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SENSITIVITY OF POWER PEAKING ANALYSIS TO LARGE REACTOR CORE MODELING

by

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ABSTRACT

Various models of large LMFBRs, based on cylindrical and hexagonal geometries, are examined in regard to application to nuclear power peaking analysis. It is shown that the general behavior of power distributions during burnup in these large reactors implies that power shaping by control rod movement is desirable for minimizing the peaking factor. Due to current limitations of available three-dimensional computer codes, one- and two-dimensional models were used to test general assumptions of power shaping. However, it is demonstrated that these models are not appropriate for more accurate power peaking analysis. Comparison of results for nuclear power peaking factors in cylindrical and hexagonal geometries shows that the use of annular rings of control poison in the cylindrical models does not provide a realistic representation of banks of control rod assemblies. Similarly, a comparison of 2-D and 3-D models makes it clear that axial smearing of control, implied in 2-D planar models, also yields misleading results for both the location and magnitudes of peaks in the power density. It is concluded that a 3-D model is necessary not only for accurate determination of power peaking factors (to better than 5 or 10% accuracy for partially inserted control rods) but also for analysis of power shaping by means of control rod movement.

I. INTRODUCTION

The usefulness of a calculational model depends in large part on the purpose for which it is intended and the cost of the computations involved. For many design considerations and/or parametric studies of large LMFBRs, one- and two-dimensional cylindrical models and two-dimensional hexagonal models have provided suitable representations. Basically, their usefulness lies in their geometric symmetries which approximate the actual symmetries of the reactor and which simplify computer calculations and thereby reduce their cost. For the analysis of nuclear power peaking factors, however, any lack of correspondence between the symmetries of the model and the actual reactor may have an important bearing on the accuracy of the results. This is because the power peaking factor, defined as the ratio of the peak to average power in the reactor core, is determined by local behavior of the power distribution, rather than properties which can be averaged over portions of the core.

For this reason, a study was undertaken of the capabilities and limitations of various geometric models of large LMFBRs in regard to power peaking analysis. The models considered were based on the following geometries:

- one-dimensional, cylindrical (r geometry);

- two-dimensional, cylindrical (r-z geometry);

- two-dimensional, hexagonal (x-y rhombic-geometry);

- three-dimensional, hexagonal (x-y rhombic -- z geometry, or "hex-z" geometry).

In evaluating the limitations of these models, it is useful to keep in mind the general behavior of power distributions in large LMFBRs.

Results from earlier studies of 1-D and 2-D models of uncontrolled cores are briefly reviewed in Section II. Based on these results the conclusion is drawn that control rod movement plays an important role in power shaping and thus in the minimization of the power peaking factor. Therefore, the simulation of control rod banks in the various models was examined.

In cylindrical models, annular rings of control poison are used to simulate banks of control rods, while in the more realistic hexagonal models, they are represented by hexagonal assembly positions. Section III contains a discussion of the sensitivity of the power peaking analysis to the use of cylindrical vs. hexagonal geometries.

The basic limitation of the "planar" models, i.e., 1-D cylindrical or 2-D hexagonal is the effective homogeneous "smearing" of control poison along the z-axis. Although discrete control rod insertion can be simulated in 3-D models, current computational limitations prevented the carrying out of desired "benchmark" calculations necessary for a detailed comparison of 2-D and 3-D models. Within the context of current 3-D computational capabilities, however, a comparison was made. The results are discussed in Section IV.

A summary of the study and resulting conclusions are contained in Section V.

II. BEHAVIOR OF POWER DISTRIBUTIONS IN LARGE LMFBRs

Previous studies^{1,2} utilizing one- and two-dimensional models have yielded information on the general behavior of power distribution in large uncontrolled LMFBRs in two respects: (1) as a function of radial enrichment zoning, and (2) changes in the distribution during burnup. These analyses were based on 5000 Mwt carbide reactors and included both reactors which gain and lose reactivity during the burn cycle. We review here the properties pertinent to power peaking analysis.

