goel et al (dates unknown) developed Bayesian plans for slightly different criteria. Blumenthal (1973) has also developed Bayesian test plan procedures. Guild (1968), (1973) has developed what he refers to as "median failure rate" (MFR) reliability demonstration plans. Other Bayesian plans have also been considered by Balaban (1969), (1975) and Ramos (1970). Joglekar (1975) discusses several of these Bayesian testing schemes. Recently, Goel and Joglekar [1976 (a)-1976 (c)] have prepared a comprehensive account of the state of the art of Bayesian reliability acceptance sampling. This five-part series provides an excellent introduction to the subject.

One of the major problems with most Bayesian test plans is the relative difficulty in obtaining a desired plan in practice. This is due to the presence of additional prior parameters. For example, most of the above Bayesian plans are derived for a constant failure-rate model and a gamma prior distribution on the failure-rate [see Waller and Martz (1975)].

The gamma prior distribution is the natural conjugate prior distribution for the constant failure-rate model. Schafer (1969) investigated data from 32 different equipments and found that in 29 cases a gamma prior distribution adequately fit the data. Others have likewise observed the suitability and versatility of the gamma prior distribution. For these reasons it is also considered here. However, it is frequently not an easy task to identify an appropriate gamma prior distribution and, once this has been done, to obtain the required test plan. The choice of test plan criteria, e.g., the consumer and producer risks that are to be controlled, can also complicate the determination of the required plan. Certain criteria yield plans that are simple to determine. One of the easiest of the above Bayesian procedures to use in practice is that given by Guild (1973). A more general version of this procedure is developed in the next section for use here. Two somewhat distinctive aspects of the procedure should be mentioned before we begin the development. First, the criterion upon which the procedure is based is simple, pertinent, and easy to grasp. Second, the procedure is straightforward and easy to apply in practice, with only a pocket calculator required for a few simple calculations. Together these provide a useful Bayesian procedure for a large variety of applications. Let us now consider the details of this procedure.

III. BAYESIAN ZERO-FAILURE (BAZE) RELIABILITY DEMONSTRATION TEST PLANS FOR TIME-DEPENDENT CHANCE FAILURE MECHANISMS

This section describes how to construct and apply Bayesian fixed time demonstration test plans of the replacement type, called BAZE plans, for systems/subsystems/components having a constant failure-rate. The BAZE procedure is developed for testing time-dependent chance failure mechanisms in this section. Section VI discusses how the procedure may also be used for testing cycle-dependent chance failure mechanisms.

To begin, consider a device, henceforth referred to as a "component," having an exponential failure time distribution with failure-rate λ . (At present we consider a test plan for one such component, while corresponding test plans for series systems of such components is considered in Section VI.) Thus, the failure-time random variable X of this component is assumed to follow the well-known exponential probability density function given by

$$f(x|\lambda) = \lambda e^{-\lambda x}, \quad x \geq 0, \quad \lambda > 0. \quad (1)$$

It is further assumed here that the prior distribution of λ is the natural conjugate gamma distribution with probability density function given by

$$f(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, \quad \lambda, a, b > 0, \quad (2)$$

where a is the prior shape parameter and b is the prior scale parameter. The parameter a can be interpreted as the number of pseudo failures in a prior life test of duration b pseudo hours. The mean and variance of (2) are given by (a/b) and (a/b^2) , respectively. Thus, the initial prior estimate of the failure-rate λ is (a/b) .

The test plans considered here assume that n identical components are tested each for a prespecified length of time t , the test duration. The quantities n and t are to be determined consistent with the following statistical test criterion. The test criterion is as follows: if no failures occur, the test is passed, while if one or more failures occur, the test is failed. Thus the test is terminated either at the prespecified test time t or at the time of the first component failure, whichever occurs first. Such "zero-failure" test plans usually require the smallest unit-hour test combination nt for a stated confidence, and are thus test resource-effective. In addition, by restricting consideration to zero-failures, such test plans are easy to obtain. Now the probability of obtaining exactly zero failures during the test is given by

$$P(0 \text{ failures}|\lambda) = e^{-nt\lambda}. \quad (3)$$

The posterior distribution of λ is also a gamma distribution with scale parameter $(b+nt)$ and shape parameter a . Thus, conditional on zero failures in nt unit-hours of testing, the posterior probability density function of λ becomes

$$f(\lambda|0\text{failures}) = \frac{(b+nt)^a}{\Gamma(a)} \lambda^{a-1} e^{-(b+nt)\lambda}. \quad (4)$$

In order to find the required unit-hour test combination nt , some criterion regarding the desired confidence level of the demonstration test must be given. The plans presented here satisfy the posterior risk criterion given by

$$P(\lambda \leq k\lambda_0 | 0 \text{ failures}) = \gamma, \quad (5)$$

where $P(\cdot)$ is a probability function. Here λ_0 is the specified failure-rate. If we define $\lambda_1 \equiv k\lambda_0$, then λ_1 is referred to as the test criterion failure-rate and k is known as the discrimination ratio. The test criterion in (5) is interpreted as follows. In a test that is passed, i.e., zero failures occur, the probability is $100\gamma\%$ that the component failure-rate does not exceed $(k\lambda_0)$. Here $(1-\gamma)$ will be referred to as the posterior risk and γ will be referred to as the posterior assurance. The word "posterior" denotes that the assurance pertains to tests which have been passed. Of course, values of γ and k are required in order to determine the required test plan. More will be said about the selection of these values later.

Recall that the test procedure requires that n items be placed on life test for t hours. If no failures occur, the test is passed. Now, if the test is passed, it may be claimed that a failure-rate not exceeding $(k\lambda_0)$ has been demonstrated with $100\gamma\%$ posterior assurance. If a single failure occurs, the test is failed, and the foregoing claim cannot be made.

The BAZE procedure described requires specification of values for the following five quantities:

- λ_0 (the specified failure-rate)
- a (the prior shape parameter)
- b (the prior scale parameter)

- γ (the posterior assurance)
- k (the discrimination ratio).

The procedure is developed by writing

$$\begin{aligned}
 p(\lambda \leq \lambda^* | 0 \text{ failures}) &= \int_0^{\lambda^*} \frac{(b+nt)^a}{\Gamma(a)} \lambda^{a-1} e^{-(b+nt)\lambda} d\lambda \\
 &= \frac{I(a, [b+nt]\lambda^*)}{\Gamma(a)},
 \end{aligned} \tag{6}$$

where $I(a, x)$ is the widely studied incomplete gamma function defined by

$$I(a, x) = \int_0^x y^{a-1} e^{-y} dy. \tag{7}$$

Tables and computer routines for evaluating this function are widely available for use in our development. Hence, when λ_0 , a , b , γ , and k are specified, the three-step procedure for obtaining the required BAZE test plan is as follows:

Step 1. Obtain the value of θ_γ from Table A1 (in the APPENDIX) for the given values of a and γ . Note: Table A1 may be used directly for $a = 0.0001$ (0.0001) 0.01 (0.001) 0.10 (0.01) 1.0 (0.1) 5.0 (0.5) 10.0 (1.0) 50.0 and $\gamma = 0.99, .975, .95, .90, .85, .80, .75, .70, .60, .50$. For other values of a and/or γ , either interpolate in Table A1 or solve the equation given by

$$\int_0^{\theta_\gamma} \gamma \lambda^{a-1} e^{-\lambda} d\lambda - \gamma \Gamma(a) = 0$$

for θ_γ . It is mentioned here that in constructing Table A1 the incomplete gamma function in the above equation was numerically calculated by use of the code INCGAM, written by D. E. Amos and S. L. Daniel of Sandia Laboratories, Albuquerque, NM, November 1974. The above equation was solved on a CDC 6600 computer by use of the root-solving code ZERCIN, written by L. F. Shampine and H. A. Watts, also of

Sandia Laboratories, September 1970.

Step 2. With the θ_γ value obtained in Step 1 solve for the required BAZE unit-hours of test $(nt)_0$ given by

$$(nt)_0 = (\theta_\gamma - bk\lambda_0)/(k\lambda_0).$$

Note: Negative values of $(nt)_0$ can occur. A negative value of $(nt)_0$ can be interpreted as a demonstration of the failure-rate $(k\lambda_0)$, at the stated assurance level, without the need for further testing. This situation occurs whenever the prior distribution satisfactorily meets the assurance requirement.

Step 3. The required test duration t_0 and number of test units n_0 is given by any pair of values satisfying $n_0 t_0 = (nt)_0$, where $(nt)_0$ is the required unit-hours of test from Step 2 and n_0 is a positive integer. The values n_0 and t_0 are selected by outside considerations, such as test time constraints and the number of test units available.

It is noted here that if $a = 1$, $b = 0$, and $\gamma = 0.50$, then the BAZE test plan is exactly the same as the classical test plan. If $a = 1$ and $\gamma = 0.50$, then the BAZE test plan will always require less unit-hours of testing, depending upon the magnitude of b .

For a specified fixed value of a , the required BAZE unit-hours of test decreases as γ decreases, b increases, or k increases. Consequently, for given values of a and b , an opportunity is present for the test designer to trade between decreasing testing costs [decreasing $(nt)_0$] and decreasing test assurance [decreasing γ and/or increasing k]. Such tradeoffs are illustrated in Section V.

Suppose that high-reliability components with failure-rate on the order of magnitude of 10^{-6} failures/hr. are being considered. If the prior mean is in this range, then it is true that as the prior variance increases both a and b will generally decrease. In fact, in situations such as this, a will frequently be less than one. This situation occurs whenever the prior variance is quite

large, i.e., whenever the prior distribution is diffuse [see Waller and Martz (1975)]. Such situations frequently occur in reliability and this fact has motivated the fine grid of a values less than one considered in Table A1.

A quantity of particular interest to the producer or contractor is the unconditional probability of passing the test when using a test plan with (nt) unit-hours of test. This probability must be sufficiently large in order that the contractor be willing to conduct the test. This probability also conveys to the consumer the likelihood that the required posterior assurance will be realized. This probability is given by

$$P(\text{Passing the Test}) \equiv P(0 \text{ failures}) = [b/(b+nt)]^a. \quad (8)$$

Related to this quantity is the conditional probability of passing the test when it is known that the true (unknown) failure-rate λ lies within a given interval $[\lambda_*, \lambda^*]$, where $0 \leq \lambda_* < \lambda^* \leq \infty$. In this case we have

$$P(\text{Passing the Test} | \lambda_* \leq \lambda \leq \lambda^*) = \left(\frac{b}{b+nt} \right)^a \left\{ \frac{I(a, [nt+b]\lambda^*) - I(a, [nt+b]\lambda_*)}{I(a, b\lambda^*) - I(a, b\lambda_*)} \right\}, \quad (9)$$

where $I(a, x)$ is defined in (7). It is also noted that, if $\lambda_* = 0$ and $\lambda^* = \infty$, then the conditional probability of passing the test given in (9) reduces to the unconditional probability given in (8). In practice, an interval $[\lambda_*, \lambda^*]$ which is certain to contain the failure-rate can frequently be identified. If this can be done, then (9) should be used in place of (8).

What posterior assurance do we have about the failure-rate if one or more failures occur during the test, i.e., if the test is failed? Suppose that failed items are replaced as they occur during the test. Then

$$P(x \text{ failures} | \lambda) = \frac{e^{-nt\lambda} (nt\lambda)^x}{x!}, \quad x = 0, 1, \dots \quad (10)$$

and the posterior probability density function of λ becomes

$$f(\lambda | x \text{ failures}) = \frac{(b+nt)^{a+x}}{\Gamma(a+x)} \lambda^{a+x-1} e^{-(b+nt)\lambda}. \quad (11)$$

Now, for any specified interval $[\lambda_*, \lambda^*]$, where $0 \leq \lambda_* < \lambda^* \leq \infty$, we have

$$P(\lambda_* \leq \lambda \leq \lambda^* | x \text{ failures}) =$$

$$\frac{1}{\Gamma(a+x)} \left\{ I(a+x, [b+nt]\lambda^*) - I(a+x, [b+nt]\lambda_*) \right\}. \quad (12)$$

In particular, we have

$$P(\lambda \leq k\lambda_0 | x \text{ failures}) = I(a+x, [b+nt]k\lambda_0) / \Gamma(a+x). \quad (13)$$

It is observed that, if $x = 0$, then (13) is equal to the specified posterior assurance γ . For the criterion failure-rate ($k\lambda_0$), as x increases (13) becomes smaller than γ . Thus, as more failures occur, we have less posterior assurance about the failure-rate not exceeding the criterion value.

Also, the unconditional probability of obtaining exactly x failures in a test of nt unit-hours duration is

$$P(x \text{ failures}) = \frac{b^a (nt)^x \Gamma(a+x)}{\Gamma(a) \Gamma(x+1) (nt+b)^{a+x}}. \quad (14)$$

The statistical performance characteristics of the chosen plan are completely summarized by means of the posterior operating characteristic (POC) curve. This curve is obtained by plotting $P(\lambda \leq \lambda^* | 0 \text{ failures})$ as a function of λ^* . Unlike classical OC curves, the POC curve is a cumulative distribution function. This probability may be computed from (6).

Example:

Let us illustrate the foregoing BAZE procedure and associated probabilities by means of an example. Suppose that it is required to obtain the BAZE test plan that demonstrates a component failure-rate of 2.0×10^{-6} failure/hr. with 70% posterior assurance. From prior experience and engineering judgement, the prior gamma distribution mean and standard deviation are both thought to be 1.67×10^{-6} failures/hr. Thus $a = 1.0$ and $b = 0.60 \times 10^{-6}$ hr. Now $(k\lambda_0) = 2.0 \times 10^{-6}$ failures/hr. and from Table A1, for $a = 1.0$ and $\gamma = 0.70$, we find $\theta_{0.70} = 1.203973$. The BAZE test plan thus requires $(nt)_0 = [1.203973 - (0.60 \times 10^{-6})(2.0 \times 10^{-6})] / (2.0 \times 10^{-6}) = 1987$ unit-hours of testing without any failures in order to

verify a failure-rate of 2.0×10^{-6} or less with 70% assurance.

The unconditional probability of passing this test is calculated from (8) as

$$P(\text{Passing the Test}) = [(0.60 \times 10^6)/(0.60 \times 10^6 + 1987)] = 0.9967.$$

Suppose that it is known before the test that λ lies in the interval $(1.0 \times 10^{-7}, 3.0 \times 10^{-5})$. Thus $\lambda_* = 1.0 \times 10^{-7}$, and $\lambda^* = 3.0 \times 10^{-5}$. The conditional probability of passing the test from (9) now becomes

$$\begin{aligned} &P(\text{Passing the Test} | 1.0 \times 10^{-7} \leq \lambda \leq 3.0 \times 10^{-5}) \\ &= \left(\frac{0.60 \times 10^6}{0.60 \times 10^6 + 1987} \right)^{1.0} \left\{ \frac{I(1.0, 18.0596) - I(1.0, 0.0602)}{I(1.0, 18) - I(1.0, 0.06)} \right\} \\ &= 0.9965. \end{aligned}$$

The probability of a single failure in 1987 unit-hours of test is

$$P(1 \text{ failure}) = \frac{(0.10 \times 10^6)^{1.0} (1987)^1 \Gamma(2.0)}{\Gamma(1.0) \Gamma(2) (1987 + 0.60 \times 10^6)^{2.0}} = 0.00329.$$

The POC curve for this plan is plotted in Figure 1.

