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Part One

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WATER RESOURCES RESEARCH PROGRAM

SURFACE THERMAL PLUMES: EVALUATION OF
MATHEMATICAL MODELS FOR THE NEAR AND COMPLETE FIELD

by

William E. Dunn, Anthony J. Policastro,
and Robert A. Paddock

GREAT LAKES PROJECT

ENERGY AND ENVIRONMENTAL
SYSTEMS DIVISION

ENVIRONMENTAL STATEMENT PROJECT

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Energy and Environmental Systems Division
Great Lakes Project

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FOREWORD

A major objective of the Great Lakes Research Program for the past few years has been to determine which mathematical models, if any, could be used to predict with reasonable reliability the characteristics of thermal plumes from surface outfalls. The present report is the final one in a series on the subject and summarizes our attempts to test the usefulness of existing mathematical models by comparing model predictions with laboratory and field data. In conducting these comparisons, we had to examine many aspects of hydrothermal prediction. These included an in-depth study of the formulation of the models being tested and a close examination of the physical processes being modeled. Comparisons of the model predictions to experimental data were the major ingredient in our evaluation of a given model. At the same time, however, our evaluation of the formulation of each model was an integral part of our judgment as to its competitiveness. The present effort to evaluate models updates previously published works in this research area, such as those of Policastro and Tokar (1972); and Benedict, Anderson, and Yandell (1974). Furthermore, we feel that this report presents a viewpoint not available in other review documents, such as those of Brady, Edinger, and Geyer (1974); and Jirka, Abraham, and Harleman (1975). This report advances the state of the art by its evaluation of existing models, its discussion of important modeling issues, and in its suggestions for further modeling improvements.

Because all the models described here were developed by persons outside Argonne National Laboratory, frequent consultation with model authors was necessary to resolve questions. We are grateful to all authors who gave generously of their time and knowledge through conversations and personal correspondence. Without their cooperation a report such as this would have been impossible. Among those researchers who provided valuable assistance were:

1. Dr. Barry Benedict, now at Tulane University, on the Motz-Benedict Model (in Sec. II.B).
2. Dr. Keith Stolzenbach and Mr. Eric Adams of the Massachusetts Institute of Technology on the Stolzenbach-Harleman Model (in Secs. II.C and IV.D).
3. Dr. Edmund Prych now with the U. S. Geological Survey on his own model (in Sec. II.D).
4. Dr. Mostafa Shirazi of the Pacific Northwest Environmental Research Laboratory of the U. S. EPA and Dr. Lorin Davis of Oregon State University on the Prych and Shirazi-Davis Models (in Sec. II.E) and Shirazi's data analysis (Sec. IV.B).
5. Dr. Donald Pritchard of The Johns Hopkins University on the two Pritchard Models (in Secs. II.F and II.G).
6. Dr. John Edinger of Edinger Associates Inc. on the Brady-Geyer Model (in Sec. II.H).

7. Dr. John Paul, now at the U. S. EPA Grosse Ile Laboratory, and Dr. Wilbert Lick of Case Western Reserve University on the Paul-Lick Model (in Sec. II.K).

8. Dr. William Waldrop (now with the Tennessee Valley Authority) and Dr. Richard Farmer of Louisiana State University on the Waldrop-Farmer Model (in Sec. II.L).

9. Mr. James Taft, Dr. Walt England (both now at Science Applications, Inc.), and Dr. John Pritchett of Systems, Science, and Software on the Pritchett-England-Taft Model (in Sec. II.I).

10. Dr. Henry Till of the Electric Power Research Institute on his own model (in Sec. II.J).

11. Dr. Frank Fang of the Virginia Institute of Marine Science on the Surry Plant field surveys (Sec. IV.B).

12. Dr. James Weil now with the Coppola Co. of San Francisco on his work verifying the Prych model at Oskarshamnsverket (in Chapter IV, Sec. E).

13. Mr. Jim Davis of the Boston Edison Company and Mr. Dan Frye of EG&G Environmental Consultants for their discussions with us on the Pilgrim Plant studies (in Sec. IV.D).

14. Dr. Heinz Stefan of the St. Anthony Falls Hydraulic Laboratory for his helpful review comments on Chapter III.

The fact that the model authors mentioned above reviewed our discussion of their work before publication should not be construed as agreement by them with the opinions expressed nor as an endorsement of the presentation.

The length of the present report necessitated its publication in two parts. The titles of the chapters contained in each part are as follows:

Part One

- I. Physical Aspects and Mathematical Modeling of Surface Thermal Plumes
- II. Summary and Evaluation of Individual Models

Part Two

- III. Discussion of Near- and Complete-field Modeling Issues
- IV. Verification Efforts for Surface-discharge Models
- V. Observations on the State of the Art
- Appendixes

The two parts of the report do not stand alone, and the reader is urged to acquire both parts because they are often cross-referenced.

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WATER RESOURCES RESEARCH PROGRAM
SURFACE THERMAL PLUMES: EVALUATION OF
MATHEMATICAL MODELS FOR THE NEAR AND COMPLETE FIELD

by

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ABSTRACT

This report evaluates mathematical models that may be used to predict the flow and temperature distributions resulting from heated surface discharges from power-plant outfalls. Part One discusses the basic physics of surface-plume dispersion and provides a critical review of 11 of the most popular and promising plume models developed to predict the near- and complete-field plume. Part Two compares predictions from the models to prototype data, laboratory data, or both. Part Two also provides a generic discussion of the issues surrounding near- and complete-field modeling.

The principal conclusion of the report is that the available models, in their present stage of development, may be used to give only general estimates of plume characteristics; precise predictions are not currently possible. The Shirazi-Davis and Pritchard (No. 1) models appear superior to the others tested and are capable of correctly predicting general plume characteristics. (The predictions show roughly factor-of-two accuracy in centerline distance to a given isotherm, factor-of-two accuracy in plume width, and factor-of-five accuracy in isotherm areas.)

The state of the art can best be improved by pursuing basic laboratory studies of plume dispersion along with further development of numerical-modeling techniques.

I. PHYSICAL ASPECTS AND MATHEMATICAL MODELING OF SURFACE THERMAL PLUMES

During the past few years there has been intense interest in the thermal effects of once-through cooling systems for steam-electric generating plants. This concern is prompted by a growing national desire to protect and preserve our natural environment, since both the number and size of steam-electric plants have been increasing at a rapid rate. A thorough understanding of thermal effects is essential input for the formulation of policies that will serve the public's need for electrical energy and, at the same time, minimize environmental abuse.

Only with a thorough understanding of the physical processes that distribute the heat to the environment can one reliably assess the biological effects. Consequently, a number of analytical models have been developed to predict the distribution of waste heat from a power plant in a receiving-water body. The models developed to predict thermal plumes range from the very simple to the very complex.

Mathematical plume models serve several purposes. First, such models assist in the design of outfall structures by assessing the possibility of recirculation. Second, they can be used to estimate the size of the mixing zone. Third, time-temperature predictions obtained from model calculations are useful in determining the temperature history of organisms as they are drawn into the plant intake or are entrained with ambient water into the plume. These temperature histories can be used in conjunction with temperature tolerance studies (usually done in the laboratory) to help assess the biological impact of the heated discharge on the organisms of interest. Of main concern are planktonic organisms, small fish in the egg or larval state, and juveniles that may have limited mobility. Fourth, models may also be used to predict the possible effects of bottom scouring by providing velocity predictions near the bottom. Fifth, dilution rates of discharged chemicals can be determined easily from model predictions of velocity and spreading. Finally, these predictive models may be used to help plan a postoperational field study of the thermal plume.

Although over 40 models have been formulated to predict surface discharge plumes, in our opinion none has been experimentally validated sufficiently enough to guarantee accurate predictions over a wide range of prototype situations. This report is devoted to qualifying this statement. It is essentially a compilation of our experience with the models. Being such, this report is not a truly definitive nor objective appraisal of the state of the art. We acknowledge and regret the following inadequacies:

1. Only a few of the over 40 available models have been adequately considered, and even among these none has been studied thoroughly enough to pass final judgment on,

2. Certain models have been particularly frustrating, with incorrect equations or inoperative computer codes plaguing a fruitful investigation. Other models contain vague or unclear procedures. Such difficulties have undoubtedly biased our view of these models.

3. Our focus has been on models appropriate to the Great Lakes, and our experience has not included extensive consideration of rivers and coastal waterways. Had time permitted, a broader base of modeling situations would have been desirable.

To compensate for the stated shortcomings, we have included as broad a spectrum of opinion as feasible by citing numerous works of other authors.

This report is divided into five chapters. This introductory chapter discusses the basic characteristics of the surface plume; the processes that disperse, diffuse, and dissipate the excess heat; and the general techniques that have been used to analytically model surface-plume hydrodynamics and heat transfer. The second chapter summarizes 11 analytical plume models, including some that have been frequently used for power-plant siting along with other new and untested models. Since the summaries are brief, some prior familiarity with the models would be helpful. A general discussion emphasizing inadequacies of existing models is given in the third chapter, which covers issues that are fundamental to a good physical understanding of plume dispersion and, hence, basic to a correct theoretical modeling of the plume. The fourth chapter summarizes several efforts at model verification, placing particular emphasis on comparisons of model predictions with prototype field data. The final chapter summarizes our conclusions concerning the present status of the problem of predicting plumes from surface thermal discharges.

A. Examples of Surface Discharges

The most common method of disposing of the heated effluent from a once-through cooling system is by means of a channel or canal discharging at the water surface. Most older plants use this type of outfall. Besides being low in cost, these types of outfalls are easily maintained and serviced. In addition, low-velocity surface discharges such as these yield minimal entrainment of ambient water and somewhat greater surface heat loss. In contrast, high-velocity discharges used in more recent designs give greater initial dilution with a corresponding decrease in average plume temperature and surface heat loss.

There is a great deal of variation in the design of surface-discharge structures. Each outfall design is individually tailored to accommodate the nature of the receiving body, the size of the plant, and many different factors not the least of which is the desire to meet local, state, and federal water-quality standards. The idealized surface discharge is that of an open rectangular channel. Most mathematical models, in fact, assume that the outfall is

a rectangular canal discharging effluent parallel to the water surface at a convenient distance offshore. The outfalls at the Point Beach Nuclear Power Station, illustrated in Fig. I.A.1, exemplify a plant with such a design. However, irregular designs for surface-discharge outfalls also exist. The Palisades Nuclear Generating Station, shown in Fig. I.A.2 uses a diverging outfall structure with condenser cooling water discharged at the shoreline. Both plants have intakes 2000-3000 ft offshore. The Waukegan Generating Station depicted in Fig. I.A.3 has a shoreline intake adjacent to its outfall and a breakwater to reduce recirculation. The individual characteristics of each outfall structure must be considered when analysis and modeling is attempted.

B. Near, Intermediate, and Far Fields

Heated effluent may pass through several regimes of flow as it is dispersed into the receiving body of water. For the purposes of analysis, the plume is divided into two regions, the near and far fields. As the name suggests, the near field is that part of the thermal discharge closest to the actual outfall. This field normally possesses a significant velocity disparity with the

This two-unit nuclear power plant has a net capacity of 497 MWe per unit. The normal maximum cooling-water flow to each unit is 885 cfs. At this flow and full electric generation, the maximum temperature rise across the condensers is 19.3 F° and the heat-rejection rate is about 3.9×10^9 Btu/hr for each unit. The discharge flumes extend in opposite directions about 150 ft into Lake Michigan at an angle of 60° to the shoreline. The outfall width and depth at discharge are 35.0 and 13.9 ft, respectively. The nominal outfall velocity of each unit is 1.82 ft/sec.

Fig. I.A.1. Schematic of Discharge at Point Beach Nuclear Power Station.
ANL Neg. No. 190-1115.

The Palisades Plant has a capacity of 714 MWe and discharges into Lake Michigan. The once-through cooling system is scheduled to be replaced by mechanical-draft cooling towers during 1974. The present cooling-water system pumps 405,000 gpm (900 cfs) from Lake Michigan. The maximum cooling-water temperature increase is 25 F°, and the heat-rejection rate is about 4.9×10^9 Btu/hr. The discharge canal is a diverging pile structure on the lake shore. It is 37 ft wide at the outlet adjacent to the building structure and diverges to a width of 100 ft at the point of discharge, 108 ft from the outlet. The average discharge velocity is less than 2 fps.

Fig. I.A.2. Schematic of Discharge at Palisades Nuclear Generating Station. ANL Neg. No. 190-1114 Rev. 1.

The Waukegan Generating Station is a seven-unit fossil-fueled plant with 1100-MWe gross capacity. The cooling water is withdrawn from Lake Michigan by way of a long canal. The heated water is discharged through a canal parallel to the intake canal and separated from it by a breakwater. The canal is about 2000 ft long and discharges approximately at the shoreline. The effective width at the outfall is 100 ft, and the depth is 5 ft. At maximum capacity, the flow rate is 1950 cfs with a temperature rise of 12.8 F°.

Fig. I.A.3. Schematic of Discharge at Waukegan Generating Station. ANL Neg. No. 190-1113.

rest of the water body and for that reason is often called the jet regime. Correspondingly, the far field is referred to as the thermal plume or simply as the plume, although the term "plume" is also used to mean any point not at ambient conditions. This rather confusing usage of terms is nevertheless quite common, the context determining which meaning is intended.

The point of separation between the near and far fields is nebulous. Consequently an intermediate field, a transition zone possessing properties of both the near and far fields, is commonly considered. The definitions of the near and the far fields are somewhat arbitrary. The generally acknowledged characteristics are listed below.

Near Field

1. Temperature and velocity excesses are greater than about 20% of their outfall values.
2. Hydrodynamics of the plume are important. Particulars of the outfall structure must be considered. Characteristic nondimensional groups are the aspect ratio, the initial densimetric Froude number, the ratio of ambient to outfall velocities, the bottom slope, and the ratio of discharge depth to initial water depth.
3. Surface heat loss is minimal.

Far Field

1. For this peripheral area of the plume, velocity and temperature excesses are small.
2. Ambient conditions are predominant in determining dilution with virtually no dependence on outfall characteristics. Diffusion by means of ambient turbulence is considered to be the prevailing mechanism.
3. Because of the large areas involved, surface heat loss is significant.

Models are generally classified as near- or far-field models, depending on whether outfall or ambient conditions are stressed. Models treating both the near and far fields are called complete-field models.

This report is confined exclusively to near- and complete-field mathematical modeling of surface thermal plumes. Purely far-field models are not considered, since we believe that models that ignore the dilution of the near field cannot be used to assess reliably the size and character of the thermal plume. Two plants with the same heat-rejection rate can have vastly different thermal plumes, depending on the amount of near-field mixing that occurs. For example, if initial dilution is maximized by using a vertical slot with a high discharge velocity, the thermal plume will be different than that resulting from a horizontal slot with a low discharge velocity where near-field dilution is small.

Furthermore, the near field is of particular interest since potential episodes of biological consequences may occur there. Moreover, the near field is the regime of greatest interest from a regulatory point of view. Within this region outfall architects must design their discharges to meet mixing-zone criteria. These criteria limit the temperature rise in the thermal plume beyond a prescribed distance from the point of discharge. Some states have adopted a restrictive mixing-zone criterion; others have none at all.

Before undertaking analytical modeling, one must decide what features properly characterize the plume. Ideally, we might seek a point-by-point specification of temperature and velocity for all times. Unfortunately, such a description is far beyond the state of the art, and we must resort to simpler conceptualizations.

The first simplification normally made is to restrict attention to the steady-state plume. As described in Chapter III, even plumes existing in so-called stagnant ambient systems exhibit transient phenomena. Some recent numerical models have made possible attempts at the treatment of unsteady thermal plumes. Startup and shutdown of plants can be simulated by solving the basic conservation equations numerically, using variable boundary conditions. Tidal effects can also be simulated in this manner. Unsteady modeling in the near and complete fields is very recent, and little validation presently exists. Steady-state predictions therefore are still of prime interest and importance, especially considering the large time scale of most unsteady effects such as ambient current or tidal changes. Even considering only the steady plume, processes are so complex that further simplifications are often made to make the mathematics tractable and the physics more interpretable.

As a result of years of theoretical and experimental work, several physical parameters have become standard measures of thermal-plume size. Some of these parameters have been borrowed from classical jet theory. Classical nonbuoyant jet theory assumes that, after an initial potential core and a transition region, velocity profiles become similar. It is only natural to attempt to extend these ideas to buoyant surface jets, even though the connection grows weaker as buoyancy becomes more important. Abstractions borrowed from classical jet theory include the jet centerline or trajectory, centerline temperature decay, centerline velocity decay, jet width, and jet thickness or depth. These items are commonly discussed in the literature, although they may not actually exist in the idealized fashion in which they are described.

Other concepts have evolved from the manner in which thermal-plume measurements are taken. Relatively speaking, it is easy to measure water temperature at many points in the plume. Even so, point-by-point data would be difficult to digest without further reduction. A good qualitative picture of the plume may be obtained by sketching vertical and lateral isotherms from the point-by-point data. These qualitative interpretations may be translated into numerical values by planimetry of the areas within specific surface isotherms. Being integral quantities, surface areas within isotherms tend to

average out experimental fluctuations, and thus provide a reliable estimate of plume size. The price one pays for this decrease in statistical variation comes in the form of loss of detail. Nevertheless, area within surface isotherms is the most common measure of plume size.

The features that relate to a good physical understanding of the plume may be poor parameters for assessing biological consequences. In view of the fact that no analytical model can reliably predict the simple physical parameters outlined above, it is not currently possible to give detailed descriptions of the phenomena that are biologically important. One parameter of biological interest, which can be easily obtained from the jet conceptualization, is the temperature history of an organism traveling along the centerline of the discharging jet. Exposure times are computed by integrating the reciprocal of velocity along the centerline.

Within the context of the jet conceptualization, there are five physical processes that are thought to control plume dilution. These are considered next.

C. Five Basic Physical Processes of Thermal-plume Dispersion

The five phenomena important to the dispersion of momentum and energy within a thermal plume are: jet entrainment, crossflow interaction, ambient-turbulence-induced diffusion, buoyant spreading, and surface heat exchange. These processes are shown schematically in Fig. I.C.1. Of the five processes, only the last, surface heat exchange, involves actual removal of heat from the water body. The other four are merely ways in which the excess heat is mixed into and moved through the water body. In the regions of greatest interest, the near and intermediate fields, only a small percentage of the excess heat is lost to the atmosphere through surface heat exchange. Consequently, the four processes of plume dilution are those of significant importance in modeling these regions.

Fig. I.C.1

Plume Dispersion Processes.
ANL Neg. No. 190-1146.

The five processes are now briefly defined and discussed.

1. Jet Entrainment

Entrainment refers to the incorporation of ambient fluid into the momentum-dominated jet. The velocity difference between the jet and the ambient fluid causes shear stresses that in turn generate turbulent eddies. These eddies, which transport jet momentum laterally and vertically, cause the gross mixing of the jet and the ambient water. Entrainment is generally assumed to be the principal mechanism for mixing in the near and intermediate fields where jet momentum dominates.

For nonbuoyant jets, entrainment is the only mechanism by which dilution occurs. As a result, a simple theory of the classical jet has been developed,¹ which compares favorably with experimental results. As noted earlier, this theory forms the basis for much of the modeling of buoyant surface jets, although other processes complicate the description considerably.

Fig. I.C.2. Schematic View of an Entraining Jet

A schematic view of the classical jet is given in Fig. I.C.2. The jet is assumed to exit with a nearly uniform velocity profile. Sharp velocity gradients existing at the jet boundaries cause intense mixing to occur with a corresponding smoothing of the velocity distribution. This mixing results in momentum transport away from the jet centerline to the ambient water. Near the point of discharge, the central region of the jet is unaffected by the mixing, which occurs only at the boundaries of the jet. Proceeding outward along the centerline of the classical jet, the width of the uniform region decreases to zero and the profile becomes completely smoothed. This point, which has been found experi-

mentally to be four to six outfall diameters from the outfall, marks the end of the potential core region. Laboratory studies indicate that the smoothed velocity profile can be accurately described as a Gaussian distribution. Beyond the potential core, the velocity distribution is similar; i.e., it retains the Gaussian shape with only the height and width of the Gaussian changing along the centerline. The classical jet actually possesses a short transition zone that separates the potential core and similarity regimes. This zone may be ignored when interest lies primarily in the fully developed jet. This jet idealization has been extended to buoyant jets by assuming an identical construction for the temperature profiles. In some cases, a correction is made to account for the fact that heat diffuses approximately twice as fast as momentum by postulating a faster spreading rate for the temperature distribution.

In 1956, Morton et al.² suggested that the amount of jet entrainment should be proportional to some characteristic local velocity. Based on these early ideas, many models postulate a fictitious entrainment velocity, which assumes the following form when no crossflow is present:

$$V_i = EU,$$

where

V_i = local-entrainment velocity,

U = jet-centerline velocity,

and

E = coefficient of proportionality, known as the entrainment coefficient.

The product of this velocity and the circumferential area of the jet below the water surface gives the net flow of ambient water into the jet. Thus, the entrainment coefficient can be considered as the constant of proportionality between the local velocity of entrainment water into the jet and the local jet-centerline velocity. If an ambient current is present, U in the above equation is typically replaced by $|\vec{U} - \vec{U}_a|$, the magnitude of the vector difference between centerline and ambient velocities.

Although the concept of entrainment aids greatly in conceptualizing the interfacial mixing between jet and ambient fluid, it nevertheless grossly oversimplifies this process and reflects a general lack of understanding concerning it.

Successful use of entrainment velocities relies heavily on the existence of flow similarity. The presence of strong buoyancy forces in heated jets may destroy similarity and thus invalidate the use of the entrainment velocities. Even when similarity exists, buoyancy can affect jet entrainment. Many models consider entrainment velocities in both the vertical and horizontal directions to account for the reduction in vertical entrainment due to buoyancy. Ellison and Turner³ have investigated the effects of buoyancy on vertical entrainment, assuming that the key parameter in evaluating the influence of buoyant forces is the Richardson number, Ri , defined as

$$Ri = \frac{\Delta\rho gh}{\rho U^2},$$

where

U = local jet velocity,

ρ = density of ambient fluid,

$\Delta\rho$ = density disparity between ambient and jet fluid,

g = acceleration due to gravity,

and

h = local thickness of the jet.

The numerator of the Richardson number is the vertical buoyant body force per unit of lateral area; the denominator is the longitudinal jet momentum flux. Thus, the Richardson number represents the ratio of buoyant to inertial terms.

Another nondimensional parameter commonly used to characterize the buoyancy effects is the densimetric Froude number, F . The densimetric Froude number is related simply to the Richardson number defined above by

$$F = Ri^{-1/2} = \sqrt{\frac{\rho U^2}{\Delta\rho gh}}.$$

Figure I.C.3 is a plot of the data obtained by Ellison and Turner indicating that the rate of vertical entrainment decreases as the densimetric Froude number decreases. In that figure, e represents the local entrainment coefficient, and e_0 the nonbuoyant entrainment coefficient. The authors concluded that for densimetric Froude numbers less than about 1.2, vertical entrainment is negligible. Thus, models that consider vertical entrainment must allow the local vertical-entrainment coefficient to be reduced by a factor that depends on the densimetric Froude number. Most often this is done by using one of several fits to the experimental data of Ellison and Turner; e.g., Fig. I.C.3 suggests that the factor $\exp(-5/F^2)$ may be used.

Fig. I.C.3. Ellison and Turner Data with Fit. ANL Neg. No. 190-1184.

Because of its dependency on the local densimetric Froude number, vertical entrainment may be suppressed in certain regions of the plume. The scenario of a typical thermal plume is shown schematically in Fig. I.C.4a. At the point of discharge (indicated by 0 in the figure), the densimetric Froude number is larger than 1.2 and the plume will entrain vertically. Rapid mixing causes the densimetric Froude number to decrease until (at position x_1 in the figure) vertical entrainment effectively ceases. The stabilizing effects of buoyancy limit mixing to lateral entrainment alone, hence the densimetric Froude number will stop decreasing and may actually begin increasing. If the densimetric Froude number again becomes great enough, vertical entrainment will resume (position x_2 in the figure). Vertical entrainment may also be limited by interaction with the bottom of the receiving reservoir. This is common occurrence for discharges into shallow water. Figure I.C.4b illustrates such effects indicating that vertical mixing is suppressed in the region where the plume is attached to the bottom.

Fig. I.C.4. Schematic View of Entrainment Process, Showing the Effects of Buoyancy and Bottom. ANL Neg. No. 190-1182 Rev. 1.

2. Crossflow Interaction

In the presence of an ambient crosscurrent, the jet bends in the downcurrent direction until it eventually follows the current. Such bending is the result of a pressure gradient across the jet produced by the ambient current motion around the jet. Customarily, an analogy is drawn between such behavior and that corresponding to the form drag experienced by a solid body in crossflowing fluid. Although this heuristic approach aids in conceptualizing crossflow interaction, there is no good experimental foundation for such a theory in water-plume interaction.

For a jet discharged into a shallow area, bending is enhanced, since the bottom restricts the ambient water from moving under the jet. On the other hand, a jet discharged into deep water allows a much freer path for the crossflow. Bending is also generally greater for higher densimetric Froude numbers. High-velocity jets entrain more vertically, creating a greater area for crossflow interaction. Low densimetric Froude number jets typically stratify into a small layer on the surface presenting a small area for crossflow interaction.

Crossflow interaction can affect dilution in several ways. For example, jet bending can partially isolate the lee side of the jet from free ambient water and can therefore reduce the effectiveness of entrainment in lowering plume temperatures. A horizontal circulation pattern may be established between the lee side of the jet and the shore. Water entrained into the plume near the discharge may first move downstream with the plume and then be brought back along the shore as part of a return current. This recirculation of plume water can adversely affect dilution. Moreover, jets in the presence of a crossflow will generally exhibit asymmetric velocity and temperature profiles and may never become fully developed. Crossflow interaction is discussed further in Chapter III.

3. Ambient-turbulence-induced Diffusion

Ambient turbulence refers to the turbulence in all natural bodies of water, the genesis of which is normally wind stress at the water surface. Turbulent eddies are produced, which result in mixing that is similar to but much larger than molecular diffusion. The scales of eddies present determine the rate of mixing. In analogy to molecular transport, turbulent dispersion is treated as a diffusion process, the diffusion coefficient being known as the eddy diffusivity. Unlike molecular diffusion, however, the eddy diffusivity is generally a strong function of the local flow field. For diffusing plumes in oceans, Richardson's $4/3$ power law is often used to calculate a lateral eddy diffusivity. Here the eddy diffusivity is assumed proportional to the $4/3$ power of the distance from the source. The assumption that all sizes of turbulent eddies can take part in plume dispersion is implicit. In more confined bodies of water, eddy sizes are restricted by the lateral and vertical boundaries. A constant lateral diffusivity may be more appropriate, due to the restricted eddy spectrum. Horizontal and vertical diffusion can be appreciably diminished in both near-shore and shallow areas.

Vertical diffusion due to ambient turbulence may be limited by the interaction of turbulence and buoyancy as well as by the shallowness of the water body. Vertical stratification, resulting from the natural distribution of heat in the water body or from stratification of the heated effluent, inhibits the mixing due to wind- or plume-induced turbulence. Vertical diffusivities are typically given as functions of the gradient Richardson number, which is an indicator of the relative ability of the vertical density gradient to inhibit vertical mixing. Vertical diffusion in large lakes has been found to be two to three orders of magnitude smaller than lateral diffusion.

Plume mixing induced by ambient turbulence is masked in the near field by jet-induced entrainment, but becomes increasingly more important as the jet momentum is dissipated.

4. Buoyant Spreading

Buoyant forces develop due to (1) the density disparity between the discharge and the receiving water and (2) any density variation within the jet.

These forces increase horizontal spreading and, as noted earlier, may inhibit vertical entrainment. The importance of buoyancy is measured by the densimetric Froude number defined earlier.

For densimetric Froude numbers greater than about 1, the inertial forces of jet momentum dominate; on the other hand, for values less than about 1, buoyancy forces are predominant. Discharges with large densimetric Froude numbers produce much jet entrainment and rapid dilution; plumes are typically long and slender, with significant vertical penetration. Discharges having low densimetric Froude numbers typically have wide isotherms and form a thin surface layer. Buoyancy restricts vertical entrainment and increases lateral spreading. If large dilutions in short distances from the outfall are desired, an outfall design with a large discharge velocity, a small outfall depth, and minimal excess temperature (outfall-ambient temperature) would produce a large initial densimetric Froude number and cause inertial forces to predominate over buoyant forces, with jet mixing prevailing.

As indicated in Fig. I.C.4 each thermal discharge eventually reaches a downstream location at which the initial velocity has decreased sufficiently to cause the local densimetric Froude number to become less than 1.2. Here vertical entrainment is suppressed and the jet maintains an approximately constant depth. Lateral spreading due to buoyancy becomes most important until the normalized density difference $\Delta\rho/\rho$ approaches zero and the local densimetric Froude number increases above 1.2. At this point, vertical mixing becomes significant again.

Depending on heat loss and mixing parameters, the discharged fluid usually enters a regime of stratified flow where buoyant forces are dominant. The rapid deceleration of fluid moving from the momentum-dominated jet regime to a stratified flow regime may produce an internal hydraulic jump. This phenomenon is discussed in Chapter III.

5. Surface Heat Exchange

There is continual heat exchange between the water surface and the atmosphere through conduction, radiation, and evaporation. At a specific temperature, T_e , known as the equilibrium temperature, there is no net exchange of heat with the atmosphere. The equilibrium temperature depends on many environmental conditions such as wind speed, cloud cover, time of day, time of year, air temperature, and humidity. The temperature of a natural body of water continually approaches but seldom reaches this equilibrium temperature, because of the long times needed to exchange large amounts of heat with atmosphere. Consequently, the ambient temperature is lower than daytime equilibrium during spring heating and higher than daytime equilibrium during fall cooling. During winter and summer months, the ambient temperature may be close to equilibrium. Normally, condenser cooling water is at a temperature higher than equilibrium and there is a net heat loss from the plume surface.

Since temperature excesses in the plume are generally small, the rate of surface heat loss is often assumed to be proportional to the difference between the plume temperature and the equilibrium temperature. This is often expressed as

$$q_s = K(T - T_e),$$

where

q_s = surface heat transfer per unit area,

K = empirical exchange parameter known as the surface heat-transfer coefficient,

T = surface temperature,

and

T_e = equilibrium temperature.

Since the processes that determine the amount of surface cooling from a heated plume are identical to those that prevail under normal conditions, heating by a discharge may be regarded as a perturbation of the normal ambient temperature variation. This observation is the justification for use of the equilibrium temperature concept for surface-heat-loss calculations for a heated discharge. Most treatments of surface heat exchange use the ambient temperature T_a as the equilibrium, even though T_e may be quite different from T_a at certain times of the year. The error incurred by making this assumption is believed to be inconsequential.

For typical values of K , the amount of heat transfer by this mechanism is small in the near field. Unless surface areas are appreciable, surface heat loss has a minimal effect on plume temperatures. Surface heat loss may, however, be significant to near-field modeling in the following respect. Stratification in the intermediate and far fields may be controlled by surface heat loss. This downstream stratification may influence near-field flow patterns by providing "boundary conditions" for the near field. This matter is treated in greater detail in Chapter III.

A major effect of surface cooling is the ultimate limitation of the areal extent of any thermal plume. Without surface heat dissipation, no thermal plume would ever reach steady state.

D. Basic Modeling Techniques

There are five broad classifications of mathematical approach for plume models. A given model may use more than one approach, particularly when the model analyzes more than one regime of flow.

1. Phenomenological Method

This technique relies on correlations derived from numerous hydraulic model and/or field studies. Correlations for jet characteristics such

as centerline temperature decay, plume half-widths (of the temperature distribution), and surface isotherm areas are made with such variables as initial densimetric Froude number, outfall aspect ratio, bottom slope, and the ratio of ambient current to initial discharge velocity. Such an approach is common in many areas of fluid mechanics and heat transfer. An advantage of the method lies in its close relationship to the phenomenon being modeled; the principal disadvantage is the often unknown range of physical situations for which the correlations are valid.

2. Differential-Numerical Method

The differential-numerical method, or more simply the numerical method, involves solving the general partial differential equations of motion and heat transport to arrive at velocity and temperature distributions, using either finite-difference or finite-element schemes. The problem of jet dispersion lies in gaining the solution to these equations once having established such requirements as boundary conditions and eddy-transport coefficients. These boundary conditions and eddy-transport coefficients distinguish one plume problem from another; the hydrodynamic and energy equations are otherwise unchanged. The equations are impossible to solve analytically while retaining all the terms describing the motion and dilution of the heated jet. For the collective class of plume problems, the very few analytical solutions to the general set of governing equations that do exist have been derived by making gross simplifications of the physical phenomena. Differential-numerical modeling has an advantage in that many complexities associated with real physical phenomena do not have to be ignored, as required in the development of other methods of solution. Fewer assumptions about the nature of the physical phenomena need be made. However, uncertainty as to the proper form of the generally unknown and variable eddy-diffusion coefficients (for momentum and energy) and the precise flow and pressure boundary conditions has hampered the development of a reliable differential-numerical model. In addition, problems such as long expensive computer runs and numerical stability have often negated the attractiveness of a numerical method (at least for the jet regime).

A note regarding the terminology is appropriate here. The references to "numerical models" may be somewhat misleading, since models based on other methods, for example the integral technique, often use numerical procedures to solve their model equations. In general, the words "numerical model" will be reserved for models using finite-difference or finite-element schemes to solve the generalized flow equations.

3. Integral Analysis

The integral analysis involves a sacrifice of the microscopic characteristics of the flow and temperature fields in favor of a solution for the macroscopic quantities. In the integral technique, one assumes the precise shape of the lateral and vertical temperature and velocity profiles based upon

intuition, hydraulic model experiment, or (quite often) both. Moreover, assumptions on the mixing of ambient fluid into the jet and on the pressure field are made by an a priori choice of an entrainment function and a pressure-drag formula, respectively. Using the assumed profiles along with the entrainment and drag functions, the multidimensional equations of mass, momentum, and energy conservation are reduced to a set of coupled nonlinear ordinary differential equations that are solved analytically or numerically.

For a large initial Froude number, a common assumption made in modeling the jet regime by the integral method is that inertial forces dominate the buoyancy forces within the jet and that, to a fair approximation, density-induced pressure gradients can be neglected in the equations of flow. Use of this assumption results in an uncoupling of the flow and energy equations, which clearly simplifies any solution. An alternative approach is to suppose that, in plumes of low initial densimetric Froude number, buoyancy forces completely inhibit vertical entrainment. The two-dimensional plume models use this rationale.

In general, the integral models solve a more simplified set of equations than do most numerical models. Several terms of advection and diffusion are dropped from the governing equations necessitated by the very nature of the integral approach, being a "whittling and fashioning" method in which small terms are dropped from the governing equations and the dependent variables are assumed to have a fixed general form.

The principal difficulty with the integral technique is that the physical processes--entrainment, crossflow interaction, buoyancy, ambient-turbulence-induced diffusion, and surface heat loss--must be simulated artificially in the context of the presumed jet structure. This produces model parameters that may not be physically measurable. Thus, the models must be fit to large amounts of data before they can be used with confidence.

4. Subchannel Balancing Method

Subchannel balancing, which is a cross between the integral approach and the differential-numerical method, can be efficiently applied to those hydrodynamic problems in which the flow is essentially one-dimensional, as in thermal discharges into rivers. In this technique, the flow field is divided into numerous subchannels aligned with the principal direction of flow. Conservation equations are developed by balancing the longitudinal, lateral, and vertical inflows of mass, momentum, and heat to each subchannel. A set of perhaps several hundred coupled ordinary differential equations must then be solved to obtain the model predictions, in contrast to the integral models in which no more than 10 equations are integrated simultaneously and to the differential-numerical models that solve multidimensional forms of the conservation equations.

The concept is extremely convenient when the flow is basically one-dimensional, since the amount of computer time required is much smaller than

that required for the differential-numerical models. Further, the conceptualization of the discharge is far less restrictive than that of the integral models. However, application of such a model to situations in which the flow is not primarily unidirectional, as for thermal discharge into a stagnant lake or low-flowing river, is questionable.

5. Stochastic Method

Stochastic models use probability theory to predict general plume behavior. Such models are empirical and require a large amount of site-dependent data for their application. This type of model is nondeterministic as well as site specific. Currently, no models that we are aware of for the near and complete field use this approach.

E. Summary of Existing Models

It is virtually impossible to review and compare to data all models available in the literature. We have chosen for study eleven models that either have been extensively used for environmental impact analyses or appear promising for future use or further development. This section enumerates the near and complete-field models that have appeared in the published and unpublished literature on heated surface discharges. Development and verification of each model is described briefly.

Some of the models in the list below are of historical interest only, though others might achieve some distinction if further development work were done. Each model makes some contribution to the methodology of thermal plume prediction, so we feel that its existence should be acknowledged in this report. Some of the models have already been critiqued by Policastro and Tokar⁴ as noted below.

Near-field Models

1. Hoopes, Zeller, and Rohlich⁵ (The University of Wisconsin, Madison)

Type of Model:	Two-dimensional (horizontal) integral, steady state
Phenomena Considered:	Jet entrainment, ambient crossflow, surface heat transfer, wind stress, drag force
Treatment of Flow Establishment:	Only indirectly through calibration of model parameters to data
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Model was calibrated to field data taken at two outfalls on Lake Monona, Wisconsin and later compared to other field data for the same sites ⁵

Computer Code: Available from authors or Argonne

[Model reviewed in Ref. 4]

2. Hayashi and Shuto⁶ (Chuo University, Tokyo, Japan)

Type of Model: Three-dimensional integral, steady state

Phenomena Considered: Buoyancy, surface heat transfer

Treatment of Flow: None

Establishment:

Method of Solution: Closed form

Verification: Authors' tank data⁶

Computer Code: Unknown but may be easily written from model equations

3. Wada (Model No. 1)⁷ (Central Research Institute of the Electric Power Industry, Tokyo, Japan)

Type of Model: Quasi three-dimensional integral (two-layer flow with fluid properties averaged vertically within each layer), time dependent

Phenomena Considered: Jet entrainment, buoyancy, bottom slope, surface heat transfer

Treatment of Flow: None

Establishment:

Method of Solution: Numerical by finite differences

Verification: Unknown

Computer Code: Unknown

4. Carter⁸ (Chesapeake Bay Institute, Johns Hopkins University)

Type of Model: Two-dimensional (horizontal) integral, steady state

Phenomena Considered: Jet entrainment, ambient crossflow, drag force

Treatment of Flow: Experimental from limited tank data

Establishment:

Method of Solution: Closed form

Verification: Limited comparisons with author's tank data⁸

Computer Code: Unknown

[Model reviewed in Ref. 4]

5. Motz and Benedict⁹ (Vanderbilt University)

Type of Model: Two-dimensional (horizontal) integral, steady state

Phenomena Considered: Jet entrainment, ambient crossflow, drag force, surface heat transfer

Treatment of Flow Establishment: Experimental from limited tank data

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: Fitting of model to tank and field data to ascertain coefficients.⁹ Verification with Point Beach and Palisades data appears in Chapter IV of this report.

