COOLANT MIXING IN LMFBR ROD BUNDLES AND OUTLET PLENUM MIXING TRANSIENTS

Progress Report

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Reports Issued Under This Contract

A. **Quarterly Progress Reports**

COO-2245-1   Period June 1, 1972 - November 30, 1972

COO-2245-2   Period December 1, 1972 - February 28, 1973

COO-2245-3   Period March 1, 1973 - May 31, 1973

COO-2245-6   Period June 1, 1973 - August 31, 1973

COO-2245-7   Period September 1, 1973 - November 30, 1973

COO-2245-8   Period December 1, 1973 - February 28, 1974

COO-2245-10   Period March 1, 1974 - May 31, 1974

COO-2245-13   Period June 1, 1974 - August 31, 1974

COO-2245-14   Period September 1, 1974 - November 31, 1974

COO-2245-15   Period December 1, 1974 - February 28, 1975

COO-2245-23   Period March 1, 1975 - May 31, 1975

COO-2245-25   Period June 1, 1975 - August 31, 1975

COO-2245-26   Period September 1, 1975 - November 30, 1975

COO-2245-28   Period December 1, 1975 - February 29, 1976

COO-2245-30   Period March 1, 1976 - May 31, 1976

COO-2245-31   Period June 1, 1976 - August 31, 1976

Reports Issued Under This Contract

B. Topical Reports

E. Khan and N. Todreas, "A Review of Recent Analytical and Experimental Studies Applicable to LMFBR Fuel and Blanket Assembly Design," COO-2245-4TR, MIT, Sept. 1973.

E. Khan, W. Rohsenow, A. Sonin and N. Todreas, "A Simplified Approach for Predicting Temperature Distribution in Wire Wrapped Assemblies," COO-2245-5TR, MIT, September 1973.

T. Eaton and N. Todreas, "Instrumentation Methods for Interchannel Coolant Mixing Studies in Wire-Wrap Spaced Nuclear Fuel Assemblies," COO-2245-9TR, MIT, June 1974.

Y. B. Chen, K. Ip, N. E. Todreas, "Velocity Measurements in Edge Subchannels of Wire Wrapped LMFBR Fuel Assemblies," COO-2245-11TR, MIT, September 1974.

E. Khan, N. Todreas, W. Rohsenow, A. A. Sonin, "Analysis of Mixing Data Relevant to Wire-Wrapped Fuel Assembly Thermal-Hydraulic Design," COO-2245-12TR, MIT, Sept. 1974.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," COO-2245-16TR, MIT, March 1975.

E. Khan, W. M. Rohsenow, A. Sonin, N. Todreas, "Input Parameters to the ENERGY Code (To be used with the ENERGY Code Manual) COO-2245-17TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III," COO-2245-18TR, MIT, May 1975.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "Manual for ENERGY Codes I, II, III Computer Programs," COO-2245-18TR Revision 1, MIT, July 1976.

P. Carajilescov and N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," COO-2245-19TR,

B. Chen and N. Todreas, "Prediction of Coolant Temperature Field in a Breeder Reactor Including Interassemly Heat Transfer," COO-2245-20TR, MIT, May 1975.

F. Carre and N. Todreas, "Development of Input Data to Energy Code for Analysis of Reactor Fuel Bundles," COO-2245-21TR, MIT, May 1975.

Reports Issued Under This Contract

B.   Topical Reports, Continued

H. Ninokata and N. E. Todreas, "Turbulent Momentum
Exchange Coefficients for Reactor Fuel Bundle Analysis,"
COO-2245-22TR, MIT, June 1975.

R. Anoba and N. Todreas, "Coolant Mixing in LMFBR Rod
Bundles and Outlet Plenum Mixing Transients,"
COO-2245-24TR, MIT, August 1975.

B. Bosy, "Fabrication Details for Wire Wrapped Fuel
Assembly Components," COO-2245-27TR, MIT, November 1975.

Ralph G. Bennett and Michael W. Golay, "Interferometric
Investigation of Turbulently Fluctuating Temperature
in an LMFBR Outlet Plenum Geometry," COO-2245-29TR, MIT,
June 1976.

N.E. Todreas, "Analysis Methods for LMFBR Wire Wrapped
Bundles," COO-2245-32TR, MIT, November 1976

K.L. Basehore and N.E. Todreas, "Development of Stability
Criteria and an Interassembly Conduction Model for the
Thermal-Hydraulics Code SUPERENERGY," COO-2245-33TR, MIT
December 1976.

Reports Issued under this Contract

C. Papers and Summaries

Yi Bin Chen, Ka-Lam Ip, Neil E. Todreas, "Velocity
Measurements in Edge Channels of Wire-Wrapped LMFBR
Fuel Assemblies," American Nuclear Society Transactions
Vol. 19, 1974, pp. 323-324.

P. Carajilescov, N. Todreas, "Experimental and Analytical
Study of Axial Turbulent Flows in an Interior Subchannel
of a Bare Rod Bundle," J. of Heat Transfer, Vol. 98,
No. 2, May 1976, pp. 262-268 (Included as Appendix to
Quarterly Progress Report, COO-2245-15).

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous
Body Model for Predicting Temperature Distribution in
Wire-Wrapped Fuel Rod Assemblies," Nuclear Engineering
and Design, 35 (1975) 1-12.

E. Khan, W. Rohsenow, A. Sonin, N. Todreas, "A Porous
Body Model for Predicting Temperature Distribution in
Wire-Wrapped Rod Assemblies Operating in Combined Forced
and Free Convection," Nuclear Engineering and Design,
35 (1975) 199-211.

Ralph G. Bennett and Michael W. Golay, "Development of
an Optical Method for Measurement of Temperature Fluc-
tuation in Turbulent Flows," American Nuclear Society
Transactions, Vol. 22, 1975, p. 581.

B. Chen and N. Todreas, "Prediction of the Coolant
Temperature Field in a Breeder Reactor Including
Interassembly Heat Transfer," Nuclear Engineering and
Design 35, (1975) 423-440 (Included as Appendix to
Quarterly Progress Report, COO-2245-23).

R. Bennett and M.W. Golay, "Interferometric Investigation
of Turbulently Fluctuating Temperature in an LMFBR Outlet
Plenum Geometry," Accepted for the ASME Winter Annual
Meeting, Dec., 1976, (Included as Appendix in Quarterly
Progress Report, COO-2245-30).

B.B. Mikic, E.U. Khan, N.E. Todreas, "An Approximate Method
for Predicting Temperature Distribution in Wire Wrapped Fuel
Assemblies of a Liquid Metal Fast Breeder Reactor," Mech.
Res. Comm., Vol. 3, 353-360 (1976).

Commencing with report COO-2245-30, a new task, TASK IV, which has been added to the contract, was reported. This TASK IV and TASK IID differ in that TASK IV is initially concentrated on thermal analyses using slug and laminar velocity profiles while TASK IID is concentrated on hydro-dynamic analyses of turbulent velocity fields.

COOLANT MIXING IN LMFBR ROD BUNDLES AND
OUTLET PLENUM MIXING TRANSIENTS

Contract AT(11-1)-2245

Quarterly Progress Report

September 1, 1976 - November 30, 1976

The work of this contract has been divided into the
following Tasks:

TASK I: BUNDLE GEOMETRY (WRAPPED AND BARE RODS)

    TASK IA: Assessment of Available Data

    TASK IB: Experimental Bundle Water Mixing
                Investigation

    TASK IC: Experimental Bundle Peripheral Velocity
                Measurements (Laser Anemometer)

    TASK ID: Analytic Model Development - Bundles

TASK II: SUBCHANNEL GEOMETRY (BARE RODS)

    TASK IIA: Assessment of Available Data

    TASK IIB: Experimental Subchannel Water Mixing
                 Investigation

    TASK IIC: Experimental Subchannel Local Parameter
                 Measurements (Laser Anemometer)

    TASK IID: Analytic Model Development - Subchannels

TASK III: LMFBR OUTLET PLENUM FLOW MIXING

    TASK IIIA: Analytical and Experimental Investigation
                  of Velocity and Temperature Fields

TASK IV: THEORETICAL DETERMINATION OF LOCAL TEMPERATURE FIELDS
IN LMFBR FUEL ROD BUNDLES

TASK I: BUNDLE GEOMETRY (WRAPPED AND BARE RODS)

TASK IA: <u>Assessment of Available Data and Codes</u> (Kerry Basehore)

No work was performed this quarter.

TASK IB.2 <u>Experimental Bundle Water Mixing Investigation</u>

Repeat of Laminar and Transition Flow - 61 Pin Fuel Mixing
Experiments (Stuart Glazer)

During the past quarter, 3 salt injection rods were built.
Initial difficulties in fabrication forced temporary delay of
the mixing tests. It was therefore decided to initiate flow
split tests while waiting for the completion of the injection
rods. Initial flow split data was taken at an approximate
Reynolds number of 10,000, the maximum capacity of the outlet
plenum. As the eventual goal of the test series is to take
flow split and mixing data well into the turbulent regime, it
was decided to disassemble the bundle, and add a second water
discharge line at the outlet plenum. It is estimated that
this will widen the range of Reynolds numbers at which data
can be taken to at least 20,000.

While the bundle was disassembled, several of the pins
which were recently wrapped and found to be faulty were re-
paired. Flow split data over a wide range of Reynolds numbers
will be taken during the next quarter. Mixing will also be
initiated, pending completion of the flow split data.

The hardware modifications to the Interdata Computer,
required for use as key component in the Automated Data
Acquisition System, were completed. Additional software,
comprising the real time operating system, was written, and
the entire system is being readied for laboratory checkout
early in the current quarter. Completion of the additional
software package to be used for analysis of data taken with
the computer based system will be completed during the next
quarter.

TASK IB.3 <u>217 Pin Mixing Experiments</u> (Stuart Glazer)

Modification of the 217 pin bundle obtained from HEDL
was nearly completed durin the past quarter. Space and
mounting considerations required the shortening of the bundle
and all pins to approximately five feet. Additional mounting
hardware, necessary to permit bundle assembly with use of the
inlet and outlet plenum of the 61 pin fuel bundle, was designed
and fabricated. The bundle is expected to be ready for testing
early in the next quarter.

TASK IB.4 61 Pin Blanket Bundle Experiment (Chong Chiu)

During this quarter, flow split and mixing experiments were run.

Flow Split - Results are shown in Figs. 1, 2 and 3 for the three subchannel types compared to theoretical flow splits. A strong dip and hump appear in the transition regime for the interior and edge channels. This trend is not apparent from the corner channel data primarily due to the larger error involved with only six corner positions to average. Additional measurements are planned to confirm the transition regime results and explore the laminar regime. Significant differences are also apparent between theoretical and measured results over the Reynolds number region tested which persist even at Reynolds number of 16×10^3 where the measurements appear to become constant with Reynolds number.

Mixing - Salt conductivity data was obtained with the injector rod 16 and 22.5 inches from the measurement plane. These data were gathered over the following test conditions

$$668 \leq Re \leq 10,650$$

for the following geometric conditions

> BUNDLE GEOMETRIC PARAMETERS:
> P/D=1.067
> NUMBER OF RODS=61
> ROD DIAMETER=0.501 INCH
> WIRE DIAMETER=0.0314 INCH
> FLAT TOLERANCE=0.022

The data was reduced using the experimental flow split presented above. Values of ε^* and C_1 which fit the data were obtained by a last mean square method utilizing SUPERENERGY. These resulting values are shown on Figs. 4 and 5 which indicate that for this geometry the flow appears to be slipping over the wire wrap more than for geometries of increased h/D.

TASK IC: Experimental Bundle Peripheral Velocity

Measurement (Laser Anemometer)

No work was performed this quarter since the equipment is being used for TASK III experiments.

TASK ID: <u>Analytic Model Development - Bundles</u>

TASK ID.1:   <u>Steady State SUPERENERGY</u> (Kerry Basehore)

1.   SUPERENERGY empirical constants

     There has been some confusion on the definition and usage of empirical constants in SUPERENERGY [1]. These difficulties stem from two sources. First, the length parameter used in nondimensionalizing the enhanced eddy diffusivity constant in SUPERENERGY is not consistent with the ENERGY definition [2, 3,4]. SUPERENERGY uses the rod diameter while ENERGY uses the interior subchannel equivalent diameter. In order to correct this inconsistency, the SUPERENERGY code has been revised by simply changing the factor CONVER on line 3 of page 2-3 in the manual. Revision 1 of the Manual [1], now being issued, contains this change along with an updated sample program. Thus, Khan's correlation [2,3] for ϵ^* anc C can now be used to provide the constants for SUPERENERGY. The constants from this correlation must, however, be adjusted by lateral and axial porosities as specified in the appendices of references 2 and 3 since SUPERENERGY is based on a subchannel arrangement and not on the porous body model used in producing the correlations. This adjustment is frequently missed and is the source of the second difficulty with the empirical constants.

     With the aid of some simplifying assumptions, it is possible to put the conversion factors in a usable form. When Khan's correlation is used, the empirical constants for SUPERENERGY are a combination of these conversion factors and the correlation values. The derivation of the two conversion factors follows:

1a.   Derivation of the Eddy diffusivity conversion factor to transform ϵ^* to ϵ_1^*

     ϵ^* = nondimensional enhanced eddy diffusivity given by Khan's correlation and used in the ENERGY porous body model.

     ϵ_1^* = nondimensionalized enhanced eddy diffusivity used in the ENERGY subchannel model and SUPERENERGY Revision 1.

$$\epsilon^* = \frac{\epsilon_H}{U \cdot D_e} \qquad \epsilon_1^* = \frac{\epsilon_H}{V \cdot D_e} \qquad (1)$$

where

     ϵ_H = enhanced effective eddy diffusivity for heat transfer
     V = average bundle velocity (subchannel model)

U = superficial average bundle velocity (porous body model)

D_e = equivalent interior channel hydraulic diameter

then from [1]

$$\varepsilon_1^* = \frac{\lambda_a}{\lambda_{L1}} \varepsilon^* \qquad (2)$$

and

λ_a = bundle average axial porosity

λ_{L1} = lateral porosity in the bundle interior region.

$$\lambda_a = \frac{\text{void area}}{\text{total area}} = \frac{\frac{\sqrt{3}}{2} D_F^2 - \frac{N\Pi}{4}(D_R^2 + D_W^2)}{\frac{\sqrt{3}}{2} D_F^2} \qquad (3)$$

$$\lambda_{L1} = \frac{\text{lateral cross flow area}}{\text{pitch length}} = \frac{P - D_R}{P} = 1 - \frac{D_R}{P} \qquad (4)$$

where

D_F = duct flat to flat distance = $\left[\sqrt{3}\, PN_{RING} + D_R + 2D_W\right]$

P = fuel rod triangular pitch

D_R = fuel rod diameter

D_W = wire wrap diameter

N = number of fuel rods in the bundle

N_{RING} = number of hexagonal rings of fuel rods in the bundle

assume that the fuel rod pitch P can be approximated by

$$P \cong D_R + D_W$$

then [3] becomes

$$\lambda_a = 1 - \frac{N\Pi}{2\sqrt{3}}\, \frac{\left[D_R^2 + (P - D_R)^2\right]}{\left[P(\sqrt{3}\, N_{RING} + 1) + (P - D_R)\right]^2} \qquad (5)$$

$$\lambda_a = 1 - \frac{N\Pi}{2\sqrt{3}} \frac{\left[(\frac{D_R}{P})^2 + (1 - \frac{D_R}{P})^2\right]}{\left[\sqrt{3} \; N_{RING} + 2 - \frac{D_R}{P}\right]^2} \tag{5}$$

replacing (4) and (5) into (2) yields

$$\varepsilon_1^* = B_1 \; \varepsilon^* \tag{6}$$

where

$$B_1 = \left[1 - \frac{N\Pi}{2\sqrt{3}} \frac{\left[(\frac{D_R}{P})^2 + (1 - \frac{D_R}{P})^2\right]}{\left[\sqrt{3} \; N_{RING} + 2 - \frac{D_R}{P}\right]}\right] \left[\frac{1}{\left[1 - \frac{D_R}{P}\right]}\right] \tag{7}$$

The conversion factor in Eq. (7) is presented graphically in Fig. 6 as a function of P/D_R and the number of fuel rods, N.

1b. Derivation of the swirl velocity conversion factor to transform C to C_1

    C = ratio of the superficial swirl velocity to the superficial bundle average axial velocity used in the ENERGY porous body model and given by Khan's correlation.

    C_1 = ratio of thw swirl velocity to bundle average axial velocity in the ENERGY subchannel model and SUPER-ENERGY

$$C = \frac{U_s}{U} \qquad C_1 = \frac{V_s}{V} \tag{8}$$

where

    U_s = superficial swirl velocity in the bundle peripheral region

    V_s = actual swirl velocity in the bundle peripheral region.

then from Eq. (8)

$$C_1 = \frac{\lambda_a}{\lambda_s} \; C \tag{9}$$

and 6: for bundle ??? . ??!

λ_a = bundle average axial porosity as derived above ly from a ?? ?? water bundle at two flow rates

λ_s = lateral porosity in the peripheral region of lengths in the bundle. ??? For the purpose of this comparison, only the turbulent flow data was considered here since most ??? of interest that satisfy the Gr_g^* criteria [2,3] enabling

$$\alpha_s = \frac{\text{lateral cross flow area}}{\text{region width}} \qquad (10)$$

??? ??? ?? ??? data analysis can be undertaken ter the validity of this new swirl model is accomplished ??? more data is available.

where

Figures 9 and 10 show twelve inch wire wrap pitch data. ?? thig = distance between the duct wall and outermost transverse fuel rod, i.e., the velocity at a particular point within the subchannel. The transverse velocity shape we seek assume that g can be approximated by ENERGY CODE. Since this code utilizes only subchannel averaged transverse velocities, some question exists regarding which data is most applicable for this purpose. The gap point R A, B and C perhaps do the best job since they contain little of the internal subchannel

$$g = \frac{P - D}{2}$$

then Eq. (10) becomes compared to points D and E) that are modeled by another empirical constant, the enhanced eddy diffusivity ??. Future work should include a more detailed experimental look into the subchannel transverse velocity flow field

$$\lambda_s = \frac{P - D_R}{P - \frac{D_R}{2}} = \frac{P - D_R}{P - \frac{D_R}{2}} \qquad (11)$$

that is true lumped or average transverse velocity can be found.

$$P - D_R + \frac{D_R}{2} \qquad P - \frac{D_R}{2}$$

Figure 11 shows a replot of Fig. 9 on a normalized scale. Also included are two averages, again normalized. The dashed line probably represents the best average of the two since it uses data points that are far from the corner and the most fully developed hydrodynamically.

$$\lambda_s = \frac{P - D_R}{P - 1}$$

Figures 12, 13 and 14 are similar to Figs. 9, 10 and 11 respectively except they are for the six inch wire wrap pitch. replacing Eqs. (5) and (11) into Eq. (9) yields

All MIT data is arbitrarily extrapolated to zero at $\alpha = 0°$, i.e., the point at which the wire wrap passes through the gap between the fuel pin and duct wall. Thus the range of ??? bility of MIT data extends from $45° < \alpha < 315°$. Future efforts should try to fill in this range of data uncertainty, $-45° < \alpha < 45°$.

$$C_1 = C_2 \frac{N\Pi}{2\sqrt{3}} \frac{\left[\left(\frac{D_R}{P}\right)^2 + (9) \left(\frac{D_R}{P}\right)^2 \right] \left[\frac{P}{D_R} - \frac{1}{2} \right]}{\left[\frac{1}{\sqrt{3}} N_{RING} + 2 + \frac{D_R}{P} \right]^2 \left[\frac{P}{D_R} - 1 \right]} \qquad (13)$$

research - 1 pin bu??

B may be ??? ??? ??? ??? ??? of the transverse velocity their six inch wrap lead bundle which ??? designed primarily in a wire ??? axial pressure ??? in a wire profile may not

The conversion factor in Eq. (13) is presented graphically in Fig. 7 as a function of P/D_R and the number of fuel rods, N.

2.   Swirl flow model (Kerry Basehore)

The present version of SUPERENERGY [1] requires the input of a correlated empirical constant, C_1 which is the ratio of the transverse velocity in the bundle periphery to the bound average axial velocity. ENERGY [2,3,4] has a similar constant C, which is a ratio of the superficial velocities encountered in the porous body model. These constants are discussed in some detail in the preceding section. Both models assume that this input parameter and therefore the transverse velocity or swirl flow is invariant with edge subchannel axial and radial positioning. Experimental studies, most notable the WARD 11:1 air flow experiments [5,6,7] and the MIT laser doppler anemometer water tests [8], have shown that this assumption is incorrect.

Work started this quarter on reducing the available swirl flow data and incorporating it into the SUPERENERGY code. Data from all sources was graphed on a common base for ease in comparison. The convention used was that suggested by the WARD 11:1 documents [5,6,7]:

$$\frac{V_T}{V_B \cdot \tan \theta} \quad \text{versus} \quad \alpha$$

where

V_T = transverse velocity in the bundle periphery

V_B = bundle average axial velocity

$\tan \theta$ = tangent of the angle of projection of the wire wrap centerline on the axis of the rod

α = wire wrap position on the rod circumference. The zero position is when the wire is crossing the gap between the fuel rod and the duct wall.

This definition implies that when

$$\frac{V_T}{V_B \; \tan \theta} = 1 \quad ,$$

then the flow is following the wire wrap tengentially. For more discussion see Ref. 5.

MIT-61 pin bundle data [8]

The MIT data was collected at several points radially
(see Fig. 8) from a 61 pin water bundle at two flow rates
(one turbulent, the other laminar) and wire wrap lead lengths
of six and twelve inches. For the purpose of data comparison,
only the turbulent flow data was considered here since most
cases of interest that satisfy the Gr_c^* criteria [2,3] enabling
use of forced convection SUPERENERGY will be in this flow
regime. Expansion to laminar data analysis can be undertaken
after the valadity of this new swirl model is accomplished
and more data is available.

Figures 9 and 10 show twelve inch wire wrap pitch data.
In this case the transverse velocity plotted is the local
transverse velocity, i.e., the velocity at a particular point
within the subchannel. The transverse velocity shape we seek
is one which can be used with the SUPERENERGY CODE. Since this
code utilizes only subchannel averaged transverse velocities,
some question exists regarding which data is most applicable
for this purpose. The gap points A, B and C perhaps do the
best job since they contain little of the internal subchannel
mixing effects (as compared to points D and E) that are modeled
by another empirical constant, the enhanced eddy diffusivity
ε_i^*. Future work should include a more detailed experimental
look into the subchannel transverse velocity flow field so
that a true lumped or average transverse velocity can be found.

Figure 11 shows a replot of Fig. 9 on a normalized scale.
Also included are two averages, again normalized. The dashed
line probably represents the best average of the two since it
uses data points that are far from the corner and the most
fully developed hydrodynamically.

Figures 12, 13 and 14 are similar to Figs. 9, 10 and 11 respec-
tively except they are for the six inch wire wrap pitch.

All MIT data is arbitrarily extrapolated to zero at
$\alpha = 0°$, i.e., the point at which the wire wrap passes through
the gap between the fuel pin and duct wall. Thus the range of
reliability of MIT data extends from $45° < \alpha < 315°$. Future
efforts should try to fill in this range of data uncertainty,
$-45° \leq \alpha \leq 45°$.

French - 19 pin bundle data [9]

Lafay et al., took some visual measurements of the trans-
verse velocity in their six inch wire wrap lead bundle which
was designed primarily to investigate radial and axial pressure
profiles in a wire wrap bundle. While these profiles may not

be as precise as laser doppler anemometer or pitot tube measurements, they do serve to point out gross data errors that may occur with the more sophisticated acquisition techniques. The French data are shown on Figs. 12 and 14.

WARD 11:1 Air Flow Tests [5,6,7]

One of the important results of the WARD tests was a detailed measurement of the cross flow profile in the gap as a function of wire wrap angle. Of particular interest in the data reduction done here is the transverse velocity profile for the edge subchannels as reported in Ref. 6 are reproduced in Fig. 15. The profile will vary to a small degree (see Ref. 10) depending on the method used to select the characteristic transverse velocity. Three possible choices are the gap averaged velocity, maximum velocity in the gap region or the gap centerline velocity. The average velocity is plotted here since it appears to be most reasonalbe choice physically. This choice in reality is not that important anyhow, since as can be seen the range of measurement is quite small, $-66.7° < \alpha < 49°$, and offers no hint as to the profile in the center region of Fig. 10.

Through comparison of the various sets of data it was decided to use the French visual data shown in Fig. 14 for the six inch lead velocity profile, and the hybrid curve shown in Fig. 16 for the twelve inch lead velocity profile. These can be updated as more reliable data becomes available. Each of these curves was numerically integrated, and the average was found by changing the coordinate system so that the average is designated as 1.00 it is possible to use the curves only as forcing functions in conjunction with the empirical constant C_1 which will specify the magnitude. Some work has been started in checking this new swirl model against existing heated pin data. These results will be reported upon their completion next quarter.

Recommendations under consideration for future MIT laser doppler anemometer measurements are:

1. Repeat existing data in an attempt to determine reproducibility, also taking more data points per lead length.

2. Extend the range of the measurements to include a four inch lead with geometry similar the CRBR blanket assembly.

3. Extend the range of measurement to include other flow rates in oreder to determine Re effects.

TASK ID 2. Analytical Model Development - Transient

2.1   SUPERENERGY (Stuart Glazer)

During the past quarter, much effort was devoted to
evaluation of the requirements for transient analysis codes,
as well as some of the numerical methods and analysis tech-
niques currently available. Following is a report listing
the results, and recommendations for future effort.

1.0   Introduction

Knowledge of primary coolant and fuel temperatures in
the core region of a liquid metal fast breeder reactor during
transient events is necessary to verify component design
temperature requirements, verify reactor safety, establish
allowable burnup limitations, and evaluate rod and assembly
bowing and associated loads on the core support structure.
Simplified assumptions and computation techniques currently
in use produce necessarily conservative estimates of these
temperatures. As fuel cost is highly sensitive to burnup
restrictions, one major goal of the core designer is to
reduce the conservation of these estimates so that limits
on fuel burnup may be safely extended. Fuel rods in LMFBR's, unlike the bare rods in PWR's,
are spirally wound with a spacer wire, and packed in arrays
in hexagonal ducts. Further, interassembly heat transfer
effects are extremely important in LMFBR's owing to the very
severe power gradients existing in the radial blanket region.
Therefore, to refine estimates of coolant and fuel tempera-
tures throughout the core, computer codes must accurately
model intraassembly heat transfer at the very detailed sub-
channel level, and must evaluate interassembly heat transfer
effects for at least a symmetrical 1/12 core sector. A large number of computer codes have been developed
to date which analyze coolant temperatures in LMFBR's under
steady state conditions. Some of these are: COBRA [11],
THI-3D [12], COTEC [13], CORTAC [14], SUPERENERGY [15], and
ENERGY I, II, III [16]. A chart summarizing the available
codes and their capabilities has been prepared by Dr. N.
Todreas of MIT and is included here as Table 1 for reference.
Codes such as COBRA and THI-3D, which solve the momentum,
continuity, and energy equations simultaneously, require long
computational running times and large amounts of memory, even
for the solution of the steady state temperature field. Use
of this type of code to predict temperature fields during
transient events requires such long computational running
time as to be totally impractical.

The ENERGY series of codes, developed initially at MIT by E. Khan, and and extended with the steady state multiassembly code SUPERENERGY by B. Chen, have exhibited very good agreement with the data [17]. In particular, the SUPERENERGY code, restricted to problems where intra-assembly bouyancy forces are negligible in relation to the viscous forces, requires relatively modest amounts of core storage for even the largest cases, and extremely fast running times. This is due to the explicit finite difference scheme used, and the fact that the ENERGY model permits uncoupling of the ENERGY, momentum, and continuity equations for those cases where the bouyant forces are unimportant. E. Khan developed a modified Grashoff number criterion specifically to determine when decoupling of the 3 equations is permissible. Flow under these conditions is termed forced convection in the context of the ENERGY model. In cases of forced convection, temperature differences caused by radial power skews are assumed not to significantly affect the flow field.

The memory and computational time requirements of the SUPERENERGY code makes its extension to cover transient events in single and many coupled assemblies in cases of forced convection highly desirable. It will be shown in section 5 that it is not possible to provide predictions of the coolant temperature field at the subchannel level for all assemblies in 1/12 core sector studies. Two different versions of a transient SUPERENERGY code are therefore proposed to deal with individual aspects of the problem.

Chapter 2 describes broad clases of transient events postulated to occur in the design life of the Clinch River Breeder Reactor Plant, and defines the capabilities and operational characteristics considered essential for a useful transient computer code. Chapter 3 describes limitations and capabilities of currently available transient analysis codes. Chapter 4 evaluates several numerical methods, with particular application to the extension of the steady state SUPERENERGY code to cover transients. Chapter 5 proposes specific modifications to the ENERGY model for each of the 2 versions of the code; one intended for evaluation of interassembly heat transfer in 1/12 core sector studies, and one intended for detailed single assembly studies. Finally, Chapter 6 discusses the work to be done and additional questions to be resolved in the process of providing a complete package of transient analysis codes applicable in the forced convection flow regime.

2.0 Requirements for Transient Analysis Codes

2.1 CRBRP Core Transients

The transient version of SUPERENERGY under development is designed for use with liquid metal fast breeder reactors. The transients defined in this section are solely representative of events considered possible at various times during the design life of the Clinch River Breeder Reactor Plant (CRBRP)

Many trasient events are postulated to occur during the life of the CRBRP. Although thermal effects, and operational, structural, and safety consequences must be determined for all of these, a small number of events have been identified as "worst case" transients, i.e., those events imposing the most severe thermal conditions on the reactor core for a large number of similar events. The list of transients contained in this section were determined with the assistance of Dr. Ron Coffield at Westinghouse Advanced Reactors Division. The events described herein are not intended to represent either a complete list of "umbrella" events, and may not reflect the final event definitions. They are intended to identify some of the major causes of significant core therml transients, as well as some transient mechanisms, such as variation of coolant flow rate, rod power production, and inlet coolant temperature with time, and to define even duration times. This data is necessary to establish the capabilities for a transient analysis code.

The list of transients and event descriptions is contained in Table 2. Since current efforts are directed toward development of a code capable of thermal analysis under conditions of forced convection as defined in section 1.0, only those transient events or parts of transient events will be considered here which result in coolant flow in the forced convection regime at all times.

It can be seen from Table 2 that a useful transient analysis code must include the capability to analyze transients resulting in either single or combined variation of coolant flow rate, increases and decreases of power production of various rates, and also variation in coolant inlet temperatures. Event duration and transient energy storage within fuel rods affect dictate that the code be able to analyze transients lasting in the range of at least 1/2-2 minutes. Changes in bundle geometry resulting from severe thermal transients are not within the scope of this analysis.

2.2 CRBRP Core Transient Analysis Requirements

The CRBRP core consists of 377 [18] fuel, blanket, and primary and secondary control assemblies. All fuel, breeding, and control rods are contained within hexagonal stainless steel ducts of uniform flat to flat dimensions. Individual fuel and blanket bundles include groupings of 61 and 217 pins. Severe thermal gradients occurring in the core during reactor operation currently call for symmetric movement of certain assemblies during the refueling cycles to assure more uniform thermal exposure. A code which can model the thermal and hydraulic effects with a 1/12 core sector is considered sufficient [19] to accurately model interassembly heat transfer during most types of transient events. As a design goal therefore, the transient analysis code under development should be able to couple a maximum of up to 39 assemblies, the number of assemblies in a 1/12 core sector, to attain good estimates of interassembly heat transfer, with at least a coarse nodal structure.

Detailed transient analysis of individual bundles requires that the code must be capable of providing temperature profiles at several axial sections throughout the bundle on a subchannel basis. The largest bundle, the 217 pin assembly, contains 438 subchannels in the coolant. In addition, to account for the transient thermal energy capacity of the fuel rods and ducts which significantly dampen the response of the coolant to rapid thermal transients, it is necessary to accurately model each fuel pin and duct wall to evaluate the transient heat flux absorbed from or released to the coolant.

In addition to the functional characteristics already discussed, the code should offer maximum flexibility to the user in terms of specification of input parameters, forcing functions, and output. It has been determined from discussions with core designers at Westinghouse that when in the multi-assembly analysis role, the code should permit the following degree of specificity:

A. For each bundle type (i.e., 61 pin fuel, 217 pin fuel, etc.)

1. Complete description of all bundle geometric parameters

2. Empirical constants ϵ^* and C_1

3. Complete description of the following forcing functions:

a. normalized rod power generation vs. time

b. normalized bundle flow rate vs. time.

3.2 COBRA

The COBRA [...] of codes were originally designed [...] for fuel assemblies in PWR's. It therefore contains extensive capability to evaluate two phase fluid flow which is not necessary in the currently identified transients occurring in [...] liquid metal fast breeder reactors. [...] code is structured to permit analysis of a wide variety [...] of materials, [...] wire wrapped pins to hexagonal ducts, and as such, is an extremely useful [...] tool in evaluating [...] bundles with new or extremely complex geometries. It does not, however, have the capability to [...] each [...] for single subchannel, or sector [...] COBRA [...] for core sector [...] Heat flux [...] altered the [...] conditions can be either adiabatic, isothermal, radial no[...] profile read in by user, profile read from [...] fuel model with an [...] previous case results on magnetic tape elements in the fuel and clad regions. This is a significant departure from previous versions [...] required fuel to coolant heat transfer as a function of time to be specified as [...]

COBRA III solves the [...] continuity equations simultaneously by using a mixture of explicit and implicit [...] spatial and [...] calculations, combined with [...] in expressive computational time and core storage requirements. It is uneconomical, and in many cases impossible to run transients of necessary duration for the full subchannel [...] in the C[...]

COBRA V has as a goal analysis of interassembly heat transfer effects and [...] information as to individual bundle thermal and hydraulic performance for a 1/12 core sector. This code [...] and lumping within individual bundles. COBRA V is still under development, so that [...] must await its formal release.

3.3 FORE-III (20)

[...] transient [...] performed to date at Westinghouse ARD were done using the code FORE-II. FORE-II is a coupled point kinetics-thermal hydraulic code with the capability [...] maps and other [...] parameters in the coolant, fuel,

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c. normalized axial power profile
d. coolant inlet temperatures vs time

B. For each individual bundle;
1. Bundle type
2. Bundle identification number
3. Neighboring bundles
4. Heat flux conditions on each boundary face, i.e., each face for single subchannel, or sector boundaries for core sector. Heat flux boundary conditions can be either adiabatic, isothermal, profile read in by user, profile read from previous case results on magnetic tape elements
5. Normalized radial pin power profile
6. Initial (time = 0) average rod power value, to be multiplied by the radial, axial, and temporal power profiles to obtain individual rod power
7. Initial (time = 0) bundle inlet coolant flow rate, to be multiplied by the temporal flow rate curve to obtain actual flow rate at any time

C. General data
1. Constant thermal, physical and heat transfer properties for the coolant, fuel, clad, wire, and duct walls.
2. Number of axial sections
3. Maximum permissible size of timestep
4. Axial locations of desired printouts of the code
5. Times at which printouts are required
6. Transient duration
7. Number of nodes per fuel pin
8. Storage device for interassembly heat flux results

In detailed single assembly studies, essentially all of the above data is also requried.

2.3 Core Storage and Computational Time Requirements

An obvious design goal is to minimize both core storage and computational running times for all cases. Specifically, to permit running on many of the large computer systems currently in use in industry and government research laboratories, the largest multiassembly problems and the largest single bundle cases should require a maximum of 2000 K bytes of core storage. Also, for practical consideration, the largest and longest cases should probably not require in excess of 2 hours of running time on a high speed computer, such as a Control Data 7600. This is a softer limit than than on core storage, as the permissible running time can be determined by each user on a case by case basis.

3.0 Available Transient Codes

In this section, brief descriptions will be given of some general computer codes currently used in the evaluation of core transients. It will be shown that none of these codes can practically be used to evaluate coolant and fuel temperatures in large bundles on a subchannel basis while considering interassembly heat transfer effects.

3.1 CORTAC

The code CORTAC [14] was developed by Argonne National Laboratories and General Electric to analyze the effects of core transients on the core restraint systems of liquid metal fast breeder reactors. Structural and thermal analysis of the core restraint system is accomplished by coupled reactor kinetics, core thermal hydraulics, and structural deformation analyses. The core region is modeled as a series of annular rings of assemblies. Each of the annular rings may be further subdivided into up to 9 additional annular regions. Up to 30 axial sections may be specified.

While the above physical model of the reactor core is adequate to perform structural analyses of the core restraint system, it is not sufficient to provide detailed thermal and hydraulic data for a single bundle on a subchannel basis. It is also not sufficient to provide data about individual assemblies during multiassembly studies. Thus, CORTAC cannot be considered sufficient to provide the core analysts the data necessary to set and evaluate fuel burnup requirements nor realistically analyze the safety implications of certain severe transients with respect to the fuel.

3.2 COBRA

The COBRA [11] series of codes were originally designed to provide detailed thermal and hydraulic data on a sub-channel basis for fuel assemblies in PWR's. It therefore contains extensive capability to evaluate two phase fluid flow which is not necessary in the currently identified transients occurring in liquid metal fast breeder reactors. The code is structured to permit analysis of a wide variety of bundle geometries, is not limited to wire wrapped pins in hexagonal ducts, and as such, is an extremely useful design tool in evaluation of bundles with new or extremely complex geometries. It does not, however, have the capability to evaluate multiple assembly clusters.

COBRA III-C/MIT is a modified version of COBRA III altered to reduce running time and allow large number of radial nodes. Both versions include a finite difference fuel model with an arbitrary number of annular elements in the fuel and clad regions. This is a significant departure from previous versions of COBRA which required fuel to coolant heat transfer as a function of time to be specified as input.

COBRA III solves the momentum, energy, and continuity equations simultaneously by using a mixture of explicit and implicit numerical methods permitting unconditional spatial and temporal stability. The complexity of the calculations, combined with the numerical techniques used, result in excessive computational time and core storage requirements. It is uneconomical, and in many cases impossible, to run transients of necessary duration for the full subchannel representation of 217 pin and 61 pin bundles in the CRBRP.

COBRA V has as a goal analysis of interassembly heat transfer effects and some limited information as to individual bundle thermal and hydraulic performance for a 1/12 core sector. This code will require some homogenization and lumping within individual bundles. COBRA V is still under development, so that further evaluation of the code capabilities must await its formal release.

3.3 FORE-III [20]

Many transient analyses performed to date at Westinghouse ARD were done using a modified version of the code FORE-II. FORE-II is a coupled point kinetics-thermal hydraulic code with the capability of providing extremely detailed temperature maps and other system parameters in the coolant, fuel,

and clad for a single fuel rod, as a function of time.
Boundary conditions which must be input are currently
evaluated at Westinghouse with the use of steady state
subchannel codes such as COTEC. Although some statistical
and other data necessary to evaluate safety and burnup
criteria can be determined from FORE-II, it is not capable
of producing temperature fields on a subchannel basis for
an entire fuel bundle or of determining inter and intra-
assembly heat transfer effects. The code cannot be expanded
to a subchannel code since core storage requirements currently
approach the capacity of the computer. The running time is
extremely fast, and runs approximately as long as the transient
event duration.

3.4 Other Codes

Other specialized or test codes exist for determination of
transient temperatures. One such code, COTECT [24], an experi-
mental transient version of COTEC, was an attempt at determining
the feasibility of expanding the CORTAC code into a 3-D
transient version. Results indicated the impracticability
of such a conversion. Hence, the COTECT code is not considered
acceptable for general use.

4.0 Comparison of Fuel and Coolant Models and Numerical Methods

4.1 Energy Model and Steady State SUPERENERGY Code Numerical Methods

The ENERGY method models interchannel heat transfer as
the result of both thermal diffusion and turbulent eddy dif-
fusion. For the specific case of forced convection flow con-
ditions, the velocity profile within the bundle is assumed
known. Flow in the interior region of the bundle is assumed
to be purely axial, and is currently approtioned to each sub-
channel type using E. Novendstern's friction drop correlations.
In the peripheral region of the bundle, consisting of all the
edge and corner subchannels immediately adjacent to the duct,
the flow is postulated to contain both the axial component,
as well as a circumferential component taken to be a percen-
tage of the axial flow.

The assumption of known velocity profile under forced
convection conditions is particularly significant, in that it
permits uncoupling of the energy equations from the momentum
and continuity equations, and causes vast simplifications in
the computational procedure. In the discussion of analytical
methods for conversion of SUPERENERGY to a transient code, it
will be assumed that forced convection flow conditions, as
defined in section 1.0, prevail. The transinet codes developed
as a result of this study will be applicable only to transients
which remain in the forced convection flow regime at all times.

4.2 Modifications for Transient SUPERENERGY Code

4.2.1 Coolant Model

4.2.1.1 Single Temperature per Node

Currently, the steady state SUPERENERGY code uses an explicit, finite difference marching scheme to determine axial and spatial temperature profiles. A single temperature is assigned to each subchannel at every axial step, representing the coolant temperature at the end of the axial step. The single temperature per node model shown in Fig. 17(a) is exemplified by the following explicit equation, describing the steady state temperature at the next axial level in an interior subchannel surrounded by 3 interior subchannels.

$$T_o^{j+1} = T_o^j + \frac{Q_o \Delta z}{C_p V_1} + \frac{(p-d)\ z}{\rho C_p V_1} \left(k + \rho C_p \epsilon_{H_1}\right) \left[\frac{T_1^j + T_2^j + T_3^j + 3T_o^j}{1}\right] \tag{4-1}$$

4.2.1.2 Bender Method

The simplest extension for the transient case of the explicit finite difference marching scheme with a single temperature per node is a finite difference marching scheme explicit both axially and temporally, with the single temperature assigned per node. Equation (4-1) extended to the transient case in this manner becomes

$$T_o^{i+1,j} = T_o^{ij} + \frac{Q_o \Delta t}{C_p} + \frac{(p-d)\Delta t}{\rho C_p A_1}\left(k + \rho C_p \epsilon_{H_1}\right) \left[\frac{T_1^{ij} + T_2^{ij} + T_3^{ij} + 3T_o^{ij}}{1}\right]$$
$$\frac{V_1 \Delta t}{z}\ (T_o^{ij} - T_o^{i,j-1}) \tag{4-2}$$

Bender [12] indicates that for the case of single isolated fuel coolant channel, in contact with only one fuel rod, the governing equation of coolant temperature, a function of position, v, and time, t, within one axial section is given as:

$$\rho C_p A \frac{\partial T_c(v,z)}{\partial t} = \frac{1}{H}\ [T_w(t) - T_c(t)] - \rho C_p VA \frac{\partial T_c(v,z)}{\partial z} \tag{4-3}$$

where Δt is the time step, and i is the timestep index.

The primary advantages of this method are extreme simplicity, minimization of core storage requirements, and speed. The major disadvantage is the stability limits which must be observed. Note that in order for equation 4-2 to be considered strictly correct, the timestep must never be taken larger than the time required for the coolant to pass completely through a node. This time period is called the node sweep time. This restriction is necessary in order that the last term in equation 4-2, the convective term, result in coolant being convected into the current node only from the coolant node immediately upstream. It would be physically incorrect to permit the coolant to be drawn from more than one node upstream of the current node, as the effect of intermediate nodes on the coolant temperature would be lost. Therefore, in order to insure the node sweep time criteria is always observed, we require that

$$V\Delta t \leq \Delta z \qquad\qquad (4-3)$$

or

$$\Delta t \leq \frac{\Delta z}{V} \qquad\qquad (4-4)$$

where Δt is the timestep

Δz is the axial step length

V is the smallest axial coolant velocity in any sub-channel of the bundle.

Note that the node sweep time criteria also applies in the circumferential direction. In the edge and corner nodes, which

lie exclusively in a flow region postulated to contain a cir-
cumferential component of velocity, an additional term would be
added to equation 4-2 representing the transient convection of
energy into the current node from the adjacent edge or corner
node. Care must be taken to insure that in addition to equation
4-4,

$$\Delta t \approx \frac{\Delta l}{V_\theta} \qquad (4-5)$$

where Δl is the lateral extent of the adjacent edge or corner
node, V_θ is the circumferential velocity component.

4.2.1.2 Bender Method

Another finite difference formulation in use in codes such
as CORTAC is one which specifies two nodal temperatures; the node
average temperature and the node outlet temperature, as shown in
Fig. 17(a). Although this formulation is potentially more accurate
than the assignment of a single temperature per coolant node, it
has a disadvantage of requiring twice as much storage.

Bender [13] indicates that for the case of single isolated
axial coolant channel, in contact with only one fuel rod, the
governing equation for determination of coolant temperature as
a function of position, z, and time, t, within one axial section
is given as:

$$\rho C_p A \frac{\partial T_c(t,z)}{\partial t} = \frac{1}{R_{wc}} [T_w(t) - T_c(t)] - \rho C_p V A \frac{\partial T_c(t,z)}{\partial z} \qquad (4-5)$$

where $T_c(t,z)$ = coolant temperature

$\overline{T}_c(t)$ = node average coolant temperature

$T_w(t)$ = node average fuel rod wall temperature

R_{wc} = thermal resistance between the clad and the coolant.

Bender further notes that

$$\overline{T}_c(t) = \frac{1}{\Delta z} \int_0^{\Delta z} T_c(t,z) \, dz \tag{4-6}$$

and

$$\frac{\partial T_c(t,z)}{\partial z} = \frac{T_L(t) - T_{in}(t)}{\Delta z} \tag{4-7}$$

where $T_L(t)$ = node outlet temperature

$T_{in}(t)$ = node inlet temperature, which equals the outlet temperature from the upstream node.

In order to determine a valid expression for $T_L(t)$, Bender solved a reduced form of equation (4-5) by Laplace transform methods, where the heat transfer from the fuel rod to the coolant is replaced by a time varying heat flux, q''. From this, the coolant temperature as functions of both position and time was explicitly determined. Evaluating this solution at $z = \Delta z$, the node outlet, and expanding the result to be applicable to a coolant subchannel in the core, Bender obtains the general solutions for the coolant nodal outlet temperature as a function of time:

$$T_L^{i+1,j} = T_L^{i,j} - 2(T_L^{i,j} - \overline{T}_c^{i,j}) \frac{V\Delta t}{\Delta z} + \sum_{k=1}^{N} \frac{1}{R_{e_k}} (\overline{T}_k^{i,j} - \overline{T}_c^{i,j})\Delta t$$

$$+ \frac{\dot{q}'''_{i+1}\Delta t}{\rho C} \quad (\text{for } \Delta t \leq \frac{\Delta z}{V}) \tag{4-8}$$

$$T_L^{i+1,j} = T_L^{i,j} + \left(\frac{T_{in}^{i+1,j} - T_{in}^{i,j}}{\Delta t}\right)\left(\Delta t - \frac{\Delta z}{V}\right) + \sum_{k=1}^{N} \frac{1}{R_{c_k}}(\overline{T}_k^{i,j} - \overline{T}_c^{i,j})$$

$$\cdot \Delta t + \frac{\dot{q}'''_{i+1}\frac{\Delta z}{V}}{\rho C} \quad (\text{for } \Delta t > \frac{\Delta z}{V}) \tag{4-9}$$

where: i is the timestep index

j is the axial step index

k is the index for neighboring subchannels

R_{c_k} = thermal resistance between node k and the current coolant node.

The general form for the coolant node average temperature is also given by an adaptation of equation 4-5 to finite difference form:

$$\overline{T}_c^{i+1,j} = \overline{T}_c^{i,j} + \sum_k^N \frac{1}{R_{wc}}(\overline{T}_k^{i,j} - \overline{T}_c^{i,j})\Delta t \tag{4-10}$$

$$+ \frac{\dot{q}'''^{i+1}\Delta t}{\rho C} - (T_L^{i,j} - T_{in}^{i,j})\frac{V\Delta t}{\Delta z}$$

Bender compared the accuracy of his particular solutions for the coolant outlet temperature as a function of time with some other heuristic algorithms commonly in use at the time, namely

$$T_L(t + \Delta t) = T_L(t) + [\overline{T}_c(t+\Delta t) - \overline{T}_c(t)] \qquad (4\text{-}11)$$

and

$$T_L(t + \Delta t) = \overline{T}_c(t + \Delta t) + [\overline{T}_c(t+\Delta t) - \overline{T}_{in}(t+\Delta t)] \qquad (4\text{-}12)$$

It was shown that in all cases considered, the exact solution obtained by Bender produced results of higher accuracy than expression (4-11) and (4-12).

In addition, Bender introduced a solution procedure of Levy [24] which permits extension of the timestep, t, in explicit finite difference equations without instability. As reported in the paper by Levy, however, this technique will only produce acceptable accuracy if the temporal stability criteria, as derived for example, by Dusinberre [25], would not normally be exceeded without the method for the majority of nodes in the bundle cross-section. Levy's procedure was derived for the specific purpose of permitting the inclusion of a small number of physically small nodes in a finite difference network containing many larger nodes. As shown by Dusinberre, the time-step stability criteria varies directly with node volume. Since overall network stability is governed by the stability of the least stable node, Levy's method was capable of reducing overall computational time by permitting selection of timesteps normally suitable for most of the larger nodes. Levy reports, however, that even though unconditional stability for all timesteps results with his method, solution accuracy is good only when the timestep selected is normally stable for most of the nodes in the network.

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The Bender method was originally investigated because of the promise of unconditional temporal stability of the explicit finite difference equations. The Levy method does not in general permit large increases in the size of the timestep with acceptable accuracy. Bender does point out the importance of the node sweep time, and develops specialized expressions which are valid for the nodal outlet temperatures for Δt less than the sweep time and approximate expressions for the case of t greater than the sweep time. Timesteps larger than the sweep time cannot strictly be used in a finite difference scheme (see section 4.2.1.1). Other methods, such as that employed by Fox and Yackle [16] have been developed which provide alternate definition of the coolant outlet temperature as a function of time. Doubling of the storage requirements, added complexity and lack of increase of stability suggest rejection of the dual temperature specification per node formulation in favor of that described in section 4.2.1.1.

The only unresolved question concerning the final selection of the single versus dual temperature per node formualtion is that of accuracy. An investigation comparing the results of both methods for various transient test cases will be carried out. Only if the dual temperature approach yields significant improvements will it be selected.

4.2.2 Fuel Models

4.2.2.1 Analytic Solutions

While a number of totally analytic methods [27] exist for determination of both temperature distributions and heat fluxes valid at all times, such tecniques are in general not applicable to the current problem. All such methods necessarily require a priori specification of the heat transfer coefficient, power generation, and coolant temperature. In a transient SUPERENERGY code, the heat transfer coefficient is determinate from the input specification of coolant flowrate and an experimental correlation for heat transfer coefficient in liquid metals as a function of local Nusselt number. Transient power generation is also specified in the input. The coolant temperature as a function of time is not known a priori. Hence, as the coolant temperature is calculated at each timestep, the analytic solution for transient temperature distribution and heat flux may be advanced at most by only one timestep as long as care is taken to insure correct initial conditions at each calculation point. Hence, even though core storage requirements are minimized, the advantages of the analytic approach are lost.

4.2.2.2 Hybrid Solution

Nijsing and Eifler have developed a hybrid solution technique [28] which permits accurate and computationally simple solutions to the transient temperature problems in fuel rods. The hybrid method breaks up the fuel pin cross section into several radial annular nodes, similar to a finite difference

nodalization. The analytic transient energy equation is written
for each node, so that the temperature of each node as a function
of time may be determined in closed form. This solution tech-
nique is not advantageous in the current problem for the same
reasons as the analytic solutions (see section 4.2.2.1).

4.2.2.3 Lumped Empirical Solution Technique

Early in the investigation into the characteristics of
various methods of solution of transient heat fluxes from the
fuel pins, it was recognized that computational time and the
storage requirements could potentially impose a more severe
restriction on code performance than the completely explicit
coolant solution scheme. It was therefore decided to investi-
gate whether a weighted lumped parameter approach would permit
rapid and efficient calculation of transient heat fluxes from
fuel pins, while requiring minimal core storage requirements.
In the previous project report []9] results of this investi-
gation were reported. The mass of the entire fuel pin over an
axial segment was lumped in two studies into one and two finite
difference nodes, respectively, and transient energy balances
written for these nodes. Empirical factors, Γ, were applied
in such a way as to alter the heat capacity of the elements.
These empirical factors were calibrated against the finite
difference fuel model of COBRA III-C-MIT in such a way as to
match the transient heat flux from the rod as well as possible.

The finite difference lumped energy balance for the single
region model is:

$$mC_p \Gamma \frac{dT}{dt} = q''' V - hA_s (T - T_c) \qquad (4\text{-}13)$$

where m = fuel rod mass

           C_p = specific heat

           T = temperature of the (single node) fuel pin

           h = heat transfer coefficient from the surface

           A_s = fuel rod surface area

           T_c = coolant temperature

           q''' = volumetric heat flux

           V = volume of fuel pin.

Similar equations used in the two region lumped model are given in [29]. Note that in the formulation of the single region lumped model, equation (4-13), the thermal resistance of the fuel clad interface does not explicitly appear. This resistance was included in the finite difference code used for calibration, and hence, its effect is included in the empirical constant, Γ. Results presented in [29] indicate that neither the single nor the dual lumped node approach provides acceptable matches in transient heat fluxes, especially in the relatively massive blanket bundle pins. Additionally, it is considered possible that the empirical constants are functions of fuel rod properties, pin geometry, and transient characteristics. The advantages of this method, including extreme computational speed and minimal core storage requirements, were offset by poor accuracy in some cases, and the need for extensive additional parametric analysis and evaluation of the empirical constants. The use of

this method in the transient SUPERENERGY code was therefore rejected, and further development beyond that reported in [29] was suspended.

4.2.2.4 Collocation Method

A new highly accurate procedure recently developed to determine the transient temperature distribution in fuel pins utilizes a collocation method with Hermite cubic splines [30], [31], [32]. The detailed formulation of the equations pertaining to the general case of transient response of the fuel and clad regions of a fuel rod with temperature dependent properties, is contained in [30] and will not be presented here. Simply, the technique involves nodalization of the fuel rod into annular circumferentially symmetric elements in the fuel and clad regions. The temperature distribution in each region is assumed to be of a specific form, namely, cubic Hermite polynomials. Substitution of the assumed temperature distribution into the governing partial differential equation and boundary and initial conditions, and then substitution of these conditions back into the governing equation yield a set of simultaneous equations for the complete temperature distribution, determinate to within a set of constants. It can be shown that the number of constants is equal to twice the number of nodes in the fuel and clad region. Therefore, all the constants may be uniquely determined by requiring the governing differential equations to be satisfied exactly with the assumed temperature distribution at two points in every node. The Gauss-Legendre quadrature points of

order two are chosen as these two collocation points so as to maximize accuracy.

It can be seen that at each step, the simultaneous solution of a set of equations is necessary in order to determine heat fluxes and temperature distributions. A transient version of SUPERENERGY, capable of determining temperature distributions in a 7 pin bundle of fixed geometry, was written as a test vehicle for evaluation of various coolant and fuel models. Comparison of running times using the simple explicit single lumped region fuel model discussed in section 4.2.2.3, with a modified version of a collocation code written by Yeung [33] with constant properties, indicates the collocation procedure is slower by an order of magnitude. Rough estimates for the running time of far larger bundles during long transients would prohibit the use of the collocation technique. Therefore, even though extremely high accuracy has been demonstrated with the collocation method using variable fuel and clad properties, the use of this procedure is tentatively rejected in favor of some other very simple and fast explicit fuel model.

4.2.2.5 - Finite Difference Method

The finite difference method, as used in codes such as COBRA III-C, assumes circumferentially symmetric nodalization of the fuel and clad into annular elements. Discussions with Dr. Ron Coffield of Westinghouse ARD have indicated that 3 nodes in the clad and 7 nodes in the fuel region would normally be sufficient for a "high accuracy" solution. Fully explicit

solutions to the finite difference model are possible, and are described, for example, by Bender [23]. While this method has not currently been programmed and tested in conjunction with the 7 pin bundle test version of transient SUPERENERGY, estimates of running time based on extending the single region explicit fuel model described in section 4.2.2.3 indicate the finite difference method will be much faster than the collocation procedure. Storage requirements will be very large, but estimates of these requirements made in section 5.1.3 and 5.2.3 indicates sufficient storage available in industrial and government computers for even the largest anticipated cases. Evaluation of the finite difference procedure in the test cose, and investigation of the effect of temperature dependent properties on solution accuracy is necessary before final selection is made. If running time and accuracy of the explicit finite difference fuel model compare favorably with the collocation procedure, the finite difference fuel model will probably be selected for use in the transient SUPERENERGY code.

4.2.3 Dynamic Storage Replacement Scheme

Initial investigation of Eq. (4-2) indicates a need to store one value of coolant temperature for each subchannel at every axial level, for both the current timestep and the previous timestep. This would seem necessary as the explicit scheme requires knowledge of the current and surrounding subchannel temperatures at the current timestep, at both the current and previous axial levels. Schematic representation

of this method of temperature storage is shown in Fig. 18a.

It may be seen from Figs. 18a that storage of temperatures at the current timestep more than one axial level upstream is unnecessary. An alternate storage method is therefore proposed shown in Figs. 18b and 18c. In this method, temperatures at the next timestep are calculated and placed in temporary storage until the calculational procedure has marched two levels downstream. Then, the next timestep temperatures are dynamically replaced into the (single) main array as shown. With estimates of 18-36 axial levels in assemblies, the reduction in core storage requirements is significant.

5.0 Analysis Method for Detailed Single Bundle and Multiassembly Studies

It is proposed to write two options within the transient SUPERENERGY code; one which provides detailed temperature fields within a single large bundle while accounting for either user defined or previously calculated interassembly heat transfer, and one version which is capable of considering a 1/12 core sector of 39 highly "lumped" assemblies while determining and storing interassembly heat transfer data. Both versions of the code are described below.