It is important to examine the sensitivity of the optimal test plan to changes in the criterion failure-rate and posterior risk. Suppose we let $\lambda_0 = 2.0 \times 10^{-6}$. In Table I we have given the required test plan as a function of a selected grid of values of k and γ . The plan $(nt)_0 = 1987$ is indicated in the table for $k = 1.0$ and $\gamma = 0.70$. It is clearly observed that the required unit-hours of test increase as γ increases and k decreases. It is observed that the optimal test plan is quite sensitive to increasing γ and also somewhat sensitive to decreasing k , for the range of k indicated. The "zeros" indicate those situations in which no testing is required, i.e., the prior distribution is sufficient to guarantee that the risk is at or below the specified level. Table II gives the unconditional probability of passing the corresponding test given in Table I. Again, the probability of passing the test when $(nt)_0 = 1987$ is indicated in Table II. In

Figure 1. The POC curve for Test Plan $(nt)_0 = 1987$ Unit-Hours of Test.

TABLE I

REQUIRED UNIT-HOURS OF TESTING FOR
SELECTED VALUES OF K AND γ

γ	k										
	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	4005176	3237642	2699407	2274231	1953429	1702595	1493259	1319021	1171219	1044704	935057
.975	3039079	2474066	2036416	1705549	1449377	1244440	1076763	937033	813900	717457	629626
.950	2390732	1896443	1539009	1272333	1064246	897866	761646	642222	552205	489904	430577
.900	1748565	1318921	1044704	834116	679214	551293	446630	359410	289014	222352	167528
.850	1297126	980933	755086	545700	453956	366560	282327	196467	129652	77543	32373
.800	1079439	741198	549544	405894	294132	204712	131553	70599	19015	0	0
.750	789244	555245	390210	266434	170165	93147	30134	0	0	0	0
.700	463473	403311	259981	152402	68874	17027	0	0	0	0	0
.600	318291	163576	54494	0	0	0	0	0	0	0	0
.500	93147	0	0	0	0	0	0	0	0	0	0

TABLE II

THE UNCONDITIONAL PROBABILITIES OF PASSING
THE CORRESPONDING TESTS IN TABLE I

γ	k										
	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
.990	.130	.156	.182	.208	.235	.261	.287	.313	.339	.365	.391
.975	.163	.195	.228	.260	.293	.325	.358	.390	.423	.455	.488
.950	.200	.240	.280	.320	.361	.401	.441	.481	.521	.561	.601
.900	.261	.313	.365	.417	.469	.521	.573	.625	.677	.730	.782
.850	.316	.380	.443	.506	.569	.633	.696	.759	.822	.886	.949
.800	.373	.447	.522	.596	.671	.746	.820	.895	.969	1.000	1.000
.750	.433	.519	.606	.692	.779	.866	.952	1.000	1.000	1.000	1.000
.700	.498	.598	.698	.797	.897	1.000	1.000	1.000	1.000	1.000	1.000
.600	.655	.786	.917	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.500	.865	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE III

RANKINE POWER CONVERSION SYSTEM RELIABILITY PROJECTION

COMPONENT	GENERIC FAILURE RATES ^{1,2}			PROJECTED PCS GENERIC FAILURE RATE ¹	
	LOWER LIMIT	MEAN	UPPER LIMIT	OPERATING	STARTING
ACCUMULATOR	0.35	7.2	19.3	--	3.35
START VALVE (SOLENOID)	1.87	4.6	7.41	--	1.87
JET PUMP	0.01	0.15	2.11	0.01	
START PUMP (MOTOR)	2.25	8.7	27.4	--	2.25
HIGH-PRESSURE PUMP	1.12	8.74	31.3	1.12	
RADIATOR	2.21	5.0	12.5	0.81	
THERMAL BY-PASS VALVE	0.24	1.9	2.2	0.24	
RADIATOR FILTER	0.011	0.3	1.8	0.01	
JET CONDENSOR	--	--	--	0.50	
WICK SCAVENGE JET PUMP	0.01	0.15	2.11	0.01	
FLOW LIMITER VALVE	0.24	1.9	2.2	0.24	
FLOW CONTROL VALVE	0.24	1.9	2.2	0.24	
TURBINE ASSEMBLY	3.33	10.9	16.67	1.0	
ALTERNATOR	0.033	0.7	2.94	0.7	
REGENERATOR	1.0	4.4	19.6	0.5	
REGENERATOR BY-PASS VALVE	0.224	3.92	32.5	0.22	
BOILER	1.0	4.4	19.6	0.5	
SCREEN FILTER	0.011	0.3	1.8	0.01	
ELECTRONIC CONTROL UNIT	--	--	--	0.36	

¹($\times 10^{-6}$ /HR.).²Failure rates are from Avco Corp. "Reliability Engineering Data Series."

practice, tables like Tables I and II should be constructed and used by both consumer and producer to select the final test plan. We cannot emphasize enough that both the consumer and producer must be willing to pay the price for increasing assurance of small failure-rates by increasing the unit-hours of testing. For illustration we note that verification of small failure-rates with even reasonable confidence requires a large amount of testing. The next section builds on the preceding general discussion to address the problem of selecting a prior distribution with application to a particular hardware system.

IV. PRIOR DISTRIBUTIONS FITTED TO COMPONENTS OF A PROPOSED RANKINE POWER CONVERSION SYSTEM

The first step in determining the BAZE test plan for a given system is to identify a prior distribution for the failure-rate. Our source of prior failure-rate information is Proposal No. 2911A-P1, RFQ No. SNS-75-2-CAO "Kilowatt Isotopes Power System" prepared by Sunstrand Energy Systems. In particular Table 4.4-B of that document, as reproduced in Table III, provides component failure-rates for the power conversion system (PCS) of the proposed Rankine system. We use the generic failure-rate data in Table III to fit a gamma prior distribution to each appropriate component of the proposed Rankine system.

The gamma prior probability density function (pdf) is given by

$$f(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}, \quad \lambda \geq 0, \quad a > 0, \quad b > 0, \quad (15)$$

where the shape and scale parameters, a and b , are determined from the data in Table III. The mean and variance of the gamma distribution above are given by (a/b) and (a/b^2) , respectively. It will be assumed here that the spread between the upper and lower limits in Table III represents a certain assumed number of standard deviations of the gamma pdf. The means listed in Table III will be taken to be the prior means. Table IV gives the prior mean, prior variance, shape parameter a , and scale parameter b for the appropriate components in the PCS. The jet condenser and electronic

control unit are not considered in Table IV due to the absence of prior information on these components. Classical reliability demonstration test plans should be used for these components. The prior variance is calculated from the expression

$$\text{Prior Variance} = \left(\frac{\text{UPPER LIMIT} - \text{LOWER LIMIT}}{m} \right)^2, \quad (16)$$

where $m > 0$ is a constant chosen to reflect the prior confidence that the range (lower limit, upper limit) contains the true failure-rate. There are no rigid rules for selecting a value of m . However, engineering judgement and statistical analysis can be used in determining a suitable value. To assist in selecting m , the following arbitrary rules of thumb are suggested. These rules were used here to select the values of m appearing in Table IV.

Rule 1. If the specified failure-rate $(\text{SFR})\lambda_0$ is well within the range of the generic failure-rate, m is taken to be 3. (This yields a somewhat concentrated prior distribution with a relatively small prior probability that the true failure-rate lies outside the given range.)

Rule 2. If the SFR λ_0 is close to either the lower or upper limits of the generic failure-rate, m is taken to be 2. (This yields a diffuse prior distribution with a rather large prior probability that the true failure-rate lies outside the given range.)

Rule 3. If the SFR λ_0 is well below the lower limit of the generic failure-rate, m is taken to be 1. (This yields an extremely diffuse prior distribution with a large prior probability that the true failure-rate lies outside the given range.)

The more diffuse the prior distribution, the less precise the prior information concerning the failure-rate. In fact, for the situation described in Rule 3 above, it may be inappropriate to consider prior information at all, due to the projected differences in the reliability of the current component compared to previous components. These differences are due to technological advances. In such a case, a Bayesian verification test procedure may be less desirable than a purely classical procedure which does not utilize prior information.

TABLE IV

GAMMA PRIOR DISTRIBUTIONS FOR THE COMPONENTS IN
THE PROPOSED RANKINE PCS

COMPONENT	GENERIC FAILURE RATES ¹			λ_0 ¹	PRIOR MEAN ¹	m	PRIOR VARIANCE ²	a	b ³
	LOWER LIMIT	MEAN	UPPER LIMIT						
ACCUMULATOR	0.35	7.2	19.3	0.35	7.2	2	89.78	0.58	8.02
START VALVE	1.87	4.6	7.41	1.87	4.6	2	7.67	2.80	60.87
JET PUMP; WICK SCAVENGER JET PUMP	0.01	0.15	2.11	0.01	0.15	2	1.10	0.02	13.64
START PUMP (MOTOR)	2.25	8.7	27.4	2.25	8.7	2	158.15	0.48	5.50
HIGH-PRESSURE PUMP	1.12	8.74	31.3	1.12	8.74	2	227.71	0.34	3.84
RADIATOR	2.21	5.0	12.5	0.81	5.0	1	105.88	0.24	4.72
THERMAL BY-PASS VALVE; FLOW LIMITER VALVE; FLOW CONTROL VALVE	0.24	1.9	2.2	0.24	1.9	2	0.96	3.80	200.00
RADIATOR FILTER; SCREEN FILTER	0.011	0.3	1.8	0.01	0.3	2	0.80	0.11	37.50
TURBINE ASSEMBLY	3.33	10.9	16.67	1.0	10.9	1	177.96	0.67	6.12
ALTERNATOR	0.033	0.7	2.94	0.7	0.7	3	0.94	0.52	74.47
REGENERATOR; BOILER	1.0	4.4	19.6	0.5	4.4	1	345.96	0.05	1.27
REGENERATOR BY-PASS VALVE	0.224	3.92	32.5	0.22	3.92	2	260.44	0.05	1.51

¹($\times 10^{-6}$ /HR.). ²($\times 10^{-12}$ /HR.²). ³($\times 10^4$ HR.).

The parameters a and b in Table IV are calculated by means of the expressions

$$a = (\text{Prior Mean})^2 / \text{Prior Variance} \quad (17)$$

and

$$b = a / \text{Prior Mean}. \quad (18)$$

Table V presents certain quantities of interest associated with the fitted gamma prior distributions in Table IV. The quantities P_1, P_2, P_3 , and P_4 in Table V represent the probabilities defined by

$$\begin{aligned} P_1 &\equiv \text{Prob.}(\lambda \leq \text{Prior Mean}), \\ P_2 &\equiv \text{Prob.}(\lambda \leq \lambda_0), \\ P_3 &\equiv \text{Prob.}(\lambda \leq \text{Lower Limit}), \\ P_4 &\equiv \text{Prob.}(\lambda \geq \text{Upper Limit}). \end{aligned} \quad (19)$$

Thus $(P_3 + P_4)$ represents the prior probability that the generic interval given in Table IV does not contain the true failure-rate. The probabilities in (19) are calculated by means of the incomplete gamma function code, INCGAM, mentioned in Section III.

It is observed that the probabilities labeled P_2 in Table V range from 0.002 to 0.89, with an average of 0.45. Such values represent a somewhat pessimistic prior view towards the projected component reliability of the KIPS PCS. Such a view is typically taken by a consumer. These values are the result of the specified component failure-rates being near or below the lower limit of the generic failure-rate range of the handbook values as reported in Table III. The direct effect of these P_2 values is an increase in the number of unit-hours of demonstration testing required. This is a reasonable conclusion supportive of the pessimistic view that, if a component failure-rate is projected to be towards the lower limit of the generic interval, correspondingly increased testing should be performed to support such a low projection.

The values of $(P_3 + P_4)$ in Table V range from 0.20 to 0.91 with an average of 0.54. For those components whose projected failure-rate lies at or below the lower limit of the historical generic interval, $(P_3 + P_4)$ is correspondingly large. This represents an optimistic point of view favoring the producer.

TABLE V

ANALYSIS OF THE GAMMA PRIOR DISTRIBUTIONS FOR THE
COMPONENTS IN THE PROPOSED RANKINE PCS

COMPONENT	GAMMA PARAMETERS		PRIOR MEAN ¹	PRIOR VARIANCE ²	P ₁	P ₂	P ₃	P ₄	P ₃ +P ₄
	a	b ³							
ACCUMULATOR	0.58	8.02	7.2	89.78	0.67	0.14	0.14	0.10	0.24
START VALVE	2.80	60.87	4.6	7.67	.58	.14	.14	.14	.28
JET PUMP; WICK SCAVENGER JET PUMP	0.02	13.64	0.15	1.10	.94	.89	.89	.02	.91
START PUMP (MOTOR)	0.48	5.50	8.7	158.13	.69	.40	.40	.08	.48
HIGH-PRESSURE PUMP	0.34	3.84	8.74	227.71	.71	.38	.38	.07	.45
RADIATOR	0.24	4.72	5.0	105.88	.74	.50	.63	.13	.76
THERMAL BY-PASS VALVE; FLOW LIMITER VALVE; FLOW CONTROL VALVE	3.8	200.00	1.9	0.96	.57	.002	.002	.32	.32
RADIATOR FILTER; SCREEN FILTER	0.11	37.50	0.3	0.80	.82	.57	.58	.05	.63
TURBINE ASSEMBLY	0.67	6.12	10.9	177.96	.66	.17	.35	.22	.57
ALTERNATOR	0.52	74.47	0.7	0.94	.68	.68	.16	.04	.20
REGENERATOR; BOILER	0.05	1.27	4.4	345.96	.89	.80	.83	.05	.80
REGENERATOR BY-PASS VALVE	0.05	1.51	3.92	260.44	.89	.77	.77	.03	.80

¹(X10⁻⁶/HR.). ²(X10⁻¹²/HR.²). ³(X10⁴HR.).

Also, it is to be noted that those gamma prior distributions in Table IV for which $a < 1.0$ are L-shaped, with an asymptote existing at the ordinate axis.

It is remarked here that these fitted prior distributions represent our suggested choices based on statistical analyses only. Other considerations may be present which have not been included here. These may yield different prior distributions from those given in Table IV.

V. BAZE TEST PLANS FOR COMPONENTS OF A PROPOSED RANKINE POWER CONVERSION SYSTEM

The purpose of this section is to propose a range of possible BAZE test plans for several components in a proposed Rankine PCS. These test plans will be obtained by means of the gamma prior distributions suggested in the preceding section. It is emphasized that the plans given here are not to be considered an exhaustive set or the sole source for the final selection. The final selection should be made by the producer with the consenting agreement of the consumer. It may be necessary or desirable to alter the prior distributions given in Table IV for reasons of which we are unaware at this time. Such alterations will result in different test plans from those proposed here.