Computer Code: Available from authors or Argonne

[Model reviewed in this report and Ref. 4]

6. Koh and Fan (Model No. 1)¹⁰ (Tetra-Tech, Inc.)

Type of Model: Two-dimensional (longitudinal, vertical) integral model, steady state

Phenomena Considered: Jet entrainment, buoyancy, interfacial shear, surface heat transfer

Treatment of Flow Establishment: None

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: None

Computer Code: Available from authors

[Model discussed in Chapter III of this report]

7. Koh and Fan (Model No. 2)¹⁰ (Tetra-Tech, Inc.)

Type of Model: Axisymmetric (radial, vertical) integral, steady state

Phenomena Considered: Jet entrainment, buoyancy, interfacial shear, surface heat transfer

Treatment of Flow Establishment:	None
Method of Solution:	Numerical Solution of coupled nonlinear ordinary differential equations
Verification:	None
Computer Code:	Available from authors

8. Barry and Hoffman¹¹ (Detroit Edison Co.)

Type of Model:	Quasi three-dimensional numerical, steady state
Phenomena Considered:	Advection and diffusion (no pressure variations), bottom interaction, ambient crossflow, surface heat transfer
Treatment of Flow Establishment:	Numerical
Method of Solution:	Numerical by finite difference
Verification:	Model is compared to field data taken at the St. Clair Power Plant on the St. Clair River ^{11c}
Computer Code:	Proprietary

9. Stolzenbach and Harleman¹² (Massachusetts Institute of Technology)

Type of Model:	Three-dimensional integral, steady state
Phenomena Considered:	Jet entrainment, buoyancy (longitudinal, lateral, vertical), ambient crossflow, surface heat transfer
Treatment of Flow Establishment:	Integral technique
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Comparison to authors' tank data; ¹² Pilgrim Plant field data comparisons; ^{13,14} Point Beach and Palisades field data and tank data of Carter et al. ^{15,16} (see Chapter IV)
Computer Code:	Available from authors or Argonne

[Model reviewed in this report and Ref. 4]

10. McLay, Hundal, Martinek, and Henson¹⁷ (University of Vermont)

Type of Model:	Two-dimensional numerical, steady state for zone of flow establishment. Two-dimensional integral, steady state for established flow region.
Phenomena Considered:	Advection and lateral diffusion (no pressure difference) for zone of flow establishment model, jet entrainment, wind stress, ambient crossflow, drag force, surface heat transfer for established flow model
Treatment of Flow Establishment:	Numerical
Method of Solution:	Numerical by finite differences for zone of flow establishment; numerical solution of coupled nonlinear ordinary differential equations in established flow region (following treatment of Hoopes, Zeller, and Rohlich ⁵ with altered coefficients)
Verification:	None
Computer Code:	Available from authors or Argonne

11. Stefan and Vaidyaraman¹⁸ (University of Minnesota)

Type of Model:	Three-dimensional integral, steady state
Phenomena Considered:	Jet entrainment, buoyancy (lateral, vertical), ambient crossflow, drag force, wind stress, surface heat transfer
Treatment of Flow Establishment:	None
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Comparison to authors' tank data ¹⁸
Computer Code:	Available from authors

[Model discussed in Chapter III of this report]

12. Benedict, Polk, Yandell, and Parker¹⁹ (Vanderbilt University)

Type of Model:	Two-dimensional integral, steady state (rapid rivers)
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Phenomena Considered: Jet entrainment, ambient crossflow, drag force

Treatment of Flow Establishment: None

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: Comparison with data at the Piacenza Plant on the Po River in Italy¹⁰

Computer Code: Available from authors

[Model discussed in Chapter IV, Section F of this report]

13. Prych²¹ (Swedish Meteorological and Hydrological Institute, Stockholm, Sweden)

Type of Model: Three-dimensional integral, steady state

Phenomena Considered: Jet entrainment, ambient diffusion, buoyancy (longitudinal, lateral, vertical) ambient crossflow, drag force, interfacial shear

Treatment of Flow Establishment: Integral technique

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: Comparison to Oskarshamnsverket data in Sweden,²² Stolzenbach-Harleman tank data,²¹ Point Beach and Palisades field data, tank data of Carter et al.^{15,16}

Computer Code: Available from author or Argonne

[Model reviewed in this report]

14. Cavanna and Folli²⁰ (Center for Computational and Experimental Studies, Milan, Italy)

Type of Model: Two-dimensional integral (rapid rivers), steady state

Phenomena Considered: Jet entrainment, ambient crossflow

Treatment of Flow Establishment: None

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: Comparison with data at the Piacenza Plant on the Po River in Italy²⁰

Computer Code: Unknown

[Model discussed in Chapter IV, Section F of this report]

15. Abramovich²³ (Massachusetts Institute of Technology)

Type of Model: Two-dimensional integral (rapid rivers), steady state

Phenomena Considered: Jet entrainment, ambient crossflow

Treatment of Flow Establishment: Experimental based upon tank data

Method of Solution: Closed form

Verification: Comparison with tank data in author's book²³ and comparison with field data taken at the Piacenza Plant on the Po River²⁰

Computer Code: Not necessary but may be easily written

[Model discussed in Chapter IV, Section F of this report]

16. Engelund and Pedersen²⁴ (University of Denmark, Copenhagen, Denmark)

Type of Model: Quasi three-dimensional integral (fluid properties are averaged vertically within the plume)

Phenomena Considered: Jet entrainment, buoyancy (lateral, vertical)

Treatment of Flow Establishment: None

Method of Solution: Closed form

Verification: Comparison to tank data²⁴

Computer Code: Unknown

17. Sill²⁵ (Virginia Polytechnic Institute and State University)

Type of Model: Two-dimensional integral (for rivers)

Phenomena Considered: Jet entrainment, ambient crossflow, shear force (between plume and bottom and plume and nearshore), bottom depth variations, surface heat transfer

Treatment of Flow Establishment:	Integral technique
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Comparison to authors' tank data ²⁵
Computer Code:	Available from authors

18. Hayashi, Miyahara, and Arita²⁶ (Chuo University, Tokyo, Japan)

Type of Model:	Three-dimensional integral, steady state
Phenomena Considered:	Jet entrainment, buoyancy (longitudinal, lateral, vertical), ambient crossflow, surface heat transfer
Treatment of Flow Establishment:	Integral technique
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Comparison to tank data ¹²
Computer Code:	Unknown

19. Shirazi and Davis²⁷ (Pacific Northwest Environmental Research Laboratory)

Type of Model:	Three-dimensional integral, steady state
Phenomena Considered:	Jet entrainment, ambient diffusion, buoyancy (lateral, longitudinal, vertical), interfacial shear, ambient crossflow, drag force, surface heat transfer
Treatment of Flow Establishment:	Experimental from limited tank data
Method of Solution:	Numerical solution of coupled nonlinear ordinary differential equations
Verification:	Comparison to assorted tank and field data; ²⁷ tank data of Carter et al. ^{15,16} and Point Beach and Palisades field data (see Chapter IV)
Computer Code:	Available from authors or Argonne

[Model reviewed in this report]

Complete-field Models

20. Wada (Model No. 2)²⁸ (Central Research Institute of Electric Power Industry, Tokyo, Japan)

Type of Model:	Two-dimensional (longitudinal and vertical) numerical, steady state
Phenomena Considered:	Jet entrainment, ambient and jet diffusion, buoyancy (vertical and longitudinal), bottom slope, surface heat transfer
Treatment of Flow Establishment:	Numerical
Method of Solution:	Numerical by finite differences
Verification:	None
Computer Code:	Unknown

[Model reviewed in Ref. 4]

21. Pritchard (Model No. 1)²⁹ (Chesapeake Bay Institute, Johns Hopkins University)

Type of Model:	Quasi three-dimensional, integral/phenomenological, steady state
Phenomena Considered:	Jet entrainment (vertical, lateral), recirculation, surface heat transfer, buoyancy effects on vertical entrainment (through vertical mixing depth), limited heuristic treatment of bottom effects
Treatment of Flow Establishment:	Empirical correlation
Method of Solution:	Closed form
Verification:	Comparison to Waukegan Plant data, ^{29b} Point Beach and Palisades data (see Chapter IV)
Computer Code:	Available from Argonne

[Model reviewed in this report and Ref. 4]

22. Sundaram, Easterbrook, Piech, and Rudinger³⁰ (Cornell Aeronautical Laboratory)

Type of Model:	Two-dimensional integral, steady state
Phenomena Considered:	Jet entrainment, ambient turbulent diffusion, ambient crossflow, surface heat transfer

Treatment of Flow Establishment:	None
Method of Solution:	Closed form
Verification:	Model was fit ³⁰ to data of the Milliken Station, Lake Cayuga
Computer Code:	Available from Argonne

[Model reviewed in Ref. 4]

23. Elliott and Harkness³¹ (Ontario Hydro, Toronto, Canada)

Type of Model:	Two-dimensional integral/ phenomenological, steady state
Phenomena Considered:	Jet entrainment alone simulated in the integral model for jet trajectory; data base for other plume characteristics includes outfall and meteorological parameters and measured plumes at the Lakeview site
Treatment of Flow Establishment:	Empirical
Method of Solution:	Numerical solution of an ordinary differential equation for the jet trajectory; the remainder of the model is closed form
Verification:	Comparison again with the same Lakeview data; ³¹ comparison to field data at the Douglas Point Generating Station ³¹
Computer Code:	Available from authors

24. Giles, Johnson, McComb, Morris, Schell, and Young³² (General Electric)

Type of Model:	Two-dimensional integral in the near field and three-dimensional numerical in the far field
Phenomena Considered:	Jet entrainment in the near-field model and diffusion, wind, waves, and surface heat transfer in the far-field model
Treatment of Flow Establishment:	None
Method of Solution:	Closed form in the near field and finite-difference solution of the heat energy equation in the far field

Verification: Comparison to field data at the
J. H. Campbell Plant³²

Computer Code: Available from authors

25. Loziuk, Anderson, and Belytschko³³ (Sargent and Lundy Engineers)

Type of Model: Two-dimensional integral and numerical,
steady state

Phenomena Considered: Jet entrainment and ambient crossflows
for flow-field integral model, surface heat
transfer, advection and diffusion for the
numerical model of the heat energy
equation

Treatment of Flow Establishment: Empirical

Method of Solution: Closed form for flow field; finite-element
method for temperature field

Verification: Comparison to field data on Horseshoe
Lake (cooling-pond),^{33b} comparison to lab-
oratory model of unspecified thermal dis-
charge into a river^{33a}

Computer Code: Proprietary

26. Brady and Geyer³⁴ (The Johns Hopkins University)

Type of Model: Three-dimensional numerical, transient

Phenomena Considered: Advection, diffusion, pressure differences
in all three directions, surface elevations,
bottom effects, ambient crossflow, surface
heat transfer

Treatment of Flow Establishment: Numerical

Method of Solution: Finite-difference scheme for continuity,
momentum, and heat energy equations

Verification: None

Computer Code: Available from authors

[Model reviewed in this report]

27. Till³⁵ (University of Missouri - Rolla)

Type of Model: Three-dimensional subchannel balancing
technique, steady state

Phenomena Considered: Advection, diffusion, pressure differences in all three directions, bottom effects, ambient crossflows, surface heat transfer

Treatment of Flow Establishment: Subchannel balancing technique

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations

Verification: Compared to data taken at Philip Sporn;^{35a} comparison to field data acquired at the Point Beach power plant^{35b} (see Chapter II of this report)

Computer Code: Available from author
[Model reviewed in this report]

28. Pritchard (Model No. 2)³⁶ (Chesapeake Bay Institute, The Johns Hopkins University)

Type of Model: Quasi three-dimensional phenomenological (from empirical correlations), steady state

Phenomena Considered: Data base includes crossflow, bottom, aspect ratio, and initial densimetric Froude number effects

Treatment of Flow Establishment: Empirical

Method of Solution: Closed form

Verification: Author's comparison³⁶ to tank data of Carter et al.,¹⁵ Pilgrim Plant field data and Ginna Plant field data; comparison to tank data from Carter et al.^{15,16} and Point Beach field data (see Chapter IV)

Computer Code: Available from Argonne
[Model reviewed in this report]

29. Pritchett, England, and Taft³⁷ (Systems, Science, and Software)

Type of Model: Three-dimensional numerical, transient

Phenomena Considered: Advection, diffusion, pressure differences in all three directions, bottom effects, ambient crossflow, surface heat transfer

Treatment of Flow Establishment: Numerical

Method of Solution: Finite difference scheme for continuity, momentum, turbulent energy, and heat energy equations

Verification: Compared to data taken at Point Beach and Pilgrim plants (see Chapter II)

Computer Code: Proprietary

[Model reviewed in this report]

30. Wu and Gallagher³⁸ (Limnetics, Incorporated)

Type of Model: Three-dimensional integral and three-dimensional numerical, steady state

Phenomena Considered: The model couples Prych's analysis of the near field to a diffusion analysis in the far field. The latter analysis involves the heat-energy equation alone and treats lateral and vertical diffusion in addition to longitudinal advection in the direction of the current.

Treatment of Flow Establishment: Integral technique (from Prych)

Method of Solution: Numerical solution of coupled nonlinear ordinary differential equations in the near field (Prych's analysis); finite-difference solution of the heat energy equation in the far field

Verification: None

Computer Code: Available from authors

31. Waldrop and Farmer³⁹ (Louisiana State University)

Type of Model: Three-dimensional transient numerical model

Phenomena Considered: Advection, diffusion, pressure difference in all three directions, surface elevations, bottom effects, ambient crossflow, surface heat transfer

Treatment of Flow Establishment: Numerical

Method of Solution: Finite-difference scheme for continuity, momentum, and heat energy equations

Verification: Comparison to Point Beach and Holston River field data^{39c, 39d} (see Chapter II)

Computer Code: Available from authors

[Model reviewed in this report]

32. Paul and Lick⁴⁰ (Case Western Reserve University)

Type of Model: Three-dimensional transient numerical model

Phenomena Considered: Advection, diffusion, pressure differences in all three directions, surface elevations, bottom effects, ambient crossflow, surface heat transfer

Treatment of Flow Establishment: Numerical

Method of Solution: Finite-difference scheme for continuity, momentum, and heat energy equations

Verification: Comparison^{40c} to two Point Beach thermal plumes (see Chapter II)

Computer Code: Available from authors

[Model reviewed in this report]

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II. SUMMARY AND EVALUATION OF INDIVIDUAL MODELS

A. Introduction

This chapter discusses and critiques the theoretical development of 11 mathematical models that can be used for predicting surface thermal plumes. The reviews are organized first by mathematical approach and then within each approach by publication order. Moreover, the first generation models that use the integral, integral/phenomenological, and the phenomenological approaches are presented before the more recent numerical models. Further, the model reviews vary in content and detail. Abbreviated reviews are given of the Motz-Benedict, Stolzenbach-Harleman, and Pritchard No. 1 models, since they were considered in an earlier publication of the Great Lakes Research Program (Policastro and Tokar, 1972); of the Pritchard Model No. 2, since little experience with it exists; and of the Brady-Geyer model, since it is not presently suitable for prototype application. Only as much detail as is required to delimit what is and what is not included in model development is provided. In contrast, the numerical models of Paul and Lick; Waldrop and Farmer; Till; and Pritchett, England, and Taft are given an expanded discussion so that readers may more fully understand these complex numerical schemes. As documentation of these models in the literature is often poor, the reviews were carried out in close consultation with the authors of the models to resolve questions and to clarify points in an effort to be as thorough as time permitted.

In addition to the criticisms leveled on a model-by-model basis in this chapter, certain generic criticisms are given an expanded discussion in Chapter III. That chapter seeks to identify the important modeling issues. In addition, the integral and phenomenological models are compared with laboratory and prototype field data in Chapter IV. These comparisons, in principle, represent true laboratory and prototype testing of the models. Given our philosophy for evaluating models, these comparisons are essential and form the basis for many of our opinions on the viability of each model. Thus, consideration of the information presented in total is necessary before deciding which models are suitable for individual applications. Comparisons of numerical model predictions with prototype data are included in the model reviews of this chapter because they are few in number and they aid in explaining model development. Some of these comparisons were carried out as mere modeling exercises or as part of model development.

An unavoidable weakness of this report in general, and this chapter in particular, is the omission of models that should have been reviewed. A good example is the model by Stefan (Ref. 18, Chapter I). This integral model is simple and easy to apply. The original computer code contained only a few easily-corrected errors. Its major shortcoming seems to be the omission of buoyancy-induced motion along the jet axis, which becomes important at low densimetric Froude numbers. This model has received less attention in the literature than have some others, but should not be discounted for that reason. Time simply did not permit an investigation of all existing models.

With readers forewarned that reviews of the models presented here focus on model construction only and hence are not complete without the additional information in Chapters III and IV, and with the imbalance in detail explained, we proceed with the model summaries.

B. Motz-Benedict Integral Model¹

1. Synopsis of Approach

This two-dimensional, steady-state model uses an integral analysis. Temperature and velocity profiles are assumed to be Gaussian in shape, equal in width, and similar for all jet cross sections in the region of established flow. These assumed profiles are used to generate ordinary differential equations for the conservation of mass flux, x- and y-momentum flux, and heat flux. Two additional equations of geometry complete the set of six ordinary differential equations that is solved by a fourth-order Runge-Kutta scheme.

The results of hydraulic model studies conducted by Motz and Benedict are used to handle the region of flow establishment and to provide appropriate values for the model parameters.

2. List of Symbols

A	Ratio of ambient velocity to initial jet velocity; U_a/U_o
b	Half-width of jet
b'_o	Half-width of outfall
b_o	Half-width of jet at end of flow-establishment zone
C	Circumference through which entrainment takes place
C_D	Drag coefficient
C'_D	Reduced drag coefficient
c_p	Specific heat
dA	Differential area of jet cross section
E	Entrainment coefficient
F_D	Drag force
F_o	Initial densimetric Froude number
K	Surface heat-transfer coefficient
M	Nondimensional momentum flux
O	Origin of the coordinate system (x, y), beginning of zone of established flow
O'	Origin of the coordinate system (x' , y'), point of jet discharge
s	Coordinate axis of jet, referenced to beginning of zone of established flow
s'	Coordinate axis of jet, referenced to point of jet discharge
s'_e	Length of flow-establishment zone

S	Nondimensional distance along jet axis, referenced to beginning of zone of established flow
T	Centerline temperature excess
T_0	Initial centerline temperature excess
T'	Local temperature excess
u	Local jet velocity
U	Centerline jet velocity
U_a	Ambient velocity
U_0	Initial jet velocity
V	Nondimensional volume flux
v_i	In-flow velocity
W_r	Ratio of the width of the ambient system to the discharge width
x	Longitudinal axis, referenced to beginning of zone of established flow; long-shore distance
X	Nondimensional longitudinal axis, referenced to beginning of zone of established flow
y	Lateral axis, referenced to beginning of zone of established flow; offshore distance
Y	Nondimensional lateral axis, referenced to beginning of zone of established flow
z	Depth of jet
Z	Nondimensional depth of jet
β	Angle between jet and ambient current
β'_0	Discharge angle
β_0	Angle of jet with current at end of flow-establishment zone
η	Horizontal distance along axis perpendicular to s axis
θ/θ_0	Excess temperature ratio used when referring to isotherms
ρ	Density
ρ_a	Ambient density

3. Basic Assumptions

The model is developed in the context of a coordinate system shown in Fig. II.B.1. One coordinate, s , lies along the jet centerline; the other coordinate, η , is perpendicular to the centerline in the horizontal plane. The basic assumptions of the model are:

a. The jet is completely turbulent when fully developed. Velocity and temperature profiles are then assumed to be given by

$$u(s, \eta) = U(s) \exp\left(-\frac{\eta^2}{b^2}\right)$$

and

$$T'(s, \eta) = T(s) \exp\left(-\frac{\eta^2}{b^2}\right),$$

where

u = local jet velocity parallel to jet centerline,

U = jet-centerline velocity,

b = characteristic width of the lateral profile,

T' = local temperature excess,

and

T = jet-centerline temperature excess.

b. The effects of ambient turbulence are negligible.

c. Buoyancy forces completely inhibit vertical entrainment, but are otherwise negligible.

d. Lateral entrainment is simulated in terms of an inflow velocity, which is assumed proportional to the difference between the jet-centerline velocity and the component of the ambient velocity along the jet centerline; thus,

Fig. II.B.1

Motz-Benedict Coordinate System.
(Adapted from Motz-Benedict, 1)
ANL Neg. No. 190-1180.

$$v_i = E(U - U_a \cos \beta),$$

where

v_i = local entrainment velocity,

U = jet-centerline velocity,

U_a = ambient velocity,

β = angle of jet trajectory with ambient velocity,

and

E = empirical entrainment coefficient.

The entrainment coefficient is to be determined by the combined use of Fig. II.B.2 and Table II.B.1. Motz and Benedict do not specifically state how this is done, other than to suggest that the coefficient should be first estimated from Fig. II.B.2 and then corrected for the effects of initial discharge angle and the finite width of the receiving body using Table II.B.1. (See Criticisms and Limitations, Sec. 5 below.)

e. Crossflow interaction may be treated as though the jet were a solid body experiencing form drag due to the acceleration of ambient fluid around the jet. The force acts perpendicularly to the jet centerline as shown in Fig. II.B.3. The drag force is assumed to be of the form

$$F_D = \frac{C_D Z U_a^2 \sin \beta}{2},$$

Fig. II.B.2. Entrainment Coefficient as Function of Ambient-to-discharge Velocity Ratio. (Adapted from Motz-Benedict.¹) ANL Neg. No. 190-1144.

TABLE II.B.1. Effects of Angle and Width Ratio on Entrainment Coefficient (from Motz-Benedict³)

Data Source	β'_0	IF_0	$\frac{U_0}{U_a}$	w_r^a	E
Lab	90°	3.4-6.9	1.37-5.2	24	avg = 0.4
Lab	45 & 60°	3.1-6.3	1.53-5.2	24	avg = 0.2
Widows Creek					
VU1	85°	4.3	2.0	7.5	0.16
VU2	85°	3.7	1.5	7.5	0.16
TVA	85°	3.7	1.33	7.5	0.16
New Johnsonville	60°	1.3	1.75	4	0.04
Stefan (lab)	--	3.7-4.1	∞	34	.06-.09
Vandy (lab)	90°	4.2-7.4	3-30- ∞	24	0.4-.04
Fitch (Lake St. Croix)	60°	1.0	3.19 ^b	20	.11-.296
Ayers (Michigan City)	90°	1.2	10 ^b	--	.04
Romberg (Waukegan)	90°	5.6-6.0	1.7-5.6	--	.046-.059

^a w_r is defined as the ratio of the width dimension of the receiving body to the width of the discharge. This value is only significant for rivers, hydraulic models and perhaps small lakes.

^bEstimated values.

Fig. II.B.3

Force Diagram of Motz-Benedict Model, (Adapted from Motz-Benedict.¹) ANL Neg. No. 190-1141.

where

F_D = drag force per unit length,

Z = depth of the jet,

and

C_D = empirical drag coefficient, normally taken to be about 0.5.

f. Surface heat exchange is small and may be treated through the use of an empirical heat-transfer coefficient as

$$q_s = KT',$$

where

q_s = rate of heat loss per unit area,

T' = local temperature excess,

and

K = empirical heat transfer coefficient.

g. The centerline temperature and velocity remain unchanged from their nominal values throughout the zone of flow establishment, which begins at the point of discharge and extends a distance s'_e downstream. In the zone of flow establishment, the width grows from its initial value of b'_0 to the value b_0 while the angle between the jet and the current changes from β'_0 at the outfall to β_0 at the end of flow establishment. Thus, the region of flow establishment is characterized by the parameters s'_e , b_0 , and β_0 . The authors of the model recommend that b_0/b'_0 be taken as a constant, 1.6. The values of s'_e and β_0 are functions of the ratio of ambient to outfall velocity. Figures II.B.4 and 5 show the results of their hydraulic model studies to correlate these parameters with the velocity ratio.

Fig. II.B.4. Length of Flow Establishment as a Function of Ambient-to-discharge Velocity Ratio. (Adapted from Motz-Benedict.³)
ANL Neg. No. 190-1155 Rev.1.

4. Equations of the Model

The authors of the model develop four basic conservation equations in terms of the integrated quantities of mass flux, momentum flux in the x direction, momentum flux in the y direction, and heat flux. The integral form of these conservation equations is given in Table II.B.2.

TABLE II.B.2. Integral Form of Conservation
Equations for Motz-Benedict Model

Conservation of Mass Flux

$$\frac{d}{ds} \int_A u \, dA = \int_C V_i \, dC$$

Conservation of Momentum Flux in the X-direction

$$\frac{d}{ds} \left[\int_A u (u \cos \beta) \, dA \right] = C V_i U_a + F_D \sin \beta$$

Conservation of Momentum Flux in the Y-direction

$$\frac{d}{ds} \left[\int_A u (u \sin \beta) \, dA \right] = - F_D \cos \beta$$

Conservation of Heat Flux

$$\frac{d}{ds} \left[\int_A u T' \, dA \right] = - \int_{-b}^{+b} \frac{K}{\rho c_p} T' \, d\eta$$

where

u = local velocity

V_i = entrainment velocity

U_a = ambient velocity

T' = local temperature

β = angle of jet trajectory with shore

F_D = drag force on jet

K = heat loss coefficient

ρ = density of effluent

c_p = specific heat of effluent

A = cross sectional area of the jet

Fig. II.B.5. Angle of Jet with Current at End of Zone of Flow Establishment as a Function of the Discharge Angle and the Ambient-to-Discharge Velocity Ratio. (Adapted from Motz-Benedict,³) ANL Neg. No. 190-1188.

The conservation of mass flux is arrived at by equating the rate of change of mass flux along the jet centerline to the net inflow due to entrainment. The two equations for the conservation of momentum flux are developed by equating the rate of change of momentum flux along the jet centerline to the rate of entrainment of ambient momentum plus the net drag force in each of the two coordinate directions. The conservation of heat flux is expressed as an equality between the rate of change of heat flux along the jet centerline and the rate of surface loss.

Using the assumed forms for the local velocity and temperature distributions, integral quantities may be determined and the conservation equations expressed in terms of a single independent variable, s . Nondimensionalizing the conservation equations and adding the two equations of geometry yield the set of Table II.B.3.

5. Analysis

The values of nine parameters must be established in order to make model calculations:

β'_0 = angle of discharge with respect to the current,

b'_0 = half-width of the outfall channel,

A = ratio of ambient to outfall velocities,

E = entrainment coefficient,

TABLE II.B.3. Nondimensionalized Equations of the Motz-Benedict Model

Conservation of Mass Flux

$$\frac{dV}{dS} = \frac{M}{V} - A \cos \beta$$

Conservation of Momentum Flux in X-direction

$$\frac{d}{dS} (M \cos \beta) = \sqrt{2} A \left(\frac{M}{V} - A \cos \beta + AC'_D \sin^2 \beta \right)$$

Conservation of Momentum Flux in Y-direction

$$\frac{d}{dS} (M \sin \beta) = -\sqrt{2} A \left(AC'_D \sin \beta \cos \beta \right)$$

Conservation of Heat Flux

$$\frac{dQ}{dS} = -\frac{Q\phi V}{Z_o M}$$

Equations of Geometry

$$\frac{dX}{dS} = \cos \beta$$

$$\frac{dY}{dS} = \sin \beta$$

where

$$V = \frac{U b}{U_o b_o} \text{ (nondimensional volume flux)}$$

$$M = \frac{U^2 b}{U_o^2 b_o} \text{ (nondimensional momentum flux)}$$

$$Q = \frac{U b T}{U_o b_o T_o} \text{ (nondimensional heat flux)}$$

$$S = \frac{2E}{\sqrt{\pi} b_o} s \text{ (nondimensional arc length)}$$

$$Z_o = \frac{2E}{\sqrt{\pi} b_o} z_o \text{ (nondimensional plume depth)}$$

$$A = \frac{U}{U_o} \text{ (nondimensional ambient current)}$$

$$C'_D = \frac{C_D}{4E} \text{ (reduced drag coefficient)}$$

$$\phi = \frac{2}{\sqrt{2}} \frac{K}{\rho C_p} \frac{1}{U_o} \text{ (reduced heat loss coefficient)}$$

C_D = drag coefficient,

K = heat-transfer coefficient,

L = ratio of length of flow establishment to the width of the outfall,

r = ratio of jet width at end of flow establishment to the width of the outfall,

and

r' = ratio of the angle the jet makes with the current at end of flow establishment to the equivalent angle at the outfall.

Among the nine parameters, only the first three are directly measurable. The others must be determined from experimental data. Because the data are sparse, the results from which the values are to be determined are inconclusive and prove to be one of the greatest inadequacies of the model. Until these parameters have been well correlated with experimental data, we cannot recommend the model as a predictive tool. Not all the parameters, however, are significant in their effect on model predictions. It is important, from both a modeling and practical point of view, to determine which parameters are most important in fitting the model predictions to data. To this end, it is instructive to examine the exact analytical solution that exists for $A = K = 0$.

$$\left. \begin{aligned} \frac{U}{U_0} &= \xi^{-1/2} \\ \frac{T}{T_0} &= \xi^{-1/2} \\ \frac{b}{b'_0} &= r\xi \end{aligned} \right\} \xi \geq 1;$$

$$\xi = \frac{4E}{\sqrt{\pi r}} \left(\frac{s}{b'_0} \right) + \left(1 - \frac{8EL}{\sqrt{\pi r}} \right), \quad s \geq 2Lb'_0;$$

$$\text{AREA}(\theta/\theta_0) = 2(b'_0)^2 \left[L + Lr + f(\theta/\theta_0) \frac{r^2}{E} \right];$$

$$f(\lambda) = \frac{\sqrt{\pi}}{16} \lambda^{-4} \int_0^{-\ln \lambda} \sigma^{1/2} e^{-Z} d\sigma \approx 0.095 \lambda^{-4} \quad (\lambda \lesssim 0.5).$$

This last formula allows a quick estimate of the area within the lower isotherms without using the computer code.

In this simplified example, the crucial parameters of the model are exposed. They are the entrainment coefficient E and the jet-width ratio r . The parameter L is significant only in determining the location of predictions with respect to the outfall. We note also from the above solution that center-line velocity and temperature decay approximately as $s^{-1/2}$, while width grows linearly. Areas are roughly proportional to r^2/E .

For the more general case of $A \neq 0$, the deviation from the above behavior is small. The effects are principally on the jet trajectory, but widths and areas are normally smaller in the presence of crossflow.

For typical values of K , the effects of surface heat loss are small and are detectable only at large distances from the outfall near the limit of the model's applicability. Areas within lower temperature isotherms are slightly reduced, and the rate of temperature decay is somewhat increased.

A computer program written by Motz and Benedict has been modified by the Argonne Energy and Environmental Systems Division to facilitate our comparisons of model predictions with prototype field data. These comparisons appear in Chapter IV. We have also prepared a sample calculation with this program; the input parameters and computer output appear in Appendix A.

6. Criticisms and Limitations

a. Generic Limitations of Integral Models. The generic limitations of integral models are summarized in the Shirazi-Davis review (Sec. II.E), the last in the sequence of integral models considered.

b. Specific Limitation of the Motz-Benedict Model

(1) Range of Initial Densimetric Froude Number. Because the model treats neither vertical entrainment nor buoyant forces, the types of discharges for which the model may be used are severely limited. For densimetric Froude numbers less than about 5, buoyant convective motions and the resulting lateral spreading are important. For densimetric Froude numbers exceeding 5, a substantial amount of vertical entrainment may be expected. Within the Froude-number ranges referred to, these effects have been neglected.

(2) Small Ambient Currents. The local jet velocity is assumed to go to zero at large distances perpendicular to the jet centerline. Such an assumption is obviously physically invalid when a nonzero, crossflow velocity exists. Motz and Benedict chose this form to obtain convenient nondimensional relationships in terms of the velocity ratio A . If the velocity inflow equation were modified to account for an asymptotic approach to the ambient cross current (or better, its component in the jet axial direction), the fitted values of E would probably differ from those presently specified by the authors of the

model. In any case, this inflow velocity assumption becomes a physically invalid approximation when any but small ambient currents are present. The assumption is especially inappropriate at distances from the outfall where the jet centerline velocity approaches the ambient current value. Moreover, for ambient velocities of the same order of magnitude as the outfall velocity, the existence of a Gaussian velocity or temperature profile is doubtful. An additional problem in model applications when an ambient current is present involves the erratic results that occur when the ambient velocity exceeds about half the outfall velocity.

(3) Correlation of E with Ambient Velocity Only. The authors of the model assume that the entrainment coefficient can be successfully correlated with the ambient-to-outfall current ratio alone. There is considerable experimental evidence that entrainment depends on many other factors such as local densimetric Froude number or local ambient turbulence. In its present form, the model does not consider these effects.

(4) Unequal Spreading Rates for Heat and Momentum. Experimental data have indicated that velocity profiles in a turbulent surface jet are about half as wide as the temperature profiles. The model cannot successfully predict this behavior, since the assumption of equal widths is incorporated into the foundation of the model.

(5) Vague Selection Procedure for Entrainment Coefficient, E . Unfortunately, Motz and Benedict do not provide a clear, definitive scheme for selecting one of the most crucial parameters of the model, namely, the entrainment coefficient. Instead, a shotgun pattern of data is given (Fig. II.B.2) from which a value of E is to be obtained, based on the current ratio, A . For 90° initial discharge angles and $A = 0$, a value of 0.04 is recommended. For the same angle and $A > 0.2$, a value of 0.4 is suggested by the authors. In between, where many prototype situations lie, there is no clearly specified value. In the past, we have used linear interpolation between the above values, but must admit that the choice is arbitrary. The correction for an oblique discharge angle requires the insight of the user. When the current is large ($A \geq 0.2$), the authors of the model recommend that a value of 0.2 be used for the entrainment coefficient if the discharge angle is more like 45° than 90° , say $15-60^\circ$. For smaller ambient currents, the correction is less certain.

The major drawback to using this model in a predictive sense is the need for the specification of the drag, and particularly the entrainment coefficients. The value of the entrainment coefficient depends on boundary effects, the relationship being unknown, except in a gross way.

The drag coefficient C_D is reasonably well behaved, comparing favorably with the empirically obtained laboratory data. As a first approximation, therefore, either an extrapolation of the laboratory coefficients to the prototype situation can be considered to obtain C_D , or a value of roughly

0.5 can be assumed, as indicated by the field data of Motz and Benedict. They recommend the value 0.5 for general use; the model is not sensitive to the value chosen, except in determining the jet trajectory.

The model predicts characteristically long, narrow isotherms for nearly all applications. For problems having lower densimetric Froude numbers, the model underestimates plume widths and surface isotherm areas.

More difficulties appear upon a close examination of Table II.B.1 and Fig. II.B.2. A simple inspection shows considerable variation in the value of the entrainment coefficient from situation to situation. In particular, for lake data, the value of E is shown to be extremely small, ~ 0.04 ; while for riverine situations, E is about an order of magnitude larger. A particularly interesting contrast is afforded in Table II.B.1 when the Motz-Benedict laboratory data for 90° are compared with the Romberg-Ayers lake data. Although most of the initial conditions are similar, the entrainment coefficients are different by a factor of about 10. The influence of ambient cross-flows on the entrainment coefficient as the velocity ratio A approaches low values may be important here. Perhaps, too, part of the problem lies with the fact that the data were taken from locations at which prominences (breakwaters, for instance) were present.

Reference

1. L. Motz and B. Benedict, Heated Surface Jet Discharged into a Flowing Ambient Stream, Department of Environmental and Water Resources Engineering, Report No. 4, Vanderbilt University, Nashville, Tenn. (Aug 1970).

C. Stolzenbach-Harleman Integral Model¹⁻⁵

1. Synopsis of Approach

This model is developed from the three-dimensional, time-averaged equations of mass, momentum, and energy conservation for a turbulent, incompressible fluid. Molecular processes such as heat conduction and viscous transfer are ignored as is mechanical dissipation. These equations are integrated using similarity forms for temperature and velocity to yield a set of nonlinear ordinary differential equations that must be solved numerically.

The model relies on nonbuoyant jet theory, wherever possible, with corrections made for buoyant effects. Accordingly, entrainment coefficients are determined first by solving the nonbuoyant jet equations and then by reducing vertical entrainment to conform to the experimental results of Ellison and Turner.⁶ Buoyant spreading is simulated by means of a lateral spreading velocity. Crossflow velocities are assumed small, with the corresponding dynamic pressure neglected.

2. List of Symbols

A	Discharge channel aspect ratio; h_o/b_o
b	Half-width of the turbulent region of the jet
$\bar{b}$	$b/\sqrt{h_o b_o}$
b_o	Half-width of the discharge channel
c_p	Specific heat of water at constant pressure
D	Ratio of local jet flow to initial discharge flow; dilution
D_o	Diameter of circular discharge
D_w	Depth of water in hydraulic tank
$\left(\frac{db}{dx}\right)_{nb}$	Nonbuoyant spreading rate
F	Densimetric Froude number; $\frac{u}{\sqrt{\frac{\Delta\rho}{\rho}gh}}$
F_o	Densimetric Froude number at the point of discharge; $\frac{u_o}{\sqrt{\frac{\Delta\rho_o}{\rho_a}gh_o}}$

F_L	Local densimetric Froude number in the jet; $\frac{u_c}{\sqrt{\frac{a\Delta T_c g h}{\rho_a}}}$
F_y, F_z	Similarity functions for jet-velocity distribution
f	Similarity function for velocity; $(1 - \zeta^{3/2})^2$
g	Acceleration due to gravity
HT	Dimensionless heat flux
h	Vertical height of turbulent region of jet
$\bar{h}$	$h/\sqrt{h_0 b_0}$
h_0	Depth of the discharge channel
I_1	$\int_0^1 f(\zeta) d\zeta = 0.4500$
I_2	$\int_0^1 f^2(\zeta) d\zeta = 0.3160$
I_3	$\int_0^1 t(\zeta) d\zeta = 0.6000$
I_4	$\int_0^1 \int_{\zeta}^1 t(\bar{\zeta}) d\bar{\zeta} d\zeta = 0.2143$
I_5	$\int_0^1 f(\zeta) \zeta^{1/2} d\zeta = 0.2222$
I_6	$\int_0^1 f^2(\zeta) \zeta^{1/2} d\zeta = 0.1333$
I_7	$\int_0^1 f(\zeta) t(\zeta) d\zeta = 0.3680$
k	Surface heat loss coefficient
p	Pressure
p_d	Dynamic pressure
r	Vertical distance from jet centerline to boundary of core region
$\bar{r}$	$r/\sqrt{h_0 b_0}$

s	Horizontal distance from jet centerline to boundary of core region
$\bar{s}$	$s/\sqrt{h_0 b_0}$
T	Local temperature
T'	Turbulent temperature fluctuation
T_a	Ambient temperature
T_o	Outfall temperature
T_y, T_z	Similarity function for jet temperature distribution
l	Similarity function for temperature; $(1 - \zeta^{3/2})$
u, v, w	Velocity components in coordinate system relative to centerline of the deflected jet
u', v', w'	Turbulent velocity fluctuations
$\tilde{u}, \tilde{v}, \tilde{w}$	Velocity components in the fixed coordinate system
$\bar{u}$	Dimensionless surface centerline jet velocity; u_c/u_o
u_o	Velocity in discharge channel (assumed uniform)
u_c	Surface centerline jet velocity
V	Ambient crossflow velocity
$\bar{V}$	Ambient to outfall velocity ratio; V/u_o
v_b	Lateral velocity in jet at $ y = s$ and $-h < z < -r$
v_e	Lateral velocity of entrained flow at jet boundary
v_s	Lateral velocity in jet at $ y = s$ and $-r < z < \eta$
w_e	Vertical velocity of entrained flow at jet boundary
w_h	Vertical velocity in jet at $z = -r$ and $s < y < b + s$
w_r	Vertical velocity in jet at $z = -r$ and $0 < y < s$
x, y, z	Coordinate directions relative to centerline of the deflected jet
$\tilde{x}, \tilde{y}, \tilde{z}$	Coordinate directions in fixed system
$\bar{x}$	Dimensionless distance along jet centerline; $x/\sqrt{h_0 b_0}$
α_y	Lateral entrainment coefficient in nonbuoyant and buoyant jets
α_z	Vertical entrainment coefficient in a nonbuoyant jet
α_{sz}	Vertical entrainment coefficient in a buoyant jet

β	Coefficient of thermal expansion; $\frac{1}{\rho} \frac{\partial \rho}{\partial T} \approx -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T}$
ΔT	Temperature excess above ambient; $(T - T_a)$
$\overline{\Delta T}$	Surface centerline temperature excess ratio; $\Delta T_c / \Delta T_o$
ΔT_c	Surface centerline temperature excess; $(T_c - T_a)$
ΔT_o	Initial temperature excess at the discharge; $(T_o - T_a)$
$\Delta \rho$	Difference between local and ambient water densities; $(\rho_a - \rho)$
$\Delta \rho_o$	Density difference at the discharge; $(\rho_a - \rho_o)$
ϵ	Spreading rate of the turbulent undeflected nonbuoyant jet (≈ 0.22)
η	Water surface elevation
θ	Angle between jet centerline and $\tilde{y}$ axis
θ_o	Angle between discharge channel centerline and $\tilde{y}$ axis
ρ	Density of water
ρ_a	Density of ambient water
ρ_o	Density of effluent at the outfall

3. Basic Assumptions and Model Development

The two coordinate systems used in the development of the model equations are shown in Fig. II.C.1. The $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ system is a rectangular Cartesian coordinate system oriented so that the $\tilde{x}$ direction is offshore, the

Fig. II.C.1

Stolzenbach-Harleman Coordinate System. (Adapted from Stolzenbach and Harleman.³) ANL Neg. No. 190-1148.

$\tilde{y}$ direction is shore parallel, and the $\tilde{z}$ direction is vertically downward. The origin is located at the midpoint and surface of the discharge channel. The second system, designated x, y, z , is a rectangular Cartesian system centered on the hypothetical jet axis as shown in Fig. II.C.1. These two systems are related by the following equations of geometry:

$$\frac{d\tilde{x}}{dx} = \sin \theta \quad (1)$$

and

$$\frac{d\tilde{y}}{dx} = \cos \theta, \quad (2)$$

where θ is the angle between the jet centerline and the $\tilde{y}$ axis. Also shown in Fig. II.C.1 is the profile of the ambient crossflow $V(\tilde{x})$ assumed in model development. The ambient flow is treated as shore parallel but is allowed to vary with offshore position.

Motivated by experimental results from nonbuoyant jet theory, Stolzenbach and Harleman divide the jet conceptually into four separate regions as shown in Fig. II.C.2. The assumed physical characteristics of the four regions are:

Fig. II.C.2. Assumed Jet Structure of Stolzenbach-Harleman Model. (Adapted from Stolzenbach and Harleman.³) ANL Neg. No. 190-1161 Rev. 1.

Region 1: An unsheared central core of half-width s and depth r .

Region 2: A vertically sheared region of thickness h below the core.

Region 3: A horizontally sheared region of width b , laterally adjacent to the core.

Region 4: A region sheared in both directions.

In the unsheared regions, the velocity and temperature profiles are assumed to be uniform, whereas in the sheared regions the profiles are assumed to decay smoothly from their maximum value at the interior edge of the region to zero at the boundary of the jet. In particular, the similarity profiles assumed for the jet are taken to be

$$u(x, y, z) = u_c(x)F_y F_z + V \cos \theta \quad (3)$$

and

$$\Delta T(x, y, z) = \Delta T_c(x)T_y T_z, \quad (4)$$

where

$$T_y = \begin{cases} 1.0 & 0 < |y| < s \\ t\left(\frac{|y| - s}{b}\right) & s < |y| < s + b \\ 0 & s + b < |y|, \end{cases} \quad (5)$$

$$T_z = \begin{cases} 1.0 & -r < z < \eta \\ t\left(\frac{-z - r}{h}\right) & -r - h < z < -r \\ 0 & z < -r - h, \end{cases} \quad (6)$$

$$F_y = T_y^2, \quad (7)$$

$$F_z = T_z^2, \quad (8)$$

and

$$t(\zeta) = 1 - \zeta^{3/2}. \quad (9)$$

It should be noted here that regions 1, 2, and 3 exist only within the initial flow-development regime commonly referred to as the zone of flow establishment. At the outfall, only region 1 exists implying that a uniform discharge profile is assumed.

Unlike the other integral models reviewed, where model equations are derived by balancing mass, momentum, and heat flows into a control volume slice of the jet, the Stolzenbach-Harleman equations are obtained by integrating the equations of flow with terms of small order omitted. To do this a number of boundary and interregion conditions must be specified. On the one hand, the control-volume methods present, in our opinion, a clearer view of the physical assumptions being made, but entail a number of confusing geometric considerations such as the vector addition of velocities at the center and edge of the jet. On the other hand, the Stolzenbach-Harleman approach tends to obscure the assumptions being made, but does allow an almost mechanical derivation of the model equations. Stolzenbach⁵ states that, with the division of the jet into four regions, the control volume approach is particularly cumbersome from the standpoint of satisfying the attendant geometrical considerations.

The concept of entrainment is introduced in developing the model equations. In the Stolzenbach-Harleman treatment, a lateral entrainment velocity v_e and a vertical entrainment velocity w_e are assumed to exist on the three submerged sides of the jet. These entrainment velocities are further assumed to be proportional to the centerline velocity $u_c(x)$. Thus,

$$v_e = \alpha_y u_c(x), \quad (10)$$

and

$$w_e = \alpha_{sz} u_c(x), \quad (11)$$

where α_{sz} and α_y are vertical and lateral entrainment coefficients, respectively. To evaluate these coefficients, the authors apply results from nonbuoyant jet theory. Lateral entrainment is assumed to be only negligibly affected by buoyancy and the above formula is applied without a correction for buoyancy. Vertical entrainment is reduced to account for the presence of buoyant forces according to the theory of Phillips⁷ and the experimental results of Ellison and Turner.⁶ Stolzenbach and Harleman write

$$\alpha_y = \begin{cases} -(I_1 - I_2)\epsilon & s > 0 \\ -\frac{I_1\epsilon}{2} & s = 0, \end{cases} \quad (12)$$

$$\alpha_z = \begin{cases} (I_1 - I_2)\epsilon & r > 0 \\ \frac{I_1\epsilon}{2} & r = 0, \end{cases} \quad (13)$$

and

$$\alpha_{sz} = \alpha_z \exp(-5.0F_L^{-2}), \quad (14)$$

where ϵ is the spreading rate of a nonbuoyant jet, normally taken to be 0.22.

The derivation of model equations is outlined in Tables II.C.1-3. An important point here is that the assumptions and conditions used in the derivation are motivated largely by the desire to achieve equations which, in their final form, state the basic conservation laws for integral quantities such as mass, momentum, and heat flux. In a sense, the end justifies the means although Stolzenbach and Harleman^{1,3} present plausibility arguments to support the specific boundary conditions given. The reader should refer to the original reports for those arguments, for they will not be repeated here.