5.1 Single Bundle Studies

5.1.1 Basic Model

The primary purpose of a single assembly study should be

to provide extremely detailed spatial and temporal coolant and
fuel temperature fields for design, analysis, and safety veri-
fication, while considering interassembly heat transfer effects.
Therefore, the basic subchannel coolant nodal structure will
not be changed in the single bundle version of the transient
SUPERENERGY code, with two exceptions; it is proposed to
eliminate the corner subchannels, and the subchannels within
the interassembly gaps between 2 neigboring assemblies. The
major purpose of these moves is to enhance axial stability
criteria, permit use of fewer axial steps, and hence reduce
computational running time.

Strict stability criteria exist for each subchannel type
in the coolant and in the fuel which specify the maximum per-
missible axial step size, and the maximum permissible timestep
size, due to the explicit nature of the proposed computation
scheme. As reported in [22], axial stability criteria of the
gap edge and corner subchannels nearly always governed the
overall stability of bundles in the steady state case. To
eliminate the need for an excessive number of very small axial
steps, K. Basehore developed an alternative procedure which
eliminates the need to calculate temperatures in the gap and
duct regions. Heat transfer between assemblies is modelled
as occurring solely by conduction between edge and corner
subchannels of adjacent bundles. The duct walls of the 2
facing assemblies, as well as the sodium in the interassembly
gap region, are taken to be effective resistances to heat

transfer.

It has been found through examination of stability criteria, that the small physical size of the corner subchannels makes them the next most limiting subchannels after the gap edge and corner subchannels. It becomes possible to significantly increase the axial step size by eliminating the corner subchannels as separate entities, and instead combine 1/2 of the areas of the corner subchannel into each of the two neighboring edge subchannels. Preliminary results from test cases run with the 7 pin bundle test code previously discussed has indicated very little loss in accuracy by this move. In many cases, the temperature of the coolant in the corner subchannel is predicted by the steady state SUPERENERGY code to be very close to the temperature of the surrounding edge channels due to both conduction and the circumferential coolant velocity component assumed to occur in the peripheral subchannels. Therefore, unless further investigation yields significant loss of accuracy, only interior and edge subchannel temperatures will be computed.

Further moves to permit even larger axial step sizes would be desirable, but are not essential at this point. Current estimates are that axial step sizes of the order of 1 inch would be permissible for a variety of bundle geometry and coolant flowrate conditions. This would result in 36 axial levels in a 3 foot bundle, and will probably be considered acceptable from an accuracy, storage, and running time standpoint. It is important to realize that owing to the explicit marching scheme used

in the code, where temperatures are determined by the temperatures of immediately adjacent nodes, it is possible for thermal effects to propagate radially inward by 1 node for every axial step taken. A 217 pin bundle would require at least 18 axial sections for the effects of an externally applied heat flux to penetrate to the center of the bundle.

5.1.2 Timestep Selection

The code will be set to automatically determine the largest permissible timestep. In all cases, the smallest of the following criteria will be used to select the timestep:

1. node sweep time

2. stability criteria of the most restrictive coolant or fuel node

3. maximum time step size as input by the user.

5.1.3 Estimates of Computational Time and Storage Requirements

Detailed estimates of running time and core storage required are currently underway and will be reported in the next quarterly progress report. Rough estimates can be made on storage requirements, however. Major arrays, which contain all coolant and fuel node temperatures at all axial sections in bundle, are assumed to impose the primary storage requirement for the purposes of this calculation. Then, the number of bytes of core storage required is determined approximately as:

$$B = [N_c + (N_p \times N_f)] \times N_a \times N_b$$

where   B = # bytes of storage required

N_c = # subchannels in the coolant

N_p = # fuel pins in the bundle

N_f = # nodes in each fuel pin cross-section

N_a = # axial sections in the bundle

N_b = # bytes/computer word.

For the largest bundle now considered for use in the CRBRP core, the 217 pin bundle,

$$N_c \approx 440, \; N_p = 217, \; N_f \approx 10, \; N_a \approx 36, \; N_b \approx 8$$

The number of bytes of core storage required is then:

$$B \approx [440 + (217 \times 10)] \times 36 \times 8$$

$$\approx 750 \text{ K bytes}$$

Allowing for all other program arrays, a total storage requirement estimated at 1000 K bytes would appear to be fairly realistic for the case of a 217 pin bundle. This estimate falls well within the design goal of 2000 K bytes as discussed in section 2.3.

5.2 Multiple Assembly Studies

5.2.1 Basic Model

Projections of core storage requirements alone listed in section 5.1.3 indicate that the design goal of 2000 K bytes would be grossly exceeded if the 39 bundles in a 1/12 core sector were modelled on a subchannel basis. Computational running times

would also be excessive. To efficiently run large transient
multiple assembly studies therefore it is necessary to greatly
reduce the detail of a subchannel and individual rod nodal
structure of individual bundles. Such homogenization or
lumping is acceptable if we realize that the primary purpose
of this multiassembly code is to provide interassembly heat
transfer rates as boundary conditions to the detailed single
assembly code. In this sense, the multiassembly study results
are used in a cascade method, enabling the designer or analyst
to select particular individual bundles for detailed examina-
tion with the single assembly code described in section 5.1.
Since the designer may not know a priori which particular
assembly will be selected for later detailed study, maximum
utility is achieved only if the multiassembly code stores on
a secondary scratch storage medium, such as magnetic tape,
all interassembly heat transfer rate data, at selected axial
levels at selected points in time during the transient. The
single assembly code can then use the stored interassembly heat
transfer data for the particular bundle selected as thermal
boundary conditions throughout the transient.

Bundle nodal structures in the multiassembly code must
be detailed enough to indicate the presence of internal
power skews, and exhibit the effect of interassembly heat
transfer from each of the six bundle faces. The nature of
the thermal exchanges in a wire wrapped bundle as incor-
porated already in the ENERGY model suggest that in the

multiassembly code, each assembly may be modelled as a lumped
7 pin bundle with no corner subchannels, as shown in Fig. 19.
It is believed that this is the coarsest nodal structure which
can exhibit both inter- and intra-assembly heat transfer effects.
However, any one or several assemblies could have a more detailed
representation as desired.

The 7 pin lumped bundle does not alter the basic ENERGY
heat transfer model, in that it retains the two region flow
field representation. The nodal structure permits the transfer
of heat through the turbulent eddy diffusivity mechanism. It
also retains the feature of energy transport around the bundle
by means of the circumferential coolant flow around the peri-
phery of the assembly. The relatively coarse nodal structure
in the interior of the bundle is believed to be acceptable, as
it has been shown that the circumferential convection of cool-
ant around the bundle has dominant effects on inter- and intra-
assembly heat transfer in most cases. The nodal structure
should prove acceptable as one of the primary purposes of
the code is to determine transient interassembly heat transfer
rates. The single edge node in the bundle also facilitates the
coupling of any size bundle to any other size bundle.

The proposed 7 pin lumped bundle model includes 12 aver-
aged nodal temperatures in the coolant, and a maximum of 10
nodal temperatures in each of the 7 lumped fuel rods. The
presence of the fuel rods is necessitated due to the important
thermal moderating heat capacity effects in determining trans-
ient heat fluxes to the coolant. Interassembly heat transfer

rates as determined from the proposed model will be compared with results from more detailed models, and will be considered acceptable if no significant differences result.

5.2.2 Timestep Selection

The procedure for the dynamic selection of timesteps are as described in section 5.1.2.

5.2.3 Estimates of Computational Time and Storage Requirements

Detailed estimates of running time and core storage requirements are currently underway and will be reported in the next quarterly progress report. A rough estimate on the amount of core storage required will be made in manner of section 5.1.3. In the multiassembly code, storage required for the major arrays is approximately given by:

$$B \approx [N_c + (N_p \times N_f)] \times N_a \times N_{bd} \times N_b$$

where B, N_c, N_p, N_f, N_a, and N_b are defined in section 5.1.3, and N_{bd} is the number of coupled bundles. For 39 coupled assemblies,

$$N_c = 12, N_p = 7, N_f = 10, N_a \approx 18, N_b \approx 8, N_{bd} = 39$$

Then:

$$B \approx [12 + (7 \times 10)] \times 18 \times 39 \times 8$$

$$\approx 460 \text{ K bytes}$$

Allowing for program and miscellaneous storage, the total storage requirement for a 1/12 core sector with the cornerless 7 pin bundle assembly idealization could be taken to be approximately 700 K bytes. This too lies well within the design goal of 2000 K bytes specified in section 2.3. Note that if accuracy demands, individual assemblies could be modelled as 19 pin assemblies. For a 19 pin assembly,

$$N_c = 36, \ N_p = 19, \ N_f = 10, \ N_a = 18, \ N_b = 8, \ N_{bd} = 39,$$

and,

$$B \approx [36 + (19 \times 10)] \times 18 \times 39 \times 8$$

$$\approx 1250 \text{ K bytes.}$$

The total storage requirement with a 19 pin bundle model would therefore be approximately 1500 K bytes.

6.0 Future Development

Work is currently progressing on determination of axial step and time step stability criteria for 61 pin and 217 pin bundles of CRBRP geometry for the single and multiple assembly code models. This data will be used to make detailed projections of core storage and computational running times for various transients listed in table 2. This in turn will enable determination of the range of transients for which each of the

two codes are practically applicable.

In the next quarter, development will begin on the single assembly transient SUPERENERGY code. Upon completion, the code will be tested against existing transient codes such as COBRA III, and compared for accuracy of solution.

Work will be initiated in the near future on development of the multiassembly transient SUPERENERGY code. Results from the code utilizing the 7 pin bundle nodal structure will be compared with those from one using a 19 pin bundle nodal structure. Solution sensitivity to the particular lumped model used will be examined. Estimates as to accuracy, as well as running times, will be compiled for various transients.

Resolution of several questions will be necessary before release of either the single bundle of multiassembly transient computer code. Among those items to be examined are: 1) physical implications of elimination of the corner subchannels as separate entitites, 2) sensitivity of coolant temperature distribution to temperature dependent properties in the fuel and clad, 3) neccessity of inclusion of heat capacity in the duct walls. Additionally, attempts will be made to include in the capabilities of transient SUPERENERGY as many of those specified in Ref. 19 as possible.

Finally, very little detailed data is available to date on mixing effects during flow transients in rod bundles. If deemed necessary at some point in the future, it will be possible to utilize the Automated Data Acquisition System to measure the effects of transients on mixing parameters in 61 pin fuel, 61 pin blanket, or 217 pin fuel bundles.

REFERENCES

[1] B.C. Chen, N.E. Todreas, "Prediction of Coolant Temeprature Field in a Breeder Reactor Including Interassembly Heat Transfer," MIT Department of Nuclear Engineering, May 1975, December 1976; COO-2245-20TR Revision 1.

[2] E.U. Khan, W.M. Rohsenow, A.A. Sonin, N.E. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," MIT Department of Mechanical and Nuclear Engineering, June 1975, COO-2245-16TR.

[3] E.U. Khan, W.M. Rohsenow, A.A. Sonin, N.E. Todreas, "INPUT Parameters to the ENERGY code (to be used with the ENERGY Cosed Manual)," MIT Departments of Mechanical and Nuclear Engineering, May 1975, COO-2245-17TR.

[4] E.U. Khan, W.M. Rohsenow, A.A. Sonin, N.E. Todreas, "Manual for ENERGY I, II, III Computer Programs," Department of Mechanical and Nuclear Engineering, May 1975, July 1976 COO-2245-18TR Revision 1.

[5] Westinghouse Electric Corp, "11:1 Scale Wire Wrapped Rod Bundle Air Flow Test, Interior Subchannels," WARD-D-0108, November 1975.

[6] Westinghouse Electric Corp, "11:1 Scale Wire Wrapped Rod Bundle Air Flow Test, Side Subchannel, WARD-D- Jan. 1976.

[7] Westinghouse Electric Corp, "11:1 Scale Wire Wrapped Rod Bundle Air Flow Test, Corner Subchannel," WARD-D-0154 July 1976

[8] Y.B. Chen, K. Ip and N.E. Todreas, "Velocity Measurements in Edge Subchannels of Wire Wrapped LMFBR Fuel Assemblies," MIT Department of Nuclear Engineering, September 1974, COO-2245-11TR.

[9] J. Lafay, B. Menant, J. Barroil, "Local Pressure Measurements and Peripheral Flow Visualization in a Water 19-Rod Bundle Compared with FLICA IIB Calculations: Influence of Helical Wire-Wrap Spacer System, April 1975, ASME 75-HT-22.

[10] W.T. Sha, R.C. Schmitt, "A Wire Wrap Model," ANS Transactions, Vol. 24, pp. 334-345.

[11] D. Rowe, "COBRA III-C: A digital Computer Program for Steady State and Transient Thermal Hydraulic Analysis of Rod Bundle Nuclear Fuel Elements," BNWL-1965, 1973.

[12] W.T. Sha and R.C. Schmitt, "THI3D - A Computer Program for Steady State Thermal Hydraulics Multichannel Analysis," ANL-8112, July 1974.

[13] M. Carelli, LMFBR Rod Bundle T&H Working Group Meeting, June 2-3, 1976 at MIT, oral presentation.

[14] P.J. Fulford, J. Fox and T. Yackle, "CORTAC - A Core Restraint Transient Analysis Code: Preliminary Users Manual," ANL-CT-76-10, July 1975.

[15] B. Chen and N. Todreas, "Prediction of Coolant Temperature. Field in a Breeder Reactor Including Interassembly Heat Transfer," COO-2245-20TR, May 1975.

[16] E. Khan, W. Rohsenow, A. Sonin and N. Todreas, "A Porous Body Model for Predicting Temperature Distributions in Wire Wrapped Fuel and Blanket Assemblies of a LMFBR," COO-2245-16TR, June 1975.

[17] N.E. Todreas, "Thermal Analysis Methods for LMFBR Wire Wrapped Bundles," COO-2245-32TR, November 1976.

[18] M. Carelli and R. Markley, "Preliminary Thermal-Hydraulic Design and Predicted Performance of the Clinch River Breeder Reactor Core," ASME technical publications, 75-HT-71, August 1975.

[19] Meeting of ERDA/RDD, October 12, 1976, Discussion of Program to Extend COBRA IV to Provide Solution to "Whole Core" Thermal Hydraulic Transients in LMFBR's.

[20] J. Fox, et al., "FORE-II: A Computational Program for the Analysis of Steady State and Transient Reactor Performance," GEAP-5273, September 1966.

[21] J.N. Fox, "COTECT - A Transient Version of COTEC," SG016-MEF-028, March 1976.

[22] K. Basehore and N. Todreas, "Development of Stability Criteria and an Interassembly Conduction Model for the Thermal Hydraulic Code SUPERENERGY," COO-2245-33TR, December 1976.

[23] D. Bender, "An Explicit Unlimited Stability Approach to the Transient Conduction Equations," Proceedings of the Effective Use of Computers in the Nuclear Industry, CONF 690401, p. 561-585.

[24]   S. Levy, "Use of Explicit Method in Heat Transfer Calcu-
       lations with an Arbitrary Time Step," (General Electric
       Technical Information Series), 68-C-282, 1968.

[25]   G. Dusinberre, "Heat-Transfer Calculations by Finite
       Differences," pp. 13-18, International Textbook Company
       1961.

[26]   T. Yackle and J. Fox, "Computational Techniques for Reactor
       Core Transient Conduction-Convection Heat Transfer,"

[27]   J. Blair and W. Selander, "Temperature Transients in a
       Cylinder Due to a Time Varying Heat Transfer Coefficient,"
       Atomic Energy of Canada Report #AECL-4126.

[28]   R. Nijsing and W. Eifler, "The Hybrid Method, A New
       Accurate Computation Method for Transient Temperature
       Distributions in Fuel Rods," Nuclear Eng. and Design,
       Vol. 32, 1975, p. 208-220.

[29]   N. Todreas, M. Golay and L. Wolf, "Coolant Mixing in
       LMFBR Rod Bundles and Outlet Mixing Transients," Progress
       Report, COO-2245-31, August 1976.

[30]   T. Chawla, G. Leaf, W. Chen and M. Grolmes, The Application
       of the Collocation Method Using Hermit Cubic Splines to
       Non-Linear Transient One-Dimensional Heat Conduction
       Problems," ANL-75-33, July 1975.

[31]   T. Chawla, G. Leaf, W. Chen, and M. Grolmes, "The Application
       of the Collocation Method Using Hermite Cubic Splines to
       Non-Linear Transient One-Dimensional Heat Conduction Problems,"
       Trans. ASME, Journal of Heat Transfer, 76-HT-B, November
       1975, p. 562-569.

[32]   T. Chawla, G. Leaf, and W. Chen, "Collocation Method Using
       B Splines for One Dimensional Heat or Mass Transfer Controlled
       Moving Boundary Problems," Nuclear Engineering and Design,
       Vol. 35, 1975, p. 163-180.

[33]   M. Yeung, "The Application of the Collocation Method Using
       Hermite Cubic Polynomials to LWR Fuel Pin Transient Conduc-
       tion Problems," Report in Nuclear Engineering Department,
       September 1975.

TABLE I

CODE APPLICATIONS

	SUBCHANNEL			SINGLE ASSEMBLY		CLUSTER OF ASSEMBLIES	WHOLE CORE
	ADIABATIC B.C.		HEAT FLUX B.C.	ADIABATIC B.C.	HEAT FLUX B.C.		
	INTERIOR	EDGE					
STEADY STATE FORCED	FATHOM-360 MIT-WOLF			COTEC ENERGY I THI-3D COBRA-IIIC	SUPER-ENERGY COBRA V	TRITON SUPERENERGY COBRA V	TRITON SUPERENERGY/ HEXFLOW COBRA V
MIXED CONVECTION				ENERGY II, III THI-3D COBRA IIIC	COBRA V	COBRA V	COBRA V
TRANSIENT FORCED				SUPER-ENERGY I COBRA V	SUPER-ENERGY I COBRA V	SUPER-ENERGY I COBRA B	SUPER-ENERGY I COBRA B
MIXED CONVECTION				COBRA B	COBRA V	COBRA V	COBRA V
WITH FLOW REVERSAL				COBRA V	COBRA V	COBRA V	COBRA V

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TABLE 2

SELECTED CRBRP CORE TRANSIENTS

Transient	Description	Type of Forcing Functions			Estimated Total Transient Duration (Minutes)
		Power Change	Flow Change	Inlet Temp Change	

U-1	Reactor Trip (Scram)
	  Occurs dur to PPS trip level exceeded
	  t = 0.0 sec: PPS trips
	  t = 0.0200 sec: control rods inserted, power
	     drop rapidly
	  t = 0.500 sec: main sodium pumps trip, flow
	     coasts down as:
	  t = 2.500 sec: flow ≈ 80% full flow
	  t = 5.500 sec: flow ≈ 50% full flow
	  t = 30.000 sec: flow ≈ 10% full flow
		✓	✓		1

U-2b	Uncontrolled Rod Withdrawal from 100% Power
	  t = 0.0 sec: step increase in reactor power
	     to 115% full power, maintained
	     for transient duration
	  t = 300.00 sec: manual reactor trip occurs
	
	NOTE: Other less severe scenarios include
	   various ramp rates of reactivity
	   insertion from 0.1¢/sec to 20¢/sec.
	   This event is umbrella event.
		✓		✓	6

U-16	Operating Basis Earthquake
	  Reactivity insertion occurs as a result
	  of core compaction
	  Also, loss of offsite electrical power is
	  assumed. Diesel generators assumed to per-
	  mit main sodium pumps to keep flow at pony
	  motor level
		✓	✓		

(Continued)

Table 2. (continued)

| TRANSIENT | DESCRIPTION | TYPE OF FORCING FUNCTIONS UP UNTIL SCRAM | | | ESTIMATED TOTAL TRANSIENT DURATION (MINUTES) |
		POWER CHANGE	FLOW CHANGE	INLET TEMP. CHANGE	
U-16 (cont.)	t = 0.0 sec: earthquake hits, electrical power to pumps lost, flow begins to coast down to pony motor flow (10% full flow) t = 0.5 sec: step reactivity insertion of 30¢. Also, automatic reactor trip initiated. Rate of control rod insertion slowed due to core misalignment	✓	✓		2
E-16	Loss of offsite Electrical Power t = 0.0 sec: all electrical power to reactor lost, including diesel power. Main sodium pumps coast down, eventual natural circulation of ≈ 3-3-1/2% full flow established t = 0.500 sec: reactor trip initiated	✓	✓		2
F-1	Safe Shutdown Earthquake Reactivity insertion occurs as a result of core compaction Also, loss of offsite electrical power is assumed. Diesel generators assumed to permit main sodium pumps to keep flow at pony motor level. t = 0.0 sec: earthquake hits, electrical power to pumps lost, flow begins to coast to pony motor flow t = 0.500 sec: step reactivity insertion of up to 60¢ occurs. Also, automatic reactor trip initiated. Rate of control rod insertion slowed due to core misalignment.	✓	✓		2

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(continued)

Table 2. (continued)

| TRANSIENT | DESCRIPTION | TYPE OF FORCING FUNCTIONS UP UNTIL SCRAM | | | ESTIMATED TOTAL TRANSIENT DURATION (MINUTES) |
		POWER CHANGE	FLOW CHANGE	INLET TEMP. CHANGE	
F-1 (cont.)	Note: This is an especially severe transient, due to both severe structural damage, and the fact that maximum power could reach 230% of full power for short time before scram. Thermal effects could be very severe.				
	Large Reactivity Insertions For purpose of PPS analysis only, large reactivity insertions of up to $2.00/sec are studied. Ramp rates this large are assumed extremely unlikely faults, and are not postulated to occur.	✓			1

Data:

Novenstern's prediction
Error bar from F=0 to F=1

BUNDLE GEOMETRIC PARAMETERS:
P/D=1.067
NUMBER OF RODS=61
ROD DIAMETER=0.501 INCH
WIRE DIAMETER=0.0314 INCH
FLAT TOLERANCE=0.022 INCH

$Re \times 10^{-3}$
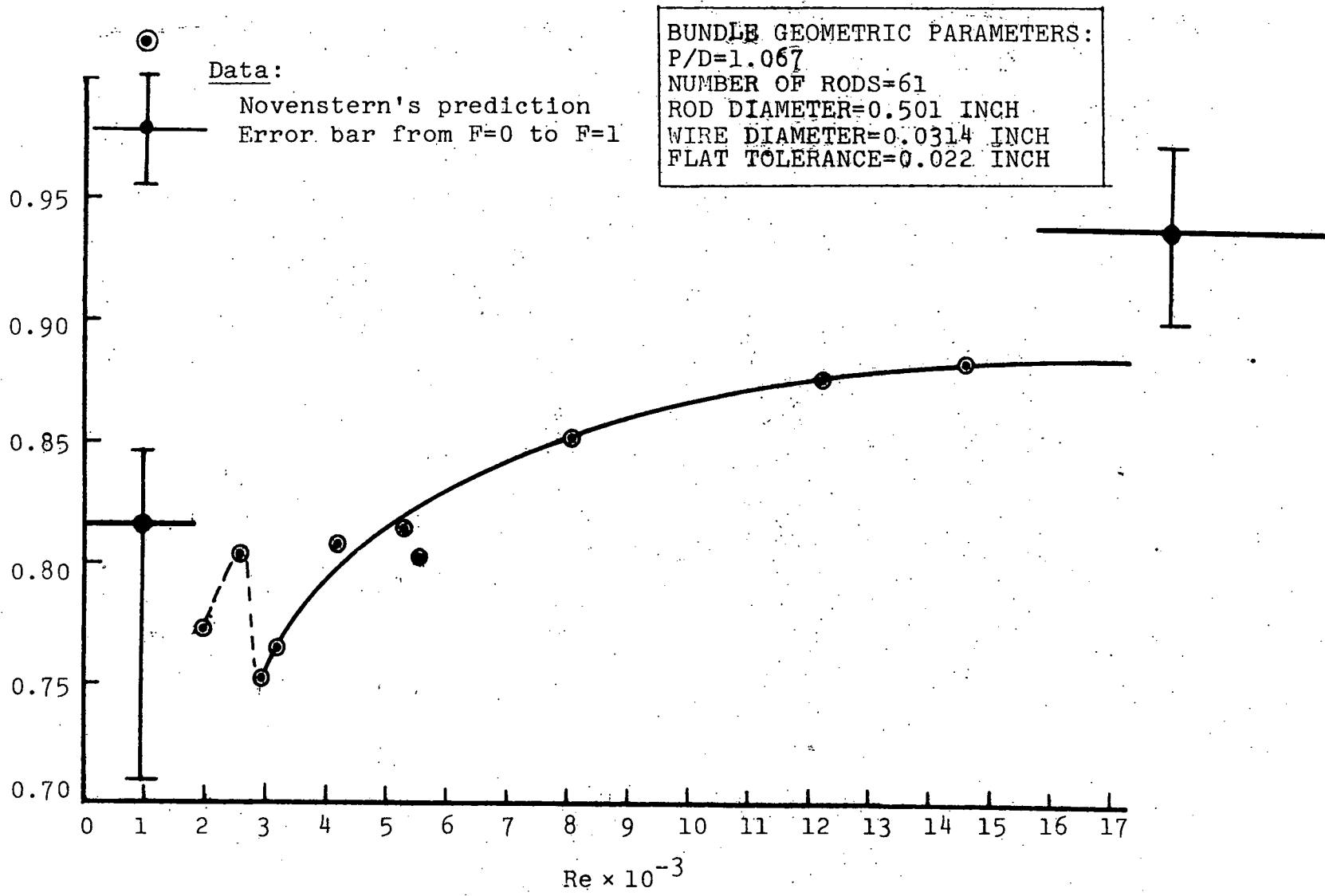
Figure 1   Interior Channel Flow Split versus Reynold Number
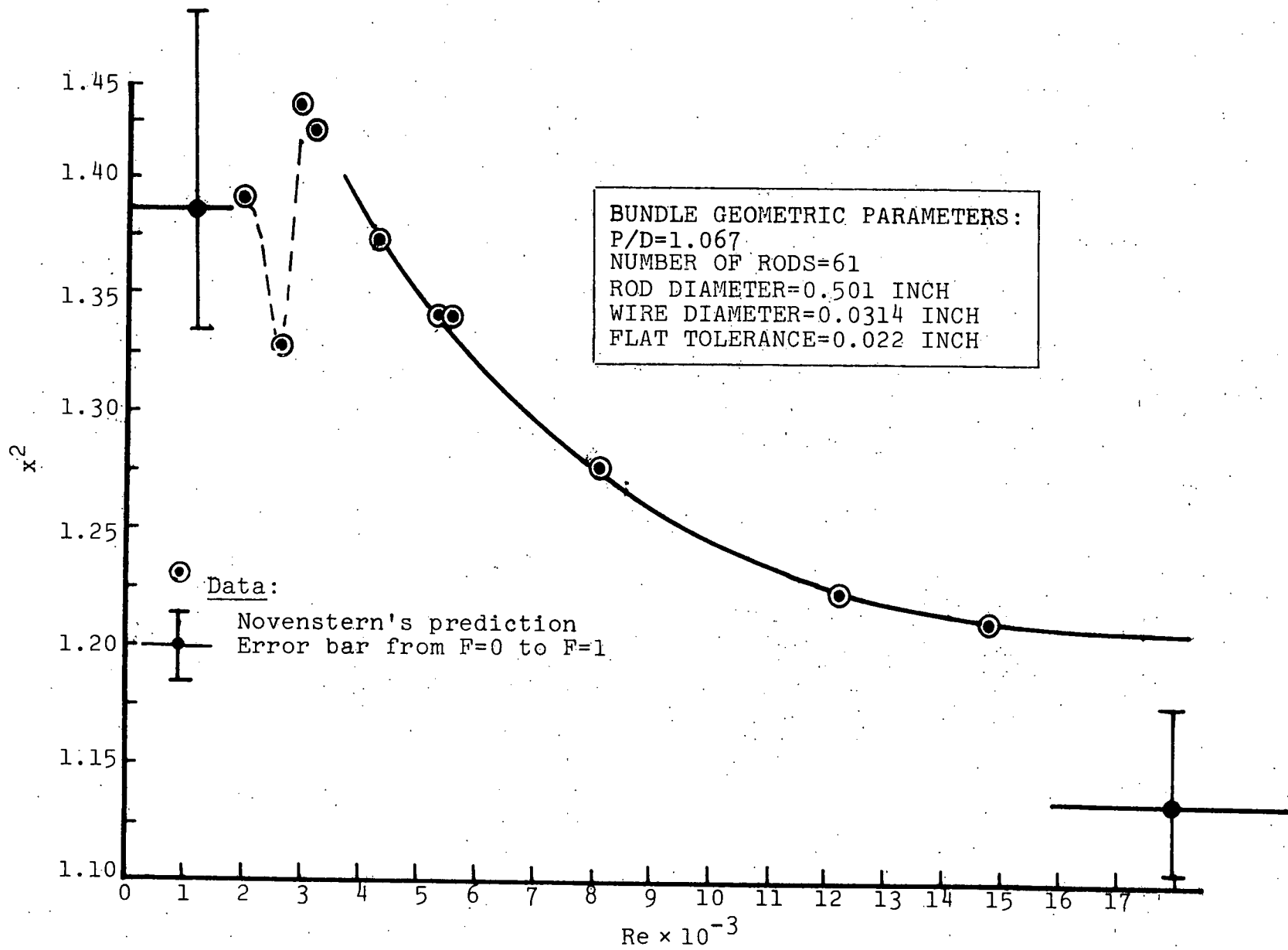
Figure 2   Edge Channel Flow Split versus Reynolds Number

50
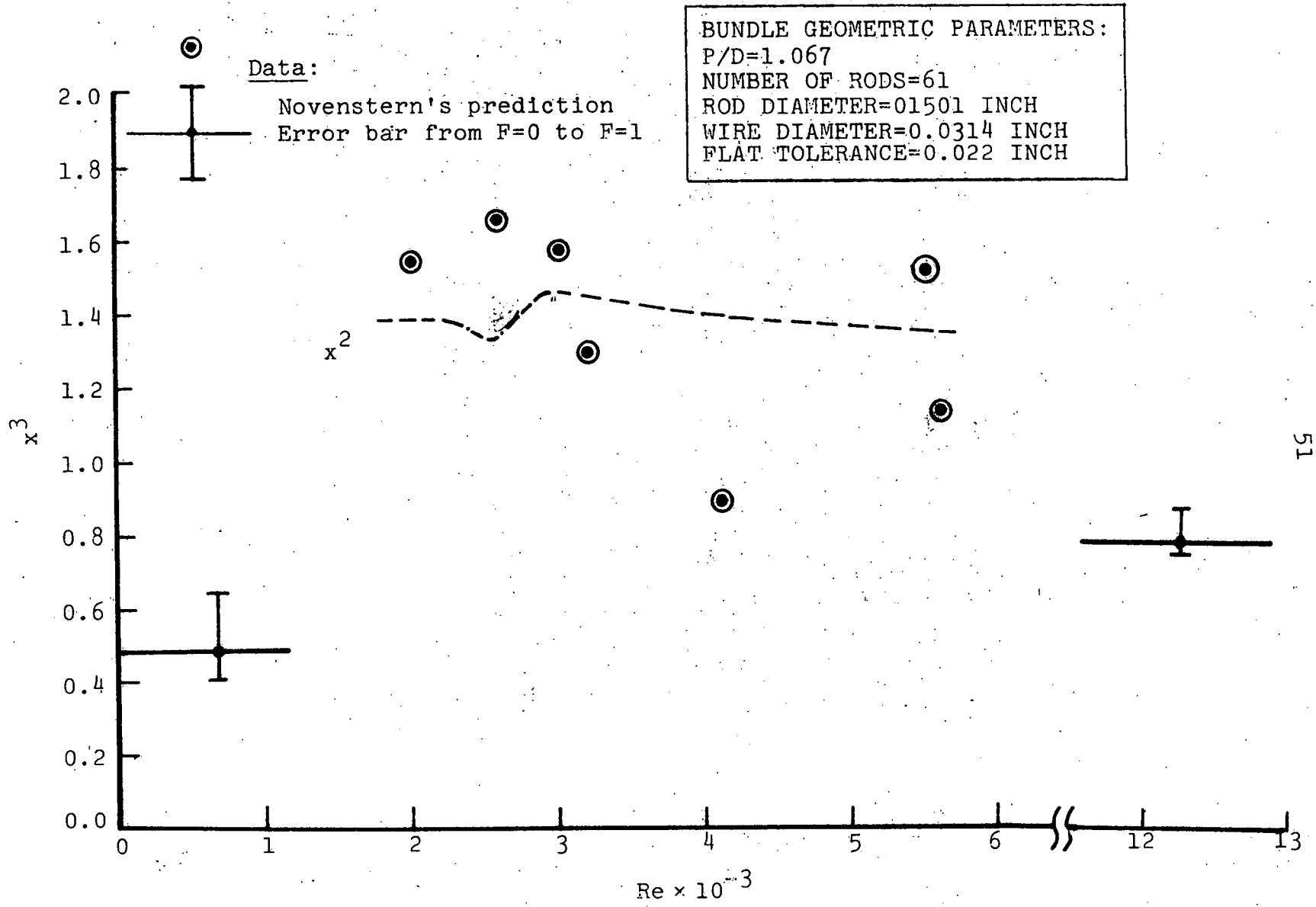
Figure 3 Corner Channel Flow Split versus Reynolds Number
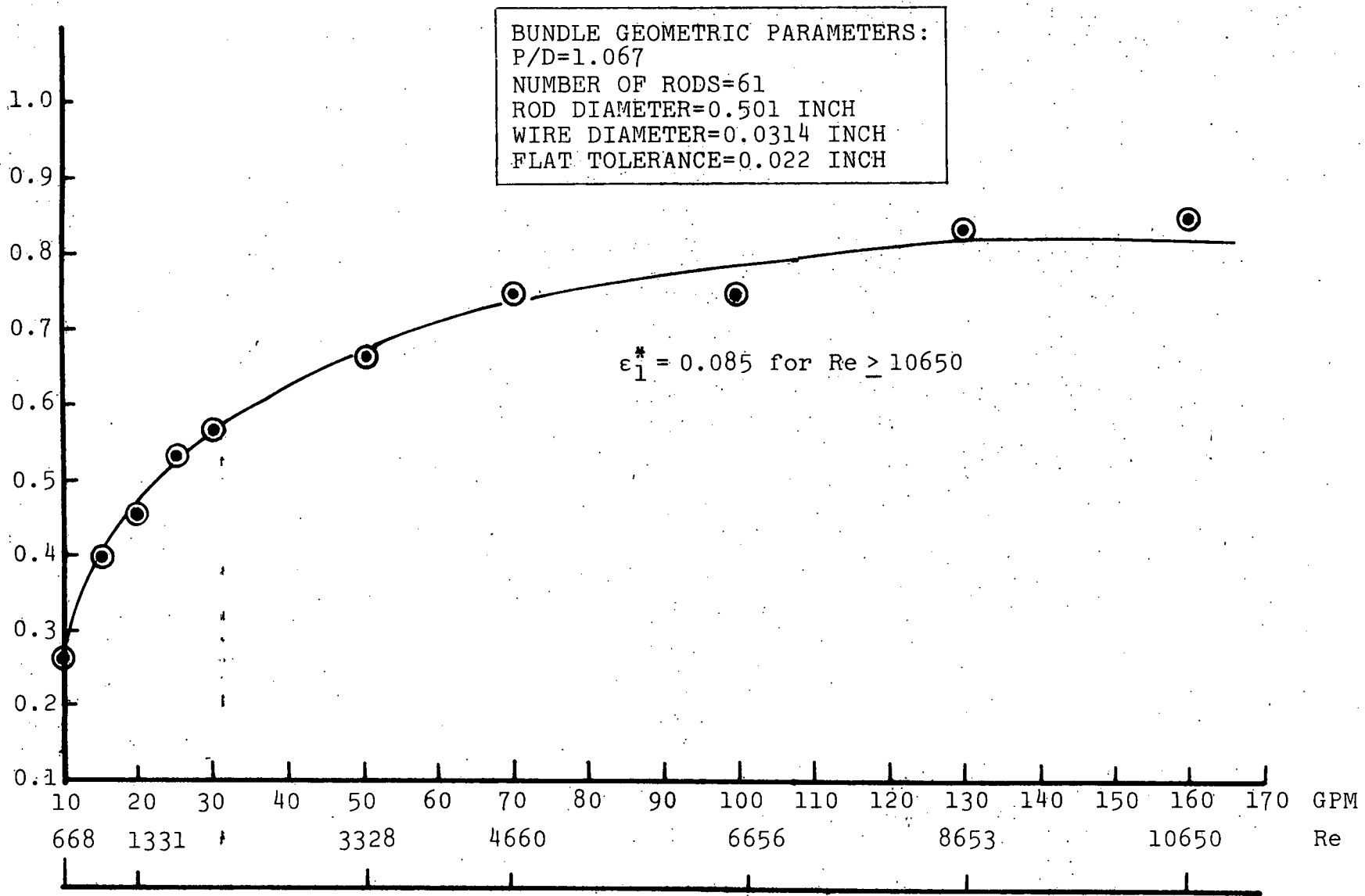
Figure 4 ϵ_1^* versus Reynolds Number

Plot axis labels and content:

Y-axis: 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1

X-axis (GPM): 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170   GPM

Re: 668, 1331, 3328, 4660, 6656, 8653, 10650   Re

BUNDLE GEOMETRIC PARAMETERS:
P/D=1.067
NUMBER OF RODS=61
ROD DIAMETER=0.501 INCH
WIRE DIAMETER=0.0314 INCH
FLAT TOLERANCE=0.022 INCH

$\epsilon_1^* = 0.085$ for Re ≥ 10650
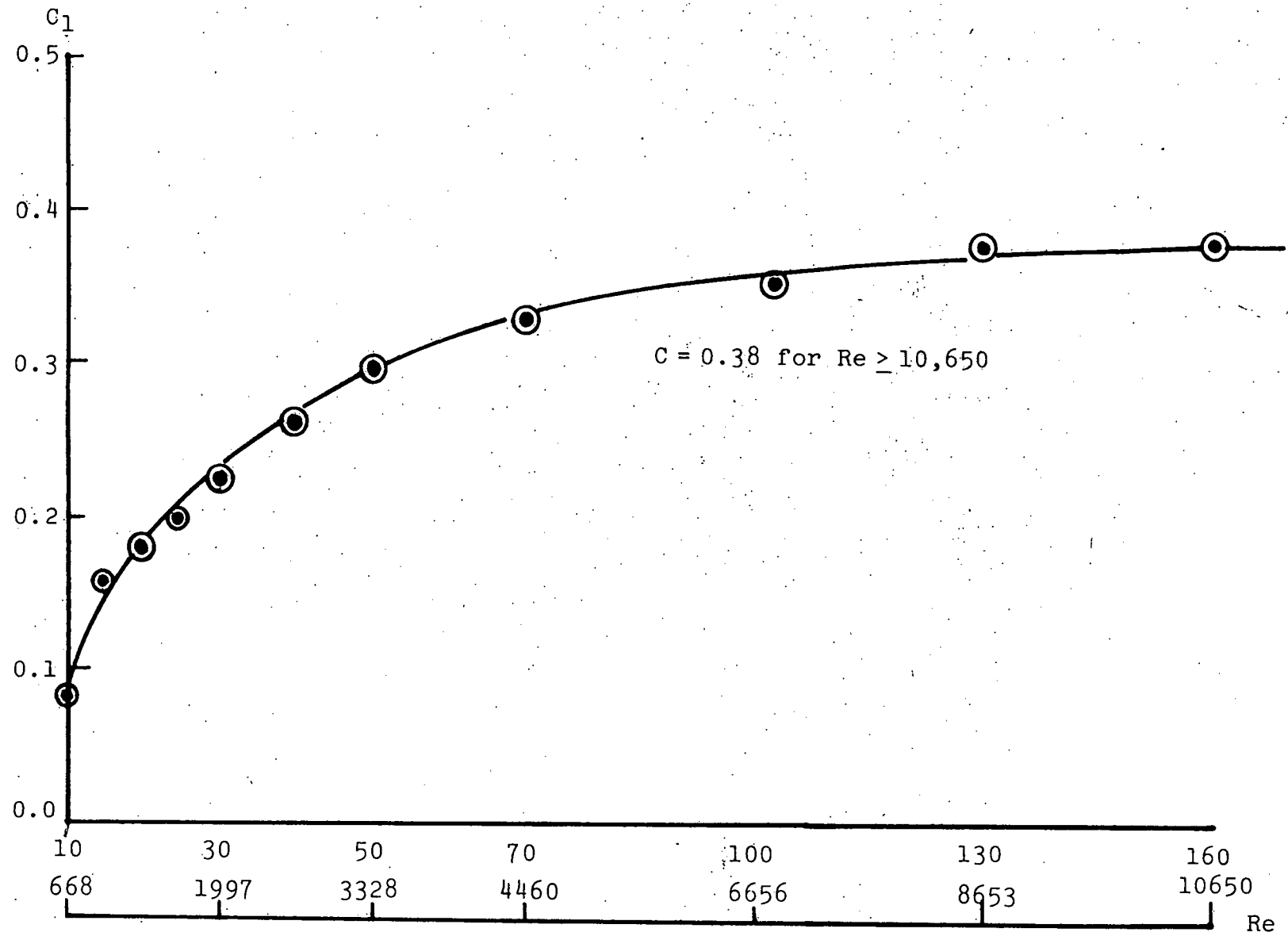
Figure 5 C_1 versus Re for Blanket Assembly

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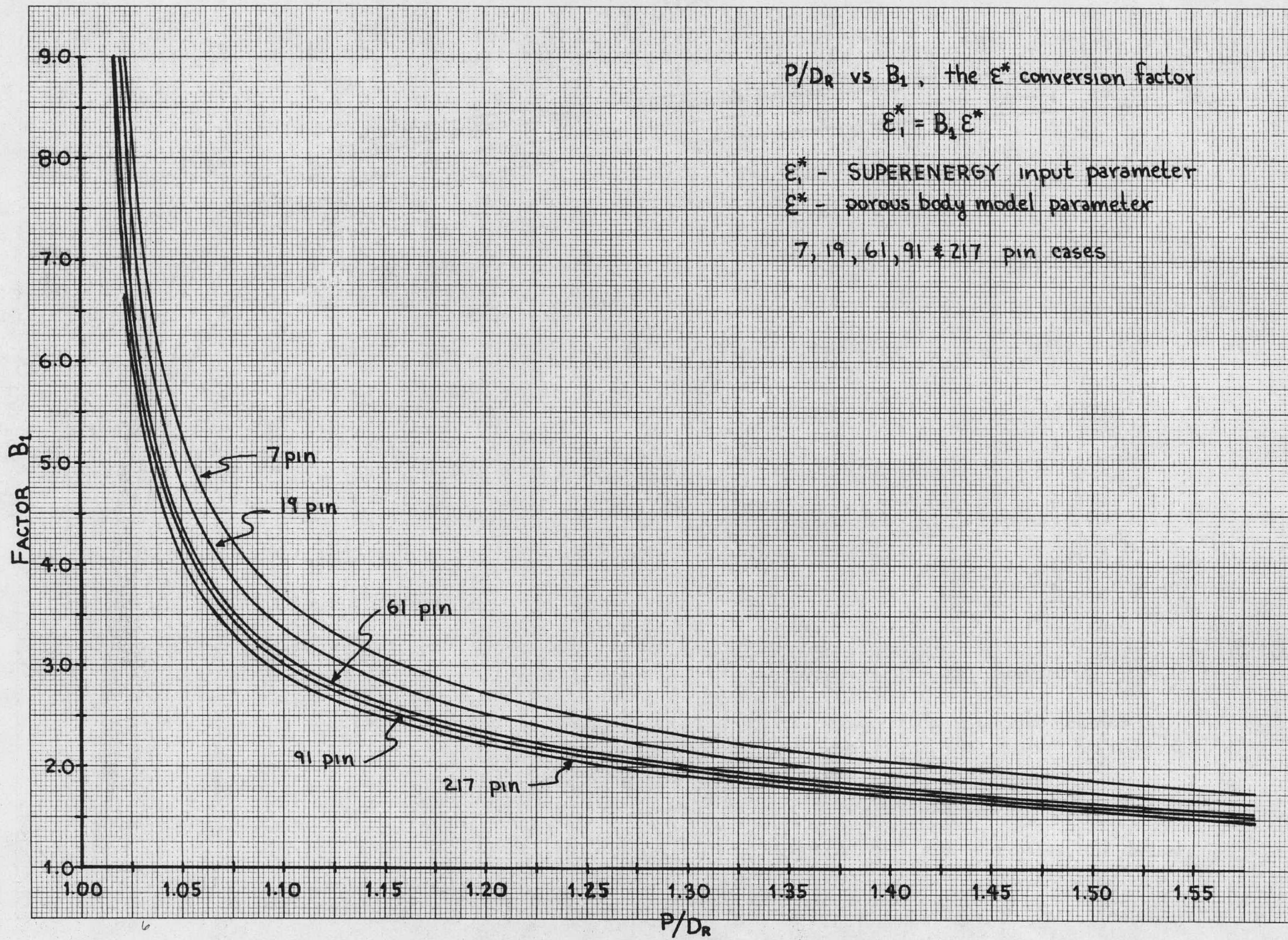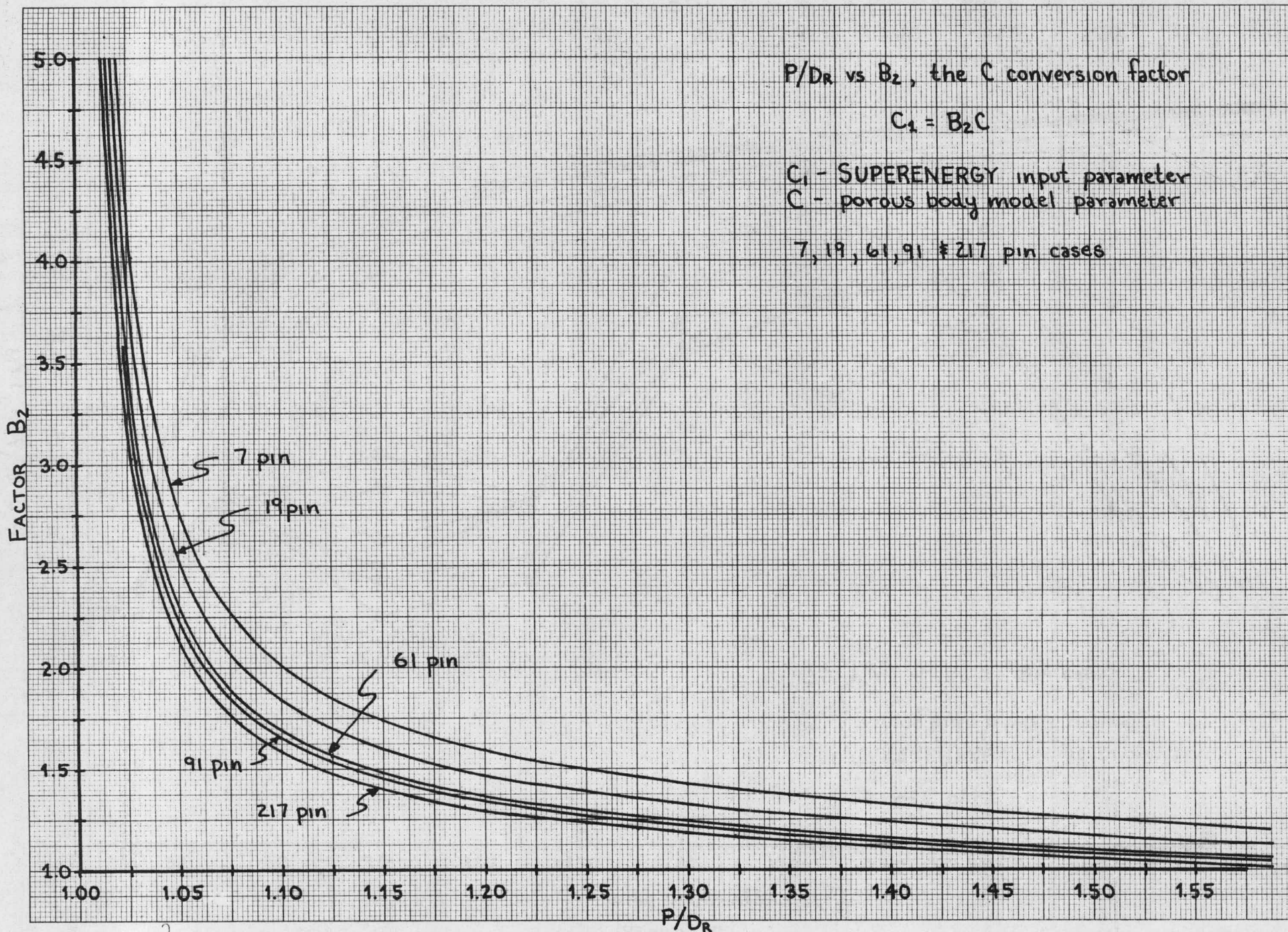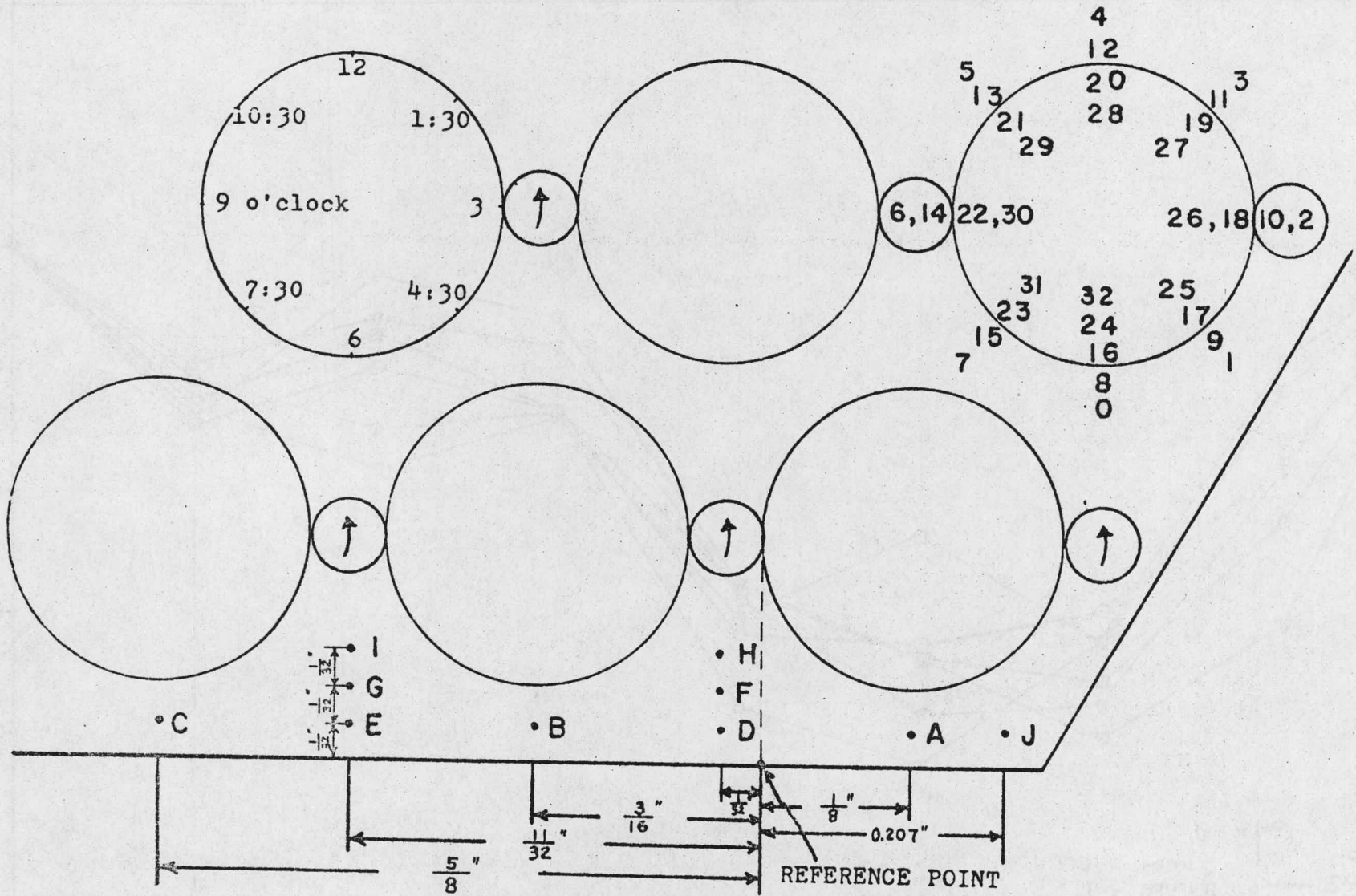
Figure 6

