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Forbidden Coupling and Inhibited Decay:**A Study of Disconnection***

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An attempt is made to correlate deviation from ideal mixing mass formula, high energy scattering behavior, and violation of the Okubo-Zweig-Iizuka rule. Pole or cut nature of the intermediary is discussed. Timelike and spacelike forbidden communications are compared. Off-shell effects are examined. Some problems for phenomenology are revealed.

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I. Introduction

In a recent series of papers¹⁻³ we have attempted to develop a consistent phenomenological scheme, with underpinnings in dual resonance models⁴ and the recent topological expansion⁵ development, for correlating various decay and production processes which are forbidden by ideal mixing and the Okubo-Zweig-Iizuka⁶ (OZI) rule. We would like now, in this longer and more discursive paper, to review some of the successes of the model, report some of its shortcomings, relax some of our initial, highly specific assumptions, and compare and contrast our picture with other efforts, such as the charmonium-asymptotic freedom approach,⁷ the "asymptotic planarity" approach of Chew and Rosensweig,⁸ the dual unitarization approach,⁹ and the classic wavefunction-mass matrix mixing model of several groups.¹⁰ Beyond the different language and pictures which dress many of these calculations, we find some significant similarities and differences which we feel useful to point out.

The specific model which was our starting point is that of Freund and Nambu⁴ (also Refs.11 and 12) in which the OZI violating transition is associated with an s-channel (timelike) Pomeron effect which is pole dominated by "particles"--"0" mesons or closed strings--identified with Pomeron or Pomeron daughter trajectories.

Our general approach is the conventional one in which the new states are bound systems of a new degree or degrees of freedom--the charm picture or suitable generalizations--and the dynamics are simply those of strong interaction physics--Regge behavior, duality, unitarity, etc.--modified by the new mass scale involved. Thus $\psi \rightarrow 3\pi$ is inhibited, for the same reason that $\psi \rightarrow 3\pi$ is inhibited, to a greater degree only because nearby states mix more

strongly than distant states and hence receive larger corrections to the OZI rule.

The basic problem, in this approach, is to give meaning to the dual diagrams which, in some Born term sense the theory must eventually specify, represents "forbidden" quark communication. For example, if a Regge exchange is represented by Fig.1a, then in the Harari-Freund^{13,14} picture the Pomeron is represented as one of the topologically equivalent diagrams of Fig.1b, in which the quarks make a forbidden "U-turn" or in which two intermediate exchanges are twisted, or as a cylinder correction in terms of the topological expansion. (In terms of the topological expansion, planar diagrams have the structure of a sphere with one hole; singly disconnected diagrams are cylinders with the structure of a sphere with two holes; doubly disconnected, three holes, and so on.) The double twist diagram can be cut into two single twist diagrams (see 1c), each of which is a line reversal of an allowed diagram, representing "s-channel" background (relatively real) which, iterated via unitarity, contributes to the Pomeron.

Similarly, the Pomeron link appears in an OZI violating decay, as in Fig.1d, which again illustrates the role that unitarity plays in requiring OZI violation, e.g., $\psi \rightarrow D\bar{D}^*$ as well as $D\bar{D}^* \rightarrow \rho\pi$ are both OZI allowed, whereas their iteration, as illustrated in the middle diagram of Fig.1d, is forbidden.

At this stage, even if values can be assigned to the forbidden links, or cylinder corrections, one is thinking in a perturbative framework, in which the real particles are represented, approximately, by quark lines of a single species (bare states) with transitions represented as overlaps with other bare states of different species (not continuously connected) with the OZI violating

connection "small." This may simply be a bad approximation when multiplets are involved which appreciably deviate from ideal mixing (e.g., 0^- and 0^+) where some sector of the perturbation theory must be summed. Additionally, one can hope to explain the deviation from the ideal mixing mass formula simultaneously with the deviations from ideal mixing in the wave functions; a propagator mass matrix formalism will be described which accomplishes this goal.

Historically, perhaps the first phenomenological treatment of the forbidden amplitudes occurred for spacelike transitions (t-channel) and is contained in the "f-dominated Pomeron" approach of Freund and Rivers¹⁵ and Carlitz, Green, and Zee.¹⁶ The Freund and Nambu model is in a sense simply the timelike implementation (s-channel) of that work.¹⁷ Figures 2a and 2b make clear that connection, where normal trajectories are mediated by the Pomeron as normal states are mediated by Pomeron-related effects. (ρ daughters, O-mesons, Sea-balls, Glue-balls, empty bags, cuts?) in the timelike forbidden transitions.

Important features of this implementation are:

(1) s- and t-channel "unitarity" effects are related. If the intermediary for timelike transitions is considered a Pomeron daughter pole, its spectrum is controlled by the Pomeron trajectory; if it is a cut, its structure is related to Pomeron-Reggeon cuts. In any case a unified s- and t-channel description is feasible.

(2) Detailed exclusive processes can be estimated (a single and asymptotic energy is not necessary) and production or decay processes for OZI violations are multiplicably related to corresponding allowed processes (see Figs. 2c and 2d, where an intermediate pole approximation has been used).

(3) Correspondingly, production and decay are related, independent of the detailed nature of the OZI violating process; e.g., in Figs 2c and 2d, $f_{0\omega}^2 f_{0\phi}^2 (m_\phi^2 - m_0^2)^{-2} (m_\phi^2 - m_\omega^2)^{-2}$ may be eliminated to predict $\phi \rightarrow 3\pi/\omega \rightarrow 3\pi$ in terms of $\pi^- p \rightarrow \omega n/\pi^- p \rightarrow \phi n$. Both (2) and (3) are possible because of the factorizing nature of OZI violation in this picture.

(4) When OZI violation is small (e.g., $c\bar{c} \rightarrow$ normal quarks or $\bar{s}s \rightarrow$ normal in 1^- channel), a perturbative approach may be valid, similar to that employed in the f-dominated Pomeron picture. In fact, we shall argue below that the Pomeron in a sense itself is small relative to Regge exchanges and appears large only because it is all that remains after Regge exchanges are no longer important.