Results of the zoning analysis (two radial zones of different plutonium enrichments) showed that the nuclear power peaking factor is relatively insensitive to the ratios of the volumes of the inner and outer cores, but highly sensitive to the ratio of enrichments of each zone. For static, or beginning-of-cycle, conditions, minimization of the power peaking factor implies equal peak power densities in the inner and outer cores, as indicated for a cylindrical core in Fig. 1. This implies that $\epsilon_2/\epsilon_1 > 1$, where ϵ_2 , ϵ_1 are the plutonium enrichments of the outer and inner core, respectively. It was found that a change of 2% away from the optimum enrichment split caused as much as a 5% variation in the power peaking factor (defined as the peak/average power in the core). This sensitivity to very small changes in average enrichments is indicative of the effects that small changes in core properties can have on the power shape in the large LMFBR.

Along this line are the changes in core properties that take place during burnup. The ratio $\epsilon_2/\epsilon_1 > 1$ generally implies that the conversion ratio of the inner core is greater than the conversion ratio of the outer

core. For the large LMFBR, the inner-core conversion ratio is greater than unity. As a result, if the total power of the reactor is held constant during burnup, the power density increases in the inner core and decreases in the outer core. This is illustrated in Fig. 2 which displays the midplane power distribution at beginning of cycle, midcycle, and at the end of cycle in a cylindrical model. At $r=0$ cm (center of core), the power density takes on the values 354, 435, and 510 watts/cm³ at BOC, midcycle, and EOC, respectively. In the outer core, the peak density is 510, 440, and 300 watts/cm³ at BOC, midcycle, and EOC, respectively. Peaks in the power density near the core center and in the midplane of the inner core, rather than flattening out with burnup (as in smaller reactors), become more pronounced.

In the present study, the effect of modeling on the shape of the static midplane power distribution was considered by comparing the 1-D cylindrical and the 2-D hexagonal models (see Section III for details). To complete our discussion of the general properties of power distributions in uncontrolled cores, we examine the midplane power distribution typical of the two models. In Fig. 1, taken from Ref. 1, minima of the radial power peaking factor are determined for a cylindrical model for several choices of the ratio of outer to total core volume. The first and second legs of the "vee" - shaped curves correspond to the power peaking in the inner and outer core, respectively. The minima occur when the midplane radial power distribution displays equal peak densities at the core center and in the ring bordering the boundary between the inner and outer core.¹

Corresponding values of the peaking factor for the core represented in the figure ranged from 1.42 to 1.49 (when the axial peaking factor is taken into account).

The azimuthal symmetry of the power distribution in the cylindrical model is severely broken in the more realistic hexagonal models in the outer regions of the core. Figure 3 displays a typical plot of the optimized midplane power distribution in the hexagonal geometry for a 5000 MWt uncontrolled carbide reactor. Plotted numbers are the ratios of the peak power in an assembly to the average power in the core. The power distribution has equal peaks at the core center and in the outer core at six symmetrical positions and yields a power peaking factor of 1.49. Although this number is in the same range as for the cylindrical model of Fig. 1 (the values cannot be compared directly for they represent different carbide reactors), the general power shape is very different in the two geometries. Azimuthally the power density of the hexagonal model displays maxima along the six axes which lie perpendicular to the sides of the hexagonally shaped reactor. For example, in row 12 (first row of the outer core), the peak/average power parameter of the individual assemblies varies from a minimum of 1.11 to a maximum of 1.49; in row 11 it ranges from 1.13 to 1.33.

The beginning-of-cycle power distribution and its behavior during burnup suggests that in order to both flatten the power distribution at BOC and maintain its shape during burnup, optimum configurations of control-rod placement and movement should be sought. There are several constraints which when taken into account restrict the number of possible configurations of control rod banks; for example, the maintenance of

hexagonal symmetry, reasonable spacing of control rods, behavior of power distribution during burnup.

For reactors which gain reactivity, the last consideration suggests that the bulk of the control rod positions should be placed in the inner core and along the six hexagonal axes perpendicular to the sides of the reactor. Since control is added during burnup, such configuration will help prevent the power density from increasing in the inner core and thus also help to maintain the power density in the outer core. This is illustrated in Table I where results are given for a 5000 MWt carbide reactor. All of the 19 control-rod positions are located in the inner core of this 2-D hexagonal model and all but six are along the hexagonal axes discussed above. The power distribution, which was optimized at BOC, is closely maintained throughout burnup, with the power peaking factor varying by less than 0.5% over the 300 day burn cycle. Inner and outer core peak power densities shift very little during the burn cycle; they are located at positions (61) and (126), respectively, at BOC and at positions (51) and (126) at EOC (see Fig. 4 for definitions of assembly positions). Another method of maintaining the power density in the outer core might include control rods in the outer core which are partially inserted at BOC and withdrawn during burnup. Schemes of this type, involving control rods included for power shaping purposes only, were not investigated in the present study.