To illustrate the BAZE test plan procedure, consider the accumulator of the proposed Rankine PCS. From Table 4, we have the SFR $\lambda_0 = 0.35 \times 10^{-6}$ f./hr., $a = 0.58$, and $b = 8.02 \times 10^4$ hr. From Table A1, we have

γ	0.99	0.975	0.95	0.90	0.85
θ_γ	3.550563	2.723769	2.112674	1.519950	1.185539
γ	0.80	0.75	0.70	0.60	0.50
θ_γ	0.955969	0.783608	0.647407	0.443461	0.297721

Table VI presents the required unit-hours of testing for selected values of the discrimination ratio (k) and the posterior assurance

$$\begin{aligned}
 (\gamma). \quad & \text{For example, for } k = 20 \text{ and } \gamma = 0.75, \text{ it would require } (nt)_0 \\
 & = [0.783608 - (8.02 \times 10^4)(20)(0.35 \times 10^{-6})] / [(20)(0.35 \times 10^{-6})] \\
 & = 31,744 \text{ unit-hours}
 \end{aligned}$$

of testing without a failure in order to demonstrate a failure-rate 7.0×10^{-6} f./hr. with 75% assurance. Thus 10 accumulators could be tested for 3,174 hr.; 5 accumulators for 6,349 hr., etc. One may question whether a multiple of 20 times the SFR is too large to be meaningful in view of the overall projected system failure-rate requirement which is 6.43×10^{-6} f./hr. For $k = 1$ and $\gamma = 0.75$, it would require 2,158,680 unit-hours of testing. This unreasonable amount of testing is largely due to the fact that the generic failure-rate interval (0.35×10^{-6} , 19.3×10^{-6}) is extremely large and that the SFR corresponds to the lower end point of this interval. Hence, in order to verify a failure-rate which is significantly smaller than the historical mean 7.2×10^{-6} would require an extremely large amount of testing without failures. This is a pessimistic result which places the burden of proof on the producer in the form of increased testing.

Table VII gives the unconditional probabilities of passing the corresponding tests given in Table VI. It is observed that the unconditional probability of passing a test having 31,744 unit-hours of testing is 0.824.

Tables VIII - XXIX similarly provide a suggested range of test plans and associated unconditional probabilities of passing the corresponding tests for the remaining components in the proposed Rankine PCS.

Again, it is emphasized that these are suggested plans only. The final choice of a BAZE component test plan must be made by the producer in cooperation with the consumer.

The selection of BAZE component test plans which adhere to an overall system failure-rate requirement will be considered in the next section.

VI. ADDITIONAL CONSIDERATIONS AND CONCLUSIONS

In this section we consider several additional aspects associated with the BAZE testing procedure. First, we consider the

TABLE VI

REQUIRED BASE TEST PLANS FOR THE ACCUMULATOR
OF THE PROPOSED RANKINE PCS

Y	k										
	10.5	12.5	14.0	16.0	18.5	21.0	22.5	24.0	26.1	28.5	30.0
.990	974247	745172	446475	553270	443701	627823	387912	342485	303472	252102	247969
.975	499127	568115	675571	476197	352144	379210	277534	244355	212114	187736	174277
.950	221927	422919	354958	297049	255145	221411	194173	171349	151462	135170	121037
.910	154477	291493	223996	171227	141542	126335	117134	100745	85927	74997	64557
.850	255225	262871	151747	131573	147331	90147	77754	60735	52373	42773	37734
.810	192376	167412	113496	90570	77467	54747	47052	33505	25552	17348	16845
.750	143499	166373	73720	59737	44102	31744	21557	13087	5911	211	11
.710	106773	73365	51524	35470	22553	12247	7873	2	2	2	2
.650	45523	25384	16352	11	11	11	11	0	0	0	0
.500	4463	1	0	1	1	1	1	0	0	0	0

TABLE VII

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BASE TESTS IN TABLE VI

Y	k										
	10.5	12.0	14.5	16.0	18.0	22.5	22.5	24.0	26.5	28.0	30.5
.990	.231	.255	.212	.301	.323	.763	.763	.781	.730	.617	.434
.975	.249	.298	.325	.352	.376	.674	.627	.665	.644	.484	.524
.950	.311	.345	.377	.417	.436	.644	.608	.575	.567	.444	.597
.910	.375	.417	.425	.493	.528	.561	.509	.424	.467	.492	.710
.850	.434	.482	.527	.570	.610	.648	.645	.720	.756	.799	.821
.810	.491	.546	.537	.645	.691	.734	.774	.816	.756	.803	.920
.750	.551	.613	.674	.724	.775	.826	.871	.914	.842	1.000	1.000
.710	.615	.685	.743	.819	.846	.921	.973	1.000	1.000	1.000	1.000
.650	.747	.853	.932	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.500	.945	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE VIII
REQUIRED BASE TEST PLANS FOR THE START VALVE
OF THE PROPOSED RANKINE PCS

y	k										
	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
.999	2344557	2044537	1773279	1547337	1345224	11457714	1044593	1351541	1245454	1124325	1114444
.995	1855757	1655183	1443354	1242407	1041717	8317613	7144474	1049757	935724	827535	844274
.990	1470137	1254444	1077211	877269	717014	604116	517724	444447	385444	327512	275534
.980	1145134	9474323	777484	647448	517433	417723	344415	277133	243442	214456	174030
.960	841344	681447	547445	447479	364754	294454	244734	175124	144441	114445	84404
.940	647447	517181	414412	327470	244474	184424	144733	107123	82744	64454	47264
.920	517744	407784	314423	244480	184444	134437	104453	744474	544449	414444	274177
.900	407744	317747	244475	184471	134451	944444	744474	517467	377434	245133	144937
.880	317744	244727	184451	134444	944474	744433	517433	37747	2444	15247	9444
.860	244724	184733	134737	94447	67744	4271	27433	2	4	4	4

TABLE IX
THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BASE TESTS IN TABLE VIII

y	k										
	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
.999	.013	.016	.014	.022	.025	.030	.037	.044	.047	.048	.044
.995	.027	.024	.024	.033	.039	.044	.051	.054	.044	.037	.044
.990	.037	.036	.042	.040	.048	.054	.074	.087	.090	.091	.094
.980	.044	.050	.043	.050	.053	.054	.074	.061	.062	.064	.061
.960	.044	.042	.037	.043	.042	.042	.074	.060	.055	.054	.054
.940	.037	.038	.034	.041	.045	.047	.074	.064	.057	.057	.054
.920	.037	.040	.035	.041	.046	.047	.074	.061	.054	.054	.054
.900	.037	.037	.035	.041	.046	.047	.074	.061	.054	.054	.054
.880	.037	.037	.035	.041	.046	.047	.074	.061	.054	.054	.054
.860	.037	.037	.035	.041	.046	.047	.074	.061	.054	.054	.054

TABLE X

REQUIRED BAZE TEST PLANS FOR THE JET PUMP
AND WICK SCAVENGER JET PUMP OF THE
PROPOSED RANKINE PCS

y	k										
	5.5	6.5	7.0	8.0	9.5	15.5	17.5	12.0	17.0	14.5	15.0
.990	1114112	9178367	7247544	6849678	4577444	5452454	4944377	4520383	4162723	3855443	3540537
.975	1714722	3376702	2417247	2273042	2444777	1791147	1614927	1469700	1345337	1247429	1148642
.950	781407	628767	519457	437478	377711	322755	287454	246183	216754	191529	169657
.925	0	0	0	0	0	0	0	0	0	0	0
.900	0	0	0	0	0	0	0	0	0	0	0
.875	0	0	0	0	0	0	0	0	0	0	0
.850	0	0	0	0	0	0	0	0	0	0	0
.825	0	0	0	0	0	0	0	0	0	0	0
.800	0	0	0	0	0	0	0	0	0	0	0
.775	0	0	0	0	0	0	0	0	0	0	0
.750	0	0	0	0	0	0	0	0	0	0	0
.725	0	0	0	0	0	0	0	0	0	0	0
.700	0	0	0	0	0	0	0	0	0	0	0
.675	0	0	0	0	0	0	0	0	0	0	0
.650	0	0	0	0	0	0	0	0	0	0	0
.625	0	0	0	0	0	0	0	0	0	0	0
.600	0	0	0	0	0	0	0	0	0	0	0

TABLE XI

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE X

y	k										
	5.5	6.0	7.5	9.0	9.0	15.5	17.5	12.0	13.5	14.5	15.5
.990	.914	.919	.922	.924	.926	.928	.935	.932	.933	.935	.935
.975	.915	.919	.917	.914	.916	.918	.925	.927	.927	.925	.926
.950	.917	.916	.915	.912	.914	.914	.921	.920	.921	.921	.921
.925	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.875	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.850	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.825	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.775	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.750	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.725	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.675	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.650	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.625	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XII

REQUIRED BAZE TEST PLANS FOR THE START PUMP
(MOTOR) OF THE PROPOSED RANKINE PCS

y	k										
	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
.070	469444	462994	443071	571441	441912	427457	307727	370547	347547	325447	394047
.075	495944	441313	423053	364947	374947	348347	284215	206143	243722	232339	217972
.080	364745	322951	291655	264874	241062	222144	204247	189557	175974	167414	142877
.085	274474	262465	197513	168444	142844	139114	124434	116143	105075	98166	96574
.090	164717	146461	123744	115551	103940	92411	87472	75422	69174	61493	55634
.095	119974	103945	91495	79497	69497	61556	44277	47843	42134	37718	32417
.740	84157	72412	61794	52417	45117	38434	32434	27443	22847	18765	15077
.750	59174	47884	39313	32944	24430	20454	14734	11574	7875	4566	1594
.800	14747	12674	5486	1754	?	?	?	?	?	?	?
.900	?	?	0	?	?	?	?	?	?	?	?

TABLE XIII

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XII

y	k										
	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
.090	.294	.304	.317	.329	.341	.343	.344	.374	.385	.405	.415
.075	.372	.348	.353	.377	.391	.474	.474	.429	.447	.452	.463
.050	.372	.396	.413	.430	.445	.444	.475	.489	.522	.515	.529
.040	.442	.471	.471	.510	.528	.544	.563	.590	.594	.612	.627
.050	.512	.536	.542	.591	.602	.622	.642	.641	.679	.697	.714
.060	.574	.601	.627	.651	.675	.697	.719	.741	.747	.791	.811
.750	.639	.668	.677	.724	.750	.774	.822	.823	.844	.849	.890
.700	.727	.740	.772	.842	.831	.840	.894	.912	.929	.942	.926
.600	.844	.909	.943	.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XIV
REQUIRED BASE TEST PLANS FOR THE
HIGH-PRESSURE PUMP

y	k										
	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
.030	2142274	1247292	792245	534660	350870	234433	157511	103324	673422	437005	284031
.075	1770274	875613	547679	316017	194124	124512	792221	500345	323544	203114	126467
.090	1506477	622264	353911	204970	124134	792221	500345	323544	203114	126467	82774
.098	1422452	541424	305044	181711	114712	792221	500345	323544	203114	126467	82774
.099	1400467	520274	274049	160977	10050	57324	35457	21264	12734	7621	4541
.400	130444	250704	121043	61142	32242	15111	70014	31374	16734	8621	5071
.750	124142	143447	93044	52721	28494	12317	57453	27160	1404	74	4
.900	114477	101214	6674	31670	17467	8130	4401	8	4	0	0
.950	127437	42443	13672	2124	7	4	0	0	0	0	0
.990	51471	6334	0	0	0	4	0	0	0	0	0

TABLE XV
THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BASE TESTS IN TABLE XIV

y	k										
	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
.000	.242	.306	.332	.348	.418	.445	.460	.491	.517	.527	.547
.075	.249	.341	.391	.432	.456	.465	.522	.545	.550	.540	.650
.090	.299	.379	.435	.430	.517	.541	.587	.577	.535	.455	.677
.098	.345	.436	.511	.532	.536	.574	.568	.520	.428	.384	.779
.099	.385	.487	.553	.516	.665	.714	.744	.780	.712	.642	.849
.400	.424	.537	.615	.590	.733	.782	.822	.840	.704	.629	.959
.750	.445	.589	.675	.745	.804	.844	.872	.844	.692	1.000	1.000
.900	.559	.645	.747	.816	.890	.927	.937	1.000	1.000	1.000	1.000
.950	.613	.776	.837	.952	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.990	.751	.949	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XVI
REQUIRED BASE TEST PLANS FOR THE RADIATOR

γ	k										
	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
.030	1129104	1203374	1142214	1087649	1036583	976311	916457	820321	772443	729272	696404
.075	344315	406176	415796	769367	632457	667111	590934	501024	423086	437866	476556
.090	478497	612415	457426	511171	651222	486545	664212	379735	155017	334735	316636
.090	394451	350112	326362	295742	271265	250114	231459	215245	203494	187458	175725
.090	256753	227303	201628	135072	148681	146152	141621	133610	121552	111723	103777
.090	166764	145685	123023	115037	106101	94539	85267	77675	70560	64351	58774
.740	107522	99325	79451	68747	47450	53221	47551	41463	35034	32127	28131
.770	69464	50313	42187	35315	29615	26350	19465	15947	12301	9255	6432
.870	5510	724	0	0	0	0	0	0	0	0	0
.900	0	0	0	0	0	0	0	0	0	0	0

TABLE XVII
THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BASE TESTS IN TABLE XVI

γ	k										
	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
.030	.479	.448	.437	.456	.475	.482	.405	.437	.524	.511	.517
.075	.477	.488	.439	.538	.517	.525	.533	.561	.540	.556	.563
.090	.519	.531	.542	.533	.563	.572	.581	.599	.598	.625	.613
.090	.583	.597	.629	.621	.632	.647	.653	.643	.672	.691	.689
.090	.661	.655	.662	.632	.694	.726	.717	.728	.738	.747	.756
.090	.697	.713	.722	.743	.756	.740	.781	.732	.783	.813	.824
.750	.757	.774	.731	.806	.820	.836	.847	.840	.871	.883	.896
.770	.821	.840	.859	.875	.890	.895	.919	.933	.944	.954	.970
.870	.974	.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.900	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XVIII

REQUIRED BASE TEST PLANS FOR THE THERMAL
BY-PASS VALVE, FLOW LIMITER VALVE,
AND FLOW CONTROL VALVE

y	k										
	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	5.0
.030	1140486	1064274	9717534	825548	664358	412925	244752	1204205	600301	344158	1447920
.075	375717	262294	173928	773742	2201015	4817412	6397017	4315217	5467046	5344194	5054271
.090	4171525	7723152	7141145	6062710	4744564	3774564	2620534	1571496	6740410	4442203	4222915
.098	4307674	6340721	5743554	5422448	4542148	3444473	2444454	173253	3343144	3557148	1744453
.099	4304672	540464	474745	406572	316543	2034434	1711134	1653346	1213305	242045	2732743
.099	4121167	6042925	6152117	4100743	3770727	3400244	3220013	2411445	2774139	2574217	2322248
.730	4743256	4300174	1272471	3641044	3744144	3074341	2444443	2415401	2416711	223784	2441533
.740	409444	3954456	1457176	124044	2972172	272444	2404432	2244143	2107447	1936133	1774031
.840	720447	3150457	2453443	255574	274470	2127144	124473	1751346	1503834	1439306	1341724
.840	2422734	2521315	2253345	2010467	1474623	1617152	1444412	1200220	1165263	1014210	843642