TABLE II.C.1. Partial Differential Equations of Stolzenbach-Harleman Model Indicating the Scale of Various Terms

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

1 1 1

Conservation of Longitudinal Momentum

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -g \frac{\partial \eta}{\partial x} - \frac{g}{\rho_a} \int_{\eta}^z \frac{\partial \Delta \rho}{\partial x} dz - \frac{1}{\rho_a} \frac{\partial p_d}{\partial x} - \frac{\partial u'^2}{\partial x} - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z}$$

1 1 1 δ F^{-2} δ δ 1 1

Conservation of Lateral Momentum

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -g \frac{\partial \eta}{\partial y} - \frac{g}{\rho_a} \int_{\eta}^z \frac{\partial \Delta \rho}{\partial y} dz - \frac{1}{\rho_a} \frac{\partial p_d}{\partial y} - \frac{\partial u'v'}{\partial x} - \frac{\partial v'^2}{\partial y} - \frac{\partial v'w'}{\partial z}$$

1 1 1 δ F^{-2} δ δ 1 1

Conservation of Vertical Momentum

$$\frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial w^2}{\partial z} = \frac{1}{\rho_a} \frac{\partial p_d}{\partial z} - \frac{\partial u'w'}{\partial x} - \frac{\partial v'w'}{\partial y} - \frac{\partial w'^2}{\partial z}$$

δ δ δ δ δ 1 1

Conservation of Heat

$$\frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} = -\frac{\partial u'T'}{\partial x} - \frac{\partial v'T'}{\partial y} - \frac{\partial w'T'}{\partial z}$$

1 1 1 δ 1 1

where

u, v, w = mean velocity components

u', v', w' = fluctuating velocity components

T = mean temperature

T' = fluctuating temperature

η = surface elevation

ρ = density of fluid

ρ_a = reference density

$\Delta \rho = \rho_a - \rho$

p_d = dynamic pressure

g = acceleration due to gravity

TABLE II.C.2. Stolzenbach-Harleman Equations with
Terms of Small Order Neglected

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of Longitudinal Momentum

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = \beta g \int_0^z \frac{\partial \Delta T}{\partial x} dz' - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z}$$

Conservation of Lateral Momentum

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = \beta g \int_0^z \frac{\partial \Delta T}{\partial y} dz' - \frac{\partial v'^2}{\partial y} - \frac{\partial v'w'}{\partial z}$$

Conservation of Vertical Momentum

$$\frac{\partial v'w'}{\partial y} + \frac{\partial w'^2}{\partial z} = 0$$

Conservation of Heat

$$\frac{\partial u \Delta T}{\partial x} + \frac{\partial v \Delta T}{\partial y} + \frac{\partial w \Delta T}{\partial z} = - \frac{\partial v'T'}{\partial y} - \frac{\partial w'T'}{\partial z}$$

TABLE II.C.3. Integration of the Stolzenbach-Harleman Equations

Continuity and x-momentum Equations

The continuity and x-momentum equations are integrated separately over the four regions of the jet using the following boundary and interregion velocities:

At the bottom of the jet ($z = -r - h$)

$$w = \begin{cases} w_e - V \cos \theta \left(\frac{dh}{dx} + \frac{dr}{dx} \right) & 0 < |y| < s \\ w_e f\left(\frac{|y| - s}{b}\right) - V \cos \theta \left(\frac{dh}{dx} + \frac{dr}{dx} \right) & s < |y| < s + b \\ 0 & s + b < |y| \end{cases}$$

TABLE II.C.3 (Contd.)

and

on the side of the jet ($|y| = s + b$)

$$v = \begin{cases} v_e + V \cos \theta \left(\frac{ds}{dx} + \frac{db}{dx} \right) & -r < z < 0 \\ v_e f\left(\frac{-z-r}{h}\right) + V \cos \theta \left(\frac{ds}{dx} + \frac{db}{dx} \right) & -r-h < z < -r \\ 0 & z < -r-h, \end{cases}$$

where

$$f(\zeta) = t^2(\zeta) = (1 - \zeta^{3/2})^2,$$

$$v_e = \alpha_y u_c(x),$$

and

$$w_e = \alpha_{sz} u_c(x).$$

At $z = r$

$$w = \begin{cases} w_r & 0 < |y| < s \\ w_h f\left(\frac{|y|-s}{b}\right) & s < |y| < s+b \\ 0 & s+b < |y|. \end{cases}$$

At $|y| = s$

$$v = \begin{cases} v_s & -r < z < 0 \\ v_b f\left(\frac{-z-r}{h}\right) & -r-h < z < -r \\ 0 & z < -r-h. \end{cases}$$

Since the interregion velocities w_r , w_h , v_s , and v_b are eliminated in the next step, only the functional form of the interregion conditions is important.

Turbulent transfer between jet regions and between the jet and ambient is eliminated by requiring that:

$$\text{At } y = 0, |y| = s, |y| = s+b, \overline{(v')^2} = \overline{u'v'} = \overline{v'w'} = 0.$$

$$\text{At } z = 0, z = -r, z = -r-h, \overline{(w')^2} = \overline{u'w'} = \overline{w'v'} = 0.$$

Two additional boundary conditions are obtained from jet symmetry

$$v = 0 \text{ at } y = 0$$

TABLE II.C.3 (Contd.)

and from the free-surface condition

$$w = 0 \text{ at } z = 0.$$

Using these boundary conditions, the following eight equations may be obtained.

Region 1, continuity

$$rs \frac{d}{dx} (u_c + V \cos \theta) + rv_s - sw_r = 0.$$

Region 2, continuity

$$s \frac{d}{dx} [h(u_c I_1 + V \cos \theta)] + (u_c + V \cos \theta) \frac{dr}{dx} + w_r - \alpha_{sz} u_c + v_b h I_1 = 0.$$

Region 3, continuity

$$r \frac{d}{dx} [b(u_c I_1 + V \cos \theta)] + (u_c + V \cos \theta) \frac{ds}{dx} - v_s + \alpha_y u_c - w_h b I_1 = 0.$$

Region 4, continuity

$$\frac{d}{dx} [hb(u_c I_1^2 + V \cos \theta)] + (u_c I_1 + V \cos \theta) \left(b \frac{dr}{dx} + h \frac{ds}{dx} \right) + (w_h - \alpha_{sz} u_c) I_1 b - (v_b - \alpha_y u_c) I_1 h = 0.$$

Region 1, x momentum

$$(u_c + V \cos \theta) \left[2rs \frac{d}{dx} (u_c + V \cos \theta) + rv_s - sw_r \right] + \beta g s \left[\frac{d}{dx} \left(\Delta T_c \frac{r^2}{2} \right) + I_3 r \frac{d}{dx} (\Delta T_c h) \right] = 0.$$

Region 2, x momentum

$$s \left\{ \frac{d}{dx} [h(u_c^2 I_2 + 2u_c I_1 V \cos \theta + V^2 \cos^2 \theta)] + (u_c + V \cos \theta)^2 \frac{dr}{dx} + w_r (u_c + V \cos \theta) \right. \\ \left. - \alpha_{sz} u_c V \cos \theta + \beta g \left(I_4 \frac{d\Delta T_c h^2}{dx} + I_3 \Delta T_c h \frac{dr}{dx} \right) \right\} + v_b h (u_c I_2 + I_1 V \cos \theta) = 0.$$

Region 3, x momentum

$$r \left\{ \frac{d}{dx} [b(u_c^2 I_2 + 2u_c I_1 V \cos \theta + V^2 \cos^2 \theta)] + (u_c + V \cos \theta)^2 \frac{ds}{dx} - v_s (u_c + V \cos \theta) \right. \\ \left. + \alpha_y u_c V \cos \theta \right\} - w_h b [u_c I_2 + I_1 V \cos \theta] + \beta g \left[\frac{I_3}{2} \frac{d\Delta T_c b r^2}{dx} + I_3^2 r \frac{d\Delta T_c b h}{dx} \right. \\ \left. + \Delta T_c \left(\frac{r^2}{2} + I_3 h r \right) \frac{ds}{dx} \right] = 0.$$

TABLE II.C.3 (Contd.)

Region 4, x momentum

$$\begin{aligned} \frac{d}{dx} [hb(u_c^2 I_2^2 + 2u_c I_1^2 V \cos \theta + V^2 \cos^2 \theta)] + (u_c^2 I_2 + 2u_c I_1 V \cos \theta + V^2 \cos^2 \theta) \left(b \frac{dr}{dx} + h \frac{ds}{dx} \right) \\ + (w_h b - v_b h)(u_c I_2 + I_1 V \cos \theta) - u_c I_1 (\alpha_{sz} b - \alpha_y h) V \cos \theta + \beta g \left(I_3 I_4 \frac{d\Delta T_c h^2 b}{dx} + I_4 \Delta T_c h^2 \frac{ds}{dx} \right. \\ \left. + I_3^2 \Delta T_c h b \frac{dr}{dx} \right) = 0. \end{aligned}$$

Eliminating the four arbitrary interregion parameters and nondimensionalizing, we obtain the following equations of continuity and conservation of x momentum in terms of the initial densimetric Froude number F_0 , the aspect ratio A , and the dimensionless variables denoted by a bar.

$$\frac{d}{d\bar{x}} [\bar{u}(\bar{s} + \bar{b}I_1)(\bar{r} + \bar{h}I_1) + \bar{V} \cos \theta (\bar{s} + \bar{b})(\bar{r} + \bar{h})] - \bar{u}[\alpha_{sz}(\bar{s} + \bar{b}I_1) - \alpha_y(\bar{r} + \bar{h}I_1)] = 0. \quad (A)$$

$$\begin{aligned} \frac{d}{d\bar{x}} [\bar{u}^2(\bar{s} + \bar{b}I_2)(\bar{r} + \bar{h}I_2) + 2\bar{u}\bar{V} \cos \theta (\bar{s} + \bar{b}I_1)(\bar{r} + \bar{h}I_1) + \bar{V}^2 \cos^2 \theta (\bar{s} + \bar{b})(\bar{r} + \bar{h}) \\ + F_0^{-2} A^{-1/2} \bar{\Delta T} (\bar{s} + \bar{b}I_3)(\frac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2 I_4)] - \bar{u}\bar{V} \cos \theta [\alpha_{sz}(\bar{s} + \bar{b}I_1) \\ - \alpha_y(\bar{r} + \bar{h}I_1)] = 0. \end{aligned} \quad (B)$$

$$\begin{aligned} (\bar{r}\bar{b} + \bar{s}\bar{h}) \left\{ (\bar{u}I_2 + I_1\bar{V} \cos \theta) \frac{d\bar{u}}{d\bar{x}} + \left[\bar{u} \left(2I_1 - \frac{I_2}{I_1} \right) + \bar{V} \cos \theta \right] \frac{d\bar{V} \cos \theta}{d\bar{x}} \right\} \\ + \bar{u}\bar{V} \cos \theta \left(I_1 - \frac{I_2}{I_1} \right) \left(\bar{s} \frac{d\bar{h}}{d\bar{x}} + \bar{r} \frac{d\bar{b}}{d\bar{x}} \right) + \bar{u} \left(1 - \frac{I_2}{I_1} \right) \frac{d}{d\bar{x}} [\bar{r}\bar{s}(\bar{u} + \bar{V} \cos \theta)] \\ + F_0^{-2} A^{-1/2} \left[\bar{s} \left(I_4 \frac{d\bar{\Delta T} \bar{h}^2}{d\bar{x}} + I_3 \Delta T \bar{h} \frac{d\bar{r}}{d\bar{x}} \right) + \frac{1}{2} I_3 \frac{d\bar{\Delta T} \bar{b} \bar{r}^2}{d\bar{x}} + I_3^2 \bar{r} \frac{d\bar{\Delta T} \bar{b} \bar{h}}{d\bar{x}} \right. \\ \left. + \bar{\Delta T} (\frac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3) \frac{d\bar{s}}{d\bar{x}} \right] + \bar{u}(\alpha_{sz}\bar{s} - \alpha_y\bar{r}) \frac{I_2}{I_1} = 0. \end{aligned} \quad (C)$$

$$(\bar{u} + \bar{V} \cos \theta) \frac{d}{d\bar{x}} (\bar{u} + \bar{V} \cos \theta) + F_0^{-2} A^{-1/2} \left(\frac{1}{2} \bar{r} \frac{d\Delta T}{d\bar{x}} + \bar{\Delta T} \frac{d\bar{r}}{d\bar{x}} + I_3 \frac{d\bar{\Delta T} \bar{h}}{d\bar{x}} \right) = 0. \quad (D)$$

Heat-energy Equation

To obtain the heat-energy equation, three conditions are imposed on the turbulent transfer of heat as follows:

$$\text{At } y = 0, |y| = s + b, \overline{v'T'} = 0.$$

$$\text{At } z = 0, \overline{w'T'} = k(T - T_a).$$

$$\text{At } z = -r - h, \overline{w'T'} = 0.$$

These conditions, coupled with the ones for velocity, give, after carrying out the necessary integrations and changing to dimensionless variables,

TABLE II.C.3 (Contd.)

$$\frac{d}{dx} [\bar{u} \bar{\Delta} \bar{T} (\bar{s} + \bar{b} I_7) (\bar{r} + \bar{h} I_7) + \bar{\Delta} \bar{T} \bar{V} \cos \theta (\bar{s} + \bar{b} I_3) (\bar{r} + \bar{h} I_3)] + \left(\frac{k}{u_0} \right) \bar{\Delta} \bar{T} (\bar{s} + \bar{b} I_3) = 0. \quad (E)$$

Conservation of y momentum:

A different set of boundary conditions is specified for the integration of the y-momentum equation.

For all z ,

$$v = \begin{cases} \pm \left(\frac{db}{dx} - \epsilon \right) u \left(\frac{|y| - s}{b} \right)^{1/2}, & s < |y| < s + b \text{ (v has sign of y),} \\ 0 & \text{otherwise,} \end{cases}$$

where db/dx and ϵ are the total and nonbuoyant spreading rates respectively.

At $z = -r - h$,

$w = 0$.

Integrating the y-momentum equation, using these conditions on v and w rather than those previously given and changing to dimensionless variables, yields

$$\frac{d}{dx} \left\{ \left(\frac{db}{dx} - \epsilon \right) [u^2 \bar{b} I_6 (\bar{r} + \bar{h} I_2) + 2 \bar{u} \bar{b} I_5 \bar{V} \cos \theta (\bar{r} + \bar{h} I_1) + \frac{2}{3} \bar{b} \bar{V}^2 \cos^2 \theta (\bar{r} + \bar{h})] \right\} - F_0^{-2} A^{-1/2} \bar{\Delta} \bar{T} \left(\frac{1}{2} \bar{r}^2 + \bar{r} \bar{h} I_3 + \bar{h}^2 I_4 \right) = 0. \quad (F)$$

The coefficient of the underscored term was incorrectly given as 1 in the Stolzenbach-Harleman model equations and computer code. It has only very recently been corrected and all calculations of which we are aware were made with the wrong value. The effects of this error on the results are not presently known.

Equation of Jet Bending

An equation of jet bending, used only for nonzero crosscurrent cases, is derived from the following integral conservation relationship.

$$\frac{d\theta}{dx} \int_0^{s+b} \int_{-r-h}^0 u^2(x, y, z) dy dz = V \sin \theta \left\{ \int_{-r-h}^{-r} v_e f\left(\frac{-z-r}{h}\right) dz + \int_{-r}^0 v_e dz + \int_0^s w_e dy + \int_s^{s+b} w_e f\left(\frac{|y|-s}{b}\right) dy \right\}.$$

When these integrations are performed and the dimensionless variables introduced, the result is

$$\frac{d\theta}{dx} + \bar{V} \sin \theta \frac{\bar{u} [\alpha_{sz} (\bar{s} + \bar{b} I_1) + \alpha_y (\bar{r} + \bar{h} I_1)]}{\bar{u}^2 (\bar{s} + \bar{b} I_2) (\bar{r} + \bar{h} I_2) + 2 \bar{u} \bar{V} \cos \theta (\bar{s} + \bar{b} I_1) (\bar{r} + \bar{h} I_1) + \bar{V}^2 \cos^2 \theta (\bar{s} + \bar{b}) (\bar{r} + \bar{h})} = 0. \quad (G)$$

Stolzenbach and Harleman begin with the time-averaged equations of turbulent flow as shown in Table II.C.1. Assuming that the scale of vertical motion is small compared to horizontal motion, that dynamic pressure (drag force) is small compared to hydrostatic pressure, and that surface fluctuations are small compared to the mean jet depth, the authors determine the scale of each term and eliminate those of order δ , as noted in Table II.C.1. This results in the somewhat simplified equation set of Table II.C.2.

Table II.C.3 shows how the equations are then integrated using the boundary conditions specified in that table and the similarity forms postulated earlier in this review. Equations A-G of Table II.C.3 and Eqs. 1 and 2, given earlier, constitute the nine basic model equations, although the jet-bending equation (Eq. G, Table II.C.3) is required only when a nonzero crossflow is treated.

There are several important features of the derivations summarized in Table II.C.3 that warrant comment. First, the boundary conditions used to integrate the continuity and x-momentum equations seem to us to be somewhat arbitrary, although Stolzenbach^{1,2,5} argues their validity based on intuitive reasoning. In particular, the functional form of the interregion velocities and the elimination of turbulent transfer between regions seems questionable to us. Second, a subtle point is obscured by the algebraic complexity of the continuity and x-momentum integrations. It appears that the two physical laws have been expanded to four independent equations. In actuality, the four equations are derived from the two physical laws plus the two equations specifying entrainment velocities. Third, a different and conflicting set of boundary conditions is used in the integration of the y-momentum equation. The contradiction between the form specified for w in the continuity and x-momentum integrations and that used in the y-momentum integrations is particularly disturbing. Also noteworthy is the similarity form specified for v when the y-momentum integrations are performed, which is where the modeling of buoyant spreading is included. We think this form is untested and to a large extent can never be tested, except through its effects on overall model performance. Stolzenbach,^{1,5} on the other hand, argues the validity of this modeling concept based on his general conceptualization of the jet and its spreading mechanisms. Finally, the equation of jet bending is formulated using only the flow due to "entrainment." If the boundary flows specified in the continuity and x-momentum integrations were used instead, an additional term would arise. Stolzenbach⁵ argues that such a term is incorrect in the jet-bending equation, because it would lead to bending even if the entrainment coefficients were set to zero. Furthermore, the model does not allow external forces, such as wind acting at the surface or the drag of ambient current, to bend the jet. Instead, only the entrainment of ambient momentum is considered. We consider this omission only a small deficiency, for our experience indicates that momentum entrainment is the major component of bending even within the integral models that allow drag force.

The model equations in their final form do present the traditional point of view in which the rate of change of a basic integral quantity is balanced against the forces or flows that are thought to produce the change. The conservation of heat equation is a simple example. In that equation, the first term represents the rate of change of jet heat flux along the jet centerline, and the second term is the rate at which heat is lost to the atmosphere at the surface. The ability to identify these integral balances in the final equation justifies, to a limited degree, the procedure outlined in Table II.C.3; but does not explain away, the problems in that derivation such as the previously mentioned need to choose different boundary conditions for different equations. In our opinion, the procedure followed by Stolzenbach and Harleman does not alleviate, and to some extent aggravates, the problems associated with the control volume method. To a certain extent, Stolzenbach⁵ agrees.

4. Coding the Equations for Numerical Solution

Since the second order derivative of b appears in the y -momentum equation, the equations of Table II.C.3 are equivalent to a set of ten first order ordinary differential equations. These equations can be integrated by any of several standard numerical techniques (the Runge-Kutta method is used in the Stolzenbach-Harleman code), provided that explicit expressions can be found for each of the ten derivatives. This is a straightforward matter for four of the derivatives; however, the solution for the other six is very difficult. Fortunately, the six difficult-to-solve ordinary differential equations form a linear algebraic system, the unknowns being the derivatives. This algebraic system can be solved numerically through matrix reduction. At times, the coefficient matrix of the system, which must be solved at each integration step along the jet axis, has a determinant less than 10^{-12} implying that the set is very nearly singular. The combination of numerical integrations and matrix reductions makes the procedure very susceptible to numerical error and caution should be exercised in interpreting results obtained from this two-step numerical procedure.

To compound the difficulties discussed above, the computer code written by the authors of the model suffers from several operational problems. To fully explain this statement, we must digress from the model development and present several facts concerning the computer program itself. First, there are several editions of the code in use:

MIT Version No. 1

A computer code was originally published by Stolzenbach and Harleman along with their hydraulic model data and sample model predictions.¹ It included a treatment of bottom interaction that was later abandoned by them and therefore that treatment was not discussed in our review of the model. The code failed in all of our attempts to use it. We were never able to get the code to run successfully, despite the fact that the deck and sample

output were supplied to us by MIT. No coding changes were needed as the program was already written in IBM FORTRAN. This program, or some modification of it, was used by MIT for predictions at Pilgrim (see discussion in Ch. IV); but, we were never able to duplicate any results of the "theory with bottom slope," nor to understand why our deck failed to run. Furthermore, a line-by-line check of the program uncovered no discrepancies. One possible explanation for the variations in code performance is some unknown difference in the two computers. If this is true, we seriously question the adequacy of the code.

MIT Version No. 2

This program is the so-called "User's Manual Code" presented by Stolzenbach et al.³ With this code we were able to duplicate all of the test cases supplied with the deck and, with a few exceptions, all of the curves presented in the User's Manual.³

Following the publication of the User's Manual Code, Harleman⁴ suggested some minor changes in the criterion used to terminate the integration of the equations; these are summarized below.

1. The variable UI is added to blank common.
2. UI is set equal to 1.0 in the main program.
3. In subroutine OUTP these statements

UI = U

IF(ABS(M-RMI)-.25) 180,180,158

are replaced by

UI = U + V*DCOS(TH)

RMX = RMI*SIN(THI)/DSIN(TH)

IF(ABS(M-RMX)-.25) 180,180,158

We made these changes in our copy of the User's Manual Code following Harleman's suggestion.

Argonne Version

It is our policy to spotcheck each computer code that we use for model calculations. Time, unfortunately, does not permit a very thorough examination of each code, because the methodical unraveling of a complex program is a painfully slow process. During the course of our routine

investigation, however, we encountered some potential errors in coding. These issues were discussed with the model developers who agreed that the errors did exist in the code but hoped that the corrections would have an insignificant effect on the calculations. Nevertheless, the program changes summarized in Table II.C.4 were incorporated into our copy of the Stolzenbach-Harleman computer code near the end of subroutine FCT. Later discussions with the model developers revealed that none of the corrections we suggested were implemented by MIT until recently (May 1975). (For example, the Pilgrim predictions prepared by MIT and discussed in Sec. IV.D were made without the code corrections.)

TABLE II.C.4. Corrections of the Stolzenbach-Harleman Computer Code

<u>Original Statements</u>	<u>Corrected Statements</u>
$\left\{ \begin{array}{l} C(5) = (U+VCT)*DT \\ GO TO 9 \\ 8 A(4, 1) = A(4, 1) + VCT*I1*(R*B+S*H) \end{array} \right.$	$\left\{ \begin{array}{l} C(5) = (U+VCT)*DT \\ 8 A(4, 1) = A(4, 1) + VCT*I1*(R*B+S*H) \end{array} \right.$
$A(4, 2) = A(4, 2) + S*U*VCT*(I1-I1/I2)$	$A(4, 2) = A(4, 2) + S*U*VCT*(I1-I2/I1)$
$C(4) = C(4) - R*U*VCT*(I1-I1/I2)*BX$	$C(4) = C(4) - R*U*VCT*(I1-I2/I1)*BX$
$132 A(6, 1) = A(6, 1) + 2*B*VCT*I5*H2*DB$	$132 A(6, 1) = A(6, 1) + 2*B*VCT*I5*H1*DB$

Two additional comments are warranted. First, the corrections presented in Table II.C.4 are those uncovered after only a cursory examination of the code, and we feel that a closer look is needed. Second, given the location and nature of the Argonne modifications, differences between the MIT (Version No. 2) and Argonne predictions, if any, should occur only in nonzero crosscurrent cases.

To determine whether any significant difference exists between the MIT and Argonne versions, we decided to run both codes for the two sites for which published model-data comparisons exist.

Comparison to Published Pilgrim Calculations

As expected, the two computer codes (MIT Version 2 and Argonne Version) agree exactly for all stagnant cases. Further, our code results agree with the published results of MIT^{8,9} with one notable exception. That exception is the case of April 19, 1973, in which the initial densimetric Froude number is 1.7. The results that we obtained agree with the MIT curve for the first hundred feet or so from outfall. At that point, our curve changes slope and then quickly terminates. The MIT curve continues on with an almost negligible change in temperature for several hundred additional feet, a result that is physically most improbable.

In the few nonzero crosscurrent cases published, the two computer codes give very similar, but not exactly equal, results. Furthermore, general agreement exists between the two codes and the published comparisons. Thus, for all practical purposes, we are able to duplicate the Pilgrim model-data comparisons with both the MIT and the Argonne versions of the model. We would say, then, that the corrections appearing in Table II.C.4 do not materially affect results in these cases.

Comparison to Published Point Beach Calculations¹⁰

Again, for stagnant lake cases, the two codes agree exactly; however, for the crosscurrent cases, only the Argonne version ran successfully. The published comparisons¹⁰ were, of course, prepared with the Argonne version of the code. The errors are quite significant in this case because the code will not run without the corrections. Furthermore, we found that one or both versions of the code may fail for some low initial densimetric Froude number cases, for some small values of aspect ratio, and for discharge angles greater than 90°.

These circumstances are quite unfortunate because they hamper the fruitful analysis of the model and becloud the results obtained so far. The potential model user is thus forewarned of the operational difficulties that have been encountered in past applications of the model. A thorough examination of the code seems warranted. The anomalous predictions discussed in the next section are most disturbing to us. It is not clear whether these difficulties are the result of some easily-corrected coding errors such as those discussed above, are the result of the two-step numerical procedure, or are the outward manifestation of errors in the basic modeling concepts used in model development. We suggest that a careful review of the code be carried out to assure that the results are not due to errors in programming or in numerical technique.

5. Criticisms and Limitations

a. Generic Limitations

As in the other integral models, Stolzenbach and Harleman assume that certain physical mechanisms are dominant in plume dilution. Clearly, the model will give incorrect or only fortuitously correct predictions if these phenomena are not actually controlling the dispersion. A strong indictment of the work so far completed in validating integral models is that the models are often applied to situations that deviate significantly from those postulated in the development of the models. These situations are, however, the ones of practical interest and the integral models are often too idealized to be of any value in treating these practical problems. An expanded discussion of these generic limitations is given in the review of the Shirazi-Davis model (Sec. II.E).

b. Specific Limitations

(1) Bottom Effects. In its present form, the Stolzenbach-Harleman model, like the other three-dimensional integral models reviewed, fails to account for the possibility of bottom contact and the corresponding cutoff of vertical entrainment. These models assume that the receiving water body is infinitely deep or that the receiving basin is deep enough that the entrainment demands of the plume are completely satisfied. An early version of the Stolzenbach-Harleman model included a treatment of bottom interference, but this treatment was found inadequate in comparisons with hydraulic tank data.¹

In our judgment, the inclusion of bottom interference in the Stolzenbach-Harleman model was an invalid extension of free-jet similarity theory. Moreover, it represented the beginning of a trend to extend

the integral approach to treat effects alien to the fundamental concepts on which the approach itself is founded. In the original documentation of the Stolzenbach-Harleman model, several methods of schematizing prototype effects such as ambient stratification, nonideal discharge geometry, and obstructions of the free discharge were advanced. Figure II.C.3 shows several of these schematization procedures (middle and bottom left of the figure). We feel these recommendations are an invalid extension of the model and further contributed to the trend toward extrapolation of the capabilities of free-jet models.

Fig. II.C.3. Model Schematization Offered by Stolzenbach and Harleman. (Adapted from Stolzenbach and Harleman.¹)

The two prototype sites where the Stolzenbach-Harleman model has been tested (Pilgrim and Point Beach) both suffer from significant bottom

effects; the Pilgrim site has bottom interference during lowtide and midtide stages (see Ch. IV). Unfortunately, bottom interference occurs, to some extent, for nearly all existing discharges. It is common practice to apply the Stolzenbach-Harleman and other three-dimensional integral models to shallow water discharge sites, although such application is risky due to the assumption of deep water in the model formulation. Application of the model to these two sites represents, in major part, an attempt to determine the severity of the limitations imposed by the restrictive idealization offered by the integral models. The conclusion generally reached is that the Stolzenbach-Harleman model is applicable only to situations that almost identically match the idealization of the model, and this in terms of centerline-decay predictions only. (See further discussions of bottom effects in Ch. III.)

(2) Large Ambient Crossflow. The authors have developed their model to treat heated surface jets in the presence of ambient currents less than about 10% of the outfall velocity. There are three areas of weakness in the formulation of the model that make its use with larger ambient currents unwise. First is the assumption that temperature and velocity profiles are symmetric about the jet centerline. Second, the possibility that different rates of entrainment may occur on the two lateral sides of the jet is omitted. Third, no consideration is made of the dynamic pressure (drag force) that may be expected when a cross current is present.

A further indictment of the model is its erratic behavior for nonstagnant cases. This behavior has been documented by Pagenkopf et al.⁹ (See discussion of Pilgrim model-data comparisons in Ch. IV.) That report well illustrates the inconsistent performance of the model as cross current is varied. Additional difficulties encountered in our use of the model are:

1. The code may not run for certain nonstagnant cases no matter how small a nonzero current is used.
2. The asymptotic behavior of the model is incorrect for some cases, i.e., the nonstagnant predictions do not approach the stagnant predictions as the current goes to zero.

These characteristics may possibly be linked to the coding errors previously discussed, but may also be due to fundamental flaws in the foundation of the model. Those concerns provide additional motivation for restricting the use of the model to stagnant cases.

(3) Low Densimetric Froude Numbers. The Stolzenbach-Harleman model exhibits a number of anomalies that seem to disappear when momentum dominates buoyancy either as a result of a high initial densimetric Froude number (greater than 6) or a large ambient current. Recall, however, that the model shows anomalies of a different kind under this latter circumstance. Specifically, three features are most distressing to us. We will consider these in their approximate order of importance.

In our opinion, the Stolzenbach-Harleman model incorrectly predicts jet-widths. A plot of a jet width as a function of centerline distance reveals that predicted widths are too small near the discharge and much too large at points farther out in the jet. The data we are familiar with have never indicated such a trend but, more typically, show a nearly linear growth with centerline distance. The problem is also clear from a plot of surface isotherms. As shown in Fig. II.C.4, the model isotherms have a mushroomlike shape with the stem of the mushroom located at the discharge. We have never seen any data that suggest such a shape for surface isotherms.

Fig. II.C.4. Isotherms Calculated by Stolzenbach-Harleman Model.
(Adapted from Stolzenbach and Harleman.¹)

Stolzenbach⁵ contends that the discrepancy may be due to the fact that most existing data were taken under conditions where ambient turbulence or boundaries tended to limit lateral spreading. We do not find this argument convincing and feel strongly that the preponderance of data (laboratory and field, buoyant and nonbuoyant jets) contradicts the trend of jet widths predicted by the model.

It is interesting that the excessive lateral spreading does not seem to contaminate the temperature-decay predictions. Except for the other anomalies noted here, the temperature-decay predictions appear to be adequate in comparison to tank studies at moderate to large densimetric Froude numbers where no bottom or ambient current effects exist. An example is given in Fig. II.C.5 where temperature predictions and data are

plotted from those presented by Koester.¹¹ The data show the effects on centerline temperature decay of decreasing the basin depth. Three model predictions are also given. One is the prediction obtained from the Stolzenbach-Harleman code. The other two curves are results of a simple interpretation of two- and three-dimensional nonbuoyant jet theory. It is readily observed that neither the data nor the Stolzenbach-Harleman model deviate significantly from three-dimensional jet theory. This is to be expected because the initial densimetric Froude number is large and the Stolzenbach-Harleman model is built on nonbuoyant jet theory. No width comparisons were given by Koester. The model compared poorly to isotherm area measurements for that data, however. As a contrast, Fig. II.C.6 compares model predictions and laboratory data presented by Stolzenbach and Harleman¹ for an initial densimetric Froude number of 1.8. Again, it seems as if the predicted centerline temperature decay compares favorably with the data, but the predicted widths are excessive over most of the distance over which the calculation was made. Even with this laboratory data where the effects of ambient turbulence are small, the widths appear to be incorrectly predicted. (In fact, if any unnatural effects are to be seen, we feel that the confined laboratory basin should increase widths over those of a limitless ambient.)

Fig. II.C.5. Centerline Temperature Decay Data for Various Receiving-basin Depths, along with Model Predictions of Stolzenbach-Harleman, 2-D Nonbuoyant Jet Theory, and 3-D Nonbuoyant Jet Theory

Fig. II.C.6. Comparison of Stolzenbach-Harleman Prediction with Hydraulic Model Data. (Adapted from Stolzenbach and Harleman.¹)

In essence, the problem with widths is particularly acute at low initial densimetric Froude numbers. Widths are slightly under-predicted near the outfall and largely overpredicted as the local densimetric Froude number approaches unity. Furthermore, the effect on centerline temperature-decay predictions is generally small. Finally, we expect area predictions to be in error since widths are typically too large at lower temperatures but centerline temperature decay is about right.

The second anomaly we find is the existence of a temperature plateau in certain predictions, particularly at low local densimetric Froude numbers. Figure II.C.7 is a plot of several jet characteristics as a function of nondimensional centerline distance for an initial densimetric Froude number of 8. The temperature decay ($\Delta T_c / \Delta T_o$) and the local densimetric Froude number curves are of particular interest. The temperature decay shows a flattening as the local densimetric Froude number approaches unity. Furthermore, the rate of dilution decreases significantly at this point. Although the plateau appears to be small in this log-log plot, a careful examination shows that it occupies approximately half the region for which predictions are made.

u_c = centerline velocity of the jet	D = dilution
u_o = outfall velocity	F_L = local Froude number
ΔT_c = centerline temperature excess	h_o = depth of discharge
ΔT_o = outfall temperature excess	b_o = half-width of discharge
HT = dimensionless heat flux	

Fig. II.C.7. Jet Behavior as Predicted by the Stolzenbach-Harleman Model. (Adapted from Stolzenbach and Harleman,³) ANL Neg. No. 190-1212 Rev. 1.

Based on predictions like those in Fig. II.C.7, Stolzenbach and Harleman suggest that the jet passes through four definable regimes of

flow. The first one encountered as one moves out from the discharge is the core region, also referred to as the region of flow establishment. The second region is characterized by good dilution due to entrainment. The third or stable region shows the plateau and in the fourth region temperature decay resumes due to surface heat transfer.

We do not feel that a stable region, as shown in Fig. II.C.4, actually exists, at least not one of the length often predicted by the Stolzenbach-Harleman model. Especially dramatic examples are afforded in the Pilgrim studies (see Ch. IV) in which a temperature plateau of several hundred feet is predicted. Furthermore, we feel that many of the assumptions made in model development are tenuous at densimetric Froude numbers near or below one and the meaning of predictions in this region is questionable. Stolzenbach⁵ suggests that ambient turbulence may be at work here and that the temperature plateau is destroyed by it. This assertion is certainly plausible but is very difficult, if not impossible, to check.

The final anomaly commonly observed at low initial densimetric Froude numbers is an increase in centerline velocity just after the point of discharge. The model often shows an increase in velocity of 5-10%. Stolzenbach⁵ states that such an increase may well be due to buoyant accelerations but that the amount of increase is insignificant. We agree that the error is quantitatively unimportant, but we are concerned that such behavior may be an outward sign of deeper trouble.

Unfortunately, to trace with certainty the characteristics that we find anomalous to particular assumptions is not possible, for none of the assumptions can be validated separately from the model as a whole; and the only judgments possible are based on the cumulative effect of all assumptions. However, given this limitation, we feel that the lateral spreading assumption is the one most open to question. Stolzenbach and Harleman treat buoyant spreading by superimposing a buoyant spreading velocity on the natural lateral spreading velocity of a classical jet. They write

$$\frac{d\bar{b}}{d\bar{x}} = \left(\frac{d\bar{b}}{d\bar{x}} \right)_{nb} + \left(\frac{d\bar{b}}{d\bar{x}} \right)_b, \quad (15)$$

where the subscripts nb and b indicate the nonbuoyant and buoyant contributions, respectively. The Prych and Shirazi-Davis models make a similar assumption.

In the Stolzenbach-Harleman formulation, the nonbuoyant spreading rate is taken as constant and equal to 0.22. The buoyant spreading rate is assumed to vary inversely with the square of the local densimetric Froude number, a choice not based on experimental data but instead on intuitive reasoning. To see how this assumption manifests itself in predictions of centerline temperature, consider the following simple result, which can be

derived from the Stolzenbach-Harleman model equations for the entrainment and stable regions of the jet (see Fig. II.C.7 and attendant discussion for definition of these regions) assuming only a stagnant ambient free of surface heat loss.

$$\overline{\Delta T} = \overline{\Delta T}_{EOCR} \exp \left\{ -\frac{\epsilon}{2} \int_{\bar{x}_{EOCR}}^{\bar{x}} \left[\frac{\exp(-5F_L^{-2})}{\bar{h}} + \frac{1}{\bar{b}} \right] d\bar{x}' \right\}. \quad (16)$$

Here the subscript "EOCR" is introduced to denote the end of the core (beginning of entrainment) region.

First, we observe that this result reduces to three-dimensional nonbuoyant jet theory $\left[\overline{\Delta T} = \overline{\Delta T}_{EOCR} (\bar{x}/\bar{x}_{EOCR})^{-1} \right]$ if we set $F_L \rightarrow \infty$, $\bar{h} = \bar{b} = \epsilon(\bar{x}/\bar{x}_{EOCR})$ and, further, reduces to two-dimensional nonbuoyant jet theory $\left[\overline{\Delta T} = \overline{\Delta T}_{EOCR} (\bar{x}/\bar{x}_{EOCR})^{-1/2} \right]$ if we set $F_L \rightarrow \infty$, $\bar{h} = \infty$ and $\bar{b} = \epsilon(\bar{x}/\bar{x}_{EOCR})$. Second, we note that the factor multiplying ΔT_{EOCR} is the reciprocal of the effective dilution (local volumetric flow along jet centerline normalized to one at the end of the core region). The result that temperature decays inversely with dilution is, of course, expected, because surface heat loss has been ignored. Thus, Eq. 16 relates dilution or, equivalently, centerline temperature decay to the local characteristics F_L , $\bar{h}$, and $\bar{b}$. An interesting feature of Eq. 16, which requires some reflection to comprehend, is that dilution does not deviate significantly from that predicted by nonbuoyant jet theory until F_L nears one. At this point, the local width, $\bar{b}$, becomes rapidly large due to the lateral spreading assumption and the contribution of vertical entrainment is cut off. As a consequence (again referring to Eq. 16), dilution rate becomes negligible and a temperature plateau is produced. Moreover, had linear growth in width been assumed instead of the choice made by Stolzenbach and Harleman, no temperature plateau would be encountered. (The Shirazi-Davis model, e.g., has nearly linear growth in widths and no temperature plateau. The Prych model also shows no plateau. Although the fundamental difference is not presently clear, it may be that the dilution due to ambient turbulence that Prych includes may be compensating.) The Stolzenbach-Harleman model apparently can calculate approximately correct centerline temperature in the entrainment region, barring such factors as bottom interference, because the volumetric inflow into the jet due to lateral and vertical entrainment is based on classical jet theory. Since dilution calculations are so based, predicted depth must decrease in order to compensate for large model widths. Comparisons with data generally show that the Stolzenbach-Harleman plume is too shallow, although the effect is difficult to observe because only small changes in depth are produced by large discrepancies in width and because the data are often not sufficiently refined to give accurate

depth estimates. The effects of the Stolzenbach-Harleman lateral spreading assumption on predicted depth lead to somewhat of a paradox since the plume is predicted to be entraining vertically yet decreasing in depth.

Returning to the lateral spreading assumption itself, some investigators have attacked the sum decomposition into buoyant and non-buoyant components as inherently incorrect, because it overlooks the nonlinear interaction between buoyancy and entrainment and the possible interaction between buoyancy and ambient turbulence. The problem is aggravated by the fact that in a typical surface discharge the local densimetric Froude number drops from its initial value to a near-unity value at some distance from the outfall. Thus, the modeling of buoyant and nonbuoyant spreading must be valid over a wide range of densimetric Froude numbers.

An additional consideration is the assumption that no turbulent transfer occurs either between regions of the jet or between the jet and the ambient water. In our opinion, the elimination of turbulent transfer through the boundaries is tantamount to elimination of the entire turbulent diffusion mechanism at the outset. However, because the model is built on the "spreading rate" idealization of nonbuoyant theory, Stolzenbach and Harleman assume that the true turbulent diffusion of momentum can be artificially treated through the entrainment coefficient in the context of similarity profiles.

As noted earlier, the Stolzenbach-Harleman model has been applied to the Point Beach and Pilgrim Nuclear Power Stations. Both of these sites have a surface discharge of condenser cooling water but neither one matches the model idealizations well. In fact, we are not aware of any field site that matches the idealizations of these integral models, even offshore-sited plants. However, many existing field data come from those two sites where the Stolzenbach-Harleman model has been extensively applied. The results and conclusions we draw from those results are summarized in Secs. IV.D and IV.F.

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D. Prych Integral Model^{1,2}

1. Synopsis of Approach

The Prych three-dimensional model builds on the conceptual framework established by Fan,³ and Stolzenbach and Harleman.⁴ In addition to the classical type of entrainment that is due to vector velocity differences between jet and ambient fluids, Prych allows "entrainment" due to ambient turbulence in the mass flux equation. The momentum flux equation is formulated to include buoyancy forces, shear forces between jet and ambient fluids, drag force due to crossflow, and entrainment of ambient momentum. The heat flux equation includes surface heat loss.

Following the treatment of Stolzenbach and Harleman, the rate of lateral spreading is written as the sum of a nonbuoyant and a buoyant component. The form of the buoyant component is derived by considering a moving density front such as exists when oil is spreading over water.

With the addition of two equations of geometry, six ordinary differential equations result. Prych provides a well-documented program to integrate these equations numerically by the Runge-Kutta algorithm due to Gill.

2. List of Symbols

A	Cross-sectional area of jet
B	Local characteristic half-width of jet
B ₀	Half-width of outfall
C _D	Empirical drag coefficient
C _F	Coefficient of interfacial shear
C _p	Specific heat of water at constant pressure
c	Celerity of a density front
c ₁	Dimensionless coefficient of order unity
d	Uniform depth of fluid behind density front
(dB/ds) _b	Buoyant spreading rate
(dB/ds) _{nb}	Nonbuoyant spreading rate
E	Two-dimensional, jet-entrainment coefficient
E _h	Multiplier used in computer program to change the assumed value of ϵ_h
E ₀	Entrainment coefficient for a two-dimensional, uniform-density jet flow
E _v	Multiplier used in computer program to change the assumed value of ϵ_v

F	Densimetric Froude number
F_D	Drag force
F_o	Initial densimetric Froude number
f	Function for reducing vertical entrainment
g	Acceleration due to gravity
g'	Reduced acceleration due to gravity; $(\Delta\rho_o/\rho_o)g$
H	Local characteristic depth of jet
H_o	Depth of outfall
J	Local kinematic heat flux in jet
k	Heat-transfer coefficient
M	Local kinematic s-momentum flux
n	Local horizontal coordinate perpendicular to jet centerline
P	Kinematic pressure force on jet cross section
P_o	Kinematic pressure force at outlet
Q	Local volume flux in jet
Q_o	Outfall volume flux
q_s	Surface heat loss
R_i	Richardson number
S_{Fx}, S_{Fy}	Kinematic shear forces in x and y directions, respectively, per unit length
S_h, S_v	Horizontal and vertical model length scales
s	Local coordinate along jet centerline
s_i	Length of zone of flow establishment
s_L	Value of s at which computer code calculation is to stop
s_o	Value to be assigned to s at start of computer code calculation
T	Centerline excess-temperature ratio; T'/T_o
T_o	Excess temperature at outlet above ambient
T'	Local surface centerline excess temperature (above ambient)
t	Local temperature excess (above ambient)
U	Centerline velocity excess
U_o	Velocity at outfall

U_{∞}	Velocity outside the boundary layer (above a flat plate)
u	Local velocity excess in s direction
$\bar{u}$	Average velocity in s direction in jet
V	Ambient current speed
XI	Index used by computer code to designate whether parameters at the end of flow establishment are to be calculated within the code or are to be input explicitly
x	Rectilinear coordinate parallel to ambient current
x_0	Value to be assigned to x at start of computer code calculation
y	Rectilinear coordinate, horizontal and perpendicular to x
y_0	Value to be assigned to y at start of computer code calculation
z	Rectilinear coordinate in vertical direction
ΔP	Increment in pressure
ΔS	Incremental length along jet centerline
Δs	Maximum length of integration step to be used in computer calculation
Δu	Velocity difference
Δu_h	Velocity difference in horizontal direction
Δu_v	Velocity difference in vertical direction
$\Delta \rho$	Average density deficiency in the jet region
$\Delta \rho_0$	Difference between outlet and ambient water densities
δ	Boundary layer thickness
ϵ	Eddy viscosity (turbulent diffusion coefficient for momentum)
ϵ_a	Ambient turbulent diffusion coefficient
ϵ_h, ϵ_v	Ambient turbulent diffusion coefficients in horizontal and vertical directions, respectively
θ	Angle between jet-centerline and ambient-current direction (s and x directions)
θ_0	Value of θ at the outfall (angle between x axis and outlet velocity)
ν	Kinematic viscosity of fluid

ρ	Density of fluid
ρ_0	Ambient fluid density
σ	Standard deviation of lateral distribution of velocity
τ	Shear stress

3. Development of Model Equations

Two coordinate systems are used in the development of the Prych model equations. One is the fixed x, y, z Cartesian system centered at the outfall as shown in Fig. II.D.1. The other is the natural coordinate system of the jet, which consists of s , the centerline distance along the jet axis, n , the horizontal distance perpendicular to the jet axis, and z , the vertical coordinate oriented as shown in Fig. II.D.1. The local angle between the jet or s axis and the x axis is θ . The coordinate systems can be related through two equations of geometry,

$$\frac{dx}{ds} = \cos \theta \quad (1)$$

and

$$\frac{dy}{ds} = \sin \theta. \quad (2)$$

Fig. II.D.1. Definition Sketch of Coordinate Systems and Jet Region for Prych Model. (Adapted from Prych.¹) ANL Neg. No. 190-1045.