For reactors which lose reactivity during burnup, the power density still increases at the core center and decreases in the outer core provided $\epsilon_2/\epsilon_1 > 1$ and the inner core conversion ratio is greater than unity. Control rods placed in the outer core will tend to maintain the

power shape as they are withdrawn during burnup. Table II contains results for a cylindrical model of a 3000 MWt oxide reactor. Control poison is present at beginning of cycle in two annular rings; one in the outer core and the second in the outer region of the inner core. The locations of the two rings and the subsequent removal of control poison during burnup help to maintain the power density in the outer core. As indicated in Table II, the BOC power distribution changes very little during the burn cycle, with a variation in the power peaking factor of only 1%.

III. CYLINDRICAL VERSUS HEXAGONAL MODELING

FOR ANALYSIS OF STATIC POWER SHAPING

In order to investigate in detail the sensitivity of static power peaking analysis to modeling, a 5000 MWt carbide reactor corresponding to 0.40 in. pin diameter and a 30 kW/ft linear heat rating was selected to represent the large LMFBR. This reactor gains reactivity during burnup so that control must be added to maintain criticality. At the beginning of cycle, it was assumed that no control poison was present, but control rod positions were taken into account. The core layout, shown in Fig. 4, consists of 14 rows (rounded) of assemblies with the inner core consisting of the first eleven rows. Other assumptions for the reactor are contained in Table III. Control rod positions consisted of banks of six assemblies each (with the central rod an exception), symmetrically placed in a given hexagonal row.

Several control-rod configurations were considered. For each case, the enrichment split between inner and outer core was chosen to minimize the power peaking factor. Each bank of control-rod positions in a given

row in the hexagonal model was simulated in the cylindrical model by an annular ring. The volume of the annular ring itself equals the volume occupied by the bank of control rods. In turn, the ring encloses a volume of the core equal in magnitude to that enclosed in the hexagonal model by the row of assemblies containing the control rod positions. Since no poison is present at BOC, the control rod positions or rings contain only structural material and coolant. Calculations were performed with the codes DIF1D and DIF2D⁽³⁾ for the cylindrical and hexagonal models, respectively. Table IV contains a comparison of the BOC peak to average power in the core for corresponding 1-D cylindrical and 2-D hexagonal models. The peak/average parameter was larger in the hexagonal model by as much as 7%. However, more pertinent is the lack of correlation of results for corresponding configurations in the two geometries. For example, a cylindrical model with n control rings can correspond to several different hexagonal models containing $6n$ control rods. The power peaking factors and locations of peaks in the power density will be different in these hexagonal models, as illustrated in Table IV where two hexagonal cores with control rods in the same rows but located in different positions differ in the peaking factor by 2%. A more striking result displayed in the Table is the following: when one of the control rings is shifted in the cylindrical model to yield an improvement in the power peaking factor, the corresponding hexagonal model peaking factor is raised by 4%. The lack of correspondence between the two models is expected to become even more pronounced when control rod insertion is considered.

The cylindrical model does not appear to be capable of predicting optimum BOC control rod configurations in the hexagonal geometry. If it is used to estimate nuclear power peaking factors, the results may be underestimated by at least 7%. However, the cylindrical model is not excluded as a useful tool for understanding very general features of power peaking analysis (see, for example, Section II).

IV. TWO-DIMENSIONAL VERSUS THREE-DIMENSIONAL MODELING OF CONTROLLED REACTORS

The diffusion code DIF3D⁽⁵⁾ is available for neutronic calculations based on hexagonal-z geometry, but it is currently limited in the total number of z-axis mesh points that can be used in the calculation. For large LMFBRs, this restriction implies a cruder mesh structure along the z-axis than in the x-y plane (which is fixed by the code to six mesh points per hexagonal assembly).⁽⁶⁾ It was felt, nonetheless, that the present version of the code could yield useful information regarding modeling and power distribution provided the results are not highly sensitive to the mesh spacing along the z-axis. A sensitivity study, described in part (A) of this section, indicated that this was true provided the average z mesh spacing is on the order of 7 cm or less in the core of the reactor.