To summarize the model, OZI violating processes, including Pomeron exchange, proceed via intermediate states which may be a pole or cut, whose spectrum is revealed both in timelike and spacelike forbidden transitions. The parameters of the intermediary are partially determined by that spectrum, with the determination completed by high energy behavior or by the size of the OZI forbidden production and decay processes. Once these parameters are determined, other processes can be calculated to check the consistency of the scheme.

This paper is organized as follows: In Sec. II we review and discuss t-channel pole implementation, determine some basic parameters, and discuss the "size" of the Pomeron. We then review the timelike production and decay results, in the pole approximation. Questions of the perturbative or non-perturbative nature of the transition in various sectors of $SU(4)$ are examined. Factorization and model independence are discussed. The problem of off-mass-shell behavior is reviewed.

In Sec.III the model is formulated in terms of a cut rather than a pole, motivated by the apparent experimental absence of 0-mesons, by rising cross-section data, and by a need for stronger damping.

In Sec.IV we discuss, both in pole and cut pictures, the propagator-mass matrix formulation in which a unified, highly constrained, and non-perturbative treatment is developed of masses, mixings, and OZI violation.

In Sec.V we comment on the topological expansion and point out some counting difficulties for our phenomenology.

Section II

A. t-Channel Pole Model Results

We first review the f-dominated Pomeron results, with a view toward estimating the size of the coupling between Regge trajectories and the Pomeron. This clarifies what we mean by the "smallness" of the Pomeron and the validity of a perturbative approach. Since these junctions, when continued to timelike t , are related to 0-meson coupling parameters, this will enable us later to compare the size of the timelike and spacelike couplings.

We remark that it is not clear how, if at all, this t-channel picture of the Pomeron as an OZI violating process can be incorporated in the Quantum Chromodynamics Picture (QCD).¹⁸

Consider first a rough fit to pp and $p\bar{p}$ total cross sections. Neglecting the A_2 trajectory, which lies below f , and is diminished by a Clebsch-Gordan coefficient of $1/9$, the leading diagrams for the sum $\sigma_T(pp) + \sigma_T(p\bar{p}) = \sigma_T^+$ are given in Fig.2, resulting in

$$\gamma_f^2 \left[\frac{F_{OT}^2}{(J - \alpha_f(t))^2 (J - \alpha_p(t))} + \frac{1}{J - \alpha_f(t)} \right] \quad (1)$$

whose Mellin transform, at $t=0$, is

$$\sigma_T^+ = \gamma_f^2 \left\{ \frac{F_{OT}^2}{(\alpha_p(0) - \alpha_f(0))^2} + s^{\alpha_f(0)-1} \right\} \quad (2)$$

A hand fit to the data points ($\sigma_T^+ \approx 120$ mb at $s = 10 \text{ GeV}^2$, 80 mb at $s = 100 \text{ GeV}^2$) yields

$$F_{OT}^2 \cong 0.1 \quad (3)$$

for the probability that the f -trajectory makes the transit to the Pomeron. Roughly speaking this is a measure of the mixture of non-valence quarks, at $t=0$, in the proton. Note F_{OT}^2 is a small dimensionless number. The major successes of this simple approach are given by¹⁹

$$\frac{\sigma_T(kp)}{\sigma_T(\pi p)} = \frac{1}{2} \left(1 + \frac{\alpha_p(0) - \alpha_f(0)}{\alpha_p(0) - \alpha_{f'}(0)} \right) \quad (4)$$

$$\frac{\sigma_T(\varphi p)}{\sigma_T(\rho^0 p)} = \frac{\alpha_p(0) - \alpha_f(0)}{\alpha_p(0) - \alpha_{f'}(0)} \quad (5)$$

$$\frac{\sigma_T(\psi p)}{\sigma_T(\varphi p)} = \frac{\alpha_p(0) - \alpha_{f_c}(0)}{\alpha_p(0) - \alpha_{f'}(0)} \quad (6)$$

where the Pomeron has been assumed to be an SU(4) singlet ($F_{\rho f} = F_{\rho f_c} = F_{\rho f}/\sqrt{2}$).

With exchange degenerate ω -f, ϕ -f', and ψ -f_c trajectories, $\alpha_f = 1 + \alpha' (t - m_\omega^2)$, $\alpha_{f'} = 1 + \alpha' (t - m_\phi^2)$, and $\alpha_{f_c} = 1 + \alpha' (t - m_\psi^2)$, the results are:

	Model	Exp.	
$\frac{\sigma_T(kp)}{\sigma_T(\pi p)} =$	0.8	0.84	
$\frac{\sigma_T(\varphi p)}{\sigma_T(\rho^0 p)} =$	0.6	0.4	
$\frac{\sigma_T(\psi p)}{\sigma_T(\varphi p)} =$	0.1	0.1	(7)

The difficulties with the small coupling of the Pomeron to normal states, within a QCD approach, seem quite serious. Since the squared mass carried by the Pomeron glue cluster is zero, asymptotic free field theory results can not be used to justify weak coupling. Even worse, the small coupling seems in direct contradiction to the large effective coupling needed to confine the quarks at large distances.

B. s-Channel Pole Model, $J^P = 1^-$

Classifying the OZI forbidden transition by the J^P of the decaying state, the $\varphi \rightarrow \rho\pi$ rate in the $J^P = 1^-$ sector is the classic problem for which the OZI rule was formulated. If the forbidden transition is mediated by a pole---the O-meson---then in QCD this pole would be a bound state of glue,^{20,21} and in the dual model this particle would be, following Freund and Nambu,⁴ a particle on

one of the Pomeron daughters. For the moment let us adopt the latter approach, which fixes the squared mass of the lowest contributing daughter at $(\alpha'_p)^{-1}$ implying $2 \leq m_0^2 \leq 3$ if the slope of the Pomeron is between 1/2 and 1/3. Interestingly enough, Bag calculations have found glue-ball states in the same range.²⁰