As with all integral models, similarity forms must be specified for temperature and velocity. The forms used by Prych are:

$$t(s, n, z) = T'(s) \exp(-n^2/B^2) \exp(-z^2/H^2), \quad (3)$$

$$u(s, n, z) = U(s)\exp(-n^2/B^2)\exp(-z^2/H^2) + V \cos \theta, \quad (4)$$

where

- t = local jet temperature excess (above ambient),
- T' = centerline temperature excess (above ambient),
- u = local velocity in s direction,
- U = local surface centerline velocity excess,
- V = ambient current velocity,
- θ = angle between ambient current and jet centerline,
- B = local characteristic width of jet,

and

H = local characteristic depth of jet.

These forms are used to relate the integral quantities of mass, momentum, and heat flux to the jet parameters $T'(s)$, $U(s)$, $B(s)$, and $H(s)$. $B(s)$ and $H(s)$ are then local $1/e$ widths and depths of the plume for a centerline distance s while $T'(s)$ and $U(s)$ are the excess temperature on the trajectory at the surface and the velocity excess (above ambient) at the same location. (All temperatures and velocities are meant to be time-averaged quantities.) Note the use of exponential functions for lateral and vertical decay instead of polynomials as used by Stolzenbach and Harleman. The advantage of one choice over the other is quite unimportant compared to other assumptions made in the models.

The Prych model is based on a number of physically plausible but untested assumptions. So that the reader may more fully understand the motivation for these assumptions, each equation is discussed individually.

a. Conservation of Mass

Because density variations are assumed negligible in the calculation of fluid masses, the conservation of mass equation reduces to the conservation of fluid volume. Prych equates the rate of increase in the local jet flow, dQ/ds , to the rate of mixing into the jet. Mixing is assumed by Prych to come from the two sources: entrainment due to shear caused by velocity differences between the plume and ambient stream, and diffusion resulting from ambient turbulence. Mass flow into the jet is written as the sum of the individual contributions from jet entrainment and ambient diffusion for both the horizontal and vertical directions as follows:

$$\frac{dQ}{ds} = \left(\frac{dQ}{ds} \right)_v + \left(\frac{dQ}{ds} \right)_h, \quad (5)$$

where

$$\left(\frac{dQ}{ds}\right)_v = \left(\frac{dQ}{ds}\right)_{j,v} + \left(\frac{dQ}{ds}\right)_{a,v} \quad (6)$$

and

$$\left(\frac{dQ}{ds}\right)_h = \left(\frac{dQ}{ds}\right)_{j,h} + \left(\frac{dQ}{ds}\right)_{a,h} \quad (7)$$

here the subscripts v, h, j, and a refer to vertical, horizontal, jet, and ambient, respectively.

The volume flux along the jet axis, $Q(s)$, can be written:

$$Q(s) = \iint_A u \, dn \, dz \quad (8)$$

or

$$Q = \iint_A U \exp(-n^2/B^2) \cdot \exp(-z^2/H^2) \, dn \, dz + \iint_A V \cos \theta \, dn \, dz, \quad (9)$$

where the elliptical jet cross section, A , is defined in Fig. II.D.1. The first term is actually integrated over the entire half-plane $z \leq 0$. This is done for convenience in evaluating the numerical value of the similarity integral. The precise area A for the integration of the $V \cos \theta$ term is arbitrary to some extent, because twice that area could have easily been chosen. If an alternate area were chosen, the numerical value of the coefficients in the model (such as entrainment and drag) would have to be altered to retain the same flow and temperature predictions.

Integrating Eq. 9 using the postulated similarity forms gives

$$Q = \pi B H \left(\frac{U}{2} + V \cos \theta \right). \quad (10)$$

Next an evaluation of the terms in Eqs. 5, 6, and 7 is undertaken. The top diagram of Fig. II.D.2 shows the jet region, A , divided into vertical slices for the calculation of vertical entrainment. Using the concept of entrainment due to Morton, Taylor, and Turner,⁵ the rate at which ambient fluid is entrained into the jet is assumed proportional to some characteristic local velocity difference between the jet and ambient fluids. This is the classical concept of entrainment. Prych then simulates vertical entrainment over the control volume slice in Fig. II.D.2 as

$$\left(\frac{dQ}{ds}\right)_{j,v} = \int_{-\sqrt{2}B}^{\sqrt{2}B} E(s, n) \Delta u_v(s, n) \, dn. \quad (11)$$

Here the entrainment coefficient $E(s,n)$ is assumed to have the form

$$E = E_0 f(R_i), \quad (12)$$

where

$$f = \begin{cases} \frac{\exp(-5R_i) - \exp(-4)}{1 - \exp(-4)} & R_i \leq 0.8, \\ 0 & R_i > 0.8 \end{cases} \quad (13)$$

following Koh and Fan⁶ and Stolzenbach and Harleman⁴ who used this form as an empirical fit to the data of Ellison and Turner.⁷ This form is meant to account in some way for the expected vertical stratification and fluid stability in the jet that results in decreased vertical entrainment. The quantity R_i is the local jet Richardson number, defined by

$$R_i(s,n) = g' \frac{t(s,n,0)}{T_0} \sqrt{2H/\Delta u_v^2}. \quad (14)$$

The value of E_0 is taken to be 0.1 in Eq. 12 above, a compromise between Fan's values of entrainment coefficient (0.1 in stagnant situations and 0.5 in large ambient current situations) for submerged round jets and

CONTROL VOLUME FOR COMPUTING ENTRAINMENT

Fig. II.D.2

Control Volume for Computing Entrainment for Mass Conservation Equation (top); Forces on Control Volume for Computing Momentum Conservation (bottom). (Adapted from Prych.¹) ANL Neg. No. 190-1174 Rev. 1.

FORCES ON CONTROL VOLUME

the values of Hoopes, Zeller, and Rohlich for surface jets of 0.07-0.78. The value Prych uses is somewhat suspect because the Fan³ and Hoopes-Zeller-Rohlich⁸ values for the entrainment coefficient were based on a fit of their models to data. More specifically, the values from Fan are based upon data for submerged jets and the Hoopes-Zeller-Rohlich values are based upon model-data fitting to site-dependent Lake Monona plume data. Thus, wide variation in the value of E_0 is possible.

The velocity difference in Eq. 11 is assumed to be the magnitude of the vector difference between the surface velocity and the velocity of the ambient fluid. Thus,

$$\Delta u_v = |\vec{u}(s, n, 0) - \vec{V}| = [U^2 \cdot \exp(-2n^2/B^2) + V^2 \sin^2 \theta]^{1/2}. \quad (15)$$

The magnitude of the vector-velocity difference is used in writing the formula for entrainment following the analysis of Fan. Fan argues that the vector difference is the controlling factor on entrainment and that such a formulation allows a consistent treatment of entrainment when a crossflow is present. The assumptions on entrainment such as those implicit in Eqs. 11 and 15 can only be verified experimentally.

Substituting Eq. 12 and Eq. 15 into Eq. 11 yields

$$\left(\frac{dQ}{ds}\right)_{j,v} = 2E_0 \int_0^{\sqrt{2}B} f(R_i) [U^2 \exp(-2n^2/B^2) + V^2 \sin^2 \theta]^{1/2} dn. \quad (16)$$

By analogy with Eq. 11, Prych writes for horizontal entrainment

$$\left(\frac{dQ}{ds}\right)_{j,h} = 2E_0 \int_{-\sqrt{2}H}^0 \Delta u_h(s, z) dz. \quad (17)$$

A constant entrainment coefficient equal to the value obtained from nonbuoyant jet theory is assumed for horizontal mixing. Horizontal stratification between plume and ambient is assumed not to affect horizontal mixing. Since the precise interaction between buoyancy, stratification, and entrainment are not well understood, the validity of a constant E_0 cannot be judged at this time.

The velocity difference used in Eq. 17 is given by

$$\begin{aligned} \Delta u_h(s, z) &= |\vec{u}(s, 0, 0) - \vec{V}| \exp(-z^2/H^2) \\ &= (U^2 + V^2 \sin^2 \theta)^{1/2} \cdot \exp(-z^2/H^2). \end{aligned} \quad (18)$$

At the surface Δu_h is equal to the vector difference between jet centerline velocity (the maximum jet velocity) and velocity of the ambient fluid.

Comparing this form of the velocity difference with that used in Eq. 15, we find that the two have an important difference. In this latter case, exponential attenuation is applied to the ambient velocity as well as to the vertical jet profile. This representation should be considered arbitrary and its validity can be tested only by comparison with experiment.

Substituting Eq. 18 into Eq. 17 yields

$$\left(\frac{dQ}{ds}\right)_{j,h} = \sqrt{\pi} E_0 (U^2 + V^2 \sin^2 \theta)^{1/2} H. \quad (19)$$

In addition to the classical type of entrainment described above, Prych considers plume dilution due to ambient turbulence. It is unclear how ambient turbulence should enter the integral formulation. The considerations here are rather subtle and need expanded explanation.

First, we feel that the equation of mass flux conservation is actually an equation of dilution and represents an attempt to account for shear- and turbulence-induced spreading within the framework of the integral formulation. In the Navier-Stokes equations (differential form of the equations of motion) entrainment is modeled both in the advection terms (shear part) and in the spatial distribution of Reynold's stresses (turbulent part). Thus, the gradients of both the mean and fluctuating velocity components can participate. In the integral formulation, the two spreading mechanisms are lumped together into a single empirically determined equation of dilution. In classical jet theory, the form depends on the mean velocity difference only with the implicit assumption that the difference in turbulence levels between the jet and ambient fluids is related to the vector difference in mean velocities. Thus, in this formulation, the effects of ambient turbulence are manifested in the value of the entrainment coefficient. Prych, on the other hand, considers each mechanism separately. We feel that the Prych formulation is untested, but we do not see that it conflicts with the integral formulation.

The point of whether and how ambient turbulence fits into an integral model is muted by the fact that many assumptions in integral models are arbitrary and the only test is with experimental data. The major justification for Prych's treatment of ambient turbulence is its effect on model predictions at lower isotherms. It seems that the greater the coefficients of ambient turbulence used in model calculations, the greater the dilution at lower isotherms, a result which agrees with our physical intuition. Moreover, the inclusion of ambient turbulence allows the model to be better fit to data.

Prych presented the reasoning behind his treatment of ambient turbulence in an earlier paper.² There, three separate analyses were compared: one considered the ambient mixing of a passive tracer in a uniform current, a second considered jet mixing due to entrainment, and the third considered jet mixing due to turbulent diffusion. The rates of dilution found from these analyses are summarized below:

For jet mixing due to entrainment,

$$\frac{dQ}{ds} = 2E\Delta u(s); \quad (20)$$

for jet mixing due to turbulent diffusion,

$$\frac{dQ}{ds} \approx \frac{3.6\epsilon(s)}{\sigma(s)}; \quad (21)$$

and for diffusion of a passive tracer due to ambient turbulence,

$$\frac{dQ}{ds} \approx \frac{3.5\epsilon_a(s)}{\sigma(s)}. \quad (22)$$

The form of Eq. 20 is the same one used in the treatment of classical entrainment discussed earlier, whereas the result of Eq. 22 is used in the treatment of dilution due to ambient turbulence. In the development of the thermal plume model, Prych¹ attempts to extend these ideas to buoyant jets. In the vertical direction, Prych approximates σ for each vertical slice by half the height of the slice from which

$$\left(\frac{dQ}{ds}\right)_{a,v} = \frac{3.5\epsilon_v}{2} \int_{-\sqrt{2}B}^{\sqrt{2}B} \frac{2f(s,n)dn}{\sqrt{2H(1 - n^2/2B^2)^{1/2}}}. \quad (23)$$

Prych uses the function f once again to reduce the ambient diffusion coefficient because of the stabilizing effect of the vertical density gradient in the jet. The precise form for f is seldom known with much accuracy so Prych uses the same f as for jet entrainment. In conflict with the previous usage of f , the function is now taken to be independent of the lateral distance, n . To do this, the Richardson number is evaluated at the jet centerline. This allows the integration to be evaluated explicitly, yielding

$$\left(\frac{dQ}{ds}\right)_{a,v} = \frac{3.5\pi B\epsilon_v}{H} f\left(\frac{g'T\sqrt{2}H}{(U + V \cos \theta)^2}\right). \quad (24)$$

The expression for horizontal mixing due to ambient turbulence is derived in a similar manner to be

$$\begin{aligned} \left(\frac{dQ}{ds}\right)_{a,h} &= 3.5\epsilon_h \int_{-\sqrt{2}H}^0 \frac{2}{\sqrt{2B(1 - z^2/2H^2)^{1/2}}} dz \\ &= \frac{3.5\pi H\epsilon_h}{B}. \end{aligned} \quad (25)$$

Here ϵ_h is the ambient horizontal turbulent diffusion coefficient. Prych does not reduce ϵ_h due to the presence of a vertical (or horizontal) density gradient.

In the computer code, E_o , E_h , and E_v are input quantities with the ambient diffusion coefficients given by

$$\epsilon_h = 1.0E_h \text{ m}^2/\text{sec}$$

and

$$\epsilon_v = 0.001E_v \text{ m}^2/\text{sec}.$$

Prych recommends that E_o be set to 0.1, E_h to 1.0, and E_v to 1.0. The recommended value for E_h comes from Okubo⁹ (p. 798 for a plume of 1 km) and the value of E_v is taken from Csanady's¹⁰ measurements in the upper few meters of large lakes. In reality, ϵ_h and ϵ_v can vary over orders of magnitude, depending upon environmental conditions.

b. Conservation of Heat Energy

The kinematic heat flux $J(s)$ through the jet cross section at centerline distance s is defined by

$$J(s) = \frac{1}{T_o \rho_o C_p} \iint_A \rho C_p u t \, dn \, dz, \quad (26)$$

where

T_o = temperature excess at outfall (above ambient),

ρ_o = density of ambient water,

and

ρ = local density.

Assuming that $\rho = \rho_o$ and that C_p is a constant, and introducing the functional forms for u and t into Eq. 26 yields (integrating over all n , and all $z \leq 0$)

$$J = (\pi/2)BH \left(\frac{U}{2} + V \cos \theta \right) \frac{T'}{T_o}. \quad (27)$$

The equation of heat conservation is derived by equating the rate of change of heat flux along the jet centerline to the surface heat loss. Thus,

$$\rho_o C_p T_o \frac{dJ}{ds} = - \int_{-\sqrt{2}B}^{\sqrt{2}B} k \rho C_p t(s, n, 0) dn, \quad (28)$$

where k is a constant heat-exchange coefficient with units of velocity.

For ρ , C_p , k constant we have,

$$\frac{dJ}{ds} = \sqrt{\pi k T B}, \quad (29)$$

where, again, for the sake of convenience the integration of Eq. 28 was carried out from $n = -\infty$ to $n = \infty$.

c. Conservation of Momentum

Figure II.D.2 illustrates the forces on a control volume slice of the jet included in the longitudinal and lateral momentum equations. The forces simulated are:

(1) Pressure due to density differences between jet and ambient fluids. Based upon a hydrostatic pressure assumption, the kinematic pressure at each longitudinal position along the jet axis is given by

$$P(s) = \frac{1}{\rho_0} \int_{-\infty}^0 \int_{-\infty}^{\infty} \left[\int_{-\infty}^z g \frac{\Delta \rho_0}{T} t(s, n, z') dz' \right] dn dz. \quad (30)$$

Substituting the similarity form for local temperature yields

$$P(s) = \frac{\sqrt{\pi}}{2} g' T H^2 B. \quad (31)$$

This longitudinal force is important whenever the densimetric Froude number approaches unity. Lateral pressure forces are simulated through the specification of the lateral spreading velocity as discussed in the next subsection. Buoyant vertical forces are simulated in the hydrostatic-pressure form of the vertical-momentum equation.

(2) Pressure drag on the jet due to the ambient current. This force is caused by the fluid as it moves under and around the jet. The fluid decelerates as it reaches the jet from the upcurrent side and builds up a slightly higher pressure there. A slightly lower pressure region exists on the lee side of the jet. These nonsymmetric pressure forces deform and bend the jet. This bending is not accounted for in the choice of similarity functions but is treated by the integration of those similarity functions and in the drag force appearing in the momentum equations:

$$F_D = \frac{1}{2} C_D (\sqrt{2} H) V |V| \sin^2 \theta. \quad (32)$$

This is the formula for form drag on a solid body. The adequacy of this commonly used simulation for fluids is unknown.

(3) Interfacial shear stress between jet and underlying ambient fluids. Defining the boundary of the jet as shown in Fig. II.D.2, Prych postulates an interfacial shear stress to account for the force on the jet from fluid below. Prych approximates the shear stress by modifying the classical expression for a turbulent boundary layer above a flat plate. The expression for this stress, taken from Schlichting,¹¹ is

$$\tau = 0.0225\rho_o\left(\frac{\nu}{U_\infty\delta}\right)^{1/4}U_\infty^2, \quad (33)$$

where

τ = shear stress,

ν = kinematic viscosity of the fluid,

U_∞ = velocity outside the boundary layer,

and

δ = boundary-layer thickness.

Modifying this expression to apply to the jet control volume, Prych assumes

$$\delta \sim \sqrt{2}H$$

and

$$U_\infty \sim \Delta u_V(s, n) = |\vec{u}(s, n, 0) - \vec{V}| = [V^2 \cdot \exp(-2n^2/B^2) + V^2 \sin^2 \theta]^{1/2}.$$

A coefficient, C_F , is assumed to multiply the right-hand side of Eq. 33 to account for the difference between solid-fluid and fluid-fluid shear. The direction of the shear force is assumed to be parallel to the vector $\Delta \vec{u}_V(s, n)$ and its magnitude per unit length of control volume becomes

$$\begin{aligned} S_{Fx} &= \int_{-\sqrt{2}B}^{\sqrt{2}B} (\tau/\rho_o)_x dn \\ &= 0.02C_F\left(\frac{\nu}{H}\right)^{1/4} \int_{-\sqrt{2}B}^{\sqrt{2}B} \Delta u_V^{3/4} [V \sin^2 \theta - U \cos \theta \exp(-n^2/B^2)] dn, \end{aligned} \quad (34)$$

$$\begin{aligned}
S_{Fy} &= \int_{-\sqrt{2}B}^{\sqrt{2}B} (\tau/\rho_o)_y dn \\
&= -0.02C_F \left(\frac{v}{H}\right)^{1/4} \sin \theta \int_{-\sqrt{2}B}^{\sqrt{2}B} \Delta u^{3/4} [V \cos \theta + U \exp(-n^2/B^2)] dn. \quad (35)
\end{aligned}$$

Inclusion of this force in the momentum equation has the expected effects on the plume. The trajectory bends more, centerline temperatures increase, the plume thickness decreases, and in general the force reduces dilution. Unfortunately, the model is not very sensitive to the value of C_F (tested from $C_F = 0.0$ to $C_F = 0.5$). In spite of the proper performance of this interfacial friction factor, our doubts about it include:

(a) The precise location where the interfacial shear force defined above should act. Based upon the flat plate analogy, the interfacial friction should act at the vertical location of maximum shear; i.e., the location of the maximum velocity gradient vertically, not at the boundary as arbitrarily defined in Fig. II.D.2. If the force acts at the position supposed, then all definitions of integral quantities should cease there, boundary conditions should be given at that boundary, and integrations should not be extended to infinity. This is also true if the force is assumed to act at the point of maximum velocity gradient in the vertical direction. The interfacial shear force will be, in fact, zero (or negligibly small) if the bottom of the jet is taken at (or near) the point where the plume velocities are at (or near) ambient values. Prych is not consistent with his definition of plume boundaries. The plume is assumed to extend to infinity when this avoids the definition of boundary velocities but is defined as finite when considering entrainment and shear forces.

(b) The use of flat plate skin friction formulae. The accuracy of the form for the shear stress with coefficient C_F is unverified for thermal plume applications. Data from Sjöberg¹² on the diffusive properties of an interfacial layer indicated a C_F value on the order of unity might be correct. Prych expects $C_F < 1$ since the shear stress for a fluid-fluid interface should be smaller than for a fluid-solid interface; on this basis, Prych recommends 0.5. The validity of this value for interfacial friction is not really known.

Finally, to derive the momentum equations, the kinematic momentum flux of the jet in the s direction into the control volume is defined by

$$M(s) = \frac{1}{\rho_o} \iint_A \rho u^2(s, n, z) dn dz. \quad (36)$$

Using the similarity form for local velocity u , Eq. 36 becomes

$$M = \pi BH \left(\frac{U}{2} + V \cos \theta \right)^2,$$

and the momentum equations for the x and y directions become

$$\frac{d}{ds}(M \cos \theta) = -\frac{d}{ds}(P \cos \theta) + S_{Fx} + F_D \sin \theta + V \frac{dQ}{ds}, \quad (37)$$

and

$$\frac{d}{ds}(M \sin \theta) = -\frac{d}{ds}(P \sin \theta) + S_{Fy} - F_D \cos \theta. \quad (38)$$

The term $V(dQ/ds)$ in Eq. 37 refers to the momentum of the ambient fluid entrained into the jet per unit length of the control volume. Note that this term acts solely in the x direction since this is the direction assumed for the ambient current.

An equation for $d\theta/ds$ is obtained by multiplying Eq. 37 by $-\sin \theta$, Eq. 38 by $\cos \theta$, then adding, and using equations for P and M as follows:

$$P(s) = (\sqrt{\pi}/2)g'TH^2B \quad (39)$$

and

$$M = \frac{Q^2}{\pi BH}, \quad (40)$$

to yield

$$\frac{d\theta}{ds} = \frac{-\pi \left(F_D - S_{Fy} \cos \theta + S_{Fx} \sin \theta + V \sin \theta \frac{dQ}{ds} \right)}{\left(\frac{Q^2}{BH} + \frac{\pi^{3/2}}{2} g' TH^2 B \right)}. \quad (41)$$

Also utilized is an equation for dH/ds obtained by multiplying Eq. 37 by $\cos \theta$ and Eq. 38 by $\sin \theta$ and adding to yield

$$\begin{aligned} \frac{dH}{ds} = & \left[\pi(S_{Fy} \sin \theta + S_{Fx} \cos \theta) + \left(\pi V \cos \theta - \frac{2Q}{BH} \right) \frac{dQ}{ds} - BH^2 \frac{\pi^{3/2}}{2} g' \frac{dT}{ds} \right. \\ & \left. + \left(\frac{Q^2}{B^2 H} - \frac{\pi^{3/2}}{2} g' H^2 T \right) \frac{dB}{ds} \right] \cdot \left(\pi^{3/2} g' THB - \frac{Q^2}{BH^2} \right)^{-1}. \end{aligned} \quad (42)$$

An important parameter in the model is the gross densimetric Froude number that Prych defines as

$$F^2 = \frac{Q^2}{\pi^{3/2} g' T B^2 H^3} \quad (43)$$

or

$$F = \frac{\pi^{1/4} \left(\frac{U}{2} + V \cos \theta \right)}{\sqrt{g' T H}}$$

by taking the ratio of the two terms in second bracketed expression of Eq. 42. In the Prych model calculation, when F drops to 1, dH/ds (Eq. 42) becomes infinite, halting further calculations with the model.

d. Lateral Spreading Assumption

At this point there are five variables, U , T' , θ , B , and H ; but only four equations, mass, x-momentum, y-momentum, and heat energy. The model development to this point was a straightforward modification of non-buoyant jet theory. In this modification a piece of information was lost and to patch up that error, knowledge of the variation of H with s or B with s is needed. Prych assumes a form for dB/ds as described below.

It is our opinion that the lateral spreading assumption is the main cause of difficulty with the Stolzenbach-Harleman, Prych, and Shirazi-Davis models. The assumption made is that

$$\frac{dB}{ds} = \left(\frac{dB}{ds} \right)_{nb} + \left(\frac{dB}{ds} \right)_b \quad (44)$$

Here the rate of increase in the width B is written as the simple sum of the contribution due to horizontal mixing and entrainment denoted as nonbuoyant with the subscript nb and the contribution due to gravitational advection, denoted as the buoyant term by the subscript b . A linear interaction between the two contributions is assumed, which is probably inadequate when the terms are of nearly equal magnitude. The approximation of Eq. 44 is probably good only when one term is much larger than the other.

To derive $(dB/ds)_{nb}$, Prych assumes

$$\frac{dB/ds}{dH/ds} = \frac{(dQ/ds)_h B}{(dQ/ds)_v H} \quad (45)$$

implying that the ratio of the nonbuoyant horizontal to vertical growth rates is directly proportional to the ratio of the horizontal to vertical entrainment rates and inversely proportional to the vertical to horizontal dimensions. Eliminating dH/ds from Eqs. 45 and 43 and replacing dB/ds by $(dB/ds)_{nb}$ yields,

$$\left(\frac{dB}{ds}\right)_{nb} = \frac{\pi(S_{Fy} \sin \theta + S_{Fx} \cos \theta) + \left(\pi V \cos \theta - \frac{2Q}{BH}\right) \frac{dQ}{ds}}{\frac{1}{2} - \frac{Q^2}{B^2 H} \left(\frac{(dQ/ds)_v}{(dQ/ds)_h} + 1 \right)} \quad (46)$$

The form of $(dB/ds)_{nb}$ seems reasonable physically, but no data exist by which to justify it. The fact that no constant of proportionality exists in Eq. 45 and that no power of B/H or $[(dQ/ds)_h]/[(dQ/ds)_v]$ is used are simply assumptions as is the form itself.

A more significant assumption lies in the derivation of $(dB/ds)_b$. Figure II.D.3 illustrates the derivation of $(dB/ds)_b$ from a vector diagram showing an outer lateral boundary of the jet, $n = \pm\sqrt{2}B$, expanding outward normal to itself at velocity $c(s)$ and at the same time moving forward

PRYCH SIMULATION FOR $\left(\frac{dB}{ds}\right)_b$

Fig. II.D.3. Method of Calculation of Buoyant Lateral Spreading

along the centerline with velocity $\bar{u}(s)$. The velocity $c(s)$ is then derived by simulating the buoyant spreading as if it were due solely to buoyancy, i.e., a layer of less dense fluid (say, oil) over a heavier fluid (water). So the velocity of the density front of depth d and uniform density difference $\Delta\rho$ is then given by

$$c = c_1 \sqrt{\frac{g\Delta\rho d}{\rho_0}}, \quad (47)$$

where

d = average thickness of jet region $(\pi H/2\sqrt{2})$,

$\Delta\rho$ = average density deficiency in the jet region $(\Delta\rho_0 T)/2$,

$\bar{u}$ = average velocity in the jet mass section $(U/2) + V \cos \theta$,

and

c_1 = dimensionless coefficient of order unity (taken as $4\sqrt{2}/\pi\sqrt{\pi} \approx 1.01$).

The values in parentheses represent the parameters as they are simulated by Prych in his jet model. When the forward and outer velocities are added vectorially, as in Fig. II.D.3, the result can be written

$$\left(\frac{dB}{ds}\right)_b = \frac{1}{\sqrt{2}\sqrt{F^2 - 1}}. \quad (48)$$

Considering the buoyant spreading as simple gravitational advection is a great oversimplification because the plume is not a moving density front at all but has a lateral and vertical distribution that is sheared due to horizontal and vertical mixing. Note in Eq. 48 that as F approaches unity, the growth in plume width (including total plume width, see Eq. 44) will approach infinity. The model breaks down somewhere before this point is reached.

4. Numerical Solution of the Governing Equations

Table II.D.1 lists the conservation equations of the model described above. These equations express the derivatives of Q , T , θ , B , H , x , and y with respect to s as functions of these variables and the constant parameters. Table II.D.2 lists the equations to be solved in the computer code. Taking these equations in order and using

$$U = 2\left(\frac{Q}{\pi BH} - V \cos \theta\right),$$

each derivative in the latter table can be expressed as an explicit function of Q , T , θ , B , and H . This procedure is not shown here. With an initial condition,

TABLE II.D.1. Equations of the Prych Model

Conservation of Mass:

$$\frac{dQ}{ds} = \left(\frac{dQ}{ds}\right)_{j,h} + \left(\frac{dQ}{ds}\right)_{a,h} + \left(\frac{dQ}{ds}\right)_{j,v} + \left(\frac{dQ}{ds}\right)_{a,v}$$

$$\left(\frac{dQ}{ds}\right)_{j,h} = \sqrt{\pi} E_0 (U^2 + V^2 \sin^2 \theta)^{1/2} H$$

$$\left(\frac{dQ}{ds}\right)_{a,h} = \frac{3.5\pi H \epsilon_h}{B}$$

$$\left(\frac{dQ}{ds}\right)_{j,v} = 2E_0 \int_0^{\sqrt{2}B} f(Ri) \cdot [U^2 \exp(-n^2/B^2) + V^2 \sin^2 \theta]^{1/2} dn$$

$$\left(\frac{dQ}{ds}\right)_{a,v} = \frac{3.5\pi B \epsilon_v}{H} \cdot f \frac{g' T \sqrt{2} H}{(U + V \cos \theta)^2}.$$

Conservation of Momentum:

x direction

$$\frac{d}{ds}(M \cos \theta) = - \frac{d}{ds}(P \cos \theta) + S_{Fx} + F_D \sin \theta + V \frac{dQ}{ds}$$

y direction

$$\frac{d}{ds}(M \sin \theta) = - \frac{d}{ds}(P \sin \theta) + S_{Fy} - F_D \cos \theta$$

$$P(s) = \frac{\sqrt{\pi}}{2} g' T H^2 B$$

$$S_{Fx} = 0.02 C_F \left(\frac{v}{H}\right)^{1/4} \int_{-\sqrt{2}B}^{\sqrt{2}B} \Delta u_V^{3/4} [V \sin^2 \theta - U \cos \theta \exp(-n^2/B^2)] dn$$

$$S_{Fy} = - 0.02 C_F \left(\frac{v}{H}\right)^{1/4} \sin \theta \int_{-\sqrt{2}B}^{\sqrt{2}B} \Delta u_V^{3/4} [V \cos \theta + U \exp(-n^2/B^2)] dn$$

$$F_D = \frac{1}{2} C_D \sqrt{2} H V |V| \sin^2 \theta.$$

TABLE II.D.1. (Contd.)

Conservation of Heat Energy:

$$\frac{dT}{ds} = -\frac{T}{Q} \left(2 \sqrt{\pi k_B} + \frac{dQ}{ds} \right).$$

Lateral Spreading Equation:

$$\frac{dB}{ds} = \left(\frac{dB}{ds} \right)_{nb} + \left(\frac{dB}{ds} \right)_b$$

$$\left(\frac{dB}{ds} \right)_{nb} = \frac{\pi \left(S_{Fy} \sin \theta + S_{Fx} \cos \theta \right) + \left(\pi V \cos \theta - \frac{2Q}{BH} \right) \frac{dQ}{ds}}{\frac{1}{2} - \frac{Q^2}{B^2 H} \left[\frac{(dQ/ds)_v}{(dQ/ds)_h} + 1 \right]}$$

$$\left(\frac{dB}{ds} \right)_b = \frac{1}{\sqrt{2} \sqrt{F^2 - 1}}; \quad F = \frac{\pi^{1/4} \left(\frac{U}{2} + V \cos \theta \right)}{\sqrt{g' TH}}.$$

Equations of Jet Geometry:

$$\frac{dx}{ds} = \cos \theta$$

$$\frac{dy}{ds} = \sin \theta.$$

Note:

$$Q = \pi BH \left(\frac{U}{2} + V \cos \theta \right)$$

$$J = \frac{QT}{2}$$

$$M = \frac{Q^2}{\pi BH}.$$

TABLE II.D.2. Equations for Computer Solution of Prych Model

Conservation of Mass:

$$\frac{dQ}{ds} = \left(\frac{dQ}{ds}\right)_{j,h} + \left(\frac{dQ}{ds}\right)_{a,h} + \left(\frac{dQ}{ds}\right)_{j,v} + \left(\frac{dQ}{ds}\right)_{a,v}$$

$$\left(\frac{dQ}{ds}\right)_{j,h} = \sqrt{\pi} E_0 (U^2 + V^2 \sin^2 \theta)^{1/2} H$$

$$\left(\frac{dQ}{ds}\right)_{a,h} = \frac{3.5 \pi H \epsilon_h}{B}$$

$$\left(\frac{dQ}{ds}\right)_{j,v} = 2 E_0 \int_0^{\sqrt{2} B} f(Ri) \cdot [U^2 \exp(-2\pi^2/B^2) + V^2 \sin^2 \theta]^{1/2} dn$$

$$\left(\frac{dQ}{ds}\right)_{a,v} = \frac{3.5 \pi B \epsilon_v}{H} f \left[\frac{g' T \sqrt{2} H}{(U + V \cos \theta)^2} \right]$$

Conservation of Heat Energy:

$$\frac{dT}{ds} = -\frac{T}{Q} (2\sqrt{\pi} k_B + \frac{dQ}{ds})$$

Jet Bending Equation:

$$\frac{d\theta}{ds} = \frac{-\pi (F_D - S_{Fy} \cos \theta + S_{Fx} \sin \theta + V \sin \theta \frac{dQ}{ds})}{\frac{Q^2}{BH} + \frac{\pi^{3/2} g'}{2} TH^2 B}$$

Lateral Spreading Equation:

$$\frac{dB}{ds} = \left(\frac{dB}{ds}\right)_{nb} + \left(\frac{dB}{ds}\right)_b$$

$$\left(\frac{dB}{ds}\right)_{nb} = \frac{\pi (S_{Fy} \sin \theta + S_{Fx} \cos \theta) + (\pi V \cos \theta - \frac{2Q}{BH} \frac{dQ}{ds})}{\frac{1}{2} - \frac{Q^2}{B^2 H} \left[\frac{(dQ/ds)_v}{(dQ/ds)_h} + 1 \right]}$$

$$\left(\frac{dB}{ds}\right)_b = \frac{1}{\sqrt{2} \sqrt{F^2 - 1}}; \quad F = \frac{\pi^{1/4} \left(\frac{U}{2} + V \cos \theta \right)}{\sqrt{g' TH}}$$

Jet-depth Equation:

$$\frac{dH}{ds} = \left[\pi (S_{Fy} \sin \theta + S_{Fx} \cos \theta) + \left(\pi V \cos \theta - \frac{2Q}{BH} \frac{dQ}{ds} \right) \right. \\ \left. - \frac{\pi^{3/2} g'}{2} \frac{dT}{ds} + \left(\frac{Q^2}{B^2 H} - \frac{\pi^{3/2} g'}{2} H^2 T \right) \frac{dB}{ds} \right] \left(\pi^{3/2} g' THB - \frac{Q^2}{BH^2} \right)^{-1}$$

Equations of Jet Geometry:

$$\frac{dx}{ds} = \cos \theta$$

$$\frac{dy}{ds} = \sin \theta$$

the seven equations are integrated numerically to give Q , T , θ , B , H , x , and y as functions of s . The model uses the Runge-Kutta scheme due to Gill. The solution, in conjunction with the similarity forms for t and u , provides the temperature and velocity distribution of the jet.

The equations of the Prych model yield explicit expressions for the derivatives of the dependent variables. The Stolzenbach-Harleman model, in contrast, has such a complicated set of equations that the derivatives of the dependent variables must be obtained numerically by solving a set of simultaneous equations. Besides the easier set of equations, which provide a better understanding of relationships, the Prych model has fewer computational difficulties. Moreover, the computer code given by Prych¹ is well documented.

In addition to the basic outfall and ambient parameters, the model requires the value of E_O , E_h , E_v , k , CD , and CF . If the model is to be compared to hydraulic model data, Prych recommends zero values for E_h and E_v due to the negligible ambient turbulence in tank environments for near-field plume predictions. The Prych code has the option of using prototype dimensions as input and output for calculations performed for a hydraulic scale model. If this option is used, calculations are done after all dimensions are scaled down by the horizontal and vertical model scales. When the calculations are completed, the results are scaled up to prototype dimensions.

Code calculations will terminate when either

- (1) the surface centerline temperature excess becomes less than 1% of the outlet temperature excess, or
- (2) the upper limit in centerline distance, specified by the user, is reached, or
- (3) five pages of output information are generated, or
- (4) the maximum integration step is halved 10 times in order to obtain the desired accuracy in integration.

It is our opinion that the first cutoff is quite liberal and that many of the assumptions of the model break down before 1% of the initial temperature excess is reached.

5. Treatment of Flow-establishment Region

This small zone near the outfall provides the initial conditions for the established flow calculations described in the previous section. A separate analysis is needed here to allow the somewhat uniform distribution of temperature and velocity at the outfall to warp into the bell-shaped distribution assumed in the previous analysis.

The approach used by Prych is to consider the zone of flow establishment (ZFE) as a control volume itself for which a mass, momentum, and energy balance is written. For example, the momentum equations balance the difference in the momentum flux between the outlet and the cross section at the end of the ZFE with the average values of the forces assumed to act upon the initial zone control volume from drag, buoyancy, interfacial shear, and the entrainment of momentum from the ambient current. Only values of the variables at the end of the flow-establishment zone are derived with no distribution within the ZFE considered. We do not discuss this formulation in any greater detail because we feel it is inadequate for the following reasons.

(1) Numerous assumptions in Prych's development are questionable and based on little or no data. For example:

(a) A dilution of 2 is assumed to occur in the initial zone independent of outfall or environmental conditions.

(b) The ratio of B to H at the end of the ZFE is derived by assuming no dilution in the initial region (inconsistent with (a)) along with lateral spreading defined by a celerity velocity similar to that defined in the established flow regime and due strictly to buoyancy alone. In most cases, jet entrainment causes the lateral spreading in the ZFE, not buoyancy. Zero entrainment in this region is a poor assumption.

(2) The flow-establishment length, defined as the centerline distance from the outfall to the point where the outfall temperature begins to drop, is assumed to be

$$\frac{s_i}{H_0} = 6.38A^{1/2},$$

where here A is the aspect ratio of the outfall, $A = 2B_0/H_0$. This value is based closely on nonbuoyant jet theory and is inadequate due to its lack of dependence on initial densimetric Froude number and ambient current. The Shirazi-Davis¹⁵ formulation for s_i is thought to be an improvement (see Ch. II, Sec. E) as some limited tank tests at EPA Pacific Northwest Environmental Research Laboratory on heated jets provided a densimetric Froude number dependence. The detailed study of the ZFE by Stefan,¹⁴ including initial densimetric Froude number, aspect ratio, and ambient current effects, in our opinion, provides the best information for this region.

In short, it is our opinion that ZFE model of Prych is inadequate due to its numerous untested and often unrealistic assumptions.

6. Sensitivity Analysis of Model

Table II.D.3 summarizes the input parameters used for Oskarshamnsverket (power plant) in Simpevarp, Sweden. Sensitivity of the predictions to the parameters E_O , C_D , E_h , and E_v is indicated in Figs. II.D.4-7. The solid lines in those figures pertain to the standard case of Table II.D.3. Each parameter is varied from the standard case for comparative purposes. The computations show that the Prych model is relatively insensitive to changes in the parameters E_O , E_v , C_D , and C_F .

The trends were as expected.

TABLE II.D.3. Input Data Summary for Sensitivity Analysis Based upon Oskarshamnsverket Data

Discharge	$Q_O = 130 \text{ m}^3/\text{sec}$
Outlet temperature	$T_O = 1.0$
Angle	$\theta_O = 90^\circ$
Width	$2B_O = 14 \text{ m}$
Depth	$H_O = 10 \text{ m}$
Coordinate	$x_O = 0.0 \text{ m}$
Coordinate	$y_O = 0.0 \text{ m}$
Ambient current	$V = 0.07 \text{ m/sec}$
Density difference	$\Delta\rho_O = 1.9 \text{ kg/m}^3$
Atm. heat-trans. coef.	$k = 0.00001 \text{ m/sec}$
Entrainment coef.	$E_O = 0.1$
Form drag coef.	$C_D = 0.2$
Shear drag coef.	$C_F = 0.5$
Horiz. amb. diff. mult.	$E_h = 0.0$
Vert. amb. diff. mult.	$E_v = 0.0$
Horiz. model scale	$S_h = 400.0 \text{ or } 1.0$
Vert. model scale	$S_v = 50.0 \text{ or } 1.0$
Starting coordinate	$s_O = 0.0 \text{ m}$
End coordinate	$s_L = 4000.0 \text{ m}$
Integration step	$\Delta s = 50.0 \text{ m}$
Index	$XI = 0.0$
Outlet velocity	$U_O = 0.93 \text{ m/sec}$
Outlet Froude No.	$F_O = 2.15$

Fig. II.D.4

Jet Trajectory for Various Choices for the Basic Parameters: E_o , E_h , E_v , C_D , C_F ; for Oskarshamnsverket. (Adapted from Prych.¹) ANL Neg. No. 190-1151.

Fig. II.D.5

Surface Centerline Excess Temperature as a Function of Centerline Distance for Basic Parameters: E_o , E_h , E_v , C_D , C_F ; for Oskarshamnsverket. (Adapted from Prych.¹) ANL Neg. No. 190-1168.

Fig. II.D.6

Characteristic Jet Width as a Function of Centerline Distance for Various Choices of the Basic Parameters: E_0 , E_h , E_v , C_D , C_F ; for Oskarshamnsvrket. (Adapted from Prych.¹) ANL Neg. No. 190-1147.

Fig. II.D.7

Characteristic Jet Thickness as a Function of Centerline Distance for Various Choices of the Basic Parameters: E_0 , E_h , E_v , C_D , C_F ; for Oskarshamnsvrket. (Adapted from Prych.¹) ANL Neg. No. 190-1150.

(1) The trajectory bent more rapidly when the rate of entrainment increased by increasing E_o , E_h , and E_v . The trajectory also turned more rapidly by increasing the drag; i.e., increasing C_D or C_F .

(2) The centerline temperature decreased more rapidly, by increasing E_o , E_h , and E_v . Reducing the drag, C_D or C_F , caused greater centerline temperature decay.

(3) Lateral spread of the jet increased slightly by increasing E_o or E_h . An extremely slight increase due to increasing E_v is unexplained; greater vertical mixing should indicate a thinner plume. Decreasing C_F and increasing C_D also decrease the lateral spread only slightly.

(4) Coefficient variation had very little effect on the jet thickness.

The effect of the atmospheric heat loss parameter, k , was also slight as calculations to 4 km from the source indicated only a 25% loss of heat to the overlying air.

The lack of sensitivity of the model to its parameters has two counterposing results. On the one hand, it is not necessary to have precise estimates of the input coefficients for a prediction because the model is not very sensitive to them. On the other hand, why are the physical phenomena of thermal plume dispersion not sensitive to, say, doubling the amount of lateral entrainment for a given problem? Also, there will be difficulty in attempting to calibrate the model to sets of data to improve performance.

An additional sensitivity check was made by Prych for the outfall and ambient parameters. A most notable result was that an increase in ambient current caused more rapid bending and a narrower jet, as expected, but the jet centerline temperature and jet thickness did not change significantly. The current was changed from 7 to 14 cm/sec while the outfall velocity was held at 93 cm/sec. It should be kept in mind that the above statements on model sensitivity apply only for problems of related initial densimetric Froude numbers, aspect ratios, and currents.