This mesh spacing was subsequently used to make a brief study of power distribution and control rod worth in the 3-D model with control rods, described in part (B) and then to make a comparison of radial power peaking factors in 2-D and 3-D hexagonal models, described in part (C).

A. Sensitivity Study of Z-Axis Mesh Structure in DIF3D

The 5000 MWt carbide reactor described in Table III and Fig. 4 was chosen as the physical model. A map of an axial cut of the reactor is shown in Fig. 5. The assembly pitch is 18.11 cm. With six mesh points per hexagonal assembly, the mesh point spacing in the x-y plane is on the order of 4 to 5 cm. If there are no control rods present in the reactor, symmetry allows us to use half-core geometry along the z-axis. It is possible, in this case, to use z-mesh-point spacing comparable to that in the x-y plane. A sensitivity study of the z mesh structure was carried out for full-core geometry along the z-axis. The eigenvalue k_{eff} was calculated and compared to the half-core base case for six choices of z-axis mesh-point spacing, as shown in Fig. 6. The convergence criterion for k_{eff} was 10^{-7} in all cases. As a practical limitation in running the DIF3D code, an upper level of 2000k of computer core was chosen. The base case corresponds to 41 z-axis mesh points in the full core geometry. In the first full-core geometry calculation, Case A of Fig. 6, 15 mesh points were taken along the z-axis (9 of which are in the reactor core). This yielded a value of k_{eff} differing from the base-case by 0.0064. This difference is approximately three times larger than the worth of a control rod placed at the center of the reactor (see Part B of this section). In subsequent calculations, mesh points were added along the z-axis until the limit of computer core discussed above was reached. Cases B, C, and D of Fig. 6 correspond to 19, 23, and 25 z-axis mesh points,

respectively, and yielded $k_{eff} - k_{eff} \text{ (base case)} = 0.0019, 0.0014, \text{ and } 0.0006$, respectively. A further slight improvement was gained by redistributing the 25 z-axis mesh points of case D in order to have a higher density of mesh points in regions where the flux has relatively greater curvature. This is illustrated in Case E and F of Fig. 6.

$k_{eff} - k_{eff} \text{ (base case)} = 0.0004$ in both of these cases. Since the values of $k_{eff} - k_{eff} \text{ (base case)}$ for cases D, E, and F are less than a third of the change in $k_{eff} \text{ (base case)}$ induced by fully inserting a central control rod, these mesh structures were used in the subsequent 3-D calculations to be described.

B. Control Rod Worth and Axial Power Distribution in 3-D Hexagonal Model

In order to make a comparison to 2-D models and make a meaningful evaluation of the results, it is necessary to have some idea of control rod worths and the power distribution in the 3-D model. Calculations were made to determine the worth of a control rod at the center of the core; the changes in k_{eff} as various banks of control rods are partially inserted; and finally, changes in the axial power distribution as control rods are inserted. The last consideration is important, for a comparison of power distributions in 2-D and 3-D models at the midplane of the reactor, say, is meaningful only if there is very little axial asymmetry introduced by insertion of control rods in the 3-D model.

Control rod banks for the model reactor are shown in Fig. 7. There are 19 control rod positions: one central rod (bank 1), six rods in each of rows 5 (bank 5), 8 (bank 8), and 9 (bank 9). The enrichment split for the reactor is close to being optimized, as indicated in the figure by the radial peak to average values (the $z = 12$ plane is about 4 cm below the

midplane of the reactor): The radial peaking factor of the inner core is 3.5% higher than the outer core peaking factor. In order to obtain an estimate of the worth of a single rod, the central rod was completely inserted. The resulting difference in k_{eff} for rod in and rod out was 0.002, as shown in Table V (the convergence criterion is 10^{-7} in all 3-D calculations). Control rod worths were not determined for other regions of the core. However, various patterns of control rod insertion were investigated. The results are given in Table V. We see that, for example, insertion of bank 5 (6 rods) 25% into the core yields the same change in k_{eff} as the single rod inserted 100% into the core. Insertion of bank 5 25% into the core along with 100% insertion of the central rod has an effective worth of 0.003. Finally, if banks 5, 8, and 9 (18 control rods) are inserted 25% into the core, the effective control rod worth is 0.006. Note that the resulting values of k_{eff} vary by 0.0002 or less within the different z-axis mesh point spacings, cases D, E, and F.