TABLE XIX

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BASE TESTS IN TABLE XVIII

y	k										
	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	5.0
.030	.001	.001	.001	.002	.002	.002	.002	.002	.002	.002	.005
.075	.001	.002	.002	.002	.003	.004	.004	.004	.004	.007	.008
.090	.002	.002	.003	.004	.005	.006	.007	.007	.008	.011	.013
.098	.003	.004	.005	.007	.008	.010	.013	.015	.017	.020	.026
.099	.005	.007	.009	.010	.013	.015	.019	.022	.024	.031	.036
.099	.007	.009	.012	.014	.019	.022	.024	.031	.037	.043	.050
.750	.010	.012	.015	.019	.024	.029	.035	.042	.049	.058	.068
.760	.013	.016	.021	.026	.031	.038	.044	.055	.065	.076	.089
.690	.021	.027	.034	.043	.052	.064	.077	.092	.109	.127	.149
.840	.035	.045	.057	.071	.087	.105	.127	.151	.174	.203	.246

TABLE XX
REQUIRED BAZE TEST PLANS FOR THE RADIATOR
FILTER AND SCREEN FILTER

y	k										
	10.5	13.5	14.5	19.5	22.5	25.5	29.5	31.5	36.5	37.5	40.5
.000	1437844	1243330	1117220	834907	713793	624554	557251	495650	452254	412546	374791
.005	115571	76435	41651	51156	43549	37373	31575	29952	24933	24450	22130
.010	43474	65353	75427	27600	25240	21602	15885	16093	16413	13375	12455
.015	24504	19502	12217	12221	12217	12217	12217	12217	12217	12217	12217
.020	12349	8403	4237	4782	3540	2673	1944	1438	973	587	264
.025	6045	2936	1491	823	255						
.030	3157										
.035											
.040											
.045											
.050											

TABLE XXI
THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XX

y	k										
	10.5	13.5	16.5	19.5	22.5	25.5	29.5	31.5	36.5	37.5	40.5
.000	.659	.678	.634	.707	.719	.729	.738	.746	.754	.741	.747
.005	.694	.714	.715	.744	.756	.767	.777	.786	.794	.811	.819
.010	.733	.754	.772	.796	.799	.815	.821	.830	.838	.846	.853
.015	.795	.818	.827	.853	.867	.879	.890	.900	.909	.919	.928
.020	.852	.877	.887	.914	.929	.943	.954	.965	.975	.984	.993
.025	.912	.938	.945	.979	.994	1.000	1.000	1.000	1.000	1.000	1.000
.030	.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.035	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.040	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.045	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XXII

REQUIRED BAZE TEST PLANS FOR THE TURBINE ASSEMBLY

y	k										
	1.5	1.9	2.6	3.4	4.2	5.5	6.8	8.6	10.4	12.2	14.0
.990	173554	204910	232300	255400	27790	299151	319411	339665	359975	380219	390650
.975	248476	157655	147271	135902	124573	112738	101409	90076	78743	67410	56077
.950	225543	122533	102300	82020	61740	41460	21180	10900	9880	8860	7840
.900	151733	84276	62312	43854	26375	17456	10756	6264	3412	1611	1250
.850	104977	64786	45641	31495	20937	13124	8224	4297	2215	1057	8457
.800	70457	55112	34221	25312	17157	10933	6597	3597	1815	7377	4132
.750	45545	44857	29172	20864	13726	8270	4997	2782	1297	5596	4076
.700	30454	35614	23481	16514	10257	6271	3534	1922	1014	5267	2432
.650	18749	24102	14320	9907	6486	3737	2111	1132	626	325	1
.600	11431	14970	8612	5045	2919	1675	925	0	0	0	0

TABLE XXIII

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XXII

y	k										
	1.5	1.8	2.5	3.4	4.2	5.5	6.8	8.6	10.4	12.2	14.0
.990	.043	.093	.110	.143	.155	.185	.214	.223	.247	.259	.274
.975	.075	.111	.141	.159	.195	.210	.242	.254	.285	.295	.325
.950	.099	.130	.155	.179	.229	.248	.285	.310	.335	.359	.382
.900	.158	.160	.205	.245	.282	.317	.350	.392	.412	.442	.470
.850	.125	.187	.233	.296	.330	.372	.409	.446	.482	.516	.549
.800	.144	.214	.273	.327	.377	.424	.468	.510	.557	.590	.628
.750	.163	.242	.303	.370	.426	.479	.529	.577	.623	.667	.711
.700	.187	.272	.349	.416	.480	.539	.595	.649	.701	.751	.799
.650	.231	.343	.439	.525	.605	.680	.751	.818	.884	.947	1.000
.600	.294	.436	.533	.658	.770	.865	.956	1.000	1.000	1.000	1.000

TABLE XXIV

REQUIRED BAZE TEST PLANS FOR THE ALTERNATOR

γ	k										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
.000	4179977	3661273	3275781	2964512	2701427	2471435	2274457	2093287	1935521	1794551	1667539
.075	2921271	2598881	2311276	2075278	1873957	1699231	1544532	1411754	1291951	1184759	1088234
.050	2149291	1813674	1549293	1319970	1124594	951224	801444	670589	556290	456348	362296
.020	1240491	1047474	815440	588678	409151	274229	181175	127983	862745	504453	281994
.000	789762	650212	533949	435617	351152	278235	214302	157890	107744	62380	22591
.075	477176	386494	323535	275274	238268	20934	18972	0	0	0	0
.050	24447	154544	79678	162	0	0	0	0	0	0	0
.020	62179	0	0	0	0	0	0	0	0	0	0
.000	0	0	0	0	0	0	0	0	0	0	0
.075	0	0	0	0	0	0	0	0	0	0	0
.050	0	0	0	0	0	0	0	0	0	0	0
.020	0	0	0	0	0	0	0	0	0	0	0
.000	0	0	0	0	0	0	0	0	0	0	0

TABLE XXV

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XXIV

γ	k										
	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
.000	.379	.398	.415	.434	.451	.467	.483	.499	.514	.529	.543
.075	.417	.459	.495	.520	.540	.559	.577	.595	.612	.628	.643
.050	.521	.526	.521	.574	.597	.619	.640	.657	.672	.689	.718
.020	.599	.630	.659	.697	.714	.747	.765	.790	.814	.837	.859
.000	.687	.722	.755	.787	.818	.848	.877	.905	.932	.959	.985
.075	.771	.812	.857	.896	.921	.944	.967	1.000	1.000	1.000	1.000
.050	.843	.907	.949	.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.020	.959	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.075	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.050	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.020	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XXVI

REQUIRED BAZE TEST PLANS FOR THE
REGENERATOR AND BOILER

y	k										
	1.1	1.4	2.6	3.4	4.2	5.1	5.9	6.6	7.4	8.2	9.0
.000	275254	119577	423936	627041	545278	422351	362344	316984	241253	252575	228995
.075	1127144	617491	123544	320912	247344	214175	182877	159171	143591	125635	113339
.050	519144	282782	191843	143731	113934	93573	79827	67885	59174	52161	46334
.020	139914	72497	45045	32192	23641	17927	13614	10426	7926	5914	4250
.040	33412	13322	5112	921							
.010	864		0	0				0			
.750			0					0			
.700			0					0			
.650			0					0			
.600			0					0			

TABLE XXVII

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XXVI

y	k										
	1.1	1.4	2.6	3.4	4.2	5.1	5.9	6.6	7.4	8.2	9.0
.000	.773	.796	.811	.822	.831	.838	.844	.850	.855	.859	.863
.075	.799	.823	.833	.849	.858	.864	.872	.878	.883	.887	.892
.050	.835	.854	.871	.892	.901	.909	.914	.919	.924	.927	.929
.020	.883	.909	.925	.939	.949	.957	.964	.970	.974	.978	.981
.040	.937	.965	.983	.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.010	.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.750	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.650	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XXVIII

REQUIRED BAZE TEST PLANS FOR THE
REGENERATOR BY-PASS VALVE

γ	k										
	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
.020	273452	808367	491151	602977	536257	479276	434733	396880	365189	338726	314494
.075	505517	414577	353195	307158	271352	242754	219269	199739	183217	169547	156771
.090	226456	186364	157593	135998	119259	105779	94789	85632	77889	71242	65495
.090	56279	42715	34456	28242	23444	19595	16434	13899	11584	9678	8026
.090	5951	2442	0	0	0	0	0	0	0	0	0
.090	0	0	0	0	0	0	0	0	0	0	0
.750	0	0	0	0	0	0	0	0	0	0	0
.700	0	0	0	0	0	0	0	0	0	0	0
.650	0	0	0	0	0	0	0	0	0	0	0
.600	0	0	0	0	0	0	0	0	0	0	0

TABLE XXIX

THE UNCONDITIONAL PROBABILITY OF PASSING THE
CORRESPONDING BAZE TESTS IN TABLE XXVIII

γ	k										
	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
.090	.811	.819	.825	.831	.836	.841	.844	.848	.851	.854	.857
.075	.839	.846	.852	.858	.863	.868	.872	.876	.879	.882	.885
.050	.871	.878	.885	.891	.896	.901	.906	.909	.913	.917	.920
.090	.927	.935	.942	.949	.954	.959	.964	.968	.972	.976	.979
.050	.994	.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.090	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.750	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.700	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.650	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

manner in which the BAZE procedure can be used to obtain test plans for testing cycle-dependent chance failure mechanisms. Second, we consider the manner in which BAZE component test plans can be developed taking into consideration a system failure-rate requirement. Third, we present a summary and give some conclusions.

A. Cycle-Dependent Chance Failure Mechanisms

Many of the components in the proposed Rankine PCS are likely to possess failure modes related to cyclic operation. Such modes are likely to be exposed by the required FMEA (failure modes and effects analysis). To demonstrate the reliability of cyclically operated components, we consider testing n items each for N cycles of operation rather than a fixed time interval as used in the preceding cases. Under certain conditions, the BAZE testing procedure may be used to test those cyclically operated chance failure mechanisms by providing an approximate number of required test cycles. In particular, the procedure is useful when the number of cycles is large and the probability of failure on any cycle is small and approximately constant from cycle to cycle. The second assumption will likely hold if the test duration is contained within the useful life of the device and wear-related phenomena are negligible or constant.

Let

λ_0 = specified failure-rate (failures/hr. of operation),

m = required component duty cycle (cycles/hr.).

Then define

$\delta_0 = \lambda_0/m$ = specified cycle failure-rate (failures/cycle).

For specified k and γ as before, Table A1 is entered to obtain θ_γ . Once θ_γ has been obtained, the number of BAZE required unit-cycles of testing is found by calculating

$$(nN)_0 = (\theta_\gamma - bk\delta_0)/(k\delta_0). \quad (20)$$

A gamma prior distribution may still be used for the cycle failure-rate; however, the prior parameters a and b are now interpreted as the number of pseudo failures in a prior cycle life test of b pseudo cycles of component operation.

Example:

Let us illustrate the cycle BAZE procedure by means of an example. Consider a component, a valve say, for which we desire to demonstrate the failure-rate associated with cyclic failure mechanisms. The SFR is $\lambda_0 = 0.7 \times 10^{-6}$ failures/hr. and the parameters of the prior gamma distribution of λ are $a = 0.75$ and $b = 1,071,429$ hr. Further suppose that the duty cycle is $m = 1000$ cycles/yr. $= 0.11416$ cycles/hr. Thus $\delta_0 = 6.132 \times 10^{-6}$ failures/cycle. Also, the value of the prior scale parameter b in the distribution of δ , the cycle failure-rate, is $b = 122,314$ cycles. We take $k = 1$ and $\gamma = 0.80$. From Table A1 we obtain $\theta_{0.80} = 1.229561$. Thus, the required BAZE cycle test plan for this component is

$$(nN)_0 = \left[1.229561 - (122,314)(6.132 \times 10^{-6}) \right] / (6.132 \times 10^{-6})$$
$$= 78,202 \text{ unit-cycles}$$

of testing. Thus some possible test plans are to cycle 10 components 7,820 times each; 5 components for 15,640 cycles; etc. If no failures occur during the test, then it can be claimed that the true cycle failure-rate is less than δ_0 with 80% assurance.

B. System Failure-Rate Considerations in Component Testing

Consider a system with N components arranged in a series configuration. The system failure-rate λ is related to the component failure-rates λ_i by

$$\lambda = \lambda_1 + \dots + \lambda_N = \sum_{i=1}^N \lambda_i.$$

It is desired to determine BAZE component test plans which together assure that, if all component tests are passed, the system failure-rate specification has been met.

The posterior distribution of λ_i , conditioned on no failures in a lifetest of duration $(nt)_i$ unit-hours, is a gamma distribution with mean $a_i / [b_i + (nt)_i]$ and variance $a_i / [b_i + (nt)_i]^2$, where a_i and b_i are the shape and scale parameters, respectively, for the i^{th} component. If the component failures are statistically independent, then the posterior distribution of λ , conditional on all

component lifetests being passed, can be approximated by a gamma distribution. The mean and variance of this distribution, in terms of component means and variances, are given by

$$E(\lambda | \text{all component tests are passed}) = \sum_{i=1}^N a_i / [b_i + (nt)_i]$$

and

$$V(\lambda | \text{all component tests are passed}) = \sum_{i=1}^n a_i / [b_i + (nt)_i]^2.$$

Now, the shape and scale parameters of the approximate posterior gamma distribution of λ are given by

$$a = \left\{ \sum_{i=1}^N a_i / [b_i + (nt)_i] \right\}^2 / \sum_{i=1}^N a_i / [b_i + (nt)_i]^2$$

and

$$b = \sum_{i=1}^N a_i / [b_i + (nt)_i] / \sum_{i=1}^N a_i / [b_i + (nt)_i]^2.$$

The objective of the system's approach taken here is to obtain the minimum cost BAZE component demonstration test plans which assure that the required system failure-rate specification has been met. Let us now identify certain costs associated with component demonstration testing. Define

$$\begin{aligned} c_i &= \text{total cost of testing the } i^{\text{th}} \text{ component per unit-hour} \\ &= c_{i0} + c_{i1} + c_{i2}, \end{aligned}$$

where

- (i) c_{i0} = component cost of the i^{th} component per unit-hour,
- (ii) c_{i1} = test facility and test instrumentation cost of the i^{th} component per unit-hour,
- (iii) c_{i2} = operation cost of the i^{th} component per unit-hour.

The problem is now to find BAZE component test plans which minimize the total testing cost while providing assurance that the component test plans satisfy the required system failure-rate specification. In mathematical terms, we wish to

find $(nt)_1, (nt)_2, \dots, (nt)_N$ which

$$\text{minimizes } \sum_{i=1}^N c_i (nt)_i$$

subject to the constraints

$$(i) (nt)_i \geq 0, i=1, \dots, N,$$

$$(ii) I(a, bk\lambda_0) - \gamma \Gamma(a) = 0,$$

where $(k\lambda_0)$ is the criterion failure-rate and γ is the probability that this failure-rate has been attained. The standard techniques of non-linear mathematical programming can be used to solve this problem. This will be the subject of a future technical report.

C. Summary and Conclusions

A Bayesian zero-failure (BAZE) reliability demonstration testing procedure is developed. The procedure assumes that the component failure-rate is a constant value of a random variable having a gamma prior distribution. The testing procedure is to place n items on lifetest with replacement for a specified length of time t . The lifetest is passed if no failures occur. The quantity of testing required, the test plan, is found by considering a single criterion. In a test that is passed, the conditional probability that the failure-rate does not exceed a prespecified value is set equal to a desired posterior assurance.

The BAZE procedure can be used to statistically verify failure-rates associated with real-time chance failure mechanisms. In certain cases, the procedure can also be used to statistically verify failure-rates for cycle-dependent chance failure mechanisms.