Referring to Fig.2c, the appropriate propagators can be written in ordinary or Regge form; the former useful for comparing with the literature, the latter useful for comparing with t-channel results or for introducing cuts or continuum states. (As before, we assume SU(4) symmetric junctions.) The connection is

$$\frac{F_{\varphi 0} F_{\omega 0}}{(1 - \alpha_{p_1}(s))(1 - \alpha_{\omega}(s))} = \frac{F_{\varphi 0} F_{\omega 0}}{(1 - \alpha_{p_1}(0) - \alpha'_{p_1}s)(1 - \alpha_{\omega}(0) - \alpha'_{\omega}s)} = \frac{f_{\varphi 0} f_{\omega 0}}{(m_0^2 - s)(m_{\omega}^2 - s)} \quad (8)$$

or

$$F_{\varphi 0} F_{\omega 0} = \alpha'_{p_1} \alpha'_{\omega} f_{\varphi 0} f_{\omega 0}. \quad (9)$$

By comparing the OZI violating rate $\varphi \rightarrow \rho\pi$ with the allowed rate $\varphi \rightarrow K_L K_S$,

$f_{OV} = f_{O\varphi} = f_{O\psi} = f_{O\omega}/\sqrt{2}$ has been determined as

$$f_{OV} = \begin{cases} .155 & \text{for } m_0^2 = 2 \\ .222 & \text{for } m_0^2 = 3 \end{cases} \quad (10)$$

indicating that

$$F_{OV}^2 = \alpha'_p \alpha'_{\omega} f_{OV}^2 = \begin{cases} .010 & \text{for } m_0^2 = 2 \\ .013 & \text{for } m_0^2 = 3 \end{cases} \quad (11)$$

which is considerably smaller than the Pomeron coupling F_{OT} determined in the previous section. This is not surprising since the 1^- multiplet is much closer to ideal mixing than the tensor multiplet, as determined from the nearness of

the masses to the ideal mixing mass formula (the connection will be made precise in a following section). This value is close to the value determined from the breaking of exchange degeneracy of the ρ and ω trajectories in Ref.17, and is also consistent with the QCD picture in which the $J^P = 1^-$ channel is mediated by 3 gluon exchange while the 2^+ channel requires only 2 gluons.⁷

Having established the $J^P = 1^-$ junction coupling from the rate $\phi \rightarrow \rho\pi/\phi \rightarrow K_L K_S$ one can make a number of predictions which test the model.

The rate ratio, for example $\sigma_{\pi^- p \rightarrow \phi n} / \sigma_{\pi^- p \rightarrow \omega n}$ has been calculated

$$\frac{\sigma_{\pi^- p \rightarrow \phi n} \text{ (on shell)}}{\sigma_{\pi^- p \rightarrow \omega n} (g_\omega^2 = m_\phi^2)} = \frac{2f_{OV}^4}{(m_\omega^2 - m_\phi^2)^2 (m_\phi^2 - m_\rho^2)^2} \cong .005 \pm .0005 \quad (12)$$

with excellent success (experimental result of Ref.24 is $.0035 \pm .001$), if the small off-shell extrapolation is ignored.

It should be noted that this test survives any tampering with the q^2 propagator suppression of the model. The best way of seeing this is to express the ratio in terms of $\Gamma_{\phi \rightarrow \rho\pi} / \Gamma_{\phi \rightarrow K_L K_S}$, in which case the explicit dependence on the form of the OZI suppression cancels out:

$$\frac{\sigma_{\pi^- p \rightarrow \phi n}}{\sigma_{\pi^- p \rightarrow \omega n}} = \frac{\Gamma_{\phi \rightarrow \rho\pi}}{\Gamma_{\phi \rightarrow K_L K_S}} \left\{ \frac{m_\rho^2}{6(m_\phi^2 - m_\omega^2)^2} \frac{(m_\phi^2 - 4m_K^2)^{\frac{3}{2}}}{(m_\phi^2 - m_\rho^2)^2} m_\phi \left[1 - \frac{m_\pi^2}{(m_\phi - m_\rho)^2} \right]^{\frac{3}{2}} \right\} \quad (13)$$

where we have used the SU(6) result $g_{\omega\rho\pi}^2 = 4g_{\rho\pi\pi}^2/m_\rho^2$. Experimentally this relation gives (ignoring the off-shell effects in the cross sections)

$$.0035 \pm .001 = .005 \pm .0005.$$

This gives a strong indication of the factorizable form of the Zweig rule suppression. We should also remark that this relationship is independent of the SU(4) structure.

To actually test the q^2 dependence of a specific model, one must go to a different regime, for example large q^2 , as in ψ decay.

The basic scale of OZI violation in the model, coming from the size of f_{OV} and the propagator effect, is tested in the model's prediction for the signal to background ratio of ψ production in the initial co-discovery experiment at Brookhaven National Laboratories. Referring to Fig.3a where e^+e^- from virtual vector dominated photon production has been used to estimate the background,²⁶ the prediction is

$$\frac{d^2\sigma(\psi)}{d^3p_+ d^3p_-} \bigg/ \frac{d^2\sigma(bg)}{d^3p_+ d^3p_-} = \frac{2f_{OV}^4}{(m_\psi^2 - m_\phi^2)^2 m_\psi^2 \Gamma_\psi} \left[\sqrt{\frac{\Gamma_{\omega \rightarrow e^+e^-} m_\psi}{\Gamma_{\psi \rightarrow e^+e^-} m_\omega}} + \sqrt{\frac{\Gamma_{\phi \rightarrow e^+e^-} m_\psi}{\Gamma_{\psi \rightarrow e^+e^-} m_\phi}} \right]^{-2}. \quad (14)$$

When this is corrected for the 20 MeV resolution, the signal to background ratio is ~ 31 (180) for $m_\phi^2 = 2$ (3) GeV, which brackets the data.²⁵ This rate clearly tests the scale of the suppression and is independent of q^2 dependence concerning off-shell ω , ρ , and ϕ production, which cancel in the ratio.