Peaks in the power distribution showed very little asymmetry along the z-axis as control rods were inserted from the top of the reactor. This is illustrated in Figs. 8 and 9 where axial plots of the power distribution are given for hex assembly positions (31) and (61) (see Fig. 4), respectively. These assemblies were chosen as representative of fuel assemblies containing near maximum midplane power densities (see, for example, Figs. 7 and 10). In Figs. 8 and 9, peak power densities are shifted downward in the reactor by less than 3 cm as control bank 5 or control rod banks 5, 8, and 9 are inserted a quarter of the way into the core. Figures 10-13 contain hex plots of the peak-

to-average power and power densities for planes symmetrically displaced from the midplane by ± 3.81 cm, respectively, when bank 5 is inserted 25% into the core. Even in assemblies near the control rods the power densities differ by less than 0.7% in the two planes. From the results it may be concluded that the axial power density distribution maintains the same general shape with the maximum close to the midplane even as control rods are inserted.

For a comparison of the radial peak/average power values in the horizontal plane at $z = 92.71$ cm (near midplane of reactor) for the case of no control versus bank 5 inserted 25% into the core, see Figs. 7 and 12. For the uncontrolled core, the inner core peak in the power density is 4% greater than the peak density in the outer core. In the case with control, the reverse is true. Thus it is seen that although control rod insertion only minimally affects the shape of the axial power density distribution, it can change the radial distribution significantly.

C. Comparison of Radial Power Distribution in Corresponding 2-D and 3-D Models

The 3-D hexagonal model with control rod bank 5 inserted 25% into the core was chosen for a comparison of the midplane power distribution in the corresponding 2-D hexagonal model. The mesh structure F of Fig. 6 was used for the 3-D calculations. In the corresponding 2-D model,⁷ the control poison was added in bank 5 until k_{eff} had the same value in both models, namely, $k_{\text{eff}} = 0.99825$. This was obtained with a volume fraction of control poison of 0.1200. The increase in control worth in the 2-D model, reflected by this change from 25% control-rod insertion in the 3-D model to 12% control-rod composition in the 2-D model, is due to the homogeneous smearing of control poison axially throughout the control rod assembly position. This smearing of poison reduces shielding

factors and also distributes control poison in regions of higher flux (at midplane). The resulting ratios of the peak power in an assembly to the power of the core appear in Fig. 14. The corresponding plot for the 3-D model is shown in Fig. 12. In order to facilitate the comparison, the difference in the peak/average parameter of the 3-D and 2-D model is plotted in Fig. 15. The parameter changes 8% over a large control region of the reactor comprised of the central 30 fuel assemblies. A 5% difference appears in and around hex position (61) which corresponds to the inner core peak in the power distribution in the 3-D model. Analyzing the plots in more detail, we find that the control rod insertion in the 2-D model has a much more pronounced effect on the midplane power distribution than in the 3-D model. The control poison in bank 5, which is near the center of the reactor, tends to push the higher power density regions toward the outer core. In the 3-D model, the partial insertion of bank 5 has depressed the power density in the inner core but the peak power of the inner core has not shifted very far, going from hex position (51) with no control to (61) in the case with control. However, in the 2-D model, the peak power of the inner has shifted quite drastically from (51) to two positions (104), indicating the control poison in the 2-D models has considerably more power shaping effect than is realistic. Thus we see that as a result of the axial smearing of control poison in the 2-D models, we can be led to completely misleading results in the power peaking analysis.

V. DISCUSSION AND CONCLUSIONS

In the previous sections, various geometric models of large LMFBRs were examined in regard to nuclear power peaking analyses. Comparison

of results for 1-D cylindrical and 2-D hexagonal geometries showed that the planar power distribution in the hexagonal model deviates strongly from the azimuthal symmetry of the cylindrical model, especially when control rod positions are taken into account. The annular rings of control poison in the cylindrical model does not provide a realistic representation of actual banks of control rod assemblies likely to be used in large LMFBRs (banks of six control rod assemblies were assumed in this study).

Comparison of power peaking factors in 2-D and 3-D models was thus limited to hexagonal geometry. It was demonstrated that the axial smearing of control in the 2-D models leads to misleading results for power peaking factors and locations of radial peaks in the power density. It is particularly important to note that although this study was limited to static power distributions, the lack of correspondence between the models may grow worse in any burnup calculation. The reason for this is that the inner core conversion ratio is greater than unity and results in an increase of the power density in the center and midplane of the inner core. In the 2-D models, control poison is added or subtracted homogeneously from each control rod region and consequently has a stronger effect on the midplane power distribution.