7. Model Verification

Prych compared his model with two sets of data. The first set consisted of two hydraulic model experiments carried out by Stolzenbach and Harleman for a rectangular heated jet into a deep basin. The second set of data was from the hydraulic model for Oskarshamnsverket; a distorted model test carried out with a vertical scaling of 1:50 and a horizontal scaling of 1:400. The quality of the latter data is in doubt because basic scaling laws were violated in the near field of the jet. Subsequent prototype data, presented by Weil,¹ showed a sharp discrepancy between laboratory and field measurements. Due to the questionable nature of the Oskarshamnsverket data, the Prych analytical model comparison to that data will not be discussed.

Data for the two experimental runs of Stolzenbach and Harleman appear in Table II.D.4. Coefficients used in the Prych run for that data appear in the lower part of the table. Figures II.D.8 and 9 compare the hydraulic model data, the Stolzenbach-Harleman predictions, and the Prych predictions for those two cases. Included in the plot are the half-width, $n_{1/2}$, and the half-depth, $z_{1/2}$, defined as the lateral or vertical distance, respectively, at which the temperature excess has dropped to half its surface centerline value. Both models predicted centerline temperature decay reasonably well. With regard to plume thickness, the Stolzenbach-Harleman model compared better to the $F_0 = 1.83$ data and the Prych model to the $F_0 = 4.40$ data. Both models gave very poor predictions for plume width, underestimating it initially, then grossly overestimating it farther from the outfall. The data show a nearly linear growth in half-width and not the exponential-like increase predicted by the models. The Prych prediction for half-width appears to be an improvement over the Stolzenbach-Harleman prediction but still gives values too large to be reasonable. Excessive widths have also been predicted in other cases (see Ch. IV).

Prych states that calculations proceeded to a centerline distance $75 \sqrt{H_0 B_0}$ for Run 3 and to a centerline distance $150 \sqrt{H_0 B_0}$ for Run 6. Calculations were halted at these points since the gross densimetric Froude number reached unity.

TABLE II.D.4. Data from Stolzenbach and Harleman Experiments for Stolzenbach-Harleman and Prych Model Comparisons

Physical Data		Run No. 3	Run No. 6
Outlet depth, cm	H_0	5.0	2.41
Outlet width, cm	$2B_0$	11.4	14.0
Outlet velocity, cm/sec	U_0	9.75	14.6
Density difference, kg/m ³	ρ_0	3.95	4.65
Outlet Froude No.	F_0	1.83	4.40
Atm. heat-trans. coef., m/sec	k	6.1×10^{-6}	6.1×10^{-6}
Ambient current vel.	V	0	0
Coefficients for Computations of Prych Model			
Form drag coef.	C_D	0.2	0.2
Shear stress coef.	C_F	0.5	0.5
Entrainment coef.	E_0	0.1	0.1
Atm. heat-trans. coef., m/sec	k	6.1×10^{-6}	6.1×10^{-6}
Horiz. ambient diff. coef.	E_h	0	0
Vert. ambient diff. coef.	E_v	0	0

Fig. II.D.8

Comparison of Prych and Stolzenbach-Harleman Models to (Tank) Data: Run No. 3, $F_0 = 1.8$. (Adapted from Prych.¹)

Fig. II.D.9

Comparison of Prych and Stolzenbach-Harleman Models to (Tank) Data: Run No. 6, $F_0 = 4.4$. (Adapted from Prych.¹)

Experience with the model has shown that:

(1) $F = 1$ is usually reached when there is a zero ambient current. However, the distance to the singularity increases with increasing values of F_0 .

(2) $F = 1$ is usually reached if $\theta_0 > 90^\circ$; i.e., there is a component of the outfall velocity (in x direction) opposing the ambient current.

(3) $F = 1$ is seldom reached when the discharge velocity has a component in the same direction as the ambient velocity or is nearly perpendicular to it.

Singularity difficulties occur less often in the Prych model than in the Stolzenbach-Harleman or Shirazi-Davis models. Increases in inertial forces, whether originating from the outfall or ambient current, tend to suppress the singularity in each of these models.

8. Criticisms and Limitations

a. Generic Limitations

The Prych model suffers from many of the same limitations as do the other integral models. These generic limitations are defined and discussed in the review of the Shirazi-Davis model (Ch. II, Sec. E).

b. Specific Limitations

Besides the generic limitations applicable to all integral models, the Prych model has certain specific limitations. Most of these limitations are described in the body of the review and are summarized here for convenience.

(1) Treatment of the Zone of Flow Establishment. The model has an inadequate, arbitrary treatment of the region of flow establishment. Of major consequence is the lack of effect of initial densimetric Froude number and ambient current on the length of the ZFE as well as a fixed dilution within the ZFE for all outfall and ambient parameters. Recently available data on the ZFE have made a more thorough analysis possible.

(2) Buoyant Lateral Spreading. The model treats lateral spreading due to buoyancy as resulting from pure gravitational advection. The assumption that

$$\frac{dB}{ds} = \left(\frac{dB}{ds} \right)_{nb} + \left(\frac{dB}{ds} \right)_b$$

is at the root of most of the problems with the model because spreading due to entrainment and spreading due to buoyancy are not additive when they are of the same order of magnitude.

(3) Functional Assumptions Based on Limited Data. Numerous assumptions such as those for the functional form of fluid-ambient velocity differences like Δu_v or Δu_h , or those for the vertical stability function f , are based on limited data at best, or on pure hunches. The only true test of these assumptions will depend on final verification of the model.

(4) Simulation of Ambient Turbulence. The simulation of ambient turbulence in the model should really involve the simulation of the coupled effects of ambient turbulence and jet-induced turbulence. The Prych simulation has the same effect, however, but the precise horizontal and vertical "diffusion coefficients" to be used need some calibration because the phenomenon is not treated in a basically correct form in the model. We feel that this limitation is not really as severe as some of the other limitations of the model ((2) and (3), for example).

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E. Shirazi-Davis Integral Model¹⁻⁵

1. Synopsis of Approach

Shirazi and Davis sought to assimilate all available laboratory and field data and then to use these data to modify or calibrate an existing three-dimensional integral model. Considered suitable for such investigation, although none was ideally so, were models by Stolzenbach and Harleman,⁶ Stefan et al.,⁷ and Prych.⁸ The Stolzenbach-Harleman model was not used because of its operational difficulties as described in Sec. C of this chapter. The model of Stefan et al., not reviewed in this report, neglected buoyant forces along the jet axis that become important at low densimetric Froude numbers and hence was not used. The Prych model has a well-documented computer code and includes the longitudinal pressure omitted by Stefan, but the model as originally formulated is insensitive to its key parameters. Shirazi and Davis discovered that changes in the Prych model's treatment of buoyant spreading made it more sensitive to its parameters and allowed it to be more successfully fit to the data.

The calibration of the Prych model was begun by gathering together as many data as could be found and reducing that data to a unified format. A regression analysis was then applied to identify important trends. Once the character of the data had been established, the model formulation was examined for theoretical weaknesses and areas of potential improvement. As a result of this study, several modifications of the Prych analysis were made. One, as noted above, involved the model's treatment of buoyant spreading. A fundamental change was made, too, in the analysis of the region of flow establishment. The remaining changes by Shirazi and Davis were the addition of a procedure for extrapolating model calculations past a singular point where the local plume densimetric Froude number equals one, an improvement in the method of calculating isotherm areas, and the addition of a time-temperature calculation. The modified Prych model was then fit to the data to obtain "recommended" values for the calibration parameters.

The contributions of Shirazi and Davis are (1) the assimilation and unification of existing data, (2) the modifications of the Prych model, and (3) the calibration of the modified Prych model. Each of these will be described in this review.

2. List of Symbols

A	Aspect ratio; $2B_0/H_0$. In Table II.E.1, $A = B_0/H_0$ where B_0 is the full width at discharge from a rectangular outfall
a	Depth of spreading density wedge
$\dot{a}$	da/dt

B	Local characteristic width of jet
B'	B/H_0
B_0	Half-width of discharge channel; in Table II.E.1 this is the full width
$B_{1/2}$	Half-width of temperature distribution
b	Half-width of spreading density wedge; also half-width of jet in initial zone analysis
$\dot{b}$	db/dt
b_0	Half-width of discharge for density-wedge analysis; also half-width of outlet in initial zone analysis
b_1	Half-width of jet in zone of flow establishment analysis
C_D	Drag coefficient
C_F	Coefficient of interfacial shear
C_p	Specific heat of water at constant pressure
C_1, C_2	Constants used in extrapolation formula
c	Celerity of the spreading density wedge
c_1	Numerical constant; $(32/\pi^3)^{1/4} \approx 1.01$
D	Diameter of semicircular or circular discharge
d	Characteristic depth of plume; $\pi H/2\sqrt{2}$
$(db/ds)_b$	Buoyant spreading rate
$(dB/ds)_{nb}$	Nonbuoyant spreading rate
E_h	Horizontal turbulent-exchange coefficient
E_0	Entrainment coefficient
E_v	Vertical turbulent-exchange coefficient
F	Local densimetric Froude number
F_D	Kinematic form drag per unit length of jet (used in Prych model)
F'_D	$F_D/U_0^2 H_0^2$
F_{Di}	Kinematic form drag per unit length of jet in region of flow establishment
F_0	Initial densimetric Froude number
$f, f_1, f_2, f_3, g, \tilde{g}, \tilde{\tilde{g}}$	Functional forms of nondimensional relations

G1, G2, G3	Groups 1, 2, and 3 for data analysis
g	Acceleration due to gravity
g'	Reduced acceleration due to gravity; $g\Delta\rho/\rho_o$
H	Characteristic depth of jet
H'	H/H_o
H_o	Depth of discharge channel
h	Depth of jet in region of flow establishment analysis
h_o	Depth of outlet in region of flow establishment analysis
h_1	Depth of jet at end of region of flow establishment
K'	Dimensionless heat-loss coefficient; $K_E/\rho C_p U_o$
K_E	Dimensional heat-loss coefficient
M	Kinematic momentum flux (used in Prych model)
M'	$M/U_o^2 H_o^2$
M_o	Kinematic momentum flux at outlet
$M(0)$	Kinematic momentum flux at end of region of flow establishment
P	Kinematic pressure force (used in Prych model)
P'	$P/U_o^2 H_o^2$
P_o	Kinematic pressure force at outlet
$P(0)$	Kinematic momentum flux at end of region of flow establishment
Q	Local volume flux in jet
Q'	$Q/U_o^2 H_o^2$
Q_o	Initial volume flux of jet
$Q(0)$	Volume flux at end of region of flow establishment
q_o	Half-strength of source in density wedge analysis
R	Ratio of ambient current to discharge velocity; U_a/U_o
Re	Reynolds number; $U_o H_o/\nu$
S, s	Coordinate along jet centerline
S'	S/H_o

S_F	Kinematic interfacial shear force (used in Prych model)
S'_F	$S_F/U_0^2 H_0^2$
S_{Fix}	X component of kinematic interfacial shear force (region of flow establishment)
S_{Fiy}	Y component of kinematic interfacial shear force (region of flow establishment)
S_{Fx}	X component of kinematic interfacial shear force (region of established flow)
S_{Fy}	Y component of kinematic interfacial shear force (region of established flow)
S_i	Length of zone of flow establishment
T	Local excess temperature ratio, $\Delta T_c/\Delta T_0$; also local excess temperature ratio at end of region of flow establishment $\Delta T_c/\Delta T_0$
t	Time
U	Local jet-centerline velocity
U'	U/U_0
U_a	Ambient current speed
U_0	Discharge velocity
$\bar{u}$	Average longitudinal velocity
V	Ambient current speed
V'	V/U_0
V_n	Lateral-spreading velocity normal to interface
W	Dimensionless full width at half maximum of temperature distribution; $2\sqrt{2} B/H_0$
X	Longitudinal coordinate of plume centerline
X'	X/H_0
$X(0)$	Rectilinear coordinate parallel to ambient current at end of region of flow establishment
X_{KI}	Lateral-spreading-rate adjustment coefficient
Y	Lateral coordinate of plume centerline
Y'	Y/H_0
$Y(0)$	Rectilinear coordinate, horizontal and perpendicular to X , at end of region of flow establishment

α	Angle of jet discharge with respect to ambient current; $\alpha = \pi - \theta_0$
ΔT_c	Excess temperature along jet centerline (above ambient)
ΔT_0	Excess temperature at the discharge (above ambient)
$\Delta \rho$	Local density disparity (from ambient)
$\Delta \rho_0$	Density disparity at outfall
ϵ	Coefficient of viscosity (density wedge analysis)
η	Nondimensional jet-centerline coordinate; S/H_0
θ	Angle of jet axis with current direction
θ_0	Angle of discharge with current direction
$\theta(0)$	Angle of jet trajectory at end of region of flow establishment
λ	Reciprocal of the bottom slope
ν	Viscosity of water
ρ	Local density
ρ_a	Ambient density
ρ_0	Discharge density

3. Study of Existing Laboratory and Field Data^{1,2}

Shirazi^{1,2} attempted to unify existing laboratory and field data^{6,7,9-17} both to provide insight into general plume behavior and to assist in validating near-field analytical models. Table II.E.1 summarizes the data included in this study and the range of parameters, such as initial densimetric Froude number, Reynolds number, and aspect ratio, considered in each experiment. The eleven studies listed in Table II.E.1 comprise the bulk of published data available at the time Shirazi undertook his analysis. Such a wide variety of data was needed to cover the range of parameters of interest. Even though Shirazi included as many sets of data as he could find, he still found them insufficiently refined to determine explicitly the trend of plume changes with each parameter.

Furthermore, Shirazi states that the data appear, at times, to be inconsistent among themselves with conflicting trends portrayed. He suggests that this inconsistency is most likely due to the following:

a. The experiments were conducted under varying physical scales. The data studied included both hydraulic model and field data; even within the hydraulic model data, different outfall configurations were used by different investigators.

TABLE 11.E.1. Data Considered by Shirazi

INVESTIGATORS	REF.	DISCHARGE REYNOLDS NUMBER (NOTE 1)	AMBIENT TO DISCHARGE CURRENT RATIO (NOTE 2)	INITIAL DENSIMETRIC FROUDE NUMBER (NOTE 3)	ASPECT RATIO (NOTE 4)	BOTTOM SLOPE	ANGLE OF DISCHARGE WITH CURRENT IN RADIAN	DISCHARGE STRUCTURE	TYPE OF STUDY	
BERGSTEDT AND STEFAN	9	$10^4 - 10^5$	0	1.9 - 19.7	2.0	"	1.57	SEMI-CIRCULAR	LAB
DORNHEIM, NOEL AND WIEGEL	10	7000	0	2.3 - 2.4	1.0	"	1.57	CIRCULAR	LAB
FRIGO AND FRYE	11	10^6	0 - 0.37	1.0 - 2.6	2.6	-	1.03 - 2.41	RECTANGULAR	LAKE
HAYASHI AND SHUTO	12	3000 - 5600	0	1.2 - 3.5	1.0	"	1.57	RECTANGULAR	LAB
NOTT AND BENEDICT	13	(a) 5200 (b) $10^5 - 10^7$	(a) .18 - .73 (b) .5 - .75	(a) 5.4 - 6.5 (b) 0.3 - 4.6	(a) 1.0 (b) 31 - 83	(a) ~ (b) -	(a) 1.57 - 2.36 (b) 1.62 - 2.06	(a) CIRCULAR (b) RECTANGULAR	(a) LAB (b) RIVER
SIMMONS, KUESER WEIGT, AND JOURIS	14	$10^5 - 10^7$	0.39 - 1.24	1.3 - 3.8	4.8 - 7.2	-	1.62	RECTANGULAR	RIVER
STEFAN, HAYAKAWA AND SOEBBE	7	1500 - 9400	0	0.62 - 4.1	3.03 - 3.23	"	1.57	RECTANGULAR	LAB
STOLZENBACH AND FARLEMAN	6	70,000 - 120,000	0 - 0.4	1.0 - 11.6	0.35 - 5.7	50 - "	1.57	RECTANGULAR	LAB
TAMAI, WIEGEL AND TORSBERG	15	5600 - 20,500	0	2.4 - 11.3	1.0	"	1.57	CIRCULAR	LAB
NEIL	16	4070 - 15,100	1.0	6.7 - 35.9	2.0	"	3.14	SEMI-CIRCULAR	LAB
WIEGEL, MOBAREK AND JEN	17	700 - 93,000	0	5.0 - 50.0	4.0 - 10.0	50 - "	1.57	RECTANGULAR	LAB

NOTES:

1. THE DISCHARGE REYNOLDS NUMBER IS DEFINED AS:
FOR RECTANGULAR DISCHARGES - $U_o H_o / \nu$ WHERE U_o IS THE NOMINAL DISCHARGE VELOCITY, H_o IS THE DEPTH OF THE DISCHARGE CANAL AND ν IS THE KINEMATIC VISCOSITY OF THE DISCHARGING FLUID.
FOR CIRCULAR OR SEMI-CIRCULAR DISCHARGES - $U_o D / \nu$ WHERE D IS THE DIAMETER OF THE DISCHARGE CANAL.
2. THE AMBIENT TO DISCHARGE CURRENT RATIO IS SIMPLY U_a / U_o WHERE U_a IS THE MEAN AMBIENT CURRENT SPEED.
3. THE INITIAL DENSIMETRIC FROUDE NUMBER IS DETERMINED BY $U_o / \sqrt{\frac{\Delta \rho}{\rho} g H_o}$ WHERE $\Delta \rho$ IS THE DENSITY DIFFERENCE BETWEEN DISCHARGING FLUID AND AMBIENT FLUID AND ρ IS THE DENSITY OF AMBIENT FLUID.
4. THE ASPECT RATIO IS GIVEN BY B_o / H_o , WHERE B_o IS THE FULL WIDTH OF THE DISCHARGE CANAL WHEN THE STRUCTURE IS RECTANGULAR. FOR CIRCULAR DISCHARGES, THE ASPECT RATIO IS ONE. FOR SEMI-CIRCULAR DISCHARGES, THE ASPECT RATIO IS THE DIAMETER OVER THE RADIUS, WHICH IS TWO.

b. A varying degree of emphasis was placed on individual parameters by the experimenters. Certain sets of data lacked a complete listing of all parameters of interest for Shirazi's trend analysis.

This diversity among the data and the absence of important information on such factors as turbulence levels and turbulence scales generally inhibited the development of a universal correlation.

Based on dimensional analysis, Shirazi writes

$$\frac{\Delta T_c}{\Delta T_o} = f_1(S/H_o, R, F_o, A, \lambda, \alpha), \quad (1)$$

$$\frac{B_{1/2}}{H_o} = f_2(S/H_o, R, F_o, A, \lambda, \alpha), \quad (2)$$

and

$$\frac{Y}{H_o} = f_3(X/H_o, R, F_o, A, \lambda, \alpha), \quad (3)$$

where

ΔT_c = centerline temperature excess (above ambient),

ΔT_o = initial temperature excess (above ambient),

$B_{1/2}$ = plume half-width,

H_o = depth of discharge,

X = longitudinal coordinate of plume centerline,

Y = lateral coordinate of plume centerline,

S = coordinate along jet centerline,

R = velocity ratio (U_a/U_o),

F_o = discharge densimetric Froude number,

A = aspect ratio ($2B_o/H_o$),

α = angle of centerline with respect to current,

and

λ = reciprocal of the bottom slope.

These correlations include all parameters that could be identified from the data.

Shirazi elected to represent the functions f_1 , f_2 , and f_3 as power-law expressions in which the functions are written as the product of powers of the parameters. This choice was made in spite of the fact that power-law forms can result in incorrect asymptotic behavior. For example, neither $F_0 \rightarrow 0$ nor $F_0 \rightarrow \infty$ can be accurately represented by a power-law form. Furthermore, the power-law correlation breaks down for stagnant cases ($R = 0$). To avoid these difficulties, Shirazi divided the data into three groups, designated G1, G2, and G3, as follows:

G1: Surface discharges into stagnant water without bottom interference.

G2: Surface discharges into stagnant water with bottom interference.

G3: Surface discharges into moving water.

Further division of G3 on the basis of bottom effects was not feasible due to the lack of information on bottom contours for these data. Table II.E.2 shows the data sets that belong to each group. Within each group, the data were correlated by including only the relevant parameters. For example, the G1 data correlation involved only S/H_0 or X/H_0 , F_0 , A , and α . For the G2 analysis, λ was added, and all parameters were considered in the G3 correlations.

TABLE II.E.2. Grouping of Data

Group G1, Stagnant with No Bottom Effects	Group G2, Stagnant with Bottom Effects	Group G3, Moving Ambient
1. Stolzenbach and Harleman ⁶	1. Stolzenbach and Harleman ⁶	1. Stolzenbach and Harleman ⁶
2. Wiegel, Mobarek, and Jen ¹⁷	2. Wiegel, Mobarek, and Jen ¹⁷	2. Motz and Benedict ¹³
3. Hayashi and Shuto ¹²	3. Motz and Benedict ¹³	3. Weil ¹⁶
4. Tamai, Wiegel, and Tornberg ¹⁵	4. Frigo and Frye ¹¹	4. Simmons et al. ¹⁴
5. Dornhelm, Nouel, and Wiegel ¹⁰		5. Frigo and Frye ¹¹
6. Motz and Benedict ¹³		
7. Frigo and Frye ¹¹		

Correlation parameters were determined by multivariable regression analysis for individual data sets and for various combinations of data sets within each basic group. In general, Shirazi¹ found that the range in correlation parameters between individual sets was so large that these individual correlations were essentially useless. For this reason, Shirazi sought to correlate two or three similar sets to obtain an "average" representation. A consistent procedure for combining data sets was not followed, however. For example, in G1 the temperature data were separated into smaller subsets than were the width data since fewer width data existed.

To evaluate the regression fit, the data were plotted on a single log-log graph. Figures II.E.1-3 illustrate how this was done for G1.

Fig. II.E.1

Selected Centerline Excess Temperature Data, Group G1. (Adapted from Shirazi.¹)

Fig. II.E.2

Selected Characteristic Half-width Data, Group G1. (Adapted from Shirazi.¹)

Fig. II.E.3

Selected Characteristic Half-depth Data, Group G1. (Adapted from Shirazi.¹)

Figure II.E.1 is a plot of $(\Delta T_c / \Delta T_o) F_o^{0.234} A^{-0.530}$ as a function of S/H_o , where the exponents are the correlation coefficients determined by regression. The solid line shown is a qualitative fit established by drawing a smooth curve through the data by eye. Figures II.E.2 and 3 are similar plots for width and depth data, respectively. A correlation line is shown for the width, but none is shown for depth because of the large scatter among the data.

4. Criticisms and Limitations of Data Analysis

One problem surrounding the use of power-law correlations has already been noted, i.e., the power-law forms can result in erroneous asymptotic behavior. Another difficulty is that predictions based on purely statistical principles probably do not satisfy the basic conservation laws because the forms chosen to fit the data are largely arbitrary. Furthermore, the correlations are based on very limited data. In some instances, only a few data were available from which to establish a trend of results with certain parameter variations. Therefore, the correlations should be restricted to interpolating and not to extrapolating the data. Associated with the sparsity of data is the fact that not all jet properties were correlated with the same group of data. This point was raised before. What the analysis shows is that the several parameters considered, F_o , A , R , λ , and α , are insufficient to fully explain the data. Despite these limitations, the correlations provide a useful base for calibrating the modified Prych model.³⁻⁵ The modifications leading to that model will now be discussed.

5. Shirazi-Davis Modification of Prych Model³⁻⁵

Two fundamental changes and three additions were made to the Prych⁸ model by Shirazi and Davis^{4,5} to better represent plume dispersion and to permit the fitting of the model to data. Each of these are now discussed individually.

a. Buoyant Spreading

Shirazi and Davis^{4,5} retain the basic ideas developed by Prych,⁸ including his assumption that lateral spreading due to buoyancy can be analyzed by considering the motion of a density wedge. Whereas Prych assumes that the velocity of such a wedge, c , is given by

$$c = c_1 \sqrt{\frac{\Delta \rho}{\rho_o} g d}; \quad (4)$$

Shirazi and Davis use

$$c = c_1 \sqrt{\frac{\Delta \rho}{\rho_o} g d} \sqrt{\frac{H}{B}}, \quad (5)$$

where H and B are, respectively, the local depth and width of the plume. A rationale for this modification comes from some physical reasoning and from an analysis of Koh and Fan.¹⁸

The basic ingredients of the Koh-Fan analysis are stated in Table II.E.3. Referring to that table, the problem considered is the time-dependent spreading of a density wedge such as an oil slick. They use an integral method in which no mixing of fluids is allowed and a hydrostatic pressure field is assumed. Both lateral-shear and drag forces are included

TABLE II.E.3. Koh-Fan Model¹⁸ for Unsteady Surface Spreading of a Buoyant Fluid

Problem: A line source of strength $2q_0$ per unit length discharges lighter fluid of density ρ_0 onto a heavier stagnant ambient fluid of density ρ_a . To be determined is the lateral and vertical motion of the pool as a whole.

Definition Sketch of Unsteady Surface Spread from Line Source of Strength $2q_0$

Definition Sketch of Half the Spreading Pool at Time t for Analysis

Assumptions:

- (1) No ambient fluid is entrained by the spreading pool.
- (2) The shape of the plume/ambient interface remains similar for all times. A rectangular shape is assumed; other shapes change only some of the numerical coefficients in the results and not the form of the resulting equations.
- (3) A hydrostatic pressure distribution exists.

TABLE II.3 (Contd.)

Equations:

Conservation of Mass:

$$ab = q_0 t \quad (A)$$

Conservation of Momentum:

$$\frac{\rho_0 d}{2} \frac{d}{dt} [ab\dot{b}] = \frac{1}{2} g (\Delta \rho_0) a^2 - C_D \rho_0 \frac{a}{2} (\dot{b})^2 - \frac{\epsilon}{a} \dot{b} (b - b_0) \quad (B)$$

inertial term: rate of change of momentum of the center of mass	pressure force induced by density differences	resistive force of shear	resistive force of hydrodynamic drag
--	---	--------------------------------	---

Solution:

Three separate time periods are considered.

- I** : Initial period in which the inertial term is important; no solution is presented as the above analysis is probably not valid.
- II** : Intermediate time in which the hydrodynamic drag is balanced by the driving pressure; the shear term is of secondary importance and the neutral term has probably become negligible by now.

$$\frac{1}{2} g' a^2 = \frac{C_D}{2} a (\dot{b})^2,$$

where

$$g' = \frac{\Delta \rho_0}{\rho_a} g$$

or

$$\boxed{\frac{db}{dt} = \left(\frac{g' a}{C_D} \right)^{1/2}} \quad (C)$$

from which

$$b = (2g' q_0)^{1/3} t \quad (\text{linear increase of } b \text{ with } t),$$

and

$$a = \frac{q_0^{2/3}}{(2g')^{1/3}} = \text{constant} \quad (\text{constant } a).$$

TABLE II.3 (Contd.)

III : Large time in which the shear term dominates

$$\frac{1}{2}g'a^2 = \frac{\epsilon}{\rho_0} \frac{b}{a} \frac{db}{dt}$$

or

$$\boxed{\frac{db}{dt} = \frac{\frac{1}{2}g'a}{\epsilon/a\rho} \frac{a}{b}} \quad (D)$$

from which

$$b = \left(\frac{5\rho_0 g' q_0^3}{8\epsilon} \right)^{1/5} t^{4/5}, \quad (E)$$

and

$$a = \left(\frac{5\rho_0 g' q_0^3}{8\epsilon} \right)^{-1/5} q_0 t^{1/5}. \quad (F)$$

Note that the horizontal extent of the density wedge has its spreading rate decreased from t to $t^{4/5}$ as viscous effects dominate, while the thickness increases slowly through the factor $t^{1/5}$.

in the momentum equation. The solution is constructed on three distinct time scales with the long-time solution based on the dominance of lateral shear. It is this long-time solution which Shirazi and Davis use to determine lateral spreading due to buoyancy. Specifically, Eq. D of Table II.E.3 is used although not directly. To show how this equation is used, we rewrite it in the nomenclature of the Shirazi-Davis model as follows:

$$V_n \sim \frac{g \frac{\Delta\rho}{\rho_a} H}{\epsilon/\rho H} \frac{H}{B}, \quad (6)$$

where V_n is the velocity of the density front normal to itself. Shirazi and Davis now argue that $\epsilon/\rho H$, which is the shear velocity, varies as V_n so that

$$V_n \sim \sqrt{\frac{\Delta\rho}{\rho_a} g H} \frac{H}{B}. \quad (7)$$

Noting that the only difference between Eq. 7 and the velocity used by Prych is the factor H/B , Shirazi and Davis conclude that the appearance of H/B accounts for the effects of lateral shear not considered by Prych. For typical

applications where $H/B < 1$, the spreading velocity is in fact reduced as compared to the form used by Prych. Because the Prych model tends to over-predict widths, the correction is in the right direction.

Shirazi and Davis add to the Koh-Fan analysis an argument based upon physical reasoning. Table II.E.4 analyzes the differences between the Prych and Shirazi-Davis formulations for lateral spreading in terms of two density wedges. According to Prych, density wedges I and II shown in Table II.E.4 will spread at the same rate because they have the same density difference and depth. According to Shirazi and Davis, density wedge I spreads faster because its aspect ratio is smaller. This is explained by Shirazi and Davis as follows: (1) the lower density of the plume causes it to rise slightly above the free surface of the surrounding water and (2) the height of rise at any point is thus proportional to the depth of the plume. Because both the

TABLE II.E.4. Prych⁸ and Shirazi-Davis^{4,5} Forms for Buoyant Lateral-spreading Velocity

H = depth of buoyant wedge

B = half-width of buoyant wedge

d = average depth of density wedge

h = height of rise of buoyant wedge

$\Delta\rho$ = density difference between lighter and heavier ambient fluids

ρ_a = density of heavier ambient fluid

g = acceleration due to gravity

V_n = lateral-spreading velocity

P_0 = atmospheric pressure

Prych:
$$V_n = c_1 \sqrt{g \frac{\Delta\rho}{\rho_a} d} \quad (\text{neglect shear})$$

Shirazi-Davis:
$$V_n = c_1 \sqrt{g \frac{\Delta\rho}{\rho_a} d(H/B)} \quad (\text{includes shear})$$

density difference and depth of the plume decrease from the center to the edge, this height varies from a maximum at the center to zero at the edge causing the plume to spread in that direction. The spreading rate due to buoyancy is related to the slope of this free surface. Because the height of rise at the center is proportional to $g\Delta\rho H/\rho_a$, the slope of the free surface and the spreading rate is a function of $g\Delta\rho H/(\rho_a B)$.

Note at this point that although the reasoning presented by Shirazi and Davis is plausible, it is hardly rigorous. The main justification for the form is that it lessens lateral spreading due to buoyancy and therefore improves predictions in most cases.

In implementing this change, Shirazi and Davis retain the constants used by Prych, simply writing

$$V_n = c_1 \sqrt{gd \frac{\Delta\rho}{\rho_a} (H/B)}, \quad (8)$$

where from Prych

$$d = \text{average thickness of the jet } (\pi H/2\sqrt{2})$$

and

$$c_1 = \text{a numerical constant } [(32/\pi^3)^{1/4} \approx 1.01].$$

Continuing the derivation due to Prych, Shirazi and Davis write

$$\left(\frac{dB}{ds}\right)_b = \frac{XK1}{[(B/H)F^2 - 1]^{1/2}}, \quad (9)$$

where the coefficient XK1 has been introduced to allow the result to be better fit to the data. In the Prych analysis, XK1 is set to $1/\sqrt{2}$.

In summary, the form for lateral spreading assumed by Prych has been modified by Shirazi and Davis in an attempt to reduce the amount of spreading that is predicted and therefore permit the model to be calibrated to data. This is accomplished by introducing a factor of H/B in the formula for the celerity of a density wedge to account for lateral shear and by using an adjustable multiplicative factor to either attenuate or amplify the resulting value. The analysis of Koh and Fan supports the use of this factor, although the form must still be considered arbitrary. As shown later, the modifications made by Shirazi and Davis predict a narrower, deeper plume.

The improvement in plume-width predictions made by Shirazi and Davis with their new lateral-spreading formulation is indicated in Fig. II.E.4. The data points plotted in the figure are from the tank study of

Stolzenbach and Harleman.⁶ The mean of measured tank and field data (from Shirazi³) with initial densimetric Froude numbers and aspect ratios (similar to those used in the tank study) is also plotted. The Stolzenbach-Harleman and Prych models grossly overpredict plume width, whereas the Shirazi-Davis model provides not only an improvement but also, and more important, a more correct trend--the trends shown in Fig. II.E.4 are typical in other-model-data comparisons as presented in Ch. IV. While the Shirazi-Davis model predictions appear to be more in line with data, the change in plume nature causes a faster decay of densimetric Froude number along the centerline and therefore results in computational difficulties in the code, as discussed later in Sec. c.

Fig. II.E.4

Comparison of Half-width Predictions from Stolzenbach-Harleman, Prych, and Shirazi-Davis Models to Stolzenbach-Harleman Tank Data⁶ as well as the Mean Data of Shirazi³

b. Zone of Flow Establishment (ZFE)

Shirazi and Davis^{4,5} modified the formulation of the zone-of-flow-establishment length assumed by Prych to include densimetric Froude-number effects. The formula for calculating this length, designated S_i , was changed from

$$S_i = 6.38H_0A^{1/2}$$

to

$$S_i = 5.4H_0(A^2/F_0)^{1/3}$$

The change was based on a limited number of experiments carried out in a stagnant hydraulic tank at the EPA Pacific Northwest Environmental Research Laboratory.⁴ In these tests, the length of flow establishment was measured for several different aspect ratios and initial densimetric Froude numbers. According to theory, three events occur at the end of flow establishment.

- (1) Turbulent fluctuations reach the centerline of the jet.
- (2) The centerline values of temperature and velocity begin to decrease.
- (3) Velocity and temperature profiles become similar.

In a real jet there are four usually distinct points that can be called the end of flow establishment:

- (1) The point at which the horizontal temperature profile becomes similar.
- (2) The point at which the vertical temperature profile becomes similar.
- (3) The point at which the horizontal velocity profile becomes similar.
- (4) The point at which the vertical velocity profile becomes similar.

Normally, points 1 and 2 are reached first, but the order in which they occur depends on the aspect ratio. The last of the four points to be reached is generally taken as the end of flow establishment, although this definition is not without its problems. One of these problems lies in the fact that the centerline temperature of the jet has already begun to decrease and the excess temperature ratio at the end of flow establishment is less than one. The effect on final plume model prediction is generally thought to be small, however.

The Shirazi-Davis form adds an additional degree of sophistication to the form used by Prych because it predicts a decreasing length with increasing densimetric Froude number. It does, however, result in incorrect asymptotic behavior. In the limit of $F_o \rightarrow \infty$, one physically expects an approach to nonbuoyant jet theory, but instead one gets $S_i \rightarrow 0$. Also, as $A \rightarrow \infty$ one should get the two-dimensional prediction, but instead one gets $S_i \rightarrow \infty$. These unrealistic predictions indicate that the Shirazi-Davis correlation for flow-establishment length should be limited to the range of densimetric Froude numbers and aspect ratios considered in the tank tests. Extrapolation beyond these ranges is risky.

A detailed study of flow establishment has been made by Stefan⁹ and is, at the time of this writing, under review for publication. Stefan carried out numerous tank tests to determine the effects of aspect ratio, densimetric Froude number, and crossflow. Initial indications are that the effects of crossflow are not too severe.

Shirazi and Davis retain the basic form of the Prych analysis for calculating jet parameters at the end of flow establishment as they substitute the revised form of S_i for determining its length. This analysis is summarized in Table II.E.5 and Fig. II.E.5. The parameters thus determined serve as initial conditions for the solution of the established flow regime.

TABLE II.E.5. Shirazi-Davis^{4,5} Analysis of the Flow Establishment Region

The Shirazi-Davis simulation follows Prych⁸ quite closely except for some minor modifications. The analysis proceeds in two steps.

Step 1.

Assume zero entrainment in ZFE to determine $B(0)/H(0)$, the initial values of B and H .

Using Prych's celerity velocity in the ZFE (not the Shirazi-Davis modification) with $\bar{u} = U_0$,

$$\frac{db}{ds} = \frac{\sqrt{g'h}}{\sqrt{U_0^2 - g'h}},$$

and using mass conservation,

$$bh = \frac{Q_0}{2U_0},$$

integration yields, for a rectangular profile at end of ZFE

$$b_1 = \frac{b_0}{F_0^2} \left\{ \left[\frac{3}{2} \frac{S_i}{b_0} F_0^2 + (F_0^2 - 1)^{3/2} \right]^{2/3} + 1 \right\}$$

and

$$h_1 = \frac{h_0 b_0}{b_1}.$$

It is assumed that

$$\frac{B(0)}{H(0)} = \frac{b_1}{h_1}. \quad (A)$$

Step 2.

Here entrainment is superimposed on the Step 1 analysis along with fully-developed temperature and velocity profiles at the end of development. Based upon conservation of energy at outfall and end of ZFE

$$Q(0) = \frac{2Q_0}{T} \quad (T = \text{excess temperature ratio at the end of development, usually taken at 1.0}) \quad (B)$$

TABLE II.E.5 (Contd.)

The X and Y momentum equations are derived as

$$\begin{aligned}
 & (M_o + P_o) \cos \theta_o + V Q_o \left(\frac{2}{T} - 1 \right) + F_{Di} \sin \theta_o + S_{Fix} \\
 & \begin{array}{cccc}
 \text{X momentum flux} & \text{entrained} & \text{X component} & \text{X component of} \\
 \text{from outlet} & \text{ambient} & \text{of form drag} & \text{interfacial} \\
 & \text{momentum} & & \text{shear}
 \end{array} \\
 & = [M(0) + P(0)] \cos \theta(0) \quad (C) \\
 & \begin{array}{c}
 \text{X momentum flux} \\
 \text{through cross} \\
 \text{section at} \\
 \text{end of ZFE}
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 & (M_o + P_o) \sin \theta_o - F_{Di} \cos \theta_o + S_{Fiy} = [M(0) + P(0)] \sin \theta(0) \quad (D) \\
 & \begin{array}{cccc}
 \text{Y momentum flux} & \text{Y component} & \text{Y component of} & \text{Y momentum flux} \\
 \text{from outlet} & \text{of form drag} & \text{interfacial} & \text{through cross} \\
 & & \text{shear} & \text{section at} \\
 & & & \text{end of ZFE}
 \end{array}
 \end{aligned}$$

Average values for drag and shear forces are used over the ZFE control volume,

$$\begin{aligned}
 F_{Di} &= \frac{C_D}{2} S_i \frac{h_o + h_1}{2} V |V| \sin^2 \theta_o \\
 S_{Fix} &= C_F S_i (b_o + b_1) |\vec{V} - \vec{U}_o| \left[\frac{2\nu}{(h_o + h_1) |\vec{V} - \vec{U}_o|} \right]^{1/4} (V - U_o \cos \theta_o) \\
 S_{Fiy} &= -S_{Fix} \frac{U_o \sin \theta_o}{V - U_o \cos \theta_o}
 \end{aligned}$$

Here $\vec{U}_o$ is the discharge velocity vector in the direction of θ_o and $\vec{V}$ is the vector of the ambient current.

An equation for θ_i is obtained from multiplying Eq. C by $\sin \theta_o$ and Eq. D by $\cos \theta_o$ and subtracting

$$\begin{aligned}
 \theta(0) &= \arctan \left\{ [(M_o + P_o) \sin \theta_o - F_{Di} \cos \theta_o + S_{Fiy}] / [(M_o + P_o) \cos \theta_o \right. \\
 & \quad \left. + Q_o V \left(\frac{2}{T} - 1 \right) + F_{Di} \sin \theta_o + S_{Fix}] \right\}
 \end{aligned}$$

TABLE II.E.5 (Contd.)

Multiplying Eq. D by $\cos \theta(0)$ and Eq. C by $\sin \theta(0)$ and adding yields

$$\begin{aligned} (M_0 + P_0) \cos[\theta(0) - \theta_0] + VQ_0 \cos \theta(0) \left(\frac{2}{T} - 1 \right) \\ + (S_{Fix} + F_{Di} \sin \theta_0) \cos \theta(0) \\ + (S_{Fiy} - F_{Di} \cos \theta_0) \sin \theta(0) = M(0) + P(0). \end{aligned} \quad (E)$$

Here

$$M_0 = Q_0 U_0$$

$$P_0 = g' b_0 h_0^2.$$

Following the treatment of Prych, Shirazi and Davis write

$$M(0) = \frac{Q^2(0)}{\pi B(0) H(0)}$$

and

$$P(0) \approx \frac{\sqrt{\pi}}{2} g' T H^2(0) B(0).$$

Eliminating $H(0)$ from Eq. A and using Eq. E

$$\text{LHS} \left(\frac{2b_1 h_1}{Q_0^2} \right) = \frac{8.0}{T^2 \pi} \left(\frac{B(0)}{b_1} \right)^{-2} + \frac{\sqrt{\pi} g' h_1}{4U_0^2} \left(\frac{B(0)}{b_1} \right)^3$$

where LHS is the left-hand side of Eq. E. This equation is solved iteratively for $B(0)$ in the computer code. Equation A is solved for $H(0)$. Equation B is solved for $Q(0)$. $T = 1.0$ is used for the S_i form used by Shirazi and Davis. Other forms for S_i may be used in which T may be less than 1.0. Finally,

$$X(0) = S_i \cos \theta(0)$$

and

$$Y(0) = S_i \sin \theta(0)$$

are assumed, replacing Prych's assumptions

$$X(0) = S_i \cos \frac{\theta_0 + \theta(0)}{2}$$

and

$$Y(0) = S_i \sin \frac{\theta_0 + \theta(0)}{2}.$$

TABLE II.E.5 (Contd.)

This change by Shirazi and Davis was found to eliminate considerable deviation between computed and mean data near the outfall during the calibration of the model.

This flow-establishment routine thus calculates $B(0)$, $H(0)$, $Q(0)$, $\theta(0)$, $X(0)$, and $Y(0)$ for initiating the established-flow solution.

Fig. II.E.5

Schematic of Zone of Flow Establishment
Used in the Shirazi-Davis Model Analysis

It should be stated here that the Prych analysis of flow establishment, which has been modified only slightly by Shirazi and Davis, is largely arbitrary with little or no justification for many of the assumptions. This treatment is essentially a stopgap measure made necessary by the dearth of good quality data on flow establishment. When more data become available, such as those of Stefan, a better model of flow establishment should be possible. Until then the analysis of Table II.E.5 will have to suffice.

c. Extrapolation of Model Predictions

A major difficulty with the Shirazi-Davis^{4,5} model is a pair of singularities in the model equations. One occurs in the equation for dH/ds when $F = 1$.

$$\frac{dH}{ds} = \left(\pi(S_{Fx} \sin \theta + S_{Fy} \cos \theta) + \left(\pi V \cos \theta - \frac{2Q}{BH} \right) \frac{dQ}{ds} \right. \\ \left. - \frac{\pi^{3/2} g'}{2} \frac{dT}{ds} - \frac{\pi^{3/2} g'}{2} H^2 T \frac{dB}{ds} \right) \left(\pi^{3/2} g' THB - \frac{Q^2}{BH^2} \right)^{-1} + \frac{F^2}{1 - F^2}$$

where F is the gross densimetric Froude number defined by

$$F^2 = \frac{Q^2}{\pi^{3/2} g' TH^3 B^2}$$

This singularity is usually the first to occur in the forward integration of the model equations. The second singularity occurs as a result of the form assumed for buoyant spreading

$$\left(\frac{dB}{ds}\right)_b = \frac{XK1}{[(B/H)F^2 - 1]^{1/2}},$$

when $F = \sqrt{H/B}$. This singularity will occur second and, in fact, is not encountered in practical applications because H/B is usually less than 1.

In the Prych model,⁸ the expression for buoyant spreading does not contain B/H and the two singularities both occur at $F = 1$. Prych recommends that the model predictions be terminated at this point even though the numerical integration will continue provided that $F = 1$ is not a point where the derivatives are calculated. Furthermore, the Prych model predicts a shallow, wide plume with a slow decay of densimetric Froude number and the singularity is not reached for considerable distance from the outfall.