The best test of the scale of the suppression mechanism, which is independent of the size of f_{OV} and depends only on the propagator damping effect, is given by the first prediction of Freund and Nambu⁴,

$$\begin{aligned}
\frac{\Gamma_{\psi \rightarrow \rho\pi}}{\Gamma_{\varphi \rightarrow \rho\pi}} &= \frac{g_{\omega\rho\pi}^2(m_\psi^2)}{g_{\omega\rho\pi}^2(m_\varphi^2)} \left[\frac{(m_\varphi^2 - m_\omega^2)}{(m_\psi^2 - m_\omega^2)} \frac{(m_\varphi^2 - m_\omega^2)}{(m_\psi^2 - m_\omega^2)} \right]^2 \left(\frac{p_{\psi \rightarrow \rho\pi}}{p_{\varphi \rightarrow \rho\pi}} \right)^3 \\
&\approx \left\{ \begin{array}{l} .02 \text{ for } m_\omega^2 = 2 \\ .11 \text{ for } m_\omega^2 = 3 \end{array} \right\} \times (g_{\omega\rho\pi}^2(m_\psi^2)/g_{\omega\rho\pi}^2(m_\varphi^2)) \\
&= .14 \times 10^{-2} \text{ experimentally (Ref.27).}
\end{aligned} \tag{15}$$

which indicates that either the off-shell effect in $g_{\omega\rho\pi}$ is large

$$\frac{g_{\omega\rho\pi}^2(m_\psi^2)}{g_{\omega\rho\pi}^2(m_\varphi^2)} = \left\{ \begin{array}{l} .07 \text{ for } m_\omega^2 = 2 \\ .013 \text{ for } m_\omega^2 = 3 \end{array} \right. \tag{16}$$

or the suppression given by the 0-poles is inadequate. We shall return to this question in the next section.

On the other hand, the ratio $\psi \rightarrow p\bar{p}/\psi \rightarrow \rho\pi$ which does not have the difficulty of the large q^2 dependence gives predictions in good agreement with the data.

$$\begin{aligned}
\frac{\Gamma_{\psi p\bar{p}}}{\Gamma_{\psi\rho\pi}} &= \frac{3}{16} \frac{m_\psi m_\rho^2}{p_\rho^3} \left(1 - \frac{4m_\rho^2}{m_\psi^2} \right)^{\frac{1}{2}} \left(1 + \frac{2m_\rho^2}{m_\psi^2} \right) = .11 \\
&= .12 \pm .03 \text{ experimentally (Ref.27).}
\end{aligned} \tag{17}$$

Another test of the model consists in its SU(4) symmetry and symmetry breaking predictions for OZI violating decays. The SU(4) general symmetric coupling is

$$\mathcal{L} = g_\mu \left(\sqrt{2} f_{\omega\psi} w^\mu + f_{\omega\varphi} \varphi^\mu + f_{\omega\psi} \psi^\mu \right) \tag{18}$$

where $f_{0\omega} = f'_{0\phi} = f_{0\psi} (\equiv f_{0V})$ in the SU(4) limit. Mass differences in propagators induce some symmetry breaking, in addition to that which may be there at the two particle junction. In the absence of the latter, ψ then decays basically as an SU(3) singlet. For example, because $\psi \rightarrow K\bar{K}$ is forbidden by G-parity in the SU(3) limit, its rate is proportional to mass differences, and is small ($\sim$ eV) when calculated using the symmetric couplings.

Other predictions of this type are the branching ratios

$$\frac{\Gamma_{\psi \rightarrow K^* K^+}}{\Gamma_{\psi \rightarrow \rho^- \pi^+}} = .84 \left\{ \frac{1}{2} + \frac{1}{2} \frac{f_{0\phi}}{f_{0\omega}} \left(\frac{m_{\psi}^2 - m_{\omega}^2}{m_{\psi}^2 - m_{\phi}^2} \right) \right\}^2$$

$$= .36 \pm .17 \text{ experimentally (Ref.27)} \quad (19)$$

which is unity in the SU(3) limit. The phase space-angular momentum barrier factor brings this down, as does the propagator effect, but not nearly enough to come into agreement with the data, which requires considerable breaking of the SU(4) relations for the f_{0V} couplings; even taking the upper limit for the data indicates $f_{0\phi}/f_{0\omega} \sim .7$. If this is the case, ψ is more ω - than ϕ -like in its OZI violating decays. (An argument which relates this to the K-yield dips at ψ and ψ' was advanced in Ref.2.)

So far we have concentrated on the $J^P = 1^-$ OZI violations in the s-channel. The best known OZI violations are here because the spectroscopy is clean and the multiplet is close to being ideally mixed, even in the SU(3) [U(3)] sector. The 0-meson parameters are small here; a perturbative approach is valid.

In the case of the $J^P = 0^+$ system, the picture is muddled by the uncertain status of the scalar spectroscopy. We believe that the scalars may be well mixed in the SU(3) sector, so that the perturbative mixing implied by simple

0-meson diagrams is not valid; one must sum the diagrams and diagonalize the propagator matrix exactly.

This is clearly not the case in the transition from the $\epsilon_c(\bar{c}c)$ to the normal quark model states because of the large energy difference in the denominators; the rate $\psi' \rightarrow \psi \pi^+ \pi^-$ may then be used to estimate f_{OS} , assuming the $\pi^+ \pi^-$ system is dominated by the s-wave (see Fig.3b).

Assuming $g_{\psi' \psi \epsilon_c} \approx \sqrt{2} g_{\rho' \rho \epsilon} [SU(4)]$ and with $g_{\epsilon \pi \pi \pi}$ determined from $\Gamma_{\epsilon \rightarrow \pi \pi \pi}$, then $\Gamma_{\rho' \rightarrow \rho \pi \pi}$ determines $\Gamma_{\psi' \rightarrow \psi \pi \pi}$ up to the f_{OS}^2 scalar 0-meson parameters. (For details, see Ref.3.) Tensor coupling has been used at the VVS vertex and chiral coupling at the PPS vertex, the former to keep the diagram gauge invariant when vector dominance is used to allow the final ψ to convert to a photon, the latter to cope with the experimentally rising $\pi^+ \pi^-$ mass spectrum. The result is

$$f_{OS} = \begin{cases} .40 & \text{for } m_o^2 = 2 \\ .48 & \text{for } m_o^2 = 3. \end{cases} \quad (20)$$

Note the OZI violation here is small, with the scale controlled by (in, for example, $\epsilon_c \rightarrow \pi \pi$)

$$\frac{f_{OS}}{(m_{\epsilon_c}^2 - m_o^2)} \cong .04. \quad (21)$$

This again is nicely consistent with the QCD picture in that the 0^+ violation of the OZI rule is expected in this picture to be the result of two gluon exchange and therefore should be larger than the 1^- transitions which go by three gluons.