Although the r - z cylindrical model was not included in the modeling studies described in Sections III and IV (the cylindrical model was eliminated as a tool for accurate analysis on the basis of 1-D calculations), for completeness it is included in Table VI. This Table contains as a summary a comparison of midplane radial power peaking factors for a particular 3-D hexagonal model, and the corresponding 2-D hexagonal, 2-D cylindrical (r - z), and 1-D cylindrical models. The 3-D hexagonal model provides the benchmark calculation. The power distribution, corresponding

to an enrichment split $\epsilon_2/\epsilon_1 = 1.136$, was not optimized. For this reason, both inner and outer core power peaking factors are given and considered separately. The control rods in bank 5 were inserted 25% into the core in the 3-D model, yielding $k_{\text{eff}} = 0.9983$, which is 0.0017 lower than the k_{eff} for the uncontrolled core. In the other three models, characterized by the same enrichment split, the amount of control in bank 5 was adjusted until the corresponding Δk_{eff} equaled the same value. Values of the peak/average parameters vary over the four models by as much as 3% in the inner core and 4% in the outer core. More importantly, however, the position of the peak power density in the inner core, occurring in the 6th hexagonal row of assemblies in the 3-D hexagonal models, is shifted to the 7th row in the rz-model and 10th row in the other two models. This inability of the less realistic models to predict the position of the inner core peak density for static conditions implies even less accurate results for power peaking analysis during burnup.

It is concluded that one- and two-dimensional models are useful for understanding general properties (see Section II) of power distributions in large LMFBRs but are not suitable for (1) accurate determination of the nuclear power peaking factor including location of the peak power within the core, or (2) determining control rod insertion patterns for maintaining flat power distributions. Since axial smearing of control rods in planar models has a stronger effect on the midplane power density than partial insertion of rods in the 3-D model, analyses of power shaping during burnup based upon planar models might lead to wrong results. Eventually, three-dimensional burnup calculations are likely to indicate that partial insertion of control rods at BOC (also in reactors which gain

reactivity) are necessary for power shaping. That is, in addition to regular burnup control rods, one might need control rods solely for power shaping. At present we can only say that the error introduced by the 2-D model in estimating the nuclear power peaking factor for partially inserted control rods may be of the order of 5-10%. The three dimensional (hexagonal-z) model is necessary for a reliable study of power peaking factors and power shaping by means of control rod movement.

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6. Further limitations in the use of the code DIF3D arise because capabilities such as buckling and enrichment searches have not yet been incorporated. The code VENTURE, although available, was not considered because of its long running time and high computer cost.
7. The correspondence between the 2-D and 3-D models of the carbide reactor with control rod positions shown in Fig. 7 but with no control rods inserted, was established the following way. The 3-D enrichment ratios, fuel volume fractions, etc., and the resulting value of k_{eff} were used in a DIF2D buckling search based on the same hexagonal core layout (see Fig. 4). The 2-D core height was taken to be the same as the 3-D model, namely 45.72 cm. The buckling search yielded an extrapolation distance which was used in the subsequent DIF2D calculation described in the text.

TABLE I. Example of Power Shaping in a Large LMFBF which Gains Reactivity During Burnup.
Control is added to inner core of 2-D hexagonal model of a 5000 Mwt Carbide Reactor.
Results are based on REBUS-2.^(a)

Control Rod Positions ^(b)				Burnup Days	% Control Rod Insertion				Peak/Avg Power in Core	Peak Fission Densities (fiss/cm ³) and Their Locations		
Bank 1	Bank 2	Bank 3	Bank 4		Bank 1	Bank 2	Bank 3	Bank 4				
(11)	(53)	(81)	(95)	0	0%	0%	0%	0%	1.464	1.619 (61)	1.624 (126)	
				75	3.90%	5.99%	13.02%	8.46%	1.465	1.621 (51)	1.623 (126)	
				150	7.21%	11.06%	24.05%	15.63%	1.469	1.610 (31)	1.625 (51)	1.619 (126)
				225	10.36%	15.89%	34.54%	22.45%	1.470	1.611 (31)	1.623 (51)	1.616 (126)
				300	13.16%	20.18%	43.87%	28.52%	1.468	1.608 (31)	1.619 (51)	1.612 (126)

^(a) In the code REBUS-2,⁽⁴⁾ the ratios of control in different control positions is held fixed throughout burnup. In this case, these ratios are (11)/(81) = 0.30, (53)/(81) = 0.46, (95)/(81) = 0.65.