Figure 7

FIGURE 4.2   LOCATIONS OF MEASURING POINTS

Figure 8

Figure 9
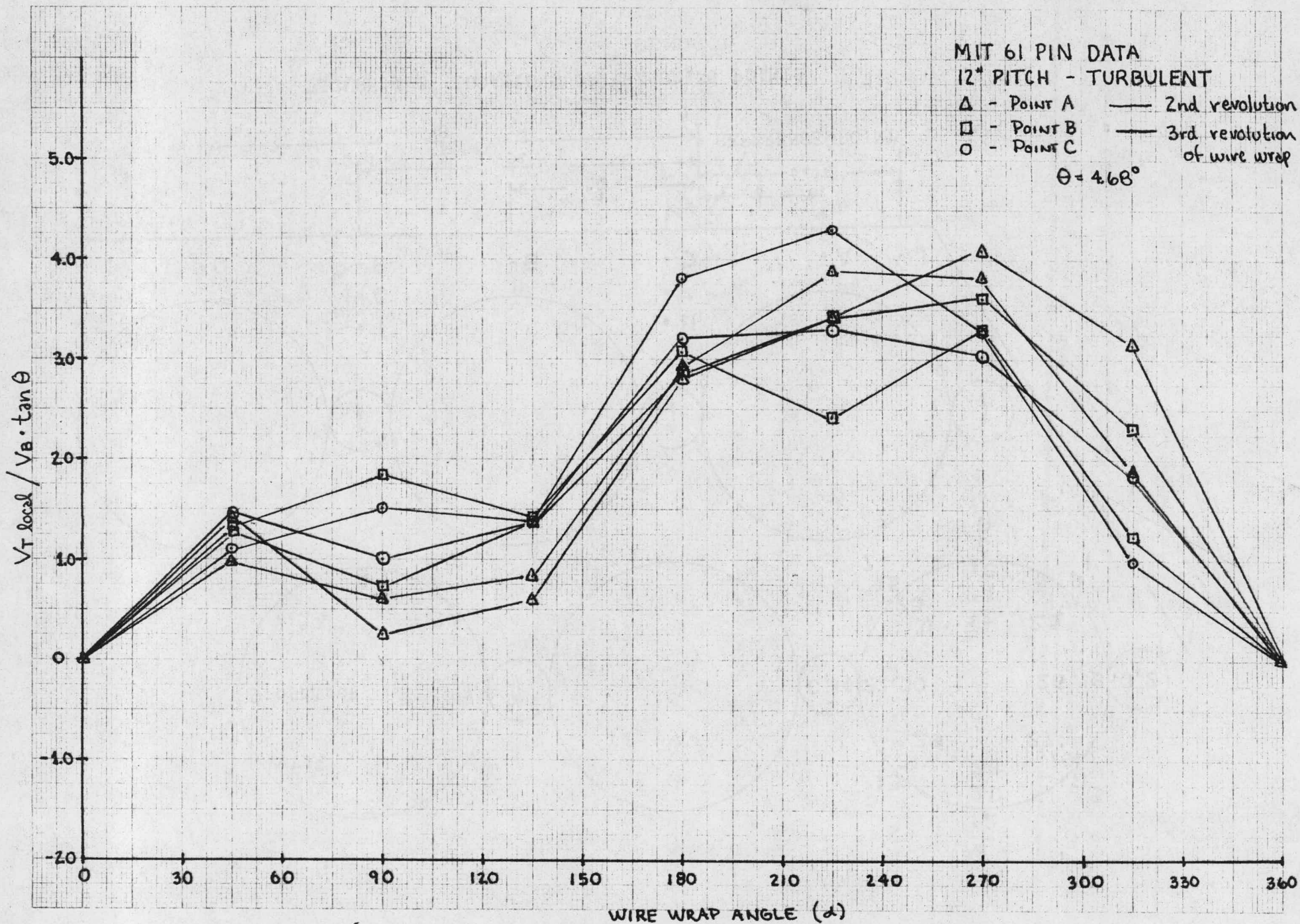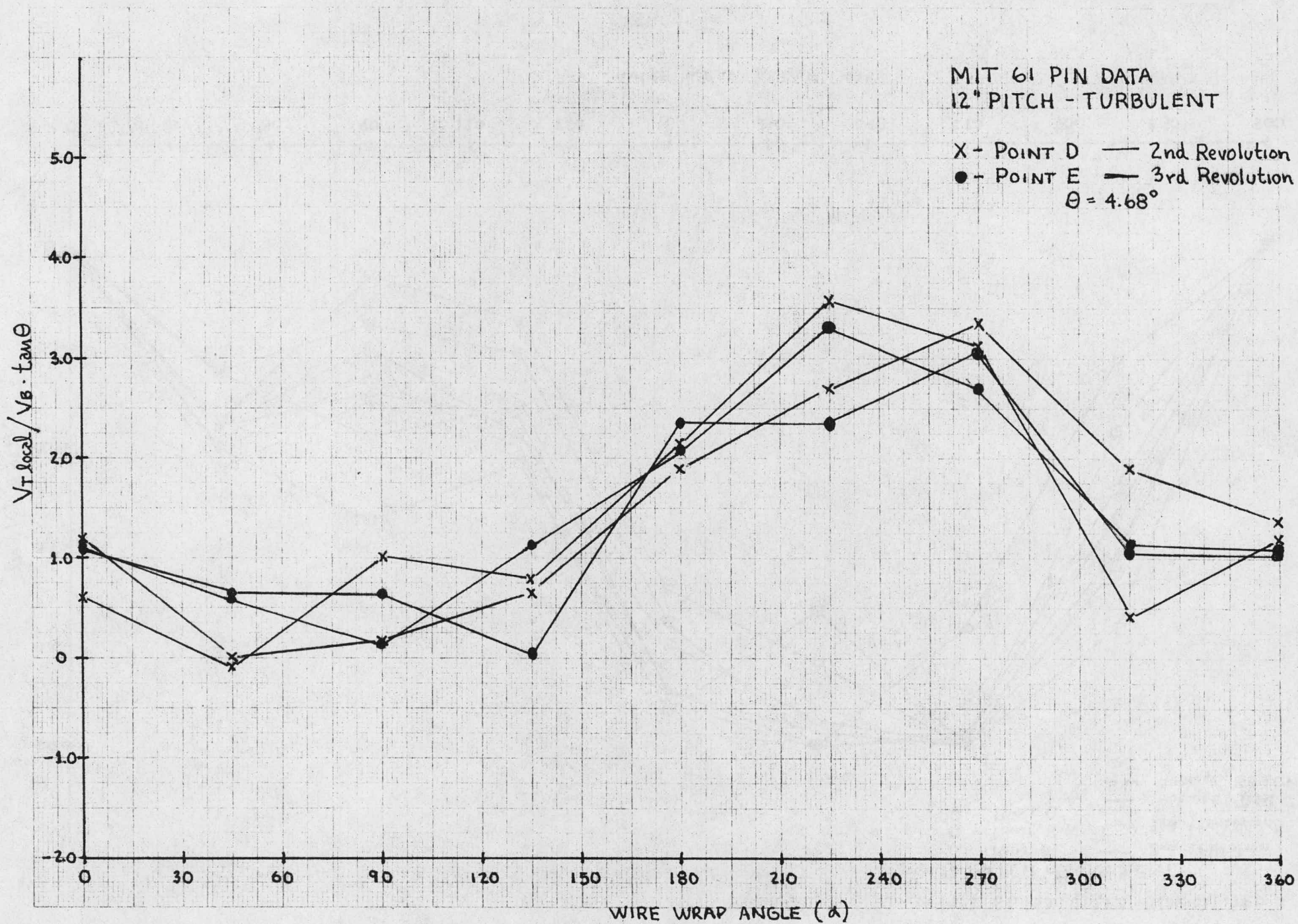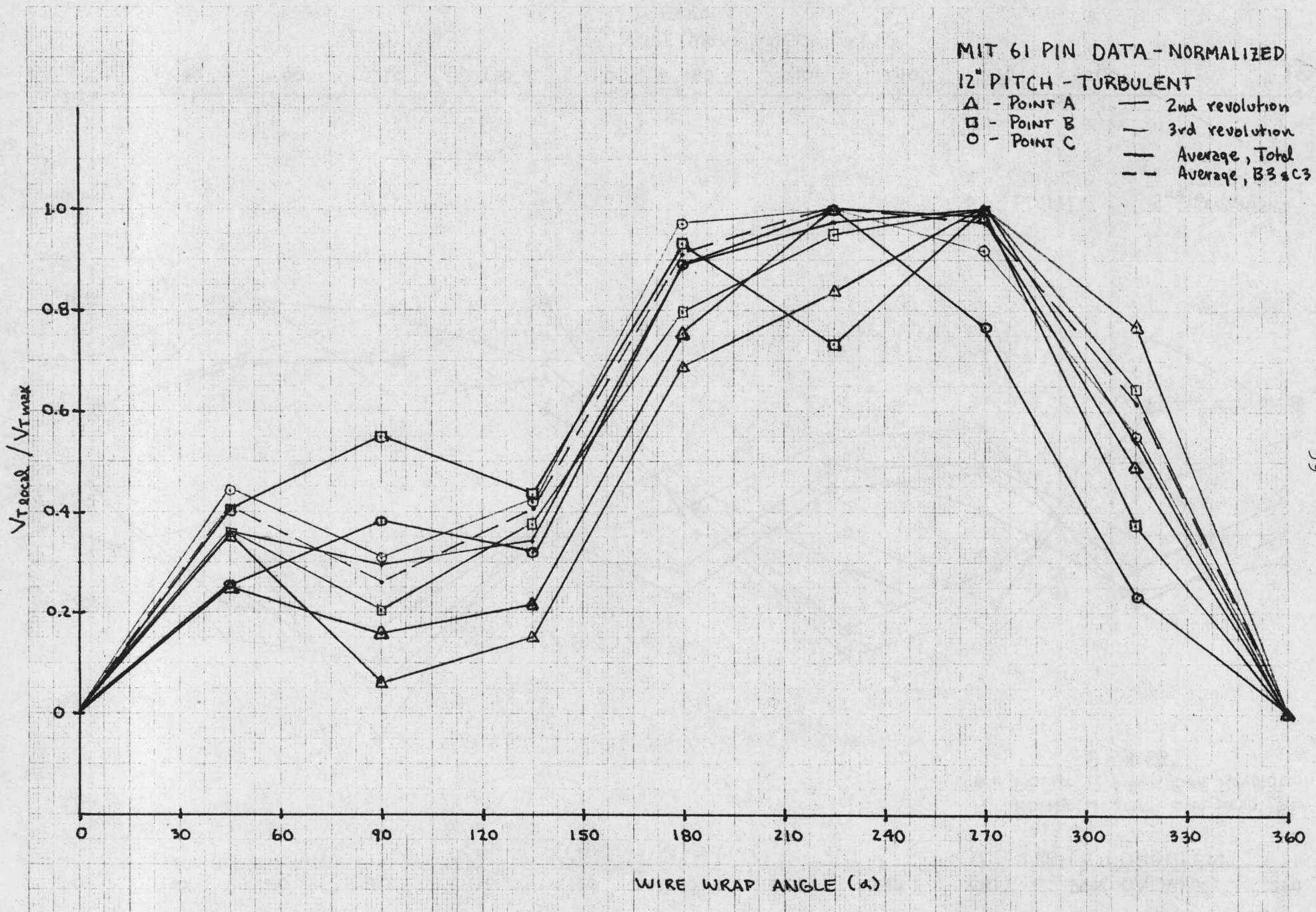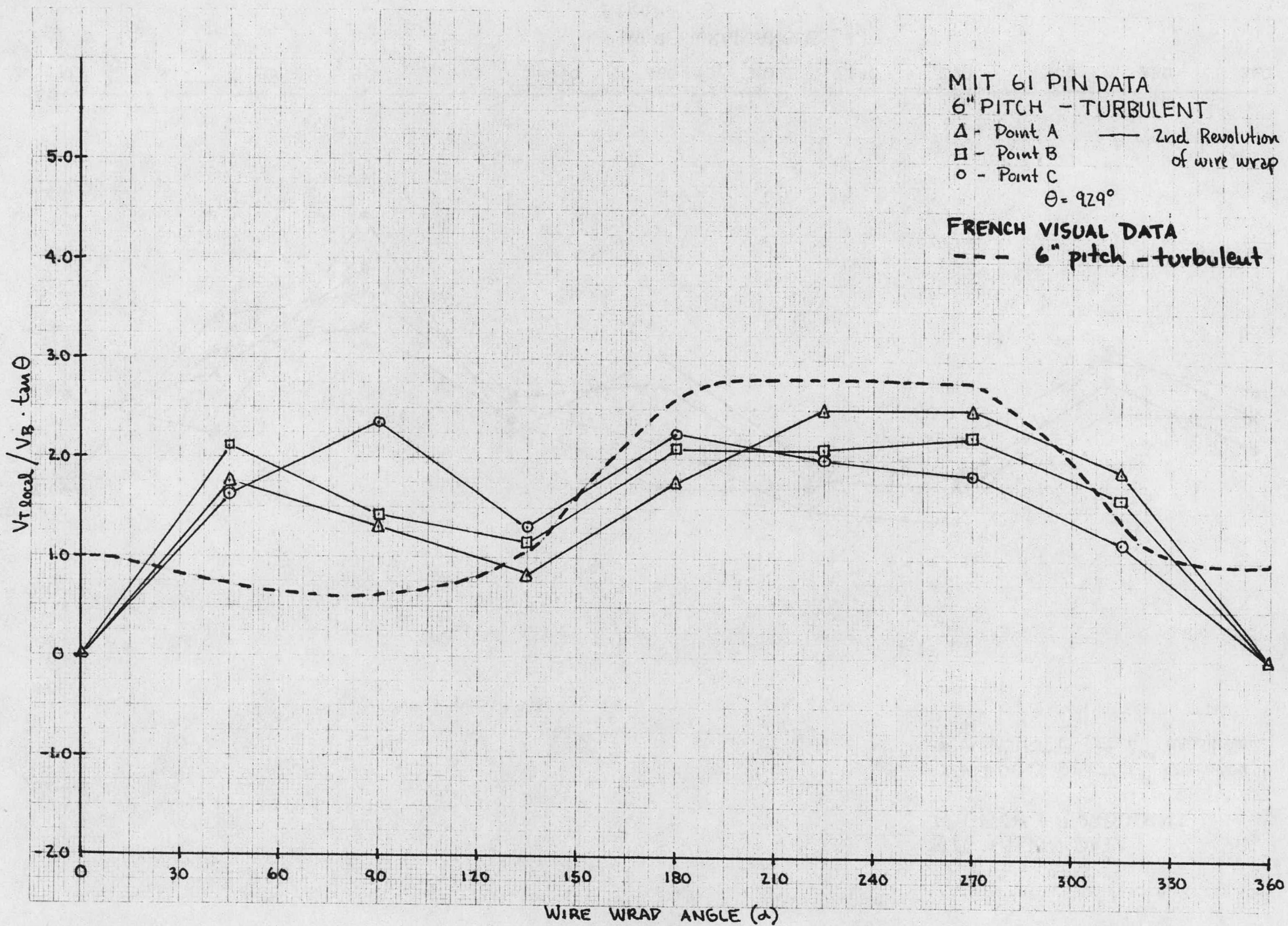
Figure 10

Fig 10

Figure 11

MIT 61 PIN DATA
6" PITCH - TURBULENT
Δ - Point A   —— 2nd Revolution
□ - Point B   of wire wrap
o - Point C
              θ = 929°
FRENCH VISUAL DATA
- - - 6" pitch - turbulent

WIRE WRAD ANGLE (α)
Figure 12

Fig 12

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MIT 61 PIN DATA
6" PITCH – TURBULENT

X – POINT D — 1st Revolution
● – POINT E — 2nd Revolution

Figure 13

MIT 61 PIN DATA - NORMALIZED
6" PITCH - TURBULENT
Δ - Point A   — 2nd revolution
□ - Point B   — AVERAGE, total
O - Point C

FRENCH VISUAL DATA
- - - 6" pitch - turbulent

62
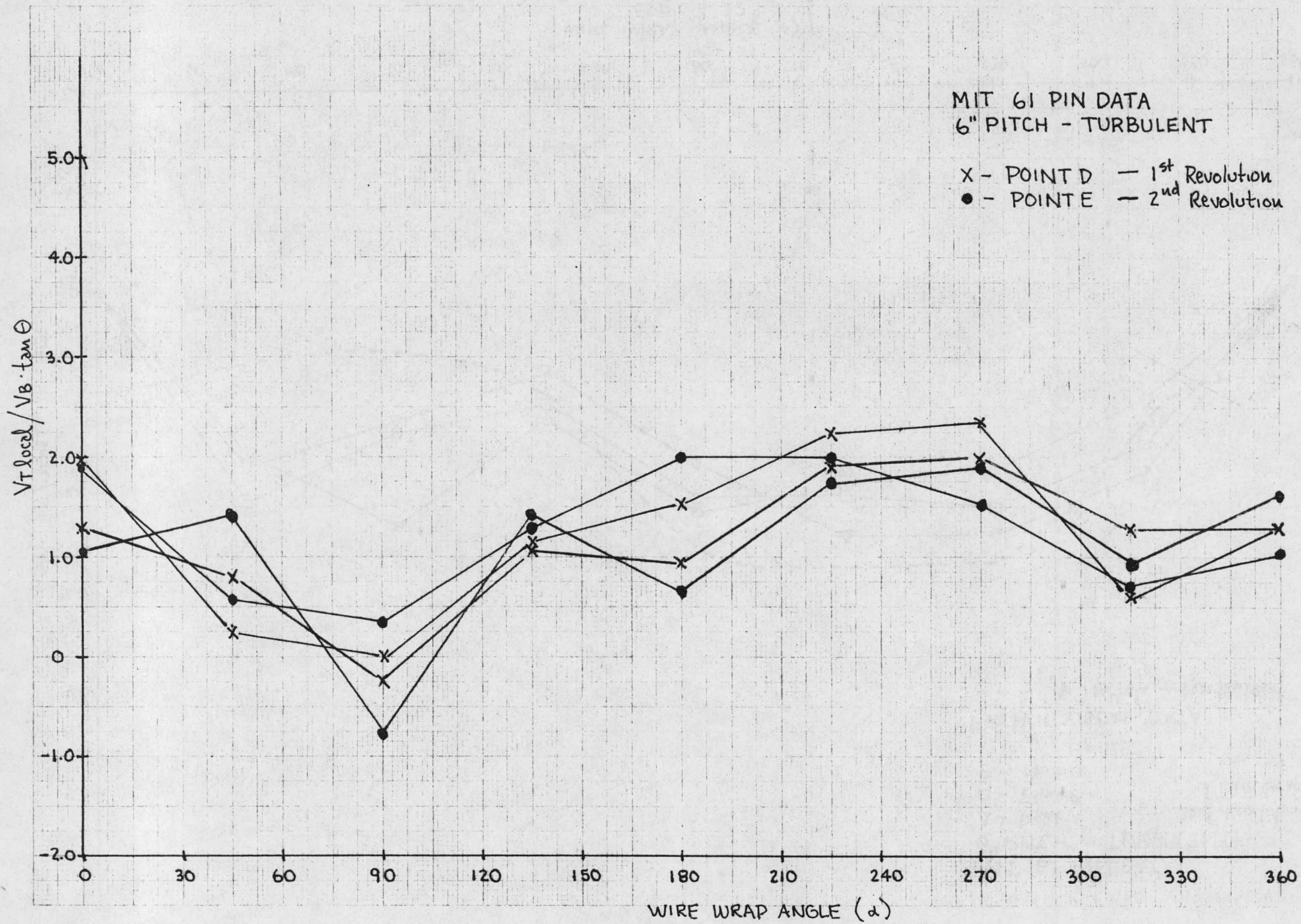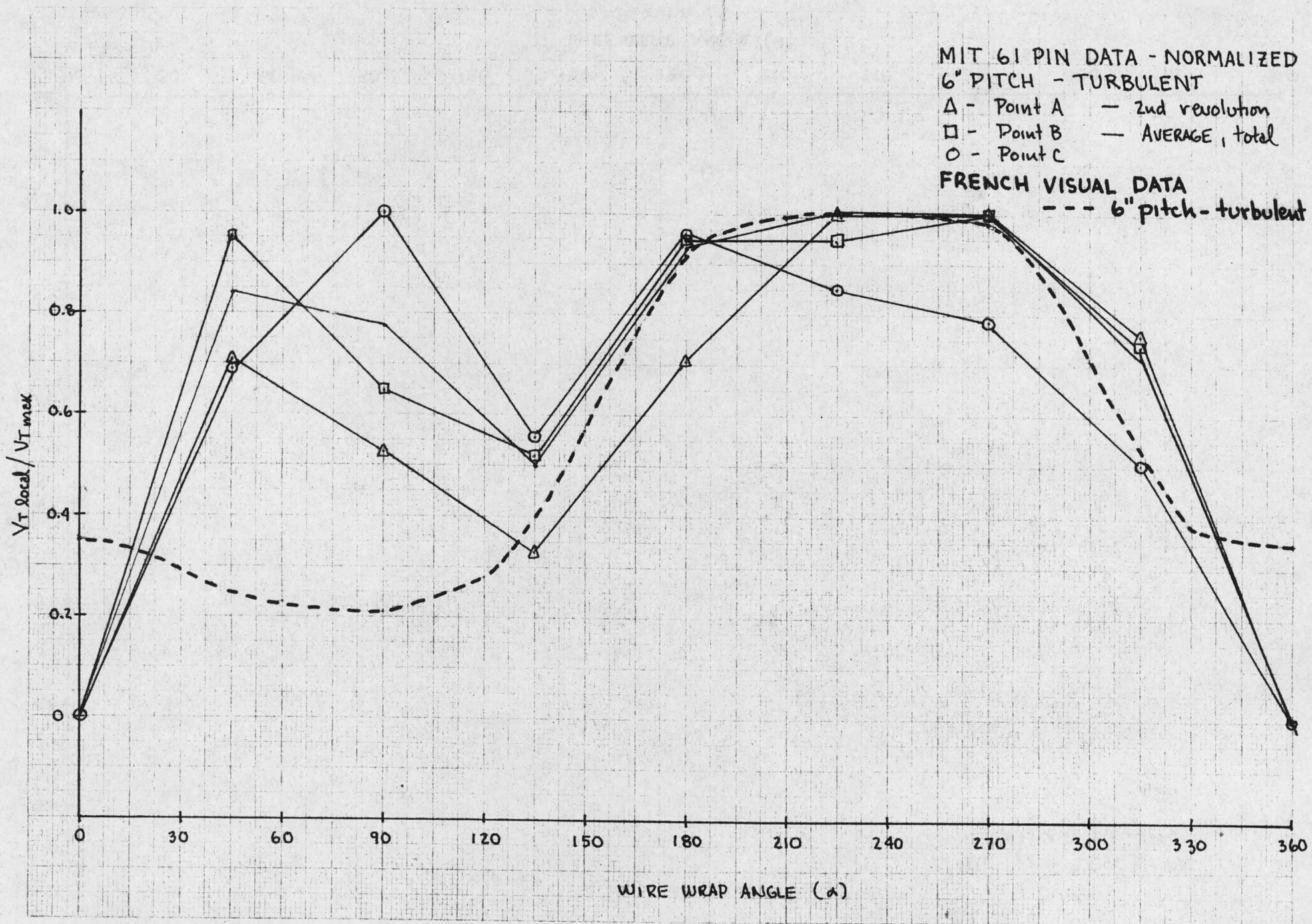
Figure 14

Fig 14.
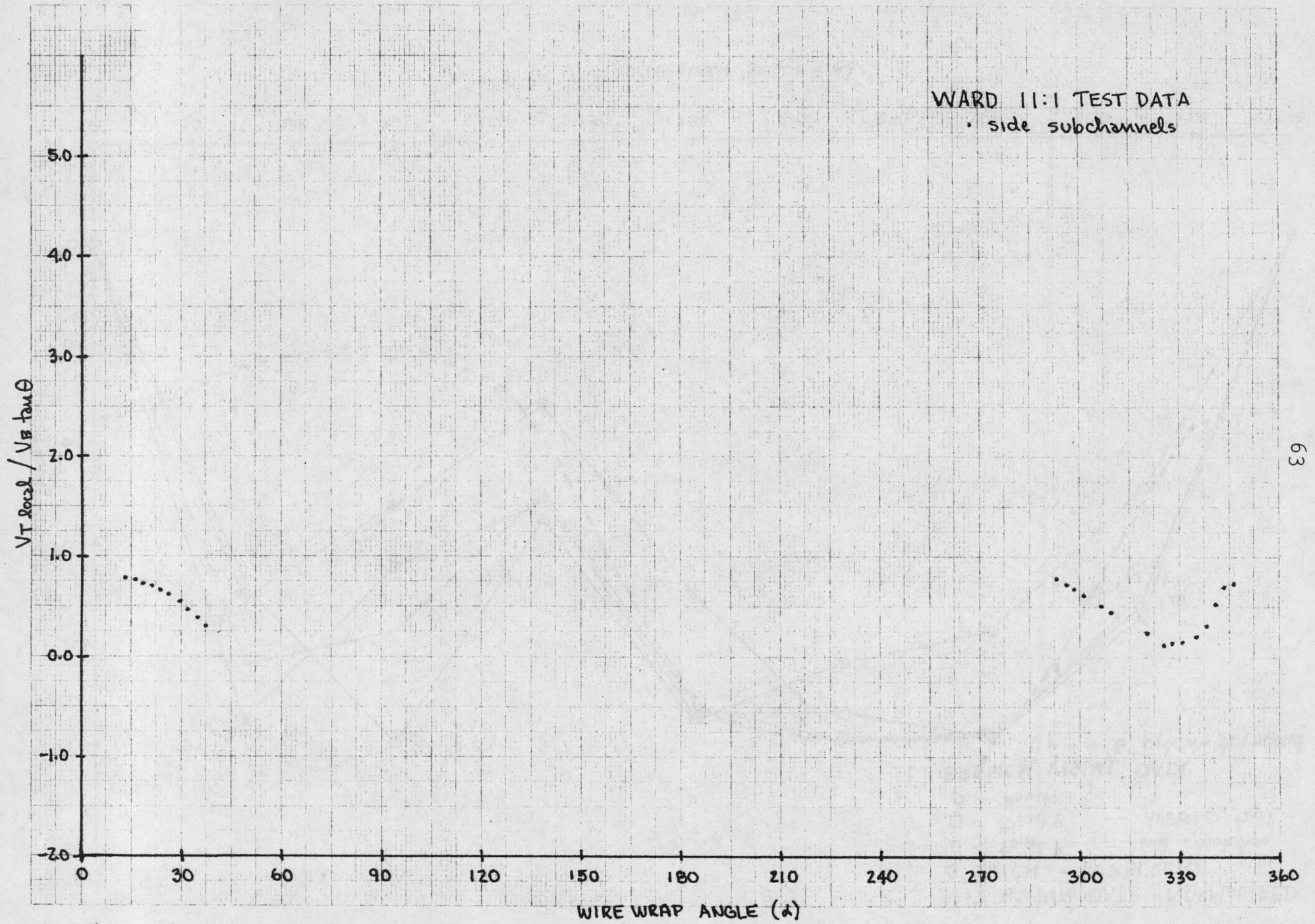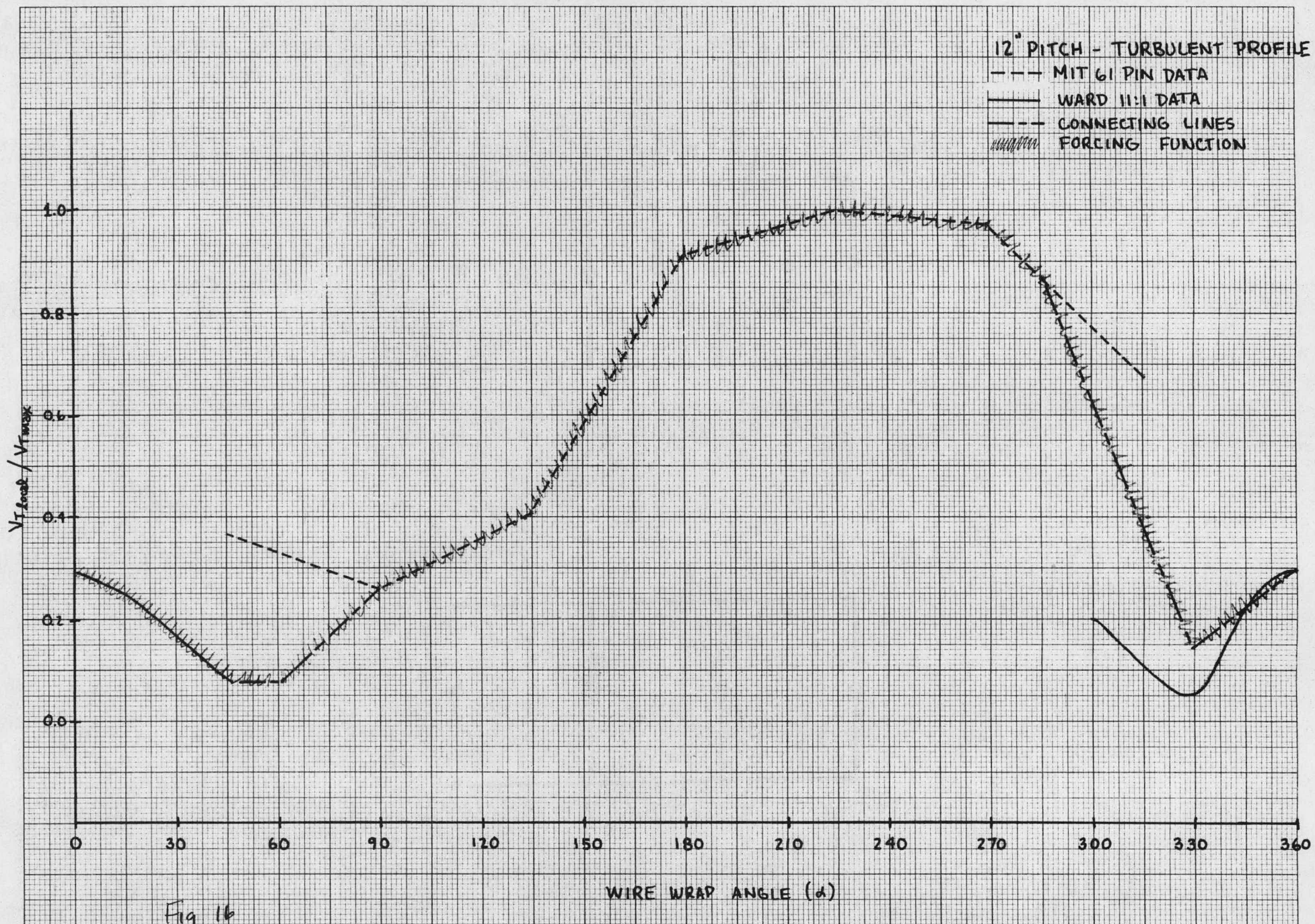
Figure 15
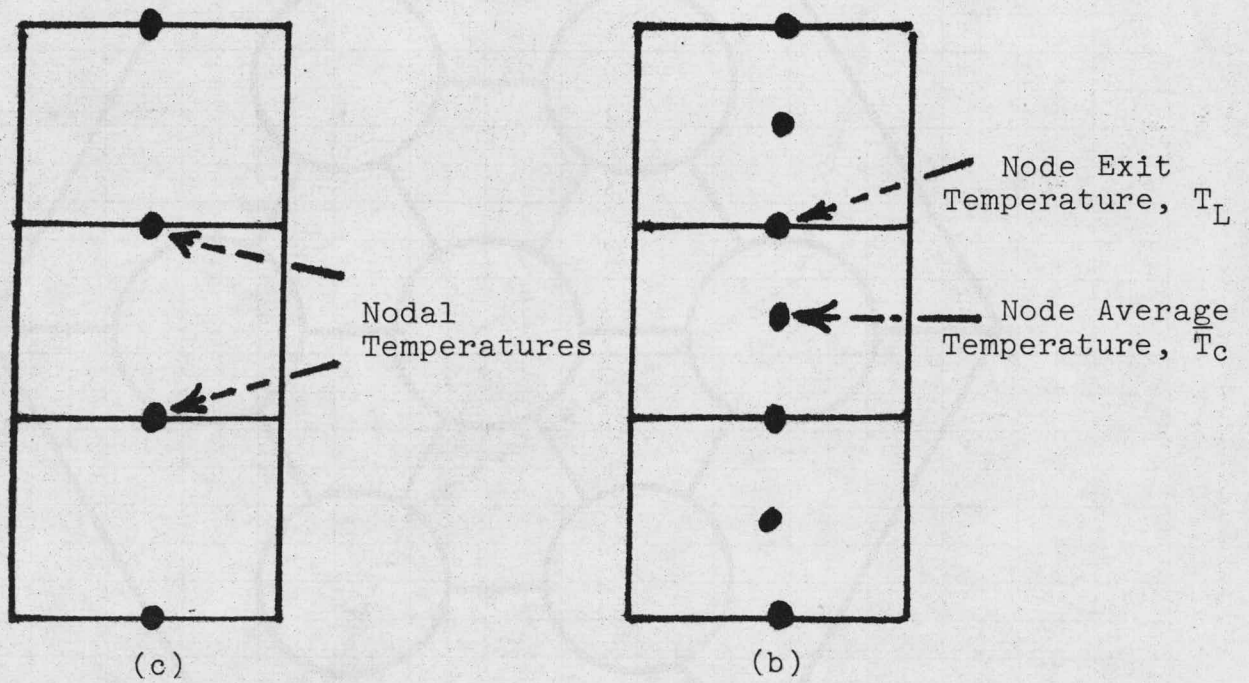
Figure 16

Figure 17

Nodal Temperatures Defined at End of Node. Entire node assumed at 1 temperature

Dual Temperature per Node Model: Node average and node exit temperatures.

Figure 18

T(t)   (a) T(t+Δt)
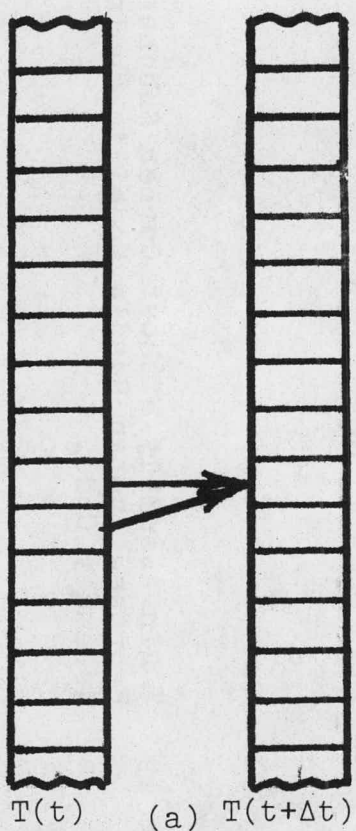
Dual Main Arrays Used
for Storage of Coolant
Temperatures
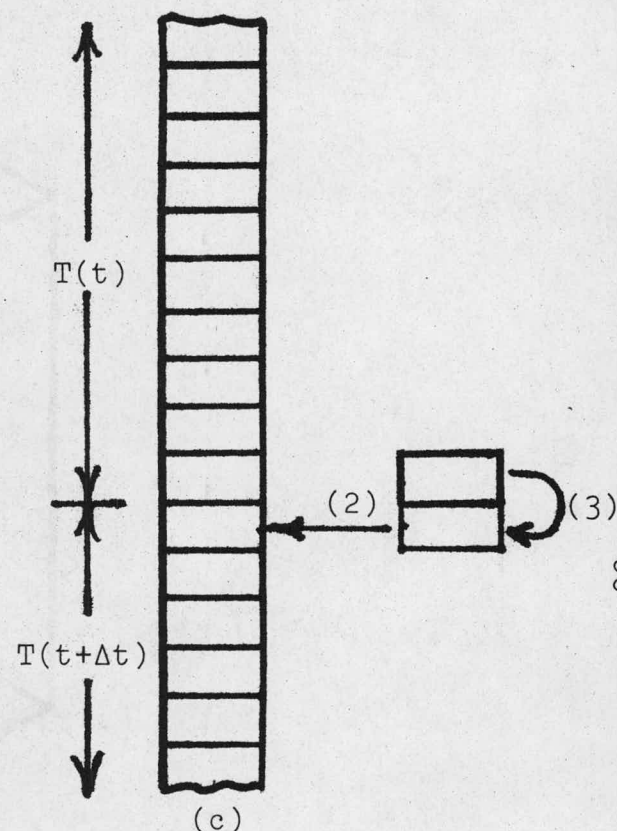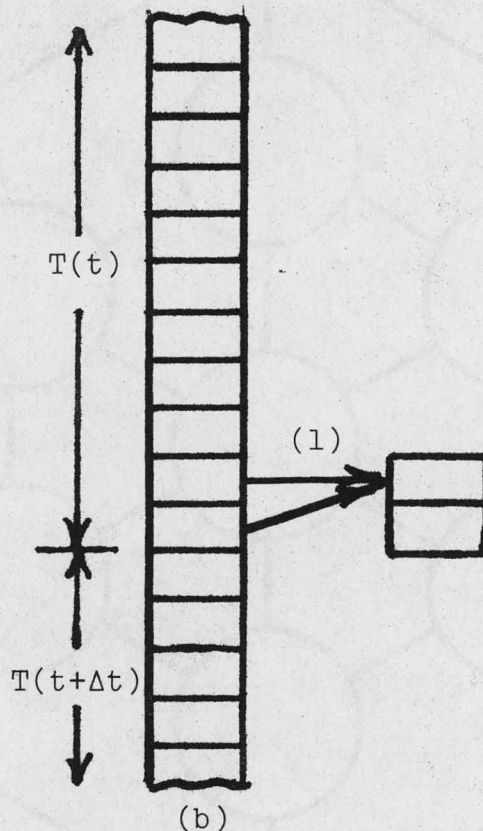
Dynamic Replacement of Storage into (Single) Main Array.
Step (1), in (b) shows temporary storage of T(t+Δt) at
the current axial step. At completion of calculation
of (T(t+ t) for entire axial step, T(t+ t) at previous
axial step is replaced into main array, (2). Step (3)
indicates "pushing down" of T(t+ t) at current axial
step in temporary storage. Calculation of T(t+ t) at
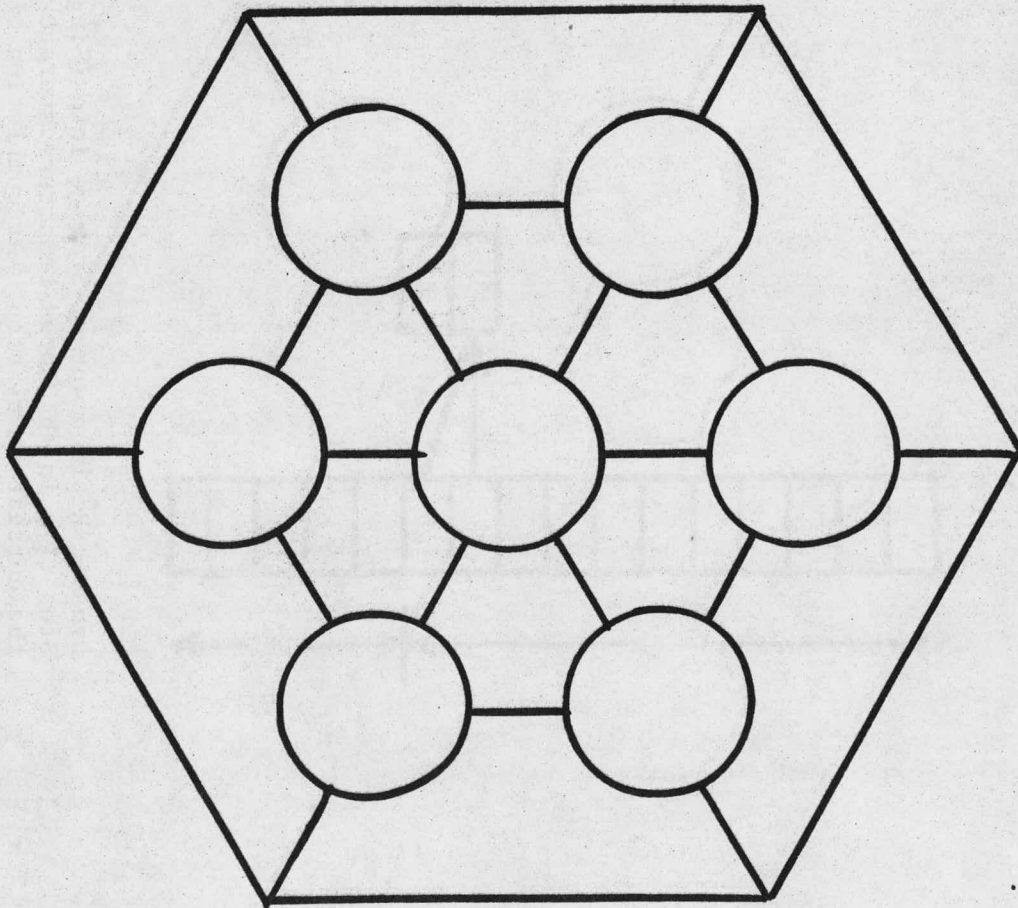next axial step now continues as procedure marches up
bundle.

Fig. 19   7 Pin Assembly Without Corner Subchannels -
Proposed Lumped Bundle Model in an Inter-
assembly Code.

TASK II: SUBCHANNEL GEOMETRY (BARE RODS)

TASK IIA: Assessment of Available Data

Nothing to report this quarter.

TASK IIB: Experimental Subchannel Water Mixing Investigation
(Joseph M. Kelly)

Bulk mixing coefficients were measured for single plane water flows in a simulated rod bundle with a pitch to diameter ratio of 1.10. A tracer technique employing Rhodamine B as the tracer and measuring fluorescence was used. Isokinetic sampling was achieved by using a pressure balance method. The results were corrected for both entrance effects and diversion cross-flows.

Our results showed a change in Reynolds number behavior as the laminar sublayer began to "choke" the turbulent mixing. This, and a review of other mixing experiments, suggested that secondary flows do not compensate for laminarization and that turbulent mixing decreases as the pitch to diameter ratio decreases for values of P/D less than 1.05 in a manner similar to that predicted by Ramm et al.

Concentration profiles were measured through the clearance gap and the values of the gradient were used to calculate the gap averaged circumferential eddy diffusivity for mass.

TASK IIC: Experimental Subchannel Local Parameter
Measurements (Wall Shear Stress Measurement
and Laser Doppler Anemometry)

TASK IIC.1:   Wall Shear Stress Measurments (Mohammed Fakory)

As it was shown in the last progress report, use of a Preston tube was selected to measure wall shear stress. In order to measure the total pressure point by point in the peripheral direction around the rod, some changes were necessary to the selected test section. These changes are done and the test section is ready to be set up. After considering several viewpoints it was decided to use air flow in the experiments. A brief summary of these considerations follows:

Water Flow Advantages

1) One of the advantages of water flow is that we have larger Preston tube differential pressure (total local pressure - static pressure) than in the case of

air flow. The ratio of the differential pressure for the same Re number is of order

$$\frac{\Delta P_{air}}{\Delta P_{H_2O}} \sim \left[\frac{\mu_{air}}{\mu_{H_2O}}\right]^2 \left[\frac{\rho_{H_2O}}{\rho_{air}}\right] = \left[\frac{3.827 \quad 10^{-7}}{1.925 \quad 10^{-5}}\right] \left[\frac{1.935}{0.00548}\right] = 0.14$$

A typical ratio using the Patel correlation is

$$\frac{P_{air}}{P_{H_2O}} = 0.1405$$

2) In the case of water flow, the laminar sublayer is thicker so it is possible to use larger diameter probe respect to air flow.

Air Flow Advantages

1) The dissolved air in the water causes creation of air bubbles at the entrance of a small diameter probe and thus causing a false differential pressure.

2) Air flow requires a simpler transducer than water flow, there is no zeroing problem, and no zero set point shift.

3) The following error estimation shows that although as above $\Delta P_{air} < \Delta P_{H_2O}$, the measured wall shear stress error in the case of air flow is less or at most is the same as in the case of water flow.

The preferred Patel correlation for finding the wall shear stress is

$$y^* = 0.8287 - 0.1381 \, \eta^* + 0.143 \, \eta^{*2} - 0.006 \, \eta^{*3}$$

where

$$\eta^* = \log_{10}(\frac{\Delta P d^2}{4\rho\nu^2}) \quad , \quad y^* = \log_{10}(\frac{\tau_w d^2}{4\rho\nu^2})$$

This formula gives τ_w within $\pm$ 1.5%. Considering both instrument error and correlation error we have in the case of water flow

$$\frac{\Delta \tau_w}{\tau_w} = 0.432\% + 1.5\% = 1.932\% \quad (Re = 25000)$$

And in the case of air flow:

$$\frac{\Delta\tau_W}{\tau_W} = 0.32\% + 1.5\% = 1.89\% \qquad \text{(Re = 25000)}$$

Therefore probes, accessory devices, and instruments were designed and calculated on the basis of air flow. Hoepfully gathering of experimental data will be initiated during January.

TASK IIC.2 Laser Doppler Anemometry Measurements (John Bartzis)

No measurements were obtained this quarter since the LDA system was being used for Task III.

TASK IID: Analytical Model Development - Subchannels (John Bartzis)

1.1 The Differential Equations

1.1.1 The Equations

Since we are going to use a 2-equation turbulent model, the equations of continuity, momenta, turbulent kinetic energy, energy diysipation are to be solved. The modeling of the above equations is discussed in Ref. 1.

The momentum and transport equations can be put in the general form

$$|{<}\text{---Convection---}*\text{---Diffusion---}\to\text{---Source>}|$$

$$\nabla \phi \underline{V} \quad + \quad \nabla \underline{T}_\phi \quad = \nabla S_\phi + S_\phi^* \qquad (1\text{-}1)$$

$$\underline{T}_\phi = \left(T_{\phi r}, T_{\phi\theta} \right)$$

$$\underline{S}_\phi = \left(S_{\phi r}, S_{\phi\theta} \right)$$

The separation of diffusion term into a T_ϕ term and an S_ϕ term which is treated as a source term has been on the ground that T_ϕ includes only the terms that depend explicitly on ϕ.

1.1.2 The Solution Domain

The equations are to be solved within the characteristic triangle of the infinite rod bundle as shown in Fig. 1.

The secondary flow quantities (i.e., v_r, v_θ, p/p) are going to be solved in the whole flow area of the characteristic

triangle whereas the v_z momentum equation and the k, ε transport equations are going to be solved only in the outer region of the flow (i.e., $y_B^+ \geq 25\text{-}30$). As pointed out in Ref. 1 (p. 56) the exclusion of the wall region is made due to the following reasons:

a) The sharp gradients of the hydrodynamic quantities within the wall region need a relatively high number of points to describe the hydrodynamic field sufficiently well.

b) The strongly anisotropic turbulent kinetic equation makes a turbulent model based on the Kolmogorov-Prandtl hypothesis questionable. However, on the other hand, the prescription of the quantities themselves within the wall region is relatively not difficult since the main parameters of dependence are the wall shear stress, the distance from the wall and the fluid viscosity.

The approach of excluding the all region from the domain of solution has been used by several authors (see e.g., [2], [3]) and in the efforts preceding this work at MIT [4].

1.1.2.1 Wall Layer Thickness Prescription

The wall layer thickness is defined from the relation

$$\frac{y_B \, \overline{u}^*}{\nu} \approx 30 \tag{1.2}$$

where $\overline{u}^*$ is the average friction velocity

$$\text{i.e} \quad \overline{u}^* = \left(\frac{\tau_w}{\rho} \right)^{1/2}_{avg}$$

y_B is the wall layer thickness

ν is the fluid kinematic viscosity.

In case the bulk velocity v_b is given instead of $\overline{u}^*$ we use the friction factor relation, i.e.,

$$\overline{u}^* = \sqrt{\frac{f}{2}} \; v_b \tag{1.3}$$

For the friction factor estimation we will use Ibragimov's expression [5] which has also been used in Ref. 6. This expression is as follows:

$$\frac{f}{f_o} = \left(0.58 + 0.42 \, e^{-\gamma} \right) \left[1 + 0.1 \left(\frac{2\overline{y}}{D} \right)^{4/3} \right]$$

$$\text{where } \quad \gamma = 0.21 \frac{P}{D} \left(\frac{A}{\overline{y}^2} \right)^{1/4} \left(\frac{1}{\sqrt{3}} - \frac{1}{2} \right) \frac{D}{\overline{y}}$$

$$\overline{y} = \frac{1}{A} \iint\limits_{A} y \, r \, dr \, d\theta = \frac{1}{2} \left(\frac{3}{\pi \sqrt{3}} \frac{P}{D} - 1 \right) D$$

(1.2)

A is the flow area within the characteristic triangle and

$$f_o = 0.046 \, Re^{-0.2} \qquad\qquad (1.4)$$

is the friction factor for circular pipe.

1.1.3 Boundary Conditions

The boundary conditions are given in Table Ib. The nature of the boundary conditions at the symmetry lines is profound. We will discuss below the boundary conditions at the near wall boundary (i.e., at the position y_B).

1.1.3.1 Near Wall Boundary Conditions

1.1.3.1.1 Axial Velocity

Along the y_B circumferential line we will make the widely used assumption of logarithmic profile, i.e.,

$$V_z (y_B, \theta) = \frac{u^*(\theta)}{\kappa} \ln E \frac{u^*(\theta) y_B}{\nu} \qquad (1.5)$$

where κ and E are constants
According to Ref. 4, $\kappa \approx 0.419$, $E \approx 9.8$

The velocity profile in the region $0 \le y \le y_B$ needed for calculation of the bulk velocity is taken as follows

$$\frac{V_z(Y,\theta)}{u^*(\theta)} = \frac{u^*(\theta)\,Y}{\nu} \qquad \text{for} \quad \frac{u^*(\theta)y}{\nu} \leq 5 \qquad (1.6\,r)$$

$$\frac{V_z(y,\theta)}{u^*(\theta)} = \alpha + \beta \ln \frac{u^*(\theta)y}{\nu} \qquad \text{for} \quad 5 \leq \frac{u^*(\theta)y}{\nu} \leq \frac{u^*(\theta)y_B}{\nu} \qquad (1.6b)$$

where

$$\beta = \frac{V_z(Y_B,\theta)\big/u^*(\theta) - 5}{\ln \dfrac{Y_B\, u^*(\theta)}{5\nu}} \qquad (1.6c)$$

$$\alpha = \frac{1}{5}\left[V_z(Y_B,\theta)\big/u^*(\theta) + 5 - \beta \ln \frac{5\,Y_B\, u^*(\theta)}{\nu}\right] \qquad (1.6d)$$

The expressions for α, β come from the velocity continuity condition at the points

$$\frac{Y\, u^*(\theta)}{\nu} = 5 \qquad \text{and} \qquad \frac{Y\, u^*(\theta)}{\nu} = 30$$

1.1.3.1.2 Turbulent Kinetic Energy

As we have pointed out in Ref. 1, p. 58 the near wall boundary lies within the dynamic equilibrium subregion where the importance of the convection and diffusion mechanism are minor.

Thus the transport equation for k can be reduced to

$$\left(-\overline{v_r' v_z'}\right)_{Y=Y_B}\left(\frac{\partial V_z}{\partial r}\right)_{Y=Y_B} \approx \varepsilon_B \qquad (1.7)$$

where ε_B is the energy dissipation at $y = y_B$.

On the other hand the axial momentum equation neglecting the lateral gradient as small compared with the radial gradient is reduce to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ \overline{v_r' v_z'}\right) - \nu\frac{\partial^2 V_z}{\partial r^2} = \frac{4}{D_H}\left(\overline{u^*}\right)^2 \qquad (1.8)$$

where D_H is the hydraulic diameter and $\overline{u^*}$ is the average friction velocity.

Integrating Eq. (1.8) over y_B and neglecting the friction effect at y_B we obtain

$$\left(-\overline{v_r' v_z'}\right)_{Y=Y_B} = \frac{r_1}{r_2}\left[u^*(\theta)\right]^2 - 2\ \frac{r_2^2 - r_1^2}{r_2 D_H}\left(\overline{u^*}\right)^2 \qquad (1.9)$$

where
$$r_1 = D/2$$
$$r_2 = D/2 + y_B$$

Since

$$\left(\frac{\partial V_z}{\partial r}\right)_{Y=Y_B} = \frac{u^*(\theta)}{\kappa\, Y_B}$$

and

$$C_\mu\frac{k_B^2}{\varepsilon_B}\left(\frac{\partial V_z}{\partial r}\right)_{Y=Y_B} = \left(-\overline{v_r' v_z'}\right)_{Y=Y_B}$$

we find finally

$$\varepsilon_B = \left(-\overline{v_r' v_z'}\right)_{Y=Y_B}\frac{u^*(\theta)}{\kappa\, Y_B} \qquad (1.10)$$

$$k_B = \frac{1}{\sqrt{c_\mu}}\left(-\overline{v_r' v_z'}\right)_B \qquad (1.11)$$

1.2 Grid

We have tried to make the grid space as uniform as possible. The reason is that although grid nonuniformity can improve the accuracy of the problem in some cases usually it has a negative effect on rate of convergence and stability [8]. On the other hand a uniform grid makes the numerical equations easier to handle. So in the θ direction a uniform spacing $\Delta\theta$ is used, i.e.,

$$\Delta\theta = \theta_j - \theta_{j-1} = \frac{\pi}{6*(JN-1)} \qquad (1.12)$$

where JN is the number of mesh points in the θ-direction. With respect to the r-direction, the spacing along the gap (i.e., $\theta=0$) is also taken uniform, i.e.,

$$r_i - r_{i-1} = \left[\frac{P}{2} - \left(\frac{D}{2} + Y_B\right)\right] / (M-2); \quad i \le M$$
$$M = IN - (JN-1) \qquad (1.13)$$

where

M is the number of points along $\theta=0$ line

IN is the number of points along $\theta=30°$ line, and

y_B is the wall boundary thickness

For i > M the spacing cannot be uniform since it is defined from the zero shear stress line as shown in Fig. 2.

In this case

$$r_{i=M+j-1} = \frac{P}{2\cos\theta} - \frac{P}{2}, \quad M < i \le IN \qquad (1.14)$$

The grid is shown in Fig. 2.

So by the above procedure the mesh points (i,j) have been defined. However at these points only the scalar parameters k, ε, and p the axial velocity v_z and auxiliary quantities (e.g., eddy viscosities, mixing length) will be calculated. The

parameters $v_r(i,j)$ and $v_\theta(i,j)$ will be defined in a slightly different position as shown in Fig. 4. This procedure is similar to that adopted in TEACH code [9].

.1.3 The Finite Difference Equations

The finite difference equations are obtained by integration of Eq. (1.1) over the control volume. The control volume surrounding the point under consideration is shown in its general form in Fig. 3.

After integration over the control volume and taking into consideration the Divergence Theorem Eq. (1.1) becomes

$$\oint_{CS} \phi \, \underline{v} \cdot \underline{n} \, dl + \oint_{CS} \underline{T}_\phi \cdot \underline{n} \, dl = \oint_{CS} \underline{S} \phi \, \underline{n} \, dl + \qquad (1.15)$$
$$+ \iint_{CV} S_\phi^* \, dA$$

where by $\int_C \int_V$ we denote the integration over the boundary of the control volume, and $\underline{n}$ is the unit vector perpendicular to the surface boundary as shown in Fig. 3. Let

A_e = length (ne, se)

A_w = length (nw, sw)

A_n = length (ne, nw)

A_s = length (se, sw)

V_p = area (ne, se, sw, nw, ne) $\qquad (1.16)^*$

The value of above quantities for the various angles ϕ are given in Table II. Equation (1.15) can be written

$$A_e \, \phi_e \, V_{re} - A_w \, \phi_w \, V_{rw} + A_n \, \phi_n \, V_{\theta n} - A_s \, \phi_s \, V_{\theta s} +$$
$$+ T_{\phi re} A_e - T_{\phi rw} A_w + T_{\phi \theta n} A_n - T_{\phi \theta s} A_s =$$
$$= S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} A_n - S_{\phi \theta s} A_s +$$
$$+ S_{\phi p}^* V_p \qquad (1.17)$$

(*)The nomenclature used in Fig. 3 is almost the same as that used in Ref. 2.

In Eq. (1.19) the following approximation has been made

$$\iint\limits_{CV} S_\phi^* \, dA \approx S_{\phi P}^* \, V_P \qquad (1.18)$$

where

S_P is the value of S_ϕ^* at point P

V_P is the control volume surface

The continuity equation gives after integration over the control volume

$$A_e V_{re} - A_w V_{rw} + A_n V_{\theta n} - A_s V_{\theta s} = 0 \qquad (1.19)$$

In Eq. (1.17) we can distinguish the Convection term, Diffusion term and Source term as follows:

Diffusion term:

$$DT = T_{\phi re} A_e - T_{\phi rw} A_w + T_{\phi \theta n} A_n - T_{\phi \theta s} A_s \qquad (1.20a)$$

Convection term:

$$CT = A_e V_{re} \phi_e - A_w V_{rw} \phi_w + A_n V_{\theta n} \phi_n - \qquad (1.20b)$$
$$- A_s V_{\theta s} \phi_s$$

Source term:

$$ST = S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} V_{\theta n} - S_{\phi \theta s} V_{\theta s} + \qquad (1.20c)$$
$$+ S_{\phi P}^* V_P$$

Thus Eq. (1.17) can be written

$$DT + CT = ST$$

1.3.1 Diffusion Term

1.3.1.1 Scalar Quantities k, ε and the Axial Velocity V_z

From the Table Ia we have

$$T_{\phi r} = - \frac{\nu_{rz}^{eff}}{\sigma_\phi} \frac{\partial \phi}{\partial r}$$

$$T_{\phi \theta} = - \frac{\nu_{\theta z}^{eff}}{\sigma_\phi} \frac{\partial \phi}{r \partial \theta} \qquad (1.21)$$

We can make the following approximations

$$T_{\phi re} = - \frac{\left(\nu_{rz}^{eff}\right)_E + \left(\nu_{rz}^{eff}\right)_P}{2\sigma_\phi} \frac{\phi_E - \phi_P}{r_E - r_P}$$

$$T_{\phi rw} = \frac{\left(\nu_{rz}^{eff}\right)_W + \left(\nu_{rz}^{eff}\right)_P}{2\sigma_\phi} \cdot \frac{\phi_P - \phi_W}{r_P - r_W} \qquad (1.22a)$$

$$T_{\phi \theta n} = - \frac{\left(\nu_{\theta z}^{eff}\right)_N + \left(\nu_{\theta z}^{eff}\right)_P}{2\sigma_\phi} \frac{\phi_N - \phi_P}{r_P \Delta \theta}$$

$$T_{\phi \theta s} = - \frac{\left(\nu_{\theta z}^{eff}\right)_S + \left(\nu_{\theta z}^{eff}\right)_P}{2\sigma_\phi} \frac{\phi_P - \phi_S}{r_P \Delta \theta} \qquad (1.22b)$$

Thus Eq. (1.7a) becomes

$$DT = (DE + DW + DN + DS)\phi_P - DN \cdot \phi_N - DS \cdot \phi_S - DE \cdot \phi_E - DW \cdot \phi_W$$

$$(1.23)$$

where

$$DE = \frac{BE}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_E + \left(\nu_{rz}^{eff}\right)_P\right] \quad (1.24a)$$

$$DW = \frac{BW}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_W + \left(\nu_{rz}^{eff}\right)_P\right] \quad (1.24b)$$

$$DN = \frac{BN}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_N + \left(\nu_{\theta z}^{eff}\right)_P\right] \quad (1.24c)$$

$$DS = \frac{BS}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_S + \left(\nu_{\theta z}^{eff}\right)_P\right] \quad (1.24d)$$

$$BE = \frac{Ae}{2(r_E - r_P)} \quad (1.25a)$$

$$BW = \frac{Aw}{2(r_P - r_W)} \quad (1.25b)$$

$$BN = \frac{An}{2r_P \Delta\theta} \quad (1.25c)$$

$$BS = \frac{A_s}{2 r_p \, \Delta\theta} \qquad (1.25d)$$

The values of DE, DW, DN, DS, BE, BW, BN, BS are given in Table III.

1.3.1.2 The Velocity V_r

From Table I we see that

$$T_{\phi r} = -2\nu \, \frac{\partial V_r}{\partial r}$$

$$T_{\phi\theta} = -\nu_{r\theta}^{eff} \, \frac{\partial V_r}{r \partial \theta} \qquad (1.26)$$

we make the following approximations

$$T_{\phi re} = -2\nu \left(V_{rE} - V_{rp} \right) / \left(r_E - r_p \right)$$

$$T_{\phi rw} = -2\nu \left(V_{rp} - V_{rW} \right) / \left(r_p - r_W \right)$$

$$T_{\phi\theta n} = -\nu_{r\theta n}^{eff} \left(V_{rN} - V_{rp} \right) / r_p / \Delta\theta \qquad (1.27)$$

$$T_{\phi\theta s} = -\nu_{r\theta s}^{eff} \left(V_{rp} - V_{rS} \right) / r_p / \Delta\theta$$

Thus the diffusion term is

$$\boxed{DT = (DE + DW + DN + DS) v_{rp} - DN \cdot v_{rN} - DS \cdot v_{rS} - DE \cdot v_{rE} - DW \cdot v_{rW}}$$

$$(1.28)$$

where

$$(1.29a)$$

$$DE = BE \cdot \nu \tag{1.29b}$$

$$DW = BW \cdot \nu$$

$$DN = BN\left[\left(\nu_{r\theta}^{eff}\right)_{Ne} + \left(\nu_{r\theta}^{eff}\right)_{e} + \left(\nu_{r\theta}^{eff}\right)_{w} + \left(\nu_{Nw}^{eff}\right)_{Nw}\right] \tag{1.29c}$$

$$DS = BS\left[\left(\nu_{r\theta}^{eff}\right)_{Se} + \left(\nu_{r\theta}^{eff}\right)_{e} + \left(\nu_{r\theta}^{eff}\right)_{w} + \left(\nu_{r\theta}^{eff}\right)_{Sw}\right] \tag{1.29d}$$

$$BE = \frac{2A_e}{r_E - r_P} \tag{1.30a}$$

$$BW = \frac{2A_W}{r_P - r_W} \tag{1.30b}$$

$$BN = BS = \frac{A_n}{4r_P\Delta\theta} \tag{1.30c}$$

The values of DE, DW, DN, DS, BE, BW, BN, BS are given in Table III.