Assuming $g_{\psi' \psi \epsilon_c} \approx g_{\psi \psi \epsilon_c}^{28}$, then $SU(4)$ implies $g_{\omega \omega \epsilon} = g_{\rho' \rho \epsilon}$, and the rate $\psi \rightarrow \omega(\epsilon \rightarrow \pi \pi)$ may be estimated (see Fig.3c) with the result

$$\Gamma_{\psi \rightarrow \omega \epsilon} / \Gamma_{\psi \rightarrow \rho \pi} \approx .60 \quad (22)$$

where to minimize the problem of the off-shell $\omega\omega\epsilon$ vertex we quote the result as a ratio, comparing it to $\psi \rightarrow \rho\pi$ in which the $\omega\rho\pi$ vertex is similarly off-shell, and should extrapolate in a qualitatively similar way. The result is in fair agreement with the data.²⁷

Returning to the magnitude of f_{OS} extracted from $\psi' \rightarrow \psi \pi^+ \pi^-$, this implies, if the SU(4) coupling is assumed

$$f_{OS} = 0(\sqrt{2} \epsilon + \epsilon' + \epsilon_c) \quad (23)$$

that mixing effects in the SU(3) sector may be considerable, with the amplitude scaled controlled by

$$\frac{f_{OS}}{(m_\epsilon^2 - m_0^2)} \cong .27. \quad (24)$$

These effects are important when considering the rates $\psi \rightarrow \omega \pi^+ \pi^-$ and $\psi \rightarrow \phi \pi^+ \pi^-$, with lowest order diagrams (Figs. 3d, 3e, 3f, 3g). Since the scalar 0-meson coupling seems large, perturbation theory seems highly questionable when working in the SU(3) sector of OZI violation. This question has been asked and answered in Ref. 3, where the SU(3) sector of the 0^{++} nonet is handled non-perturbatively. The calculation suffers from the usual ambiguity of the scalar spectroscopy, but for a variety of plausible assignments it is fair to conclude from the detailed calculation of Ref. 3 that the ratio $\psi \rightarrow \phi \pi^+ \pi^- / \psi \rightarrow \omega \pi^+ \pi^-$ is expected to lie between 0.1 and 0.4, consistent with the data. This is despite the fact, as has often been noted, that $\psi \rightarrow \phi \pi^+ \pi^-$ is doubly suppressed, with respect to the OZI Rule, as is evident from the

diagrams in Figs. 3e and 3g, whereas $\psi \rightarrow \omega \pi^+ \pi^-$ is singly suppressed, as seen in the diagram of Fig. 3d.

It is interesting to note that these same types of considerations will seriously constrain models with more than four quarks that have a ψ' made mainly of the higher-quark state.⁷ In that case the decay of the $\psi' \rightarrow \psi \pi \pi$ will be a doubly OZI rule suppressed process completely analogous to $\psi \rightarrow \phi \pi \pi$. The ratio of the rates, however, is of the order of 10^3 , and it is difficult to see how one can accomplish this in essentially identical processes, especially with phase space working in the wrong direction.

Finally, we mention some preliminary information on f_{OT} , the "Pomeron" coupling, as measured in the s-channel decay $f' \rightarrow 2\pi$, indicating

$$f_{OT} \leq \begin{cases} .185 & \text{for } m_0^2 = 2 \\ .28 & \text{for } m_0^2 = 3 \end{cases} \quad (25)$$

with the notation

$$\mathcal{L}_{OT} = f_{OT} O_{\mu\nu} (\sqrt{2} f^{\mu\nu} + f'^{\mu\nu} + f_o^{\mu\nu}) \quad (26)$$

implying that the Regge form is

$$F_{OT} \leq \begin{cases} .13 & m_0^2 = 2 \\ .16 & m_0^2 = 3 \end{cases} \quad (27)$$

as compared with $F_{OT} \approx .1$ in the t-channel, as estimated in II-A; the variation from the s-channel value (measured at $q^2 \approx 1 \text{ GeV}$) to the t-channel value (measured at $q^2 \approx 0$) does not seem unreasonable.

Within the QCD picture the 2^+ coupling fits seem to be nicely between the 1^- and 0^+ couplings as expected, with the 2^+ coupling via two gluons, with an angular momentum suppression.

III. Cuts or Poles?

Nothing which has been calculated so far prevents f^2 from being negative, that is, that the "O-meson" is a ghost or non-particle state. [In fact, in certain mass matrix/wave function or trajectory mixing calculations, precisely this occurs.¹⁷ In such a case it appears that the O-meson pole is merely being used to approximate and parametrize a cut in the OZI forbidden amplitudes. If, however, one is willing to forego the O-mesons as real particles, there are much more general cut trajectories which one may entertain in the OZI violating and Pomeron transitions. One loses the simple picture of an intermediate particle state and the Pomeron as a simple pole, but is this picture plausible, in the absence of conspicuous O-meson states in the experimental spectrum and with respect to the asymptotic freedom arguments for multiple gluon exchange discontinuities? These two questions will be returned to in the next section; now we will be content with more mundane motivations for a cut rather than pole structure:

(a) A simple pole does not seem responsible for all the OZI damping. Off-shell effects in the OZI allowed couplings at the end of the sequential pole decay chain may be the culprit. But is this not double counting?

(b) The Pomeron itself does not seem to be a simple pole, for many reasons. Thus (like father, like daughter) the daughter trajectories below it should also be cuts, or alternatively, particle Pomeron cuts.

(c) The total cross section is rising at high energies; can this rise be attributed to the same cut structure which succeeds in the stronger damping of the OZI mechanism?