The BAZE procedure is used here to obtain sample test plans for demonstrating the real-time dependent failure effects of the components of a proposed Rankine power conversion system. For most of these components, large quantities of testing are required to demonstrate the small specified failure-rates with even reasonable assurance. This is largely the result of fitted prior distributions whose means are significantly greater than the

specified failure-rates. This is a pessimistic result which requires the producer to support projected low component failure-rates with significant quantities of testing. This insures adequate consumer protection against accepting component hardware with unacceptably high failure-rates.

Several aspects of reliability demonstration testing are not considered here. No attempt has been made to identify specific methodologies for testing classes of components such as valves, filters, etc. Before the BAZE procedure can be used in practice several things must be considered. First, the life cycle of each component must be identified. This includes establishing the environments and operating conditions to which each component is exposed during the various phases of its existence. This also includes defining the duty cycle for the operational phases. Second, the possible failure mechanisms need to be identified. This can be accomplished by means of an FMEA. Third, the BAZE procedure can be used to demonstrate either individual or collective failure modes by carefully defining the criteria which describe component failure. In other words, the practical details of implementing the BAZE procedure have not been addressed in this report. This would ordinarily occur as part of the contractor's reliability program plan.

Finally, the main advantage of the BAZE procedure over other Bayesian demonstration test procedures is its ease of application. In most instances, Table A1 and a pocket calculator are sufficient for calculating the BAZE test plans. The most laborious tasks in using the procedure are determining a satisfactory prior distribution and stating values of the posterior assurance and discrimination ratio. However, all Bayesian procedures share similar steps in their application.

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APPENDIX

TABLE A1. VALUES OF θ_y FOR SELECTED VALUES OF PRIOR SHAPE PARAMETER (α) AND POSTERIOR ASSURANCE (γ)

Q	0.99	0.975	0.95	0.90	0.85	Y	0.80	0.75	0.70	0.60	0.50
0.001	1.24775-44	6.2447-111	8.1261-149	7.6904-149	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.7043-149
0.002	1.4715-42	5.9205-56	8.3301-149	7.8948-149	7.4707-149	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.7043-149
0.003	1.5952-15	1.2530-37	3.1245-75	7.6904-149	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.7043-149
0.004	1.6772-12	1.4833-28	2.2665-115	7.2707-149	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.7043-149	4.2769-149
0.005	1.6465-09	1.5737-23	1.5737-23	1.7160-92	3.8668-142	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.006	2.9844-07	2.6533-16	4.1849-38	3.0549-77	1.3014-118	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.007	3.5668-04	1.1012-16	4.3624-33	2.4064-66	3.3073-102	6.8431-149	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.008	1.2641-06	1.0125-16	8.9134-29	3.5704-58	3.3343-89	4.0934-122	4.154-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.009	7.2649-05	3.4608-13	9.9501-26	8.0947-52	2.1198-74	1.1799-108	6.8431-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.010	2.9254-05	5.6793-12	2.9735-23	9.8217-47	1.6744-71	6.9129-98	6.4154-149	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.011	6.4945-05	5.6747-12	3.1510-22	1.4149-62	3.5646-65	4.6437-80	4.3756-114	5.9877-149	5.5599-149	5.1321-149	4.2769-149
0.012	4.2954-04	3.8633-10	1.5349-16	4.1544-19	3.0454-66	9.8040-82	1.4054-79	4.3756-114	5.9877-149	5.5599-149	4.2769-149
0.013	2.4487-04	1.9573-07	4.1124-14	3.5624-34	2.4414-54	1.5087-75	4.3940-97	3.9037-120	3.1321-149	2.7043-149	2.2769-149
0.014	2.9847-04	7.8700-09	7.8822-17	1.1634-31	2.1414-51	3.3755-70	3.2016-90	3.2016-90	3.2016-90	3.2016-90	3.2016-90
0.015	4.9224-04	2.4678-08	7.8822-17	1.1634-31	2.1414-51	3.3755-70	3.2016-90	3.2016-90	3.2016-90	3.2016-90	3.2016-90
0.016	1.5242-03	1.9147-07	7.1155-15	1.4137-26	4.3231-45	1.5171-61	1.8053-74	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.017	1.9242-03	1.9147-07	4.2745-14	6.8195-24	1.7047-42	5.5471-61	1.8053-74	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.018	2.1131-03	3.3194-07	2.3664-12	2.1337-24	3.5534-38	8.1483-55	2.1855-70	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.019	4.4447-03	1.1233-07	1.0471-08	4.6643-24	4.0001-30	5.5553-52	4.8368-67	1.8053-74	4.3756-114	5.9877-149	5.5599-149
0.020	3.7044-03	1.1233-07	4.0471-08	7.4331-24	2.8047-30	1.9725-49	1.9044-85	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.021	7.7164-03	3.2672-06	1.3747-11	9.1366-23	1.3804-33	4.0032-47	1.8053-74	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.022	5.7701-04	5.5202-04	4.2114-11	6.9381-22	4.6453-34	5.0130-47	9.1152-58	4.3756-114	5.9877-149	5.5599-149	4.7043-149
0.023	7.1497-03	3.3227-05	1.1404-10	7.1710-21	1.1554-31	4.1243-43	2.6850-55	5.2503-68	1.8053-74	4.3756-114	4.2769-149
0.024	8.0144-03	1.4774-05	2.9400-10	4.8367-16	2.1948-30	2.3448-41	4.9245-53	1.6322-65	2.0645-93	2.0942-104	2.0942-104
0.025	1.0207-02	2.2442-05	6.9124-10	2.8044-19	3.9944-29	9.6474-40	5.9245-53	1.6322-65	2.0645-93	2.0942-104	2.0942-104
0.026	1.9337-02	3.3213-06	1.5220-05	1.4165-18	4.0143-32	3.4000-34	4.9741-49	1.6322-65	2.0645-93	2.0942-104	2.0942-104
0.027	1.3707-02	4.7636-05	3.1636-06	6.3564-18	4.0453-27	7.2002-37	2.9945-47	2.3643-58	3.9374-83	1.8101-112	1.8101-112
0.028	1.5744-02	4.6593-05	6.2294-09	2.5607-17	3.6497-25	1.3791-34	1.3445-44	4.0433-58	1.3005-80	1.7342-103	1.7342-103
0.029	7.9045-02	4.0964-05	1.1710-08	9.3740-17	2.5825-25	2.1533-34	4.6502-44	2.1471-54	1.9415-77	8.8492-105	8.8492-105
0.030	2.0143-02	1.2717-05	2.1131-08	3.1408-16	1.7225-24	2.7993-33	1.7710-52	1.3047-52	6.3213-75	2.6530-101	2.6530-101
0.031	2.2494-02	1.5941-04	3.6495-08	9.7711-16	5.0814-24	3.0842-32	2.8029-61	6.0441-51	1.5357-72	4.4053-98	4.4053-98
0.032	2.8951-02	1.9410-04	6.1530-08	8.2203-15	4.9417-23	2.0242-31	5.0945-61	2.0646-69	2.9468-70	4.7707-95	4.7707-95
0.033	4.7521-02	4.4223-04	1.0002-07	7.6664-15	2.3030-22	2.4201-30	7.7673-39	4.4414-48	3.3749-68	3.3429-92	3.3429-92
0.034	3.0141-02	3.2043-04	1.5400-06	7.0004-14	9.0037-22	1.7646-29	1.0009-37	1.5220-46	3.1488-66	1.6306-92	1.6306-92
0.035	4.2037-02	4.0633-04	2.3151-07	4.7533-14	4.9414-21	1.1543-28	1.1348-36	3.1107-45	2.3145-64	5.5207-47	5.5207-47
0.036	3.5764-02	4.6721-04	3.6545-07	1.0970-13	4.7279-20	3.6229-27	1.1102-35	5.2575-43	1.3346-62	1.3524-94	1.3524-94
0.037	3.4494-02	6.0141-04	5.3702-05	2.2749-12	4.7050-20	7.9921-24	9.6252-35	7.7779-43	6.1475-61	2.4414-92	2.4414-92
0.038	4.1474-02	7.2022-04	7.7451-07	5.1174-13	1.5621-19	1.7703-24	7.4443-34	9.7051-42	2.3411-59	3.4051-80	3.4051-80
0.039	4.7313-02	6.5425-04	1.0735-06	1.0242-12	4.9749-19	7.9274-26	5.1845-33	1.0771-40	7.3521-58	3.4051-80	3.4051-80
0.040	4.7863-02	1.0754-03	1.0735-06	1.0242-12	4.9749-19	7.9274-26	5.1845-33	1.0771-40	7.3521-58	3.4051-80	3.4051-80
0.041	5.1051-02	1.1735-03	2.0774-06	3.4944-12	3.4347-16	1.9903-31	1.9403-31	9.3786-53	4.3784-55	2.1327-72	2.1327-72
0.042	5.2947-02	1.3573-03	2.7984-06	7.1404-12	4.9247-14	1.7503-24	1.0041-30	9.3786-53	4.3784-55	2.1327-72	2.1327-72
0.043	5.7504-02	1.5449-03	3.7179-06	1.2970-11	2.1704-17	6.3155-23	4.9443-30	5.7151-37	1.9422-52	5.5447-71	5.5447-71
0.044	6.0931-02	1.7849-03	4.0761-06	2.2642-11	5.1258-17	5.3197-23	2.2646-39	5.7151-37	1.9422-52	5.5447-71	5.5447-71
0.045	6.3147-02	2.0341-03	6.3145-06	3.8242-11	1.1444-16	1.6442-26	2.2646-39	5.7151-37	1.9422-52	5.5447-71	5.5447-71
0.046	6.7747-02	2.2994-03	6.0954-06	5.0364-11	2.5541-16	4.4257-22	2.2646-39	5.7151-37	1.9422-52	5.5447-71	5.5447-71
0.047	7.1213-02	2.5864-03	1.0205-05	6.3047-10	5.1444-16	1.3447-24	1.4743-27	6.2105-34	3.3446-49	2.0401-66	2.0401-66
0.048	7.5713-02	2.8444-03	1.2804-05	1.6547-10	1.1134-15	3.6442-21	5.2743-27	3.0145-37	3.0145-37	1.6875-61	1.6875-61
0.049	8.0444-02	3.2244-03	1.6024-05	2.5074-10	2.2221-15	9.4084-26	1.7441-26	1.3546-32	2.0010-46	2.0714-62	2.0714-62
0.050	8.1804-02	3.5774-03	1.9762-05	3.0774-10	4.3144-16	2.7394-20	5.7440-24	5.9404-32	5.9404-32	3.5094-66	3.5094-66

TABLE A1 (CONTINUED)

γ	0.99	0.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
0.051	.005393	.003953	2.4167E-05	6.0129E-10	4.1004E-15	5.6127E-20	1.7920E-25	2.3800E-31	1.7410E-44	5.3167E-60
0.052	.005003	.004351	2.9327E-05	8.9455E-10	1.5062E-14	1.3021E-19	5.1020E-25	9.1923E-31	1.2245E-43	7.2547E-59
0.053	.004235	.004772	3.5320E-05	1.3113E-09	2.7166E-14	2.9262E-18	1.5001E-24	3.3527E-30	7.0154E-43	8.9742E-58
0.054	.002886	.005216	4.2267E-05	1.8931E-09	4.7937E-14	6.3816E-19	4.1477E-24	1.1833E-29	4.6874E-42	1.0115E-56
0.055	.009954	.005583	5.0240E-05	2.7023E-09	8.2842E-14	1.9529E-18	1.0843E-23	1.8842E-29	2.4010E-41	1.0616E-55
0.056	.103438	.006173	5.9149E-05	3.4046E-09	1.4044E-13	2.7922E-18	7.7593E-23	1.2410E-28	1.3660E-40	9.9073E-55
0.057	.107135	.006687	6.9701E-05	4.2943E-09	2.3372E-13	5.6181E-18	6.7943E-23	7.7433E-28	4.7480E-40	8.6910E-54
0.058	.111043	.007223	8.1407E-05	7.2845E-09	3.8215E-13	1.1035E-17	1.6220E-22	1.1707E-27	3.1730E-39	7.0719E-53
0.059	.114761	.007783	9.4580E-05	9.9053E-09	4.4515E-13	2.1163E-17	1.7615E-22	3.1946E-27	1.4124E-38	5.3367E-52
0.060	.118489	.008366	1.0914E-04	1.3334E-08	9.7262E-13	3.9791E-17	8.4708E-22	6.5493E-27	5.9795E-38	3.8800E-51
0.061	.122223	.008971	1.2560E-04	1.7794E-08	1.5164E-12	7.3218E-17	1.8490E-21	2.7899E-26	2.4144E-37	2.5253E-50
0.062	.125963	.009599	1.4409E-04	2.3512E-08	2.3307E-12	1.3203E-16	3.9021E-21	5.8295E-26	9.3212E-37	1.5787E-50
0.063	.129708	.010249	1.6432E-04	3.0799E-08	3.5316E-12	2.3882E-16	8.1100E-21	1.4884E-26	1.4674E-36	6.3114E-49
0.064	.133456	.010921	1.8463E-04	3.9994E-08	5.2882E-12	4.9683E-16	1.6497E-20	3.5327E-25	1.2230E-35	5.1952E-48
0.065	.137207	.011615	2.1115E-04	5.1575E-08	7.8146E-12	6.9683E-16	3.3907E-20	8.3284E-25	4.1988E-35	2.7502E-47
0.066	.140959	.012331	2.3793E-04	6.5814E-08	1.1217E-11	1.1704E-15	4.6330E-20	1.9128E-24	1.3743E-34	1.3943E-46
0.067	.144711	.013068	2.6729E-04	8.3604E-08	1.6490E-11	1.9308E-15	6.3307E-20	4.2550E-24	4.3647E-34	6.6327E-46
0.068	.148463	.013827	2.9918E-04	1.0531E-07	2.3608E-11	3.1644E-15	8.7190E-19	9.3765E-24	1.3945E-33	3.5027E-46
0.069	.152214	.014606	3.3374E-04	1.3190E-07	3.3314E-11	5.4128E-15	1.2710E-19	2.0855E-23	1.9915E-33	1.3312E-44
0.070	.155963	.015405	3.7122E-04	1.6407E-07	4.6544E-11	8.0823E-15	8.001E-19	4.1672E-23	1.4559E-32	5.5915E-44
0.071	.159708	.016224	4.1163E-04	2.0243E-07	6.4424E-11	1.2653E-14	1.4204E-18	8.6246E-23	1.2628E-32	2.2564E-43
0.072	.163450	.017064	4.5513E-04	2.4927E-07	8.8911E-11	1.8506E-14	2.4077E-18	1.7897E-22	8.7004E-32	8.7531E-43
0.073	.167189	.017922	5.0184E-04	3.0463E-07	1.2114E-10	2.9962E-14	4.3363E-18	3.4078E-22	2.7994E-31	3.2729E-42
0.074	.170922	.018800	5.5191E-04	3.7026E-07	1.6347E-10	4.5290E-14	7.3848E-18	6.5054E-21	5.9210E-31	1.1809E-41
0.075	.174650	.019696	6.0543E-04	4.4770E-07	2.1317E-10	6.7740E-14	1.2402E-17	1.2543E-21	1.8645E-30	4.1177E-41
0.076	.178372	.020611	6.6251E-04	5.3804E-07	2.8172E-10	1.0018E-13	2.1555E-17	2.3452E-21	3.6245E-30	1.3849E-40
0.077	.182088	.021544	7.2332E-04	6.4475E-07	3.8420E-10	1.6667E-13	3.3593E-17	4.3144E-21	8.7208E-30	4.5621E-40
0.078	.185797	.022494	7.8793E-04	7.6889E-07	5.0444E-10	2.1208E-13	5.4218E-17	7.8139E-21	2.0415E-29	1.6405E-39
0.079	.189498	.023462	8.5645E-04	9.1210E-07	6.5738E-10	3.0552E-13	8.6517E-17	1.3641E-20	4.6775E-29	4.4346E-39
0.080	.193192	.024446	9.2900E-04	1.0777E-06	8.4022E-10	4.3692E-13	1.3640E-16	2.6414E-20	1.0495E-28	1.3237E-38
0.081	.196878	.025447	1.0057E-03	1.2630E-06	1.0827E-09	6.3701E-13	1.7263E-16	4.2111E-20	2.7892E-28	3.8722E-38
0.082	.200556	.026464	1.0865E-03	1.4481E-06	1.3957E-09	8.8890E-13	1.2795E-16	7.2737E-20	4.9832E-28	1.0997E-37
0.083	.204225	.027497	1.1719E-03	1.7331E-06	1.7723E-09	1.1942E-12	5.0051E-16	1.2985E-19	1.0554E-27	3.0451E-37
0.084	.207885	.028546	1.2614E-03	2.0193E-06	2.2377E-09	1.6421E-12	7.6629E-16	2.0931E-19	2.1046E-27	8.2300E-37
0.085	.211536	.029609	1.3547E-03	2.3344E-06	2.8099E-09	2.2442E-12	1.1315E-15	3.3744E-19	4.6025E-27	2.1729E-36
0.086	.215178	.030687	1.4548E-03	2.7023E-06	3.5077E-09	3.0463E-12	1.4773E-15	5.6201E-19	9.0767E-27	5.6011E-36
0.087	.218809	.031780	1.5648E-03	3.1112E-06	4.3816E-09	4.1052E-12	2.4600E-15	8.8327E-19	1.7888E-26	1.4177E-35
0.088	.222431	.032886	1.6845E-03	3.5795E-06	5.3335E-09	5.5947E-12	3.6881E-15	1.4124E-18	3.4867E-26	3.5034E-35
0.089	.226042	.034007	1.7795E-03	4.0830E-06	6.3784E-09	7.3060E-12	5.1811E-15	2.2773E-18	6.6934E-26	8.4934E-34
0.090	.229643	.035140	1.8975E-03	4.6537E-06	7.3145E-09	9.5542E-12	7.4206E-15	3.7669E-18	1.2645E-25	2.0172E-34
0.091	.233234	.036287	2.0205E-03	5.2940E-06	8.4913E-09	1.2688E-11	1.0546E-14	5.3748E-18	2.3436E-25	4.7074E-34
0.092	.236814	.037446	2.1619E-03	6.0111E-06	1.0243E-08	1.6554E-11	1.4809E-14	8.2404E-18	4.1111E-25	1.0743E-33
0.093	.240383	.038618	2.3119E-03	6.7794E-06	1.4563E-08	2.1408E-11	2.0814E-14	1.2884E-17	7.9054E-25	2.4270E-33
0.094	.243941	.039801	2.4703E-03	7.6749E-06	1.7540E-08	2.7735E-11	2.8927E-14	1.8918E-17	1.6182E-24	5.3480E-33
0.095	.247488	.040997	2.5640E-03	8.6532E-06	2.1043E-08	3.5618E-11	3.9925E-14	2.8045E-17	2.5195E-24	1.1621E-32
0.096	.251023	.042203	2.7129E-03	9.6991E-06	2.5150E-08	4.5649E-11	5.6737E-14	4.1111E-17	4.0005E-24	2.4457E-32
0.097	.254548	.043421	2.8672E-03	1.0852E-05	2.9948E-08	5.7815E-11	7.6458E-14	6.0422E-17	7.6164E-24	5.2330E-32
0.098	.258061	.044650	3.0269E-03	1.2125E-05	3.5835E-08	7.3118E-11	1.0091E-13	8.8046E-17	1.3037E-23	1.0851E-31
0.099	.261563	.045889	3.1920E-03	1.3517E-05	4.2018E-08	9.2034E-11	1.3576E-13	1.2770E-16	2.2074E-23	2.2170E-31
0.100	.265053	.047138	3.3626E-03	1.5036E-05	4.9518E-08	1.1531E-10	1.8155E-13	1.8105E-16	3.6988E-23	4.6555E-31