1.3.1.3 The Velocity v_θ

From Table I we have

$$T_{\phi r} = -\nu_{r\theta}^{eff}\, r^2\, \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) \tag{1.31a}$$

$$T_{\phi\theta} = -2\nu\, \frac{\partial v_\theta}{\partial\theta} \tag{1.31b}$$

We make the following approximations

$$T_{\phi re} = \nu_{r\theta e}^{eff} \; r_e^2 \left[\frac{\frac{v_{\theta E}}{r_E} - \frac{v_{\theta P}}{r_P}}{r_E - r_P} \right]$$

$$T_{\phi rw} = \nu_{r\theta w}^{eff} \; r_w^2 \left[\frac{\frac{v_{\theta P}}{r_P} - \frac{v_{\theta W}}{r_W}}{r_P - r_W} \right]$$

$$T_{\phi \theta n} = -2\nu \left[\frac{v_{\theta N} - v_{\theta P}}{\Delta \theta} \right]$$

$$T_{\phi \theta s} = 2\nu \left[\frac{v_{\theta P} - v_{\theta S}}{\Delta \theta} \right]$$

$$(1.32)$$

Thus Eq. (1.20a) becomes

$$DT = (DN + DS + DE \cdot \frac{r_E}{r_P} + DW \; \frac{r_W}{r_P}) v_{\theta P} - DE \cdot v_{\theta E} - DW \cdot v_{\theta W} - DN \cdot v_{\theta N} - DS \cdot v_{\theta S}$$

$$(1.33)$$

where

$$DE = BE \left[\left(\nu_{r\theta}^{eff} \right)_{nE} + \left(\nu_{r\theta}^{eff} \right)_{sE} + \left(\nu_{r\theta}^{eff} \right)_{s} + \left(\nu_{r\theta}^{eff} \right)_{n} \right]$$

$$DW = BW \left[\left(\nu_{r\theta}^{eff} \right)_{nW} + \left(\nu_{r\theta}^{eff} \right)_{sW} + \left(\nu_{r\theta}^{eff} \right)_{s} + \left(\nu_{r\theta}^{eff} \right)_{n} \right] \quad (1.34)$$

$$DN = DS = BN \cdot \nu$$

$$BE = \frac{r_e^2\, A_e}{4 r_E \left(r_E - r_P \right)}$$

$$BW = \frac{r_w^2\, A_w}{4 \sqrt{r_W} \left(r_P - \sqrt{r_W} \right)}$$

$$BN = BS = \frac{2 A_n}{\Delta \theta}$$

(1.35)

The values of DE, DW, DN, DS, BE, BW, BS are given in Table III.

1.3.2   The Convection Term

Recall that the convection term is given by the Eq. (1.20) i.e.,

$$CT = A_e\, V_{re}\, \phi_e - A_w\, V_{rw}\, \phi_w + A_n\, V_{\theta n}\, \phi_n - A_s\, V_{\theta s}\, \phi_s \qquad (1.20b)$$

where

ϕ is given in Table Ia

From Eq. (1.20b) it is evident that the common substitution

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

etc., can produce a finite difference equation with negative coefficients which can produce nonconvergence in the solution. Since our ultimate purpose is to obtain a finite difference equation with positive coefficients and on the other hand as much accurate as possible we proceed as follows:

Let

$$\phi_e = \phi_P + \left(\frac{\partial \phi}{\partial r}\right)_P (r_e - r_P) \qquad (1.36a)$$

$$\phi_n = \phi_P + \left(\frac{\partial \phi}{\partial \theta}\right)_P \frac{\Delta\theta}{2} \qquad (1.36b)$$

$$\phi_W = \phi_P - \left(\frac{\partial \phi}{\partial r}\right)_P (r_P - r_w) \qquad (1.36c)$$

$$\phi_S = \phi_P - \left(\frac{\partial \phi}{\partial \theta}\right)_P \frac{\Delta\theta}{2} \qquad (1.36d)$$

Substituting Eqs. (1.26) into (1.20b) and taking into consideration continuity Eq. (1.19) we end up with

$$CT = \left[A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w)\right]\left(\frac{\partial \phi}{\partial r}\right)_P +$$

$$+ \frac{A_n \Delta\theta}{2}(v_{\theta n} + v_{\theta s})\left(\frac{\partial \phi}{\partial \theta}\right)_P \qquad (1.37)$$

In Eq. (1.24) we have taken into consideration the fact that $A_n = A_s$

We further approximate

$$\left(\frac{\partial \phi}{\partial r}\right)_P = \begin{cases} \dfrac{\phi_E - \phi_P}{r_E - r_P} & \text{if } A_e v_{re}(r_e - r_P) + A_w v_{rw}(r_P - r_w) \le 0 \\[4mm] \dfrac{\phi_P - \phi_W}{r_P - r_w} & \text{otherwise} \end{cases} \qquad (1.38)$$

$$\left(\frac{\partial \phi}{\partial \theta}\right)_P = \begin{cases} \dfrac{\phi_N - \phi_P}{\Delta \theta} & \text{if } v_{\theta P} \leq 0 \\[2mm] \dfrac{\phi_P - \phi_S}{\Delta \theta} & \text{otherwise} \end{cases} \tag{1.39}$$

Under these circumstances the convection term can be written

$$CT = (CE + CW + CN + CS)\phi_P - CE\,\phi_E -$$
$$- CW\,\phi_W - CN\,\phi_N - CS\,\phi_S \tag{1.40}$$

where

$$CE = 0.5\left\{\left| A_e V_{re}(r_e - r_P) + A_w V_{rw}(r_P - r_w) \right| - \right.$$
$$\left. - A_e V_{re}(r_e - r_P) - A_w V_{rw}(r_P - r_w) \right\} \Big/ (r_E - r_P) \tag{1.41a}$$

$$CW = 0.5\left\{\left| A_e V_{re}(r_e - r_P) + A_w V_{rw}(r_P - r_w) \right| + \right.$$
$$\left. + A_e V_{re}(r_e - r_P) + A_w V_{rw}(r_P - r_w) \right\} \Big/ (r_P - r_W) \tag{1.41b}$$

$$CN = An \frac{|V_{\theta n} + V_{\theta s}| - V_{\theta n} - V_{\theta s}}{4}$$

(1.41c)·

$$-CS = A_s \frac{|V_{\theta n} + V_{\theta s}| + V_{\theta n} + V_{\theta s}}{4}$$

(1.41d)

The value for CE, CW, CN, CS at a particular point (i,j) and for $\phi = k$, ε, v_z, v_r are given in Table IV

The convection term for the component v_θ itself is slightly different (*) than (1.40) as in the case of the diffusion term, i.e.,

$$CT = \left(CE \frac{r_p}{r_E} + CW \frac{r_p}{r_W} + CN + CS \right) v_\theta P -$$

$$- CE \cdot v_{\theta E} - CW \cdot v_{\theta W} - CN v_{\theta N} - CS v_{\theta S}$$

The values of CE, CW, CN, CS for v_θ are given also in Table IV.

1.3.3 Source Term

Recall that the source term is given by (1.20c) i.e.,

$$ST = S_{\phi re} A_e - S_{\phi rw} A_w + S_{\phi \theta n} A_n - S_{\phi \theta s} A_s + S_\phi^* V_p$$

(1.20c)

The form of the source term for each quantity is different in character so we examine ST for each quantity separately.

1.3.3.1 Axial Velocity v_z

From Table I we see that

$$ST = S_p^* V_p = \frac{4}{DH} \left(\frac{\tau_w}{\rho} \right)_{avg} V_P$$

(*) Eq (1.40) is valid for $-v_\theta$ instead of v_θ as Table Ia shows

where

$(\frac{\tau_W}{\rho})_{avg}$ is the average shear stress, and

DH is the hydraulic diameter.

1.3.3.2 Kinetic Energy k

From Table Ia

$$ST = S_P^* V_P = \left[\left(\nu_{rz}^T \right)_P \left(\frac{\partial v_z}{\partial r} \right)_P^2 + \left(\nu_{\theta z}^T \right)_P \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right.$$

$$\left. - \, \epsilon_P \right] V_P \qquad\qquad (1.44)$$

For reasons of numerical stability as we will see later the energy dissipation term can be transferred as follows:

Since

$$\nu_{rz}^T = C_\mu \frac{k^2}{\epsilon}$$

$$\epsilon = \frac{C_\mu k^2}{\nu_{rz}^T}$$

Thus

$$ST = \left[\left(\nu_{rz}^T \right)_P \left(\frac{\partial v_z}{\partial r} \right)_P^2 + \left(\nu_{\theta z}^T \right)_P \left(\frac{\partial v_z}{r \partial \theta} \right)_P^2 \right] V_P$$

$$- \, SP \cdot k_P \qquad\qquad (1.45)$$

where

$$SP = \frac{C_\mu k_P}{(\nu_{rz}^T)_P} \qquad\qquad (1.46)$$

1.3.3.3 Energy Dissipation

From Table Ia

$$ST = \left\{ C_1 \frac{\varepsilon}{k} \left[\nu_{rz}^T \left(\frac{\partial v_z}{\partial r} \right)^2 + \nu_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)^2 \right] - C_2 \frac{\varepsilon^2}{k} \right\} \tag{1.47}$$

Since

$$\frac{\varepsilon}{k} = C_\mu \frac{k}{\nu_{vz}^T} \tag{1.48}$$

Then

$$ST = C_1 \frac{C_\mu k_P}{(\nu_{rz}^T)_P} \left\{ (\nu_{rz}^T)_P \left(\frac{\partial v_z}{\partial r} \right)_P^2 + \right.$$
$$\left. + \nu_{\theta z}^T \left(\frac{\partial v_z}{r \partial \theta} \right)_P^2 \right\} - SP \, \epsilon_P \tag{1.49}$$

where

$$SP = c_2 c_\mu \frac{k_P V_P}{(\nu_{rz}^T)_P} \tag{1.50}$$

1.3.3.4 Radial Velocity v_r

From Table I we have

$$S_{\phi r} = -\left(\overline{v_r'^2} + \frac{p}{\rho} \right)$$

$$S_{\phi \theta} = \nu_{r\theta}^{eff} \, r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right)$$

$$S_\phi^* = \frac{v_\theta^2 + \overline{v_\theta'^2}}{r} - \frac{2\nu}{r} \left(\frac{\partial v_\theta}{r \partial \theta} + \frac{v_r}{r} \right) \approx \frac{\overline{v_\theta'^2}}{r}$$

Thus Eq. (1.20c) becomes

$$ST = A_w \left[\overline{v_{rw}'^2} + (p/\rho)_w \right] - A_e \left[\overline{v_{re}'^2} + (p/\rho)_e \right]$$
$$+ \left(\nu_{r\theta}^{eff} \right)_n r_p \left[\frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]_n A_n - \nu_{r\theta}^{eff} r_p \left[\frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]_s A_s$$
$$+ \frac{\left(\overline{v_\theta'^2} \right)_p}{r_p} V_p \qquad\qquad (1.51)$$

The ST value for the point (i,j) is given in Table IV.

1.3.3.5. Velocity v_θ

From Table I we have

$$S_{\phi r} = -\nu_{r\theta}^{eff} \frac{\partial v_r}{\partial \theta} \qquad\qquad (1.52)$$

$$S_{\phi\theta} = 2\nu v_r - r\overline{v_\theta'^2} - \frac{rp}{\rho} \approx -r(\overline{v_\theta'^2} + \frac{p}{\rho}) \qquad\qquad (1.53)$$

Thus Eq. (1.20c) can be approximated

$$ST = \nu_{r\theta e}^{eff} \left(\frac{\partial v_r}{\partial \theta} \right)_e A_e - \nu_{r\theta w}^{eff} \left(\frac{\partial v_r}{\partial \theta} \right)_w A_w$$

$$+ r_p A_n \left[(\overline{v_\theta'^2})_s + (p/\rho)_s - (\overline{v_\theta'^2})_n - (p/\rho)_n \right] \qquad\qquad (1.54)$$

The ST value for the point (i,j) is given in Table IV.

1.3.4 General Form of FDE

The general form of the differential equations of all the quantities under consideration, has as follows

$$(AP + SP)\phi_P = AE + \phi_E + AN \cdot \phi_N + AW \cdot \phi_W + AS \cdot \phi_S + SU \qquad (1.55a)$$

where

$$
\left.
\begin{aligned}
AE &= CE + DE \\
AW &= CW + DW \\
AN &= CN + DN \\
AS &= CS + DS
\end{aligned}
\right\} \qquad (1.55b)
$$

$$
AP = \begin{cases}
\dfrac{r_P}{r_E} CE + \dfrac{r_E}{r_P} DE + \dfrac{r_P}{r_W} CW + \dfrac{r_W}{r_P} DW + AN + AS \quad \text{for } v_\theta \\[2mm]
AE + AW + AN + AS \qquad\qquad\qquad\qquad\qquad \text{Otherwise}
\end{cases} \qquad (1.55c)
$$

where CE, CW, CN, CS, DE, DW, DN, DS, SU, SP are given in Tables III, IV and V

1.3.5 The Boundary Conditions

1.3.5.1 Quantities k, ε, v_z

1.3.5.1.1 Boundary I and II (Fig. 1)

According to Table Ib

$$\left(\frac{\partial \phi}{\partial \theta}\right)_{I,II} = 0 \qquad\qquad (1.56)$$

where

$$\phi = k, \ \varepsilon \text{ or } v_z$$

Numerically Eq. (1.55) is translated to

$$\phi_{i,j} = \phi_{i,j+1} \qquad \text{for Boundary I}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1.57)$$

$$\phi_{i,j} = \phi_{i,j-1} \qquad \text{for Boundary II}$$

1.3.5.1.2 Boundary III

From Table Ib we have

$$(\frac{\partial \phi}{\partial n})_{III} = 0 \qquad (1.58)$$

Integrating over the triangular control volume shown in Fig. 5 and taking into consideration Eq. (1.58) we end up with

$$(AP + SP')\phi_P = AW \cdot \phi_W + AN \cdot \phi_N + SU' \qquad (1.59)$$

$$AW = DW + CW'$$

$$AN = DN + CN'$$

$$AP = AW + AN$$

Where DW, DN are given from Table III and IV respectively

$$CN' = -AN(v_\theta)_{i,j+1}/2$$

$$CW' = A_W(v_r)_{i,j}/2$$

$$SU' = SU \frac{V_P'}{V_P} \qquad (1.60)$$

$$SP' = SP \frac{V_P'}{V_P}$$

and A_e, A_n, V_P are given from Table II. SU, SP are given from Table V.

V_P' is the area of the triangular control volume under consideration, i.e.,

$$\left(V_P'\right)_j = \left[\left(r_{i+1} + r_i\right) \sin \Delta\theta - \left(r_i + r_{i-1}\right) \Delta\theta\right] \frac{r_i + r_{i+1}}{8} \qquad (1.61)$$

It is evident from Eq. (1.60) that we have substituted for

$$\phi_e = \frac{\phi_E + \phi_P}{2}$$

and similarly for ϕ_w, ϕ_n, ϕ_s.

1.3.5.1.3 Wall Region

1.3.5.1.3.1 Axial Velocity

At the y_B line we will demand continuity of the velocity and the velocity gradient.

The velocity gradient can be calculated numerically with accuracy $0(\Delta r^2)$ from the relation

$$\left(\frac{dv_z}{dr}\right)_{2,j} = \frac{r_4 - r_2}{(r_3 - r_2)(r_4 - r_3)}(v_z)_{3,j} - \frac{r_3 - r_2}{(r_4 - r_2)(r_4 - r_3)}(v_z)_{4,j}$$

$$- \frac{r_4 + r_3 - 2r_2}{(r_3 - r_2)(r_4 - r_2)}(v_z)_{2,j} \tag{1.62a}$$

since

$$(v_z)_{2,j} = \frac{u^*_j}{K} \ln E \frac{u^*_j y_B}{\nu} \tag{1.62b}$$

and

$$\left(\frac{dv_z}{dr}\right)_{2,j} = \frac{u^*_j}{K y_B} \tag{1.62c}$$

we can solve Eqs. (1.62) for u^*_j and $(v_z)_{2,j}$ numerically

1.3.5.1.3.2 Turbulent Kinetic Energy and Energy Dissipation

$k_{2,j}$ and $\varepsilon_{2,j}$ are calculated from the algebraic relations given in Table Ib

1.3.5.2 v_r Velocity

1.3.5.2.1 Boundary I and II (Fig. 1)

According to Table IB

$$\left(\frac{\partial v_r}{\partial \theta}\right)_{I,II} = 0 \tag{1.63a}$$

thus numerically Eq. (1.63) is translated

$$\left.\begin{array}{l} (v_r)_{i,j} = (v_r)_{i,j+1} \quad \text{for Boundary I} \\[2em] (v_r)_{i,j} = (v_r)_{i,j-1} \quad \text{for Boundary II} \end{array}\right\} \tag{1.63b}$$

1.3.5.2.2 Boundary III Fig. 1)

From Table Ib we have

$$V_{sec} \; n = 0 \tag{1.64}$$

Numerically Eq. (1.64) can be expressed as we can see from Fig. 5b

$$(A_e)_{i-1}(v_r)_{i,j} = (A_n)_{i-1}(v_\theta)_{i-1,j} \tag{1.65}$$

where A_e and A_n are given in Table III

1.3.5.2.3 Wall

From Table 1b we have

$$(v_r)_{i,j} = 0 \tag{1.66}$$

from the continuity equation, since

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad \text{at the wall}$$

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$$\frac{\partial rv_r}{\partial r} = 0$$

which gives

$$(v_r)_{i,j} = 0 \qquad (1.67)$$

as well

1.3.5.3 v_θ Velocity

1.3.5.3.1 Boundaries I and III

From Table 1b we have

$$(v_\theta)_{I,II} = 0 \qquad (1.68)$$

Numerically we can approximate[*] (with accuracy of $0(\frac{\Delta\theta}{2})$)

$$\left.\begin{array}{l}(v_\theta)_{i,2} = 0 \\[2ex] (v_\theta)_{i,N} = 0\end{array}\right\} \qquad (1.69)$$

where N is the maximum number of points for v_z along the tangential direction.

1.3.5.3.2 Boundary III

From Table Ib

$$\frac{\partial V_{sec}}{\partial n} = 0 \qquad (1.70)$$

This relation leads to (see Ref. 6)

$$\frac{\partial(rv_\theta)}{\partial r} = \frac{\partial v_r}{\partial \theta} \qquad (1.70a)$$

Numerically Eq. (1.70) as we can see from Fig. 5c can be expressed

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[*]More accurate schemes for Eq. (1.68) have led to instabilities.

$$\frac{r_i (v_\theta)_{i,j} - r_{i-1}(v_\theta)_{i-1,j}}{r_i - r_{i-1}} = \frac{(v_r)_{i,j} - (v_r)_{i,j-1}}{\Delta\theta} \qquad (1.71)$$

1.3.5.3.3 <u>Wall</u>

From Table Ib

$$(v_\theta)_{1,j} = 0 \qquad (1.72)$$

REFERENCES

[1] John Bartzis and N. Todreas, "Coolant Mixing in LMFBR Rod Bundles and Outlet Mixing Transients," Quarterly Progress Report, Task IID, COO-2245-28.

[2] B.E. Launder and Ying, "Prediction of Flow and Heat Transfer in Ducts of Square Corss Section," The Institution of Mech. Eng. Proceedings, Vol. 187, 37/93, pp. 455-461, (1972).

[3] D. Naot, et al., "Numerical Calculations of Reynolds Stresses in a Square Duct with Secondary Flow," Wärme-Stoffubertragung 7 (1975) pp. 151-

[4] P. Carajilescov and N. Todreas, "Experimental and Analytical Study of Axial Turbulent Flow in an Interior Subchannel of a Bare Rod Bundle," 75-WA/HT-57 (1976).

[5] M.U. Ibrajirmov et al., "Calculation of Hydraulic Resistivity Coefficients for Turbulent Fluid Flow in Fluid Flow in Channels of Noncircular Cross Section," Soviet Atomic Energy Vol. 23, No. 4, pp. 1042-1047 (1967).

[6] P. Carajilescov, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," MIT Ph.D. Thesis, 1975.

[7] Gosman, et al., "Heat and Mass Transfer in Recirculating Flows," Academic Press, London 1969.

[8] J.G. Bartzis, "Partially Blocked Flow between Parallel Plates," Euratom Internal Report, Ispra 1975.

[9] A.D. Gosman, "Listing of the TEACH-T Computer Program and Sample Output," The Pennsylvania State University, August 1975.

Table Ia THE DIFFERENTIAL EQUATIONS

General Form:

$$\nabla \phi \underline{V} + \nabla T\phi = \nabla S\phi + S_\phi{}^*$$

ϕ	$T\phi$		$S\phi$		S^*
	$T\phi r$	$T\phi\theta$	$S\phi r$	$S\phi\theta$	
v_r	$-2\nu\dfrac{\partial v_r}{\partial r}$	$-\dfrac{\nu_{r\theta}^{eff}}{r}\dfrac{\partial v_r}{\partial\theta}$	$-\overline{v_r'^2} - p/\rho$	$+\nu_{r\theta}^{eff}\, r\dfrac{\partial}{\partial r}\left(\dfrac{v_\theta}{r}\right)$	$\dfrac{v_\theta^2 + \overline{v_\theta'^2}}{r} - \dfrac{2\nu}{r}\left(\dfrac{\partial v_\theta}{r\partial\theta} + \dfrac{v_r}{r}\right)$
$r v_\theta$	$-\nu_{r\theta}^{eff}\, r^2\dfrac{\partial}{\partial r}\left(\dfrac{v_\theta}{r}\right)$	$-2\nu\dfrac{\partial(rv_\theta)}{r\partial\theta}$	$+\nu_{r\theta}^{eff}\dfrac{\partial v_r}{\partial\theta}$	$+2\overline{v_r'v_\theta'} - r\overline{v_\theta'^2} - \dfrac{rp}{g}$	0
v_z	$-\nu_{rz}^{eff}\dfrac{\partial v_z}{\partial r}$	$-\nu_{r\theta}^{eff}\dfrac{\partial v_z}{r\partial\theta}$	0	0	$-\dfrac{d\,(p/\rho)}{dz} = \dfrac{4\,\overline{u}^{*2}}{D_H}$
k	$-\dfrac{\nu_{rz}^{eff}}{\sigma_\kappa}\dfrac{\partial k}{\partial r}$	$-\dfrac{\nu_{r\theta}^{eff}}{\sigma_\kappa}\dfrac{\partial k}{r\partial\theta}$	0	0	$\nu_{rz}^T\left(\dfrac{\partial v_z}{\partial r}\right)^2 + \nu_{r\theta}^T\left(\dfrac{\partial v_z}{r\partial\theta}\right)^2 - \varepsilon$
ε	$-\dfrac{\nu_{rz}^{eff}}{\sigma_\varepsilon}\dfrac{\partial\varepsilon}{\partial r}$	$-\dfrac{\nu_{r\theta}^{eff}}{\sigma_\varepsilon}\dfrac{\partial\varepsilon}{r\partial\theta}$	0	0	$C_1\dfrac{\varepsilon}{k}\left[\nu_{rz}^T\left(\dfrac{\partial v_z}{\partial r}\right)^2 + \nu_{r\theta}^T\left(\dfrac{\partial v_z}{r\partial\theta}\right)^2\right] - C_2\dfrac{\varepsilon^2}{k}$

PARAMETER	I	II	III	NEAR WALL REGION	
v_z	$\dfrac{\partial v_z}{r\partial\theta} = 0$	$\dfrac{\partial v_z}{r\partial\theta} = 0$	$\dfrac{\partial v_z}{\partial r} = \dfrac{\partial v_z}{r\partial\theta}\,tg\,\theta$	Logarithmic Profile	
v_r	$v_r = 0$	$v_r = 0$	$\underline{n}\cdot\underline{V}sec = 0$	$v_r\,|wall = 0$	
$v\theta$	$v_\theta \neq 0$	$v_\theta = 0$	$\dfrac{\partial Vsec}{\partial n} = 0$	$v_\theta\,|wall = 0$	
k	$\dfrac{\partial k}{r\partial\theta} = 0$	$-\dfrac{\partial k}{r\partial\theta} = 0$	$\dfrac{\partial k}{\partial r} = \dfrac{\partial k}{r\partial\theta}\,tg\,\theta$	$k = \dfrac{\overline{v'_r v'_z}}{\sqrt{c_\mu}}$	
ε	$\dfrac{\partial\varepsilon}{r\partial\theta} = 0$	$\dfrac{\partial\varepsilon}{r\partial\theta} = 0$	$\dfrac{\partial\varepsilon}{\partial r} = \dfrac{\partial\varepsilon}{r\partial\theta}\,tg\,\theta$	1-EQ. $\varepsilon = c_D\dfrac{k^{3/2}}{\ell}$ Algebraic	2-EQ. $\varepsilon = \dfrac{u_*}{ky}(\overline{v'_r v'_z})$

CONSTANTS
$$c_D \approx 0.4$$
$$c \approx 0.08$$
$$k \approx 0.4$$

$$r_2 = r_1 + y_B$$

$$(*) \qquad \overline{-v'_r v'_z} = \frac{r_1}{r_2}\,u*(\theta)^2\,\frac{r_2^2 - r_1^2}{r_2 D_H}\,\left(\frac{\tau\omega}{\rho}\right)_{avg}$$

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ϕ	A_e	A_w	A_n	A_s	V_p
v_z k ε rv_θ	$\dfrac{r_i + r_{i+1}}{2}\,\Delta\theta$	$\dfrac{r_i + r_{i-1}}{2}\,\Delta\theta$	$\dfrac{r_{i+1} - r_{i-1}}{2}$	$\dfrac{r_{i+1} - r_{i-1}}{2}$	$r_i\Delta\theta\,\dfrac{r_{i+1} - r_{i-1}}{2}$
v_r	$r_i\,\Delta\theta$	$r_{i-1}\,\Delta\theta$	$r_i - r_{i-1}$	$r_i - r_{i-1}$	$\dfrac{r_i + r_{i-1}}{2}(r_i - r_{i-1})\,\Delta\theta$

Table II   Control volume (Fig 1)   Lengths and surface

TABLE III: THE DIFFUSION COEFFICIENTS

	$\phi = k, \varepsilon, v_z$	v_r	v_θ
BE	$0.25 \dfrac{r_{i+1}+r_i}{r_{i+1}-r_i}\,\Delta\theta$	$\dfrac{4r_i}{r_{i+1}-r_{i-1}}\,\Delta\theta$	$\dfrac{(r_i+r_{i+1})^3}{32r_{i+1}(r_{i+1}-r_i)}$
BW	$0.25 \dfrac{r_i+r_{i-1}}{r_i-r_{i-1}}$	$\dfrac{4r_{i-1}}{r_i-r_{i-2}}\,\Delta\theta$	$\dfrac{(r_i+r_{i-1})^3}{32r_{i-1}(r_i-r_{i-1})}$
BN BS	$0.25 \dfrac{r_{i+1}-r_{i-1}}{r_i}\,\Delta\theta$	$\dfrac{r_i-r_{i-1}}{2(r_i+r_{i-1})\,\Delta\theta}$	$\dfrac{r_{i+1}-r_{i-1}}{\Delta\theta}$
DE	$\dfrac{BE}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_{i+1,j} + \left(\nu_{rz}^{eff}\right)_{i,j}\right]$	$BE \cdot \nu$	$BE\left[\left(\nu_{r\theta}^{eff}\right)_{i+1,j} + \left(\nu_{r\theta}^{eff}\right)_{i+1,j-1} + \left(\nu_{r\theta}^{eff}\right)_{i,j-1} + \left(\nu_{r\theta}^{eff}\right)_{i,j}\right]$
DW	$\dfrac{BW}{\sigma_\phi}\left[\left(\nu_{rz}^{eff}\right)_{i+1,j} + \left(\nu_{rz}^{eff}\right)_{i,j}\right]$	$BW \cdot \nu$	$BW\left[\left(\nu_{r\theta}^{eff}\right)_{i-1,j} + \left(\nu_{r\theta}^{eff}\right)_{i-1,j-1} + \left(\nu_{r\theta}^{eff}\right)_{i,j-1} + \left(\nu_{r\theta}^{eff}\right)_{i,j}\right]$
DN	$\dfrac{BN}{\sigma_\phi}\left[\left(\nu_{\theta z}^{eff}\right)_{i,j+1} + \left(\nu_{rz}^{eff}\right)_{i,j}\right]$	$BN\left[\left(\nu_{r\theta}^{eff}\right)_{i,j+1} + \left(\nu_{r\theta}^{eff}\right)_{i,j} + \left(\nu_{r\theta}^{eff}\right)_{i-1,j} + \left(\nu_{r\theta}^{eff}\right)_{i-1,j+1}\right]$	$BN \cdot \nu$

TABLE III: THE DIFFUSION COEFFICIENTS (Continued)

	$\phi = k, e, v_z$	v_r	v_θ
DS	$\dfrac{BS}{\sigma_\phi}\left[\left(v_{\theta z}^{eff}\right)_{i,j-1} + \left(v_{\theta z}^{eff}\right)_{i,j}\right]$	$BS\left[\left(v_{r\theta}^{eff}\right)_{i,j-1} + \left(v_{r\theta}^{eff}\right)_{i,j} + \left(v_{r\theta}^{eff}\right)_{i-1,j} + \left(v_{r\theta}^{eff}\right)_{i-1,j-1}\right]$	$BS \cdot v$
σ	0	0	1

Diffusion Term $\quad DT = \left[DE\left(\dfrac{r_{i+1}}{r_i}\right)^\sigma + DW\left(\dfrac{r_{i-1}}{r_i}\right)^\sigma + DN + DS\right]\phi_{i,j} - DE\ \phi_{i+1,j} - DW\ \phi_{i-1,j} - DN\ \phi_{i,j+1} - DS\ \phi_{i,j-1}$

(Concluded)

TABLE IV: THE CONVECTION COEFFICIENTS

Coeff	$\phi = k,\varepsilon,v_z$	$\phi = v_r$	$\phi = v_\theta$
A_1	$\left[(r_i+r_{i+1})(r_{i+1}-r_i)(v_r)_{i+1,j} + (r_i+r_{i-1})(r_i-r_{i-1})(v_r)_{i,j}\right] \cdot \frac{\Delta\theta}{8}$	$\left\{r_i(r_i-r_{i-1})\left[(v_r)_{i+1,j}+(v_r)_{i,j}\right] + r_{i-1}(r_i-r_{i-1})\left[(v_r)_{i-1}+(v_r)_{i,j}\right]\right\} \cdot \frac{\Delta\theta}{4}$	$\left\{(r_i+r_{i+1})(r_{i+1}-r_i)\left[(v_r)_{i+1,j}+(v_r)_{i+1,j-1}\right] + (r_i+r_{i-1})(r_i-r_{i-1})\left[(v_r)_{i,j}+(v_r)_{i,j-1}\right]\right\} \cdot \frac{\Delta\theta}{16}$
A_2	$\frac{(r_{i+1}-r_{i-1})}{8}\left[(v)_{i,j}+(v)_{i+1}\right]$	$\frac{r_i-r_{i-1}}{8}\left[(v_\theta)_{i,j+1}+(v_\theta)_{i-1,j+1}+(v_\theta)_{i,j}+(v_\theta)_{i-1,j}\right]$	$r_i\frac{r_{i+1}-r_{i-1}}{16}\left[(v_\theta)_{i,j+1}+2(v_\theta)_{i,j}+(v_\theta)_{i,j-1}\right]$
CE	$\left(|A_1|-A_1\right)/(r_{i+1}-r_i)$	$\left(|A_1|-A_1\right)/(r_{i+1}-r_{i-1})$	$\left(|A_1|-A_1\right)\frac{r_{i+1}}{r_{i+1}-r_i}$
CW	$\left(|A_1|+A_1\right)/(r_i-r_{i-1})$	$\left(|A_1|+A_1\right)/(r_i-r_{i-1})$	$\left(|A_1|+A_1\right)\frac{r_{i-1}}{r_{i+1}-r_i}$
CN	$\left(|A_2|-A_2\right)$	$|A_2|-A_2$	$|A_2|-A_2$
CS	$|A_2|+A_2$	$|A_2|+A_2$	$|A_2|+A_2$
σ	0	0	1

Convection term $CT = \left[CE\left(\dfrac{r_i}{r_{i+1}}\right)^\sigma + CW\left(\dfrac{r_i}{r_{i+1}}\right)^\sigma + CN + CS\right]\phi_{i,j} - CE\,\phi_{i+1,j} - CW\,\phi_{i-1,j} - CN\,\phi_{i,j+1} - CS\,\phi_{i,j-1}$

TABLE V: SOURCE TERMS

ϕ	SU	SP
v_z	$\dfrac{4V_p}{DH}\left(\dfrac{\tau_w}{\rho}\right)_{avg}$	0
k	$V_p(\nu_{rz}^T)_{i,j}\left[\left(\dfrac{\partial v_z}{\partial r}\right)_{i,j}\right]^2+(\nu_{\theta z}^T)_{i,j}\left[\left(\dfrac{\partial v_z}{r\partial\theta}\right)_{i,j}\right]^2$	$\dfrac{C_\mu k_{i,j} V_p}{(\nu_{rz}^T)_{i,j}}$
ε	$C_1 C_\mu \dfrac{k_{i,j} V_p}{(\nu_{rz}^T)_{i,j}}\left\{(\nu_{rz}^T)_{i,j}\left[\left(\dfrac{\partial v_z}{\partial r}\right)_{i,j}\right]^2+(\nu_{\theta z}^T)_{i,j}\left[\left(\dfrac{\partial v_z}{r\partial\theta}\right)_{i,j}\right]^2\right\}$	$\dfrac{C_2 C\, k_{i,j} V_p}{(\nu_{rz}^T)_{i,j}}$
v_r	$r_{i-1}\Delta\theta\left[\overline{(v_r'^2)}_{i-1,j}+(p/\rho)_{i-1,j}\right]-r_i\left[\overline{(v_r'^2)}_{i,j}+(p/\rho)_{i,j}\right]$ $+\dfrac{(\nu_{r\theta}^{eff})_{i,j}+(\nu_{r\theta}^{eff})_{i,j+1}+(\nu_{r\theta}^{eff})+(\nu_{r\theta}^{eff})_{i-1,j}}{8}$ $\cdot(r_i+r_{i-1})\left[\dfrac{(v_\theta)_{i,j+1}}{r_i}-\dfrac{(v_\theta)_{i-1,j+1}}{r_{i-1}}\right]$ $-\dfrac{(\nu_{r\theta}^{eff})_{i,j}+(\nu_{r\theta}^{eff})_{i,j-1}+(\nu_{r\theta}^{eff})_{i-1,j-1}+(\nu_{r\theta}^{eff})_{i-1,j}}{8}$ $\cdot(r_i+r_{i-1})\left[\dfrac{(v_\theta)_{i,j}}{r_i}-\dfrac{(v_\theta)_{i-1,j}}{r_{i-1}}\right]$ $+\dfrac{2}{r_i+r_{i-1}}V_p\left[\dfrac{\overline{(v_\theta'^2)}_{i,j}+\overline{(v_\theta'^2)}_{i-1,j}}{2}\right]^2$	0

TABLE V: SOURCE TERMS (Continued)

ϕ	SU	SP
v_θ	$\dfrac{(\nu_{r\theta}^{eff})_{i+1,j}+(\nu_{r\theta}^{eff})_{i+1,j-1}+(\nu_{r\theta}^{eff})_{i,j-1}+(\nu_{r\theta}^{eff})_{i,j}}{4}\dfrac{r_i+r_{i+j}}{2}\left[(v_r)_{i+1,j}-(v_r)_{i+1,j-1}\right]$ $-\dfrac{(\nu_{r\theta}^{eff})_{i-1,j}+(\nu_{r\theta}^{eff})_{i-1,j-1}+(\nu_{r\theta}^{eff})_{i,j-1}+(\nu_{r\theta}^{eff})_{i,j}}{4}\dfrac{r_i+r_{i-1}}{2}\left[(v_r)_{i,j}-(v_r)_{i,j-1}\right]$ $+\dfrac{r_i(r_{i+1}-r_{i-1})}{2}\left[\overline{(v_\theta'^2)}_{i,j-1}+(p/\rho)_{i,j-1}-\overline{(v_\theta'^2)}_{i,j}-(p/\rho)_{i,j}\right]$	0

Source Term   ST = SU − SP ϕ_P
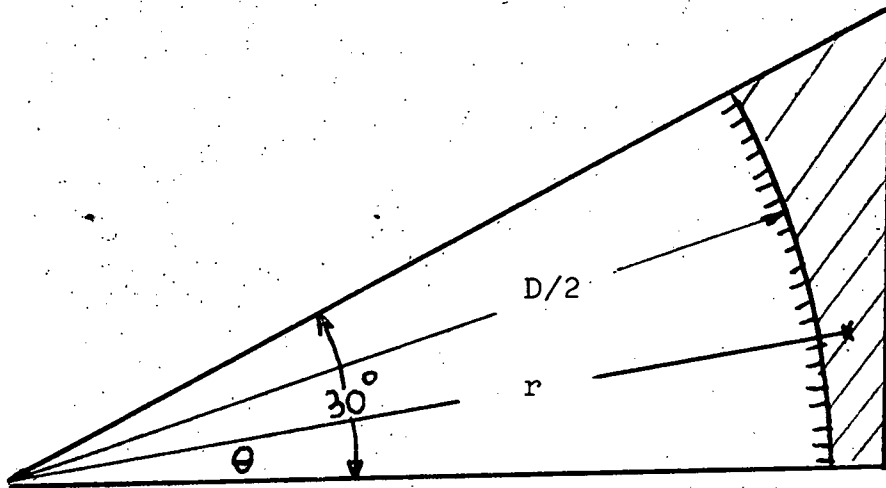
Fig. 1 Characteristic triangle for the study of
        fully developed flow in an infinite
        triangular array of bare rods
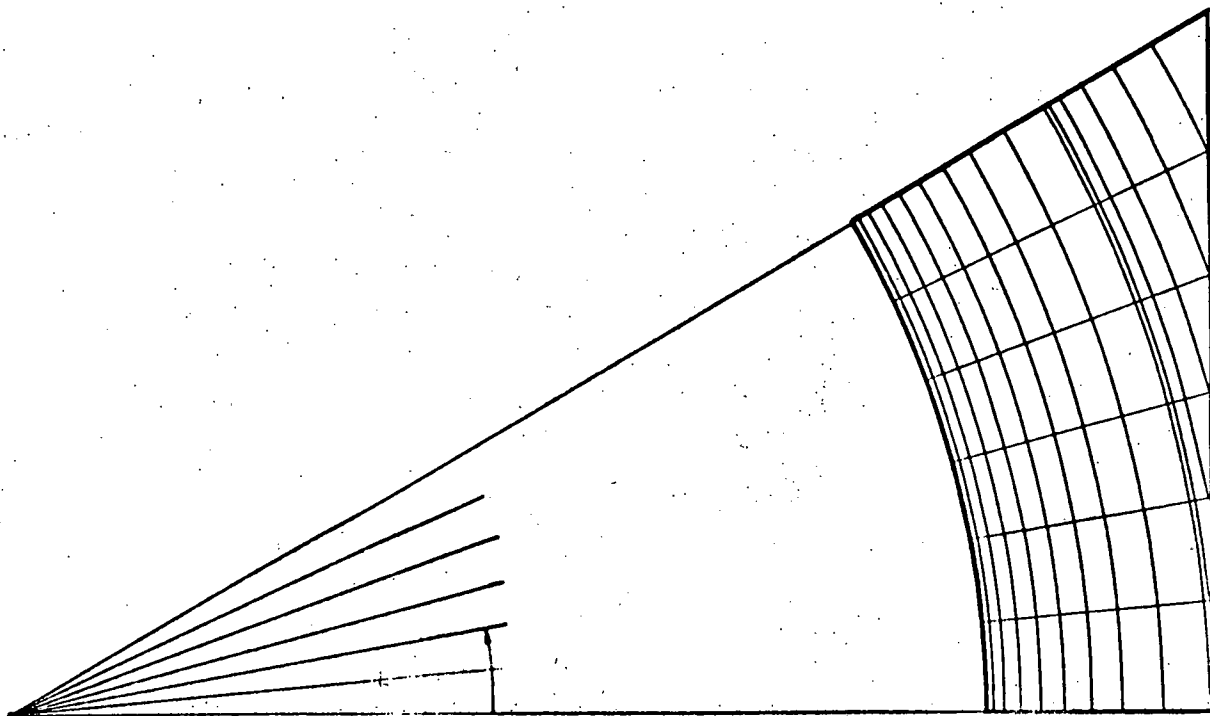
Fig. 2　　　Grid for Numerical Procedure
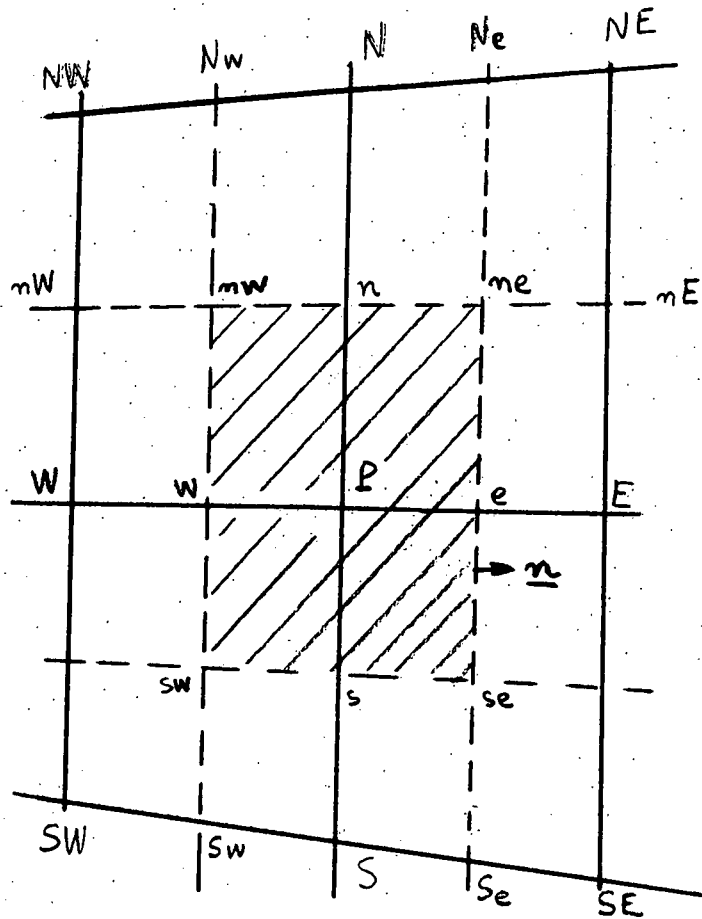
Fig. 3 Control Volume
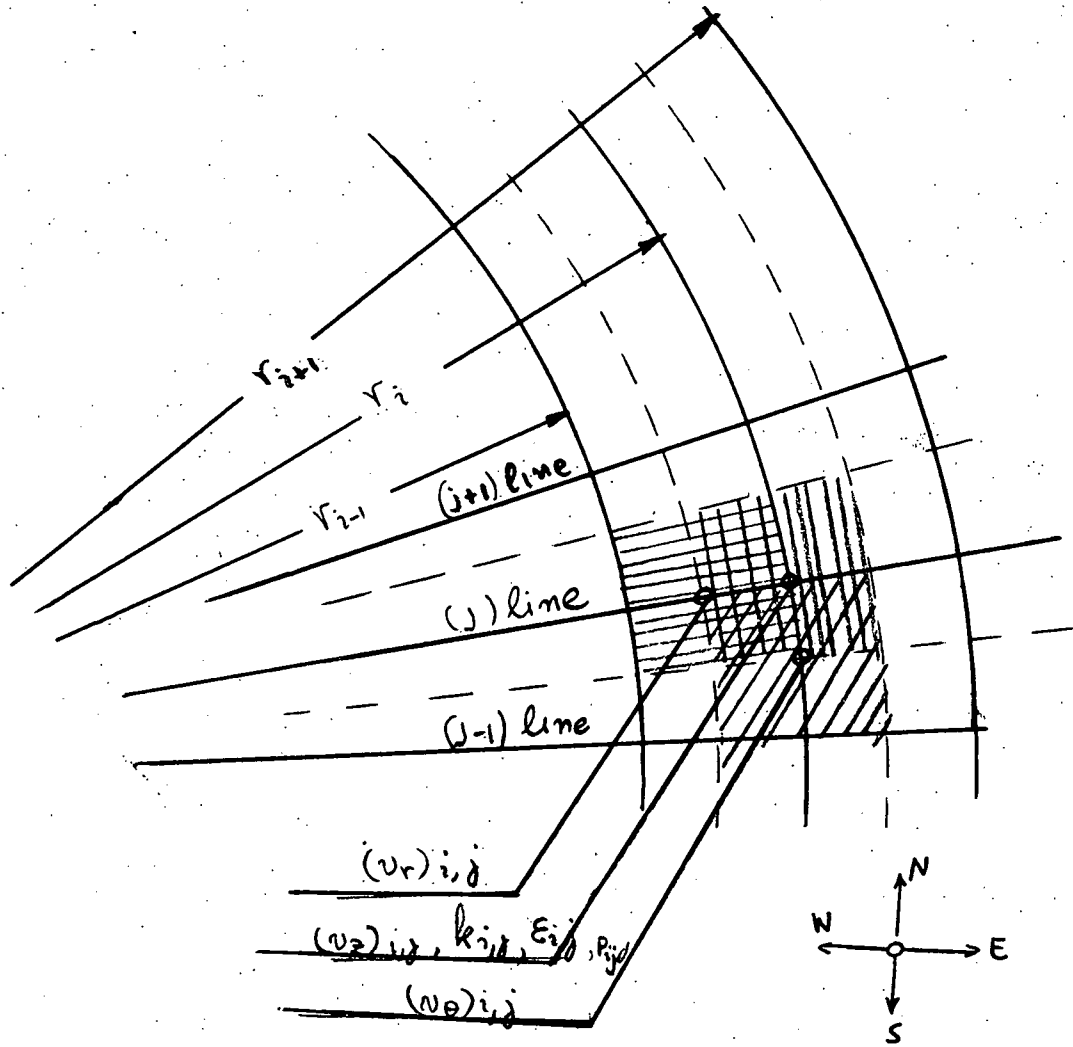
Fig. 4 Relative positions of the control volumes
at the points P=(i,j)
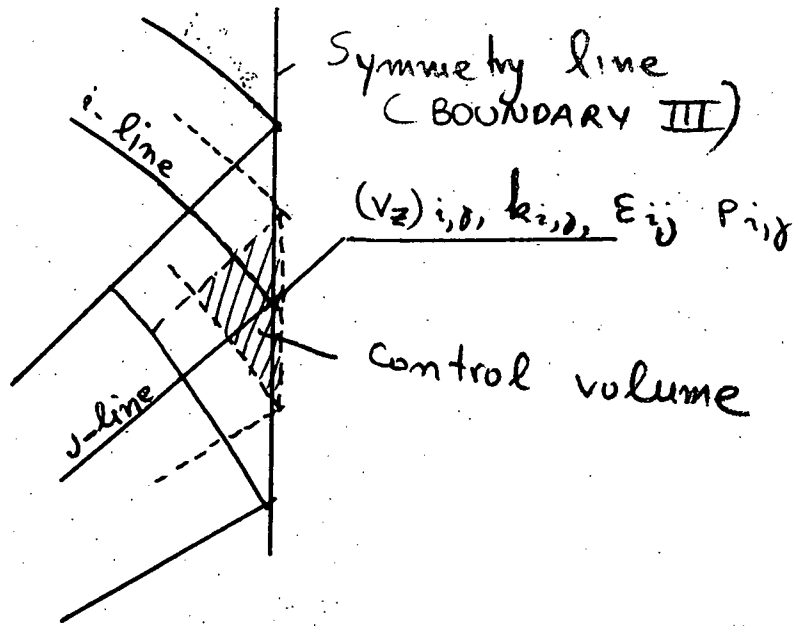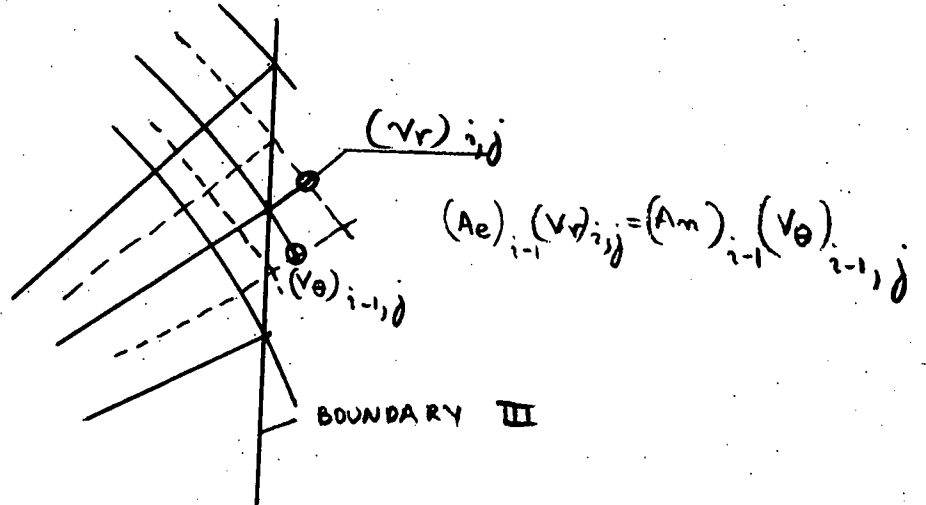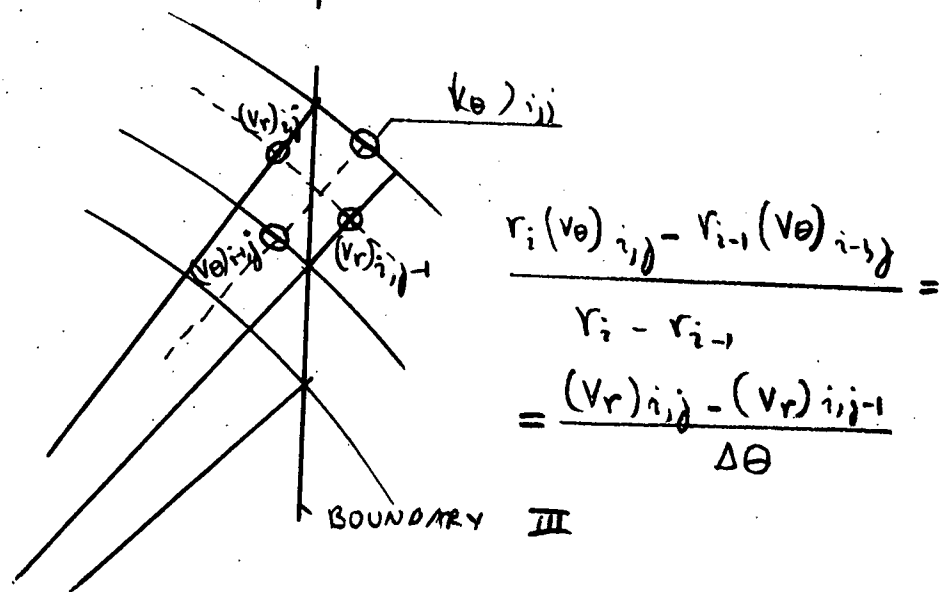
Fig. 5 Boundary Conditions in Boundary III

TASK III.   LMFBR OUTLET PLENUM FLOW MIXING

III.A. Velocity Field Analysis (Yi Ben Chen)

The planned program of steady state velocity field measurements has been completed. · The measurements ·involve obtaining steady state mean flow velocity and turbulence field mass in small-scale LMFBR outlet plenum flow cells, using water. Candidate turbulence model computer codes are used to predict the experimental results, and their success in doing so is taken as a measure of their respective validities. The results of the measurements and the validating calculations are presented and analyzed in Appendix III.A., in the paper, "Validation of Turbulence Models for LMFBR Outlet Plenum Flows" which will be presented at the 1977 National Heat Transfer Conference, and which has been submitted for publication by the Journal of Fluids Engineering.

III.B.   Combined Temperature and Velocity Measurements
(Vincent P. Manno)

The efforts of the last quarter centered on acquiring all the required components of the integrated measurement system. Currently, the acquisition procedures have been completed and all needed equipment is on hand. Among the major items purchased and received are the following: mirrors, a beamsplitter, LDA (Laser Doppler Anemometer) transmitting optical unit and LDA frequency shifter optical unit. The transmitting optics unit purchased is a model 910 assembly manufactured by Thermo-Systems, Inc. (TSI). The unit is a transducer which splits an incoming light beam (the light source being utilized is a He-Ne laser) into two beams which are focused at a particular point in the flow where the velocity is to be measured. These intersecting beams are the "probes" of the anemometer and the Doppler frequency shift caused by the flow velocity is recorded by a photodetector which is a photomultiplier tube in this case. The unit has the versatility of three different intersection focal lengths (120mm, 250mm, and 600mm) which allows greater ease in alignment procedures.

The frequency shifter purchased adds a significant feature to the velocity measurement system. Inherent in the LDA system is a directional ambiguity in the output signal of the apparatus. This is caused by the fact that flows perpindicular to the beam intersection yet moving in opposite directions will produce the same output signal. In other words, if the system is aligned to measure the vertical velocity component, it will produce the same signal whether the flow direction is up or down. The frequency shifter (TSI model 980) shifts the frequency of one of the LDA measurement beams by a specified frequency as to alleviate the directional ambiguity. Its mode of operation consists of an acousto-optic cell (Bragg cell) with an RF driver

and associated frequency downmixing electronics. The cell is mounted in front of the transmitting optics before the focusing lense which directs the beam intersection. Associated with the model 980 optical unit is the model 985 control and power unit which is already part of the equipment inventory of this task and is shared among the various sub-tasks using LDA systems. Other shared LDA equipment used in this experiment include the photomultiplier tube and power source, frequency tracker, and control unit all of which when combined with the laser source and transmitting optics comprise the complete LDA system.

At present the complete temperature measurement system is in place and operable including the photodiode. The flow circuit is also complete and operable. The experimental area's electrical system has been upgraded to provide proper power for the various electronic components and laser. The LDA transmitting system has been assembled and positioned on the optical bench. The next quarter's work will consist of installing the LDA detection and control units and aligning the system. The system alignment is a crucial step in the experiment since the velocity and temperature measurements must be made at the same point in the flow. The alignment procedure while being straightforward is time consuming and has just begun.

As was specified in the previous progress report, the development of a proper flow seeding scheme is being investigated. The flow seed is a suspended particle in the flow which provides light scattering centers which produce the Doppler frequency shift characteristic of a particular velocity. The desired qualities of the seed are the ability to follow the flow truly and the ability to significantly scatter light. The natural contamination of air is sometimes sufficient for production of good LDA data but whether that is the case in this experiment has not been ascertained. The reason for the uncertainty is that the criteria are empirical in nature; that is, one must perform some initial LDA measurements and examine the results in order to evaluate the performance of the particular seed. Since the LDA system is still not operable and the possibility of a seeding system exists, some preliminary investigations have been undertaken. The first tested was ordinary tobacco smoke injected at the air blower intake but this scheme was immediately seen to be inadequate for many reasons. First of all, the large volume flow rate (@ 30 cubic feet per minute) necessitated the presence of great amounts of smoke which seemed beyond the capacity of typical sources and secondly, the system would have called for a ventilation system-which was not available. The second seed tested was powder (actually commercial baby powder) which also had unwelcomed qualities. The most negative aspect of this particular seed was the fact that it adhered to the test section walls rendering them occluded, which interfered greatly with the optical measurement system. Also, the power was injected into the system at the air blower intake where the only natural suction of the flow circuit exists. As can be ascertained from

the flow circuit description presented in the previous progress report, the flow exits the blower and after being divided into two flow legs (one to be heated and the other to remain ambient) is directed into a heater. Therefore this seeding scheme had the unfortunate quality of depositing powder on the heater filaments and it was feared this contamination would degrade the heater's continued operation. Therefore, the powder seed concept was abandoned.

The final seeding scheme tested used plastic micro-balloons which are basically hollow plastic spheres with diameters on the order of ten microns. These spheres (product name is Phenolic Microballons) have the attractive quality of very little adherence to the test cell walls. Also, their chemical composition would be less detrimental to the heater's operation. The only negative feature is that they burst at elevated temperatures (@400F). This characteristic could cause substantial attrition in seed density in the flow leg that passes through the heater. Two possible solutions are being investigated. One possible answer would be to create a suction in the circuit after the heater by using a "Venturi" or reduced flow area section and the other avenue of investigation will be to actually measure the attrition rate through the heater in order to determine whether the depletion can be compensated for by overseeding the flow. Finally, concurrent with these tests an air filtering system was installed and successfully tested which consists of directing the two exiting jets from the test section into a crude air filter which utilizes a standard ventilation system filter. The goal of the next quarter in regard to the seeding problem is to establish whether a seeding system is actually needed and to install a workable one if called for.

Therefore, the experimental efforts of the next quarter will be directed toward aligning and testing the complete measurement system. After testing the system the actual measurements will be made using the 1/15 scale FFTF outlet plenum test section described in the previous progress report. The output of the measurement system is the product of the DC analog voltages of the temperature and velocity systems. Represented mathematically, the output arise as:

$$\text{output} = \int_0^\tau (\overline{u} + u')(\overline{T} + T') \, d\tau \qquad (1)$$

where $\overline{U}$ = mean velocity, U' = velocity, fluctuation about the mean

$\overline{T}$ = mean temperature, T' = temperature fluctuations about the mean, and

τ = resolution time of system

The integral can be evaluated with the result

$$\int_0^\tau (\overline{u} + u') (\overline{T} + T') \, d\tau = <\overline{u}\ \overline{T}> + <u'\overline{T}> + <\overline{u}\ T'> + <u'T'> = (\overline{u \cdot T}) + <u'T'>$$

(2)

since the second and third terms both equal to zero-by definition- and the first term is merely the product of the two time averaged values of T and U. Now we note that the eddy diffusivity of heat is defined in the following manner:

$$\varepsilon_H \equiv \frac{<u'T'> + <u'T'>}{\left(\frac{\partial \overline{T}}{\partial y} + \frac{\partial \overline{T}}{\partial x}\right)}$$

(3)

Therefore the quantity <u'T'> which is the desired value can be easily computed from the system's output signal by subtracting the product of the time averaged temperature and velocity DC analog voltages from it. Mathematically, that is (using Eqs (1) and (2)) the result is obtained

$$<u'T'> = \int_0^\tau (\overline{u} + u') (\overline{T} + T') \, d\tau - (\overline{u \cdot T})$$

(4)

Hence, the final measurement scheme must include evaluating the time averaged values of the temperature and velocity outputs using their respective measurement systems. These measurements will commence once the system is completely assembled and tested.