^(b) For hex assembly position, see Fig. 4.

TABLE II. Example of Power Shaping in a Large LMFBR which Loses Reactivity During Burnup. Control poison is withdrawn from outer regions of the core of the cylindrical model of a 3000 MWt oxide reactor. Results are based on REBUS-2.

Core Radius (cm)	Inner Core Radius (cm)	Burnup Days	Percent of Control Poison ^(a)				Peak/Avg Power in Core ^(b)	Peak Fission Densities (fiss/cm ³ -sec)	
			Ring 1 (0.0-8.7 cm)	Ring 2 (38.1-43.7 cm)	Ring 3 (88.6-91.2 cm)	Ring 4 (138.9-140.5 cm)		Inner Core (×10 ¹³)	Outer Core (×10 ¹³)
179.2	138.9	0	0%	0%	36.8%	36.8%	1.441	1.343	1.339
		100	0%	0%	23.6%	23.6%	1.435	1.328	1.315
		200	0%	0%	10.5%	10.5%	1.438	1.320	1.293
		300	0%	0%	0.0%	0.0%	1.450	1.320	1.269

(a) Control poison is distributed homogeneously in each annular ring in this model.

(b) Enrichment split was optimized at BOC.

TABLE III. Model Carbide Reactor for Modeling Studies.

Model corresponds to 0.4 in. pin diameter, 30 kW/ft linear heat rating.

Power	5000 MWt
% Power in core*	90
% Power in axial blanket*	8
% Power in radial blanket*	2
Active core height	3 ft.
Axial blanket thickness	15 in.
Axial reflector thickness	5 in.
Unit cell area	44.018 in. ²
Assembly pitch	7.129 in.
Number of fuel assemblies and control rod positions	529
Number of rows in core	14 (R)
Core volume	13737 liters
Core radius	218.68 cm
Volume fractions	
fuel	.397338
coolant	.486335
structural	.116327
Pellet density	95% T.D.
Fuel pin bond	Sodium

*Assumed for 1-D cylindrical and 2-D hexagonal models.

TABLE IV. Comparison of BOC Peak/Avg Power in Core
in Cylindrical and Hexagonal Geometries

5000 MWt Carbide Reactor [ϵ_2/ϵ_1 optimized in each case]

Number of Control Rods	Control* Rod Positions HEX GEOM	Peak/Avg Power in Core HEX GEOM	Corresponding Ring Positions CYL GEOM		Peak/Avg Power in Core CYL GEOM
18	53	1.447	57.83	- 62.35 cm	1.411
	95		123.60	-125.77 cm	
	137		189.44	-190.86 cm	
18	51	1.476	57.83	- 62.35 cm	1.411
	95		123.60	-125.77 cm	
	137		189.44	-190.86 cm	
18	53	1.504	57.83	- 62.35 cm	1.409
	95		123.60	-125.77 cm	
	111		171.40	-172.98 cm	
12	53	1.458	57.8328	- 62.3458 cm	1.405
	95		123.60	-125.77 cm	
0	--	1.493	--	--	1.419

*Assembly and control rod positions in the hex geometry are defined in Figure 4.

TABLE V. Eigenvalue k_{eff} for Various Control Rod Insertions in 3-D Model

z Mesh Structure*	Percent Insertion of Control in Core			k_{eff}
	Bank 1	Bank 5	Bank 8-9	
Base Case	0%	0%	0%	0.9999
E	0%	0%	0%	1.0004
D	0%	0%	0%	1.0006
F	100%	0%	0%	0.9984
D	0%	25%	0%	0.9984
E	0%	25%	0%	0.9983
F	0%	25%	0%	0.9982
E	0%	25%	25%	0.9944
F	0%	25%	25%	0.9944
E	100%	25%	0%	0.9967
F	100%	25%	0%	0.9967

*See Fig. 6 .