TABLE A1 (CONTINUED)

σ	0.99	0.975	0.95	0.90	0.85	γ	0.75	0.70	0.60	0.50
0.110	.299307	.060131	5.3753E-03	3.9217E-05	2.1715E-07	8.7746E-10	2.4840E-12	4.4904E-15	3.847E-21	2.4370E-24
0.120	.332376	.073864	7.4546E-03	8.7172E-05	7.4443E-07	4.7615E-09	2.1980E-11	6.9994E-14	1.445E-19	4.6531E-26
0.130	.366275	.088128	1.1074E-02	1.1414E-04	5.1116E-06	6.9921E-08	1.3908E-10	6.8924E-13	4.882E-18	3.968E-24
0.140	.395062	.102761	1.4771E-02	3.0621E-04	5.1616E-06	6.7922E-08	1.3908E-10	4.8824E-13	4.882E-18	1.781E-22
0.150	.424787	.117634	1.8950E-02	5.0624E-04	1.2000E-05	1.9478E-07	2.6631E-09	2.6782E-11	9.219E-16	4.4524E-21
0.160	.453166	.132646	2.3591E-02	7.8611E-04	2.2064E-05	4.9022E-07	4.8373E-08	1.1447E-10	7.752E-15	8.7220E-20
0.170	.481876	.147719	2.8449E-02	1.1594E-03	4.0136E-05	1.1344E-06	8.8373E-08	4.401E-10	5.074E-14	1.111E-18
0.180	.507754	.162790	3.4043E-02	1.6349E-03	6.8320E-05	2.3540E-06	1.6321E-08	4.401E-10	5.074E-14	1.111E-18
0.190	.533774	.177815	3.9849E-02	2.2311E-03	1.0988E-04	4.5240E-06	1.6147E-08	4.0115E-09	2.663E-13	1.0740E-17
0.200	.558866	.192756	4.5910E-02	2.9497E-03	1.6882E-04	8.1451E-06	3.2219E-07	1.0264E-08	4.612E-12	5.0687E-16
0.210	.584237	.207587	5.2228E-02	3.7909E-03	2.4880E-04	1.3867E-05	6.4162E-07	2.4414E-08	1.567E-11	2.6623E-15
0.220	.608873	.222286	5.8770E-02	4.7779E-03	3.5401E-04	2.2494E-05	1.1969E-06	5.2011E-08	4.7104E-11	1.1655E-14
0.230	.629435	.236837	6.5506E-02	5.8959E-03	4.8455E-04	3.4993E-05	2.1151E-06	1.0634E-07	1.2937E-10	4.6684E-14
0.240	.652160	.251230	7.2409E-02	7.1472E-03	6.5644E-04	5.2466E-05	3.5649E-06	2.0146E-07	3.264E-10	1.6401E-13
0.250	.673484	.265454	7.9454E-02	8.5397E-03	8.6152E-04	7.6168E-05	5.7623E-06	3.6481E-07	7.659E-10	5.2112E-13
0.260	.695041	.279506	8.6620E-02	1.0069E-02	1.1074E-03	1.0746E-04	8.9775E-06	6.301E-07	1.6821E-09	1.5150E-12
0.270	.715660	.293382	9.3888E-02	1.1724E-02	1.3974E-03	1.4779E-04	1.3530E-05	1.0413E-06	3.4851E-09	4.0699E-12
0.280	.735770	.307079	.101240	1.3511E-02	1.7345E-03	1.9870E-04	1.9820E-05	1.6865E-06	6.9551E-09	1.0189E-11
0.290	.755396	.320597	.108642	1.5423E-02	2.1213E-03	2.6177E-04	2.8269E-05	2.6187E-06	1.2470E-04	2.3944E-11
0.300	.774564	.333936	.116138	1.7457E-02	2.5401E-03	3.3860E-04	3.9380E-05	3.9490E-06	2.3170E-04	5.3155E-11
0.310	.793296	.347097	.123652	1.9608E-02	3.0528E-03	4.3880E-04	5.3699E-05	5.7959E-06	4.014E-08	1.1259E-10
0.320	.811612	.360083	.131212	2.1869E-02	3.6011E-03	5.3996E-04	7.1824E-05	8.3156E-06	6.7264E-08	2.2524E-10
0.330	.829433	.372895	.138788	2.4239E-02	4.2049E-03	6.7933E-04	9.4394E-05	1.0464E-05	1.0270E-07	4.3531E-10
0.340	.847076	.385536	.146381	2.6705E-02	4.8485E-03	8.1520E-04	1.2208E-04	1.4645E-05	1.7231E-07	8.0804E-10
0.350	.864559	.398009	.153981	2.9274E-02	5.5822E-03	9.8334E-04	1.5550E-04	2.1470E-05	2.6490E-07	1.4480E-09
0.360	.881098	.410317	.161582	3.1933E-02	6.3685E-03	1.1763E-03	1.9567E-04	2.8743E-05	3.9784E-07	2.5120E-09
0.370	.897607	.422463	.169179	3.4673E-02	7.2084E-03	1.3962E-03	2.4304E-04	3.7451E-05	5.8197E-07	4.2302E-09
0.380	.913901	.434451	.176768	3.7504E-02	8.1024E-03	1.6333E-03	2.9848E-04	4.8662E-05	8.4047E-07	6.9311E-09
0.390	.929493	.446284	.184342	4.0407E-02	9.0573E-03	1.9007E-03	3.6273E-04	6.1824E-05	1.1473E-06	1.1073E-08
0.400	.945295	.457965	.191899	4.3348E-02	1.0049E-02	2.1953E-03	4.3655E-04	7.7717E-05	1.4484E-06	1.7282E-08
0.410	.960818	.469498	.199435	4.6340E-02	1.1138E-02	2.5181E-03	5.2073E-04	9.6744E-05	2.528E-06	2.6393E-08
0.420	.975875	.480886	.206947	4.9541E-02	1.2264E-02	2.8697E-03	6.1595E-04	1.1610E-04	3.033E-06	3.9504E-08
0.430	.990474	.493242	.214433	5.2714E-02	1.3444E-02	3.2508E-03	7.2249E-04	1.4523E-04	4.028E-06	5.8032E-08
0.440	1.005026	.503242	.221890	5.5942E-02	1.4678E-02	3.6621E-03	8.4249E-04	1.7550E-04	5.2801E-06	8.3758E-08
0.450	1.019140	.512616	.229316	5.9224E-02	1.5966E-02	4.1039E-03	9.750E-04	2.1132E-04	6.840E-06	1.1899E-07
0.460	1.033425	.525058	.236710	6.2533E-02	1.7305E-02	4.5767E-03	1.1215E-03	2.5007E-04	8.716E-06	1.6644E-07
0.470	1.047288	.535772	.244069	6.5933E-02	1.8695E-02	5.0471E-03	1.2823E-03	2.9417E-04	1.1104E-05	2.2953E-07
0.480	1.060938	.546361	.251394	6.9395E-02	2.0135E-02	5.6162E-03	1.4581E-03	3.401E-04	1.393E-05	3.1213E-07
0.490	1.074482	.556827	.258682	7.2811E-02	2.1622E-02	6.1833E-03	1.6494E-03	4.0301E-04	1.7334E-05	4.191E-07
0.500	1.087427	.567174	.265932	7.6311E-02	2.3184E-02	6.7820E-03	1.8561E-03	4.684E-04	2.1364E-05	5.5739E-07
0.510	1.100880	.577404	.273145	7.9834E-02	2.4735E-02	7.4124E-03	2.0803E-03	5.3706E-04	2.6130E-05	7.3204E-07
0.520	1.113547	.587520	.280319	8.3420E-02	2.6359E-02	8.0744E-03	2.3213E-03	6.1490E-04	3.1705E-05	9.5144E-07
0.530	1.126235	.597525	.287454	8.7019E-02	2.8075E-02	8.7674E-03	2.5793E-03	7.047E-04	3.8192E-05	1.2245E-06
0.540	1.138747	.607422	.294549	9.0644E-02	2.9732E-02	9.4971E-03	2.8552E-03	7.9413E-04	4.5690E-05	1.5613E-06
0.550	1.151092	.617213	.301605	9.4249E-02	3.1480E-02	1.0244E-02	3.1488E-03	8.9424E-04	5.304E-05	1.9733E-06
0.560	1.163273	.626900	.308620	9.7971E-02	3.3266E-02	1.1035E-02	3.4607E-03	1.0072E-03	6.154E-05	2.4732E-06
0.570	1.175294	.636486	.315595	.101664	3.5089E-02	1.1853E-02	3.7913E-03	1.1272E-03	7.535E-05	3.0754E-06
0.580	1.187163	.645973	.322530	.105384	3.6944E-02	1.2700E-02	4.1445E-03	1.2568E-03	8.8014E-05	3.7958E-06
0.590	1.198882	.655364	.329425	.109122	3.8833E-02	1.3577E-02	4.5047E-03	1.3901E-03	1.0224E-04	4.6517E-06
0.600	1.210456	.664660	.336278	.112874	4.0770E-02	1.4484E-02	4.8940E-03	1.5455E-03	1.1827E-04	5.6622E-06

TABLE A1 (CONTINUED)