APPENDIX III.A

VALIDATION OF TURBULENCE MODELS FOR

LMFBR OUTLET PLENUM FLOWS

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ABSTRACT

Small scale experiments involving water flows are used to provide mean flow and turbulence field data for LMFBR outlet plenum flows. Measurements are performed at Reynolds number (Re) values of 33000 and 70000 in a 1/15 - scale FFTF geometry and at Re = 35000 in a 3/80-scale CRBR geometry. The experimental behavior is predicted using two different two-equation turbulence model computer programs, TEACH-T and VARR-II. It is found that the qualitative nature of the flow field within the plenum depends strongly upon the distribution of the mean inlet flow field, importantly also upon the degree of inlet turbulence, and also upon the turbulent momentum exchange model used in the calculations. It is found in the FFTF geometry that the TEACH-T predictions agree well with the experiments, while the agreement of the VARR-II prediction is poorer, and in the CRBR geometry neither code provides a good prediction of the observed behavior.

In the Liquid Metal-cooled Fast Breeder Reactor (LMFBR), sodium
is used as coolant to remove heat from the reactor core. Since
sodium is a good heat transfer medium and has low heat capacity,
the temperature difference between inlet and outlet of the core is
much higher than that of the current design light water reactor
(LWR). A typical value for this temperature rise is approximately
300 °F. During a reactor scram with (or without) flow coastdown,
the cold sodium will issue from the reactor core, and will mix with
hot sodium which had previously filled the reactor outlet plenum.
Predicting the transient thermal response of the sodium in the
outlet plenum is an important problem, since this thermal regime
will dictate the thermal fatigue environment for the outlet nozzles,
instrument trees, and other mechanical components which will be
exposed to the reactor coolant flow. Two-equation turbulence model
calculations are being used to predict the thermal histories which
such components will experience. Thus, it is necessary, in order to
insure the adequacy of the design of these components, that the
design numerical simulations be both accurate and economical.
Experimental verification of the accuracy of the resulting predic-
tions is an essential component of that insurance.

The detailed analytical treatment of the coolant mixing in the
outlet plenum is very difficult due to the complex nature of the
resulting turbulent flow. The Navier-Stokes equation cannot be
solved numerically because of small scale of turbulence, and due
to the limited storage capacity and speed of existing computers.

Therefore, most of the problems in the turbulent flow are solved by using the time-averaged Navier-Stokes equations (or Reynolds equation). Due to the nonlinearity of the Navier-Stokes equations, one additional term, known as Reynolds stress, appears in the Reynolds equation. Much of the attention has been concentrated on how to model this parameter in terms of known quantities. This is the so-called turbulence model approach. In general the LMFBR outlet plenum will display a recirculating flow pattern. The simplest model (e.g., Prandtl's mixing length theory) has been found to be inadequate in providing accurate predictions of this behavior.[1] Therefore current design work has adopted the use of the two-equation turbulence model. This decision is based on the need for accuracy and for reasonably short computation times.

In our work two computer codes, namely TEACH-T[2] and VARR-II[3] are used. TEACH-T is a steady-state two-dimensional code which adopts turbulent kinetic energy and turbulent energy dissipation as two additional dependent variables. VARR-II is a time-dependent two-dimensional thermal hydraulic code. Different from TEACH-T, it solves turbulent kinetic energy and turbulent viscosity conservation equations in differential form. In addition, a heat conservation equation is also incorporated into the code for temperature prediction, and in order to provide buoyant force feedback to the vertical momentum equation. (See Appendix A for a more complete discussion of the two codes). The code TEACH-T has been modified to calculate flows in the reactor plenum geometry, and has been used to generate a prediction of the flows which would be observed in the validation experiments of this work. The code VARR-II is currently in use as a design tool in

the U.S. LMFBR demonstration reactor (Clinch River) program.
Its predictions have not been verified experimentally in the
flow geometry of interest. It is also being used in this work
to predict the flow in the experimental test cell. Thus, by
intercomparison of the results from these two competing turbulence
model codes and the observed experimental data, one may identify
sources of error in the predictions and make appropriate improve-
ments in the turbulence models for the outlet plenum application.

In the experiments, small-scale Cartesian geometry test models
of a diametrical section of the prototypic outlet plenum geometries
are used. The experiments consist of measurements of appropriate
turbulence model parameters in steady-state water flows. These
data are then compared to predictions of the behavior of the
experiment by each of the two-equation turbulence model codes.

A variable geometry outlet plenum test cell which stimulates
two reactor cases, namely those of the Fast Flux Test Facility
(FFTF) and the Clinch River Breeder Reactor Project (CRBRP), has
been fabricated, and is shown in Fig. 1. Measurements of velocity,
turbulent kinetic energy, and Reynolds stresses in the two perpendi-
cular directions are obtained simultaneously by use of a DISA Mark-II
two-channel Laser Doppler Anemometer[4] operating in the reference
beam mode as shown in Fig. 2. Both velocity component signals have
40 MHz preshift devices and frequency down mixing to remove the
ambiguity regarding the flow direction in low speed measurements.
A 2-watt Spectra Physics Argon Laser is the light source.

The results obtained have shown significant discrepancies in the

agreements between the measurements and the code predictions
in a spectrum of cases. The experimental errors are typically
20 percent for the Reynolds stress, 5 percent for the turbulence
kinetic energy, and 2 percent for the mean velocity data. In
all of the TEACH-T calculations the mean velocity field and the
turbulence kinetic energy field at the plenum inlet are set equal
to the values measured in the case of interest. In all of the
VARR-II calculations the inlet mean velocity distributions are
set equal to the measured values; however, the turbulence kinetic
energy and Reynolds stress values are calculated internally in the
program. In two cases (Figs. 11 and 22) a VARR-II data option
is used which permits matching the turbulence kinetic energy at
a single mesh point to the measured value.

The measured and predicted velocity field data for the cases
investigated are shown in Figs. 3 through 18 for the 1/15 scale
FFTF geometry and in Figs. 19 through 26 in the 3/80-scale CRBR
geometry. In each of the Figures the data are normalized in terms
of the maximum velocity value in the field. The range of cases
examined is summarized in Table 1. It is seen consistently in
examining the data that both the measured and predicted flow
fields depend strongly upon both the inlet flow conditions (velocity
distribution and turbulence level) and the turbulent momentum
exchange in the flow. The results obtained are discussed in order
of ascending Figure number.

The first set of data applies to the FFTF geometry at a
Reynolds number (Re) value of 33000 (defined with respect to the
inlet duct width). As with the data obtained in other geometries

and at higher Re values it is seen that both codes predict velocity fields with inlet jets which persist spatially with greater momentum than are observed experimentally. Similarly the predicted vortex centers are significantly displaced from the observed vortex location and the predicted mean velocity values near the cell boundaries are significantly greater than those observed experimentally, while the opposite comparison is observed in the low flow speed region near the vortex center. The agreement with the experiment of the TEACH-T prediction is generally superior to that of the VARR-II prediction. This is because the former calculations are performed by matching the inlet velocity and turbulence kinetic energy values to those observed experimentally as is done in all of the TEACH-T calculations. It is seen in Fig. 6 and 7 that the turbulence parameter fields predicted by TEACH-T agree reasonably well at most positions with those observed experimentally (i. e., within the same order of magnitude), and those predicted by VARR-II are generally lower than the measured values by as much as an order of magnitude, typically. It is seen that the Reynolds stress agreement is poorer for both codes than is that for the turbulence kinetic energy, since in neither calculation are the inlet Reynolds stress values matched to the experimental values. The generally better agreement of the TEACH-T prediction reflects the value of specifying accurately the inlet velocity, and turbulence kinetic energy. Although it has not been tested it is expected that further improved agreement would be obtained by matching also the inlet Reynolds stress values in the calculations to those observed

experimentally.

In Figs. 3 through 13 data for the FFTF geometry at Re=70,000 are shown. It is seen that the velocity field comparison between TEACH-T and the experiment is excellent throughout the flow field, reflecting the value of matching inlet flow and turbulence conditions, and reflecting the expected improvement in turbulence model predictions at higher Re values. As in the case at Re = 33,000 the VARR-II calculation (Fig. 10) tends to predict too - little turbulent momentum exchange, with the resulting values of inlet jet and near-wall flow rates being higher than those observed experimentally by approximately 20 percent, and with the overall vortex shape being distorted from that observed experimentally. In Fig. 11 a calculation is presented which is identical to that in Fig. 10 except that the VARR-II data option of matching the turbulent kinetic energy at one inlet point is exercised. It is seen that the mean flow fields predicted in the two cases are virtually identical, and that the predicted turbulence fields (see Figs. 12 and 13) are affected by this turbulence data matching only in the inlet flow region. Thus, a more complete prescription of the inlet turbulence state is seen to be required in order to provide accurate turbulent momentum exchange predictions throughout the entire flow field. Even with the limited inlet flow turbulence kinetic energy matching it is seen throughout most of the flow field that the VARR-II predicted Reynolds stress and turbulence kinetic energy are lower than the experimental values by approximately an order of magnitude.

The qualitative dependence of the observed and predicted flow fields upon the inlet mean velocity distribution is shown in a startling fashion in Figs. 14 through 18. In this case in the FFTF geometry the inlet mass flow rate is maintained for Re = 70,000; however, a partial inlet flow blockage is used to provide a velocity distribution which has a maximum near the outside edge of the inlet duct (See Fig. 14). This results in a mean flow map which is qualitatively different from that observed in the previous cases. The main flow is observed to pass through the lower portion of the plenum, with an upper region being occupied by a counter-rotating secondary flow. This is due to the high rate of mean flow shear above the inlet orifice with rapid entrainment of fluid in the plenum, accompanied by strong degredation of the inlet jet momentum. It is notable that both of the codes are able to predict the qualitative features of this flow, although neither code is successful in describing the experimentally observed flow field in detail, particularly in regard to the location and rate of circulation of the secondary flow vortex.

As with the previous cases the TEACH-T prediction of the turbulence parameters is much more successful than that of VARR-II, reflecting the more accurate inlet turbulence specification in the former case. The importance of this case lies in demonstrating that the qualitative nature of the mean flow field is strongly dependent upon the inlet mean velocity field. The implications for design calculations of the need for an accurate knowledge of these inlet conditions is clearly evident.

Data similar to those obtained in the 1/15-scale FFTF test cell have also been obtained for steady state water flows in a 3/30-scale Clinch River Breeder Reactor (CRBR) outlet plenum geometry (see Fig. 19). In this geometry the inlet orifice (i.e., the region above the reactor core) is much wider, and penetrates to a much greater height into the plenum, so that relative to the inlet orifice width the outlet plenum is much shorter and narrower than in the FFTF case. A mass flowrate equal to that at a Re value of 70,000 in the FFTF geometry results in a Re value of 35,000 in the CRBR geometry.

The CRBR data with the "normal" inlet mean flow distribution are shown in Figs. 19 through 23. Effectively in this geometry the inlet jet impacts against the upper plenum boundary in a much shorter distance than in the FFTF case and then rebounds to the outlet duct due to high upper plenum stagnation pressures, while in the FFTF geometry the mean flow is able to follow streamlines from the entrance to the exit in a fashion as would be encountered in a laminar flow.

This greater mean flow chaos is seen in the comparison of the measured mean flow field (Fig. 19) to the TEACH-T (Fig. 20) and VARR-II (Figs. 21 and 22) predictions. It is seen in the lower half of the plenum that the measurements and code predictions agree reasonably well, with the TEACH-T prediction agreeing excellently and with the VARR-II prediction being approximately 20% too high in the high velocity regions. In the upper half of the plenum it is seen that neither of the codes is able to predict the flow field qualitatively. As in previous cases the

VAPR-II prediction is insensitive to the matching of the tur-
bulence kinetic energy to the experiment at a single inlet point,
and the predicted turbulence kinetic energy field is typically
too low by approximately an order of magnitude throughout most
of the flow field (see Fig. 22).

In Figs. 24 through 26 CRBR flow data for Re = 35,000 are
shown for the case with a partial inlet flow blockage, resulting
in a distorted inlet mean flow distribution (see Fig. 24). The
striking feature is that neither code is able to predict the
qualitative features of the mean flow anywhere except in the
neighborhood of the inlet and outlet ducts. In a situation such
as this one the highly chaotic mean flow distribution (with little
net mass flow) in the upper plenum region is translated by the
codes into a stagnant, or alternatively into a recirculating flow
region. Thus, the net flow into and out of the plenum is pre-
dicted correctly, but the flow pattern within the plenum is
described very inaccurately.

Conclusions

From the FFTF cases having normal inlet flow conditions it is seen that excellent agreement is obtained between the TEACH-T predictions and the measurements. The inferior quality of the VARR-II predictions is attributable to the inability to match the inlet turbulence conditions to those in the experiment, as well as to possible errors in the turbulence model. The more important conclusions arise from the inability of either program to predict the qualitative flow behavior in either geometry when the distorted inlet conditions prevail. This indicates that the turbulence model used to describe turbulent momentum transfer within the flow may be less important than an accurate knowledge of the detailed inlet mean flow and turbulence field. It also implies that codes such as those used in this study should be used with great caution in which abrupt changes in shape situations arise (i.e., CRBR flows) or in which strong mean flow gradients are imposed. Within these limitations codes of the type used in this work can provide predictions which are useful for design purposes, although further work is required to determine the degree of accuracy of turbulent momentum exchange within a simple geometry well-specified flow.

Acknowledgement

References

1. Rodi, W., and Spalding, D.B.,"A Two-Parameter Model of Turbulence and its Application to Free Jets,"Warme-und Stoffübertra gung, Vol. 3, 1970, pp. 85-95.

2. Notes on "Turbulent Recirculating Flow-prediction and Measurement," College of Engineering, The Pennsylvania State University. July 28-August 1, 1975.

3. Cook, J. L., P.I. Nakayama, "VARR-II. A Computer Program for Calculating Time Dependent Turbulent Fluid Flows with Slight Density Variation," WARD-D-0106. July 1975.

4. The DISA Mark II Laser Doppler Anemometer, DISA Electronics, Franklin Lakes, N. J., 1974.

5. Harlow, F. H. and Welch, J.E., "Numerical Calculation of Time-Dependent Viscous Incompressible Flow," Phys. Fluids, Vol. 8, 1965, pp. 182.

6. Welch, J.E., Harlow, F.H., Shannon, J. P., and Daly, B.J., "The MAC Method. A Computing Technique for Solving Viscous, Incompressible, Transient Fluid Flow Problems Involving Free Surfaces, LA - 3425, 1966, Los Alamos Scientific Laboratory.

TABLE 1

Geometry	Reynolds Number	Inlet Velocity Distribution	Figure Number					
			Measurement	TEACH-T Prediction	VARR-II Prediction No Turbulence Match	VARR-II Prediction With Turbulence Match	Reynolds Stress Comparison	Turbulence Kinetic Energy Comparison
FFTF	33,000	Normal	3	4	5	---	6	7
FFTF	70,000	Normal	8	9	10	11	12	13
FFTF	70,000	Distorted	14	15	16	---	17	18
CRBR	35,000	Normal	19	20	21	22	---	23
CRBR	35,000	Distorted	24	25	26	---	---	---

CRBRP

FFTF

Chimney

Outlet

Outlet

Grid

Inlet
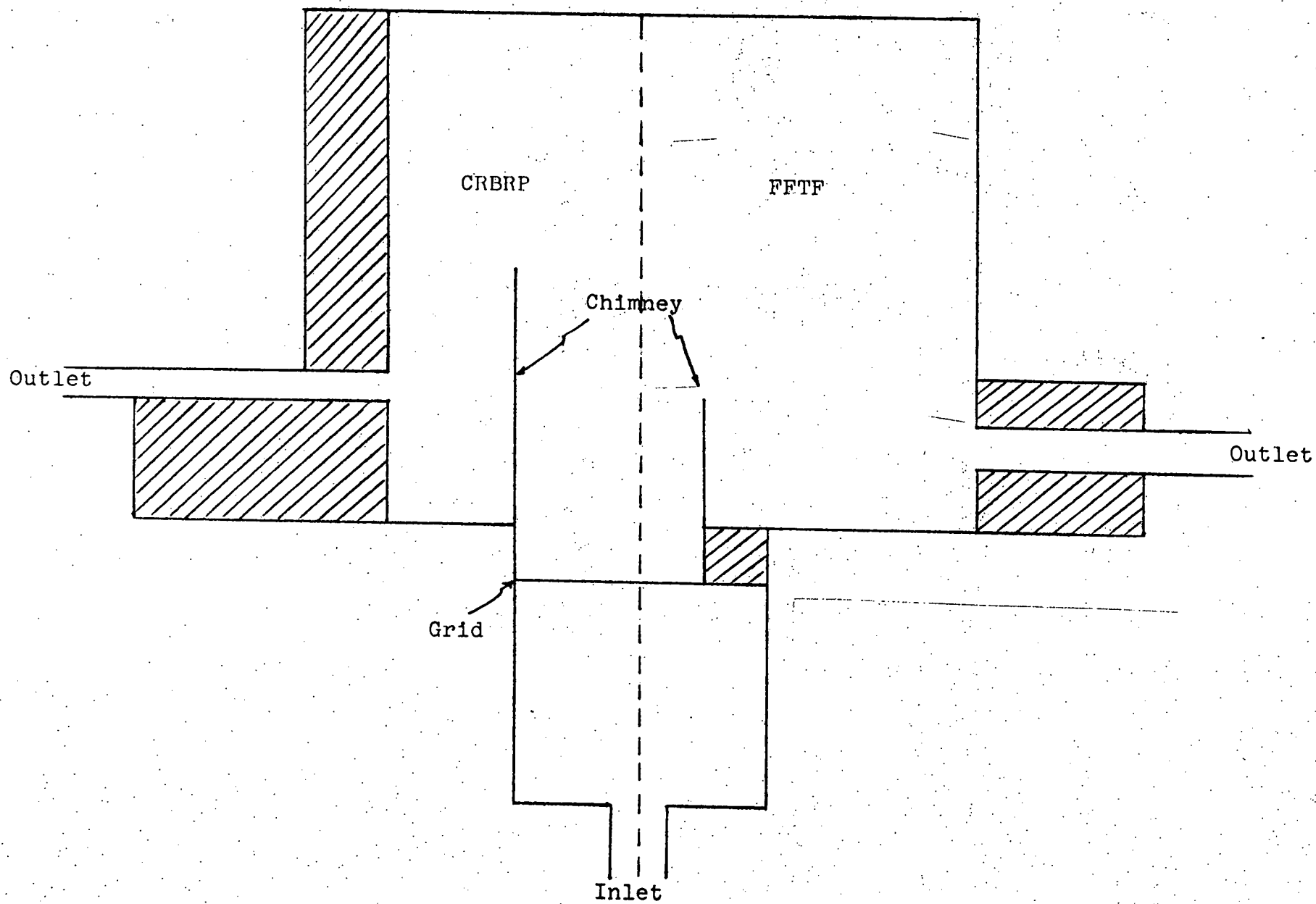
Fig. 1. Variable Geometry Outlet Plenum Test Cell
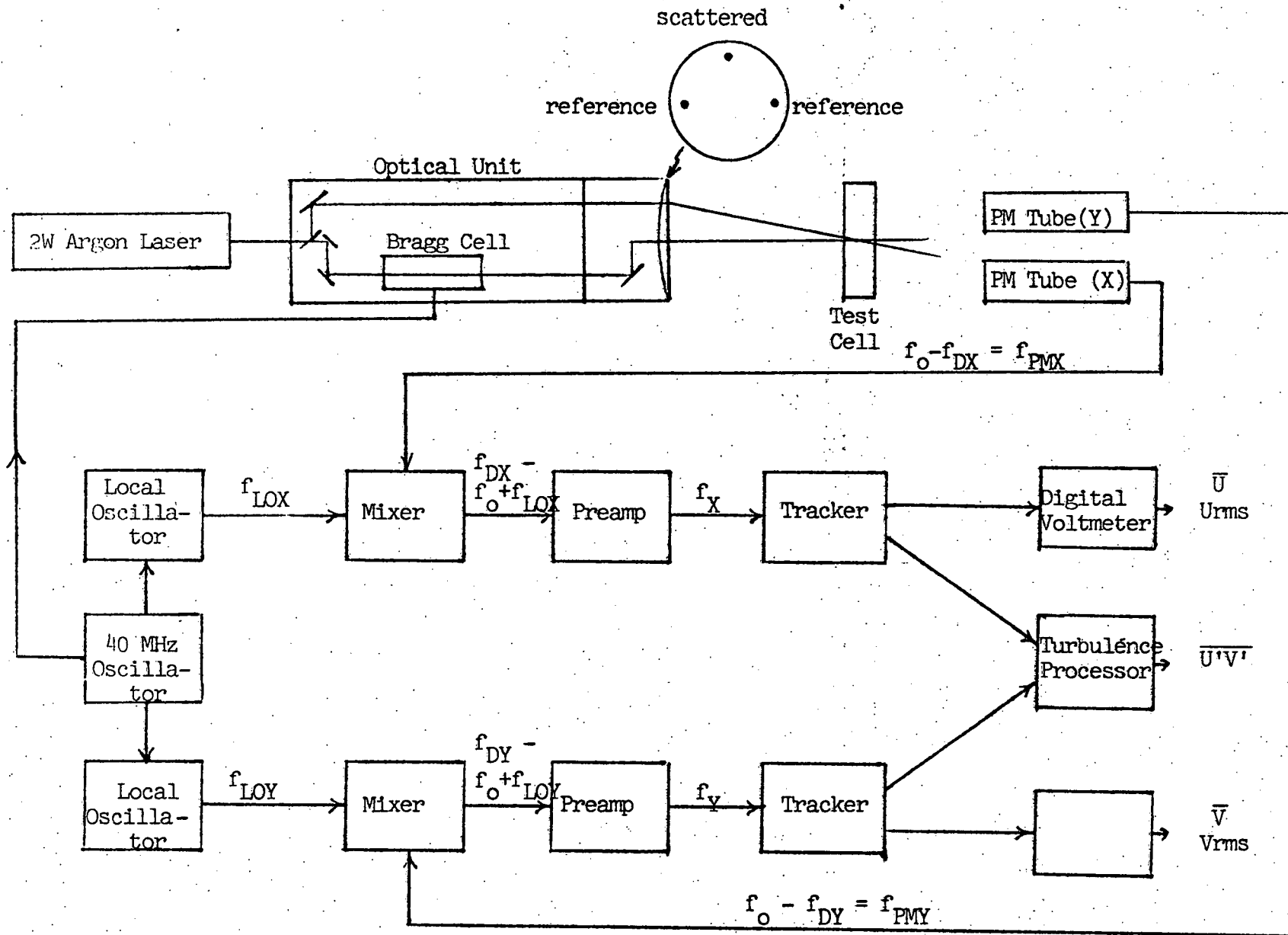
Fig.2. Layout of Instrument and Optical Unit
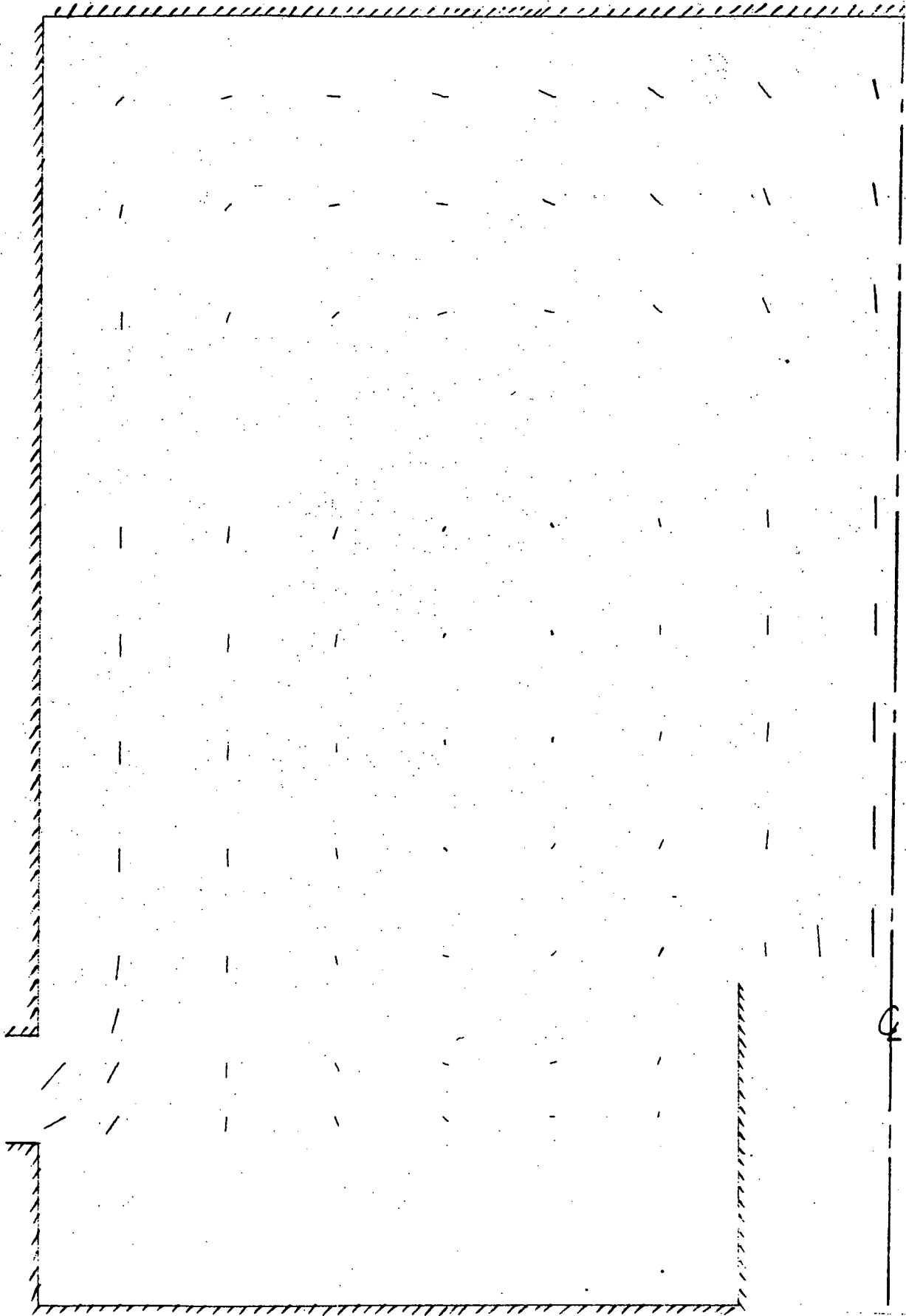
Fig. 3 Measured Mean Flow Field, FFTF Geometry, Re = 33000
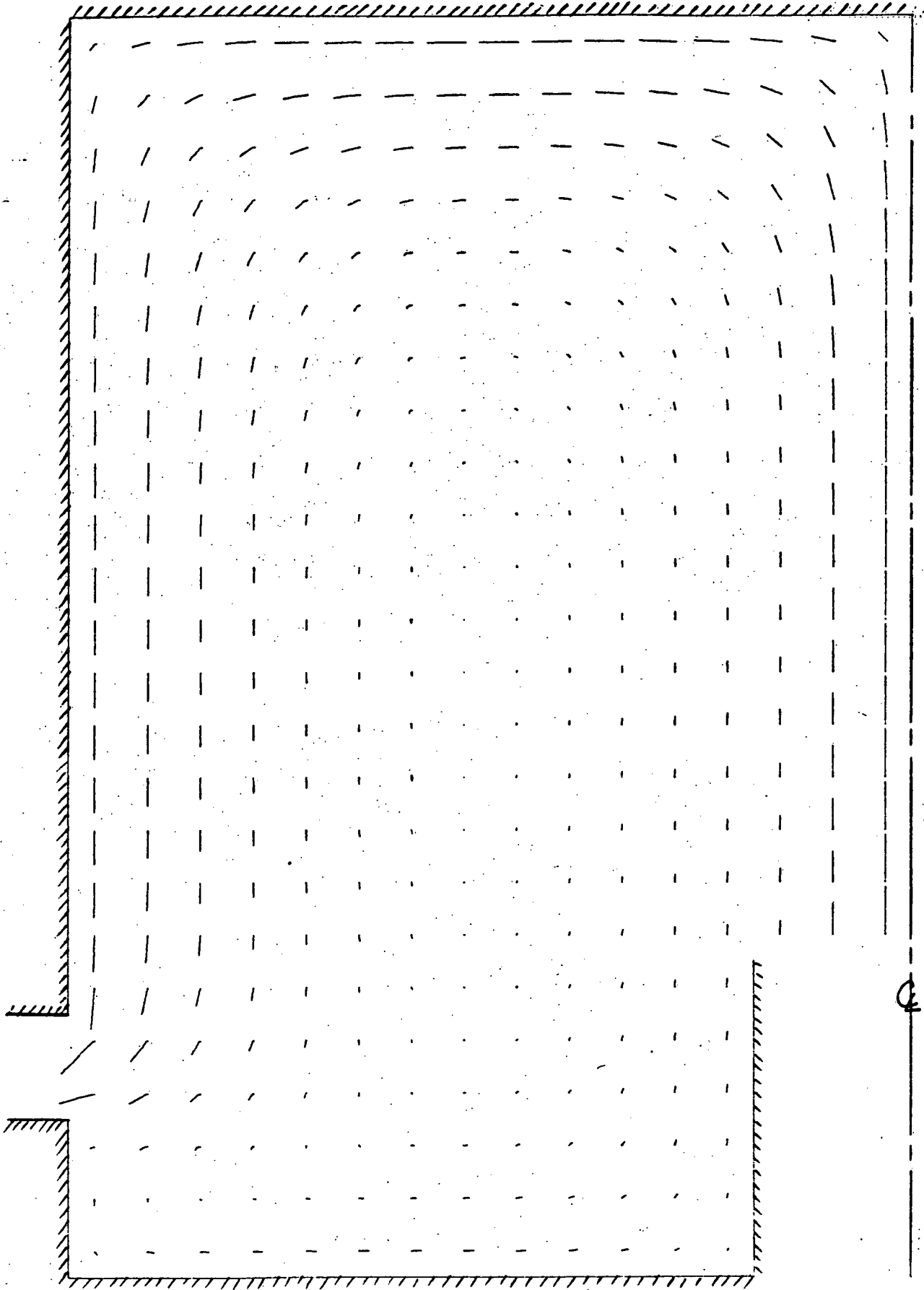
Fig. 4 TEACH-T Prediction, FFTF Geometry, Re = 33000
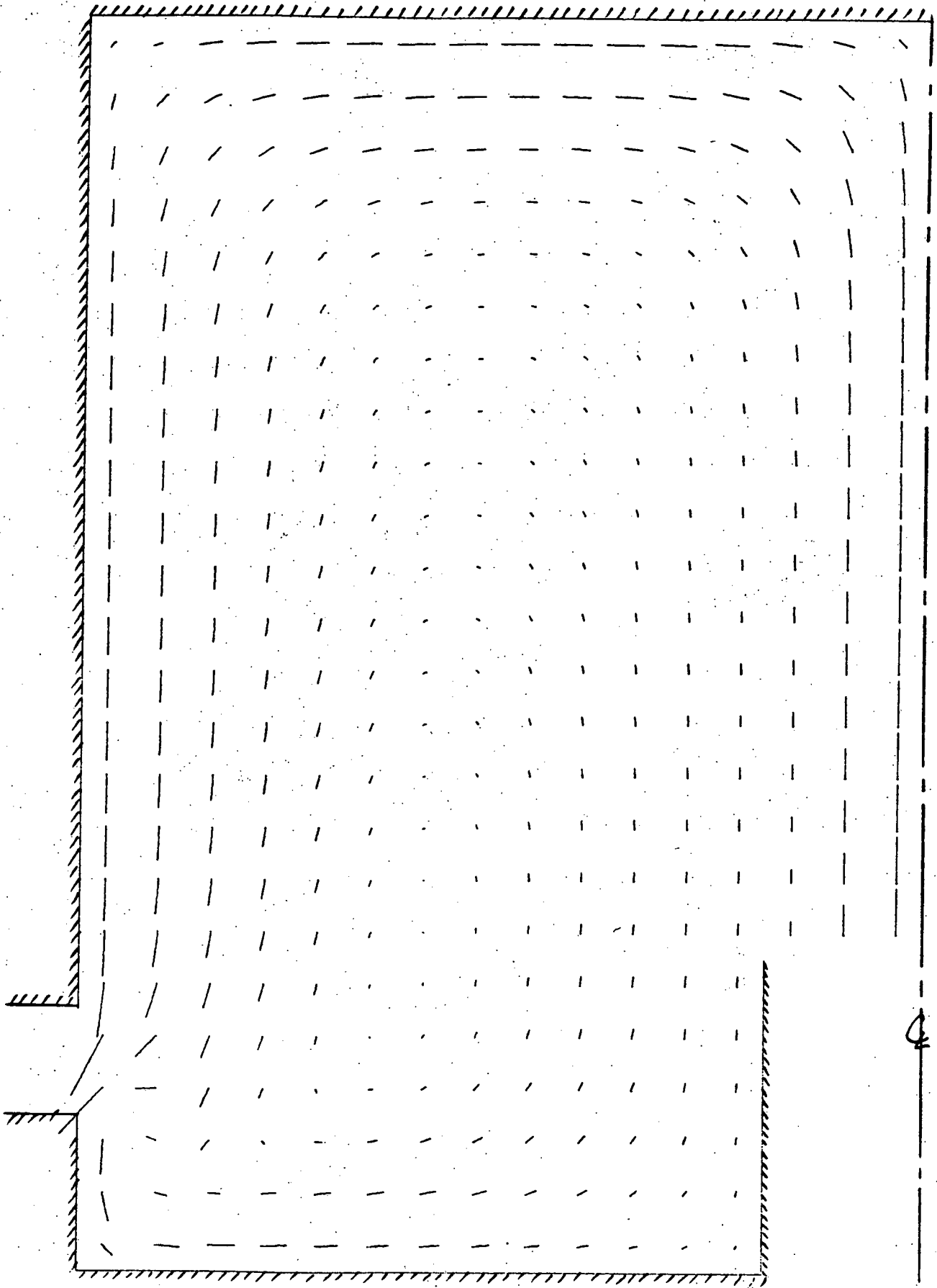
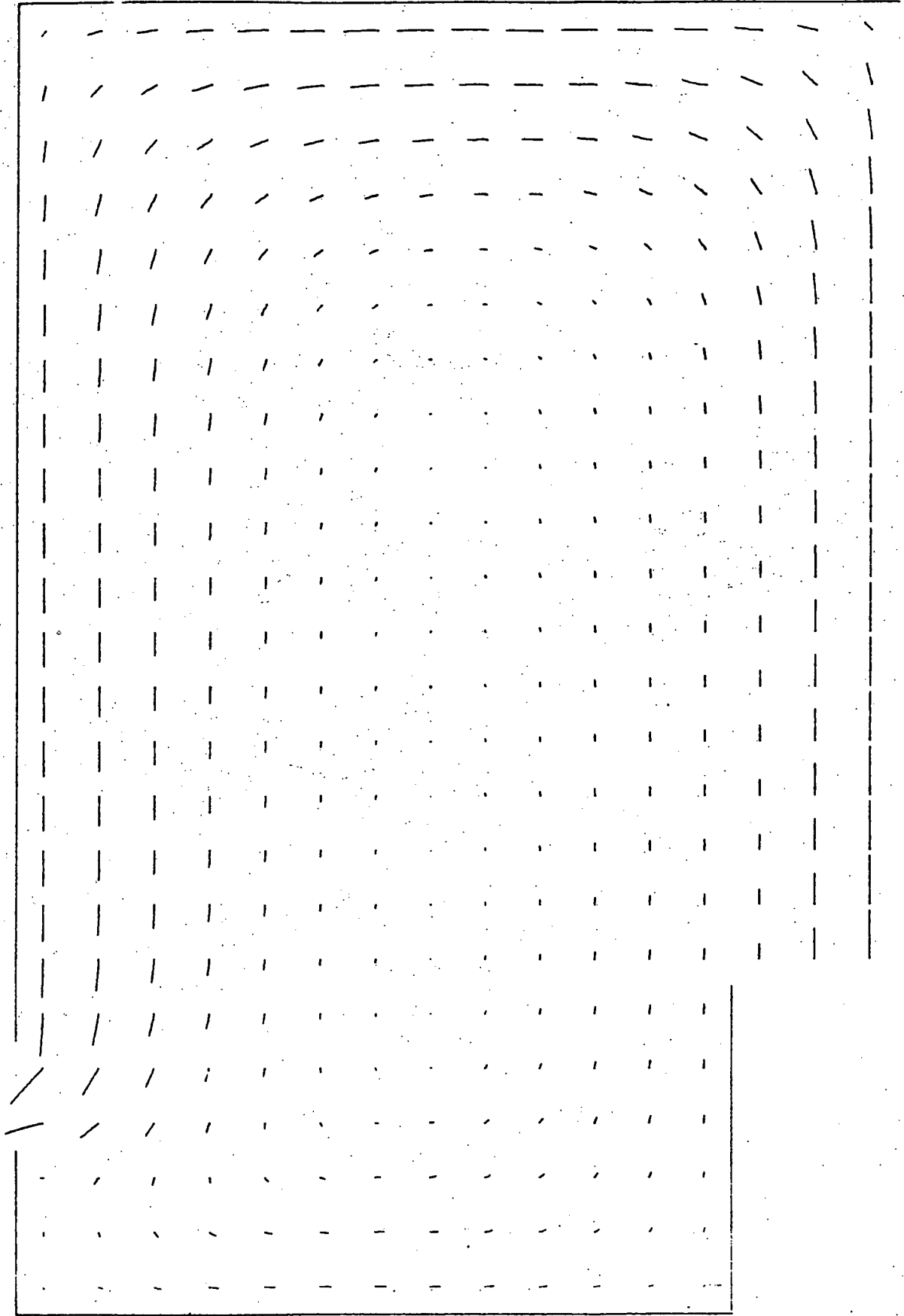Fig. 5 VARR-II Prediction, FFTF Geometry, Re = 33000

Fig. 6 Compared Calculated and Measured Turbulence Kinetic Energy Fields, FFTR Geometry, Re = 33000

.096 .083	.028	.029	.034	.038	.035		.018	.011	.0047
[.089] [.058]	.0081	.0078	.008	.0085	.0097		.0040	.0035	.0065
[.175] [.286]									
[.081] [.044]	.037	.046	.033	.028	.033		.047	.045	.048
.027 .020	.017	.019	.024	.030	.035		.026	.018	.0096
.056 .027	.0061	.0053	.0050	.0047	.0052		.0027	.0026	.0050
.051 .052	.035	.044	.045	.044	.036		.047	.049	.043
.015 .010	.0096	.012	.018	.026	.033		.028	.021	.013
.030 .018	.0094	.0077	.0063	.0046	.0052		.0026	.0024	.0040
.047 .049	.046	.043	.050	.047	.055		.055	.055	.044
.013 .0091	.0080	.0084	.017	.027	.035		.026	.021	.014
.023 .017	.012	.011	.0087	.0053	.0055		.0024	.0025	.004
.043 .046	.046	.039	.044	.052	.060		.063	.065	.063
.014 .012	.0096	.011	.022	.034	.040		.021	.017	.012
.023 .020	.016	.015	.010	.0060	.0061		.0020	.0025	.0030
.041 .042	.046	.035	.037	.039	.059		.098	.093	.087
.013 .013	.018	.025	.039	.048	.049		.013	.012	.0090
.030 .027	.024	.020	.010	.0062	.0063		.0017	.0024	.0030
.039 .036	.037	.030	.035	.043	.064		.110	.120	.094

		.0084	.072	.064	.045		.082	.0085	.0063
.104		.011	.0069	.0076	.0016		.0018	.0026	.0035
.119		.113	.089	.095	.102		.110	.117	.091
.034									
.106		.040	.022	.032	.011		.0062	.0065	.0042
.015		.0023	.0019	.0033	.0011		.0020	.0081	.0044
.085		.080	.086	.073	.031		.110	.105	.085

← TEACH
← VARR
← Exp. results

Fig. 7   Compared Calculated and Measured Reynolds Stress Fields, FFTP
Geometry, Re = 33000

Row 1:
202.   -255.   -99.8   -27.8   -11.1   -11.8   -19.6   -9.73   4.38   -1.36
428.   -418.   -11.70   -3.38   4.53   4.40   2.35   5.65   2.88   .99
-28.6   -63.6   -28.6   -21.0   -38.2   -25.4   -25.4   -31.8   3.18   -22.3

Row 2:
4.72   -42.3   -45.0   -25.8   -14.4   -21.8   -46.9   -56.6   -35.8   -18.4
15.76   -78.50   -28.73   -8.85   -.75   -5.01   -11.46   -2.80   -3.57   -3.93
-24.2   -28.6   -47.1   -33.7   -53.4   -38.1   -25.4   -49.5   -25.4   +9.54

Row 3:
-2.99   -14.4   -19.8   -15.3   -10.4   -26.2   -58.2   -74.4   -48.9   -23.1
7.32   -22.57   -30.10   -8.38   -1.20   -9.53   -18.59   -9.91   -9.44   -6.87
-17.8   -18.7   -28.8   -22.3   -24.8   -25.4   -25.4   -25.4   ~0   +12.7

Row 4:
.104   -8.92   -18.8   -8.03   -5.56   -28.8   -65.4   -73.0   -44.6   -24.3
14.42   -.17   -11.82   9.43   1.97   -12.73   -21.34   -11.31   -10.42   -7.15
-14.6   -3.82   -5.72   -20.4   -28.6   -29.6   -50.9   -40.8   -25.4   +6.36

Row 5:
-5.49   -12.1   -14.7   -1.51   -5.88   -34.4   -66.2   -48.3   -24.2   +2.1
+6.69   -1.62   +10.05   +23.68   -3.98   -13.71   -19.96   -7.66   -9.21   -6.20
-7.63   -18.4   -15.3   -11.4   -17.8   -15.9   -19.1   -95.4   -70.0   ~0

Row 6:
-19.2   -26.4   -17.1   -18.0   -28.0   -36.6   -37.3   -6.05   -6.02   -8.41
-12.8   -34.0   -2.22   -14.88   -11.04   -7.68   -8.40   -2.95   -5.85   -4.64
-7.00   -12.1   -19.81   -10.2   -7.63   -25.4   -38.2   -70.0   -38.2   ~0

Row 7:
67.4   97.4   101.   57.8   8.43   2.64   -1.70
+6.12   14.5   13.0   5.60   -.33   -2.54   -2.65
-42.6   -14.0   -146.   -6.36   38.2   +28.6   12.7

Legend:
← TEACH)
← VARR   ×10⁻¹
← EXP. Results

Row 8:
37.2   29.5   26.3   8.91   3.67   1.67   -.003
-.020   -3.93   -9.22   -.13   .22   -.17   -.35
-26.7   17.8   76.3   50.9   57.2   +6.36   6.36
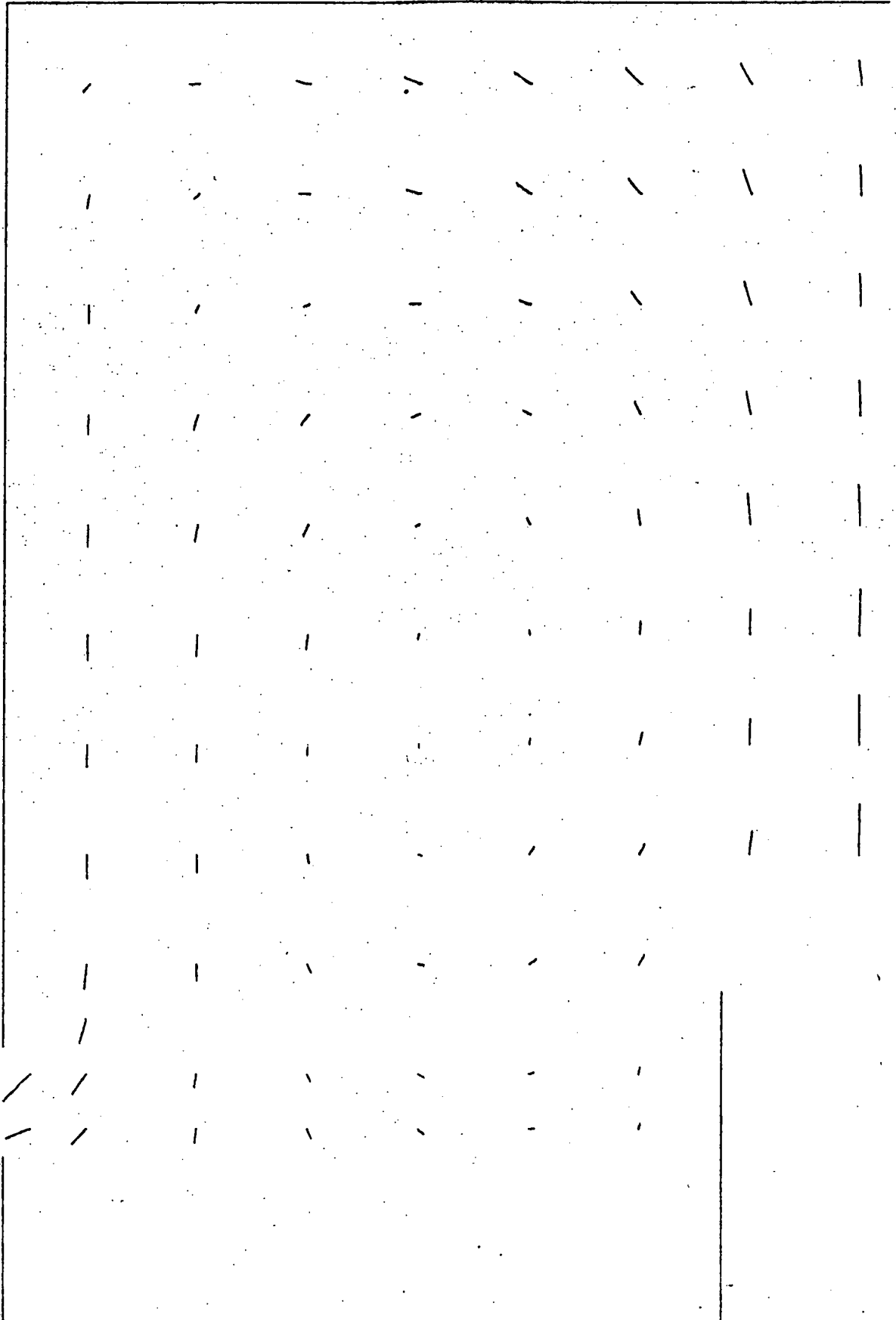
Fig. 8 Measured Mean Flow Field, FFTF Geometry, Re = 70000
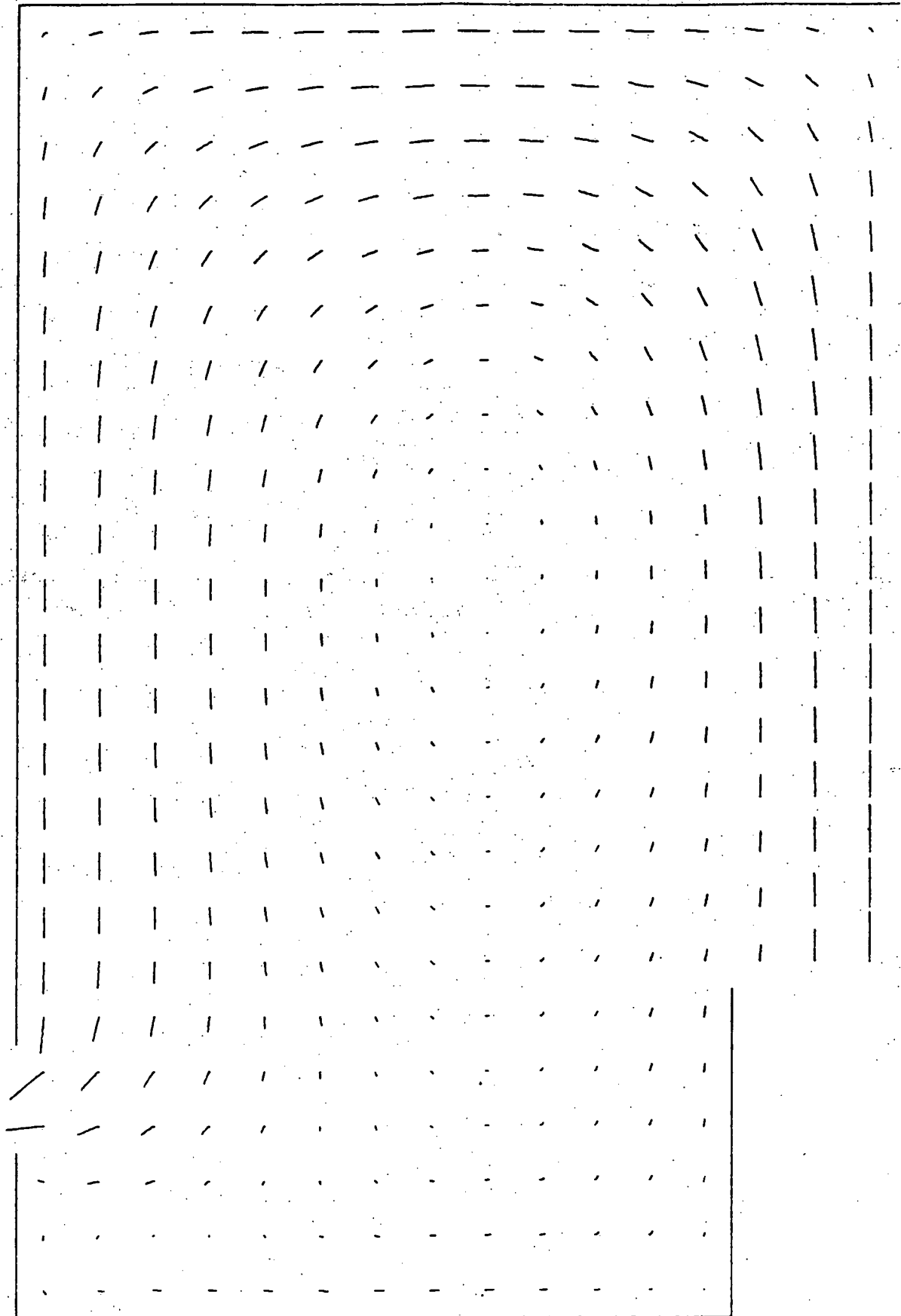
Fig. 9   TEACH-T Prediction, FFTF Geometry, Re = 70,000
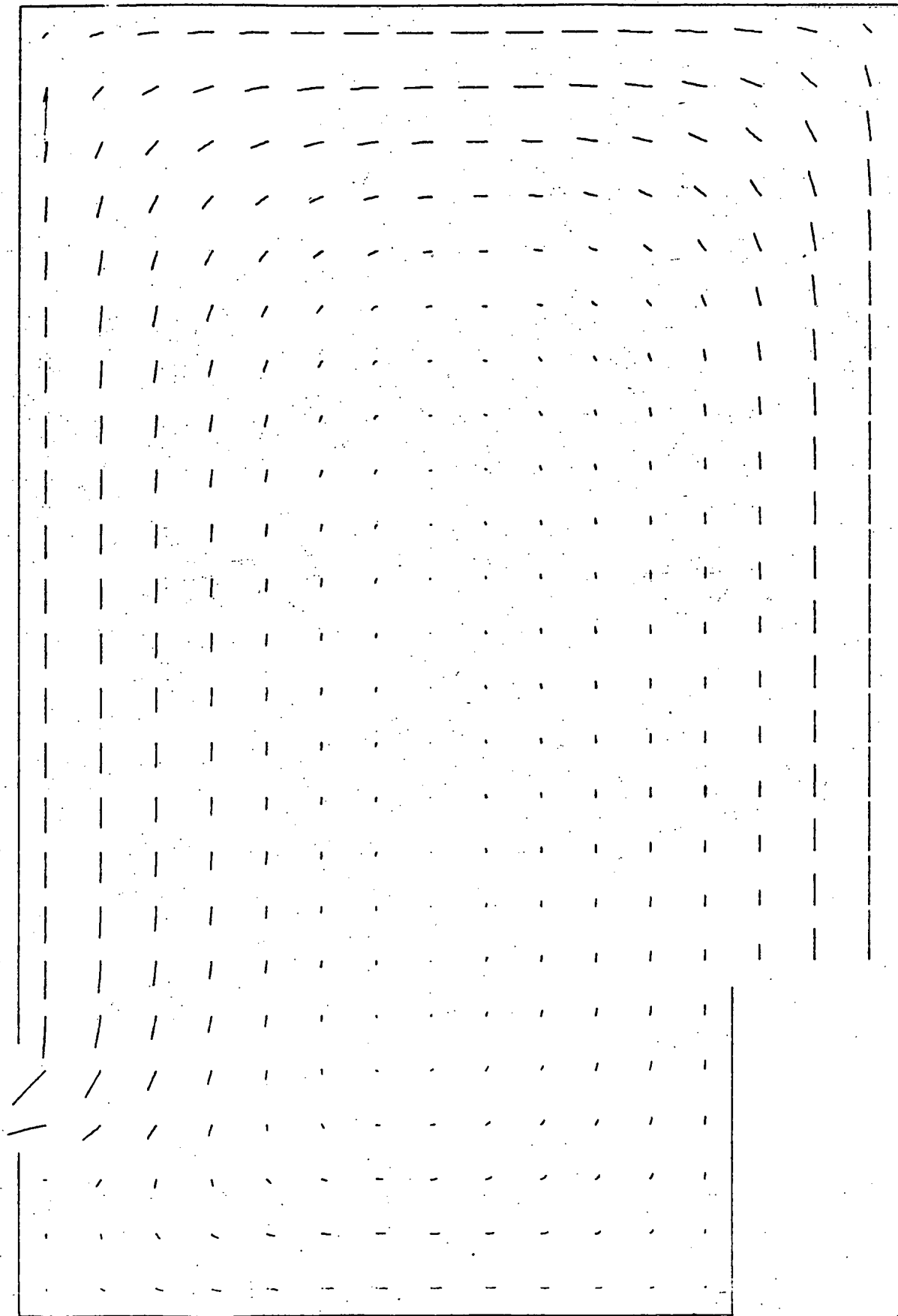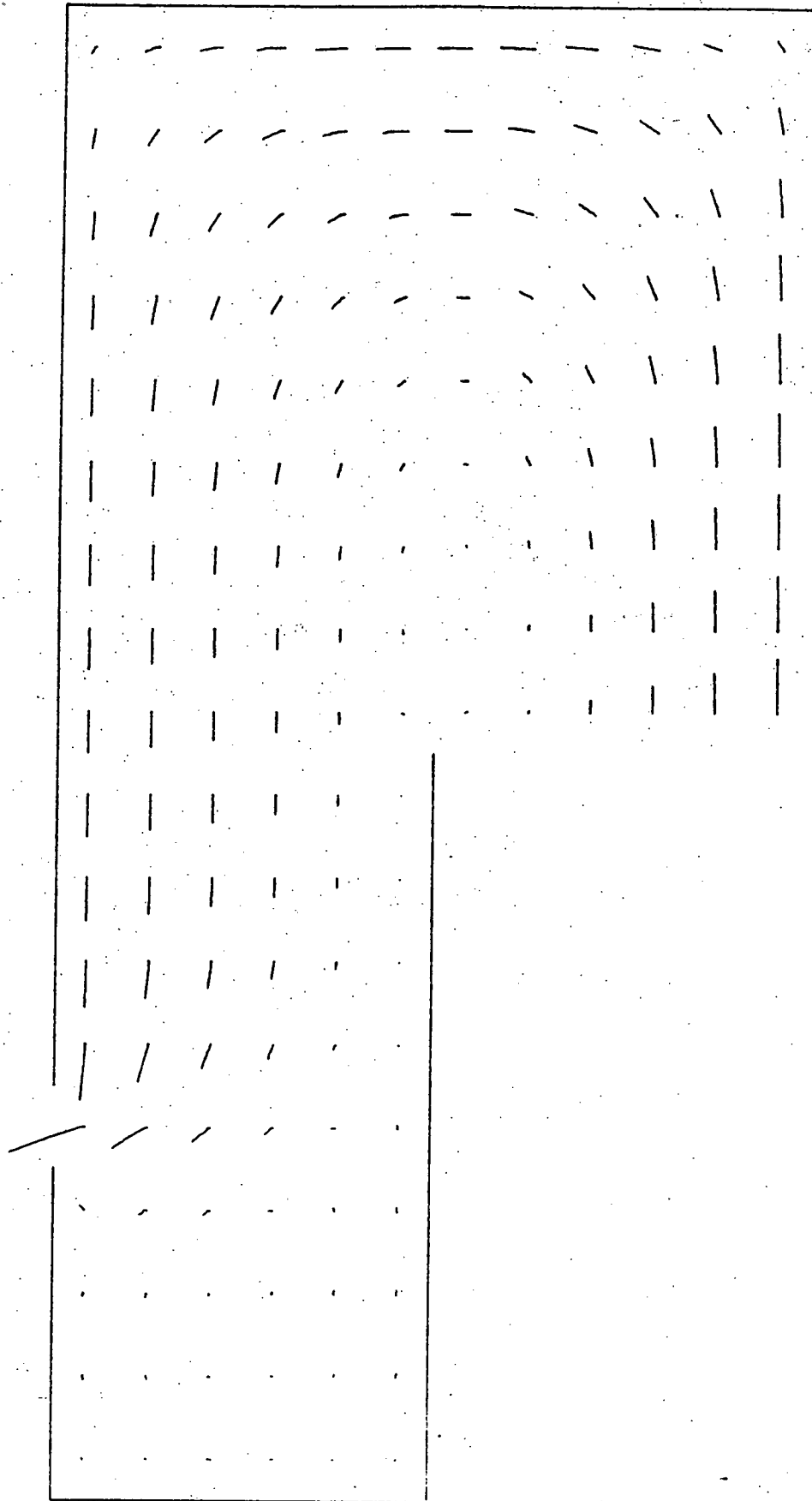
Fig. 10   VARR-II Prediction, FFTF Geometry, Re = 70000