We choose a simple form to test a cut hypothesis, namely a branch point at the conventional pole position, with linear trajectories as before, with power ν to parametrize the cut. The branching ratio $\varphi \rightarrow \rho\pi\varphi \rightarrow K_L K_S$ is then given by

$$\frac{\Gamma_{\varphi \rightarrow \rho\pi}}{\Gamma_{\varphi \rightarrow K_L K_S}} = 3 \left(\frac{g_{\omega\rho\pi}^2}{g_{\varphi K\bar{K}}^2} \right) \frac{F_{OV}^4}{(\alpha_{P_1}(m_\varphi^2) - 1)^{2\nu} (\alpha_\omega(m_\varphi^2) - 1)^2} \times \frac{1}{m_\varphi} \left[(m_\varphi^2 - m_\rho^2 - m_\pi^2)^2 - 4m_\pi^2 m_\rho^2 \right]^{\frac{3}{2}} (m_\varphi^2 - 4m_\pi^2)^{\frac{3}{2}} \quad (28)$$

and the ratio $\psi \rightarrow \rho\pi/\varphi \rightarrow \rho\pi$ is given by

$$\frac{\Gamma_{\psi \rightarrow \rho\pi}}{\Gamma_{\varphi \rightarrow \rho\pi}} = \left[\frac{\alpha_{P_1}(m_\varphi^2) - 1}{\alpha_{P_1}(m_\psi^2) - 1} \right]^{2\nu} \left[\frac{\alpha_\omega(m_\varphi^2) - 1}{\alpha_\omega(m_\psi^2) - 1} \right] \left(\frac{m_\varphi}{m_\psi} \right)^3 \times \left[\frac{(m_\psi^2 - m_\rho^2 - m_\pi^2)^2 - 4m_\rho^2 m_\pi^2}{(m_\varphi^2 - m_\rho^2 - m_\pi^2)^2 - 4m_\rho^2 m_\pi^2} \right]^{\frac{3}{2}} \quad (29)$$

where as before we make an SU(4) assumption for the trajectory couplings. α_{P_1} refers to the Pomeron first daughter. Equation (29) enables us to calculate the power ν

$$\begin{aligned} \nu = -1.63 \quad m_0^2 &= 2 \\ \nu = 2.78 \quad m_0^2 &= 3 \end{aligned} \quad (30)$$

where $\nu = -1$ in a simple pole model. Then Eq.(28) determines F_{OV} :

$$F_{OV} = .08 \quad \begin{matrix} m_0^2 = 2 \\ m_0^2 = 3 \end{matrix} \quad (31)$$

Thus, the model is completely determined and one can see if the other $J^P = 1^-$ predictions remain intact. The results are

$$\frac{\sigma_{\pi^- p \rightarrow \varphi n}}{\sigma_{\pi^- p \rightarrow \omega n}} = 4.8 \times 10^{-3} \quad (\text{exp. } (5 \pm .5) \times 10^{-3}) \quad (32)$$

$$\Gamma_{\psi \rightarrow \bar{p}p} = .08 \text{ keV (exp. } .106 \pm .032 \text{ keV)}. \quad (33)$$

Alternately rather than parametrizing the cut via

$$\left[\frac{1}{J - \alpha(s)} \right]^v \quad (34)$$

with $\alpha(s)$ determined by the daughter structure described above, one can regard $\alpha(s)$ as the Pomeron (as before) for $J^P = 2^+$ transitions, which yields

$$\left[\frac{1}{2 - \alpha_p(0) - \alpha_p' s} \right]^v = \left[\frac{1/\alpha_p'}{1/\alpha_p' - 1} \right]^v = \left[\frac{1/\alpha_p'}{m_0^2 - s} \right]^v \quad (35)$$

indicating $m_0^{(2^+)} = \sqrt{1/\alpha_p'} = \sqrt{2}$ or $\sqrt{3}$, whereas for other J^P transitions one can regard $\alpha(s)$ as an effective Pomeron-Reggeon cut, with trajectories described by the simple rules

$$\alpha_c = \alpha_p + \alpha_R - 1 \quad \text{at } t = 0 \quad (36)$$

$$\alpha_{c'} = \frac{\alpha_{p'} \alpha_{R'}}{\alpha_{p'} + \alpha_{R'}} \quad (37)$$

where $\alpha_{c'}$ is the slope of the effective P-R cut.

Then for $J^P = 1^{--}$ ($\rho + \rho$ cut) we have (for simplicity we calculated only the case $\alpha'_P = \frac{1}{2}$ corresponding to $m_0 = \sqrt{2}$ above)

$$\begin{aligned}\alpha_c &= \frac{1}{2} \\ \alpha_{c'} &= \frac{1}{3} \\ m_0 &= \sqrt{\frac{3}{2}} \text{ GeV}\end{aligned}\tag{38}$$

whereas for $J^P = 0^-$ ($\rho + \pi$ cut) we have

$$\begin{aligned}\alpha_c &= 0 \\ \alpha_{c'} &= \frac{1}{3} \\ m_0 &= 0\end{aligned}\tag{39}$$

We merely wish to emphasize here that when one divorces oneself from the strict Freund-Nambu model, both the location and the power of the branch point become negotiable, with indication from this simple Pomeron and Reggeon cut model that this location can vary from one J^P multiplet to another. Moreover, if $\alpha_P(0)$ is taken to be greater than one [e.g., $\alpha_P(0) = 1.1$] then

$$\alpha_{\pi\rho}(0) = .1$$

and

$$m_0^2 = -.3 \quad \text{at } \alpha_{P'} = \frac{1}{2}.\tag{40}$$

The moral here is that a negative "0" mass is certainly respectable, even predicted in simple cut models, when the literal interpretation of the "0" as a particle pole is given up. We make these remarks here to prepare the way for the next section, where we discuss the propagator-mass matrix formalism. In that model the simple particle pole picture must be given up, if the OZI violation above explains the deviation from ideal mixing in the mass formula.

IV. Mass-Propagator Matrix Formalism and Results

A more ambitious approach, and one implied by certain pictures, is to relate directly the deviation from ideal mixing mass formula (or deviation from exact exchange degeneracy in a Regge approach) to the violation of OZI rule in transition rates. (The corresponding modifications implied for the exact Harari-Freund duality picture have been discussed in Ref.17.)