α	0.99	0.975	0.95	0.90	0.85	γ	0.80	0.75	0.70	0.60	0.50
0.610	1.221890	.673865	.343092	.116644	4.2731E-02	1.5620E-02	5.1025E-03	1.753E-03	1.853E-03	1.360E-04	6.8481E-06
0.620	1.231870	.682379	.349866	.120427	4.6722E-02	1.5384E-02	5.1728E-03	1.857E-03	1.857E-03	1.360E-04	6.8481E-06
0.630	1.241351	.692005	.356599	.124222	4.6743E-02	1.5377E-02	5.1730E-03	1.857E-03	1.857E-03	1.360E-04	6.8481E-06
0.640	1.250386	.700944	.363293	.128028	4.8794E-02	1.4397E-02	6.6379E-03	2.2594E-03	2.019E-04	2.019E-04	1.1692E-05
0.650	1.259395	.709800	.369947	.131845	5.0872E-02	1.3455E-02	7.1219E-03	2.4532E-03	2.453E-04	2.453E-04	1.3823E-05
0.660	1.277081	.718573	.376562	.135671	5.2974E-02	1.2519E-02	7.6253E-03	2.6484E-03	2.648E-04	2.648E-04	1.6260E-05
0.670	1.297749	.727265	.383137	.139504	5.5107E-02	1.1619E-02	8.1481E-03	2.8454E-03	2.845E-04	2.845E-04	1.9034E-05
0.680	1.298700	.735878	.389674	.143354	5.7262E-02	1.0745E-02	8.6902E-03	3.0434E-03	3.043E-04	3.043E-04	2.2179E-05
0.690	1.308739	.744414	.396172	.147192	5.9441E-02	9.897E-03	9.2516E-03	3.2433E-03	3.243E-04	3.243E-04	2.5729E-05
0.700	1.319067	.752874	.402632	.151044	6.1643E-02	9.2073E-03	9.8432E-03	3.4451E-03	3.445E-04	3.445E-04	2.9723E-05
0.710	1.329288	.761260	.409054	.154900	6.3866E-02	8.5273E-03	1.0435E-02	3.6484E-03	3.648E-04	3.648E-04	3.4198E-05
0.720	1.339404	.769574	.415438	.158760	6.6111E-02	7.8405E-03	1.1080E-02	3.8539E-03	3.853E-04	3.853E-04	3.9194E-05
0.730	1.349417	.777816	.421785	.162623	6.8375E-02	7.1736E-03	1.1688E-02	4.0612E-03	4.061E-04	4.061E-04	4.4754E-05
0.740	1.359332	.785988	.428095	.166488	7.0659E-02	6.5013E-03	1.2344E-02	4.2722E-03	4.272E-04	4.272E-04	5.0920E-05
0.750	1.369148	.794093	.434369	.170354	7.2962E-02	5.8304E-03	1.3019E-02	4.4851E-03	4.485E-04	4.485E-04	5.7739E-05
0.760	1.378870	.802130	.440607	.174221	7.5242E-02	5.1617E-03	1.3712E-02	4.6984E-03	4.698E-04	4.698E-04	6.5255E-05
0.770	1.388499	.810102	.446809	.178084	7.7619E-02	4.4951E-03	1.4433E-02	4.9122E-03	4.912E-04	4.912E-04	7.3517E-05
0.780	1.398037	.818009	.452975	.181954	7.9972E-02	3.8204E-03	1.5152E-02	5.1262E-03	5.126E-04	5.126E-04	8.2575E-05
0.790	1.407486	.825854	.459106	.185821	8.2341E-02	3.1461E-03	1.5899E-02	5.3412E-03	5.341E-04	5.341E-04	9.2477E-05
0.800	1.416849	.833636	.465203	.189684	8.4725E-02	2.4655E-03	1.6654E-02	5.5562E-03	5.556E-04	5.556E-04	1.0324E-04
0.810	1.426127	.841358	.471266	.193549	8.7123E-02	1.7849E-03	1.7446E-02	5.7712E-03	5.771E-04	5.771E-04	1.1503E-04
0.820	1.435323	.849020	.477294	.197410	8.9534E-02	1.1041E-03	1.8285E-02	5.9862E-03	5.986E-04	5.986E-04	1.2778E-04
0.830	1.444437	.856624	.483289	.201268	9.1959E-02	4.3434E-03	1.9061E-02	6.2012E-03	6.201E-04	6.201E-04	1.4159E-04
0.840	1.453472	.864171	.489251	.205123	9.4396E-02	3.6624E-03	1.9894E-02	6.4162E-03	6.416E-04	6.416E-04	1.5651E-04
0.850	1.462429	.871661	.495181	.208974	9.6844E-02	2.9815E-03	2.0744E-02	6.6312E-03	6.631E-04	6.631E-04	1.7249E-04
0.860	1.471311	.879096	.501077	.212822	9.9304E-02	2.2971E-03	2.1610E-02	6.8462E-03	6.846E-04	6.846E-04	1.8931E-04
0.870	1.480119	.886477	.506942	.216666	.101775	1.6145E-03	2.2491E-02	7.0612E-03	7.061E-04	7.061E-04	2.0831E-04
0.880	1.488852	.893805	.512775	.220506	.104256	9.8801E-03	2.3349E-02	7.2762E-03	7.276E-04	7.276E-04	2.2844E-04
0.890	1.497515	.901080	.518577	.224331	.106746	9.1424E-03	2.4302E-02	7.4912E-03	7.491E-04	7.491E-04	2.4977E-04
0.900	1.506107	.908304	.524348	.228151	.109246	8.3994E-03	2.5230E-02	7.7062E-03	7.706E-04	7.706E-04	2.7256E-04
0.910	1.514632	.915477	.530088	.231974	.111755	7.6574E-03	2.6174E-02	7.9212E-03	7.921E-04	7.921E-04	2.9687E-04
0.920	1.523089	.922601	.535799	.235815	.114272	6.9154E-03	2.7133E-02	8.1362E-03	8.136E-04	8.136E-04	3.2274E-04
0.930	1.531480	.929676	.541479	.239629	.116797	6.1734E-03	2.8106E-02	8.3512E-03	8.351E-04	8.351E-04	3.5025E-04
0.940	1.539807	.936703	.547130	.243437	.119330	5.4314E-03	2.9093E-02	8.5662E-03	8.566E-04	8.566E-04	3.7945E-04
0.950	1.548070	.943684	.552752	.247239	.121870	4.6894E-03	3.0085E-02	8.7812E-03	8.781E-04	8.781E-04	4.1039E-04
0.960	1.556271	.950617	.558345	.251034	.124416	3.9474E-03	3.1110E-02	8.9962E-03	8.996E-04	8.996E-04	4.4315E-04
0.970	1.564412	.957506	.563909	.254825	.126969	3.2054E-03	3.2182E-02	9.2112E-03	9.211E-04	9.211E-04	4.7777E-04
0.980	1.572492	.964349	.569445	.258604	.129528	2.4634E-03	3.3212E-02	9.4262E-03	9.426E-04	9.426E-04	5.1431E-04
0.990	1.580514	.971149	.574954	.262384	.132093	1.7214E-03	3.4305E-02	9.6412E-03	9.641E-04	9.641E-04	5.5283E-04
1.000	1.588478	.977905	.580435	.266155	.134663	1.0000E-03	3.5380E-02	9.8562E-03	9.856E-04	9.856E-04	5.9338E-04
1.010	1.605165	1.043251	.633006	.303456	.160598	8.9902E-02	4.6657E-02	2.4322E-02	2.432E-03	2.432E-03	1.1237E-03
1.200	1.777082	1.104905	.684762	.375720	.186791	.105245	5.9065E-02	3.2473E-02	3.247E-03	3.247E-03	1.9143E-03
1.300	1.849937	1.163423	.733565	.375720	.213060	.126197	7.2338E-02	3.4273E-02	3.427E-03	3.427E-03	3.0102E-03
1.400	1.869291	1.219187	.780442	.410651	.239282	.143589	8.6317E-02	5.1175E-02	5.117E-03	5.117E-03	4.4431E-03
1.500	1.930593	1.272528	.825587	.447794	.265371	.163287	1.0086E-01	6.1404E-02	6.140E-03	6.140E-03	6.2348E-03
1.600	1.989215	1.323719	.869170	.481811	.291271	.183190	1.1587E-01	7.2645E-02	7.264E-03	7.264E-03	8.3974E-03
1.700	2.055460	1.372991	.911335	.510863	.316944	.203222	1.3124E-01	8.4151E-02	8.415E-03	8.415E-03	1.0935E-02
1.800	2.099584	1.420538	.952211	.542844	.342364	.223320	1.4690E-01	9.6101E-02	9.610E-03	9.610E-03	1.3844E-02
1.900	2.151805	1.466528	.991908	.574183	.373518	.244439	1.6279E-01	1.0841E-01	1.084E-02	1.084E-02	1.7119E-02
2.000	2.202305	1.511103	1.030526	.604909	.392398	.265544	1.7845E-01	1.2101E-01	1.210E-02	1.210E-02	2.0746E-02

TABLE A1 (CONTINUED)

α	0.99	.975	0.95	0.90	0.85	γ	0.80	0.75	0.70	0.60	0.50
.2100	2.251248	1.554368	1.408149	.635041	.617000	.283405	.139222	.195062	.139222	.080630	.024714
.2200	2.298758	1.596493	1.440856	.654630	.641326	.303505	.147027	.211366	.147027	.085859	.029907
.2300	2.344969	1.637511	1.474012	.673699	.661326	.323526	.156319	.227743	.156319	.090857	.033609
.2400	2.389980	1.677528	1.507581	.692274	.680194	.343555	.165619	.244169	.165619	.095405	.038503
.2500	2.433485	1.716618	1.541016	.710393	.700087	.363085	.174968	.260626	.174968	.099406	.043674
.2600	2.475677	1.754847	1.574367	.728068	.719594	.382709	.184267	.277094	.184267	.103236	.049105
.2700	2.516697	1.792275	1.607677	.745327	.738075	.402222	.193519	.293571	.193519	.106780	.054780
.2800	2.556743	1.828986	1.640938	.762119	.756272	.421422	.202721	.310034	.202721	.110292	.060686
.2900	2.595963	1.864938	1.674104	.778479	.772228	.440906	.211821	.326480	.211821	.113500	.066807
.3000	2.634409	1.900264	1.707167	.794311	.787228	.460074	.220821	.342899	.220821	.116514	.073131
.3100	2.671311	1.934974	1.740316	.809604	.799332	.479126	.229721	.359288	.229721	.119426	.079685
.3200	2.707671	1.969104	1.773351	.824311	.803328	.498062	.238521	.375640	.238521	.122236	.086336
.3300	2.743570	2.002688	1.806349	.838411	.817228	.516884	.247221	.391951	.247221	.125046	.093194
.3400	2.779033	2.035755	1.839384	.851911	.831128	.535593	.255921	.408220	.255921	.127856	.100208
.3500	2.814084	2.068333	1.872419	.864911	.844128	.554190	.264621	.424442	.264621	.130666	.107369
.3600	2.848743	2.100447	1.905454	.877411	.857128	.572677	.273321	.440666	.273321	.133476	.114686
.3700	2.883003	2.132122	1.938489	.889911	.869328	.591057	.282021	.456890	.282021	.136286	.122096
.3800	2.916863	2.163379	1.971524	.902411	.881528	.609457	.290721	.473114	.290721	.139096	.129645
.3900	2.950323	2.194237	1.634621	.914911	.893728	.627802	.299421	.489338	.299421	.141906	.137308
.4000	2.983389	2.224716	1.666162	.927411	.905928	.645814	.308121	.505562	.308121	.144716	.145078
.4100	3.016049	2.254833	1.697702	.939911	.918128	.663814	.316821	.521786	.316821	.147526	.152899
.4200	3.048309	2.284604	1.729237	.952411	.930328	.681814	.325521	.538010	.325521	.150336	.160910
.4300	3.080169	2.314045	1.760772	.964911	.942528	.699814	.334221	.554234	.334221	.153146	.169021
.4400	3.111615	2.343168	1.792307	.977411	.954728	.717814	.342921	.570458	.342921	.155956	.177132
.4500	3.142661	2.371988	1.823842	.989911	.966928	.735814	.351621	.586682	.351621	.158766	.185243
.4600	3.173307	2.400517	1.855377	.100000	.979128	.753814	.360321	.602906	.360321	.161576	.193354
.4700	3.203553	2.428767	1.886912	.102500	.991328	.771814	.369021	.619130	.369021	.164386	.201465
.4800	3.233409	2.456747	1.918447	.105000	.100000	.789814	.377721	.635354	.377721	.167196	.209576
.4900	3.262865	2.484470	1.949982	.107500	.102500	.807814	.386421	.651578	.386421	.169996	.217687
.5000	3.291921	2.511943	1.981517	.110000	.105000	.825814	.395121	.667802	.395121	.172806	.225798
.5100	3.320577	2.539177	1.965359	.112500	.107500	.843814	.403821	.684026	.403821	.175616	.233909
.5200	3.347724	2.566180	1.969794	.115000	.110000	.861814	.412521	.699990	.412521	.178426	.242020
.5300	3.374204	2.592959	1.994042	.117500	.112500	.879814	.421221	.715994	.421221	.181236	.250131
.5400	3.400930	2.619524	2.018103	.120000	.115000	.897814	.429921	.732018	.429921	.184046	.258242
.5500	3.426928	2.645880	2.041999	.122500	.117500	.915814	.438621	.748042	.438621	.186856	.266353
.5600	3.452185	2.672036	2.065720	.125000	.120000	.933814	.447321	.764066	.447321	.189666	.274464
.5700	3.477441	2.697997	2.089276	.127500	.122500	.951814	.456021	.780090	.456021	.192476	.282575
.5800	3.502697	2.723769	2.112674	.130000	.125000	.969814	.464721	.796114	.464721	.195286	.290686
.5900	3.527953	2.749360	2.135918	.132500	.127500	.987814	.473421	.812138	.473421	.198096	.298797
.6000	3.553209	2.774773	2.159162	.135000	.130000	.100000	.482121	.828162	.482121	.200906	.306908
.6100	3.578465	2.800016	2.182406	.137500	.132500	.102500	.490821	.844186	.490821	.203716	.315019
.6200	3.603721	2.825091	2.206769	.140000	.135000	.105000	.499521	.860210	.499521	.206526	.323130
.6300	3.628977	2.850006	2.227441	.142500	.137500	.107500	.508221	.876234	.508221	.209336	.331241
.6400	3.654233	2.874764	2.249980	.145000	.140000	.110000	.516921	.892258	.516921	.212146	.339352
.6500	3.679489	2.899370	2.272389	.147500	.142500	.112500	.525621	.908282	.525621	.214956	.347463
.6600	3.704745	2.923827	2.294673	.150000	.145000	.115000	.534321	.924306	.534321	.217766	.355574
.6700	3.729999	2.948141	2.316835	.152500	.147500	.117500	.543021	.940330	.543021	.220576	.363685
.6800	3.755255	2.972314	2.338878	.155000	.150000	.120000	.551721	.956354	.551721	.223386	.371796
.6900	3.780511	2.996352	2.360806	.157500	.152500	.122500	.560421	.972378	.560421	.226196	.379907
.7000	3.805767	3.020256	2.382620	.160000	.155000	.125000	.569121	.988402	.569121	.229006	.388018

TABLE A1 (CONTINUED)