Fig. 11 VARR-II Prediction, FFTF Geometry, Re=70000 Turbulent Kinetic Energy
Matched at Inlet Centerline

Fig. 12 Compared Calculated and Measured Turbulence Kinetic Energy Fields,
Re = 70000

VARR-MODIFIED
VARR
DATA

Fig. 13 Compared Calculated and Measured Reynolds Stress Fields, FFTF Geometry, Re = 70000

VARR-MODIFIED
VARR
DATA
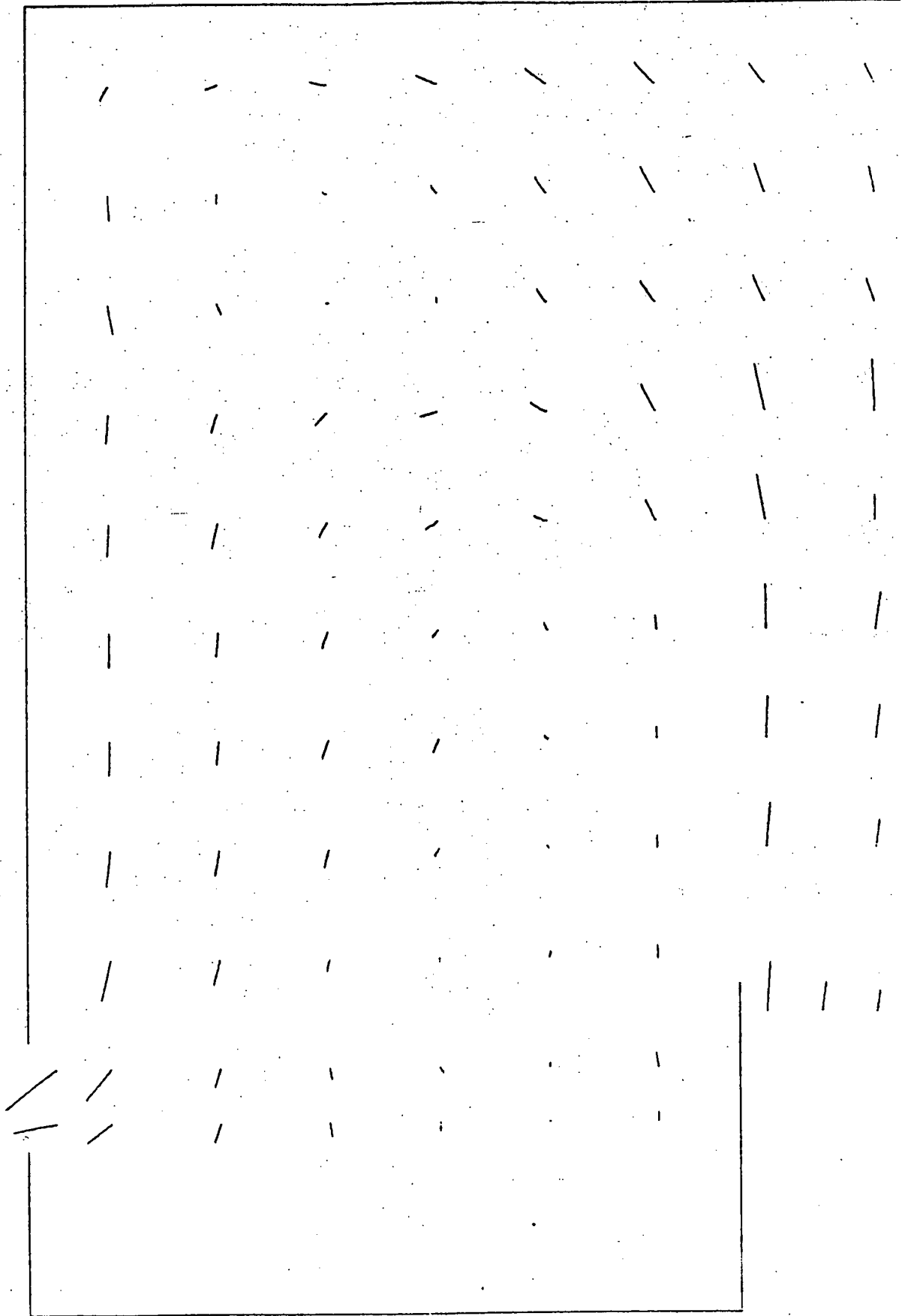
Fig. 14 Measured Mean Flow Field, FFTF Geometry, Re = 70000 Distorted Inlet
Mean Flow Distribution
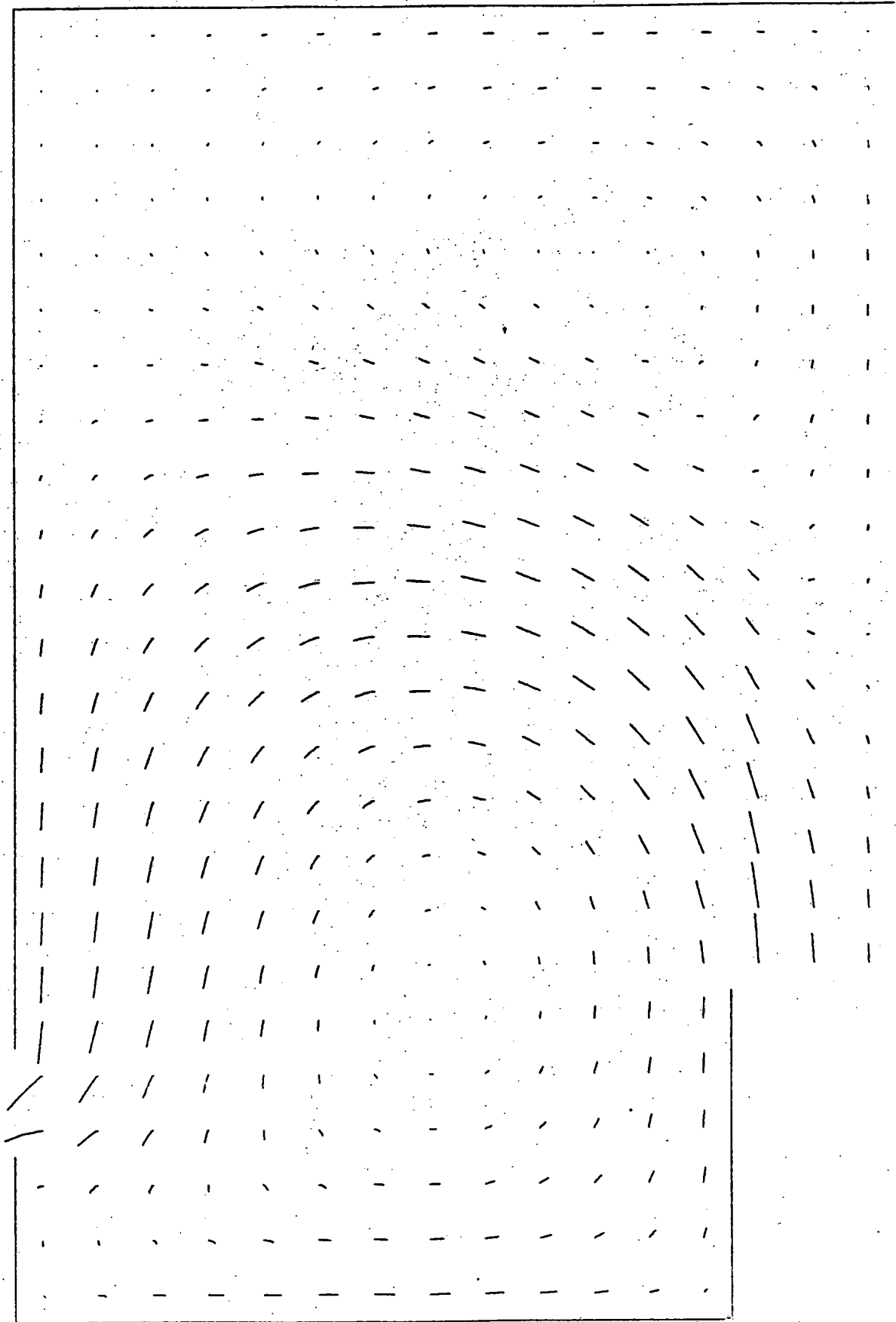
Fig. 15   TEACH - T Prediction, FFTF Geometry, Re = 70,000 Distaled Inlet
Mean Flow Distribution
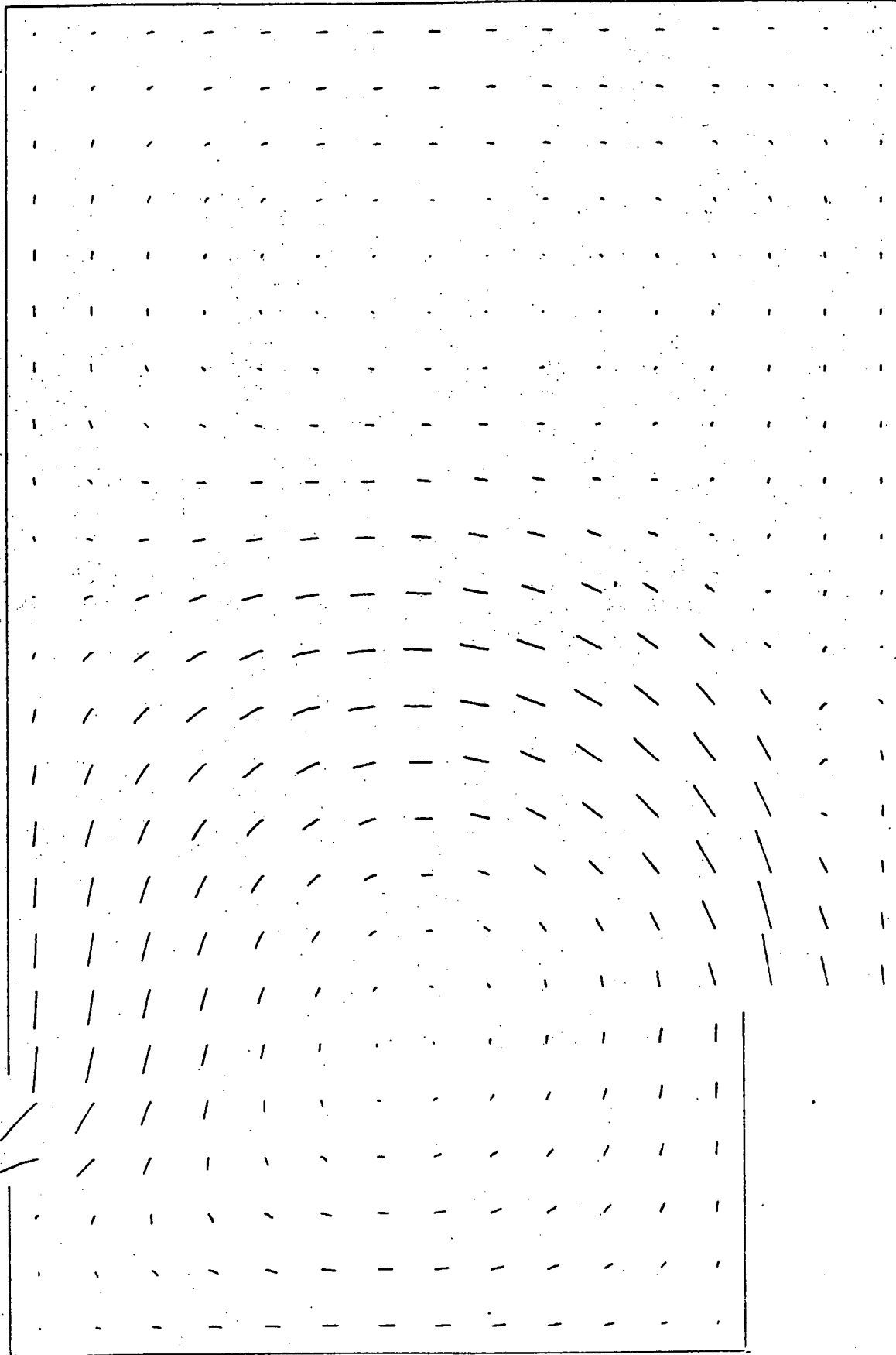
Fig. 16 VARR-II Prediction, FFTF Geometry, Re=70000 Distorted Inlet Mean
         Flow Distribution

Fig. 18 Compared Calculated and Measured Reynolds Stress Fields, FFTF Geometry, Re = 70000 Distorted Inlet Mean Flow Distribution

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← TEACH
← VARR   × 10⁻¹
← EXP. Results

(Each grid location lists three values: TEACH / VARR / EXP. Results)

Row 1:
202. / 428. / -28.0   -255. / -418. / -63.6   -99.8 / -11.70 / -28.6   -27.8 / -3.38 / -21.0   -11.1 / 4.58 / -38.2   -11.8 / 4.40 / -25.4   -19.6 / 2.35 / -25.4   -4.73 / 5.65 / -31.8   -4.34 / 2.88 / 2.18   -1.36 / .99 / +22.3

Row 2:
4.72 / 15.76 / -24.2   -42.3 / -78.50 / -28.6   -45.0 / -28.73 / -47.1   -25.8 / -8.85 / -33.7   -14.9 / -.75 / -53.4   -21.8 / -5.01 / -38.1   -44.9 / -11.46 / -25.4   -56.6 / -2.80 / -44.5   -35.8 / -3.57 / -25.4   -13.4 / -3.93 / +9.54

Row 3:
-2.99 / 7.32 / -17.8   -14.4 / -22.57 / -12.7   -19.8 / -30.10 / -24.8   -15.3 / -8.38 / -22.3   -10.4 / -1.20 / -24.8   -26.2 / -9.53 / -25.4   -58.2 / -18.59 / -25.4   -74.4 / -9.91 / -25.4   -48.9 / -9.24 / ~0   -23.1 / -6.87 / +12.7

Row 4:
.104 / 14.42 / -14.6   -8.92 / -.17 / -3.82   -18.8 / -11.82 / -5.72   -8.03 / 9.43 / -20.4   -5.50 / 1.97 / -28.6   -28.8 / -12.73 / -28.6   -65.4 / -21.34 / -50.9   -73.0 / -11.31 / -45.8   -44.6 / -10.92 / -25.4   -24.3 / -7.15 / +6.36

Row 5:
-5.40 / +6.69 / -7.63   -12.1 / -1.62 / -18.4   -14.7 / +10.05 / -15.3   -1.51 / +23.68 / -11.4   -5.88 / -3.88 / -17.8   -34.4 / -13.71 / -15.9   -66.2 / -19.96 / -19.1   -45.3 / -7.66 / -95.4   -24.2 / -9.21 / -70.0   -17.1 / -6.20 / ~0

Row 6:
-19.2 / -12.0 / -7.00   -26.4 / -34.0 / -12.1   -17.1 / -2.22 / -19.1   -18.0 / -14.88 / -10.2   -28.0 / -11.04 / -7.63   -36.6 / -7.68 / -25.4   -37.3 / -8.40 / -38.2   -6.05 / -2.95 / -70.0   -6.02 / -5.85 / -38.2   -8.41 / -4.64 / ~0

Row 7:
67.4 / +6.12 / -42.6   97.4 / 14.5 / -14.0   104 / 13.0 / -146.   57.8 / 5.60 / -6.36   8.43 / -.33 / 38.2   2.64 / -2.54 / +28.6   -1.70 / -2.65 / 12.7

Row 8:
32.2 / -.020 / -26.7   29.5 / -3.93 / 17.8   26.3 / -9.22 / 76.3   8.91 / -.13 / 50.9   3.67 / .22 / 57.2   1.67 / -.17 / +6.36   .003 / -.35 / 6.36

Fig. 19 Measured Mean Flow Field, CRBR Geometry, Re = 35000
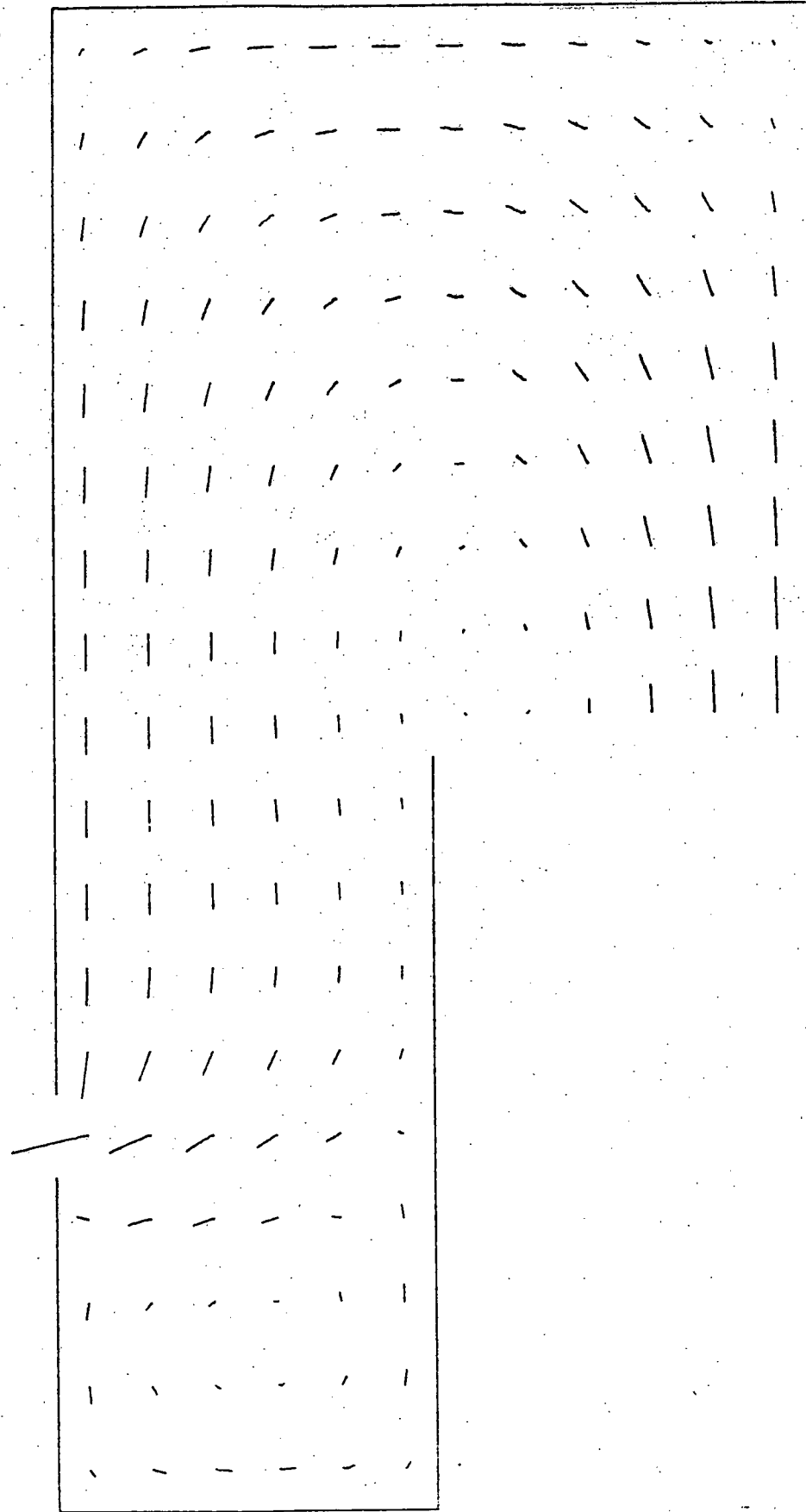
Fig. 20   TEACH-T Prediction, CRBR Geometry, Re - 35000
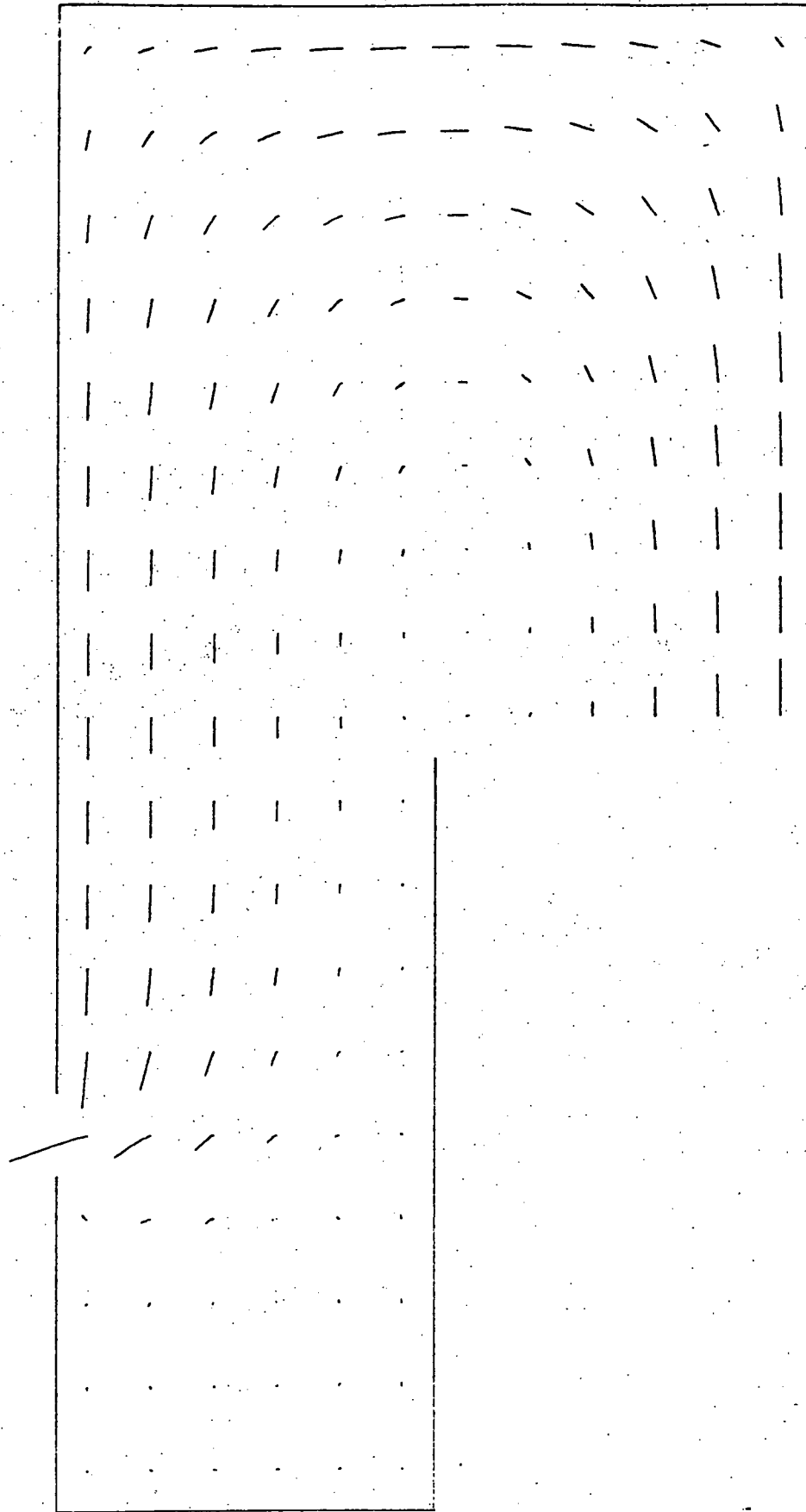
Fig. 21 VARR-II Prediction, CRBR Geometry, Re=35000

Fig. 22 VARR-II Prediction, CRBR Geometry, Re = 35000 Turbulence Kinetic
Energy Matched at Inlet Centerline
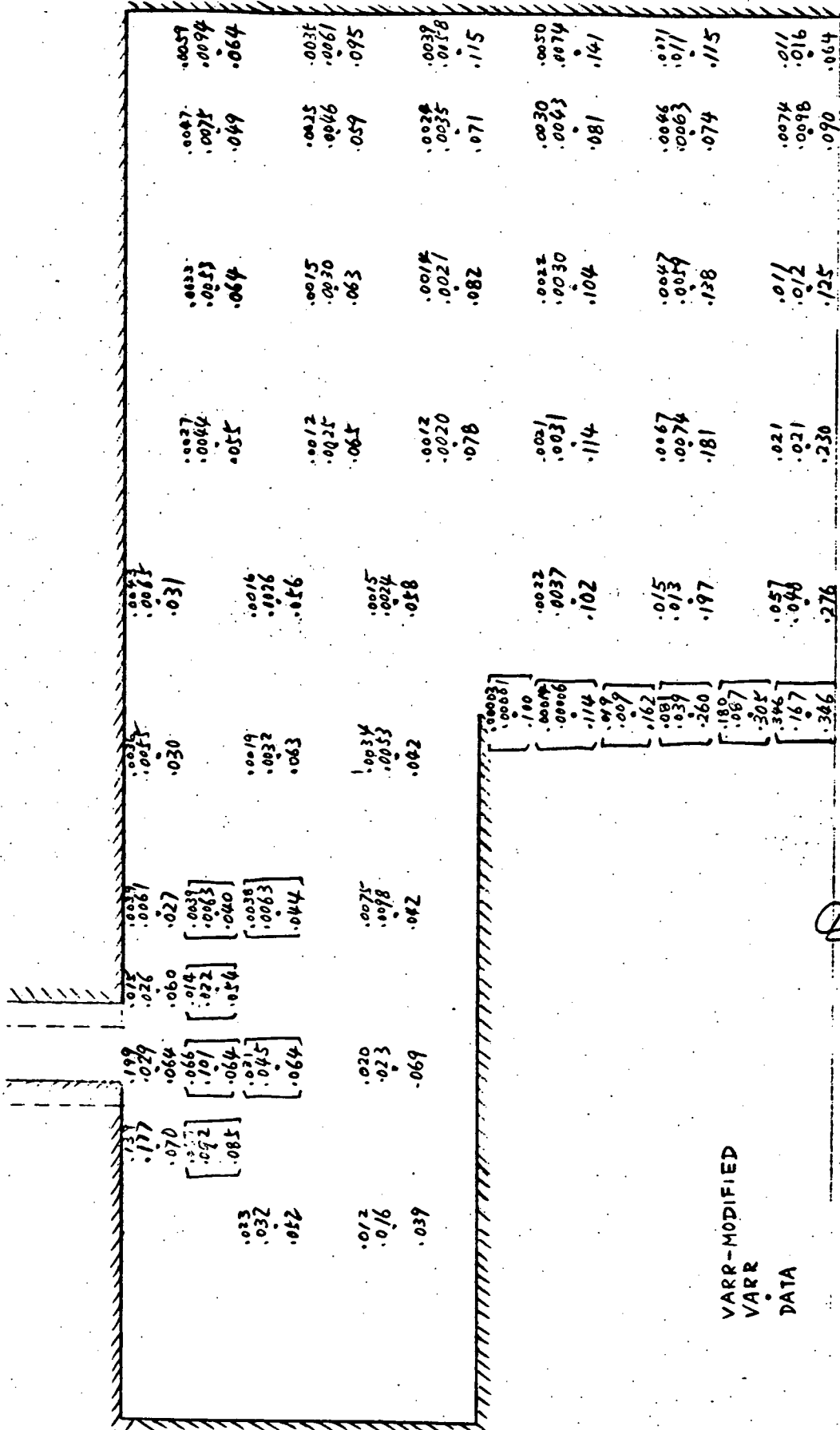
Fig. 23 Compared Calculated and Measured Turbulence Kinetic Energy Fields, CRBR Geometry, Re = 35000

Fig. 24   Measured Mean Flow Field, CRBR Geometry, Distorted Inlet Mean Flow
         Distribution
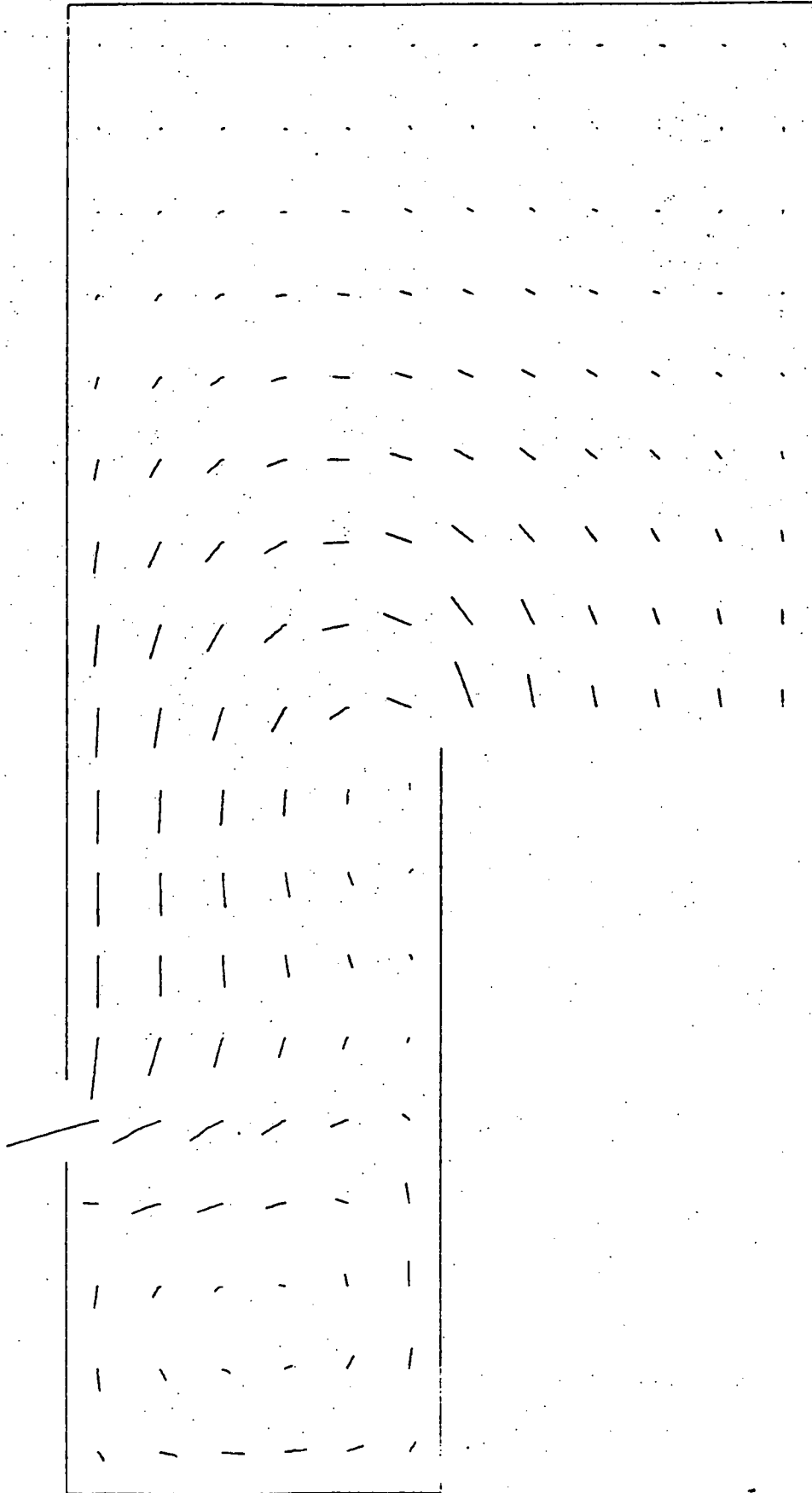
Fig. 25 TEACH-T Prediction, CRBR Geometry, Re = 35000 Distorted Inlet Mean
         Flow Distribution
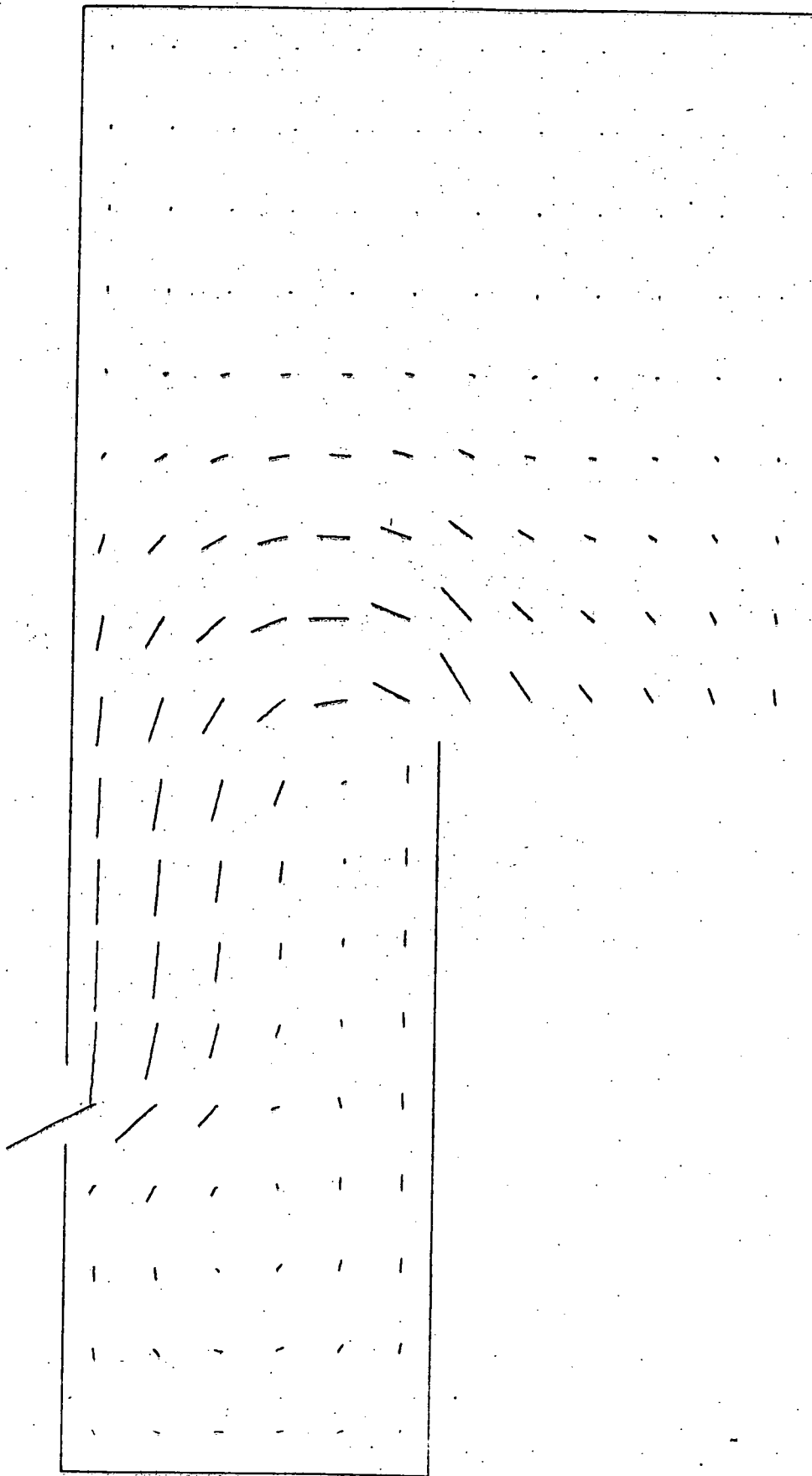
Fig. 26 VARR-II Prediction, CRBR Geometry, Re = 35000 Distorted Inlet Mean
Flow Distribution.

Appendix A - <u>Turbulence Model Code Descriptions</u>

The fundamental equations of the various turbulence model codes are outlined in this Appendix. These codes are used to provide the detailed analysis of the experiments, and the theoretically predicted velocity and turbulence distributions.

The basic conservation equations are the following:

Mass:
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(u_j \rho) = \frac{\partial}{\partial x_j} \epsilon \frac{\partial \rho}{\partial x_j} \tag{A.1}$$

Momentum:
$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_i u_j = -\frac{\partial P}{\partial x_i} + \rho g_i - \frac{\partial}{\partial x_i} R_{ij} \qquad \text{and (in VARR II)} \tag{A.2}$$

Heat:
$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\alpha + E \sigma_k\right)\frac{\partial T}{\partial x_j} + \frac{\overset{\prime\prime\prime}{\text{g}}}{\rho C}, \quad \text{where} \tag{A.3}$$

α = thermal diffusivity

$\rho \equiv \rho_{local} - \rho_0$ = local gas density deficit;

ρ_{local} = local gas density

ρ_0 = ambient gas density,

X_j = j-th spatial coordinate,

t = time,

u_j = j-th velocity coordinate,

P = pressure,

E = Ratio $\left(\dfrac{\text{eddy heat diffusivity}}{\text{eddy momentum diffusivity}}\right)$

T $\quad$ = temperature,

g_j $\quad$ = gravitational acceleration component in j-th direction

$$R_{ij} \equiv \frac{2}{3} k \delta_{ij} - \epsilon \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \qquad \text{or Reynolds Stress (A.4)}$$

$$k \equiv \frac{1}{2} \langle u_i' u_i' \rangle \qquad \text{or turbulence kinetic energy (A.5)}$$

u_i^1 $\quad$ = i-th turbulent fluctuating velocity component,

$$\epsilon \equiv - \frac{\langle u_i' u_j' \rangle}{\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}} \qquad \begin{array}{l} \text{or eddy diffusivity (A.6)} \\ \text{of momentum} \end{array}$$

μ_t $\quad$ = turbulent viscosity,

σ $\quad$ = turbulent momentum diffusivity,

k $\quad$ = molecular momentum diffusivity,

l_m $\quad$ = mixing length,

Υ $\quad$ = shear stress,

y $\quad$ = displacement normal to a boundary,

R $\quad$ = duct radius,

a, b, c, c_1, c_2 = free parameters, and

δ_{ij} $\quad$ = the Kronecker delta function

Equations A.1 and A.2 are solved via coupled iteration upon the velocity divergence as in the marker and cell (MAC) method (55). Equation A.3 is solved explicitly at each time step in terms of variables known from the preceding time step and from Eqs. A.1 and A.2.

In order to solve Eqs. A.1 through A.3 knowledge of the turbulent diffusivities for heat (Ev_T) and momentum (v_T) is required. For this a variety of phenomenological "turbulence

models" have been devised in which appropriate time-averages of Eq. A.2 (weighted by velocities or velocity gradients) are used to provide conservation equations for the turbulence kinetic energy, viscous energy dissipation rate, etc. The order of a turbulence model is denoted by the number of differential conservation equations which must be solved in addition to Eqs. A.1 - A.3. In Table A.1 are summarized a spectrum of zero, one, and two-equation turbulence models which are currently in common use - including those used in TEACH-T and VARR-II. In the formulation of the turbulence conservation equations because of the quadiatic nature of the convective terms of Eq. A.2 there are always created more statistically - defined variables than there exist equations for their solution. Thus, relationships between a sufficient set of extra variables must be postulated in order to permit solution of the equations. Thus, a combination of empirical knowledge and intuition is required in the formulation of a turbulence models, and it is the requirement for the arbitrary additional closure equations which accounts for the difference between turbulence models.

The TEACH-T program uses the turbulence kinetic energy - viscous dissipation rate two-equation model (K, ε), and the VARR-II program uses the turbulence kinetic energy - turbulent viscosity two-equation model (K, σ) as described in Table A.1.

Several Proposed Turbulence Models

MODEL	EQUATIONS (proposer)	ASSUMPTIONS (proposer)	APPLICATION	ADVANTAGES	DISADVANTAGES
ZERO EQ.	1. Mixing Length Theory (Prandtl) $$-\rho \overline{u_1' u_2'} = \mu_t \frac{\partial \overline{u_1}}{\partial x_2}$$ where $\mu_t = \rho \ell m^2 \left\|\frac{\partial \overline{u_1}}{\partial x_2}\right\|$ $\ell m = \ell m(x_1, x_2)$, the mixing length, must be pre-scribed	1. $\ell m \approx \delta$, where $\delta \equiv$ width of 1/2 Jet 2. $\ell m/R = 0.14 - 0.08(1 - x_2/R)^2 - 0.06(1 - x_2/R)^4$	Jet Pipe	1. Simple, no additional differential equations need to be solved. 2. For boundary-layer flow, this theory made very good predictions. 3. Much experience has been accumulated	In situation where more than one characteristic length and time are involved (e.g., recirculating flow) the relation between the stress and velocity gradients are too complicated. This theory does not give a good result.
		3. $\ell m \propto \left\|\frac{\partial \overline{u_1}}{\partial x_2} \middle/ \frac{\partial^2 \overline{u_1}}{\partial x_2^2}\right\|$ (Von Kármán)		It removes the necessity to pre-scribed the mixing length profile.	1. The prediction is not in agreement with measurement except in the vicinity of a wall 2. For certain flows the velocity profiles have inflection points (i.e., $\frac{\partial^2 \overline{u_1}}{\partial x_2^2} \propto 0$) this formula gives infinite mixing length.

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Several Proposed Turbulence Models

MODEL	EQUATIONS (proposer)	ASSUMPTIONS (proposer)	SUCCESSFUL APPLICATION	ADVANTAGES	DISADVANTAGES
ONE EQ.	1. Turbulent Kinetic Energy Model (Prandtl) $$\overline{u'_j u'_i} = -\frac{\mu_t}{\rho}\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)+\frac{2}{3}\delta_{ij}\,k$$ $$\underbrace{\frac{Dk}{Dt}}_{\text{convection}} = -\underbrace{\frac{\partial}{\partial x_j}\left(\overline{u'_j k'}+\overline{u'_j \phi'}-\nu\frac{\partial k}{\partial x_j}\right)}_{\text{diffusion}}$$	diffusion term $= \frac{\mu_t}{\sigma_k}\cdot\frac{\partial k}{\partial x_j}$ dissipation term $= \epsilon$			1. Transport of length scale is not accounted for 2. In practice, the model only offer very small advantages over the mixing length model
	$$\underbrace{\overline{u'_i u'_j}\frac{\partial \bar{u}_i}{\partial x_j}}_{\text{generation}} - \underbrace{\nu\overline{\left(\frac{\partial u'_i}{\partial x_j}\right)^2}}_{\text{dissipation}}$$ where $k \equiv \frac{1}{2}\overline{u'_i u'_i}$, $\mu_t \equiv \rho k^{1/2}\ell$ $$\ell \equiv \ell(x_1,x_2,x_3)\text{ prescribed}$$ algebrically	dissipation $= \frac{\rho k^{3/2}}{\ell}=\epsilon$ diffusion $=-\overline{v'k'}=Bk\sqrt{\frac{\tau_{max}}{\rho}}$ $B \equiv \frac{\tau_{max}}{\rho u_G^2}\,g(y/\delta)$ $\tau = a\cdot\rho k \ (a=0.3)$ (BRADSHAW)	boundary-layer flow	local shear stress is not related to the local mean velocity gradient	1. Transport of length scale is not accounted for 2. Shear stress ∿ energy relation is of limited validity. In practice, applicable only to external boundary layers without velocity maxima.

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Table A.1

Several Proposed Turbulence Models

MODEL	EQUATIONS (proposer)	ASSUMPTIONS (proposer)	SUCCESSFUL APPLICATION	ADVANTAGES	DISADVANTAGES
TWO EQ.	**2. k-ε Model** $\frac{Dk}{Dt} =$ diffusion term + generation term + dissipation term (same as above) $\frac{D\varepsilon}{Dt} = -2\nu \overline{\frac{\partial \bar{u}_i}{\partial x_k}\left(\frac{\partial u_i'}{\partial x_\ell}\frac{\partial u_k'}{\partial x_\ell} + \frac{\partial u_\ell'}{\partial x_i}\frac{\partial u_\ell'}{\partial x_k}\right)}$ $\underbrace{\qquad}_{a}$ $\left(\underbrace{2\nu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_\ell}\frac{\partial u_k'}{\partial x_\ell}} - 2\overline{(\nu\frac{\partial^2 u_i'}{\partial x_k \partial x_\ell})^2}}_{b}\right)$ $-\frac{\partial}{\partial x_k}\left(\underbrace{\overline{u_k' \varepsilon'} + \frac{2\nu}{\rho}\overline{\frac{\partial p'}{\partial x_\ell}\frac{\partial u_k'}{\partial x_\ell}} - \nu\frac{\partial \varepsilon}{\partial x_k}}_{c}\right)$ where $\varepsilon \equiv \nu \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2}$ $\mu_\varepsilon = \rho k^2/\varepsilon$	$a = (C_1 \frac{\overline{u_i' u_k'}}{k} + C_1' \delta_{ik})\varepsilon$ $b = C_2 \varepsilon^2/k$ $c = \frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_k}$	boundary-layer flow recirculating flow	1. The exact equation for ε can be derived from Navier-Stokes equations and ε appears directly as an unknown in the equation for k. 2. Gives satisfactory predictions for both boundary-layer flow and recirculating flow. 3. It does not greatly increase the cost of computation over that with simple model	The empirical constants are difficult to predict.

Several Proposed Turbulence Models

MODEL	EQUATIONS (proposer)	ASSUMPTIONS (proposer)	SUCCESSFUL APPLICATION	ADVANTAGES	DISADVANTAGES
TWO EQ.	2. k-σ Model (Stuhmiller) $$\frac{Dk}{Dt} = \frac{1}{2}\,\sigma\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)^2 + \frac{\partial}{\partial x_j}\,\Gamma\sigma\,\frac{\partial k}{\partial x_j} - \alpha\,\frac{(2k)^2}{\sigma}$$ $$\frac{D\sigma}{Dt} = \frac{\sigma^2}{4k}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)^2 + \frac{\sigma}{k}\frac{\partial}{\partial x_j}\,\Gamma\sigma\,\frac{\partial k}{\partial x_j}$$ $$-\frac{\sigma^3}{k^2}\frac{\partial}{\partial x_j}\,\Gamma_1 k\,\frac{\partial}{\partial x_j}\left(\frac{k}{\sigma}\right) - \alpha k$$ $$\rho\,\overline{u'_i u'_j} = \frac{2}{3}\,\delta_{ij}k - \sigma\left(\frac{\partial \bar{u}_i}{\partial x_j}\quad\frac{\partial \bar{u}_j}{\partial x_i}\right)$$ where Γ, α, Γ_1 are constants		pipe flow recirculating flow	1. The exact equation for σ can be derived from Navier-Stokes equation and σ appears directly as an unknown in the equation for k. 2. Gives satisfactory predictions for both boundary-layer flow and recirculating flow. 3. It does not greatly increase the cost of computation over that with simple model.	The empirical constants are difficult to predict.

Several Proposed Turbulence Models

MODEL	EQUATIONS (proposer)	ASSUMPTIONS (proposer)	SUCCESSFUL APPLICATION	ADVANTAGES	DISADVANTAGES
TWO EQ.	3. k-kℓ Model (Rodi and Spalding) $$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \frac{1}{y1}\frac{\partial}{\partial y}\left(y1\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + \nu_t\left(\frac{\partial u}{\partial y}\right)^2$$ $$- C_p\frac{k^{3/2}}{\ell}$$ $$U\frac{\partial k\ell}{\partial x} + V\frac{\partial k\ell}{\partial y} = \frac{1}{y1}\frac{\partial}{\partial y}\left[y1\frac{\nu_t}{\sigma k\ell}\left(\frac{\partial k\ell}{\partial y}\right.\right.$$ $$\left.\left. + C_{k\ell}\ell\frac{\partial k}{\partial y}\right)\right] + C_B\nu_t\ell\left(\frac{\partial u}{\partial y}\right)^2$$ $$- C_S k^{3/2}$$ where C_D, C_S, C_B, σ_k, C_{kL} and $C_{k\ell}$ are constants		Free Jet boundary-layer near wall	1. The exact equation for kℓ can be derived from Navier-Stokes equation and kℓ appears directly as an unknown in the equation for k. 2. Gives satisfactory predictions for both boundary-layer flow and free jet. 3. It does not greatly increase the cost of computation over that with simple model.	The empirical constants are difficult to predict.

TASK IV: THEORETICAL DETERMINATION OF LOCAL TEMPERATURE
                FIELDS IN LMFBR FUEL ROD BUNDLES

TASK IVA: Code Development for Solving the 2-D Multicell,
                Multiregion Energy Equations (Man Kit Yeung)

Summary

     During the last quarter emphasis has been primarily
focused on the application of the iterative analytical
solution to determine 2-D temperature field in finite
bundles of both square and hexagonal geometry. Further
progress has been made to predict the "effective mixing
length" for conduction heat transfer, a parameter which
enters the subchannel formulation and application for
LMFBRs. Both clad temperature distribution and the effect-
ive mixing length for conduction yield excellent agreements
when compared with results in the open literature for the
square array bundle. In addition parametric studies of
conductivities and channel wall effects have been also
investigated. A calculation with the accurately predicted
effective mixing length for a 7-pin LMFBR bundle is performed
with COBRA-IIIC. The calculational results reveal the excess
conservatism in current design calculations due to the un-
certainty of the effective conduction mixing length.

1.   Clad Temperature Distributions in Hexagonal Rod Bundles

     For subchannel calculations, it is generally assumed that
the heat flux is uniform around the clad surface and the clad
temperature is calculated by using the simple heat transfer
coefficient method. However, this procedure is highly over-
simplistic when applied to the side and corner cells because
the high degree of geometric irregularities of the cells
greatly enhance the azimuthal variations of the clad temper-
atures and heat fluxes. Information about a correct treatment
of these phenomena is only available for single side and
corner cell calculations with artifically imposed adiabatic
boundary condition at the imaginary cell boundary [1]. The
present study gives detailed clad and coolant temperature
distributions in a finite hexagonal rod bundles.

     A 30° symmetry section of a typical 7-pin fuel bundle
as shown in Fig. 1 is studied in the first place. Analytical
solutions of the temperature fields in all regions are obtained
for thermally fully developed slug flow condition. Thermal
and geometric parameters used in the analysis are presented
in Table 1. The results of the calculation of the clad tempera-
ture distributions as function of the azimuthal angle for both
the internal and corner cell are given in Fig. 2 at the axial

plane where the bulk coolant temperature is 1150°F, which represents a typical core exit temperature. Case 1 denotes the clad surface temperature distribution with the commonly imposed condition of uniform heat flux at the outside clad surface and case 2 denotes the multiregion analysis. It is evident that the latter indeed removes excess conservatism as compared to the single region analysis by reducing the clad temperature variation of the corner cell from approximately 185°F to 100°F. On the other hand the temperature variation of the internal cell is much less sensitive to the boundary condition because of its higher degree of symmetry for this 7-pin bundle case.

2. Calculation of Effective Conduction Mixing Length in Hexagonal Rod Bundles

Subchannel codes usually "lump" the assembly into a finite grid of subchannels, each of which is characterized by a bulk temperature, flow rate and pressure. Energy transport between adjacent channels is governed by the mechanisms of diversion cross-flow, turbulence mixing and conduction mixing. For LMFBR design calculations the energy transport by conduction constitutes a major contribution to the overall energy transport. This effect is even more pronounced for flows which are characterized by low Reynolds numbers. These flow situations especially arise in the case of a loss-of-flow accident and the like. Despite this importance only scattered information is available for internal cells but nothing is known for the bundle-wall-near regions of finite bundles.

The heat transfer rate due to conduction between adjacent subchannel i and j as calculated in subchannel codes is given by: [2]

$$Q_{ij} = k \frac{S_{ij}(\overline{T}_i - \overline{T}_j)}{l_{ij}} = k \frac{S_{ij}}{l_{ij}^*} (\overline{T}_i - \overline{T}_j) \frac{1}{L_{ij}} \qquad (1)$$

Where S_{ij} is the length of the common boundary, l_{ij} is the effective conduction mixing length, l_{ij}^* is the centroid-to-centroid distance and L_{ij} is a dimensionless correction factor which represents the ratio of the effective conduction mixing length to the centroid-to-centroid distance ($L_{ij} = l_{ij}/l_{ij}^*$). In subchannel calculations, L_{ij} is usually set to be unity due to the lack of knowledge of the local temperature field.

It is obvious that the lumped parameter subchannel codes will give reliable results only if reasonably correct effective mixing lengths are known. The calculation of the local temperature field thus provides a tool for the determination of the L_{ij}. With some rearrangements Eq. 1 can be written as:

$$L_{ij} = k \; \frac{S_{ij}}{l_{ij}^{*}} \cdot \frac{(\overline{T}_i - \overline{T}_j)}{\displaystyle\int_{S_{ij}} q'' \, ds} \tag{2}$$

where the channel averaged temperature and the heat transfer at the channel boundary can be calculated explicitly from the local temperature field of the coolant, and the ratio S_{ij}/l_{ij}^{*} which appears in Eq. (1) and Eq. (2) can be simply expressed by geometric parameters as:

$$\frac{S_{ij}}{l_{ij}^{*}} = \frac{\frac{1}{2}\left(\frac{P}{D}-1\right)}{\left[\frac{\frac{1}{16}\left(\frac{2W}{P}\right)^2\left(\frac{P}{D}\right)+\frac{1}{24\sqrt{3}}\left(\frac{2W}{D}\right)^3-\frac{1}{16}\quad\frac{1}{2}\left(\frac{P}{D}\right)^3-\frac{\sqrt{3}}{12}\pi\left(\frac{P}{D}\right)}{\frac{1}{4}\left(\frac{P}{D}\right)\left(\frac{2W}{D}\right)+\frac{1}{8\sqrt{3}}\left(\frac{2W}{D}\right)^2-\frac{1}{12}\pi}+\frac{1}{\sqrt{3}\left(\frac{P}{D}\right)^2-\frac{1}{2}\pi}\right]} \tag{3}$$

Fig. 3 shows L_{ij} as a function of the dimensionless wall distance for the corner region in case of P/D = 1.20. The results of both the single region and the multiregion analyses indicate that L_{ij} generally deviates quite substantially from unity, what is usually employed in subchannel codes such as COBRA-IIIC. [2]. The singularity indicates zero integral net heat transfer rate by conduction between the cells. The behavior of the curves for large 2W/D values implies that the effect of the bundle wall diminishes and only P/D governs the asymptotic regime. The domain of the curve to the left of the singularity which results in $L_{ij} > 1$ indicates a reversal of the heat flow direction.

For this regime of 2W/D the net heat transfer is obviously from the corner channel into the internal channel due to the

decrease of the wall distance and reduction of flow area in the corner channel.

In order to demonstrate the effect of L_{ij} on subchannel calculations, three cases with different effective conduction mixing lengths were performed with COBRA-IIIC to compute the channel averaged temperatures for a bundle with geometric parameters of P/D=1.20 and 2W/D=1.24. The channel averaged temperatures at 36" from the core entrance for the cases with no conduction ($L_{ij}= \infty$), the centroid-to-centroid distance as effective conduction mixing length ($L_{ij}=1.0$), and the effective mixing length calculated from the analysis ($L_{ij}=0.65$) are tabulated in Table 2. The case without the conduction mixing effect is essentially an isolated channel problem with respect to conduction and naturally yields the highest difference of averaged channel temperatures. Using the centroid-to-centroid distance without any correction leads to a reduction in ΔT of about 20°F and using the analytically derived L_{ij} from Fig. 3 further decreases the temperature difference by another 9°F. Thus the advantage of an accurately determined L_{ij} becomes obvious.

It should be mentioned at this point that most ongoing efforts have been devoted to the diversion mixing area which obtains mixing effects of similar magnitude and it is felt that an equally significant gain in design margin is possible by properly accounting for the conduction effect.

3. Clad Temperature Distributions in Square Array Configurations

In addition to the analysis of hexagonal arrangements, a two cell section of a square cell arrangement as shown in Fig. 4 has been analyzed in parallel because the only concise information in the literature is available for the latter. Thus this case serves in order to check the validity of the developed code. The clad temperature distribution of the two cell calculation is compared to the result reported by France and Ginsberg [3]. In addition, the effect of changes in the flow split as well as multiregion analysis are also investigated to provide full understanding of the problem on hand.

As a first step only identical power source densities are considered in the fuel bundle. Slug flow is assumed within each cell and the variation in the velocity from cell to cell is given by [4].

$$\frac{U_i}{U_j} = \left(\frac{D_i}{D_j}\right)^{5/7} \tag{4}$$

where U_i and D_i are the cellular velocity and hydraulic diameter of cell i, respectively. This accounts approximately for the friction effect due to the presence of the channel wall and approximates the flow split. It should be noticed that, by virtue of Eq. 3 discontinuities in the velocity field at the interfaces of the cells are introduced. However, this is of no consequence for this analysis because the momentum equation is not solved at all. Furthermore, it should be recognized that instead of Eq. (4) Novendstern's approximation for the flow split in wire-wrapped bundles can be introduced or experimentally determined quantities may be used. This will be done in the next step of the code's development.

Calculational results are presented in Fig. 5 for a particular case with geometric parameters of P/D=1.20 and 2W/D = 1.24. Curve I denotes a special case with no flow split and uniform heat flux at the outside clad surface. The clad temperature distributions of both cells are compared to those of a finite difference solution by using the THTB code as given in [3]. It can be seen that the solution given by the present method has an accuracy which is comparable to that resulting from the finite difference method with 720 nodes in the coolant region. The excellent agreement adds further confidence to the accuracy of the iterative technique presently developed.