For this discussion, applicable to any J^{PC} nonet, let us generally denote the pure states by the quark content

$$\begin{aligned}\bar{\epsilon} &= \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \\ \bar{\epsilon}' &= \bar{s}s \\ \bar{\epsilon}_c &= \bar{c}c\end{aligned}\tag{41}$$

with $\delta = \bar{\delta}$ and $\kappa = \bar{\kappa}$ the usual isovector and strange partners. The corresponding true states will be denoted by ϵ , ϵ' , ϵ_c , δ , and κ . Then the unmixed states satisfy the ideal mixing mass formula

$$\begin{aligned}m_{\bar{\epsilon}}^2 &= m_{\delta}^2 \\ 2m_{\kappa}^2 &= m_{\bar{\epsilon}'}^2 + m_{\bar{\epsilon}_c}^2.\end{aligned}\tag{42}$$

With the O-meson a real, independent particle state, corresponding to a quarkless or glue-like pole, as in the original scheme of Freund and Nambu, then implied by the SU(4) invariant interaction

$$\mathcal{L} = f O(\sqrt{2} \epsilon + \epsilon' + \epsilon_c)\tag{43}$$

is a contribution to the propagator matrix

$$\begin{aligned}\pi &= P + PQP + PQQP + \dots = P(1 - QP)^{-1} \\ &= (1 + PQ)P(1 - OP)^{-1} = (1 + PQ)(P^{-1} - O)^{-1}\end{aligned}\tag{44}$$

where

$$P^{-1} = \begin{bmatrix} (s - \frac{m_0^2}{\epsilon})^2 & & & \\ & (s - \frac{m_{\epsilon_1}^2}{\epsilon})^2 & & \\ & & (s - \frac{m_{\epsilon_c}^2}{\epsilon})^2 & \\ & & & (s - \frac{m_0^2}{\epsilon})^2 \end{bmatrix};$$

$$Q = \begin{bmatrix} 0 & 0 & 0 & \sqrt{2} f \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & f \\ \sqrt{2} f & f & f & 0 \end{bmatrix} \quad (45)$$

and

$$O = QPQ = \begin{bmatrix} \frac{2f^2}{(s - \frac{m_0^2}{\epsilon})} & \frac{\sqrt{2} f^2}{(s - \frac{m_{\epsilon_1}^2}{\epsilon})} & \frac{\sqrt{2} f^2}{(s - \frac{m_0^2}{\epsilon})} & 0 \\ \frac{\sqrt{2} f^2}{(s - \frac{m_0^2}{\epsilon})} & \frac{f^2}{(s - \frac{m_0^2}{\epsilon})} & \frac{f^2}{(s - \frac{m_0^2}{\epsilon})} & 0 \\ \frac{\sqrt{2} f^2}{(s - \frac{m_0^2}{\epsilon})} & \frac{f^2}{(s - \frac{m_0^2}{\epsilon})} & \frac{f^2}{(s - \frac{m_0^2}{\epsilon})} & 0 \\ 0 & 0 & 0 & \frac{2f^2}{(s - \frac{m_0^2}{\epsilon})} + \frac{f^2}{(s - \frac{m_{\epsilon_1}^2}{\epsilon})} + \frac{f^2}{(s - \frac{m_{\epsilon_c}^2}{\epsilon})} \end{bmatrix}$$

where rows and columns are labeled by $\bar{\epsilon}$, $\bar{\epsilon}'$, $\bar{\epsilon}_c$, $\bar{0}$.

The shifted masses are roots of $|P^{-1} - 0| = 0$.

From the direct sum structure of $0 = QPQ$ it is easy to show that the problem is equivalent to diagonalizing the 3×3 subsector. There are four independent mass relations among the nine variables f , ϵ , ϵ' , ϵ_c , 0 and $\bar{\epsilon} = \delta$, $\bar{\epsilon} = 2\kappa - \delta$, $\bar{\epsilon}_c$, and $\bar{0}$; if the physical masses (ϵ , ϵ' , ϵ_c , δ , κ) are known, then the remaining four unknowns (f , 0 , $\bar{0}$, $\bar{\epsilon}$) can be determined, fixing all mixing parameters. Since

$$\pi_{ij} = \sum_{\alpha} \frac{V_i^{\alpha} V_j^{\alpha}}{\lambda_{\alpha}(s)} \quad (47)$$

where V_i^{α} are the normed eigenvectors of $(P^{-1} - Q)^{-1} = (S - m_0^2 - Q)^{-1}$ and $\lambda_{\alpha}^{-1}(s)$, the corresponding eigenvalues, we have

$$(S - m_0^2 - Q)V^{\alpha} = \lambda_{\alpha} V^{\alpha}$$

or

$$(m_0^2 + Q)V^{\alpha} = (s - \lambda_{\alpha})V^{\alpha} \quad (48)$$

and diagonalizing the propagator matrix is equivalent to diagonalizing the mass matrix

$$m_0^2 + Q = \begin{bmatrix} \frac{m_0^2}{\epsilon} & 0 & 0 & \sqrt{2}f \\ 0 & \frac{m_0^2}{\epsilon'} & 0 & f \\ 0 & 0 & \frac{m_0^2}{\epsilon_c} & f \\ \sqrt{2}f & f & f & \frac{m_0^2}{0} \end{bmatrix}. \quad (49)$$

In this simple pole case V^{α} is s -independent and one has the simple form

$$\pi_{ij} = \sum_{\alpha} \frac{v_i^{\alpha} v_j^{\alpha}}{s - m_{\alpha}^2} \quad (50)$$

with no "continuum" terms, a special property of the 0-pole interaction. The rank 1 residue matrix yields, in this non-perturbative formulation, the decomposition of the $\alpha = (\epsilon, \epsilon', \epsilon_c, 0)$ pole in terms of pure channels $(\bar{\epsilon}, \bar{\epsilon}', \bar{\epsilon}_c, \bar{0})$ whose lowest order expansion in f yield the 0-meson factors used in earlier calculations. Thus orthogonality and completeness is realized in a 4×4 sense; this is equivalent to a classic function-mass matrix mixing model, like that studied in this context by Kazi and Kramer,¹⁰ but with an extra channel. A particle is then conceived as decomposable into various quark species plus a quarkless piece. The prescription for a transition like $\psi \rightarrow \rho\pi$, which in the perturbative formalism is suppressed by the factor

$$\frac{2f_{OV}^4}{(m_{\psi}^2 - m_0^2)^2 (m_{\psi}^2 - m_{\omega}^2)^2} \quad (51)$$

is to replace that by $|v_{\omega}^{\psi}|^2$ whose lowest order approximation is

$$\frac{2f_{OV}^4}{(m_{\psi}^2 - m_0^2)^2 (m_{\phi}^2 - m_{\omega}^2)^2} \quad (52)$$

Note that since f^2 and m_0^2 are parameters determined by the positions of the physical poles in the various particle multiplets, they may be either positive or negative numbers. In the latter case, the interpretation of the 0-meson pole as a particle pole clearly breaks down and it may be that the 0-pole is approximating a more complicated singularity, such as a multi-gluon state or some other quarkless intermediary which does not resonate.