α	0.99	0.975	0.95	0.90	0.85	γ	0.75	0.70	0.60	0.50
.7100	3.901764	3.044032	2.404325	1.776289	1.416732	1.166522	.976091	.823473	.589776	.416733
.7200	3.927444	3.067681	2.425922	1.795368	1.434024	1.182351	.990641	.836860	.601053	.426062
.7300	3.953383	3.091207	2.447415	1.814364	1.451254	1.198133	1.005157	.850226	.612331	.435412
.7400	3.978985	3.114614	2.468806	1.833286	1.468424	1.213859	1.019640	.863109	.623609	.444780
.7500	4.004452	3.137904	2.490098	1.852130	1.485533	1.229561	1.034091	.876895	.634968	.454167
.7600	4.029787	3.161080	2.511292	1.870899	1.502585	1.245208	1.048511	.890199	.646166	.463571
.7700	4.054995	3.184144	2.532392	1.889594	1.519580	1.260811	1.062899	.903493	.657444	.472993
.7800	4.080078	3.207100	2.553399	1.908218	1.536519	1.276373	1.077256	.916740	.668721	.482430
.7900	4.105039	3.229949	2.574315	1.926773	1.553403	1.291892	1.091583	.929991	.679996	.491883
.8000	4.129879	3.252695	2.595144	1.945254	1.570234	1.307371	1.105881	.943215	.691271	.501351
.8100	4.154603	3.275339	2.615885	1.963677	1.587013	1.322810	1.120149	.956420	.702545	.510834
.8200	4.179214	3.297884	2.636542	1.982031	1.603741	1.338209	1.134388	.969607	.713816	.520330
.8300	4.203712	3.320331	2.657117	2.000321	1.620419	1.353570	1.148600	.982774	.725086	.529840
.8400	4.228102	3.342684	2.677610	2.018547	1.637046	1.368894	1.162784	.995923	.736355	.539462
.8500	4.252384	3.364944	2.698024	2.036713	1.653626	1.384180	1.176940	1.009054	.747621	.548897
.8600	4.276562	3.387113	2.718360	2.054814	1.670159	1.399429	1.191070	1.022164	.758884	.558444
.8700	4.300638	3.409192	2.738621	2.072864	1.686644	1.414843	1.205173	1.035261	.770146	.568003
.8800	4.324613	3.431185	2.758807	2.090852	1.703086	1.429822	1.219250	1.048334	.781405	.577572
.8900	4.348490	3.453092	2.778920	2.108784	1.719483	1.444966	1.233302	1.061398	.792661	.587152
.9000	4.372270	3.474915	2.798961	2.126660	1.735835	1.460076	1.247328	1.074441	.803815	.596743
.9100	4.395956	3.496656	2.818932	2.144482	1.752145	1.475153	1.261330	1.087467	.815166	.606344
.9200	4.419550	3.518317	2.838835	2.162250	1.768412	1.490198	1.275308	1.100474	.826415	.615954
.9300	4.443052	3.539898	2.858670	2.179966	1.784638	1.505210	1.289261	1.113448	.837660	.625574
.9400	4.466466	3.561402	2.878439	2.197630	1.800823	1.520190	1.303191	1.126445	.848902	.635202
.9500	4.489793	3.582830	2.898144	2.215244	1.816969	1.535140	1.317098	1.139406	.860142	.644839
.9600	4.513034	3.604184	2.917784	2.232804	1.833075	1.550058	1.330981	1.152350	.871378	.654485
.9700	4.536191	3.625464	2.937362	2.250323	1.849142	1.564947	1.344843	1.165278	.882611	.664139
.9800	4.559264	3.646673	2.956879	2.267790	1.865172	1.579806	1.358682	1.178190	.893841	.673801
.9900	4.582257	3.667811	2.976355	2.285211	1.881154	1.594636	1.372499	1.191090	.905067	.683470
1.0000	4.605170	3.688879	2.995732	2.302585	1.897120	1.609438	1.386294	1.203974	.916291	.693147
1.0000	4.830207	3.886009	3.186665	2.473953	2.054791	1.755974	1.523133	1.332016	1.028337	.794275
1.2000	5.046610	4.097368	3.372663	2.641460	2.209386	1.900089	1.658130	1.458746	1.140034	.887936
1.3000	5.261312	4.293777	3.554429	2.805648	2.361325	2.042103	1.791513	1.584312	1.251383	.985999
1.4000	5.469554	4.485882	3.732515	2.966944	2.510947	2.162274	1.924766	1.708839	1.362394	1.084372
1.5000	5.672434	4.674202	3.907365	3.125694	2.658524	2.320814	2.054173	1.832435	1.473083	1.182987
1.6000	5.871938	4.859161	4.079340	3.282184	2.804280	2.478988	2.183735	1.955190	1.583463	1.281796
1.7000	6.067967	5.041111	4.248742	3.436646	2.948404	2.636755	2.312733	2.077183	1.693566	1.380762
1.8000	6.260862	5.220348	4.415828	3.589282	3.091050	2.782655	2.438884	2.198480	1.803395	1.479857
1.9000	6.450908	5.397120	4.580807	3.740258	3.232358	2.921780	2.566647	2.319141	1.912973	1.579058
2.0000	6.638354	5.571643	4.738866	3.889720	3.372440	2.994308	2.692635	2.439216	2.022313	1.678347
2.1000	6.823411	5.744103	4.905160	4.037792	3.511401	3.125932	2.817907	2.558751	2.131432	1.777712
2.2000	7.006262	5.914661	5.064831	4.184584	3.649324	3.256717	2.942518	2.677785	2.240341	1.877141
2.3000	7.187075	6.083457	5.223002	4.330193	3.786287	3.386729	3.068517	2.796353	2.349954	1.974625
2.4000	7.365991	6.250616	5.379776	4.474698	3.922359	3.516020	3.189945	2.914488	2.457581	2.074157
2.5000	7.543136	6.416252	5.535249	4.618178	4.057600	3.646373	3.312840	3.032216	2.565934	2.175730
2.6000	7.718626	6.580461	5.689507	4.760700	4.192063	3.772829	3.435236	3.149562	2.674120	2.275339
2.7000	7.892568	6.743333	5.842625	4.902326	4.325799	3.900030	3.557164	3.266552	2.782151	2.379980
2.8000	8.065047	6.904947	5.994674	5.043103	4.458850	4.026877	3.676552	3.383203	2.890033	2.474650
2.9000	8.236147	7.065379	6.145709	5.183088	4.591255	4.153199	3.799724	3.499536	2.997774	2.574344
3.0000	8.405951	7.224687	6.295794	5.322320	4.723050	4.279030	3.920403	3.615563	3.105379	2.674061

TABLE A1 (CONTINUED)

D	0.99	.975	0.95	0.90	0.85	γ	0.80	0.75	0.70	0.60	0.50
3.1000	6.574519	7.382939	6.444977	5.460843	4.854271	4.440393	4.060710	3.731314	3.421857	3.12857	2.77377
3.2000	6.741913	7.540188	6.593303	5.590691	4.984944	4.552312	4.160664	3.846189	3.532213	3.23951	2.87351
3.3000	6.908206	7.696491	6.740820	5.725901	5.115098	4.653811	4.260279	3.946207	3.627453	3.32452	2.95732
3.4000	7.073433	7.851864	6.887563	5.872497	5.244759	4.777909	4.395579	4.076978	3.753582	3.441604	3.07317
3.5000	7.237654	8.006381	7.033568	6.008518	5.373928	4.902425	4.518574	4.191715	3.868593	3.554604	3.17297
3.6000	7.400913	8.160075	7.178875	6.139965	5.502689	5.026977	4.637277	4.306429	3.983524	3.668524	3.27217
3.7000	7.563252	8.312978	7.323508	6.278921	5.631001	5.147980	4.755703	4.420330	4.095346	3.779528	3.37258
3.8000	7.724709	8.465125	7.467498	6.413355	5.758900	5.272650	4.873863	4.534521	4.2082074	3.882074	3.47230
3.9000	7.885325	8.616545	7.610876	6.547301	5.886407	5.393001	4.991767	4.648519	4.3068712	3.972211	3.57201
4.0000	8.045116	8.767274	7.753557	6.680763	6.013537	5.513066	5.109481	4.762229	4.4582629	4.175263	3.67201
4.1000	8.204140	8.917333	7.895974	6.813819	6.140305	5.634796	5.226851	4.875757	4.572728	4.281728	3.77198
4.2000	8.362412	9.066731	8.037546	6.948426	6.268724	5.755266	5.344051	4.968110	4.681118	4.386118	3.87193
4.3000	8.519905	9.215520	8.178891	7.078618	6.392611	5.879458	5.461033	5.102295	4.794426	4.490559	4.07153
4.4000	8.676814	9.363754	8.319335	7.210415	6.518569	6.000392	5.578805	5.215319	4.896594	4.600659	4.17153
4.5000	8.832998	9.511386	8.459489	7.341826	6.644020	6.121073	5.694376	5.328916	4.998820	4.706820	4.27147
4.6000	8.988535	9.658461	8.599174	7.472874	6.769170	6.241509	5.810751	5.449305	5.128905	4.818911	4.37135
4.7000	9.143445	9.805005	8.738404	7.603556	6.894031	6.351711	5.926936	5.553474	5.149937	4.818937	4.47120
4.8000	9.297752	9.951026	8.877196	7.733902	7.018612	6.461687	6.042944	5.658904	5.262892	4.918101	4.57102
4.9000	9.451474	10.096550	9.015564	7.863905	7.142921	6.581438	6.158772	5.778199	5.378786	5.027092	4.67099
5.0000	9.604626	10.241591	9.153519	7.993589	7.266984	6.7070979	6.274431	5.890336	5.489336	5.136618	4.77099
5.5000	12.352466	10.860025	9.837569	8.637505	7.883548	7.315710	6.850347	6.449336	5.764916	5.170499	4.57049
6.0000	13.18487	11.66833	10.513032	9.274674	8.494657	7.905993	7.422702	7.005551	6.791919	5.70102	4.57012
6.5000	13.44121	12.367804	11.181017	9.905965	9.100988	8.492397	7.991953	7.559261	7.334666	6.169878	4.57012
7.0000	14.570621	13.059460	11.842396	10.532072	9.703121	9.075384	8.558467	8.111049	7.734666	6.669636	4.57012
7.5000	15.288966	13.744197	12.497896	11.170915	10.301504	9.655329	9.12543	8.660847	8.208447	7.166430	4.57012
8.0000	16.744332	15.095506	13.146115	12.384520	11.094529	10.242544	9.284431	8.755507	8.382768	7.166430	4.57012
8.5000	17.402656	15.763192	14.434346	12.994712	11.868514	10.877736	10.244338	9.755507	9.433952	8.669636	4.57012
9.0000	18.085438	16.426164	15.071766	13.601792	12.664427	11.950209	11.358903	10.844563	9.955096	9.166827	4.57012
9.5000	18.763123	17.084810	15.705213	14.265991	13.249793	12.518752	11.913846	11.387274	10.475684	9.664716	4.57012
10.0000	19.446684	17.790357	16.365222	14.936642	14.111228	13.650727	13.019633	12.446598	11.515331	10.668524	4.57012
11.0000	20.144684	18.390357	16.965222	15.618123	14.811228	14.170661	14.120579	13.547985	12.553176	11.66833	4.57012
12.0000	21.449914	19.62034	17.781566	16.581823	16.147434	15.879311	15.217283	14.623163	13.580441	12.66829	4.57012
13.0000	22.820845	20.961593	19.444570	18.957964	17.857499	17.013283	16.310247	15.629537	14.62309	13.66815	4.57012
14.0000	24.139129	22.303098	20.666571	20.18012	18.995128	18.115309	17.398979	16.765117	15.457928	14.668016	4.57012
15.0000	25.446105	23.489644	21.684488	21.252374	20.128149	19.253157	18.486495	17.832955	16.497930	15.667930	4.57012
16.0000	26.742691	24.740213	22.3097125	22.42374	21.57004	20.339396	20.651808	19.960991	18.7274915	16.667854	4.57012
17.0000	28.030459	25.982992	23.4301184	22.451580	22.8606231	21.533396	21.730954	21.022595	20.411097	19.66763	4.57012
18.0000	29.309600	27.218645	24.499231	23.666091	24.076929	22.503396	22.808003	22.082338	20.811097	19.66763	4.57012
19.0000	30.581062	28.447777	25.6691771	24.756292	25.30269	23.631261	22.808003	22.082338	20.811097	19.66763	4.57012
20.0000	31.845381	29.670688	27.072242	25.992540	26.421929	24.814261	22.808003	22.082338	20.811097	19.66763	4.57012
21.0000	33.103128	30.883343	28.062421	26.1845102	27.537296	25.814261	22.808003	22.082338	20.811097	19.66763	4.57012
22.0000	34.354764	32.100744	29.062421	26.1845102	28.642910	26.814261	22.808003	22.082338	20.811097	19.66763	4.57012
23.0000	35.600707	33.306282	30.062421	26.1845102	29.748556	27.814261	22.808003	22.082338	20.811097	19.66763	4.57012
24.0000	36.841330	34.511301	31.062421	26.1845102	30.854202	28.814261	22.808003	22.082338	20.811097	19.66763	4.57012
25.0000	38.076954	35.710101	32.062421	26.1845102	31.959854	29.814261	22.808003	22.082338	20.811097	19.66763	4.57012
26.0000	39.307887	36.904948	33.062421	26.1845102	33.065506	30.814261	22.808003	22.082338	20.811097	19.66763	4.57012
27.0000	40.534399	38.096095	34.062421	26.1845102	34.171158	31.814261	22.808003	22.082338	20.811097	19.66763	4.57012
28.0000	41.756736	39.283566	35.062421	26.1845102	35.276810	32.814261	22.808003	22.082338	20.811097	19.66763	4.57012
29.0000	42.975095	40.467809	36.062421	26.1845102	36.382462	33.814261	22.808003	22.082338	20.811097	19.66763	4.57012
30.0000	44.189717	41.648842	37.062421	26.1845102	37.488114	34.814261	22.808003	22.082338	20.811097	19.66763	4.57012

TABLE A1 (CONTINUED)

α	γ									
	0.99	.975	0.95	0.90	0.85	0.80	0.75	0.70	0.60	0.50
31.0000	45.400776	42.826876	40.690505	38.315117	36.766887	35.562657	34.552154	33.661009	32.090473	30.667311
32.0000	46.608444	44.002030	41.837637	39.429823	37.857699	36.638052	35.612575	34.707970	33.113172	31.667291
33.0000	47.812869	45.174458	42.982466	40.542744	38.949060	37.712247	36.672057	35.754229	34.135515	32.667272
34.0000	49.014226	46.344288	44.125085	41.653939	40.039020	38.785329	37.730619	36.799770	35.157496	33.667258
35.0000	50.212588	47.511599	45.265616	42.763521	41.127677	39.857328	38.788329	37.844639	36.179174	34.667230
36.0000	51.408187	48.676511	46.404120	43.871508	42.215054	40.948296	39.845226	38.888883	37.200535	35.667222
37.0000	52.601029	49.839183	47.540735	44.978029	43.301215	41.998266	40.901293	39.932480	38.221599	36.667205
38.0000	53.791297	50.999632	48.675488	46.083084	44.384209	43.087307	41.956629	40.975497	39.242375	37.667198
39.0000	54.979045	52.157977	49.808471	47.186744	45.470082	44.135427	43.011231	42.017968	40.262888	38.667178
40.0000	56.164393	53.314291	50.939746	48.289090	46.552879	45.202675	44.065116	43.059856	41.283126	39.667166
41.0000	57.347484	54.468656	52.069372	49.390155	47.634639	46.299059	45.118356	44.101228	42.303114	40.667153
42.0000	58.528289	55.621137	53.197424	50.490013	48.715385	47.334670	46.170926	45.142096	43.322840	41.667142
43.0000	59.706962	56.771796	54.323951	51.588637	49.795186	48.339482	47.222881	46.182505	44.342367	42.667131
44.0000	60.883584	57.920722	55.449005	52.686107	50.874063	49.423514	48.274212	47.222420	45.361649	43.667121
45.0000	62.058173	59.067953	56.572638	53.782505	51.952057	50.546864	49.324967	48.261888	46.380712	44.667110
46.0000	63.230868	60.213551	57.694899	54.877815	53.029132	51.589485	50.375165	49.300905	47.399561	45.667100
47.0000	64.401640	61.357533	58.815828	55.972092	54.105397	52.651420	51.424800	50.339508	48.418206	46.667091
48.0000	65.570640	62.500050	59.935454	57.065355	55.186848	53.712694	52.473897	51.377691	49.436650	47.667082
49.0000	66.737841	63.641014	61.053858	58.157640	56.255501	54.773339	53.522507	52.415459	50.454918	48.667073
50.0000	67.903375	64.780609	62.171059	59.249005	57.329410	55.833358	54.570622	53.452881	51.472973	49.667065