TABLE VI. Comparison of Power Peaking Factors for Different Geometric Models of a 5000 MWt Carbide Reactor

Model	Radial Power Peaking Factor		Locations of peak power densities ^(a)		% insertion of control rods in bank 5 ^(b)
	Inner Core	Outer Core	Inner Core	Outer Core	
3-D hexagonal	1.162	1.206	Assembly (61) in row 6 ($r \approx 74.3 - 90.7$ cm)	Assembly (126) in row 12 ($r \approx 173.0 - 189.4$ cm)	25%
2-D hexagonal ^(c)	1.144	1.235	Assembly (104) in row 10 ($r \approx 140.1 - 156.5$ cm)	Assembly (126) in row 12 ($r \approx 173.0 - 189.4$ cm)	12.0%
2-D cylindrical (rz)	1.173	1.192	Ring 7 ($r \approx 90.7 - 107.42$ cm)	Ring 12 ($r \approx 173.0 - 189.4$ cm)	21.04%
1-D cylindrical (r)	1.140	1.192	Ring 10 ($r \approx 140.1 - 156.5$ cm)	Ring 12 ($r \approx 173.0 - 189.4$ cm)	12.60%

(a) See Fig. 4 for definition of hexagonal assembly positions. nth ring in cylindrical geometry corresponds to nth row in the hexagonal geometry.

(b) Control rods in bank 5 were inserted 25% into the core in the 3-D model, yielding $\Delta k_{eff} = 0.0017$, where Δk_{eff} is the change in k_{eff} relative to the uncontrolled core. In the other three models, the amount of control in bank 5^{eff} was adjusted until Δk_{eff} equaled the same value. In the planar models, % insertion refers to the volume fraction of control composition.

(c) In the planar models, the extrapolation distance d in the axial direction was adjusted to yield $k_{eff} = 1.0000$ for the uncontrolled core.

ϵ_2/ϵ_1 (BEGINNING-OF-CYCLE CONDITIONS)

Fuel Vol. Fr. = 0.45; $k_{\text{eff}} = 1.0078$

Fig. 1. Radial power peaking factor as a function of enrichment split for beginning-of-cycle conditions, cylindrical carbide core. First and second legs of the "vees" correspond to the power peak occurring in the inner and outer core, respectively. V_2/V is the ratio of the outer core to total core volume; ϵ_2/ϵ_1 the ratio of outer core to inner core plutonium enrichment.

Fig. 2. The power density vs. radius of the core at beginning of cycle (BOC), midcycle, and end of cycle (EOC) in an uncontrolled cylindrical core. The beginning-of-cycle enrichment split ϵ_2/ϵ_1 (BOC) = 1.389 corresponds to the optimum power peaking factor for the 300 day burn cycle.

Fig. 3. Optimized power distribution in a hexagonal model with no control rods. Numbers are the ratios of peak power in each assembly position to average power of core in the midplane of a 5000 Mwt uncontrolled carbide reactor BOC conditions. Enrichment split between inner and outer core has been optimized.

Fig. 4. Core layout and assembly identification in hexagonal geometry of 5000 MWt carbide reactor; pin diameter = 0.40 in., linear heat rating = 30 kW/ft.

Fig. 5. Map of 5000 MWt carbide reactor, 0.40 in. pin diam, 30 kw/ft.

Fig. 6. Sensitivity study of z mesh-point spacing in DIF3D (no control rods inserted). Δk is the change in k_{eff} from the base case half-core spacing of 4-5 cm. $\Delta k \equiv k_{eff} - k_{eff}(\text{base case})$.

Fig. 7. 5000 MWt sodium bonded carbide reactor, peak to average power in core, no control, $z = 12$ plane.

Fig. 8. Axial power distribution in 5000 MWt carbide reactor at hex position (31).

Fig. 9. Axial power distribution in 5000 MWt carbide reactor at hex position (61).

Fig. 10. 5000 MWt sodium bonded carbide reactor, power density (watts per cc); control bank 5 inserted to 119.38 cm, $z = 12$ plane.

Fig. 11. 5000 MWt sodium bonded carbide reactor, power density (watts per cc); control bank 5 inserted to 119.36 cm, $z = 13$ plane.

Fig. 12. 5000 MWt sodium bonded carbide reactor, peak to average power in core; control bank 5 inserted to 119.38 cm, $z = 12$ plane.

Fig. 14. 5000 MWt sodium bonded carbide reactor, peak to average power in core, midplane in 2-D geometry; corresponds to 25% insertion of control rod bank 5 in 3-D model.