Because of the modification made in the lateral-spreading rate, the Shirazi-Davis model predicts a narrower, deeper plume producing a much earlier encounter with the singularity. The model may calculate only a few hundred feet before reaching the singularity, and of this distance, a significant portion is the flow establishment region. The greatest difficulty in this respect occurs for low values of R or F_0 , or large values of A . Although the first singularity can be bypassed by a judicious choice of step size and error criterion, it is probably incorrect to do so, although the numerical results appear to be a reasonable extension of model predictions. The major anomaly is the appearance of a small oscillation in plume depth due to the singularity in the dH/ds equation. Shirazi and Davis recommend that code calculations of trajectory be kept past the first singularity but that predictions of other characteristics not be used.

Shirazi and Davis consider whether the singularity is a physical reality or whether it is the outgrowth of poor modeling. The first thought that comes to mind is that the singularity corresponds to the appearance of an internal hydraulic jump as seen in two-dimensional plume studies in tanks (see the discussion of this phenomenon in Ch. III). It is currently unknown whether a three-dimensional analog exists in prototype plumes. Since no discontinuities in three-dimensional prototype plumes have ever been observed, it is more likely that the singularity is an artifact of model simulation and does not represent a physical reality. Shirazi and Davis state that the uncoupling of the hydrostatic pressure force in the longitudinal and lateral directions is probably at fault and conclude that this uncoupling gives the plume a false two-dimensional character. The singularity can be eliminated by removing the longitudinal pressure force, but this revision also greatly affects the predictive capabilities of the model.

To allow predictions to be made beyond the point of singularity Shirazi and Davis present a scheme in which temperatures and widths can be obtained by extrapolating from cases of higher currents (where the model is operational) to lower current cases (where the model fails).

Taking centerline temperature decay as an example, Shirazi and Davis write

$$\Delta T_c / \Delta T_o = f(F_o, A, R, K, \theta_o, S/H_o)$$

or

$$S/H_o = g(F_o, A, R, K, \theta_o, \Delta T_c / \Delta T_o).$$

For fixed F_o , A , R , K , and θ_o , one can state

$$S/H_o = \tilde{g}(R, \Delta T_c / \Delta T_o).$$

For a particular temperature, $\Delta T_c / \Delta T_o$ is constant, so that

$$S/H_o = \tilde{\tilde{g}}(R).$$

For a fixed temperature ratio, the centerline distance to that temperature is a function of R . For small changes in R , Shirazi and Davis use

$$S/H_o = C_1(1 - R)^{C_2}$$

as an approximation for $\tilde{\tilde{g}}$.

The extrapolation is effected by first running the computer code for several values of R . Then for each value of $\Delta T_c / \Delta T_o$ of interest, S/H_o is plotted as a function of $(1-R)$ on a log-log scale. Figure II.E.6 gives an example of this procedure showing model predictions for Point Beach Unit 1. The angle between the discharge channel and the southshore is 60° , the initial densimetric Froude number of this case is 2.25, and the outfall aspect ratio is 2.55. The right half of the plot shows the results for $\theta_o = 60^\circ$, i.e., for southerly currents, while the left half shows the results for northerly currents. Computer code calculations are indicated by filled circles connected by either solid line segments when the local densimetric Froude number is larger than 1 or by broken line segments when the local densimetric Froude number is less than or equal to 1. Also shown are extrapolation results to zero current indicated by open circles and dotted line segments. Shirazi and Davis sanction the extrapolation of model predictions to any positive value of R if $\theta_o \leq 90^\circ$, but state that no model results should be extrapolated for $R < 0.1$ if $\theta_o > 90^\circ$. Thus, there is a region in which the model is not applicable even by extrapolation.

Fig. II.E.6. Extrapolation of the Shirazi-Davis Model^{4,5}

d. Two Minor Changes in the Computer Code

Shirazi and Davis^{4,5} improved upon the model's algorithm for integrating isotherm areas and added a calculation of time-temperature history. The integration of areas was improved by requiring that a smaller step size be used in the trapezoidal rule algorithm. The Prych⁸ model integrates areas using, at times, steps along the jet axis that correspond to larger than 0.05 drops in excess-temperature ratio, a rather coarse step size.

The time-temperature calculation attempts to determine the temperature history that an organism passing through the plant cooling system would encounter. This is accomplished by integrating the reciprocal velocity along the jet centerline thusly

$$t(T) = \int_0^{S(T)} \frac{dS'}{U(S')}.$$

This time represents the minimum time for an organism to reach a particular excess-temperature ratio, because paths not along the jet centerline are presumably less direct.

6. Fitting the Modified Prych Model to Data

The Prych model⁸ as modified by Shirazi and Davis^{4,5} contains six undetermined coefficients: E_o the entrainment coefficient, E_h and E_v the turbulent exchange coefficients, C_D the drag coefficient, C_F the interfacial shear coefficient, and XKl the coefficient in the lateral buoyant-spreading relationship. The objective of the fitting was to relate the model performance to available experimental data, and the attempt revealed the difficulties and limitations of obtaining exact agreement between the model and data. Tank data from different sources over a narrow range of jet parameters differed, at times, by an order of magnitude. Field data, on the other hand, showed enough scatter sometimes to fill two orders of magnitude. Fitting the model to a mean set of data became most practical. The concept of exact agreement was quickly rejected. Part of the problem is the requirement of representing a set of data by a pre-determined set of parameters, often neglecting transient effects, bottom effects (in the field), difference in ambient and jet turbulence levels, etc., that often occur and contribute to making one set of data different from another. Much of the scatter in the data could be explained by these additional unknown factors. In short, it was not practical to determine model coefficients by fitting to individual data sets. Rather, the results discussed in Sec. 2 were used.

The calibration parameters are handled somewhat differently by Shirazi and Davis than they are by Prych, because Shirazi and Davis non-dimensionalize the equations before coding, which is desirable because it makes the model calculations independent of any particular set of units. The forms used are given in Table II.E.6. Shirazi and Davis argue further that this procedure makes the model parameters more physically meaningful and provides at least some justification for having a scaling process and the type of data fitting outlined in a-d below. In our opinion, it is difficult to assess the precise advantage of this scaling of variables and model coefficients, as not all the dimensionless groups of parameters affecting the jet flow are included in the model.

To eliminate the need to fit the model by simultaneously varying all six parameters, the calibration was carried out in four steps as follows.

a. Determining the Values of E_h , E_v , and XKl

The first coefficients determined were E_h and E_v based on the data of Weil,¹⁶ who studied the effects of turbulence on plume spreading. Weil's experiments consisted of a high initial densimetric Froude number, semicircular, surface jet discharged parallel to and at the same velocity as

the ambient flow. The effects of the semicircular discharge structure were deemed unimportant beyond the ZFE. Dilution is largely due to jet and ambient turbulence effects, since the relative velocity between the plume and ambient water is zero and since buoyancy effects are small due to the high densimetric Froude number. In calibrating the model to this data, Shirazi and Davis set E_0 and C_F to zero specifically because of the zero relative velocity between jet and ambient. The drag coefficient C_D was also set to zero as the jet is parallel to the ambient current and consequently, the pressure distributions on either side of the jet are equal.

TABLE II.E.6. Nondimensionalizations Made in the Shirazi-Davis Model

Dimensionless Variables:

$U' = U/U_0$	$X' = X/H_0$	$F'_D = F_D/U_0^2 H_0^2$
$B' = B/H_0$	$Y' = Y/H_0$	$Q' = Q/U_0^2 H_0^2$
$H' = H/H_0$	$V' = V/U_0$	$A = 2B_0/H_0$
$T' = \Delta T_C / \Delta T_0$	$P' = P/U_0^2 H_0^2$	$R = V/U_0$
$K' = K_E / \rho C_p U_0$	$M' = M/U_0^2 H_0^2$	$Re = U_0 H_0 / \nu$
$S' = S/H_0$	$S'_F = S_F / U_0^2 H_0^2$	

The above dimensionless terms are introduced into the governing differential equations with the primes then dropped. The established flow equations then become identical to those of Prych if one identifies,

$\frac{1}{F_0^2} \text{ as } g'$	$\frac{K_E}{U_0} \text{ as } K$
$\frac{1}{Re} \text{ as } \nu$	$\frac{\epsilon}{U_0 H_0} \text{ as } \epsilon.$

The Weil data, after multiple-variable regression methods were applied, is plotted in Fig. II.E.7. Figures II.E.8 and 9 compare several model calculations to the average data. Many more similar comparisons were made but these are not presented here. By visual inspection, Shirazi and Davis selected E_h to be 0.02, E_v/E_h to be 0.2, and XK_1 to be 1.4, as best representing the local mean temperature decay and width of the data.

Fig. II.E.7

Correlation of Temperature Data for a Jet Initially Discharged into a Co-flowing Channel with Zero Relative Velocity. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.8
Comparison of Calculated Temperatures with Mean Data from Fig. II.E.6. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.9

Comparison of Calculated Widths with Mean of Data for a Jet Initially Discharged into a Coflowing Channel with Zero Relative Velocity. (Adapted from Shirazi and Davis.⁴)

b. Determination of E_O and C_F

Shirazi and Davis next evaluated E_O and C_F by grouping several sets of laboratory and field experiments under conditions of zero or very small cross current. In these near-stagnant cases, a value of $C_D = 0$ was used, while the values of E_V , E_h , and $XK1$ were assumed to be the same as those determined above. Figures II.E.10 and 11 give, respectively, the temperature decay and widths for the data used in this step. Figures II.E.12 and 13 show the comparison of many model calculations with the mean of the data in a similar presentation to that given in Figs. II.E.8 and 9. Shirazi and Davis felt that $E_O = 0.05$ and $C_F = 0$ best represented the mean data. The interfacial shear coefficient C_F was set to zero because it was found to have little effect on predictions.

Fig. II.E.10. Correlation of Selected Temperature Data for Zero or Negligibly Small Ambient Current. (Adapted from Shirazi and Davis,⁴)

Fig. II.E.11

Correlation of Selected Width Data for Zero or Negligibly Small Ambient Current. (Adapted from Shirazi and Davis,⁴)

Fig. II.E.12. Comparison of Calculated Temperatures with Mean of Data Shown in Fig. II.E.9. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.13. Comparison of Calculated Widths with Mean of Data. (Adapted from Shirazi and Davis.⁴)

c. Determination of C_D

The last parameter, C_D , was evaluated by grouping together data for plumes in the presence of an ambient current. Previously determined values of E_h , E_v , $XK1$, E_0 , and C_F were maintained and not varied. Following

the presentation above, Fig. II.E.14 gives the data and Fig. II.E.15 shows the comparison of several model predictions to the mean of the data. On the basis of the comparisons in Fig. II.E.15, Shirazi and Davis chose $C_D = 1$ as the best fit.

Fig. II.E.14. Correlation of Selected Trajectory Data.
(Adapted from Shirazi and Davis.⁴)

Fig. II.E.15. Comparison of Calculated Trajectory with Mean
of Data. (Adapted from Shirazi and Davis.⁴)

d. Test of Model Predictions

In order to evaluate the calibration obtained by a, b, and c above, the model was checked against data from different sources and with a wide range of parameters. Figures II.E.16 and 17 illustrate the comparison of the calibrated model with five sets of raw data. Both centerline temperature decay and width predictions appear to be in good agreement.

Fig. II.E.16. Comparison of Calculated Temperatures with Selected Data. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.17. Comparison of Calculated Widths with Selected Data. (Adapted from Shirazi and Davis.⁴)

Based upon these comparisons, Shirazi and Davis⁵ did not feel the need to readjust the coefficients for an additional refinement. It was not clear at the beginning whether one can isolate all the dispersion phenomena separately and fit the model by the inclusion of one phenomenon at a time, because, in actuality, all these dispersion processes occur at once. This final check was made to see if the building procedure for evaluating the coefficients needed further iteration before satisfactory comparisons would be produced.

7. Model Usage: Computer Code and Nomograms

There are two methods of obtaining Shirazi-Davis^{4,5} model predictions. If a precise prediction is required, then the computer code must be

used. The listing of the program originally given by Shirazi and Davis⁴ contains an incorrect statement as noted below:

Line 233 should be replaced by:

GN=0.

IF(RE.GT.0.)GN=SQRT(SQRT(1./((RE*H))) *0.206*S2*B/10.0

Shirazi and Davis^{4,5} describe the code as well as the correct method of preparing input. A sample output of the computer code appears in Appendix A of this report.

An alternative method of obtaining model curves is to use one or more of the more than 300 nomograms presented by Shirazi and Davis.⁴ These nomograms are useful for design work when identifying trends is important. Furthermore, the nomograms also spare the user the effort of manually extrapolating calculations to low ambient currents. For a given value of F_0 , A , R , θ_0 , and K , the correct nomogram is found by reading a summary table. Figures II.E.18-21 are sample nomograms that illustrate the

Fig. II.E.18

Plume Trajectory, Temperature, Width, and Depth for Surface Jet Showing Effects of Jet Densimetric Froude Number. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.19

Plume Trajectory, Temperature, Width, and Depth for Surface Jet Showing Effects of Jet Aspect Ratio. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.20

Plume Trajectory, Temperature, Width, and Depth for Surface Jet Showing Effects of Ambient Current. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.21

Plume Trajectory, Temperature, Width, and Depth for Surface Jet Showing Effects of Discharge Angle. (Adapted from Shirazi and Davis.⁴)

effect of initial densimetric Froude number, aspect ratio, ambient current, and discharge angle, respectively. To see how these graphs are interpreted, consider Fig. II.E.18, which illustrates the effect of initial densimetric Froude number for an aspect ratio of 5.0, an ambient current speed that is 10% of the discharge velocity, and a discharge angle of 90°. The three dashed lines labeled $F = 2$, $F = 4$, $F = 10$ show the trajectory predicted for each of these initial densimetric Froude numbers. Greater bending is predicted for larger densimetric Froude numbers due to increased entrainment of ambient momentum. Furthermore, the length of each dash along the trajectory is made proportional to the local plume depth. (The depth scale is indicated on the nomogram.) Two sets of solid lines intersect the trajectory curves. One set labeled $T = 0.25$, $T = 0.3$, and $T = 0.4$ intersects the trajectory at the point where that value of the temperature-excess ratio is reached, while the set labeled $W = 100$, $W = 150$, and $W = 200$ indicates the point at which this quantity reaches these values. Additional nomograms showing area and time-temperature predictions are given in Figs. II.E.22 and 23, respectively.

Fig. II.E.22. Temperature-Area Nomogram for Surface Jet. (Adapted from Shirazi and Davis.⁴)

Fig. II.E.23. Temperature-Time Nomogram for Surface Jet. (Adapted from Shirazi and Davis.⁴)

The nomograms such as those presented in Figs. II.E.18-23 are very useful in identifying key trends in plume behavior. Shirazi² cites five such trends.

(1) A weak ambient current as small as 10% of the initial jet velocity is sufficient to bend the plume substantially. An ambient current on the order of 50% of the jet velocity bends the plume totally in the direction of the current, thereby preventing plume penetration but possibly causing the plume to hug the shoreline.

(2) A small initial densimetric Froude number counters the effect of an ambient current, thus requiring a substantially stronger current to bend such a plume over and prevent a deep penetration into the moving water.

(3) A strong ambient current and large densimetric Froude number both can cause narrow and cool plumes. Conversely, a small ambient current and a small jet densimetric Froude number cause very wide and hot plumes.

(4) The effect of a large aspect ratio is generally to reduce interaction with the ambient current and cause greater penetration. One can think of a wide plume as riding over the ambient current but the narrow one trying to block and thus strongly interacting with the ambient current. Plume width is naturally greater for an initially wide jet, but the temperature of such a plume is also greater due to reduced interaction and the tendency of the heated water to float more rapidly over the cooler ambient water.

(5) A shallow angle of discharge (i.e., nearly a coflowing discharge) naturally offers the least resistance to the cross current. When compared with a steep discharge angle ($\theta_0 = \pi/2$), it is associated with a wider plume and relatively higher temperature. One can think of the nearly coflow case as one where the ambient turbulence and not the interaction with current dominates the mixing process. Another physical explanation for a wide and hot plume for shallow angle of discharge is the real effects of the shoreline (and shallow bottom) that inhibit mixing. In any event, the results must be used with caution.

In addition to the above "working nomograms," Shirazi and Davis provide a set of supplementary nomograms to better understand the sensitivity of the model to its coefficients and for application to special situations in which one or more coefficients are expected to deviate substantially from those recommended. For example, the value of E_h or E_v might be increased substantially when a plume is discharged into an ambient current of high turbulence. In this way, the effect of choosing "non-fitted" E_h and E_v values can be better understood. No recommendations are given, however, on how to vary parameters for such special applications.

8. Criticisms and Limitations

a. Generic Limitations of Integral Models

The generic limitations are those applicable to all the integral models including the Motz-Benedict, Stolzenbach-Harleman, Prych, and Shirazi-Davis ones. Among them are the requirements for:

- (1) A rectangular channel outfall structure.
- (2) Deep water so that the jet is unrestricted by bottom boundaries; a free jet, not a confined jet, is idealized in the model.
- (3) Wide water to assure that the jet is free and dilution water is not constrained by lateral or longitudinal boundaries.

(4) Homogeneous, uniform, and steady ambient and outfall conditions. (This implies no stratified ambient receiving body, a uniform current, and constant outfall conditions. No plume transients can be simulated.)

(5) No wind momentum imparted to the plume.

Integral models are often applied to prototype cases in which a number, if not all, of the above idealizations are violated. The more violations, the greater the chance of inaccurate predictions.

Not all violations of the idealized concepts are serious, however. The integral models can probably be used for nonrectangular discharges, because the rectangular nature of the outfall is lost beyond 10-13 outfall depths from the discharge point. Therefore, one may simulate a nonrectangular outfall if desired.

The deep water assumption of the model does not really require an infinite body of water, just one that is large compared to the dimensions of the plume. An outfall depth of 1 m can often be considered to be in deep water if the water is 4-5 m deep. Some limited data on bottom depth for a jet to be considered in "infinite water" is given by Koester.¹⁹ A qualitative indication of the effect of lake depth on the plume can be obtained for the comparison of lake bottom profiles to predicted plume depth.

Lateral and longitudinal boundaries may be considered unimportant if the calculated plume trajectory does not interact with them. There is no practical way of "adjusting" the model when boundary interference occurs.

Start-up or plume recirculation is a time-dependent problem. If the heat transferred from the area of the predicted plume is of the same order of magnitude as that being discharged, then the entrainment of recirculated water is probably not significant; the ambient water will disperse the heat.

Should the ambient body be stratified due to solar radiation, it may not be of great significance if the thermocline or plane of maximum temperature gradient is much deeper than the maximum plume depth. At times the nearshore area is of uniform temperature, though stratification may appear offshore, or else the plume may be discharged into the nearly constant-temperature, epilimnetic region of the receiving body. Stratification of the ambient may, in fact, be included in integral models easily by accounting for the entrainment of different temperature water. However, the resolution of the model itself may not be good enough to make a significant difference. It is assumed in these integral models, therefore, that the sun heats the lake uniformly.

The accuracy of these integral models depends on how accurately plume physics is simulated as well as how appropriate the model is for a particular application. The hope is that, with all the idealizations and with the application to nonideal situations, the model, if it errs, errs conservatively. This is a big hope that has yet to work out for any integral model. The results for all integral models have generally been discouraging.

b. Specific Limitations of Shirazi-Davis Model

The limitations specific to the Shirazi-Davis model are essentially the same as those of the Prych model, except for those areas where changes were made. Many of these concerns have already been touched upon. By way of summation, we will briefly consider each again.

(1) Lateral Spreading. The lateral-spreading assumption of the model is a major problem since spreading due to entrainment and spreading due to buoyancy are not additive when they are of the same order of magnitude. The buoyant and nonbuoyant components of spreading represent asymptotic solutions and there is no reason to believe that an intermediate solution will be well represented by their sum. The modification made by Shirazi and Davis in the form of the buoyant-spreading rate generally improves width prediction, but is by no means rigorous. Its justification is mostly in that it aids in calibrating the model to data. The precise method of handling lateral spreading can be developed only by avoiding overly-simplified model simulations and carrying out basic tests, probably in a hydraulic model tank, of the mechanism itself.

(2) Flow Establishment. Again, the modification made by Shirazi and Davis seems to be an improvement on the analysis of Prych, but the treatment of flow establishment still leaves much to be desired. The correlation used to compute the length of flow establishment lacks the refinement that numerous tests under varying conditions would provide. In its present form, the correlation does not include the effects of ambient current, nor does it result in the proper asymptotic behavior. Recent tank data by Stefan⁹ will allow more fruitful work to be done in ZFE modeling. Furthermore, the methodology advanced by Prych and retained by Shirazi and Davis for calculating jet parameters at the end of flow establishment is a weak point in the model.

(3) Calibration. The Shirazi-Davis calibration procedure has several undesirable features. First, data amenable to calibration are generally lacking. To be sure, not enough data exist to develop an accurate simulation of all physical phenomena. The wide scatter in the data cannot be accounted for in terms of measured parameters, a fact that makes calibration difficult. Second, trying to calibrate a model that may contain physically invalid assumptions is difficult. Such a model may not provide a reliable vehicle for extrapolating and interpolating the data. Finally, there is the problem of fitting the model one parameter at a time. No convincing evidence exists that

model parameters determined in one limited experiment will apply to another experiment of a different configuration. Shirazi and Davis argue that the scaling processes carried out in nondimensionalizing model equations provide some rationale for the calibration outlined above.

(4) Extrapolation. A major drawback of the model is the short range of distances over which it is directly applicable. For cases often of practical interest, the model fails before reaching the lower isotherms where standards are normally set. This occurrence made necessary an extrapolation procedure to obtain the desired result from the limited model calculations. Obviously, such a procedure is not scientifically satisfying. Such dissatisfaction stems from an important basic flaw. The extrapolation procedure is not based upon conservation laws and probably does not satisfy them; such extrapolations should be used very cautiously.

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F. Pritchard (Model No. 1) Integral/Phenomenological Model¹⁻⁵

1. Synopsis of Approach

This integral/phenomenological model is basically a synthesis of previous theoretical and physical modeling results for buoyant and nonbuoyant jets, complemented with results Pritchard has gleaned from field data obtained from some existing power plants sited on bays, estuaries, and large lakes. Both the development and application of the model are simple, yet Pritchard claims that model predictions will be conservative in many respects.

The model considers plume dispersion to be governed solely by jet entrainment, turbulent diffusion, and surface heat loss. An integral technique is used in which the plume velocity and excess temperature (above ambient) are assumed to have, at each longitudinal position, a "tophat" distribution laterally and vertically. Buoyancy-induced convective motions are not explicitly considered in the model. No ambient crosscurrent is assumed to exist in the theoretical development; yet, Pritchard expects that the predicted centerline temperature decay and areas within isotherms are still accurate in the presence of a shore-parallel current less than 10% of the initial discharge velocity. In the model, environmental changes are reflected solely in the surface-heat-transfer coefficient K . Entrainment is accounted for by the specification of an "inverse-spreading-rate" parameter. The lake bottom is assumed to have no effect other than upon the author's choice of a plume depth and, when necessary, upon the length and depth of an initial region of vertical entrainment.

The model handles both the jet and far-field regions; it predicts a two-dimensional temperature field and areas within isotherms down to a 1 F° temperature excess.

The theoretical development is carried out in four consecutive stages:

a. Horizontal spreading is considered, neglecting vertical diffusion and surface heat loss to the atmosphere. A two-dimensional temperature field is determined for the jet and far-field regions.

b. Vertical entrainment is then considered independent of horizontal spreading and surface heat loss. The depth is assumed constant, except possibly for a small region in the vicinity of the outfall, where the depth grows slowly in a linear fashion. When vertical spreading is assumed to occur, a correction of the two-dimensional temperature field is made to account for the additional dilution.

c. A second correction of the temperature field may be needed, depending upon the temperature of the water entrained into the plume. Due to

possible recirculation of condenser cooling water, the temperatures of the diluting water mixed into the plume may be above ambient. Once this additional correction on the temperature field is made, the areas within isotherms are calculated for the condition of mixing alone.

d. Surface heat losses are then included in the analysis as a correction to the areas derived in stage c. This surface-heat-loss correction yields the final two-dimensional temperature field and the isotherm areas.

The parameters required for the application of this model are:

b_o = width of rectangular outfall,

h_o = depth of rectangular outfall,

Q_h = excess heat-rejection rate of power plant based on θ_o (Q_h would be identically equal to the total condenser heat-rejection rate if the condenser intake temperature were identically equal to the ambient temperature),

θ_o = initial excess temperature of the jet,

and

K = surface-heat-loss coefficient.

As with some others (e.g., Motz and Benedict), this model relies heavily on the engineering judgment of the user. In many applications these models are used to give only order-of-magnitude type or conservative predictions. In applying their models, Motz and Benedict often employ engineering judgments in the selection of key parameters. For example, if the site of interest is thought to exhibit bottom interference, some qualitative restrictions on vertical entrainment may be imposed. In past applications of his model, Pritchard has made special allowances for such site-dependent characteristics. These considerations are difficult to formalize. In essence, Pritchard describes a methodology or a set of guidelines within which the user is supposed to work.

2. List of Symbols

A_k Surface area within a selected isotherm θ_k , for
 $k = 1, \dots, n$

A_{θ_n} Surface area with excess temperature equal to or greater
than θ_n

b Local width of velocity and temperature distribution in jet

b_o Full width of outfall

b_θ Mean width of θ isotherm

$\bar{b}_\theta$ Maximum width of θ isotherm

c_p	Specific heat at constant pressure
D	Characteristic "critical" mixing depth of receiving body of water
D'	Local depth of plume
D_o	Equivalent diameter of a rectangular outfall
D_θ	Thickness of plume at a distance ξ_θ where $\xi_\theta < \eta_1$ (when vertical mixing occurs)
h_o	Depth of outfall
K	Surface-heat-transfer coefficient (a function of excess temperature θ)
n	Inverse-spreading rate
n_1	θ_o/θ_1
n_2	θ_o/θ_2
Q_h	Excess heat-rejection rate of power plant based on θ_o ; $(\rho c_p u_o b_o h_o \theta_o)$
Q_o	Volumetric flow rate of power plant at discharge
u	Centerline velocity, a function of ξ
u_a	Velocity of transverse crosscurrent (assumed constant)
u_o	Initial velocity of the jet at discharge
$\Gamma(1^\circ)$	Rate of heat loss from the area contained within the $1^\circ F$ isotherm
$\Delta A_{\bar{\theta}_n}$	Incremental area between θ_n and θ_{n-1} isotherms
$\Delta A_{0.5}$	Area having <u>mean</u> excess temperature $0.5^\circ F$
$\Delta \Gamma_{\bar{\theta}_n}$	Rate of heat loss from the area contained between the θ_n , θ_{n-1} isotherms
ζ	$\xi/(nb_o)$
η_1	Longitudinal distance at which vertical entrainment ceases
θ	Local excess temperature
θ_{11}	Excess temperature of a particular isotherm
$\bar{\theta}$	Local excess-temperature correction of the horizontal temperature distribution due to additional dilution resulting from vertical entrainment
θ_o	Initial excess temperature at discharge
θ'	Superposed excess temperature at a given position resulting from plume No. 1 and 2

θ_1	Excess temperature at a given position resulting from plume No. 1
θ_2	Excess temperature at a given position resulting from plume No. 2
$\bar{\theta}_n$	Mean excess temperature between the nth and (n-1)st isotherm
$\theta_{\varphi'}$	Corrected excess temperature corresponding to the distance ξ_{θ} along the plume axis due to nonzero background temperature
ξ	Longitudinal distance measured along plume centerline
ξ_v	Distance from the virtual source of the jet to the end of the region of flow establishment
ξ_{θ}	Centerline distance out to a given excess temperature θ
ρ	Density of plume water (assumed constant)
φ	Background excess temperature

Notation

$()_m$	Denotes that the parameter inside the parentheses has been determined assuming dilution (mixing) alone
$()_{m,c}$	Denotes conditions after correction of the excess temperature distribution for surface cooling

3. Model Derivation and Discussion

In this first model of Pritchard, he separates dispersion into distinct vertical and horizontal processes to facilitate the model development. Each of the four steps described above is considered in some detail. First, the dominant processes involved in plume dispersion are described briefly as conceptualized by Pritchard.

Pritchard distinguishes between three primary processes, which lead to a decrease in excess temperature in a thermal plume. First, there is the initial dilution, which occurs through mechanical mixing of the heated water with the receiving water as a result of the excess momentum in the discharge stream. The second process involves natural mixing or turbulent diffusion of the heated effluent into the cooler ambient water. The third process is a loss of heat with a consequent reduction in excess temperature from the water body to the atmosphere.

According to Pritchard, jet entrainment is the primary dilution process within the range between the plume's initial excess temperature θ_0 and a value corresponding to about $0.2\theta_0$. This value is based mainly upon

experiments done at the Chesapeake Bay Institute, where it was found that, when the centerline temperature decays to about $0.2\theta_0$, the temperature distribution begins to act like the asymptotic temperature spread from a point source in a stagnant medium. Thus the temperature reduction beyond $0.2\theta_0$ decays essentially inversely with distance. Pritchard found this relationship to be generally valid in the field when ambient turbulence dominates the dispersion.

Due to the large volume of the source, natural turbulent diffusion is a secondary process in the jet regime, initially acting only at the edges of the plume with only little cooling until the transverse distribution of temperature has been altered from an initial "tophat" form to a bell-shaped form. Also in these initial regions, surface cooling contributed only a second-order correction to the distribution of excess temperature for temperatures greater than about 10% of the excess temperature at the discharge. Compared to the other two processes, surface heat loss seems to become increasingly more important when the excess temperature has decreased considerably. It then becomes the dominant process for excess temperatures less than about 1°F , due to the relatively large plume areas involved at these low excess temperatures. The above conclusions concerning the loss of heat to the atmosphere were reached from comparisons of experimental data (no surface losses) with actual field data obtained when surface heat exchange was significant.

Each step in the development of the Pritchard model (No. 1) will now be discussed individually.

a. Horizontal Dispersion

Pritchard considers horizontal dispersion (no vertical spreading or surface heat loss) due to jet entrainment and turbulent diffusion first. Vertical dilution is considered in Sec. b below as a correction. Surface heat loss is considered separately in Sec. d as a final correction.

The geometry illustrating horizontal spreading is shown in Fig. II.F.1. Pritchard assumes the spreading of the jet to be given by

$$b = \xi/n, \quad \xi > \xi_v = nb_0, \quad (1)$$

where b is the width of the jet, ξ is the distance measured normally along the jet centerline, n is the inverse spreading rate of the jet taken as 6, and ξ_v is the length of the flow-establishment region in which no spreading is assumed to occur and where the jet maintains its initially tophat profiles of velocity and temperature. For $\xi > \xi_v$, the jet begins

Fig. II.F.1. Horizontal Spreading of Jet
(mixing but no surface cooling)

to spread horizontally. For simplicity, Pritchard assumes that the excess temperature has a tophat profile laterally at each longitudinal position downstream with the realization that, in the established flow region, the profiles are more bell-shaped. Integral equations of momentum and heat-energy conservation are

$$\rho \int_{-b/2}^{b/2} u^2 dy = \rho u_0^2 b_0, \quad \xi > \xi_v, \quad (2)$$

and

$$\rho c_p \int_{-b/2}^{b/2} u \theta dy = \rho c_p u_0 \theta_0 b_0, \quad \xi > \xi_v. \quad (3)$$

Since $u = u(\xi)$, $\theta = \theta(\xi)$, for any position (ξ, y) within the jet, Eqs. 2 and 3 may be solved to yield

$$u(\xi) = u_0 \left(\frac{\xi_v}{\xi} \right)^{1/2} \quad (4)$$

and

$$\theta_\xi \equiv \theta(\xi) = \theta_0 \left(\frac{\xi_v}{\xi} \right)^{1/2}. \quad (5)$$

The equation of mass conservation is replaced essentially by Eq. 1, which gives the rate of spreading of the jet, or, equivalently, the rate of ambient fluid entrainment.

Pritchard's choice of $n = 6$ as the inverse spreading rate is based upon values found in the literature for the spreading of two-dimensional jets and upon plume experiments carried out by the Chesapeake Bay Institute. From the tank studies of Jen, Wiegel, and Mobarek,^{6,7} an inverse spreading rate of seven was obtained. Their study involved the horizontal discharge of a heated jet at the surface of a large body of water at uniform density with no bottom interferences. For that situation, the jet experienced three-dimensional entrainment. The inverse vertical spreading was also observed to be seven. The relationship between excess temperature and distance along the plume axis was

$$\frac{\theta}{\theta_0} = 7.0 \frac{D_0}{\xi}, \quad \xi/D_0 < 100, \quad (6)$$

where $D_0 = (1/\pi)\sqrt{h_0 b_0}$ for a rectangular slot orifice. Other experimenters (Refs. 8-10, for example) have found inverse spreading rates ranging from five to seven for various submerged and surface buoyant and nonbuoyant jets.

Pritchard states that when a bottom inhibits vertical mixing and/or when naturally existing thermal stratification is present, vertical mixing and vertical entrainment are generally inhibited to the point that entrainment appears to be primarily horizontal. For this reason, Pritchard assumes a two-dimensional jet. In the literature, the number $n = 6$ was found to be most prevalent for two-dimensional nonbuoyant jets. The spreading rate $n = 6$ was also seen in hydraulic model experiments at the Chesapeake Bay Institute. We show later that this two-dimensional assumption for plume dispersion yields more conservative results with respect to isotherm areas and centerline temperature decay than the assumption of three-dimensional dispersion.

The integral approach used above to derive the temperature distribution follows the technique first used by Albertson et al.¹¹ for submerged jets. Albertson et al. used Gaussian, rather than tophat, profiles and considered horizontal entrainment to occur right from the outfall rather than from the downstream longitudinal distance ξ_v .

The determination of $\xi_v = 6b_0$ comes from the choice of the inverse spreading rate $n = 6$ and from Pritchard's decision to allow the "virtual" source to coincide with the mouth of the orifice. In the literature, the point of intersection of the two diverging lines representing the spread of the jet defines the position of a "virtual" source. The length of the zone of flow establishment ξ_v would then be the distance from that virtual source to the point at which the two diverging lines become separated by a distance equal to the width of the actual orifice. At present, there is no clear agreement among experimenters as to the actual location of the virtual source. If the virtual source is located inside the actual orifice, part of the region of flow establishment would be within the outfall; if the diverging lines intersect outside the orifice, then there would be a longer region of flow establishment entirely within the receiving body of water. As stated, Pritchard assumes the virtual source to be coincident with the mouth of the actual orifice. Because the inverse spreading rate of the jet width is taken as six, $\xi_v = 6b_0$; the region of flow establishment will begin at the outfall and continue for six outfall widths downstream.

Clearly, the actual location of the virtual source is not critical. However, upon comparison of his model with data, Pritchard found that his model predicted higher temperatures than are generally observed near the outlet, indicating that there is entrainment closer to the source than he allowed for in his model. The discrepancy may be due in part, or in total, to the meandering of the jet as it issues from the outfall, creating difficulties in measuring the centerline temperature excess accurately, especially for a narrow outfall. The choice of the actual position of the virtual source and the neglecting of any spreading or entrainment in the flow-establishment region are not expected to lead to significant differences for temperatures at distances remote from the outfall.

The temperature distribution given by Eq. 5 is assumed to be valid for longitudinal distances ξ_θ where $0.2\theta_0 \leq \theta \leq \theta_0$. From Eq. 5,

$$\xi_\theta = \left(\frac{\theta_0}{\theta}\right)^2 nb_0, \quad 0.2\theta_0 \leq \theta \leq \theta_0. \quad (7)$$

Now, beyond the point where $\theta = 0.2\theta_0$, Pritchard assumes that dilution of the excess heat in the plume is primarily the result of natural mixing processes. In this region of the plume, the excess temperature along the centerline (uncorrected for surface cooling) would, as somewhat expected, decrease as the inverse first power of distance (natural diffusion from a continuous point source); thus,

$$\frac{\theta}{\theta_0} = 30b_0\left(\frac{1}{\xi}\right), \quad \theta \leq 0.2\theta_0. \quad (8)$$

The choice of the constant $30b_0$ (for $n = 6$) was made so that Eqs. 5 and 8 match for $\theta = 0.2\theta_0$. A tophat lateral distribution for temperature is assumed, again for simplicity, with the plume width defined by Eq. 1. In terms of θ , the distance ξ_θ may be written from Eq. 8 as

$$\xi_\theta = \frac{30b_0\theta_0}{\theta}. \quad (9)$$

Plume studies at Cape Kennedy¹² have shown that, in a truly three-dimensional situation, natural diffusion will cause an initial concentration to decay as ξ^{-2} ; for pseudo two-dimensional motion, the concentration drops as $\xi^{-5/2}$. Again the assumption of two-dimensional dispersion would yield conservative results for the temperature field.

Equations 1, 5, and 8 represent the two-dimensional temperature distribution considering horizontal dilution alone (no vertical mixing or surface cooling).

b. The Vertical-spreading Correction

A major assumption made by Pritchard is that for each body of water there is a "critical depth of mixing" peculiar to the natural processes of dispersion for that body. If the depth of the shoreline discharge (initial depth of the thermal plume) is less than this mixing depth, Pritchard states that the plume entrains vertically from its initial depth to that mixing depth. The plume depth then remains constant from that point on. Pritchard assumes a linear growth of plume depth during vertical entrainment (see Fig. II.F.2). If the initial depth is equal to or greater than the critical mixing depth, the plume thickness remains constant from the discharge exit to the far-field position, where the plume is sufficiently diluted to mix into the deeper layers of the lake or bay.

Fig. II.F.2

Vertical Spreading of Jet (mixing,
but no surface cooling)

Pritchard developed this plume-depth model upon examining data from large water bodies in the Midwest and east. He recommended that the critical depth be determined by examining data taken from existing thermal plumes on the lake or bay of interest. For Lake Michigan, Pritchard used 10 ft, mainly surmised from the data taken at the Waukegan Power Station¹³ situated on the lake. By examining plume data taken on the lake and other similar water bodies, Pritchard found that the excess heat is generally contained in a depth of between 10 and 15 ft. The plume temperature changes slowly with depth down to a point where it begins to decrease sharply to temperatures that are not measurably larger than ambient temperature.

Pritchard chose the critical depth as the midpoint of that region of rapid temperature decline. Thus, a plume with a critical depth of 10 ft may well contain measurable heat down to 15 ft. In general, according to Pritchard, natural stratification in a lake or bay combines with a sloping bottom to maintain a certain stability, which keeps the plume at its critical depth. Even for cases void of a great deal of stratification, Pritchard asserted that his field evidence indicated only a small variation in vertical thickness of the jet. If no evidence is available for a particular body of water, Pritchard suggested 10 ft for the critical depth. In an application to the Zion Nuclear Power Plant on Lake Michigan, where the outfall thickness is 3 ft, vertical entrainment was allowed until the jet occupied a layer 10 ft thick. No further vertical entrainment was assumed.

In an application to the Waukegan Power Station, a bar confined the initial jet thickness to about 5 ft for an offshore distance of about 500 ft. As the bottom dropped off at the end of the bar, Pritchard allowed for an additional 5-ft growth of the jet to a total 10-ft thickness. For Lake Erie, however, he used 6 ft for the critical depth in modeling the Davis-Besse Power Plant discharge because of previous plume studies on that lake and because at the plant the lake water is about 6 ft deep with a shelf-like bottom that does not increase in depth very rapidly.

These examples should indicate that each lake and outfall situation should be considered separately whenever possible. It will become apparent later that the value of the mixing depth chosen is not a critical parameter in the model, except where a small initial jet thickness requires vertical entrainment for its depth to reach that critical value.

Assuming an essentially constant depth plume and a two-dimensional temperature field is a conservative approach. If a three-dimensional or a quasi-two-dimensional distribution were assumed, a smaller area plume would result with a more rapid temperature decay with distance.

Once the critical mixing depth has been determined for a given outfall situation, vertical entrainment will then occur if $h_0 < D$, where D is the critical depth. When $h_0 \geq D$, the plume maintains a constant depth of h_0 , according to Pritchard.

Two situations are distinguished by Pritchard. For the first, such as for Waukegan (beyond the 500-ft bar) and for some shoreline conditions off the California coast, the bottom slopes off quite rapidly. From field evidence, Pritchard suggested the use of a linear growth rate for the vertical plume thickness of 0.2 ft for every 100 ft measured longitudinally along the plume centerline.

The other situation of a slowly sloping bottom is more difficult to handle and is treated theoretically as described below. As before, the plume thickness is assumed to grow linearly, but at a rate to be determined by the method of reflection. The distance between the outfall of depth h_0 and the first longitudinal position of the plume at which the plume thickness is the critical depth D is denoted η_1 . The quantity η_1 is calculated first. Both the

Fig. II.F.3. Reflection Method to Calculate η_1

water surface and the bottom act as reflecting boundaries for the vertical spread of the jet. For this derivation, Pritchard assumed a fairly small slope in the lake bottom, which may be approximated to be constant. He placed a pseudosource of height h_0 above the water surface to cancel the flux there and a second pseudosource at an equal distance below the lake bottom to cancel the flux of the surface sources at the bottom. The geometry of the source placements is shown in Fig. II.F.3. Assuming an inverse

spreading rate of six as in horizontal spreading, Pritchard calculated the superposition of the temperature fields of the actual source and the two pseudosources. The distance η_1 was calculated as that distance at which the superimposed temperature fields yield a temperature equivalent to completely mixing the actual source over a layer of depth D .

Pritchard superimposed his plumes in a unique manner. In general, the derivation is as follows: If

$$\theta_1 = \text{excess temperature at a given position resulting from one thermal plume}$$

and

θ_2 = excess temperature at the same position resulting from the other thermal plume,

and if θ_0 is the initial temperature at each of the two sources, then

$$\theta_1 = n_1\theta_0; \quad \theta_2 = n_2\theta_0, \quad (10)$$

where

n_1 = the fraction of the unit volume at the subject position which is heated water from the first source

and

n_2 = the fraction of the unit volume at the subject position which is heated water from the second plume.

Then

$1 - n_1$ = the fraction of the unit volume at the subject position that has zero excess heat under the conditions of the first source

and

$1 - n_2$ = the fraction of the unit volume at the subject position that has zero excess heat under the conditions of the second source.

The product

$(1 - n_1)(1 - n_2)$ = the fraction of the unit volume at the subject position with zero excess temperature under the conditions of superposition of the two plumes.

Thus, if the superposed excess temperature is θ' , then

$$\theta' = [1 - (1 - n_1)(1 - n_2)]\theta_0$$

or

$$\theta' = \theta_1 + \theta_2 - \frac{\theta_1\theta_2}{\theta_0}. \quad (11)$$

For large distances from each source, the superposition formula reduces approximately to the simple sum of the two plume temperatures, which is the case for the superposed plume from point sources. Using

this formula and the tophat horizontal temperature field derived in Sec. a above and applying it vertically, the condition on η_1 is given by

$$\frac{h_o}{D}\theta_o = \theta_o \left[\frac{6(2h_o)}{\eta_1} \right]^{1/2} + \theta_o \left[\frac{6(2h_o)}{\eta_1} \right]^{1/2} - \frac{\theta_o \left[\frac{6(2h_o)}{\eta_1} \right]^{1/2} \theta_o \left[\frac{6(2h_o)}{\eta_1} \right]^{1/2}}{\theta_o} \quad (12)$$

The solution of the above quadratic equation for η_1 is

$$\eta_1 = \frac{12h_o}{(1 - \sqrt{1 - h_o/D})^2} \quad (13)$$

Now, assuming that the jet thickens by vertical entrainment in a linear manner from h_o at the source to D at a distance η_1 , the vertical thickness D_θ of the plume at a point where the excess temperature is θ is

$$D_\theta = \begin{cases} h_o + \frac{D - h_o}{\eta_1} \xi_\theta, & \xi_\theta \leq \eta_1, \\ D, & \xi_\theta > \eta_1, \end{cases} \quad (14)$$

where ξ_θ is the distance along the axis of the plume to the excess temperature θ . If the plume is assumed to be uniformly mixed at each cross section, the dilution ratio of the plume, which is also spreading by horizontal entrainment, is assumed to be augmented by the factor h_o/D_θ .