Curve 2 denotes the clad temperature distribution for a case with uniform heat flux at the outside clad surface and a flow split as calculated by Eq. (3). As can be seen from Fig. 5, the flow split due to the presence of the channel wall has completely reversed the temperature distribution. However, none of these curves represent the realistic clad temperature distribution due to the unrealistic boundary condition imposed at the outside clad surface. Only a multiregion analysis with flow split and accounting for the additional effects of the fuel and clad regions is capable to remove excess conservatism. Curve 3 summarizes the result of this calculation with some typical conductivity ratios ($\kappa = 0.8327$, $\kappa_W = 0.1538$). It is obvious that the heat flux and temperature redistributions result in a smaller circumferential temperature variation at the clad surface, i.e. a reduction of about 60°F to 80°F for normal operating conditions. Thus it becomes clear that an analytical tool should handle multiregion analysis together with the flow split in order to give realistic results.

4. Calculation of Effective Conduction Mixing Lengths in Square Array Configuration

It is desirable to determine the effective conduction mixing length for the square cell bundle and compare the results to those given by France and Ginsberg in [4]. Again this will serve as an additional check for the validity of the present

calculational procedure. For the square array configuration the ratio $S_{ij}/l_{ij}{}^*$ in Eq.(2) becomes:

$$\frac{S_{ij}}{l_{ij}{}^*} = \frac{\frac{P}{D}}{\frac{3}{2}\left[\frac{P}{D}\right] + \frac{1}{2}\left[\frac{2W}{D}\right] + 2\left[\frac{\frac{\Pi}{\delta}\left[\frac{2W}{D} - \frac{P}{D}\right]}{\frac{P}{D}\left[\frac{P}{D} + \frac{2W}{D}\right] - \frac{\pi}{2}}\right]} \qquad (5)$$

All other quantities in Eq. (2) are again evaluated from the local temperature field for the square array configuration. The results are presented in Fig. 6 with the effective conduction mixing length L_{ij} plotted vs. the quantity μ, which is defined as:

$$\mu = \frac{\dot{M}_{wall}}{\dot{M}_{internal}} \qquad (6)$$

where $\dot{M}_{wall}$ and $\dot{M}_{internal}$ are the mass flow rates for the cell next to the wall and the internal cell, respectively. Curve 2 in Fig. 6 denotes the result given by France and Ginsberg who used a boundary integral method for an eight cell configuration. For both cases the boundary condition of uniform heat flux at the clad outside surface has been used for comparison. The singular points as shown in Fig. 6 indicate that the net heat transfer between adjacent cells approaches zero. It can also be seen from Fig. 6 that L_{ij} generally deviates substantially from unity and exhibits strong dependence upon μ, which itself is a function of geometry. However, the L_{ij} values for a 2-cell configuration indicate a fairly good agreement with those resulted from the 8-cell calculation except for the range where $\mu \leq 0.95$. This is because the effect on the flow in the interior cell due to the presence of the wall is more pronounced in a 2-cell configuration than in an 8-cell arrangement. On the other hand, the results indicate that even for the extreme case, a two cell model gives reasonable solutions as compared to the eight cell model. What this means is that a compromise between the gain in accuracy and additional computational efforts should be made.

4. Conclusion and Discussion of Future Efforts

The iterative analytical method has demonstrated its applicability of both the calculation of the local temperature field and the determination of the effective conduction mixing length. The analysis serves as a tool to abridge the gap between the distributed parameter and lumped parameter analyses.

Moreover, it should be pointed out that the functional dependence of L_{ij} vs. wall distance for the hexagonal bundle is of particular interest because for the first time the effect of bundle wall displacement due to thermal and mechanical loads during its lifetime can be studied consistently with better knowledge of the variation of L_{ij} as a function of time.

Future effort will be directed towards the extension of the present analysis from a 7-pin bundle to larger bundles. However, by virtue of the results indicated by the clad temperature distributions given in Fig. 2, the circumferential temperature variations of the clad for most interior cells in a large bundle may be quite small due to their higher degree of symmetry. This suggests that the channel wall effect probably does not go beyond the second or third row and from the thermal stress analysis point of view only the first row from the wall is of prime interest. Thus the implication is that future work should be directed also to coupling subchannel and distributed parameter codes together.

TASK IVB: 3-D COUPLED TWO CELL SLUG FLOW HEAT
TRANSFER ANALYSIS (Chung Nin Wong)

Summary

During the last quarter, the analytical method which
has been described in the former progress reports has been
extended in order to analyze the coupled corner and internal
cells in three dimensions. To the author's knowledge this
is the first time that results are presented for this geometry.
All information hitherto available for 3-D analyses are
limited to the coupling of two internal cells. Due to the
similarity of the internal cells, these analyses are of no
special engineering interest. The analysis of coupled corner
and internal cells is of much more interest though because
two cells with completely different thermal entrance behavior
are considered.

Results are given for the outside clad temperature vari-
ations and are compared to the results given in previous
reports. The heat flux distribution along the common boundary
between the two cells is given as a function of the axial
coordinate.

Furthermore, a sensitivity study has been performed to
show the effect of the number of collocation points, the
number of eigenvalues and the number of harmonies upon the
solution.

In a next step, the effective conduction mixing length
will be presented for the thermal entrance region and compared
to that given in the preceding section for thermally fully
developed conditions.

1. Short Outline of the Method

Before this problem has been attacked, the decision was made not to use the iterative scheme as outlined in the preceding section but instead to extend the already available analytical tool as described in the former progress reports. The only change which had to be made concerns the geometry of the coolant regime. Instead of studying the cells individually, the geometry as shown in Fig. 7 has to be considered; that is the two cells are treated as a whole new unit. This concept works well as long as the coolant channel geometry can be easily described by a rod centered polar-coordinate system located in one of the rods. Although this limits the application of the method to smaller regions it is felt that the most important bundle-wall-near regions together with the next row of interest cells can be effectively analyzed.

In order to make the analysis in its initial stage as simple as possible, the input parameters for both cells are assumed to be the same. Furthermore, the heat fluxes are assumed to be axially and azimuthally uniform in both pins.

With the assumptions made above the solution to the problem can still use the superposition of two solutions, namely, the fully developed and the thermal entrance solution, which have been outlined earlier.

2. Results

The same data, as already used in the last report, have been chosen here again for the test case:

$$P/D = 1.3 \; ; \quad 2W/D = 1.38 \qquad D = 9 \text{ mm}$$

$$P_e = 372 \; ; \quad T_{in} = 415°C$$

$$q'' = 104 \text{ W/cm}^2$$

Uniform heat flux in the axial and azimuthal directions. It should be recalled that these data reflect one of the experiments conducted at Karlsruhe.

For the presentation of the results the following abbreviations are used for the dimensionless groups:

$$\Theta (\rho, \phi, \xi) = \frac{T(r,\phi,z) - T_{in}}{q'' \, r_1/k}$$

$$\rho = \frac{r}{P/2}$$

$$\xi = \frac{z \, D_n}{\left(\frac{P}{2}\right)^2 P_e}$$

The first results of this study are summarized in Figs. 8 to
5. Fig. 8 shows the comparison between the results of the
single and two cell calculations for the clad temperature
distribution of the corner cell under thermally fully developed
conditions. Surprisingly, the two cell calculation results
in even larger temperature variations around the clad as pre-
dicted by the single cell calculation. This complies perfectly
with what has been already discovered with the 2-D analysis.
The implication of this result is that under certain geometrical
conditions the temperature variations are not getting smoothed
out as one would intuitively think in the first place but
instead become more pronounced.

Figs. 9 and 10 show the axial development of the peri-
pheral temperature distributions for corner and internal cells.
Whereas some remarkable changes in the profile can be noticed
for the former the distribution in the latter does not change
that much. What is interesting to see is that around $\phi=180°$
the temperature distribution around the corner and the internal
pin are the same, because both channel geometries are the same
in that regime. For $\phi < 180°$ the specific geometry of the
coolant channel and the thermal entrance effect dominate the
development of the temperature profile in the corner cell. At
this point it must be mentioned that the temperature distribution
in the neighborhood of $\phi = 180°$ is quite different from that
predicted by the isolated single cell analysis. In the coupled
two cell analysis this behavior is pretty much governed by the
internal cell even at small z, i.e. there is a remarkable
difference between the results obtained from the simplistic
adiabatic boundary condition as compared to the realistic one.
A good feeling about what happens along that imaginary boundary
between the two cells can be obtained by calculating the heat
flux along that line as a function of the axial distance. This
is explicitely shown in Fig. 11. These results indicate that
in the inlet regime of the coolant thermal heat is transferred
from the corner to the internal cell. However about midway
through the core this behavior reverses and heat flows from
the internal cell towards the corner cell. This trend increases
with increasing length of the channel. Naturally, the highest
amount of energy is transferred across a line segment of this
imaginary boundary which is characterized by the interval
$150<\phi<170$, whereas for $\phi>170°$ the transfer rate drops due to
the similarity and symmetry of the coolant channel cross section
in that regime. This figure shows that the assumption of zero
heat flux along the interface between the cells as applied
in the adiabatic single cell calculation is not valid in a
strict sense. On the other hand, the curves indicate that the
local transfer rate up to about midcore height is not very
large either which suggests that at least in a certain axial
regime the assumption of an adiabatic boundary condition can be
somewhat justified. The change of the heat flux distribution
as function of the axial coordinate can be explained in terms
of the vastly different thermal entrance behaviors of the two
cells.

3.   Results of the Sensitivity Analysis

In order to assure the validity of the results shown in the previous section a sensitivity study has been performed. At this point it should be recalled that the complete analytical solution contains the summations over three integers ranging from either zero or one to infinity. It is obvious that the upper limit has to be cut down to a suitably chosen finite number of summation terms for computational purposes. This raises automatically the question what are the appropriate limits for these numbers for a specific problem on hand. With respect to the fully developed part of the solution it is felt that its validity has been already shown by comparing it to the iterative solution technique which itself was tested and compared favorably to a finite-difference solution. What remains then is to check the thermal entrance solution. For convenience the general form of this part of the total solution is repeated below

$$\Theta_e(\rho,\phi,\xi)=\sum_{h=0}^{\infty}\sum_{m=1}^{\infty}[C_{n,m}]_n(\beta_m\rho)+D_{n,m}Y_n(\beta_n\rho)]\,E_m e^{-\beta_m^2\xi}\cos n\phi$$

As can be seen this equation contains a double summation. The first one sums over the number of harmonies kept in the solution and the second one sums over the number of eigenvalues.

Figs. 12 and 13 show the results for different upper limits in the above summation terms upon the peripheral clad temperature distribution. This comparison shows that selecting N_{eig} and/or N_n over a wide range does not greatly affect the clad temperature. Only for the fairly extreme case of $N_{eig}=4$ and $N_n=5$ are some small differences noticeable. However, the effects are much more pronounced for the coolant temperature. This becomes obvious from Fig. 14 where the heat flux distributions along the common boundary between the cells are compared. The selection of $N_{eig}=4$ and $N_n=5$ leads clearly to a fairly different distribution as compared to the standard $N_{eig}=16$ and $N_n = 15$. Obviously the strongest effect is introduced by the wrong selection of the harmonies to be kept in the solution. However, it should be noticed that due to the differentiation of the coolant temperature in order to get the heat flux, differences in the solutions are especially accentuated. This means that the heat flux as shown in Fig. 14 is indeed a very tight criterion to judge upon the accuracy of a solution.

In addition to the above mentioned selections there exists one more possibility to affect the solution, namely by choosing the number and distribution of points directly at the inlet of the coolant channel to satisfy the inlet boundary condition.

This selection mainly determines the coefficients E_m. The effects of two different selections upon the clad temperature is shown in Fig. 15. The solid line results by setting 5 points in the radial direction in the range $0° \leq \phi \leq 100$ and 7 in the range $101° \leq \phi \leq 180°$ which gives a total of 269 points. The dotted line represents the results as obtained by setting 5 points in the range $0° \leq \phi \leq 140°$ and 8 for $141° < \phi < 180°$ which gives a total of 320 points. The azimuthal point distributions are indicated directly in the figure itself. The results therein show how sensitive the clad temperature is with respect to these two choices. At this point there is no direct way to decide what is the more accurate solution. However, experience tells us that it is appropriate to put more points in the region where the two cells are coupled together. Further increase of the number of points may lead to difficulties in the solution of the linear set of equations if no precautionary measures are taken. Overall this raises some doubts as to what limits the boundary least square method can be finally driven. Future studies will show whether the observed inaccuracies are indeed introduced by the solution method used for solving the set of linear equations. If this should be the case, a better method will be chosen out of the variety of available methods solving the general boundary least square problem.

4.   Future Work

Efforts in the next quarter will concentrate upon the evaluation of the effective conduction mixing length as a function of the axial distance. Curves will be fitted to these results and then implemented into COBRA-IIIC. The outcome of this study will then be compared to the results obtained from COBRA-IIIC by using the effective conduction mixing lengths resulting from the study of thermally fully developed conditions as derived in subtask IVA.

TASK IVC: FULLY DEVELOPED LAMINAR MIXED CONVECTION (Jong-Yue Kim)

Summary

Work has been completed on this subject during the last
quarter [5]. A topical report will be issued soon which in
addition to the analytical results obtained in [5] will
contain a review of experimental and other yet simplified
analyses.

The undertaken efforts have been concentrated upon
solving the laminar mixed convection problem in isolated
single cells. The clad region has been considered, whereas
the fuel region remained neglected. With respect to the
peripheral heat flux distribution at the inside clad surface,
two options were examined, namely

a)   a constant distribution

and   b) an azimuthally varying distribution

The latter served to simulate power tilts across the fuel
region. The following parameters were changed in order to
study their effects upon local and integral thermal and hydro-
dynamic quantities:

1) pitch-over-diameter ratio, P/D

2) wall distance-to-diameter ratio, 2W/D

3) Rayleigh number, Ra

4) clad-to-coolant thermal conductivity ratio

5) tilt parameter in the heat flux distribution

During the course of the analysis all efforts were undertaken
to compare its results against publically available information.
These comparisons turned out to show excellent agreements.
Based upon these observations it is hoped that all the other
results obtained in this study are valid, too.

1. Results

    In what follows, only a small part of the overall results
given in [5] will be presented. Although the internal cell
has been as thoroughly examined as the edge (side)-cell,
results will be shown only for the latter here.

    Fig. 16 and 17 show the dimensionless outside clad
temperature distributions for P/D = 1.08 and P/D = 1.2,
respectively, for different values of 2W/D and two values of
the Ra-number, namely Ra=1 and Ra=10^4. The values for 2/W/D
were chosen such that they span a certain design range for
the P/D under consideration. Furthermore, it should be
noticed that Ra=1 very closely resembles the forced laminar
convection case as has been shown in [5] by comparing the
results with those given by Hsu [6].

    By comparing Figs. 16a and 17a it becomes apparent
that designs with small P/D are much more sensitive against
changes in the wall distance, which might occur during the
operation due to bowing of the pin as well as the subassembly
wall. That is, the temperature variations around the clad
can become remarkably high if 2W/D assumes values which
deviates from the design values. Naturally, this effect is
more pronounced for tighter pitches.

    Independently of P/D Figs. 16b and 17b show that by
increasing Ra always leads to a nearly flat temperature
distribution at the surface which essentially means that
free convection has a favorable effect upon the local distri-
bution.

    Fig.18 shows the additional effect of a variable heat
flux distribution. In the cases examined, the heat flux
assumed its maximum towards the inside of the bundle and has
its minimum adjacent to the subassembly wall. As can be
observed these distributions lead to even higher temperature
variations compared to the constant heat flux boundary con-
dition and yet at Ra=10^4 the variations have been totally
eliminated.

    Fig. 19a and 19b show the dimensionless heat flux ratios
for the constant and the variable heat flux boundary cases,
respectively. For Ra=1, the maximum of $q''/\bar{q}''$ is about 1.5
which steadily decreases when Ra increases. As shown in
Fig. 19b for Ra=10^5 all fluctuations have been smoothed out
and the distribution follows essentially the boundary con-
dition at the clad inside surface.

    Fig. 20 shows how the direction of the flux tilt affects
the dimensionless heat flux ratio for different Ra-numbers.

    As with regard to the average Nusselt number, $\bar{Nu}$, as

function of Ra, Figs. 21 to 23 depict quite interesting behaviors which have not been reported in the open literature thus far. Over a certain range of the Ra-number, $\overline{Nu}$ increases more or less dependent upon $2W/D$. However, at some critical Ra-number, $\overline{Nu}$ sharply drops off and may assume even negative values before it asymptotically recovers to values around zero at very high Ra-numbers. This general behavior is explainable in terms of the differences of the average clad and coolant temperatures. The latter are strongly affected by the velocity field an example of which is given in Fig. 24. This figure discloses a very complicated structure of the velocity field for higher Ra-numbers. It is evident that regions exist with local downflow. This will naturally affect the overall pressure-drop characteristics of the coolant channel as shown in Fig. 25. Detailed discussions of these findings will be given in the forthcoming topical report.

Finally, Fig. 26 shows the feedback effect of the variable heat flux upon the velocity field. Naturally, for higher tilt parameters in the heat flux distribution the effects will become even more pronounced.

In conclusion, it should be recalled that all of the aforementioned results were obtained with the assumption of fully developed thermal and hydrodynamic conditions. Whether this is indeed justifiable under realistic conditions remains to be seen. However, it can be reasonably argued that the foregoing results establish upper limits and therefore are conservative.

2. Future Work

Dependent upon the future funding situation it is planned to couple several cells together in order to derive effective convection mixing lengths which are then inputted into the especially prepared COBRA-IIIC code together with the effective conduction mixing lengths as derived in subtasks IVA and IVB.

REFERENCES

1. Hsu, C.J., "Multiregion Analysis of the Effect of Rod Displacement on Heat Transfer in Slug Flow Through Closely Packed Rod Bundles," Nucl. Sci. Engr., 47, 380 (1972).

2. Rowe, D.S., "COBRA-IIIC: A Digital Computer Program for Steady State and Transient Thermal-Hydraulic Analysis of Rod Bundle Nuclear Fuel Elements," BNWL - 1695. May 1973.

3. France, D.M. and Ginsberg, T., "Improved Point-Matching Techniques Applied to Multi-Region Heat Transfer Problems," ASME Journal of Heat Transfer, Vol. 94, May 1972.

4. France, D.M. and Ginsberg, T., "Comparison of Analytic and Lumped-Parameter Solutions for Steady-State Heat Transfer in Fuel-Rod Assemblies," Progress in Heat and Mass Transfer, Vol. 7.

5. J-Y. Kim: "Fully-Developed Mixed Convection Heat Transfer in the Finite Hexagonal Bundles," M.S. Thesis, Dept. Nuclear Engineering, MIT, Feb. 1977.

6. C-J. Hsu: "Laminar-and Slug Flow Heat Transfer Characteristics of Fuel Rods Adjacent to the Fuel Subassembly Walls," Nucl. Sci. Engrg.

TABLE 1

Thermal and Geometric Parameters Used in Clad Temperature Calculations

Case 1 and 2   Pitch to diameter ratio $(^P/D) = 1.30$

Wall distance to radius ratio $(^{2W}/D) = 1.38$

Fuel Rod Radius $(b) = 0.177"$

Fuel Pellet Radius $(a) = 0.159"$

Power Density $(q''') = 0.547 \times 10^8 \; ^{BTU}/ft^3\text{-}hr.$

Coolant conductivity $(k_c) = 15.5 \; ^{BTU}/ft\text{-}hr\,^\circ F.$

Case 2 only   Clad Conductivity $(k) = 13.0 \; ^{BTU}/ft\text{-}hr\,^\circ F.$

Fuel Conductivity $(k_f) = 2.0 \; ^{BTU}/ft\text{-}hr\,^\circ F.$
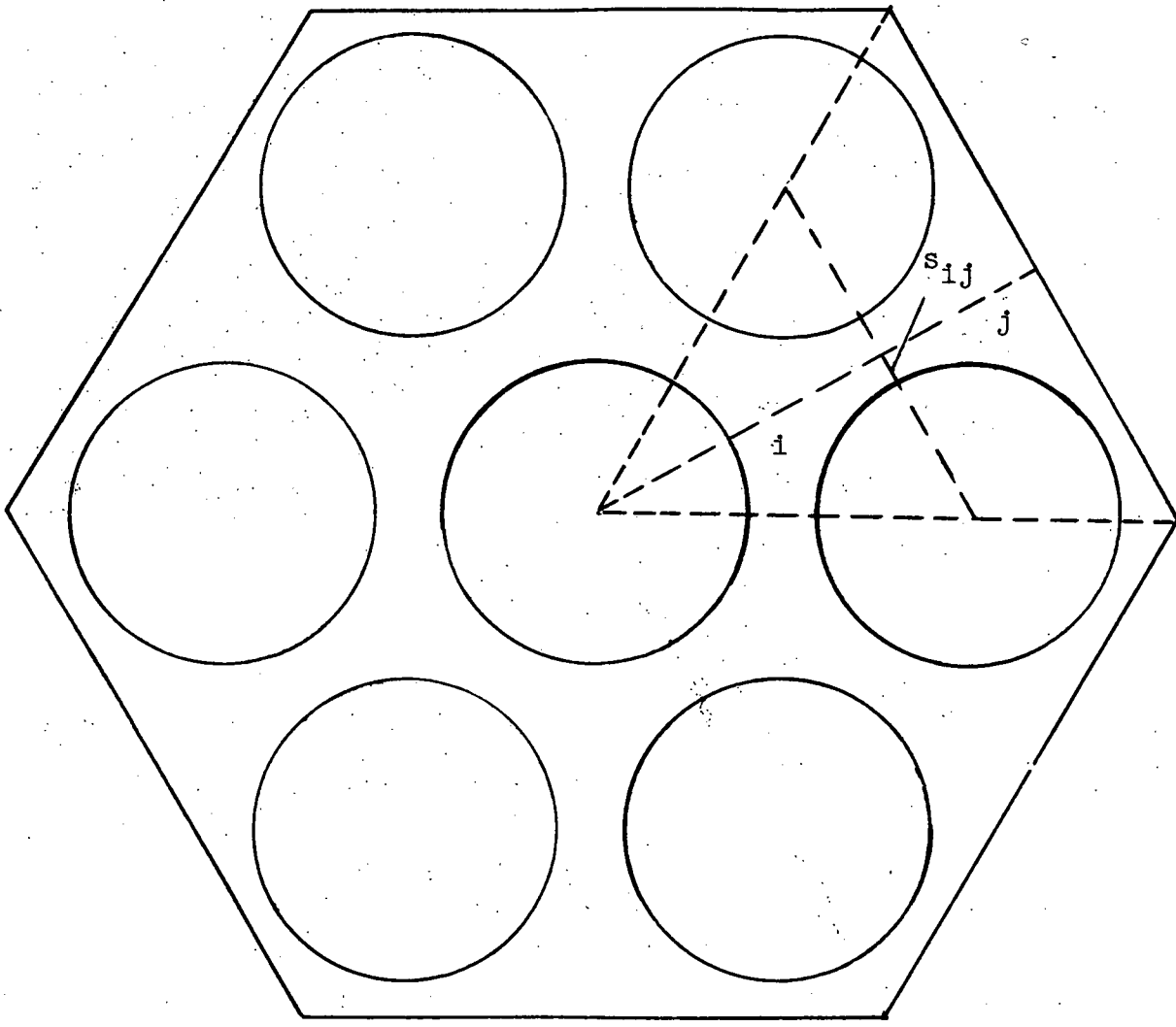
Fig. 1 Configuration of a 7 Pin Bundle
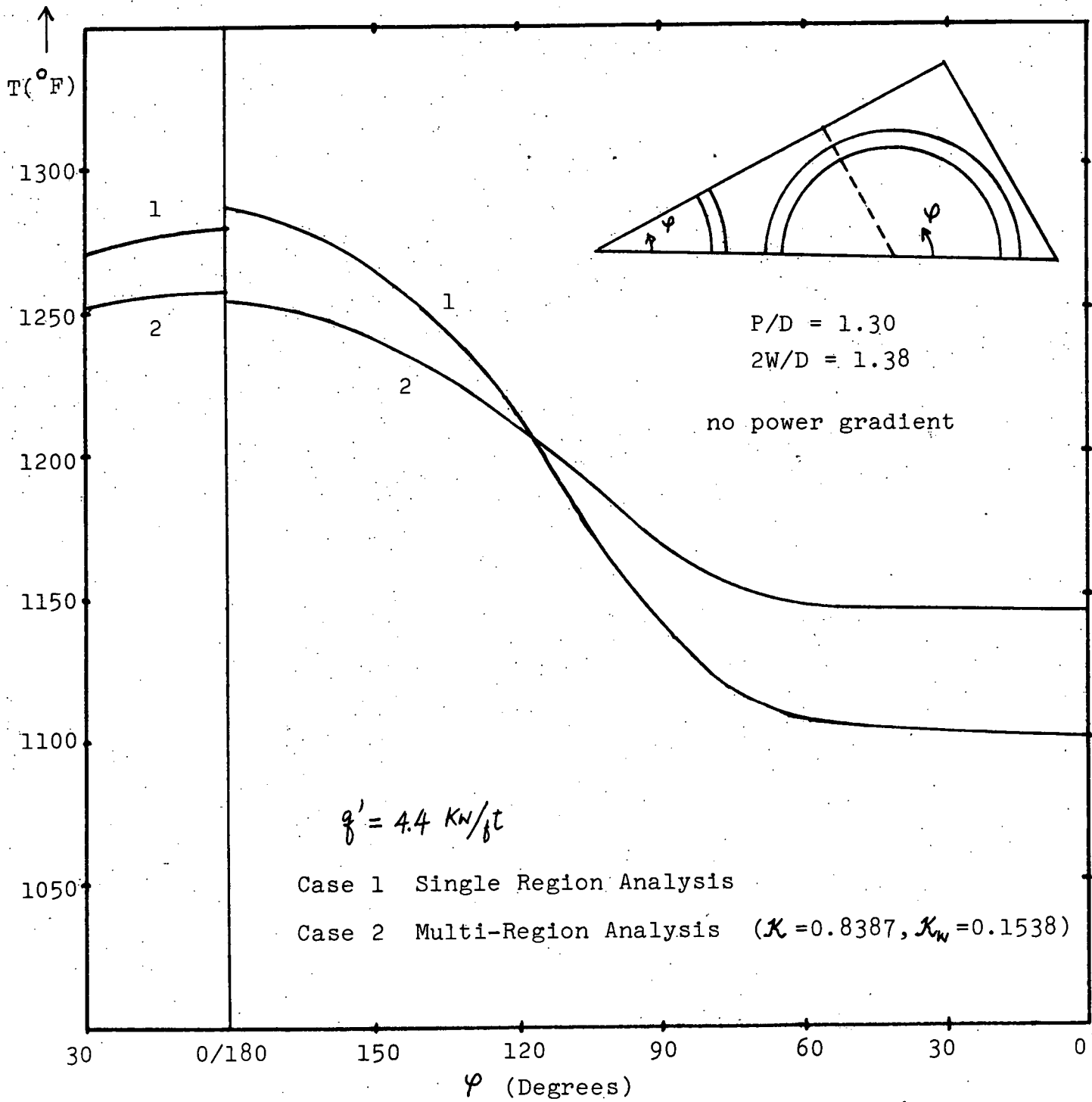
Fig. 2   Clad Temperature Distributions for a 7 Pin Bundle
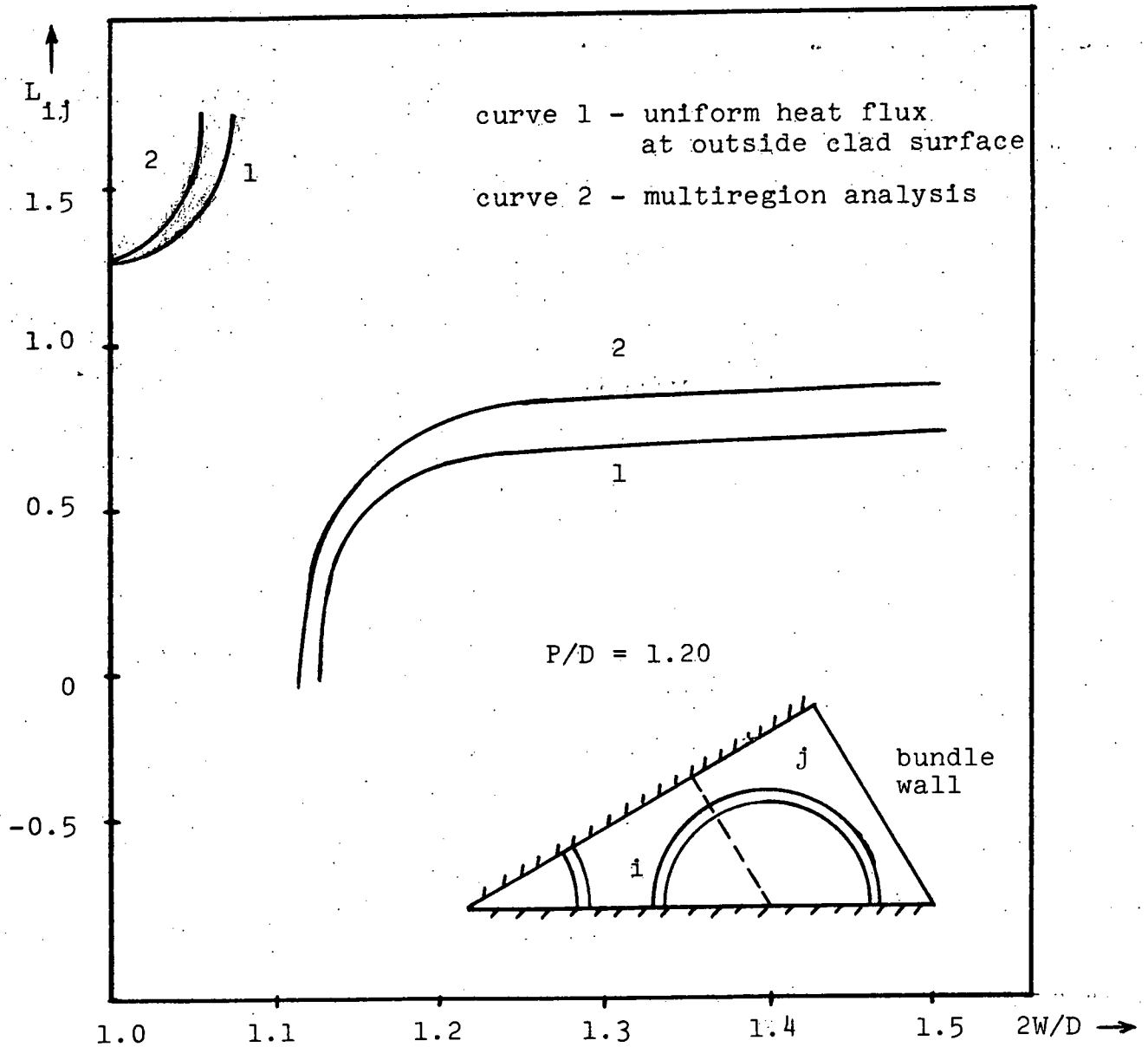
Fig. 3 Normalized Effective Conduction Mixing Lengths vs.
Wall Distance in The Range of Interest

Table 2

Effect of different L_{ij}'s upon average
cell temperatures and their differences
as calculated by COBRA-IIIC at core exit
P/D = 1.2, 2W/D = 1.24 $\overline{q}$"=0.3x10^6 BTU/ft^2-hr

	$\overline{T}_{internal}$ °F	$\overline{T}_{corner}$ °F	ΔT °F
without conduction	1025.42	924.37	101.05
L_{ij} = 1.0	1011.74	931.46	80.28
L_{ij} = 0.65	1006.08	934.39	71.69

cell 1　　　　　　　　　cell 2
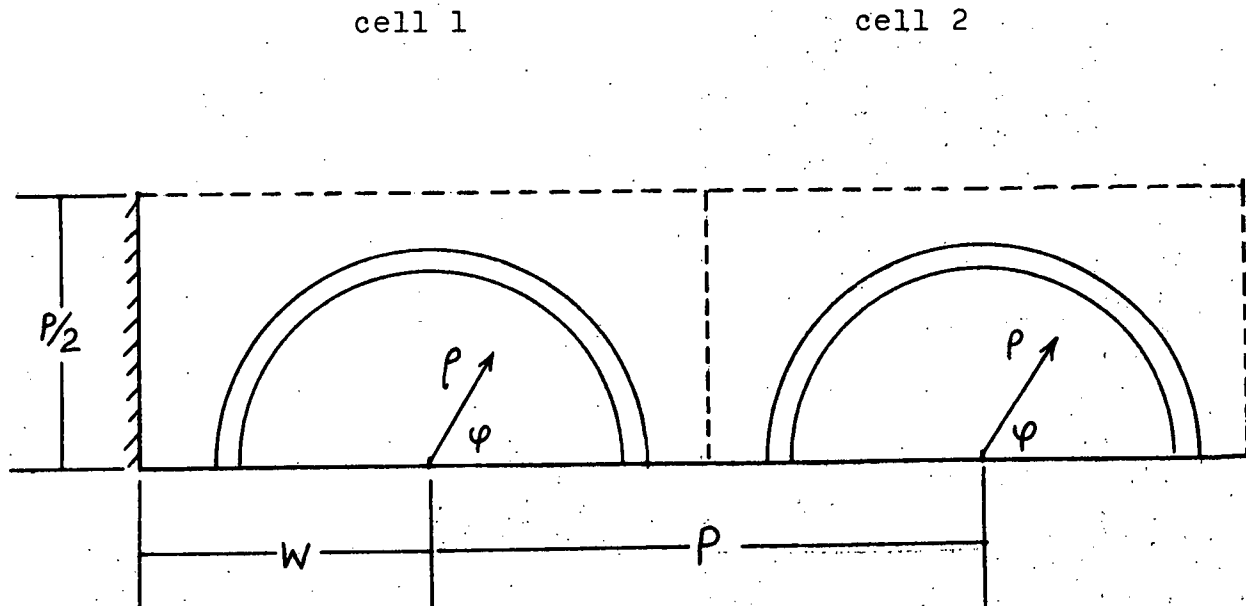
Fig. 4:　A Two Cell Section of a Square Array Configuration
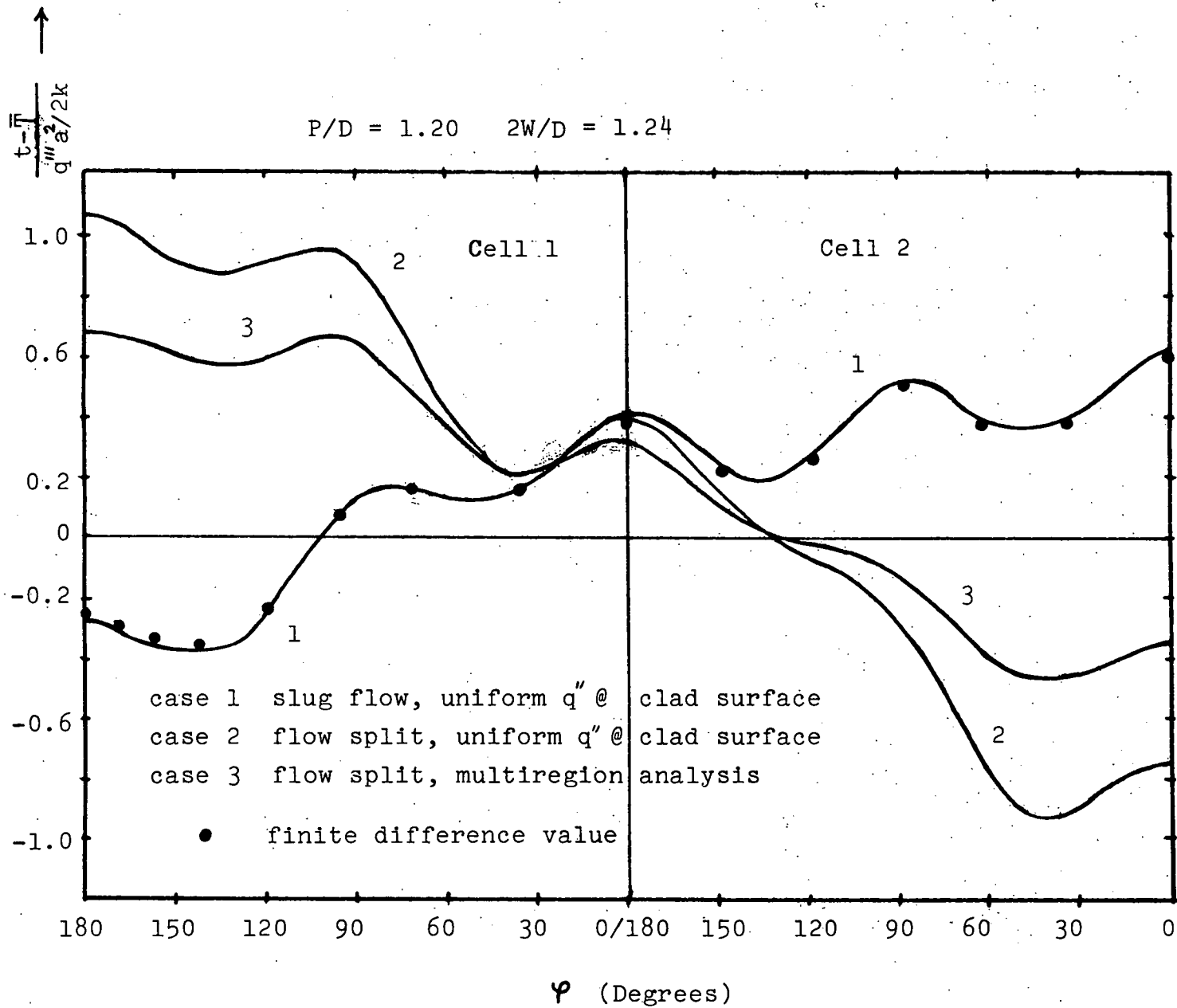
Fig.5 Clad Temperature Distributions of the Two-Cell Configuration

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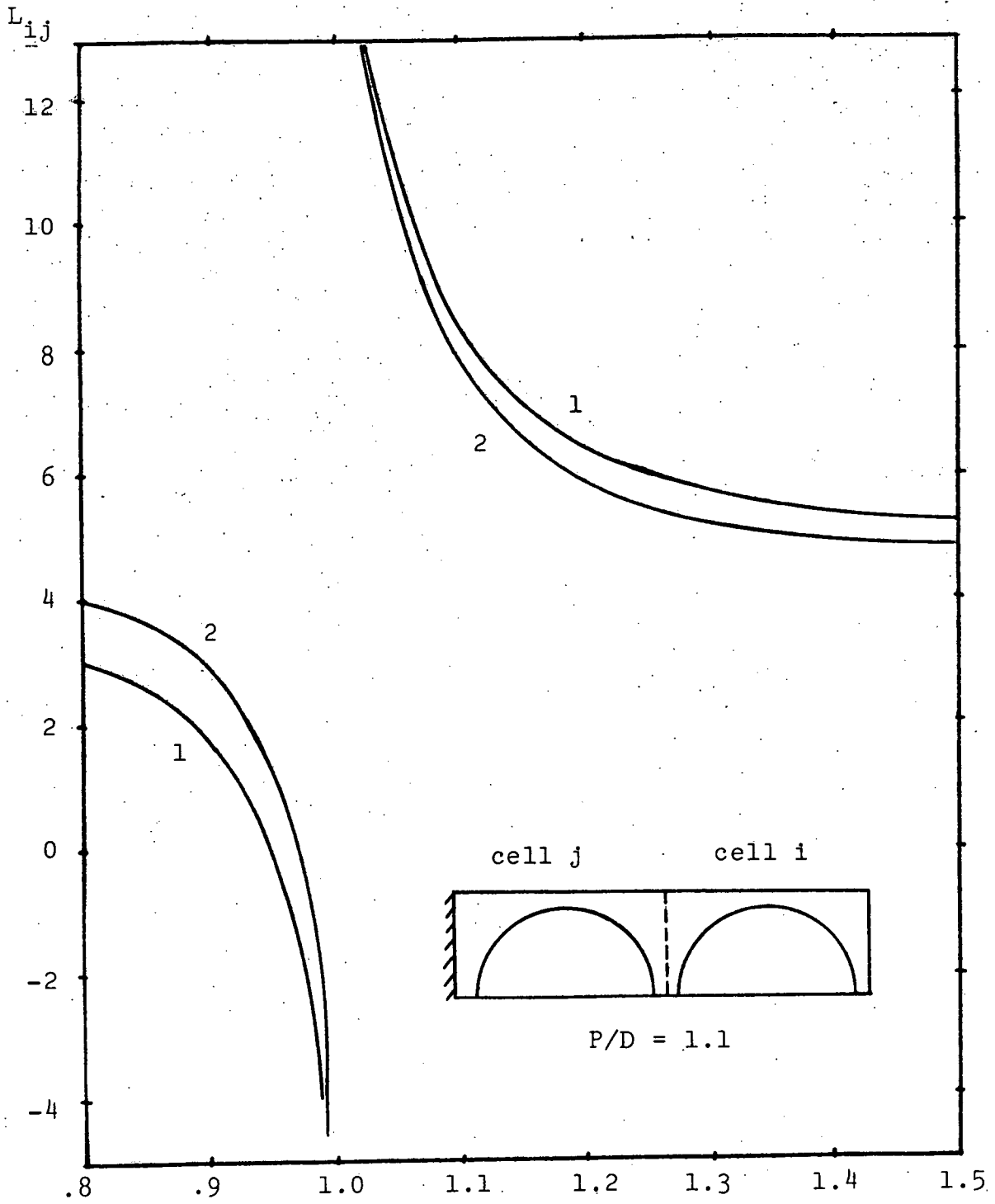
Fig. 6 L_{ij} in Square Cell Configuration
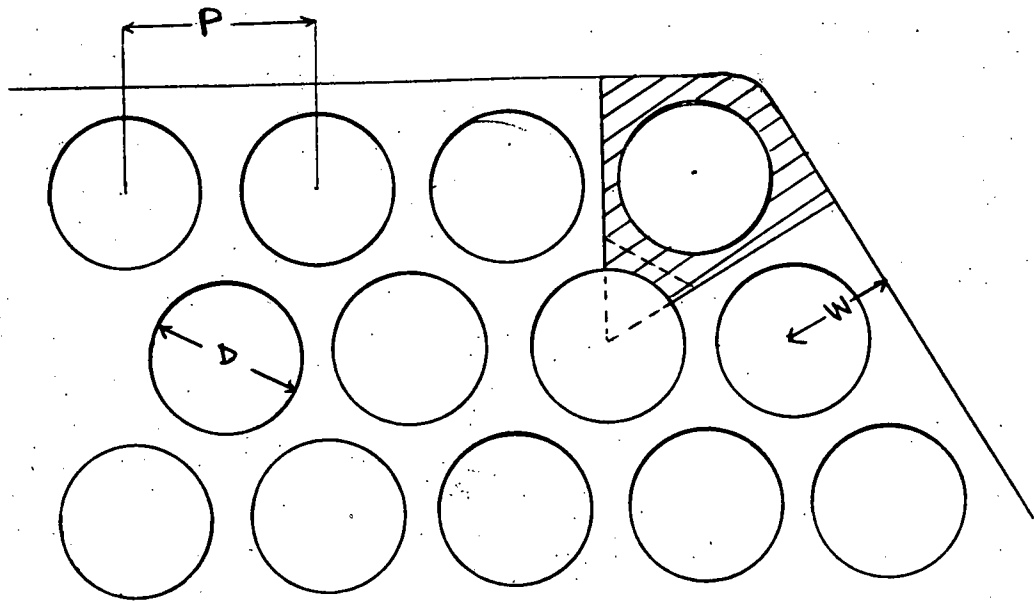
Fig. 7a: Geometry of the Coupled Two-Cell Analysis for the Corner Region.
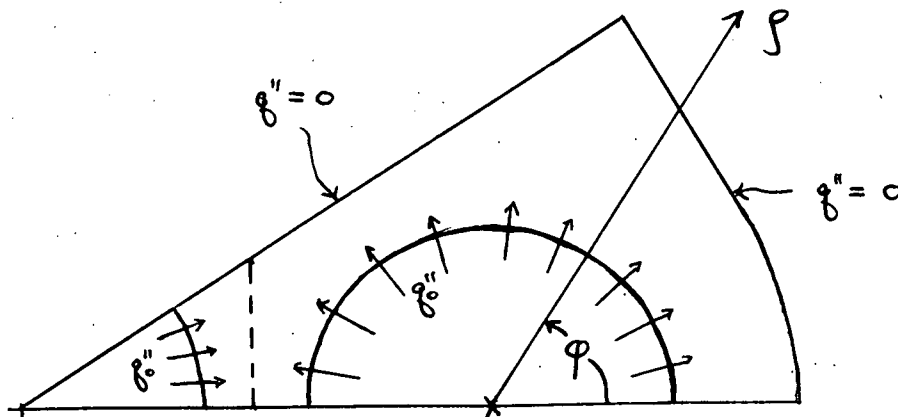
Fig. 7b: Thermal Boundary Conditions for the Coupled Two-Cell Analysis

$$\Theta(\varsigma.\varphi.\zeta) = \frac{T(r.\varphi.z) - T_{in}}{q'' r_o / k}$$

Conditions : $P/D = 1.3$, $2W/D = 1.38$

$D = 9$ mm. , $q'' = 104$ W/cm^2

$T_{in} = 415°C$, $Pe = 372$

—— coupled cell

- - - - isolated cell

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Fig. 8: Comparison of Corner Pin Clad Temperature Distributions for One and Two-Cell Analyses

Fig. 9: Axial Development of Corner Pin Clad Temperature Distribution as Predicted by Coupled Two-Cell Analysis

Fig. 9 (Continued)

Fig. 10: Axial Development of Internal Pin Clad Temperature
Distribution as Predicted by Coupled Two Cell Analysis
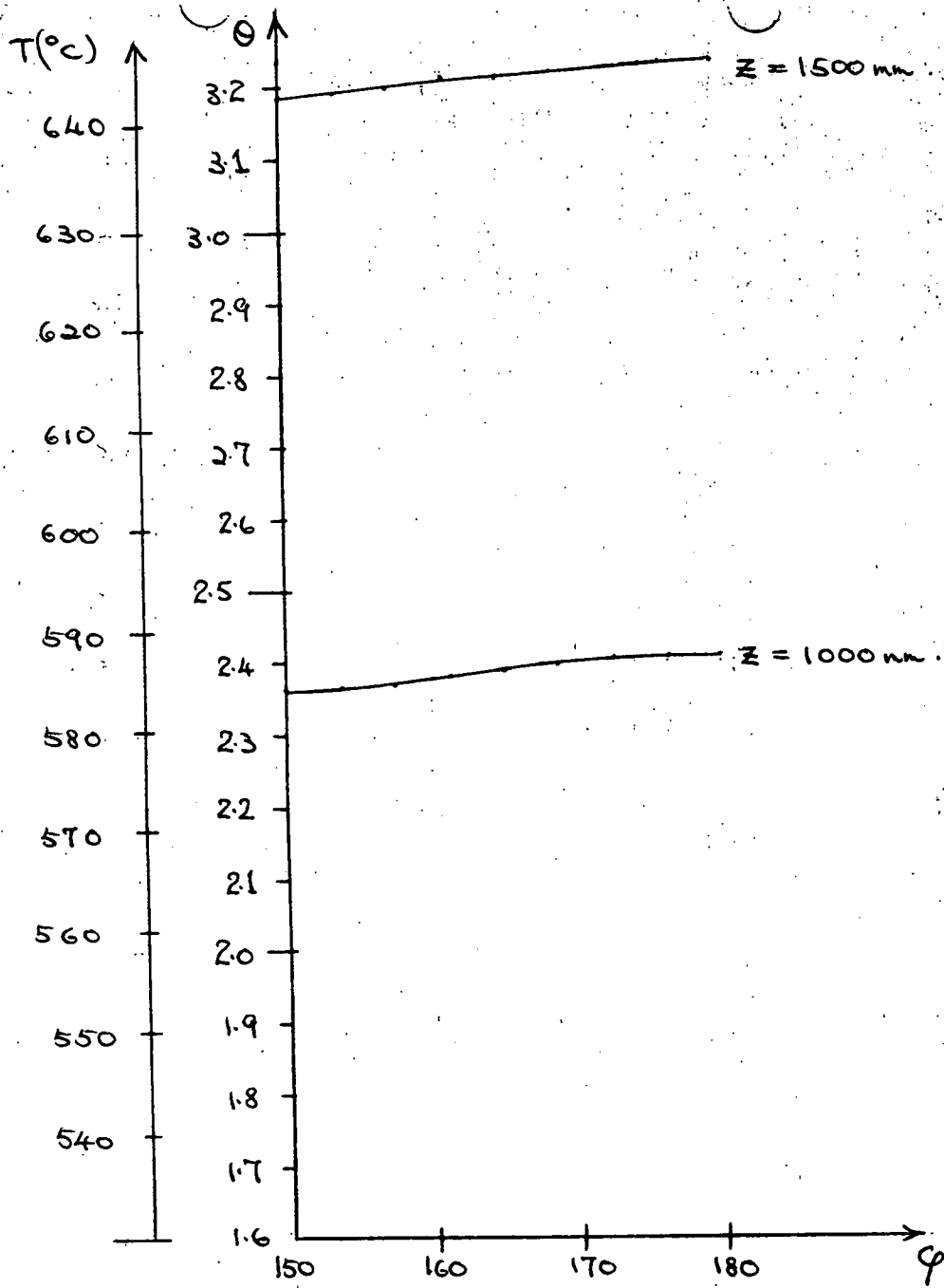
Fig. 10 (Continued)
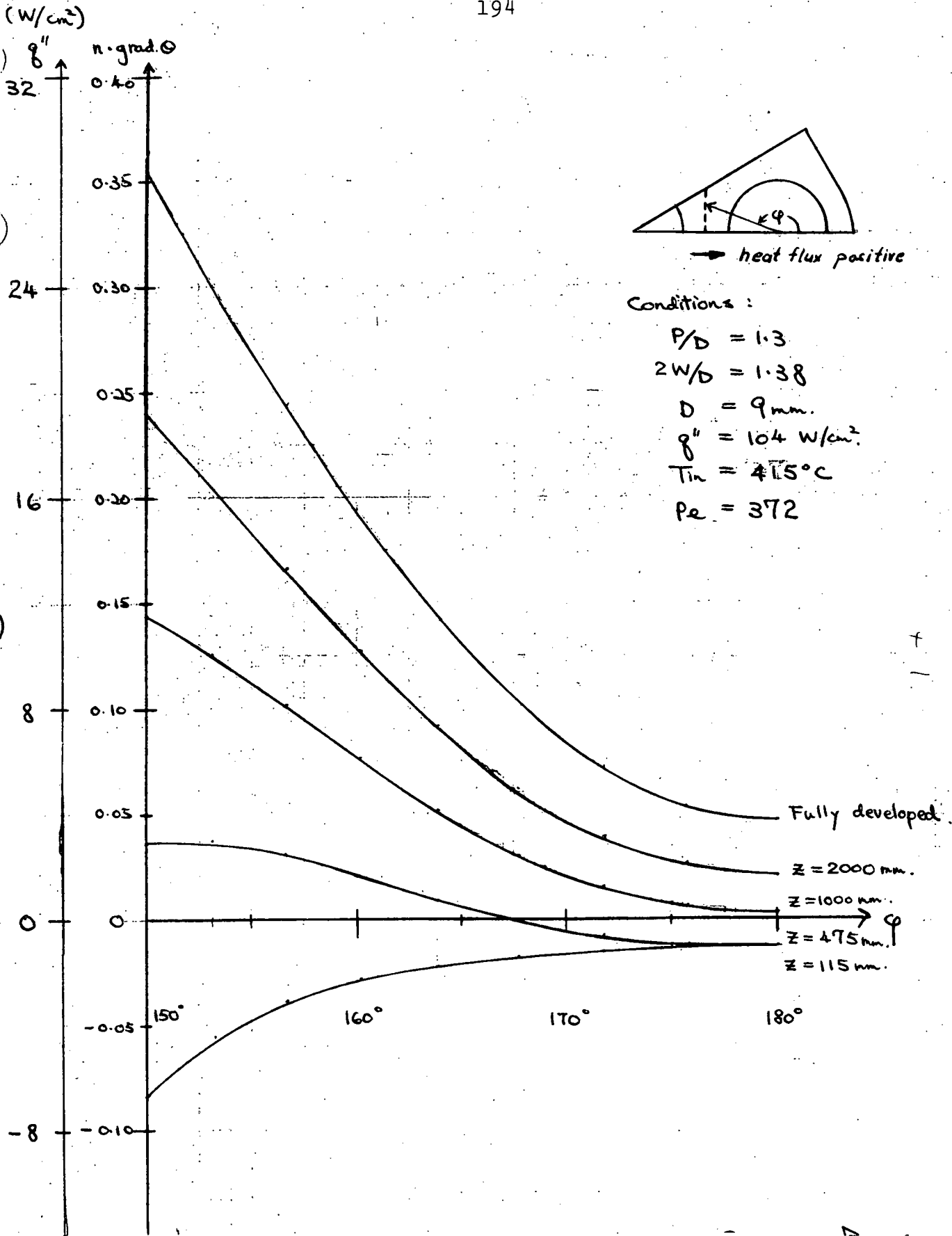
Fig. 11: Axial Development of the Heat Flux Distribution Along
the Imaginary Common Boundary Between the Two Cells
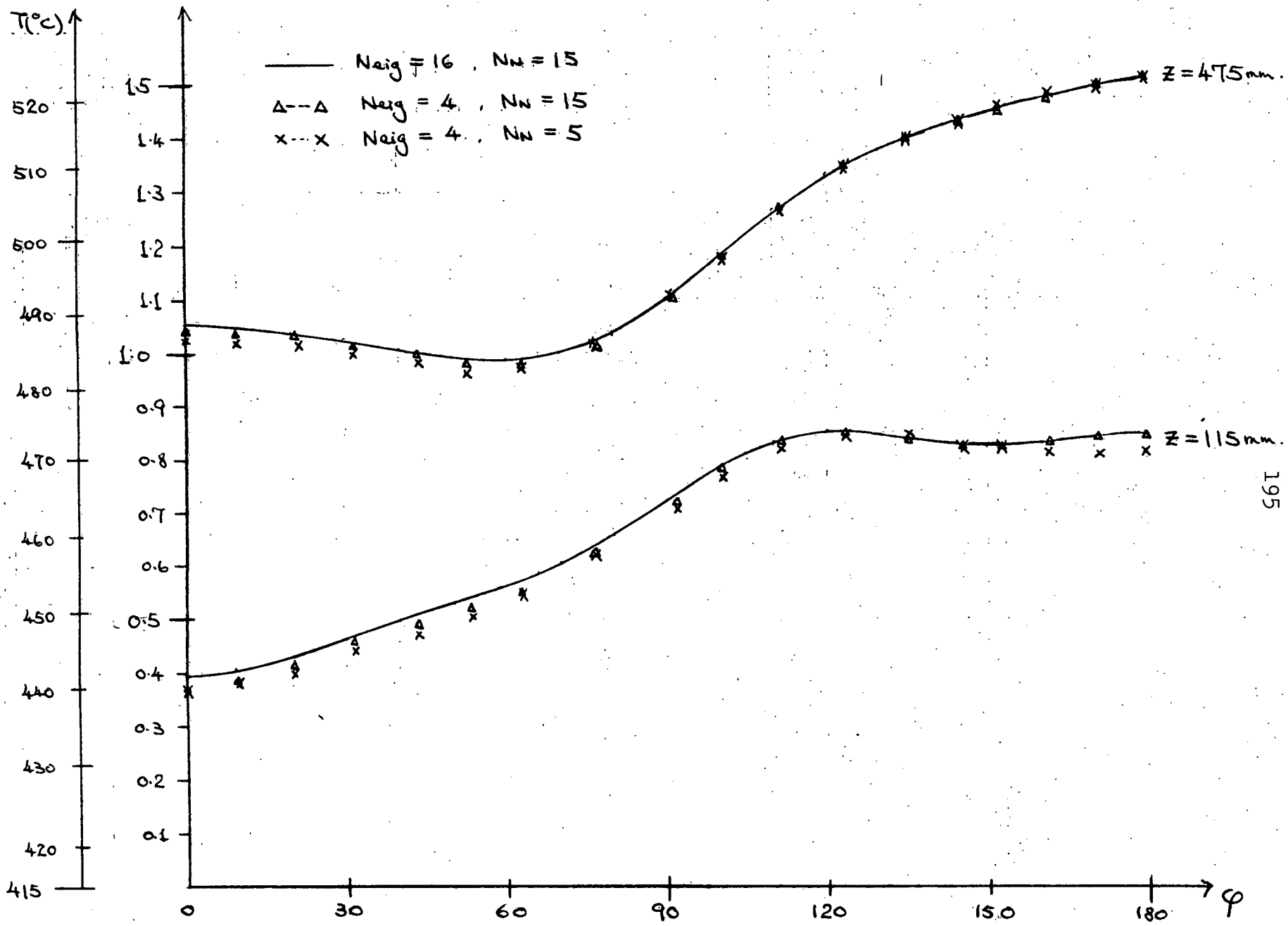
Fig. 12: Effect of Upper Limit in the Eigenvalues Upon the Corner Pin
Clad Temperature Distribution