Alternatively, one can formulate the problem in a general way by mixing the three bare channels via a second order interaction

$$0 = h(s) \begin{vmatrix} 2 & \sqrt{2} & 1 \\ \sqrt{2} & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = h(s) g g^\dagger. \quad (53)$$

For $h(s) = f^2/(s - m_0^2)$ with $f^2 > 0$ we recover the 0-meson picture of Freund and Nambu. When we normalize our eigenvector in the 4×4 sense, the problem is identical to the previous 4×4 formalism. For a general function $h(s)$, the mass equation constraints become complicated; it will be shown elsewhere that one can normalize the eigenvectors in the 3×3 sense, with a consistent interpretation for $f^2 < 0$ and $m_0^2 < 0$. Moreover, it can be shown that there exists an $h(s)$ which generates precisely those masses which appear in the 4×4 problem, but with completeness now implemented in a 3×3 sense, and no 0-pole. The procedure is lengthy and technical and will be described elsewhere. The moral is that one can live comfortably with negative "0" meson mass and negative " f^2 ." It is only the particle interpretation of the 0-meson which fails.

V. $O\left(\frac{1}{N}\right)$ OZI Violation in Vertices

In the previous development an OZI violating decay or amplitude has been calculated by inserting an intermediate cut or pole which joins single particle or trajectory states of different quark species. In terms of the topological expansion^{5,30} for an n-point function we have

$$A_n(g, n) = n^{1-n/2} \sum_{b=1}^n N^{1-b} (1/N^2)^b A_{n,b,h}(\lambda^2) \quad (54)$$

with $\lambda^2 = g^2 N$, fixed, where N is the number of quarks in $U(N)$, b the number of boundaries to which external lines are attached, and h the number of handles of the two-dimensional surface ($h = 0$, sphere; $h = 1$ torus, etc.) in which the graph can be imbedded. These planar vertices have $h = 0$, $b = 1$, and are of order N^0 (ignoring the fixed $N^{1-n/2}$ factor). Single cylinder corrections have $h = 0$, $b = 2$, and are of order N^{-1} . The diagrams of Figs.4a and 4b represent two topologies, with the dashed line the surface $S^{(h)}$, and the solid closed circles, with external lines attached, the boundaries of S^h . To the right of each we draw the corresponding quark diagram. In Fig.4c we have the double cylinder or "pair of pants" diagram $O(1/N)$ down from the single cylinder and $O(1/N^2)$ from the planar diagram. It is this diagram which is important for "double" OZI violation as in $\psi \rightarrow \phi \pi^+ \pi^-$.

At this point a problem of principle occurs when the topologically equivalent forms are saturated by single particle or single trajectory states. For example, when applied to $\psi \rightarrow \phi \pi^+ \pi^-$, one has the three alternatives Figs.5a, 5b, and 5c. Are there separate independent diagrams each of which contributes to the topological graph, or rather is each a single term estimate to three different alternative expansions of the single topological graph? If the former is the case, one should add the diagrams in the amplitudes; if the latter is the case, one should choose one of the diagrams, hopefully the one which best estimates the single topological graph. In Ref.3, in the spirit of the topological expansion, we took the latter point of view. We still believe this to be the correct one. It is possible to argue that the topology above does not imply the equivalence we demanded, and this question seems not to be resolved at this time.³¹

VI. Summary

We have reviewed, expanded, modified, and commented on our previous approaches to OZI violation. A good deal of data from both the timelike and spacelike transits have been correlated. Difficulties with the simple pole approach, off-shell behavior, and the possible disease of double counting have been revealed. More work remains to be done. In particular, we are currently studying the $J^P = 0^-$ and 2^+ OZI violation, the latter intimately connected with the Pomeron singularity, allowing the most direct test of the conjecture that OZI violation and the Pomeron are one and the same phenomenon.

In addition we are considering more general 3×3 propagator-mass matrix mixings, in which the $h(s)$ of Eq. (53) plays the role of an OZI violating potential which can be fit to reproduce the particle masses. General formulae emerge for the particle mixings which depend only on observed or observable masses, and reveal the model independence of many of the "0"-meson results while dispensing with the literal particle pole interpretation. Results of these efforts will be reported shortly.

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Figure Captions

Figure 1(a) Dual diagram for Regge exchange. (b) Dual diagram for Pomeron exchange.
 (c) Pomeron as iteration of twisted diagrams. (d) Pomeron-like effects in
 S-channel decays.

Figure 2(a) f-dominated Pomeron Picture. (b) Corresponding decay diagrams.
 (c) Diagram for ϕ -decay. (d) Diagram for ϕ production.

Figure 3(a) Signal to background diagrams ϕ or ψ production in hadronic collisions.
 (b) - (g) OZI violating decay diagrams for various rates.

Figure 4(a), (b), (c). $O(N^0)$, $O(N^{-1})$, and $O(N^{-2})$ vertex dual diagrams in terms of
 quark lines (right) and in terms of a cut sphere (left) with particles
 attached to (solid lines) holes in sphere (dotted lines).

Figure 5(a), (b), and (c). "0"-meson diagrams corresponding to Figure 4(c), (b),
 and (b), respectively.

(a)

(b)

(c)

(d)

(a)

(b)

(d)

(d)

=

(a)

=

(b)

=

=

(c)

(a)

(b)

(c)

(a)

(b)

(c)

(d)

(e)

(f)

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J. S. 10/1/69