Once the dilution ratio h_o/D_θ is determined, the two-dimensional temperature distribution obtained for horizontal entrainment and mixing only

$$\xi_\theta = \begin{cases} 6b_o \left(\frac{\theta_o}{\theta} \right)^2, & \theta \geq 0.20\theta_o, \\ 30b_o \frac{\theta_o}{\theta}, & \theta < 0.20\theta_o, \end{cases} \quad (15)$$

can now be corrected. For each θ used to compute ξ_θ , a corrected value $\bar{\theta}$ is found from

$$\bar{\theta} = \frac{h_o}{D_\theta} \theta. \quad (16)$$

The table of values for $\bar{\theta}$ versus ξ_0 is interpolated to find the corrected distance along the plume axis to the excess temperature θ . The corrected values for each θ now represent the two-dimensional temperature field due to horizontal and vertical mixing, neglecting surface cooling.

A few comments need to be made here concerning the correction for vertical mixing. First, the reflection method used to calculate η_1 removes heat flux from the bottom surface by the placement of the two pseudosources, but does not remove the heat flux at the surface from the bottom pseudosource. The heat flux can be removed from the surface and the bottom only by the usual technique of positioning an infinite number of sources above the surface and below the lake bottom at equal distances from one another. We surmise that Pritchard assumed the contribution of these terms to be of second order and therefore negligible. The use of an inverse spreading rate of six for the vertical spreading is assumed here. For three-dimensional diffusion at the surface of a large body of water, Wiegand, Mobarek, and Jen⁷ found an inverse vertical spreading rate of seven with no bottom interference. Pritchard thus assumed that his jet entrained vertically at the same rate it entrained laterally. Concerning his general method of superposition of overlapping thermal plumes, the method is purely analytical. It was not derived with any experimental data at hand, nor has it since been compared with any field or hydraulic data. Until that is done, the method must be considered unverified. The above superposition technique was used by Pritchard in his analysis and prediction of the temperature distribution resulting from the Zion discharge,¹³ where two offshore discharges have been constructed which yield intersecting plumes.

Pritchard's model for vertical diffusion is approximate at best. He does not have a three-dimensional model that includes a diffusion term or a vertical-entrainment term that results in a solution for the vertical thickness. He asserts, however, that no three-dimensional model now available can predict vertical plume thickness that matches his observational data. Any accurate model of vertical spreading should involve some assessment of the effects of vertical shear in the velocity field and vertical density gradients that control vertical entrainment.

c. Background Temperature Correction

As previously discussed, the model implicitly assumes that the receiving body of water is an infinite sink from the standpoint of dilution and surface cooling. In some practical situations, the lake or bay is not sufficiently large or the natural exchange mechanisms within the body do not adequately supply a sufficient quantity of "new" dilution water to the inshore region at which the plant is sited. Under such circumstances, Pritchard determined a "background temperature" and considered the entraining water to have some excess temperature. He estimated "background temperature" by calculating the excess temperature that would exist if the heated effluent were

completely mixed into the available rate of supply of diluting water to the in-shore area where the heated effluent is being discharged.

For a discharge into a unidirectional flowing stream such as a river, the available supply of dilution water would simply be the volumetric flow rate of the river. In an estuary, the ocean and river inflow provide two sources of new water. For an estuary system, Pritchard used a mathematical model of circulation, with the known distribution of salinity as a conservative tracer, to compute how much new water was available. The background temperature there was essentially the excess temperature that was derived from assuming that the total heat flux of the plant was mixed into the available new river and ocean water. The value obtained was then corrected for surface cooling. Pritchard used this procedure for the Surry Plant discharge on the James River⁵ and the Calvert Cliffs Nuclear Power Plant on the Chesapeake Bay.⁴ At Calvert Cliffs, for example, Pritchard found that the total volumetric flow of new water was about 90,000 cfs throughout the year, due to the relative balancing between the river outflow and sea water coming in upstream at different times of the year. From that discharge, the excess temperature of the entraining water was calculated as 0.6 F° from mixing alone and adjusted to 0.4 F° by allowing for surface cooling over the area of the tidal segment. This latter value was used as the background temperature of the new water mixed into the plume for that site.

For a lake or bay situation, the calculation is not so straightforward. Short of building a hydraulic model or undertaking an extensive field program, Pritchard knew of no general method for calculating or estimating the background temperature. At the Davis-Besse Nuclear Power Station on Lake Erie, recirculation of heated water was expected, due to the presence of islands and shoals found in the relatively sheltered and limited western basin of the lake where the heated discharge would enter. There, Pritchard estimated the background excess temperature as 0.5 F°, based upon what he concluded to be the volume rate of flow of water through the entire western basin of that lake and the rate of heat discharge of the power plant. For power plants on Lake Michigan, Pritchard recommended no background temperature correction, due to the open and fairly straight shoreline and the expected abundant supply of dilution water predicted from hydraulic-model tests and from the examination of lake circulation and bottom topography data. Although no method for estimating the background temperature was singly recommended by Pritchard, an estimate should, however, be made when required. Once the background temperature is determined, the two-dimensional temperature distribution is corrected as follows. For a given ξ_0 , θ is replaced by

$$\theta'_{\varphi} = \left(\frac{\theta}{\theta_0}\right)\theta_0 + \left(1 - \frac{\theta}{\theta_0}\right)\varphi, \quad (17)$$

where

φ = background temperature (above ambient),

$\frac{\theta}{\theta_0}$ = fraction of mixture that is condenser cooling water,

$1 - \frac{\theta}{\theta_0}$ = fraction of mixture that is diluting water,

and

θ'_φ = corrected excess temperature corresponding to the distance ξ_θ along the plume axis.

For a fixed θ , a corrected distance corresponding to θ may be determined by interpolating values of ξ_θ .

With corrections made for possible vertical entrainment and background temperature, the areas within isotherms (mixing alone, no cooling) may now be calculated. The theoretical width of the tophat distribution was assumed to have the form of Eq. 1. Because the actual lateral distribution of temperature is more bell-shaped, Pritchard used some empirical results in his calculation of areas within isotherms. After examining a great deal of experimental data, both from physical modeling and field measurements, Pritchard developed an empirical formula for calculating areas within isotherms and a series of guidelines for the schematic representation of the horizontal distribution of excess temperature in the thermal plume (see Fig. II.F.4).

Fig. II.F.4. Surface Isotherm as Conceptualized by Pritchard Model. ANL Neg. No. 190-1167.

(1) The maximum width of the area having an excess temperature equal to or greater than θ is

$$\bar{b}_\theta = \frac{1}{4} \xi_\theta. \quad (18)$$

The area within the θ isotherm, A_θ , is

$$A_\theta = 0.86 \bar{b}_\theta \xi_\theta = 0.215 \xi_\theta^2. \quad (19)$$

(2) The region within a given isotherm θ increases smoothly in width from the outfall, attaining a maximum width $\bar{b}_\theta$ at a distance along the plume axis equal to $0.33\xi_\theta$.

(3) The region defined by isotherm θ has a constant width $\bar{b}_\theta$ from longitudinal distance $0.33\xi_\theta$ to the point $0.75\xi_\theta$.

(4) The shape of the isotherm between longitudinal distance $0.75\xi_\theta$ and the maximum distance ξ_θ is "tongue"- or "bullet"-shaped.

These approximate formulas and guidelines were developed from examination of the results of hydraulic studies at the Chesapeake Bay Institute, from model studies done for the Pilgrim Nuclear Power Station on Cape Cod Bay, and from data taken at small existing plants in the Chesapeake Bay region such as the Chalk Point Plant and the Chesterfield Plant on the James River. The hydraulic modeling done at Vicksburg and the experimental data obtained from the Waukegan Plumes¹³ were also included. Pritchard found that these plume relationships were approximately valid, even for a non-negligible cooling coefficient, because the effect of surface heat transfer seemed to produce a nearly uniform shrinking in plume dimensions and areas.

d. The Surface-cooling Correction

To correct the areas within isotherms calculated from mixing processes above, Pritchard defined

$$A_{\theta_n} = \text{area of the plume inside isotherm } \theta_n \text{ (i.e., area with excess temperature } > \theta_n \text{)}.$$

Then

$$\Delta A_{\bar{\theta}_n} = A_{\theta_n} - A_{\theta_{n-1}} = \text{area between two adjacent isotherms,}$$

where

$$\bar{\theta}_n = \frac{\theta_n + \theta_{n-1}}{2} = \text{mean excess temperature for that incremental area.}$$

The rate of loss of excess heat from the area between two isotherms is given by

$$(\Delta \Gamma_{\bar{\theta}_n})_m = K \bar{\theta}_n (\Delta A_{\bar{\theta}_n})_m \quad (20)$$

or

$$(\Delta \Gamma_{\bar{\theta}_n})_{m,c} = K \bar{\theta}_n (\Delta A_{\bar{\theta}_n})_{m,c}, \quad (21)$$

where the subscript m refers to the value of the parameter determined from mixing only and m,c refers to the parameter value after the correction for surface cooling has been made. The quantities $(\Delta\Gamma_{\theta_n})_m$ are known and are used to derive $(\Delta\Gamma_{\theta_n})_{m,c}$ and $(\Delta A_{\theta_n})_{m,c}$.

Pritchard considers the rate of excess heat loss to the atmosphere as a decrease in source strength of the heated effluent. Thus heated water distributed at a given isotherm responds as if the cooling process were equivalent to reducing the source strength by the amount of heat that has been lost between the source and that isoline of excess temperature. This is represented by

$$(\Delta A_{\theta_n})_{m,c} = (\Delta A_{\theta_n})_m \left[\frac{Q_h - \sum_{i=1}^n (\Delta\Gamma_{\theta_i})_{m,c}}{Q_h} \right], \quad (22)$$

where the numerator of the bracketed term represents the effective source strength applicable to the θ_n isotherm. The bracketed correction factors are illustrated in Fig. II.F.5 for the first four isotherms. Using the relationship

$$(\Delta\Gamma_{\theta_n})_{m,c} = \frac{(\Delta A_{\theta_n})_{m,c}}{(\Delta A_{\theta_n})_m} (\Delta\Gamma_{\theta_n})_m, \quad (23)$$

Fig. II.F.5

Surface-area Correction due to Heat Loss to the Atmosphere

$$\begin{aligned} (\Delta A_1)_{m,c} &= (\Delta A_1)_m \left[\frac{Q_h - (\Delta\Gamma_1)_{m,c}}{Q_h} \right] \\ (\Delta A_2)_{m,c} &= (\Delta A_2)_m \left[\frac{Q_h - (\Delta\Gamma_1)_{m,c} - (\Delta\Gamma_2)_{m,c}}{Q_h} \right] \\ (\Delta A_3)_{m,c} &= (\Delta A_3)_m \left[\frac{Q_h - (\Delta\Gamma_1)_{m,c} - (\Delta\Gamma_2)_{m,c} - (\Delta\Gamma_3)_{m,c}}{Q_h} \right] \\ (\Delta A_4)_{m,c} &= (\Delta A_4)_m \left[\frac{Q_h - (\Delta\Gamma_1)_{m,c} - (\Delta\Gamma_2)_{m,c} - (\Delta\Gamma_3)_{m,c} - (\Delta\Gamma_4)_{m,c}}{Q_h} \right] \\ (\Delta\Gamma_i)_{m,c} &= \frac{(\Delta A_i)_{m,c}}{(\Delta A_i)_m} (\Delta\Gamma_i)_m, \quad i = 1, \dots, 4. \end{aligned}$$

derived from Eqs. 20 and 21, the simultaneous linear equation set 22 may be solved to yield

$$(\Delta A_{\theta_n})_{m,c} = (\Delta A_{\theta_n})_m \frac{Q_h - \sum_{i=1}^{n-1} (\Delta \Gamma_{\theta_i})_{m,c}}{Q_h + (\Delta \Gamma_{\theta_n})_m} \quad (24)$$

Equation 24 is applied in steps starting with $n = 1$. Here the summation term is zero, and hence the corrected incremental area for $n = 1$ is obtained immediately from Eq. 24 using $(\Delta \Gamma_{\theta_1})_m$ computed from Eq. 20.

The corrected rate of heat loss within that innermost isotherm is calculated from Eq. 23 setting $n = 1$. Now, for $n = 2$, the value of $(\Delta \Gamma_{\theta_1})_{m,c}$ is used in the summation term of Eq. 22 to allow the computation of the incremental area bounded between the first two isotherms. Equation 23 is again used to calculate $(\Delta \Gamma_{\theta_2})_{m,c}$. The first two corrected heat-loss terms are used in Eq. 22 to yield $(\Delta A_{\theta_3})_{m,c}$, from which the corresponding heat-loss term is calculated from Eq. 23. In this manner, all corrected incremental areas and heat-loss terms are calculated. Once this is accomplished, the total corrected areas $(A_{\theta_n})_{m,c}$ may be calculated by summing the incremental areas

$$(A_{\theta_n})_{m,c} = \sum_{i=1}^n (\Delta A_{\theta_i})_{m,c}$$

Using Eqs. 18 and 19 for the surface-cooling corrected parameters $(A_{\theta_n})_{m,c}$, $(\xi_{\theta})_{m,c}$, and $(\bar{b}_{\theta_n})_{m,c}$ yields

$$(\xi_{\theta_n})_{m,c} = (\xi_{\theta_n})_m \left[\frac{(A_{\theta_n})_{m,c}}{(A_{\theta_n})_m} \right]^{1/2} \quad (25)$$

and

$$(\bar{b}_{\theta_n})_{m,c} = (\bar{b}_{\theta_n})_m \left[\frac{(A_{\theta_n})_{m,c}}{(A_{\theta_n})_m} \right]^{1/2} \quad (26)$$

From these relationships, the final areas within isotherms and the final two-dimensional tophat temperature distribution are determined.

Reflecting upon the argument just presented for the surface-cooling correction to the areas within isotherms, it would seem that the relationship between uncorrected and corrected isotherm areas would more accurately be written

$$(\Delta A_{\bar{\theta}_n})_{m,c} = (\Delta A_{\bar{\theta}_n})_m \left[\frac{Q_h - \sum_{i=1}^n (\Delta \Gamma_{\bar{\theta}_i})_{m,c}}{Q_h} \right]^2,$$

rather than as postulated by Pritchard in Eq. 22. The temperature excess at a given distance is directly proportional to the source strength. Consequently, the area (length squared) within a given isotherm should be proportional to the square of the source strength. At this point, it would seem that this may be a correction that should be applied to Pritchard's model. This modification of the model increases the cooling correction; as it stands, the model is conservative in this respect.

In general, for lake situations Pritchard carried out his predictions based upon the above model to excess temperatures equal to about 1 F° above ambient, because one can seldom determine the ambient temperature distribution to within 0.5 F°. Consequently, any predictions of the temperature distribution would be purely speculative. In fact, temperatures in the natural environment can vary several degrees in space and in time, and to select the ambient temperature within 0.5 F° from the field data would be extremely difficult. A purely theoretical estimate of the area having a mean excess temperature of 0.5 F° can be computed from

$$\Gamma(1^\circ) = \sum_{i=1}^n (\Delta \Gamma_{\bar{\theta}_i})_{m,c} = \sum_{i=1}^n K \bar{\theta}_i (\Delta A_{\bar{\theta}_i})_{m,c}$$

and

$$Q_h - \Gamma(1^\circ) = K(0.5 \text{ F}^\circ) 0.5 \Delta A_{0.5^\circ},$$

where

$\Gamma(1^\circ)$ = the total heat lost to the atmosphere from areas having excess temperature equal to or greater than 1 F°,

$Q_h - \Gamma(1^\circ)$ = amount of surface heat loss from areas having excess temperatures less than 1 F°,

and

$\Delta A_{0.5^\circ}$ = the area having mean excess temperature of 0.5 F°.

The above analysis assumes that the flux of excess heat from the water to the atmosphere must ultimately be equal to the flux of heat rejected to the lake in the heated discharge. In the above analysis, the heat beyond the 1 F° isotherm is thought of as being lost entirely at the 0.5 F° isotherm. The area beyond the 1 F° isotherm is, in reality, infinite for a semi-infinite lake acting as heat sink. The quantity $\Delta A_{0.5}$ is thus only a fictional quantity, defined by the above equation, to assess the relative size of the area influenced by the heated discharge beyond the 1 F° isotherm.

The procedure for model calculations is summarized in Table II.F.1. A computer code for the model was developed at Argonne National Laboratory.

TABLE II.F.1. Summary of Calculations for Pritchard Model No. 1

A. Horizontal Dispersion

Horizontal spreading due to lateral entrainment and turbulent diffusion is treated by the use of an inverse spreading rate, n , as follows

$$b/b_0 = u/u_0 = \theta/\theta_0 = 1 \quad 0 < \zeta < 1$$

$$\left. \begin{aligned} b/b_0 &= \zeta \\ u/u_0 &= \zeta^{-1/2} \\ \theta/\theta_0 &= \zeta^{-1/2} \end{aligned} \right\} \quad 1 < \zeta < 25$$

$$\theta/\theta_0 = 5\zeta^{-1} \quad 25 < \zeta,$$

where

$$\zeta = \xi/(nb_0),$$

ξ = distance along centerline from outfall,

b = lateral width,

u = velocity,

θ = temperature excess,

h_0 = depth of outfall,

b_0 = width of the outfall,

u_0 = outfall velocity,

and

θ_0 = outfall temperature excess.

The value of n is normally taken to be about 6, although Pritchard stated that it may be as low as 4 or as large as 8.

B. Vertical-spreading Correction

Each receiving system is characterized by a critical mixing depth, D . If the depth of the shoreline discharge, h_0 , is less than this mixing depth, the plume entrains vertically until that depth is reached. Plume thickness grows linearly during vertical entrainment and remains constant after entrainment ceases. If the initial depth is greater than or equal to the critical depth, plume depth remains constant from the point of discharge to the far-field position where the plume is sufficiently diluted to mix into the deeper layers of the lake. The two-dimensional temperature distribution determined from the assumptions in A must be corrected for vertical dilution as follows:

TABLE II.F.1. (Contd.)

1. Find plume depth, D'

$$\frac{D'}{h_0} = 1 \quad h_0 \geq D$$

$$\frac{D'}{h_0} = 1 + \frac{D/h_0 - 1}{\eta_1} \left(\frac{\xi}{h_0} \right) \quad \begin{cases} h_0 < D \\ \xi/h_0 < \eta_1 \end{cases}$$

$$\frac{D'}{h_0} = \frac{D}{h_0} \quad \begin{cases} h_0 < D \\ \xi/h_0 \geq \eta_1, \end{cases}$$

where

$$\eta_1 = \frac{12}{(1 - \sqrt{1 - h_0/D})^2}.$$

2. Find temperature θ from A.
3. Find corrected temperature, $\bar{\theta}$

$$\bar{\theta} = \frac{\theta}{D'/h_0}.$$

C. Background Temperature Correction

During jet entrainment, warmer-than-ambient far-field plume water may be used for mixing. This effect may be accounted for by the use of a background temperature excess, ϕ . The correction, which is normally small, must be determined independently of the model. Once its value is established, the temperatures are corrected as follows:

$$\theta_\phi = \left(\frac{\bar{\theta}}{\bar{\theta}_0} \right) \theta_0 + \left(1 - \frac{\bar{\theta}}{\bar{\theta}_0} \right) \phi$$

Once the temperature decay along the centerline has been determined as in A, B, and the above part of C, isotherms are constructed by the following procedure. An isotherm is shown schematically in Fig. II.F.4 and explained below.

1. Find ξ_θ , the centerline distance to excess temperature θ .
2. Compute $\bar{b}_\theta$, the maximum width of isotherm θ , from

$$\bar{b}_\theta = 0.25\xi_\theta.$$

The width of the isotherm increases smoothly from the outfall attaining its maximum width $\bar{b}_\theta$ at a distance of $0.33\xi_\theta$. The shape of the isotherm between longitudinal distance $0.75\xi_\theta$ and the maximum distance ξ_θ is "tongue"- or "bullet"-shaped.

3. Based on the above isotherm shape, the area within the isotherm is given approximately by

$$A_\theta = 0.86\bar{b}_\theta\xi_\theta = 0.215\xi_\theta^2.$$

D. The Surface-cooling Correction

The areas computed in C must be reduced due to surface heat loss. The philosophy behind Pritchard's procedure for accomplishing this is that surface heat loss effectively reduces the heat rejection rate of the plant, Q_h . The procedure is:

1. Select a decreasing sequence of isotherms $\theta_0, \theta_1, \dots, \theta_k$. A difference of about 0.2°F between successive values is appropriate.

2. Compute the areas $(A_0)_m, (A_1)_m, (A_2)_m, \dots, (A_k)_m$, within these isotherms using C.

TABLE II.F.1. (Contd.)

3. Compute the change in the effective heat source, $\Delta\Gamma_n$, by

$$\Delta\Gamma_n = K\bar{\theta}_n(\Delta A_n)_m, \quad n = 1, 2, \dots, k,$$

where

K = heat-loss coefficient,

$$\bar{\theta}_n = \text{mean temperature of the area } \left(\frac{\theta_n + \theta_{n-1}}{2} \right),$$

and

$$(\Delta A_n)_m = \text{area between isotherms } [(A_n)_m - (A_{n-1})_m].$$

4. Compute the reduced incremental area for the first isotherm using

$$(\Delta A_1)_{m,c} = (\Delta A_1)_m \frac{Q_h}{Q_h + \Delta\Gamma_1}.$$

5. Compute the reduced incremental area for the second through the k th isotherm by the following recursive procedure:

$$(\Delta\Gamma_{n-1})_{m,c} = K\bar{\theta}_{n-1}(\Delta A_{n-1})_{m,c},$$

$$(\Delta A_n)_{m,c} = (\Delta A_n)_m \frac{Q_h - \sum_{i=1}^{n-1} (\Delta\Gamma_i)_{m,c}}{Q_h + \Delta\Gamma_n}.$$

6. The reduced areas are found by summing the incremental areas.

$$(A_n)_{m,c} = \sum_{i=0}^n (\Delta A_i)_{m,c}.$$

7. The centerline distance ξ_θ to isotherm θ and the maximum width $\bar{b}_\theta$ of isotherm θ must be corrected for the effects of surface cooling. Reversing the formulas for area within isotherm, one obtains

$$\frac{(\xi_\theta)_{m,c}}{(\xi_\theta)_m} = \frac{(\bar{b}_\theta)_{m,c}}{(\bar{b}_\theta)_m} = \sqrt{\frac{(A_\theta)_{m,c}}{(A_\theta)_m}}.$$

4. Criticisms and Limitations

a. Limited Experience with the Model

Despite the five years or so that the model has been available for use, it has been compared with data from only a few power plants. The verification work done with the Point Beach, Palisades, and Pilgrim plants is reviewed in Ch. IV of this report. In each case, the model was superior to the others tested for the limited verification done at each site. The state of the art is still such that no model seems to give more than general estimates of plume characteristics. A few points cloud the apparent "success" of the Pritchard model.

(1) The sites at which the model was tested do not represent a wide range of outfall and environmental conditions. The Point Beach and

Palisades sites represent plants with low initial densimetric Froude numbers and significant bottom interaction. The low-tide case at Pilgrim has a high initial densimetric Froude number and significant bottom effects; the high-tide case has a low initial densimetric Froude number with minimal bottom interference. Moreover, both situations at Pilgrim are under the effects of a transient tidal current. One should expect similar model-data comparisons only if the new site is similar in outfall and environmental conditions to the situations already tested. Use of the model in situations differing widely from those analyzed (say, a case with large vertical entrainment expected) is presently uncertain.

(2) As stated above, the Pritchard model (No. 1) is an engineering-judgment type model in which the rules given for plume calculation are essentially guidelines for application. Use of the model in a particular application is most reliably done after consultation with Pritchard himself.

(3) The model has its limitations in terms of its empirical nature. The "success" of the model is most probably due to the careful calibration of its integral skeleton to selected laboratory and field data. A tremendous amount of data has not been used in the development of the model, and it is uncertain whether the integral skeleton is sufficiently based upon physical principles to allow successful predictions over wide ranges of parameters. As an example, buoyancy is neglected in the direct development of the model. Pritchard therefore recommended that the model be used only for initial densimetric Froude numbers greater than two. In addition, the model does not treat the effects of a shallow bottom or wind. Our point here is that the model is too simple to be all-encompassing. Also, some of the major complications that may well be important in certain field cases are not included. The model is basically an intuitive one and should be regarded that way.

b. Restriction to Small Ambient Currents

Because the model ignores crossflow interaction, it is limited to nearly stagnant situations. Pritchard suggested that his correlations were generally applicable for crosscurrents less than about 10% of the outfall velocity. Pritchard's model implicitly assumes zero ambient current. If, however, the shore-parallel current is sufficiently large compared to the initial jet velocity, a number of complexities arise that cannot be handled by Pritchard's model. First, the trajectory of the jet is bent over into a direction parallel to the current. Second, the entrainment and consequent dilution may be significantly reduced on the inshore side of the plume. Application of the model to the case is expected to yield significantly smaller areas and centerline temperatures than would actually occur. Carter¹⁴ and Pritchard concluded from experimental evidence in the laboratory and in the field that, for an ambient current velocity greater than about 10% of the initial jet velocity, these complexities are significant enough to require a new analysis. Even with a small ambient current present ($u_a < 0.1u_0$), the model does not predict the plume trajectory.

c. No Dependence on Densimetric Froude Number

Buoyancy effects are not directly modeled. There is no dependence of the temperature distribution on the initial densimetric Froude number. If the initial discharge velocity is not high, the additional lateral spreading due to buoyant convective motions would be expected to distort the simple linear spread assumed in the model. In fact, the model does not directly consider the outfall velocity in its calculations, because of the implicit assumption of a well-defined jet region. Therefore, as stated above, Pritchard suggested that his model be applied only to outfalls with initial densimetric Froude numbers greater than about 2. This mitigates, only somewhat, the omission of buoyancy-induced currents. Also, with lower initial densimetric Froude numbers, the lateral temperature distribution at a given longitudinal position would not really be tophat; it would be more bell-shaped and, vertically, probably somewhat J-shaped.

d. Approximate Treatment of Vertical Entrainment

Pritchard's treatment of vertical entrainment is heuristic in nature. The critical mixing depth is only qualitatively known and difficult to know a priori. Pritchard suggested a value of 10-15 ft for large lakes. Furthermore, the effects of buoyancy on vertical entrainment are not considered.

e. Approximate Treatment of Recirculation

The analysis of recirculation in the model relies heavily on the judgment of the user, because the specification of a site-dependent background temperature is required. The complexities that far-field effects and boundaries may have on the near field are neglected (see Ch. III).

f. Limited Data Base of the Model

As with any phenomenological model, to know upon what data the correlations are based is essential. The above model guidelines were developed from examination of the results from hydraulic model studies performed at the Chesapeake Bay Institute, from model studies made for the Pilgrim Nuclear Power Station by MIT, and from field data taken at the Waukegan, Chalk Point, and Chesterfield Plants. Of these plants, only Waukegan is located on a large lake. These data are necessarily limited in scope, restricting the range of problems that one has hopes of predicting.

The algorithmic nature of Pritchard's analysis lends itself to easy programming, although Pritchard himself normally prefers hand calculations. As noted above, the Energy and Environmental Systems Division of ANL has written a computer code based upon the above guidelines to perform the many repeated calculations necessary to obtain centerline temperature decay, plume widths, and areas within isotherms. A sample output of this code appears in Appendix A.

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G. Pritchard (Model No. 2) Phenomenological Model¹

1. Synopsis of Approach

In developing his second model, Pritchard retains the same basic idealization of the thermal plume used in his first model. Surface isotherms are assumed to be roughly bullet-shaped and the plume is assumed to grow linearly in depth from the point of discharge until a certain critical mixing depth is reached. Empirical correlations are provided to compute the critical depth and linear growth rate in terms of outfall characteristics. Building on a broader collection of hydraulic model studies and field measurements, the model includes crossflow and bottom effects.

The model is based on Pritchard's observation that, when the temperature ratio is plotted versus centerline position on a log-log scale, the resulting pattern of data may be closely approximated by a series of connected straight line segments. Pritchard further observed that the slopes of the segments depend on whether the plume is interacting with the bottom of the lake or river as well as on outfall and ambient conditions. He has attempted to correlate these slopes with certain physically measurable parameters.

Pritchard further observed that a plot of the maximum isotherm width versus centerline position on a linear scale can also be approximated by connected line segments. He has also derived correlations for these slopes in terms of measurable parameters. In some cases, the correlations of the model can be written in an explicit, computable form, though in other cases numerical values must be obtained from a table.

When the length and maximum width of a given isotherm have been computed using the appropriate correlation, areas are calculated and then corrected for surface cooling following essentially the same procedure as given for Pritchard's first model. The application of the model becomes clearer as the algorithm is discussed.

2. List of Symbols

A_n	Area within a specific isotherm of excess temperature θ_n
A_{nC}	Corrected area within a specific isotherm of excess temperature θ_n
A_r	Aspect ratio; h_o/b_o
A_θ	Area within isotherm of excess temperature θ
$A_{\theta C}$	Corrected area within isotherm of excess temperature θ
b_o	Width of outfall

b_{θ}	Maximum width of isotherm of excess temperature θ
$b_{\theta C}$	Corrected maximum width of isotherm of excess temperature θ
D_{wo}	Depth of water at point of discharge
dh/dx	Rate of vertical plume growth
IF_o	Initial densimetric Froude number
h_o	Depth of outfall
$h_{1/2}$	Critical mixing depth
m_{BL}	Rate of lateral growth when bent and depth-limited
m_{BU}	Rate of lateral growth when bent but not depth-limited
m_L	Rate of lateral growth when depth-limited
m_U	Rate of lateral growth when not depth-limited
n_{BL}	Spreading factor when bent and depth-limited
n_{BU}	Spreading factor when bent and not depth-limited
n_L	Spreading factor when depth-limited
n_U	Spreading factor when not depth-limited
Q_H	Heat rejection rate
q	Local jet velocity
q_o	Discharge velocity
S_x	Bottom slope
S_{θ}	Length of isotherm of excess temperature θ
$S_{\theta C}$	Corrected length of isotherm of excess temperature θ
R	Ratio of discharge velocity to ambient current; q_o/U_a
U_a	Ambient current
x	Centerline distance along jet
α	Angle between jet centerline and ambient current direction
β_{BL}	Decay rate when bent and depth-limited
β_{BU}	Decay rate when bent but not depth-limited
β_L	Decay rate when depth-limited
β_U	Decay rate when not depth-limited
ΔA_n	Incremental change in area between isotherms of excess temperatures θ_n and θ_{n-1}

ΔA_{nC}	Corrected incremental change in area between isotherms of excess temperatures θ_n and θ_{n-1}
$\Delta \Gamma_n$	Incremental change in effective heat source
$\Delta \Gamma_{nC}$	Corrected incremental change in effective heat source
ξ_1, ξ_2	Transition points
θ	Temperature excess
θ_n	Temperature excess of a specific isotherm
$\bar{\theta}_n$	Mean temperature excess of isotherms of excess temperatures θ_n and θ_{n-1}
θ_0	Initial temperature excess
$\theta_1, \theta_2, \theta_3, \theta_t$	Transition temperatures

3. Basic Assumptions and Model Development

The first step in preparing model predictions is to determine the length and width of individual isotherms of a particular temperature excess.

a. The plume is characterized by a critical mixing depth $h_{1/2}$, which is related to initial discharge parameters through the correlation given in Table II.G.1.

b. If $h_{1/2} > h_0$, the depth of the outfall, the half-depth of the plume will grow from h_0 at the outfall until reaching $h_{1/2}$ some distance downcurrent. The rate of vertical growth dh/dx is given by another correlation also appearing in Table II.G.1. In the case where $h_{1/2} < h_0$, it is assumed that the plume attains the half-depth $h_{1/2}$ within the zone of flow establishment. Discharges into flowing ambient systems require an analysis separate from that of discharges into stagnant systems. Consider first the case of an undeflected jet.

Undeflected Jet

c. Whenever the depth of the water is less than 1.5 times the depth of the plume, interaction with the bottom will occur. Based on this rather simple principle, Pritchard has developed a complicated set of criteria to determine over what distances bottom interaction occurs. To simplify the analysis, the receiving water body is idealized as having depth D_{w0} at the point of discharge and a smooth bottom sloping away at rate S_x .

(1) If $D_{w0} > 1.5h_{1/2}$, there is no possibility of bottom interaction, since the water is always deeper than the maximum depth of the plume.

(2) If $D_{w0} > 1.5h_0$ and $S_x \geq dh/dx$, there is no bottom interaction since the bottom slopes away faster than the depth of the plume grows.

TABLE II.G.1. Correlations for Pritchard Model No. 2

Basic Definitions:

$$IF_0 = q_0 / [g h_0 (\Delta\rho/\rho)]^{1/2}$$

$$Ar = h_0/b_c$$

$$R = q_0/U_a$$

$$\beta' = 1 + (3.584/IF_0) + (6.524/IF_0^2) + (5.84/IF_0^3)$$

$$f_1 = \begin{cases} 3/(IF_0 + 15) & Ar < 1 \\ 1.5/(IF_0 + 15) & Ar > 1 \end{cases}$$

Critical Mixing Depth

$$h_{1/2} = 0.3 \sqrt{h_0 b_0} (Ar)^{1/4} IF_0^{3/4}$$

Rate of Vertical Growth

$$\frac{dh}{dx} = 0.4 (Ar)^{1/8} \exp(-6.4 IF_0^{-0.4})$$

a. Unbent, but Not Depth-Limited Plume

$$n_U = 7.57 - 3.638 IF_0^{-0.245}$$

$$m_U = n_U^{-1} [1.152 + 3.945 \exp(-0.1 IF_0) - 2.999 \exp(-0.2 IF_0) - 1.933 \exp(-0.3 IF_0)]$$

$$\beta_U = \beta' (Ar)^{-f_1}$$

b. Unbent, Depth-Limited Plume

$$n_L = n_U \{1 - 0.340 [1 - \exp(-0.0135/S_X)] [1 - IF_0^{-1}]\}$$

$$m_L = m_U \{0.4 + 0.6 \exp(-0.0196/S_X)\}$$

$$\beta_L = \beta_U \{[1 - 1.15 \exp(-0.0135/S_X)] [1 - IF_0^{-1}] + 1\}$$

c. Bent, but Not Depth-Limited Plume

$$n_{BU} = n_U [1 - 0.333 \exp(-0.05 R)]$$

$$m_{BU} = m_U [1 - 0.333 \exp(-0.05 R)]$$

$$\beta_{BU} = \begin{cases} 4 & R \leq 6.5 \\ \beta' \cdot (\text{value interpolated from table below}) & R > 6.5 \end{cases}$$

	R							
	7.0	8.0	9.0	10.0	12.0	15.0	20.0	∞
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.5	1.726	1.680	1.565	1.495	1.426	1.346	1.207	1.000
2.0	1.940	1.781	1.602	1.533	1.450	1.371	1.219	1.000
3.0	2.137	1.838	1.640	1.565	1.480	1.395	1.232	1.000
4.0	2.249	1.874	1.659	1.591	1.498	1.408	1.244	1.000
5.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
6.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
7.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
8.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
9.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
10.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
12.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
15.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
20.0	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000
∞	2.300	1.910	1.690	1.610	1.510	1.420	1.250	1.000

d. Bent, Depth-Limited Plume (R < 10 Only)

$$n_{BL} = \begin{cases} 2.08 (Ar)^{-1/2} & \theta/\theta_0 \geq 0.429 \\ 11.3 (Ar)^{-1/2} & \theta/\theta_0 < 0.429 \end{cases}$$

$$m_{BL} = m_U [\text{value interpolated from table below}]$$

R	1.0	2.0	4.0	6.0	8.0	10.0
	0.250	0.237	0.221	0.212	0.230	0.265

$$\beta_{BL} = \begin{cases} 4 & \theta/\theta_0 \geq 0.429 \\ 2 & \theta/\theta_0 < 0.429 \end{cases}$$

(3) If

$$1.5h_{1/2} < \frac{D_{wo} \frac{dh}{dx} - 1.5h_o S_x}{\frac{dh}{dx} - S_x},$$

then there is no bottom interaction since the bottom slopes away fast enough that the maximum plume depth is reached before the plume intersects the sloping bottom. The above criterion is not intuitively obvious, but will become clear in (4) below.

(4) If (1-3) are not satisfied, then bottom interaction will occur from $x = \xi_1$ to $x = \xi_2$. The value of ξ_1 is obtained by equating the depth of water at ξ_1 to the depth of the plume at ξ_1 . Thus,

$$1.5h_o + \frac{dh}{dx} \xi_1 = D_{wo} + S_x \xi_1.$$

Solving for ξ_1 yields

$$\xi_1 = \frac{D_{wo} - 1.5h_o}{\frac{dh}{dx} - S_x}.$$

The value of ξ_2 is obtained by equating the depth of the water at point ξ_2 to 1.5 times the critical mixing depth giving

$$1.5h_{1/2} = D_{wo} + S_x \xi_2.$$

Solving for ξ_2 yields

$$\xi_2 = \frac{1.5h_{1/2} - D_{wo}}{S_x}.$$

The criterion specified in (3) can be obtained by requiring

$$\xi_2 < \xi_1$$

and simplifying.

(5) It can happen that either ξ_1 or ξ_2 or both fall within the region of flow establishment, i.e., before the temperature begins to decay. The model assumes the length of this region is approximately $n_U \sqrt{h_o b_o}$ where b_o is the initial width of the outfall and n_U is obtained by the correlation given in Table II.G.1. If ξ_1 and ξ_2 are less than $n_U \sqrt{h_o b_o}$, then there will be no

apparent effect of bottom interaction. If ξ_1 is less than $n_U \sqrt{h_o b_o}$ but ξ_2 is not, the bottom interaction will begin its effect at the end of flow establishment and continue until ξ_2 is reached.

Criteria (1-5) can be distilled into a single expression as follows: Bottom interaction will occur from $x = \xi_1$ to $x = \xi_2$, where

$$\xi_1 = \max \left[n_U \sqrt{h_o b_o}, \frac{D_{wo} - 1.5h_o}{\max \left(0, \frac{dh}{dx} - S_x \right)} \right]$$

$$\xi_2 = \frac{1.5h_{1/2} - D_{wo}}{S_x}$$

whenever $\xi_2 > \xi_1$.

Corresponding to the transition lengths ξ_1 and ξ_2 are transition temperatures θ_1 and θ_2 .

d. In addition to the transition temperatures θ_1 and θ_2 , there is a temperature at which the jet begins a "far-field" decay. In the far field, the temperature decays inversely proportional to the centerline distance. Far-field decay begins at $\theta = 0.2\theta_o$ in a plume which is not depth-limited. If the plume is depth-limited at $\theta = 0.2\theta_o$, then far-field decay begins at $\theta = \theta_2$. Thus, bottom interaction is assumed to suppress far-field decay. For convenience, we define

$$\theta_3 = \min(0.2\theta_o, \theta_2).$$

e. If no interaction with the bottom occurs, then the length S_θ and width b_θ of the isotherm of excess θ are given by

$$S_\theta = n_U \sqrt{h_o b_o} (\theta/\theta_o)^{-\beta_U} \quad 1 \geq \theta/\theta_o \geq \theta_3/\theta_o$$

$$S_\theta = S_{\theta_3} (\theta/\theta_o)^{-1} \quad \theta_3/\theta_o \geq \theta/\theta_o$$

$$b_\theta = b_o + m_U S_\theta,$$

where

S_θ = distance to isotherm of excess temperature θ ,

b_θ = maximum width of isotherm of excess temperature θ ,

θ = excess temperature of isotherm,

θ_o = initial temperature excess,

and n_U , m_U , and β_U are given by the correlations appearing in Table II.G.1.

f. If the plume is depth-limited from the end of flow establishment out to $\theta = \theta_2$, then isotherm lengths and widths are given by

$$S_\theta = n_L \sqrt{h_o b_o} (\theta/\theta_o)^{-\beta_L} \quad 1 \geq \theta/\theta_o \geq \theta_2/\theta_o$$

$$S_\theta = S_{\theta_2} (\theta/\theta_2)^{-\beta_U} \quad \theta_2/\theta_o \geq \theta/\theta_o \geq \theta_3/\theta_o$$

$$S_\theta = S_{\theta_3} (\theta/\theta_3)^{-1} \quad \theta_3/\theta_o \geq \theta/\theta_o$$

$$b_\theta = b_o + m_L S_\theta \quad 1 \geq \theta/\theta_o \geq \theta_2/\theta_o$$

$$b_\theta = b_{\theta_2} + m_U (S_\theta - S_{\theta_2}) \quad \theta_2/\theta_o \geq \theta/\theta_o,$$

where n_L , m_L and β_L are given by the correlations appearing in Table II.G.1.

g. If the plume is depth-limited from $\theta = \theta_1$ to $\theta = \theta_2$, then isotherm lengths and widths are given by

$$S_\theta = n_U \sqrt{h_o b_o} (\theta/\theta_o)^{-\beta_U} \quad 1 \geq \theta/\theta_o \geq \theta_1/\theta_o$$

$$S_\theta = S_{\theta_1} (\theta/\theta_1)^{-\beta_L} \quad \theta_1/\theta_o \geq \theta/\theta_o \geq \theta_2/\theta_o$$

$$S_\theta = S_{\theta_2} (\theta/\theta_2)^{-\beta_U} \quad \theta_2/\theta_o \geq \theta/\theta_o \geq \theta_3/\theta_o$$

$$S_\theta = S_{\theta_3} (\theta/\theta_o)^{-1} \quad \theta_3/\theta_o \geq \theta/\theta_o$$

$$b_\theta = b_o + m_U S_\theta \quad 1 \geq \theta/\theta_o \geq \theta_1/\theta_o$$

$$b_\theta = b_{\theta_1} + m_L (S_\theta - S_{\theta_1}) \quad \theta_1/\theta_o \geq \theta/\theta_o \geq \theta_2/\theta_o$$

$$b_\theta = b_{\theta_2} + m_U (S_\theta - S_{\theta_2}) \quad \theta_2/\theta_o \geq \theta/\theta_o.$$

Using the principles developed in a-g above, one can determine the lengths and widths of isotherms in an undeflected jet. Now we consider the case of a deflected jet.

Deflected Jet

h. Bottom interaction will occur in a deflected jet if the initial depth of water is less than 1.5 times the critical mixing depth, i.e.,

$$1.5h_{1/2} \geq D_{wo}.$$

Otherwise, no bottom interaction will occur.

i. If no bottom interaction occurs, the length and width of isotherms of excess are given by

$$S_{\theta} = n_{BU} \sqrt{h_o b_o} (\theta/\theta_o)^{-\beta_{BU}} \quad (\theta/\theta_o) < (\theta_t/\theta_o)$$

$$S_{\theta} = S_{\theta_t} (\theta/\theta_t)^{-\beta_U} \quad (\theta_t/\theta_o) > (\theta/\theta_o)$$

and

$$b_{\theta} = b_o + m_{BU} S_{\theta},$$

where θ_t/θ_o is given by

$$\theta_t/\theta_o = 1 - \exp(-0.327R),$$

and n_{BU} , m_{BU} , and β_{BU} are given by the correlations in Table II.G.1.

j. If bottom interaction does occur, then the length and width of isotherms are given by

$$\left. \begin{aligned} S_{\theta} &= 2.08b_o(\theta/\theta_o)^{-4} \\ b_{\theta} &= b_o + m_{BL} S_{\theta} \end{aligned} \right\} \quad \theta/\theta_o \geq 0.429$$

$$\left. \begin{aligned} S_{\theta} &= 11.3b_o(\theta/\theta_o)^{-2} \\ b_{\theta} &= b_o + 61.4b_o(m_{BL} - m_L) + m_L S_{\theta} \end{aligned} \right\} \quad \theta/\theta_o \leq 0.429,$$

where the correlations for m_{BL} are again found in Table II.G.1. These correlations should be used only for $R > 10$. (See Criticisms and Limitations in Sec. 4 below.)