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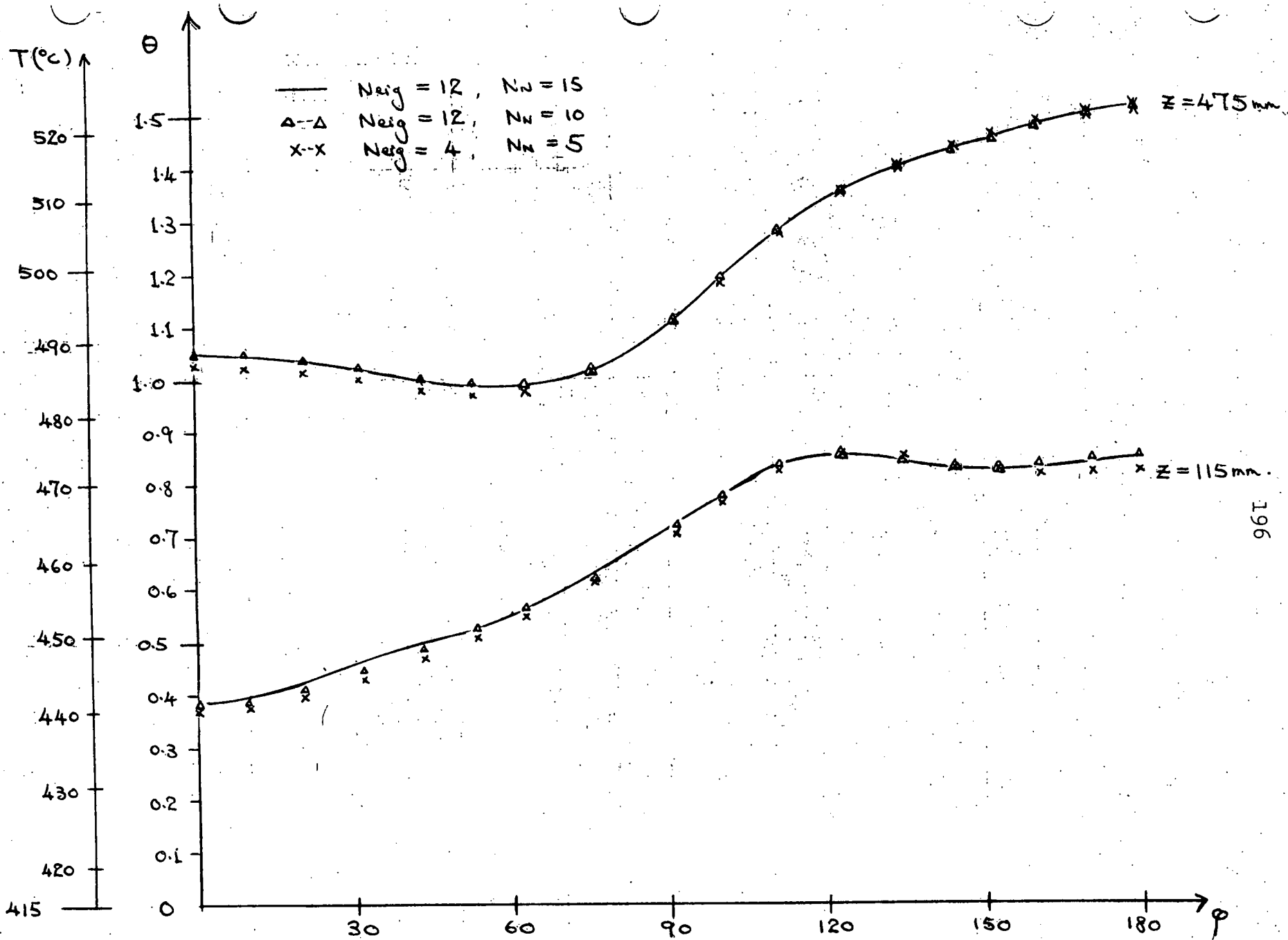
Fig. 13: Effect of Upper Limit in the Harmonics Upon the Corner Pin
Clad Temperature Distribution

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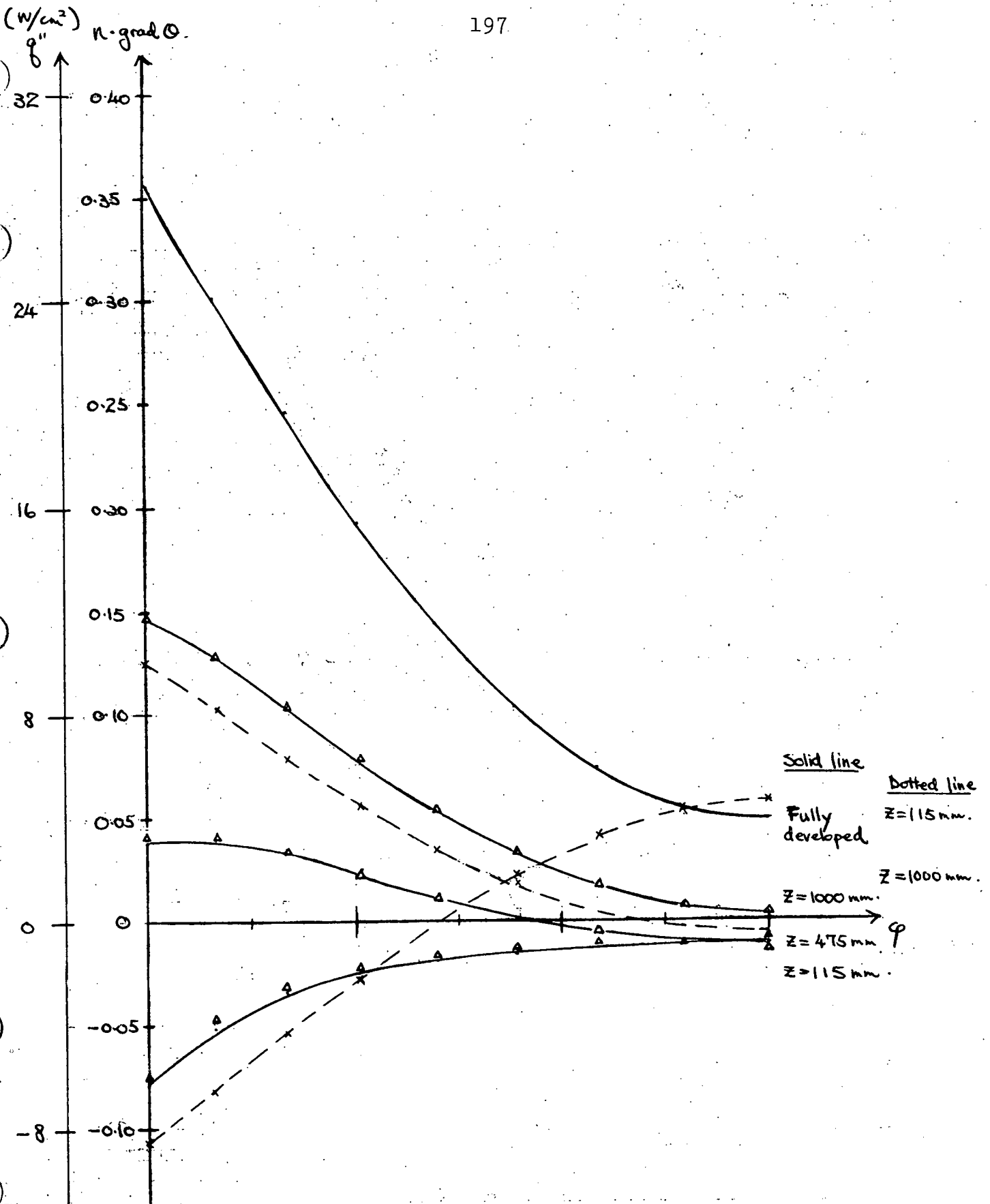
Fig. 14:   Effects of Upper Limits in the Eigenvalues and
Harmonics upon the Axial Development of the
Heat Flux Distribution
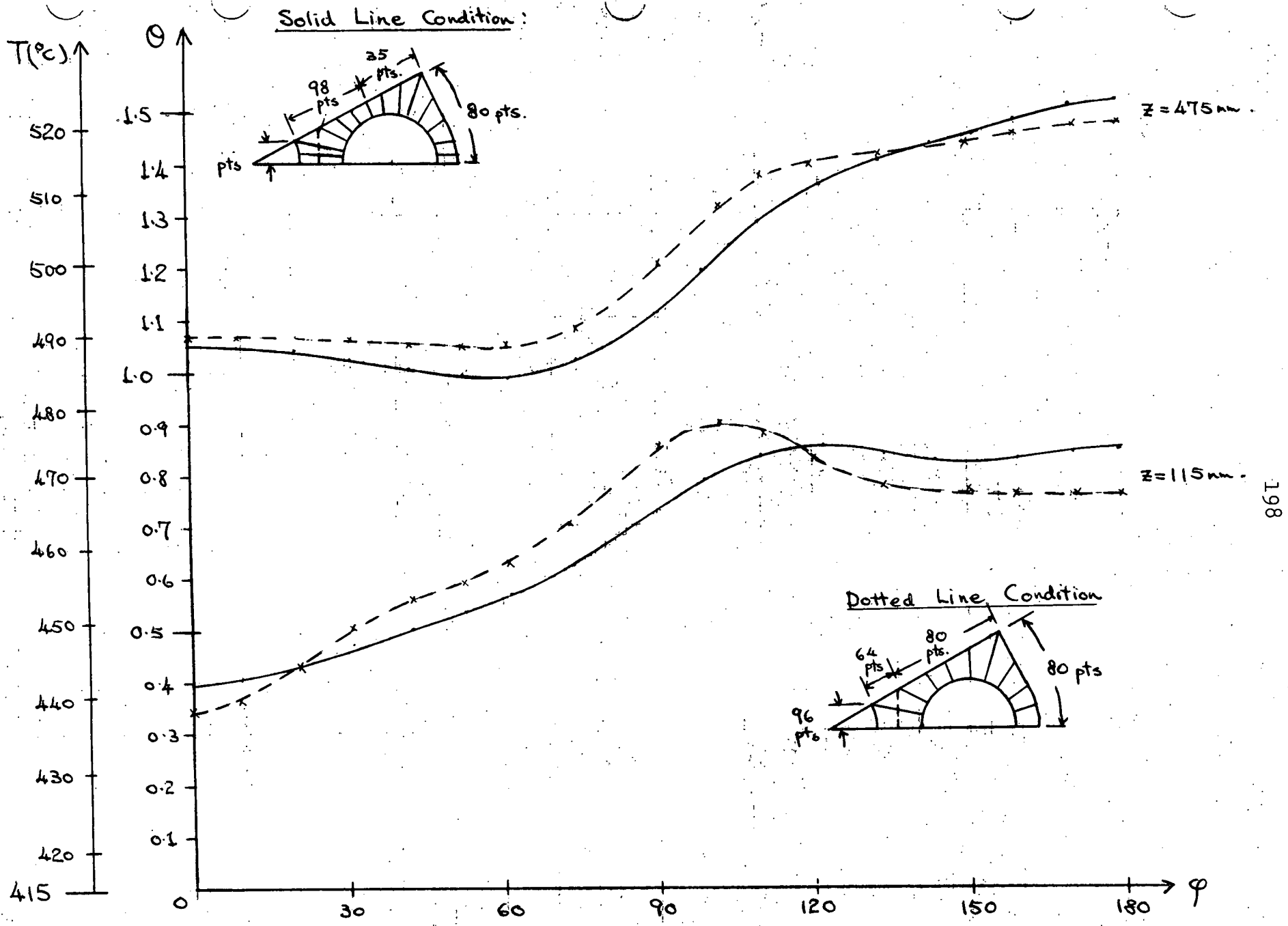
Fig. 15: Effect of Different Sets of Points at the Inlet Cross-Section
Upon the Corner Pin Clad Temperature Distribution
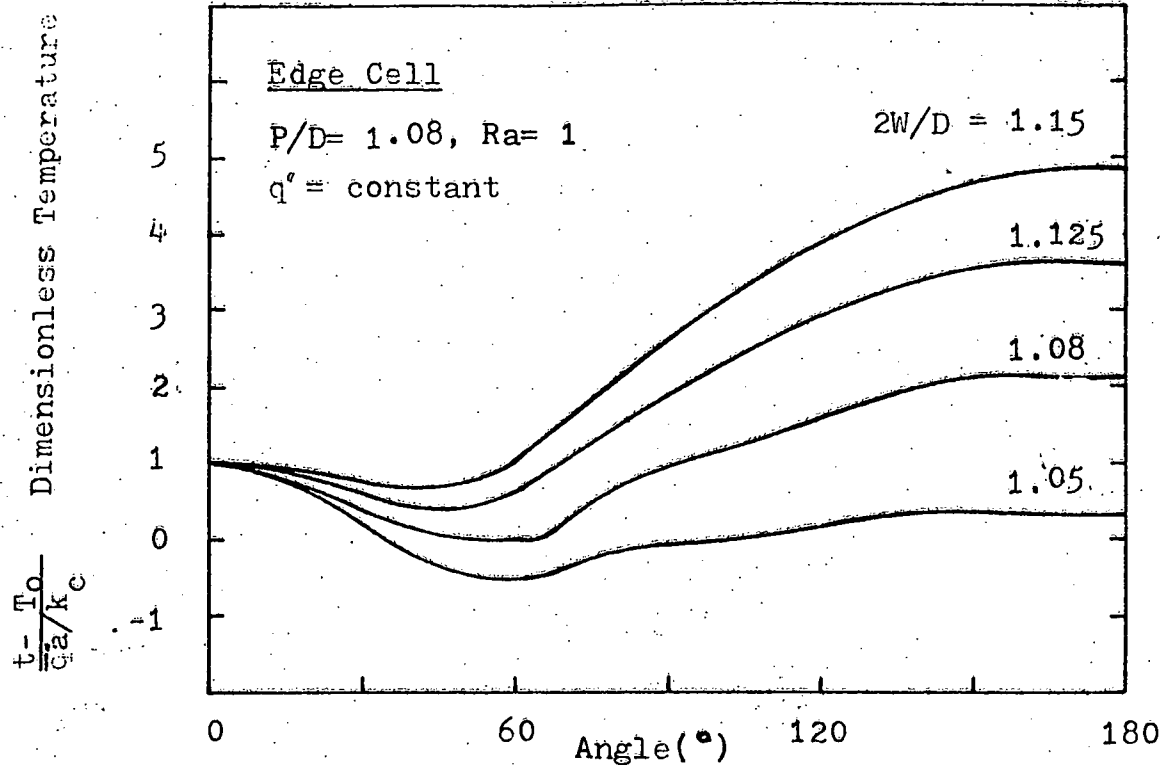
Fig. 16   Peripheral Temperature Variation
along outside Clad Surface   (a) for P/D= 1.08 and
Ra= 1, and   (b) for P/D= 1.08 and Ra= 10^4 in Edge Cell.
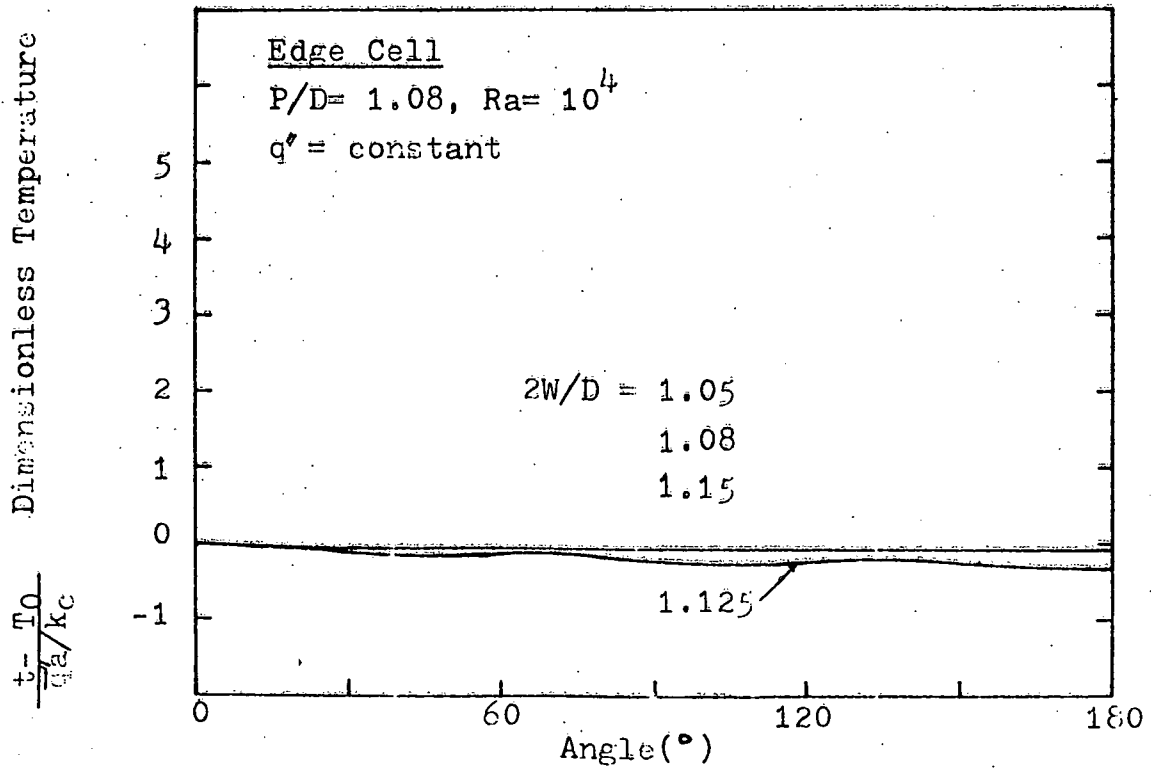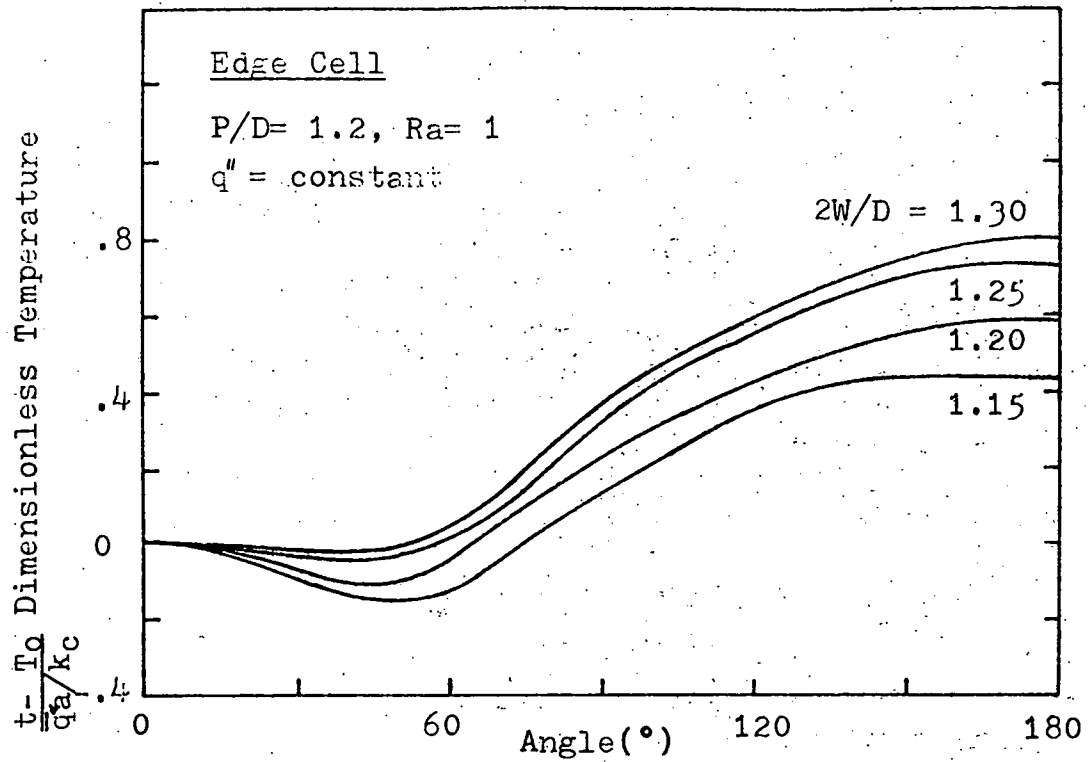
Fig. 17   Peripheral Temperature Variation
along outside Clad Surface for P/D = 1.20
(a) Ra = 1
(b) Ra = 10^4
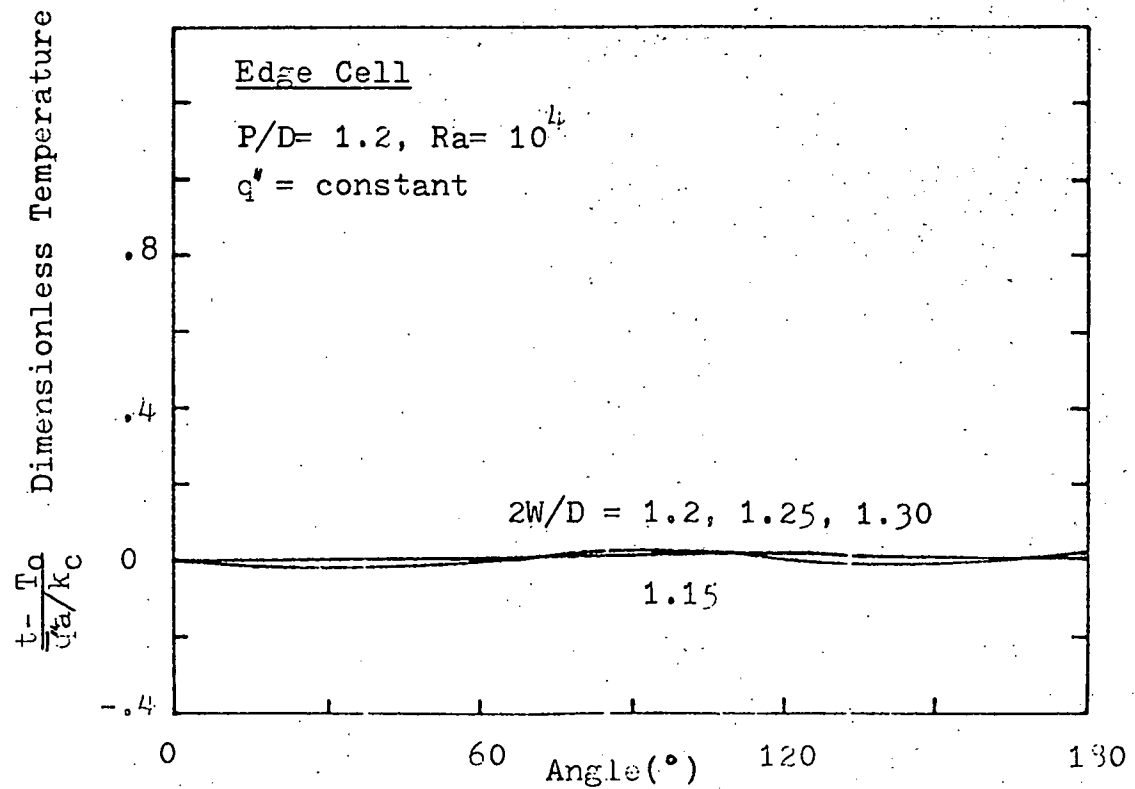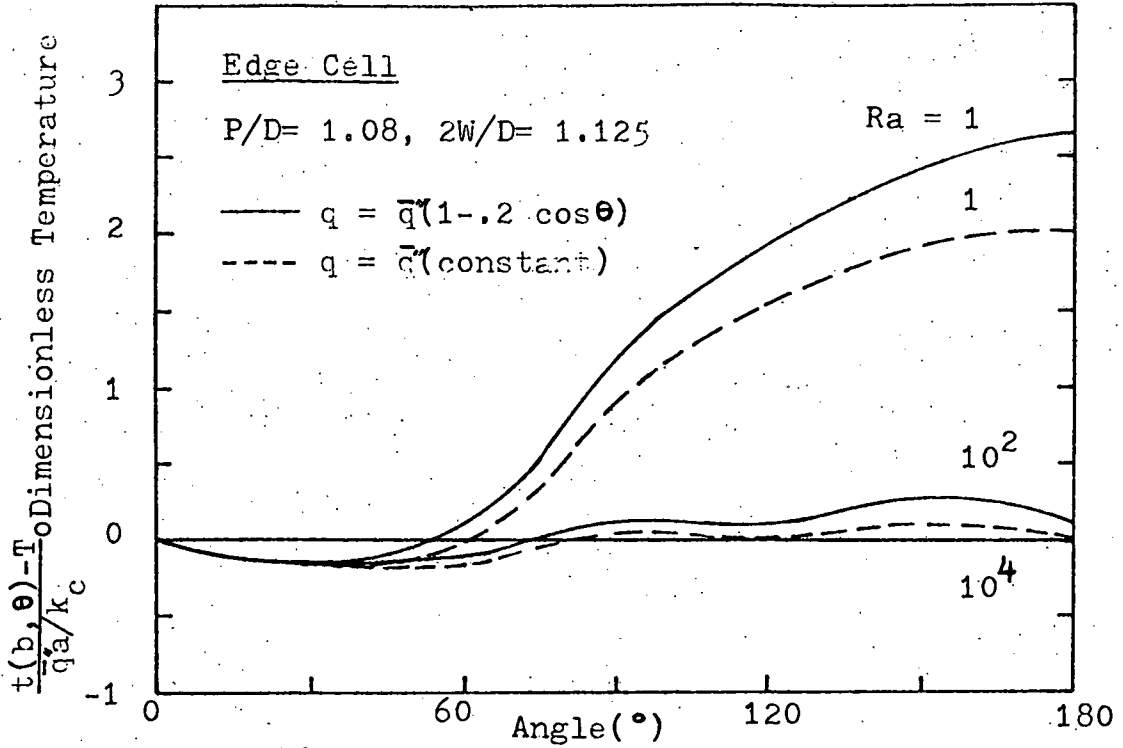
Fig. 18 Peripheral outside Clad Surface
Temperature Variation for both constant and varying
heat flux at (a) P/D=1.08 and 2W/D= 1.125
(b) P/D=1.20 and 2W/D= 1.30
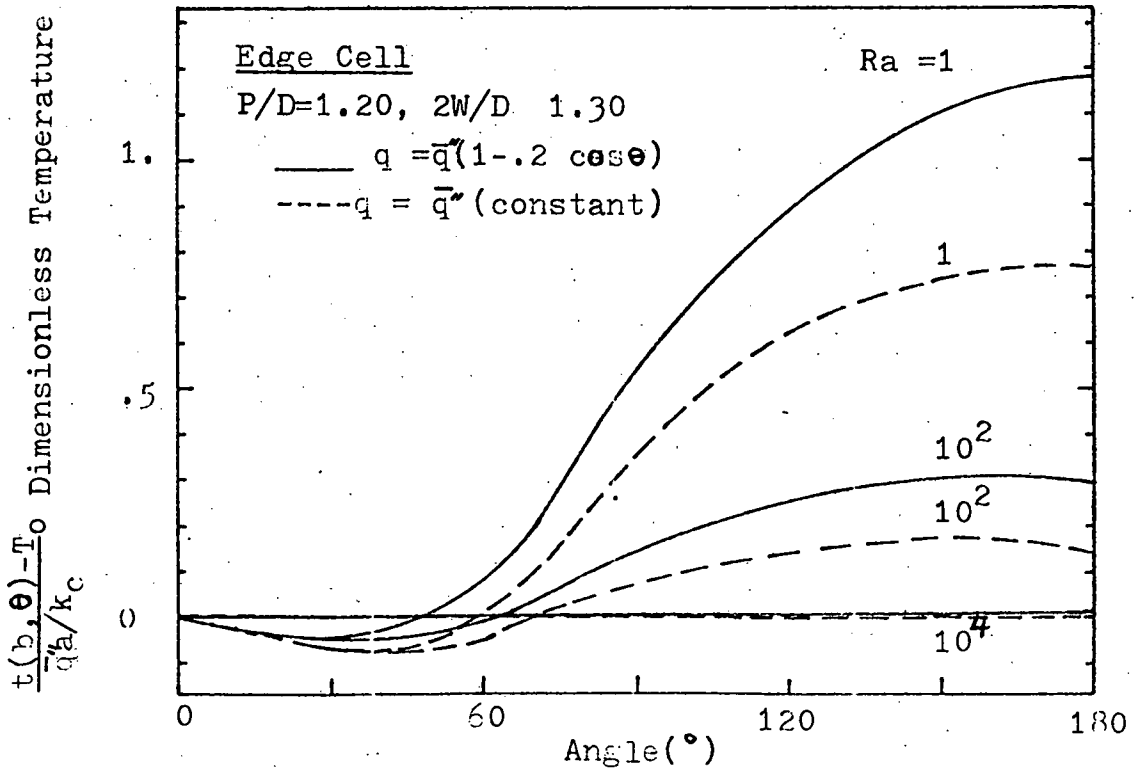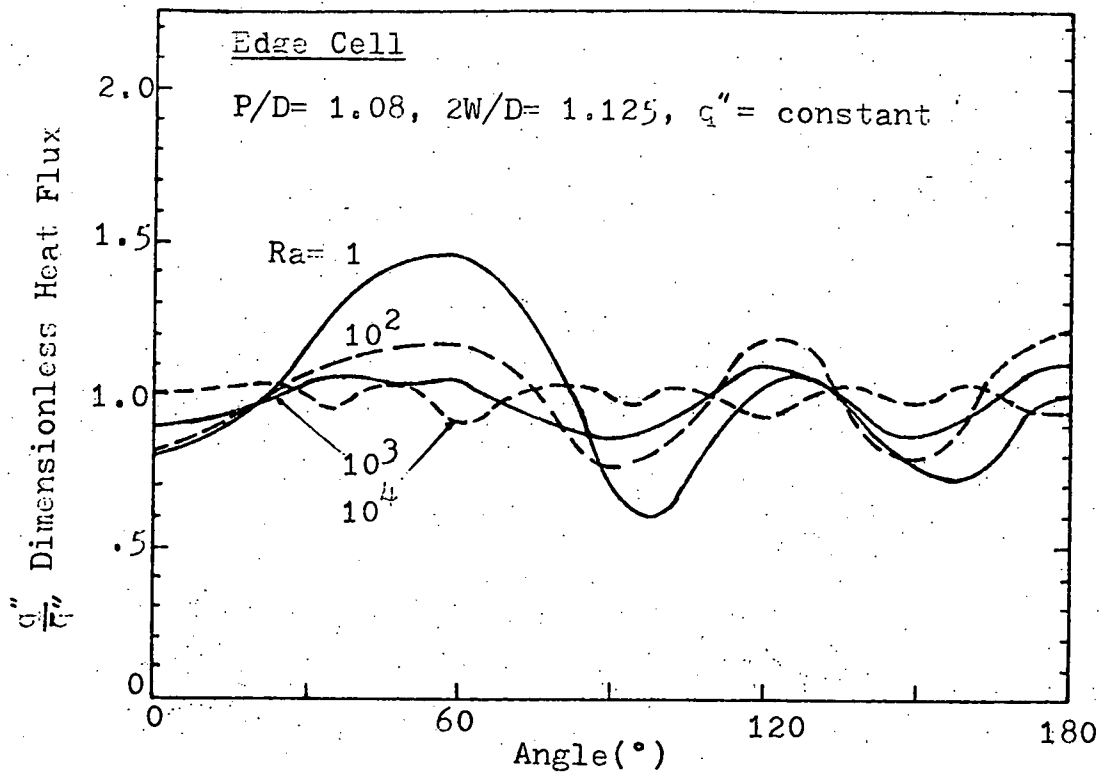
Fig. 19   Peripheral Heat Flux Variation
for P/D = 1.08 and 2W/D = 1.125 at
(a) q″= constant
(b) q″ = q̄(1-.2 cosθ)
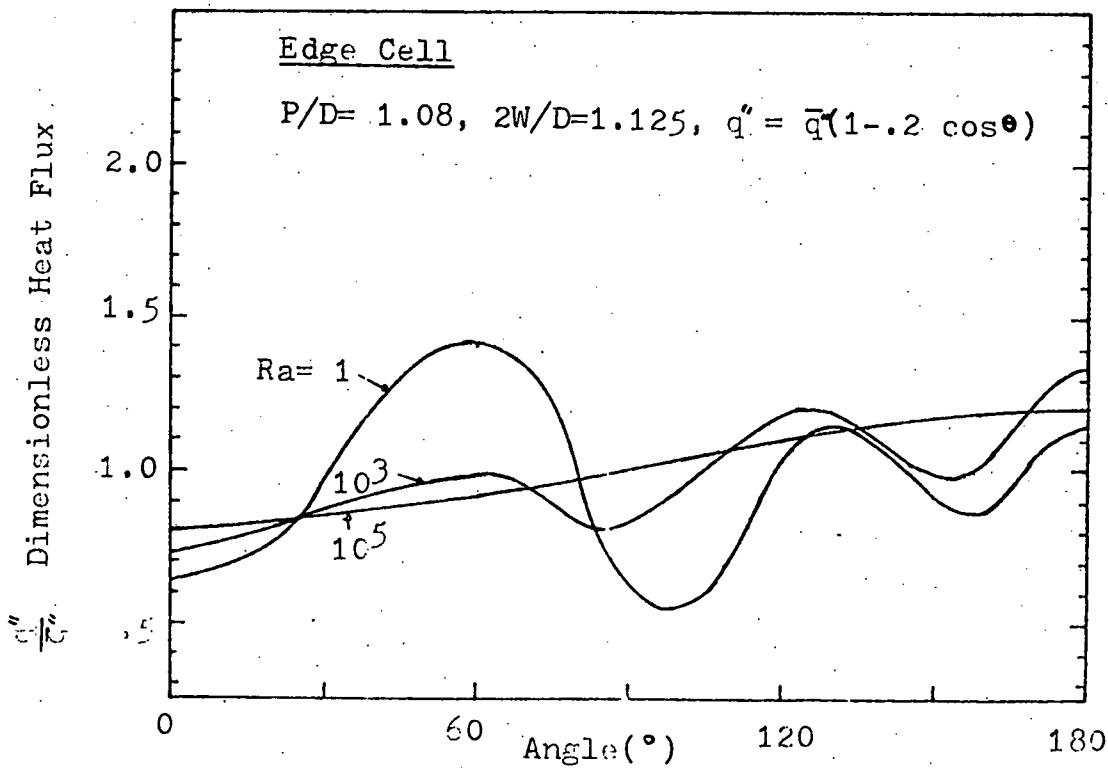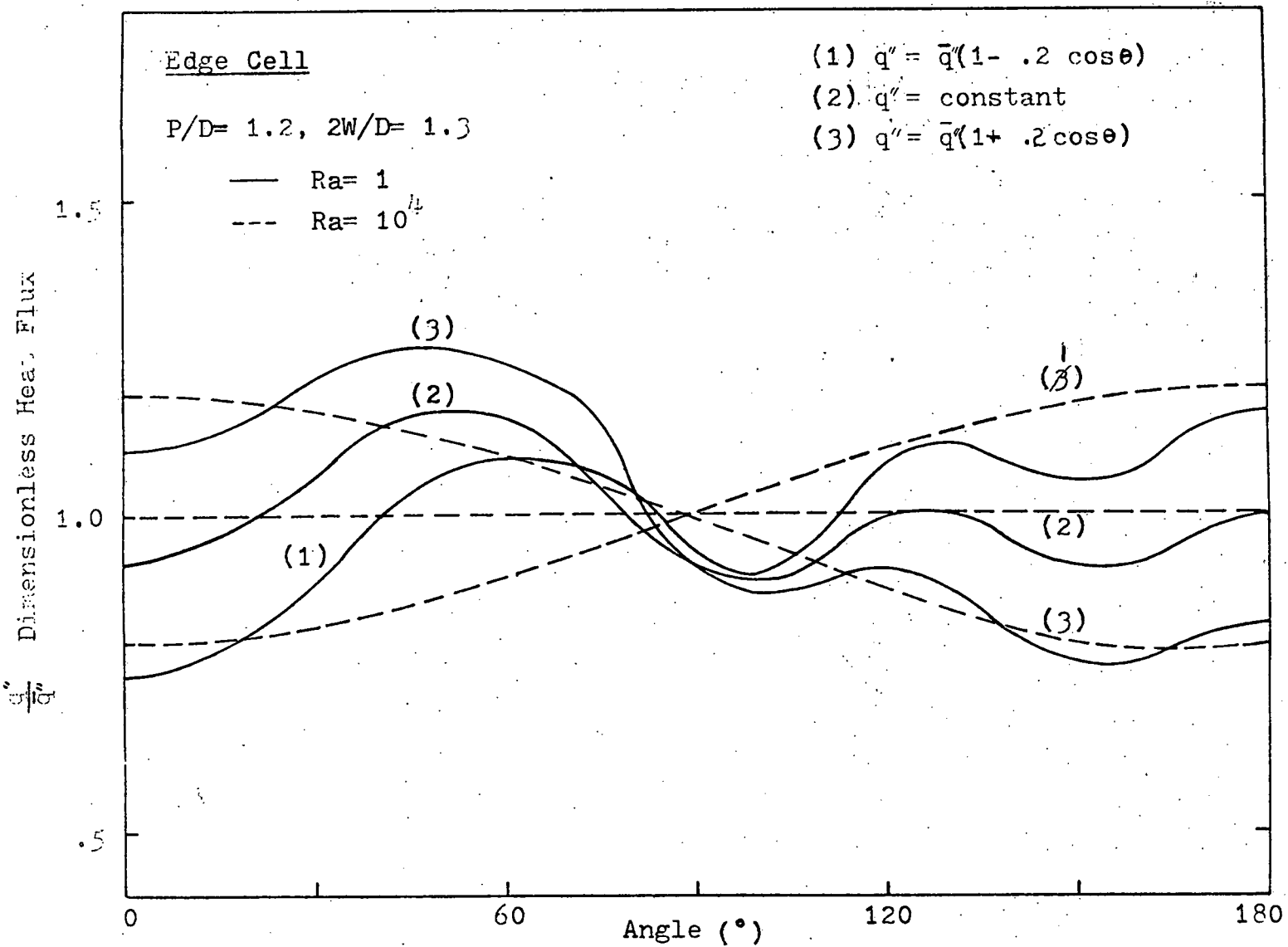
Fig. 20 Peripheral Heat flux Variation for P/D= 1.2 and
2W/D=1.3 at three different Heat Fluxes around inside surface.
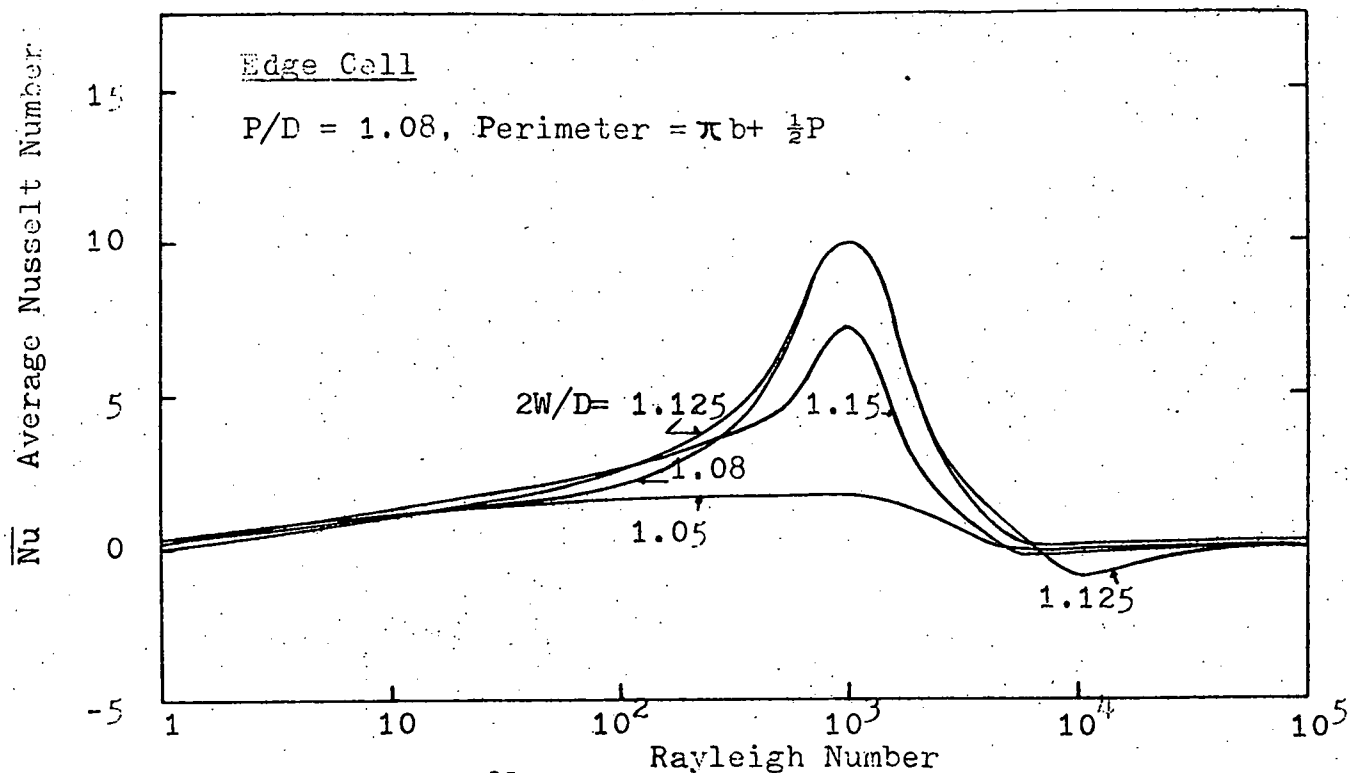
Fig. 21   Average Nusselt Number vs Rayreigh
Number for P/D =1.08 based on Perimeter b+ ½P.
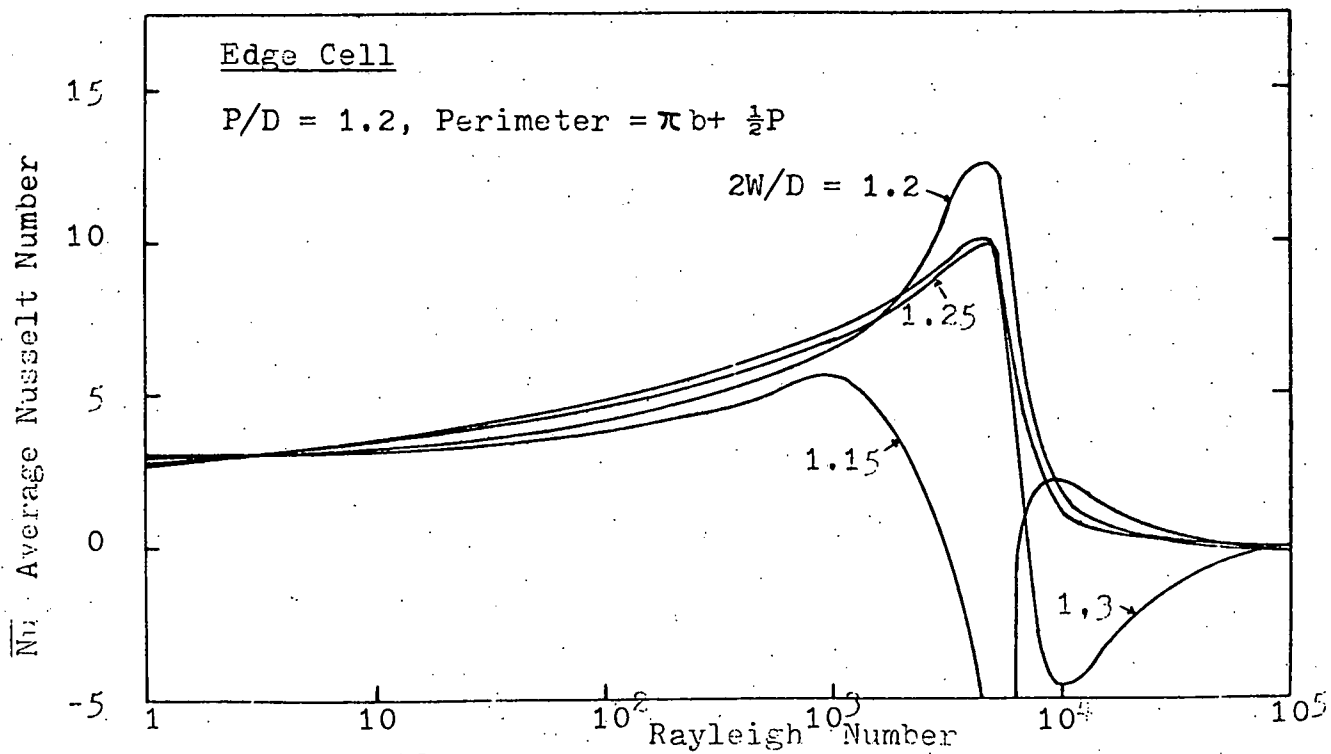
Fig. 22   Average Nusselt Number vs Rayleigh Number
for P/D = 1.2 and Perimeter = b + ½P.
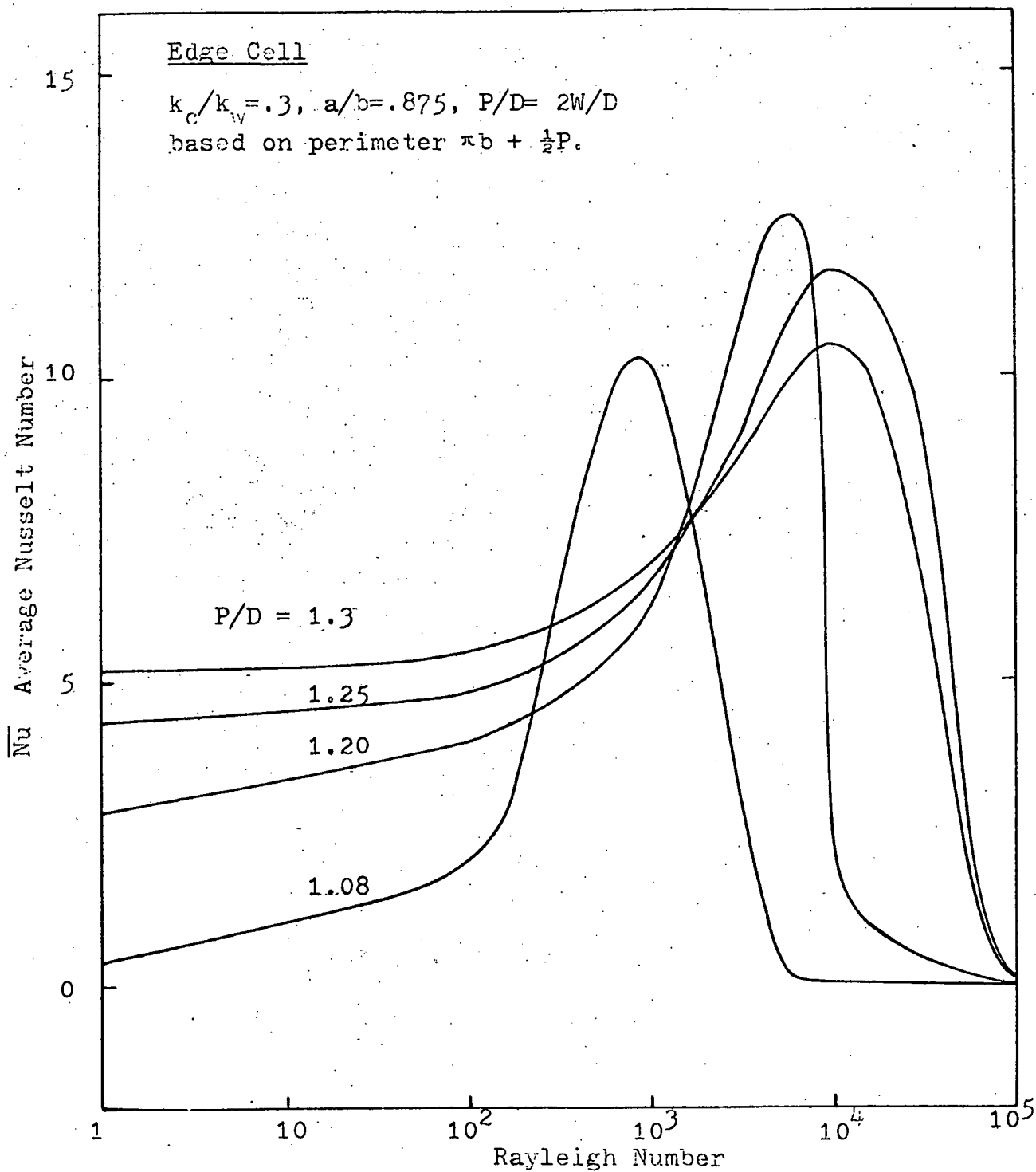
Fig. 23. Average Nusselt Number vs Rayreigh Number
For P/D= 2W/D based on Perimeter $\pi b + \frac{1}{2}P$.

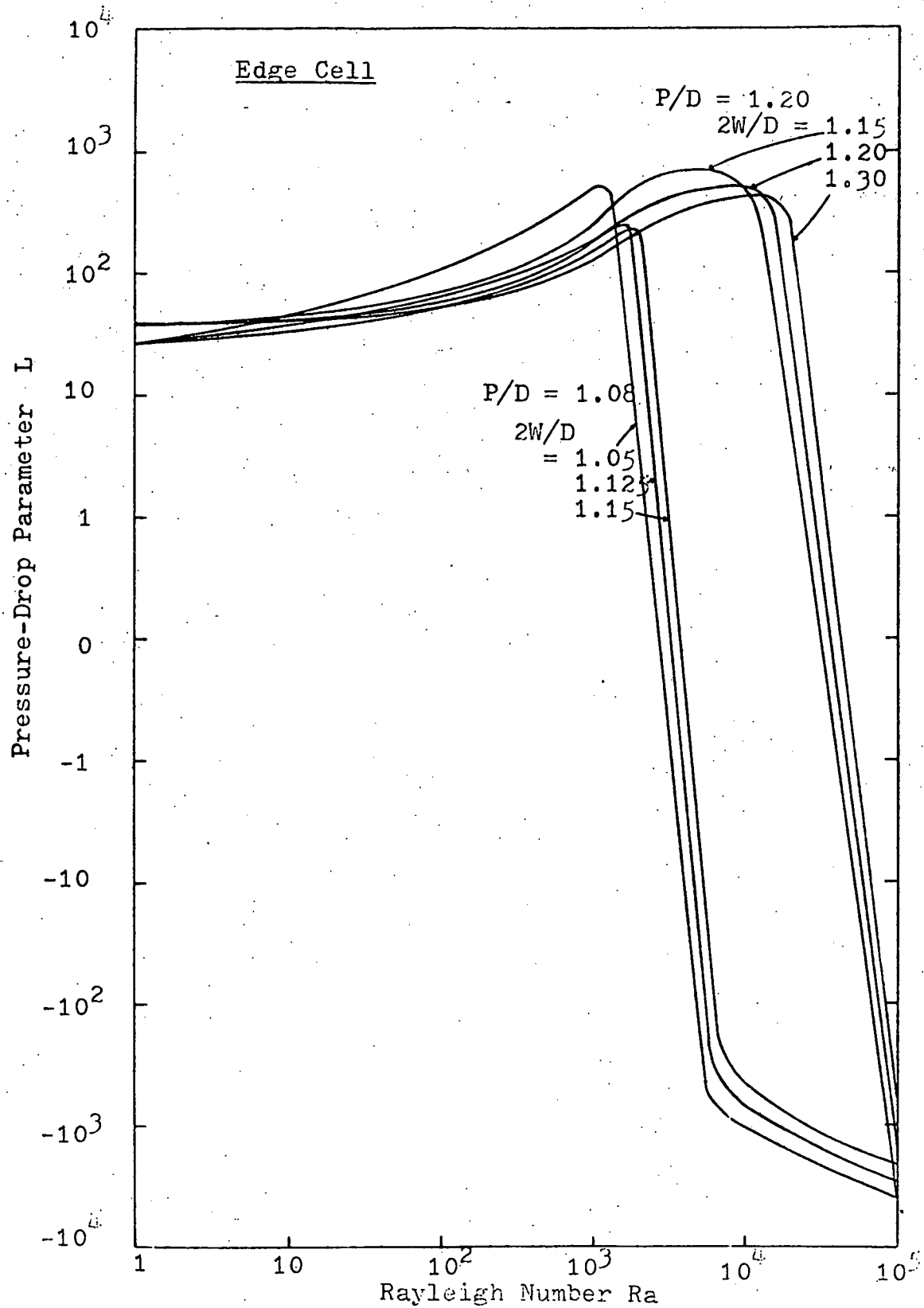Fig. 24 Velocity Profiles at several Angles for two Geometries, P/D= 1.08 1nd 2W/D= 1.125, P/D= 1.20 and 2W/D= 1.30.
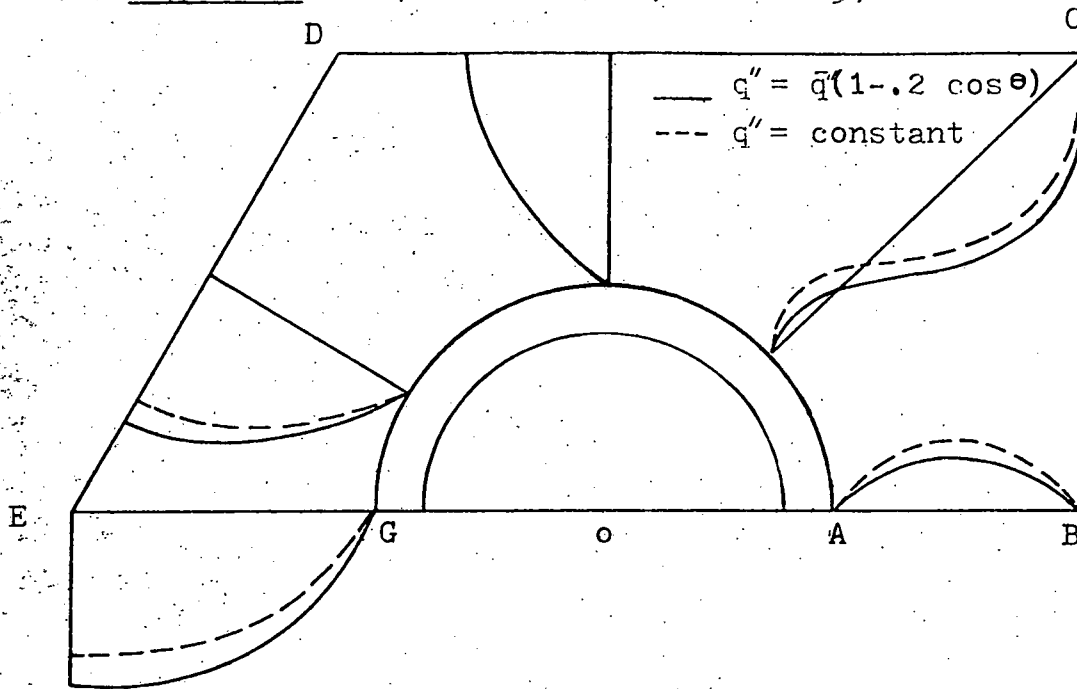
Fig. 25 Pressure-drop Parameter vs
Rayleigh Number for P/D = 1.2 and 1.08.

(a) <u>Edge Cell</u>   P/D = 1.08, 2W/D = 1.125, Ra = 10³
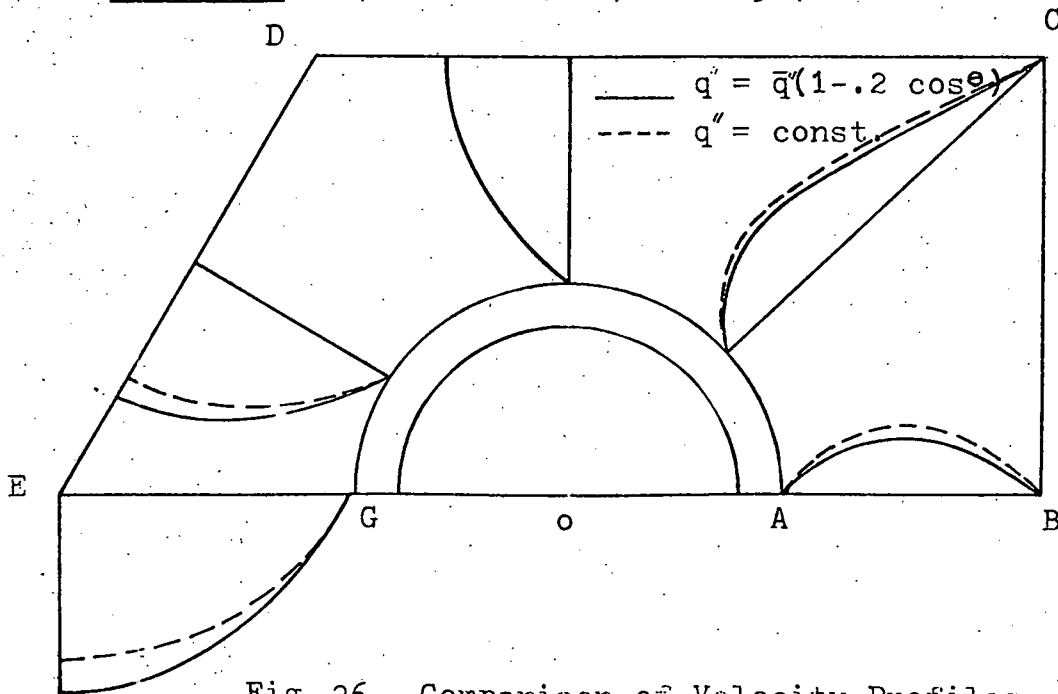
(b) <u>Edge Cell</u>   P/D = 1.20, 2W/D = 1.30, Ra = 10³

Fig. 26   Comparison of Velocity Profiles
in Coolant between q″ = q̄″(1-.2 cosθ) and q″ = constant.