Computing and Correcting Areas

k. Once the length, S_{θ} , and width, b_{θ} , of individual isotherms have been determined as above, the area is computed by the formula

$$A_{\theta} = 0.86S_{\theta}b_{\theta}.$$

1. Areas must be corrected for surface cooling as follows:

(1) Select a decreasing sequence of isotherms $\theta_0, \theta_1, \dots, \theta_k$. A difference of 0.2°F between successive isotherms is appropriate.

(2) Compute the areas within these isotherms, $A_0, \dots, A_k$, using a-h above.

(3) Compute the change in effective heat source, $\Delta\Gamma_n$, for each isotherm

$$\Delta\Gamma_n = K\bar{\theta}_n\Delta A_n, \quad n = 1, \dots, k.$$

where

K = the heat-loss coefficient,

$\bar{\theta}_n$ = geometric mean temperature of the area

$$= \sqrt{\theta_n \theta_{n-1}},$$

and

ΔA_n = area between isotherms

$$= A_n - A_{n-1}.$$

(4) Compute the reduced incremental area for the first isotherm

$$\Delta A_{1C} = \Delta A_1 \frac{Q_H}{Q_H + \Delta\Gamma_1}.$$

(5) Compute the reduced incremental areas of the other isotherms using the following recursive procedure:

$$\Delta\Gamma_{n-1,C} = K\bar{\theta}_{n-1}\Delta A_{n-1,C},$$

$$\Delta A_{nC} = \Delta A_n \frac{Q_H - \sum_{i=1}^{n-1} \Delta\Gamma_{iC}}{Q_H + \Delta\Gamma_n},$$

for $n = 2, \dots, k$.

(6) Compute reduced areas by summing the incremental areas:

$$A_{nC} = \sum_{j=1}^n \Delta A_{jC}, \quad n = 1, \dots, k.$$

m. Finally, the corrected lengths, $S_{\theta C}$, and widths, $b_{\theta C}$, of the isotherms are determined by

$$S_{\theta C} = S_{\theta} \sqrt{\frac{A_{\theta C}}{A_{\theta}}}$$

and

$$b_{\theta C} = b_{\theta} \sqrt{\frac{A_{\theta C}}{A_{\theta}}}.$$

n. The local plume velocity is assumed to be

$$q = U_a \cos \alpha + q_o(\theta/\theta_o),$$

where α is the angle between the jet centerline and the direction of the ambient current. Since the model doesn't explicitly consider the trajectory, the value of α is not known.

4. Selected Comparisons with Data

Pritchard has compared his phenomenological model to tank data taken by Carter et al.² and to prototype data from the Pilgrim and Zion plants. Examples of comparisons made with the tank data are shown in Figs. II.G.1 and 2 as given by Pritchard.¹ These plots are illustrative since they show the segmented line nature of the temperature decay.

Fig. II.G.1. Comparison of Predicted vs Observed Excess Temperature for a Bent Plume That Is Not Depth-limited. (Adapted from Pritchard.¹)

Fig. II.G.2. Comparison of Predicted vs Observed Excess Temperature for a Bent Plume That Is Depth-limited. (Adapted from Pritchard.¹)

A computer code has been written at Argonne National Laboratory to evaluate the empirical correlations and to determine the length, width, and area of plume isotherms predicted by the model. Using this code, a sample calculation has been prepared. The results appear in App. A.

5. Criticisms and Limitations

a. Cases in Which the Critical Mixing Depth is Less than the Discharge Depth

The data used for developing the model were not sufficiently refined to allow a determination of the rate at which plume thinning occurs. Pritchard states that the adjustment in depth takes place rapidly and well within the region of flow establishment. Any bottom interaction that occurs during the adjustment zone is assumed to have negligible effects on isotherm lengths and widths.

b. Correlations for the Unbent Depth-limited Jet

The values of n_L and m_L are smaller than n_U and m_U , respectively, and β_L is larger than β_U . Thus, the model predicts a depth-limited plume with smaller areas for the highest isotherms and larger areas for the middle and lower isotherms when compared to an unbent plume that is not depth-limited. The data are not sufficiently detailed to ascertain whether this behavior is observed in prototype plumes.

c. Depth-limited Slightly Bent Jet

Ideally the results for a bent jet should approach those of an unbent jet as the parameter $R = q_o/U_a$ goes to infinity. Again, the data were not sufficient to allow a detailed determination as R varies continuously over its entire range. Pritchard recommends that the correlations for a bent, depth-limited jet be used only for $R < 10$. For $R > 10$, the author suggests that bounds on the isotherm areas can be obtained by making model calculations

for the cases $R = 10$ and $R = \infty$. A detailed treatment of the case $R > 10$ is not included in the present form of the model.

d. Limited Nature of the Correlations

The model incorporates many purely empirical numbers. These numbers were extracted from several sets of data (see Sec. e below). It is not presently clear what uncertainties are associated with various numerical coefficients. Pritchard continually improves the correlations as more and better data become available. Therefore, the correlations given in Table II.G.1 are subject to change.

e. Intuitive Nature of the Model

The empirical correlations developed by Pritchard are based on six sources of data.

- (1) Twenty-five hydraulic model studies conducted by Stolzenbach and Harleman at MIT.
- (2) Seventeen hydraulic studies of crossflow interaction conducted by Carter at Chesapeake Bay Institute.
- (3) Three hydraulic studies conducted by the Corps of Engineers at Vicksburg, Mississippi.
- (4) Four field surveys of the thermal plume at the Waukegan Power Station made by Bio-Test.
- (5) A field survey of the plume at Turkey Point, Biscayne Bay, Florida.
- (6) Two field surveys of the plume at Ginna Nuclear Power Station, Lake Ontario.

References

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- 1b. J. Aniywo, Boston Edison Co., Boston, Mass., personal communication (Nov 1973).
- 1c. D. Pritchard, Chesapeake Bay Institute, The Johns Hopkins University, Baltimore, Md., personal communication (Feb 8, 1974).
2. H. H. Carter, E. W. Schiemer, and R. Regier, The Buoyant Surface Jet Discharging Normal to an Ambient Flow of Various Depths, Chesapeake Bay Institute, The Johns Hopkins University, Baltimore, Md., Technical Report 81 (Nov 1974).

H. Brady-Geyer Numerical Model¹

1. Synopsis of Approach

Brady and Geyer have developed a numerical model that solves eight coupled nonlinear partial differential equations governing the behavior of a buoyant surface plume discharged into salt water. The set of equations consists of the three equations of momentum conservation for incompressible flow, three equations stating the conservation of mass, heat, and salt concentration, the equation of state, and the hydrostatic pressure approximation.

The authors use classic arguments to eliminate certain terms in the equations and to produce the necessary boundary conditions. To effect a finite difference solution of the equation set several ingenious numerical devices are used.

Unfortunately, the model is not sufficiently developed at present to allow a thorough analysis. The model is reviewed since it is an early attempt at numerical thermal-plume modeling and since it offers a somewhat different approach than any of the other numerical models considered.

2. List of Symbols

B, C	Boundary conditions
C_p	Specific heat at constant pressure
D_{xt}, D_{yt}, D_{zt}	Thermal eddy diffusion coefficients
$D_o, D_{xv}, D_{yv}, D_{zv}$	Eddy diffusion coefficients
E	Equilibrium temperature of water body
F	Fluid status
f	Coriolis acceleration parameter
g	Acceleration due to gravity
h	Depth of the surface layer
K	Surface-heat-loss coefficient
p	Mean pressure
q_s	Surface heat loss
$\vec{q}_T$	Heat-transport vector
S	Salt concentration (salinity)
T	Mean temperature
$\bar{\bar{T}}_s$	Generalized stress tensor

t	Time
U_a	Ambient current velocity
U_o	Outfall velocity
u, v, w	Mean velocity components
$\vec{v}$	Mean velocity vector
$\vec{X}$	Body force
x, y, z	Spatial coordinates
η	Height of free surface
Φ	Dissipation function
φ	Latitude of site
ρ	Density of fluid
ρ_o	Reference density

3. Basic Assumptions

Among the 12 assumptions made in formulating the model, seven concern the equations themselves and five concern the specification of boundary conditions. For brevity, the assumptions are given in the outline form below:

a. Turbulent velocity and pressure fluctuations are stochastic in nature and hence may be treated separately from their respective mean transient behavior. This assumption leads to the so-called "time-averaged" equations of turbulent flow, shown in Table II.H.1.

TABLE II.H.1. Time-averaged Equations of Turbulent Flow

1. Continuity	where
$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0$	C_p = specific heat
2. Conservation of Linear Momentum	p = mean pressure
$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \nabla \cdot \bar{\bar{T}}_S + \vec{X}$	$\vec{q}_T$ = heat-transport vector
3. Conservation of Thermal Energy	T = mean temperature
$\rho C_p \frac{DT}{Dt} = \nabla \cdot \vec{q}_T + \Phi$	$\bar{\bar{T}}_S$ = generalized stress tensor
	$\vec{v}$ = mean velocity vector
	$\vec{X}$ = body force
	Φ = dissipation function
	ρ = mean density

b. The components of the terms resulting from the generalized stress tensor are written as

$$\left(\nabla \cdot \bar{\bar{T}}_s\right)_x = \rho \left(D_{xv} \frac{\partial^2 u}{\partial x^2} + D_{yv} \frac{\partial^2 u}{\partial y^2} + D_{zv} \frac{\partial^2 u}{\partial z^2} \right),$$

$$\left(\nabla \cdot \bar{\bar{T}}_s\right)_y = \rho \left(D_{xv} \frac{\partial^2 v}{\partial x^2} + D_{yv} \frac{\partial^2 v}{\partial y^2} + D_{zv} \frac{\partial^2 v}{\partial z^2} \right),$$

and

$$\left(\nabla \cdot \bar{\bar{T}}_s\right)_z = \rho \left(D_{xv} \frac{\partial^2 w}{\partial x^2} + D_{yv} \frac{\partial^2 w}{\partial y^2} + D_{zv} \frac{\partial^2 w}{\partial z^2} \right).$$

In determining the diffusion coefficients, Brady and Geyer follow the Prandtl mixing-length hypothesis, which assumes that diffusion rates are proportional to local velocity gradients. In particular, Brady and Geyer assume

$$D_{xv} = D_0 \left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \right),$$

$$D_{yv} = D_0 \left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial y} \right),$$

and

$$D_{zv} = D_0 \left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} \right),$$

where

D_0 is taken as constant.

c. Molecular and turbulent heat transfer may also be treated as a single diffusion process:

$$\nabla \cdot \vec{q}_T = \rho \left(D_{xt} \frac{\partial^2 T}{\partial x^2} + D_{yt} \frac{\partial^2 T}{\partial y^2} + D_{zt} \frac{\partial^2 T}{\partial z^2} \right).$$

The coefficients D_{xt} , D_{yt} , and D_{zt} are assumed to be constant.

d. Both laminar and turbulent dissipation are negligible ($\Phi = 0$).

e. To allow the model to be applied to large water bodies such as bays or any of the Great Lakes, the effects of Coriolis acceleration are included in the momentum equations.

f. The Boussinesq approximation is valid, i.e., density variations may be ignored except insofar as they influence pressure.

g. The pressure field is hydrostatic, i.e., the pressure at any point is determined by its depth and by the density of water above it. Thus,

$$p(x, y, z) = \int_z^{\eta} \rho(x, y, z') g \, dz'.$$

h. Velocities must vanish at all solid boundaries. This is the familiar "no-slip" condition.

i. There is no transfer of either temperature or salt at any solid boundary. Mathematically expressed, this requires normal gradients of temperature and salinity to be zero at fluid-solid interfaces.

j. Momentum flux at the air-water interface equals the wind stress as determined from an empirical correlation with wind speed.

k. Surface heat loss q_s may be related to the surface temperature T and the equilibrium temperature E through an empirical coefficient K as follows

$$q_s = K(T - E).$$

The authors then set q_s equal to

$$h\rho C_p \left. \frac{\partial T}{\partial t} \right|_{\text{surface}}.$$

l. Boundary conditions at fluid-fluid interfaces created by the limited extent of the finite-difference grid must be specified by the user.

4. Numerical Technique

Based on the above assumptions, the authors obtain a simplified set of equations that must be solved numerically. This set, including an equation of salt conservation, is given in Table II.H.2.

The development of any numerical model requires the creation of a stable, efficient scheme to solve the equation set. Brady and Geyer elect a finite-difference formulation supplemented with devices to reduce storage, increase efficiency, and improve flexibility. Three of these devices warrant special attention:

a. To handle air-water and solid-water interfaces in the context of the finite-difference grid, partly empty and partly solid cells are allowed. This technique overcomes many of the traditional difficulties of a rigid structure. The status of a given cell is recorded as number between 0 and 2 as illustrated in Fig. II.H.1.

TABLE II.H.2. Equations of Brady-Geyer Model

Conservation of longitudinal momentum

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + D_{xv} \frac{\partial^2 u}{\partial x^2} + D_{yv} \frac{\partial^2 u}{\partial y^2} + D_{zv} \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho_o} \frac{\partial p}{\partial x} - fv \sin \phi.$$

Conservation of lateral momentum

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - w \frac{\partial v}{\partial z} + D_{xv} \frac{\partial^2 v}{\partial x^2} + D_{yv} \frac{\partial^2 v}{\partial y^2} + D_{zv} \frac{\partial^2 v}{\partial z^2} - \frac{1}{\rho_o} \frac{\partial p}{\partial y} + fu \sin \phi.$$

Conservation of vertical momentum

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - v \frac{\partial w}{\partial y} - w \frac{\partial w}{\partial z} + D_{xv} \frac{\partial^2 w}{\partial x^2} + D_{yv} \frac{\partial^2 w}{\partial y^2} + D_{zv} \frac{\partial^2 w}{\partial z^2} - \frac{1}{\rho_o} \frac{\partial p}{\partial z} - g.$$

Conservation of incompressible fluid mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Equation of state

$$\rho = \rho(\rho_o, S, T).$$

Hydrostatic approximation

$$\frac{\partial p}{\partial z} = -\rho g.$$

Conservation of salt

$$\frac{\partial S}{\partial t} = -u \frac{\partial S}{\partial x} - v \frac{\partial S}{\partial y} - w \frac{\partial S}{\partial z} + D_{xs} \frac{\partial^2 S}{\partial x^2} + D_{ys} \frac{\partial^2 S}{\partial y^2} + D_{zs} \frac{\partial^2 S}{\partial z^2}$$

Conservation of heat

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - w \frac{\partial T}{\partial z} + D_{xt} \frac{\partial^2 T}{\partial x^2} + D_{yt} \frac{\partial^2 T}{\partial y^2} + D_{zt} \frac{\partial^2 T}{\partial z^2}$$

where

 u, v, w = components of velocity p = pressure T = temperature S = salt concentration ρ = density of fluid ρ_o = reference density D_{xv}, D_{yv}, D_{zv} = diffusion coefficients for momentum D_{xt}, D_{yt}, D_{zt} = diffusion coefficients for heat D_{xs}, D_{ys}, D_{zs} = diffusion coefficients for salt f = Coriolis acceleration parameter ϕ = latitude of the site g = acceleration due to gravity

CELL LOCATION	CELL STATUS	VALUE
TOP OF GRID
EMPTY	0.000
TOP OF FLUID
PART-EMPTY	0.001-0.999
FLUID-FILLED	1.000
BOTTOM
PART-SOLID	1.001-1.999
SOLID	2.000

Fig. II.H.1

Cell Status for Brady-Geyer Model.
(Adapted from Brady and Geyer.¹)
ANL Neg. No. 190-1176.

b. The continuity equation is incorporated into the solution by requiring that the net inflow to each nodal point is zero. This type of balancing technique takes advantage of the physical expression of continuity rather than its mathematical counterpart.

c. A novel technique is employed by the authors to reduce the storage requirements of the program. Results are computed and stored as three-digit integers ranging from 0 to 999. This computation is made possible by scaling all variables with appropriate scaling factors. Examples of these scaling factors for various variables in the model are given in Table II.H.3. Once variables are scaled to three-digit integers, three variables are stored in a single integer location using the following scheme:

$$N = 1000000 * N1 + 1000 * N2 + N3.$$

Unfortunately, the reduction in storage comes at the expense of program precision and increased computation time due to required packing and unpacking steps. The authors have optimized this scheme by storing values used proximately in the same location as illustrated in Table II.H.4.

A conceptual outline of the model is given in Table II.H.5.

Within the abbreviated format of this review, it is not possible to discuss the finite-difference technique in detail. Therefore, only the salient features will be mentioned. Essentially, central differencing is used for spatial derivatives and forward differencing for time derivatives. However, during the early stages of execution mixed differencing is employed to empirically determine the largest time step that will yield a stable solution. The use of classical central differencing in the hydrodynamics equation is undesirable since it is generally thought that upwind differencing, where the values are evaluated in the direction of flow, is preferred. Forward differencing in time allows the completely explicit integration of model equations. The procedure followed here is similar to that followed by other numerical modelers (e.g., see Waldrop and Farmer) except for one important difference. Brady and Geyer always use the latest value of the variable that has been calculated and

TABLE II.H.3. Typical Scaling Scheme for Brady-Geyer Model
(from Brady and Geyer¹)

Variable Parameter	Units of Expression	Example Values ^a	Scale Factor	Scaled Integer	Base Value	Packable Mode
Fluid Status F	0 = empty 1 = full 2 = solid	0. 1.0 2.0	IFF = 1000	0 1000 2000	0	0 1000 ^a 2000 ^b
Pressure p	lb ft ⁻¹ sec ⁻²	2.000 ^c 0.000	IFP = 1000	2000	0	2000 ^b
Salinity S	ppt	40.0 0.0	IFS = 10	400 0	0	400 0
Temperature T	F	99.9 32.0 0.0	IFT = 10	999 320 0	0	999 320 0
Longitudinal velocity u	ft sec ⁻¹	15.00 0.00 -5.00	IFU = 100	1500 0 -500	500	2000 ^b 500 0
Lateral velocity v	ft sec ⁻¹	4.99 0.00 -5.00	IFV = 100	499 0 -500	500	999 500 0
Vertical velocity w	ft sec ⁻¹	0.499 0.000 -0.500	IFW = 1000	499 0 -500	500	999 500 0

^aIncluding extreme values of the ranges in each case.

^bCertain values packed in the first position may exceed 999, but, on a 32-bit machine, could not exceed 2146.

^cThe value of pressure that is scaled is actually the difference between the actual pressure and the hydrostatic pressure. Scaling and storing pressure in this fashion greatly improves accuracy as the deviation from hydrostatic pressure is small, but nevertheless significant. Figure II.H.2 illustrates typical deviation of pressure from hydrostatic.

Fig. II.H.2

Pressure Reduction for Brady-Geyer Model. (Adapted from Brady and Geyer.¹) ANL Neg. No. 190-1208.

TABLE II.H.4. Integer-packing Scheme for Brady-Geyer Model
(from Brady and Geyer¹)

Packed Integer				
No.	Name	Packing Positions		
(1)	NP	Pressure p	Salinity S	Temperature T
(2) ^a	NU	Velocity u	Velocity v	Velocity w
(3)	NF	Fluid status F	Boundary condition B	Boundary condition C

^aVelocity components are nearly always used proximately.

TABLE II.H.5. Outline of the Operational Concept of
Brady-Geyer Model

1. Define run specifications and initial values of all fluid characteristics, including boundary conditions.
2. List initial values of all parameters.
3. Determine relative pressures at all fluid cells by integrating vertically downwards from the free surface.
4. Search for solid boundaries adjacent to fluid cells and set pressures equal to adjacent fluid pressures.
5. Compute momentum changes for each fluid cell due to pressure gradients, advection, turbulent mixing, and wind stresses.
6. Consult boundary conditions on velocities, and adjust computed values where necessary to achieve conformity.
7. Apply the principle of continuity to each fluid cell, and distribute any surplus inflow by adjusting velocities of adjacent fluid cells.
8. Repeat (6) and (7) above until all cells balance satisfactorily.
9. Compute salinity and temperature changes at each fluid cell due to advection, turbulent mixing, and surface heat transfer.
10. Consult boundary conditions on salinity and temperature, and adjust computed values where necessary to achieve conformity.
11. Compute surface elevation changes from vertical velocities.
12. List updated values of fluid characteristics.
13. Repeat from (3) above, until run specifications are met.

do not retain the entire solution from the previous time step. This intermediate updating process makes the stability of the solution dependent on the order in which the equations are integrated and may necessitate a smaller time step than if the solution from the previous time step were used alone.

5. Sample Results

The authors have tested their model in three dimensions by applying it to a river-discharge situation. The authors base their example on the discharge of the Philip Sporn Plant located on the Ohio River near Racine, Ohio. A map of the river site taken from Brady and Geyer¹ is given in Fig. II.H.3, and the basic input parameters are summarized in Table II.H.6. The river is divided into a three-dimensional grid consisting of 14 downriver sections, 14 crossriver sections, and 9 vertical sections as shown in Fig. II.H.4. The spacing of grid cells is summarized in Table II.H.7. Downstream velocity and temperature profiles are plotted in Fig. II.H.5, as predicted by the model.

TABLE II.H.6. Input Data for Sample Calculation of Brady-Geyer Model

Discharge Parameters:

Discharge flow rate	1400 cfs
Width of discharge	80 ft
Depth of discharge	20 ft
Discharge velocity (mean)	0.85 ft/sec
Discharge temperature excess	12.1 F°

Ambient Conditions:

Ambient current speed	0.72 ft/sec
Ambient temperature	77.5 °F
Average relative humidity	91 %
Average wind speed	5 mph
Average air temperature	64 °F
Average gross solar radiation	1200 BTU/(ft ² -day)

Nondimensional Parameters

Initial densimetric Froude number	7.9
Discharge aspect ratio	0.25
Ambient current ratio U_a/U_o	1.16

Fig. II.H.3. Site of Philip Sporn Plant. (Photographically reproduced with permission of Brady and Geyer.¹)

Fig. II.H.4
River Grid of Brady-Geyer Model.
(Adapted from Brady and Geyer.¹)

TABLE II.H.7. Grid Spacing for Brady-Geyer Sample Calculation

<u>Section No.</u>	<u>Length (ft)</u>	<u>Mid-point (ft)</u>	<u>Total Length (ft)</u>
<u>Longitudinal Spacing</u>			
upstream			
1	40	20	40
2	40	60	80
3	40	100	120
4	60	150	180
5	80	220	260
6	100	310	360
7	140	430	500
8	180	590	680
9	220	790	900
10	300	1050	1200
11	400	1400	1600
12	500	1850	2100
13	620	2410	2720
14	720	3080	3440
downstream			
<u>Lateral Spacing</u>			
near shore			
1	40	20	40
2	40	60	80
3	40	100	120
4	50	145	170
5	50	195	220
6	60	250	280
7	70	315	350
8	80	390	430
9	90	475	520
10	100	570	620
11	120	680	740
12	140	810	880
13	160	960	1040
14	180	1130	1220
far shore			
<u>Vertical Spacing</u>			
surface			
9	0.1	0.05	0.1
8	0.6	0.40	0.7
7	1.2	1.30	1.9
6	2.2	3.00	4.1
5	2.5	5.35	6.6
4	6.0	9.60	12.6
3	5.5	15.35	18.1
2	7.0	21.60	25.1
1	7.0	28.70	32.1
bottom

Fig. II.H.5. Brady-Geyer Predictions. (Adapted from Brady and Geyer,¹) ANL Neg. No. 190-1215.

6. Criticisms and Limitations

a. Eddy Diffusion Coefficients

The authors use the Prandtl mixing-length hypothesis to determine the necessary coefficients for momentum transfer. The spatial dependence of these diffusion coefficients is not well known and the adequacy of the author's approach cannot be assessed. The problem of handling turbulent transport is generic to numerical modeling and is discussed in greater detail in the reviews of other numerical models.

The mixing-length hypothesis employs an empirical constant with units of length squared, which according to theory represents the square of the mixing length. The authors denote this empirical constant by D_0 and use known channel-velocity profiles to determine its value. On the basis of this analysis they state its value to be about 1.0 ft^2 . Caution should be exercised in applying the model to situations different from those analyzed by the authors. For example, one may expect diffusion coefficients in a large lake to be vastly different from those of an open channel. Furthermore, the model does not consider the effects that buoyancy may have on vertical diffusion. Clearly, this omission is a deficiency in the model's formulation.

b. Limited Precision

Precision is restricted to a maximum of three significant digits when the integer-packing scheme described above is used (see Table II.H.4). The authors believe this precision is adequate stating that for most engineering problems, slide-rule accuracy is sufficient. However, the desirability of using this limited precision for the execution of numerical programs is doubtful.

Storage problems are becoming less and less severe with the trend toward larger computer memories and with the advent of disk and extended core storage. Management of computer resources is still an important consideration in developing numerical techniques, but we do not advise the use of integer-packing schemes such as the one described by Brady and Geyer.

c. Boundary and Initial Conditions

As in the case with many numerical models, the problem of specifying open-water boundary conditions and the initial condition for numerical solution is not adequately treated by Brady and Geyer. The implicit assumption is made that any intelligent guess will do. We feel that this point needs substantiation and that care should be exercised in selecting boundary conditions, at least until their significance is ascertained by a suitable sensitivity study.

d. Expensive to Use

The algorithm used to obtain the solution is complicated and, consequently, large amounts of computer time are required. The authors have minimized computer storage through their integer-packing scheme, but have done so at the expense of run time. Due to the high cost of use, the model has not been sufficiently tested and it is presently impossible to discuss limitations in more detail.

e. Preliminary Nature of the Model

According to the authors of the model, the model requires at least a year of additional work before it becomes competitive. The finite-differencing scheme is primitive in its formulation, and problems of stability are significant. Moreover, the prototype application made to date, the river-plume problem discussed above, was run for only a meager 4 min of prototype time. The computer code for the model is also rather complex and slows up the progress on improvements.

Reference

1. D. Brady and J. Geyer, Development of a General Computer Model for Simulating Thermal Discharges in Three Dimensions, Report No. 7, Dept. of Geography and Environmental Engineering, The Johns Hopkins University, Baltimore, Md. (Feb 1972).

I. Pritchett-England-Taft Numerical Model¹⁻⁷

1. Synopsis of Approach

Pritchett, England, and Taft of Systems, Science, and Software (S³) have developed a generalized three-dimensional numerical model to treat the laminar and turbulent motion of stratified fluids. The model entitled STUFF, an acronym for Stratified Turbulent Unsteady Fluid Flow, is presently proprietary because most of the funding for its development came from private sources. The original work on the model was done by Pritchett under contract to the Defense Advanced Research Projects Agency to handle two-dimensional dispersion of pollutants in air (STUFF2). This code was later extended to three dimensions (STUFF3) to allow for buoyant convective motions.

The two- and three-dimensional versions of the model solve the equations of hydrodynamic flow as well as the time-dependent advection/diffusion equation for both the fluid density and the concentration of a conservative pollutant. A major thrust of the model in its present form is the use of the Gawain-Pritchett heuristic turbulence model³ from which eddy viscosities and eddy thermal diffusivities are calculated, with time, at each difference cell based upon the present velocity field. This turbulence scheme is built into the code but in a modular way; thus it may be easily removed and replaced by another method of calculating viscosities and diffusivities, if desired. The hydrodynamic and scalar variable equations are solved using a combination of the Chorin⁸ and SMAC⁹ methods. In the flow equations, the calculation of pressure is avoided through definition of a velocity corrector potential, and the rigid-lid approximation is made at the free surface.

Pritchett, England, and Taft note that numerical diffusion can result from the finite difference representation of the advection terms, be they in the flow, energy, solute, or turbulent energy equations. A purely Eulerian finite difference representation of these terms may artificially smear the distribution of field variables. Errors in the finite difference representation of the momentum equations due to numerical diffusion can be of the same order of magnitude as the true viscous terms and can dominate the calculation for larger Reynolds number cases. In some problems, the artificial diffusion may actually become negative. The STUFF code uses the Crowley technique¹⁰ to help prevent significant loss of accuracy due to differencing the advection terms in the momentum equations.

To solve the equations of fluid density, passive solute, and turbulent energy, the STUFF model uses an Eulerian/LaGrangian technique. Use of both an array of LaGrangian particles and an Eulerian grid eliminates numerical diffusion from the advection terms. The LaGrangian fluid particles are superimposed on the Eulerian grid and move through the grid as the calculation proceeds. Values of all the scalar field variables, whether they be density, pollutant concentration, turbulent energy, or thermal energy, are carried by

the LaGrangian particles through the Eulerian grid. All spatial derivatives of the scalar field variables are determined using the Eulerian grid; their calculation proceeds from differencing the average of the values held by the LaGrangian particles within the finite-difference cell at the particular time step of the integration. Determining derivatives from the Eulerian grid avoids the distortion problem that can occur in a purely LaGrangian procedure: High-amplitude waves or vortices in a flow distort a purely LaGrangian grid, making meaningless the finite-difference approximations based on such a grid. The LaGrangian/Eulerian method is not used in solving the momentum equations because spatial gradients, and hence numerical diffusion errors for velocity calculations, were not expected to be as significant as for other variables.

The finite difference equations for momentum, fluid density, passive solute, and turbulent energy are solved in the following way:

a. The equations of motion are solved in two steps. Given values at the previous time step, a "tentative" value for the flow field at the next time step is derived by differencing the diffusion and source terms and neglecting the pressure terms. Final values at the next time step are obtained by adding to the tentative flow field, a vector field derived by solving a Poisson's equation for a velocity corrector potential Ψ .

b. The remaining equations (fluid density, passive solute, and turbulent energy) are solved by the LaGrangian/Eulerian technique discussed above. From this result a procedure is carried out to determine values of the variables within each Eulerian grid cell. These cell-based values are then used to calculate future values of the scalar derivatives.

2. List of Symbols

b_o	Outfall width
D	Turbulent-plus-molecular diffusion coefficient
E	Local turbulent kinetic energy
$\dot{E}_D$	Turbulent-energy production due to turbulent diffusion of energy
E_H	Turbulent-energy dissipation to heat
g_i	Gravitational acceleration in x_i direction
h_o	Outfall depth
I	Spatial function defined by integrating a function of w and Ω over all space
I_n	n th approximation of I
J	Spatial function defined by integrating a function of w , Ω , and Ω' over all space
n	Time level in numerical-integration scheme

P	Total pressure
Q	Boussinesq parameter
S	Momentum source
S	Centerline distance
T	Plume temperature
T_a	Ambient temperature
T_o	Reference or outfall temperature
t	Time
U_1, U_2, U_3	Components of the mean velocity in x_1, x_2, x_3 directions
u, v, w	Components of velocity vector
$\vec{u}$	Velocity vector
$\tilde{u}, \tilde{v}, \tilde{w}$	Intermediate (tentative) velocities between time step n and $n + 1$
$\vec{\tilde{u}}$	Intermediate velocity vector
u_i	Fluctuating component of velocity field in the x_i direction
u_o	Outfall velocity
V	Velocity at the intake or outfall
w, W	Weighting function defined over space
w_n, W_n	n th approximation to w, W
x, y, z	Longshore, offshore, and vertical directions in space
x_i	Components of spatial vector $\vec{x}$
$\vec{x}$	Spatial vector
y	Distance to nearest fixed boundary
α, β, γ	Slowly varying functions determined from data (β is also used as the volumetric expansion coefficient)
Γ_{ij}	Mean flow stream rate
$\Delta x, \Delta y, \Delta z$	Longshore, offshore, and vertical spacings of grid
δ_{ij}	Kronecker delta function; $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ij} = 1$ for $i = j$
ϵ	Eddy kinematic viscosity
θ/θ_o	Ratio of temperature excess of isotherm (above ambient) to outfall temperature excess
θ_c/θ_o	Ratio of centerline temperature excess (above ambient) to outfall temperature excess

λ	Macroscale length
λ^*	Correlation length of local turbulence
λ_n	nth approximation to λ
ν	Molecular kinematic viscosity, also turbulent-plus-molecular kinematic viscosity
π_Q	Energy-source term
ρ	Plume density
ρ_0	Reference density
φ	Fluctuating component of the departure of the local pressure from the hydrostatic reference pressure divided by the density
Φ	Mean departure of the local pressure from the hydrostatic reference pressure divided by the density
Ψ	Velocity-corrector potential
Ω	Generalized strain rate

3. Derivation of Model Equations and the Model for Fluid Turbulence

Table II.I.1 lists the continuity, momentum, and energy equations as classically derived for turbulent incompressible flow. The solution of those three-dimensional equations is carried out by the STUFF model. An auxiliary equation for the turbulent energy distribution is also solved in the model to aid in evaluating the Reynolds stress terms in the momentum equations; the auxiliary equation (see Table II.I.1) can be deduced rigorously from the original equations of motion. However, the turbulent energy equation itself introduces two additional unknowns into the problem. The STUFF model employs the Gawain-Pritchett heuristic turbulence model³ that solves the turbulent energy equation through a sequence of heuristic and empirical assumptions concerning the unknowns.

The terms on the right-hand side of Eq. D in Table II.I.1 represent, respectively, the turbulent energy production corresponding to the work done by the mean flow against the Reynolds stresses, dissipation of turbulent energy to heat, turbulent diffusion of energy, and molecular diffusion. The last term is very small and can be neglected in problems of high Reynolds number such as thermal plume applications. To solve Eq. D in space and time, assumptions are made about the eddy viscosity function, the diffusive term, and the turbulent energy dissipation.

Gawain and Pritchett cite theory and experiment to state that the eddy viscosity, dissipation, and diffusion functions depend not only on the turbulent energy itself but on a local length scale parameter that can be associated

TABLE II.I.1. Conservation Equations for Mean Flow in the STUFF Model

(mean-flow continuity)

$$\frac{\partial}{\partial x_j}(U_j) = 0 \quad (A)$$

(mean-flow momentum)

$$\frac{\partial}{\partial t}(U_i) + \frac{\partial}{\partial x_j}(U_i U_j) = \frac{\partial}{\partial x_j} \left\{ v \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \overline{u_i u_j} \right\} + Q g_i - \frac{\partial \phi}{\partial x_i} + S \quad (B)$$

(energy)

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x_j}(U_j Q) = \frac{\partial}{\partial x_j} \left[D \frac{\partial}{\partial x_j}(Q) \right] + \pi Q, \quad (C)$$

where

 U_1, U_2, U_3 = components of the mean velocity in x_1, x_2, x_3 directions, u_i = fluctuating component of the velocity field in x_i direction, S = momentum source, Q = Boussinesq parameter = $(\rho/\rho_0 - 1) = -\beta(T - T_0)$, β = volumetric expansion coefficient, ρ = density, g_i = gravity acceleration component in x_i direction, πQ = energy-source term, v = turbulent-plus-molecular kinematic viscosity, D = turbulent-plus-molecular diffusion coefficient,

and

 ϕ = mean departure of local pressure from hydrostatic reference pressure divided by density.

(turbulent-energy equation)

$$\begin{aligned} \frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x_k}(U_k E) = & - \frac{\overline{u_j u_k}}{2} \left(\frac{\partial U_j}{\partial x_k} + \frac{\partial U_k}{\partial x_j} \right) - \frac{v}{2} \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \\ & - \frac{\partial}{\partial x_k} \left[\overline{u_k (E + \phi)} \right] + v \frac{\partial}{\partial x_k} \left[\frac{\partial}{\partial x_k}(E) + \frac{\partial}{\partial x_j}(\overline{u_j u_k}) \right] \end{aligned} \quad (D)$$

where

$$E = \frac{\overline{u_j u_j}}{2},$$

 E = local turbulent kinetic energy,

and

 ϕ = the fluctuating component of the departure of the local pressure from the hydrostatic reference pressure divided by the density.

with each space-time point in the flow field. Expressions for the above three functions are written to include the turbulent energy E , the local length parameter (macroscale λ), and the distance to the nearest fixed wall (if any) y . In this way a determinate set of equations is developed that can conceptually be applied to any incompressible turbulent flow field. Only the boundary conditions differ for each application.

The heuristic (from dimensional analysis, empiricism, etc.) assumptions made by Gawain and Pritchett are summarized below.

The Reynolds stresses are related to the strain rates of the mean flow by

$$-\overline{u_i u_j} = -\frac{1}{3} \overline{u_k u_k} \delta_{ij} + \epsilon \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \quad (1)$$

where δ_{ij} is the Kronecker delta function and ϵ is the eddy kinematic viscosity. Under this assumption and with the omission of the last term of Eq. D, the mean flow momentum and turbulent energy equations become

$$\frac{\partial}{\partial t} (U_i) + \frac{\partial}{\partial x_j} (U_i U_j) = \frac{\partial}{\partial x_j} \left[(\nu + \epsilon) \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] - \frac{\partial P}{\partial x_i},$$

(mean-flow momentum) (2)

and

$$\begin{aligned} \frac{\partial E}{\partial t} + \frac{\partial}{\partial x_k} (U_k E) = & \epsilon \Omega^2 - \nu/2 \overline{\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right)} \\ & - \frac{\partial}{\partial x_k} \left[u_k \left(\frac{u_j u_j}{2} + \varphi \right) \right], \quad (\text{turbulent energy}) \end{aligned} \quad (3)$$

where

$$\Gamma_{jk} = \frac{\partial U_j}{\partial x_k} + \frac{\partial U_k}{\partial x_j} \quad (\text{mean-flow strain rate}), \quad (4)$$

$$\Omega^2 = (1/2) \Gamma_{jk} \Gamma_{jk}, \quad (5)$$

$$E = \frac{\overline{u_j u_j}}{2} \quad (\text{mean turbulent kinetic energy}), \quad (6)$$

and

$$P = \Phi + (2/3)E \quad (\text{total pressure}). \quad (7)$$

The last term in Eq. 3 is further represented by

$$\overline{u_k \left(\frac{u_j u_j}{2} + \varphi \right)} = -\gamma \epsilon \frac{\partial E}{\partial x_k}, \quad (8)$$

which is developed from dimensional and physical arguments. Here ϵ is again the kinematic eddy viscosity and γ is some slowly varying function that must be determined empirically. Thus the diffusion term in the turbulent energy equation becomes

$$\dot{E}_D = \frac{\partial}{\partial x_k} \left(\gamma \epsilon \frac{\partial E}{\partial x_k} \right). \quad (9)$$

Before the expressions for the turbulent energy dissipation and the kinematic eddy viscosity are developed, Gawain and Pritchett define a scale of turbulence, or macroscale λ , at a space-time point in terms of the statistical properties of the turbulence in the vicinity of the point. A formulation for the macroscale became necessary because a plausible model for ϵ should really depend upon the turbulent flow field and its interaction with the mean flow. Hence the authors of the model consider the establishment of scales appropriate to the description of the mean flow and/or the turbulent field.

In the derivation of a form for λ , the connection between the local turbulence field and the local mean flow field is defined by Eq. 1. In this way the local length scale is reflected in the local characteristics of the mean flow and is therefore deducible from that field. The calculation of λ is carried out in an iterative process as described in Table II.I.2. Since the definition of λ should emphasize the mean flow conditions in the immediate neighborhood of the point, a weighting function W is introduced to give progressively less weight to remote points.

Once the macroscale λ is determined, the turbulent energy dissipation to heat is modeled from

$$\dot{E}_H = \beta (2E)^{7/6} J^{1/3},$$

where J has been determined in the analysis of Table II.I.2 and β is a slowly varying function as given below.

TABLE II.I.2. Calculation of Macroscale Length λ in
Gawain-Pritchett Heuristic Turbulent Model

$$\lambda^2(\vec{x}) = \lim_{n \rightarrow \infty} \lambda_n^2(\vec{x})$$

The $(n + 1)$ st approximation to λ is defined from

$$\lambda_{n+1}^2(\vec{x}) = \frac{I_{n+1}^2(\vec{x})}{J_{n+1}^2(\vec{x})},$$

$$I_{n+1}^2(\vec{x}) = \int_{\text{all space}} w_n(\vec{x}, \vec{x}') \Omega^4(\vec{x}') dv',$$

and

$$J_{n+1}^2(\vec{x}) = \int_{\text{all space}} w_n(\vec{x}, \vec{x}') [\Omega(\vec{x}')]^2 dv'.$$

The weighting function W_n is defined from

$$W_n(\vec{x}, \vec{x}') = \exp \left[- \frac{(\vec{x}' - \vec{x}) \cdot (\vec{x}' - \vec{x})}{\lambda_n^2(\vec{x})} \right],$$

from which

$$w_n(\vec{x}, \vec{x}') = \frac{W_n(\vec{x}, \vec{x}')}{\int_{\text{all space}} W_n(\vec{x}, \vec{x}') dv'}$$

From previous definitions

$$\Omega^2 = \frac{1}{2} \Gamma_{ij} \Gamma_{ij}, \quad \Omega' = \frac{\partial \Omega}{\partial x_i}.$$

As an initial guess, choose $\lambda_0 = \infty$ everywhere. In computing λ_1 , the weighting function $W_0 = 1$ everywhere, from which

$$\lambda_1^2 = \frac{\int_{\text{all space}} \Omega^4 dv}{\int_{\text{all space}} (\Omega \Omega')^2 dv}$$

is independent of location. It follows that λ_2 is the first nonconstant approximation to λ .

Simple applications of the model have shown that the convergence of the λ_n 's is extremely rapid. Absolute accuracy in convergence is not essential due to the gross simplifications already inherent in the model. However, Gawain and Pritchett recommend³ that the chosen order of approximation be maintained in the calculations.

The eddy viscosity is next written in the form

$$\epsilon = \alpha \lambda (2E)^{1/2},$$

with α also a slowly varying function. Note that ϵ , at a given point and time, is a function of the local turbulent energy density and the macroscale at that point. The forms for $\dot{E}_D$, $\dot{E}_H$, and ϵ were motivated by physical and dimensional reasoning.

Based upon some limited data, α , β , and γ were determined to be

$$\alpha = 0.065 \{1 + \exp[-(y/\lambda - 1)^2]\},$$

$$1/\beta = 3.7 \{1 + \exp[-(y/\lambda - 1)^2]\},$$

and

$$\gamma = 1.4 - 0.4 \exp[-(y/\lambda - 1)^2].$$

Here y is the distance to the nearest fixed boundary. As the boundary is approached $y \rightarrow 0$ but $\lambda \rightarrow 0$ also, so that y/λ remains finite. The limit of y/λ is arbitrarily assigned the value 1. Far from a fixed boundary, y is set equal to ∞ whereupon α , β , and γ reduce to constants. Consequently, the variation in the above functions is not large. The usefulness of the Gawain-Pritchett formulation is based on whether the form for ϵ reproduces experimental data.

Table II.I.3 provides the final equations for neutrally stable stratification. When density stratification is present, the authors of the STUFF model make the Boussinesq approximation, add a buoyancy term to the momentum equation in the vertical direction, and add a term to the turbulent energy equation to simulate the effects of fluid stratification in turbulent energy dissipation.

Note that the conservation equation for a passive pollutant could easily have been included in Table II.I.3 and would assume the same form as the conservation of heat energy equation, the only difference being the replacement of the Boussinesq parameter Q by a pollutant concentration variable. Note also that the source/sink terms in the equations of Table II.I.3 can be used to include the effects of wind stress, radiation losses, and bottom stress for the two-dimensional version of the model. For the three-dimensional version, these effects must be included as boundary conditions.

Verification does exist for the Gawain-Pritchett model of turbulence but only for two simple cases. The cases tested were for flow in a two-dimensional channel dominated by boundary layer effects and in the far-downstream region of an axisymmetric jet in free turbulent flow. Computed and observed distributions of turbulent energy and Reynolds shear stress were compared. The results were quite good indicating promise for the model. The only disappointing factor in these two comparisons is that the coefficients α , β , and γ were determined precisely from examination of data from the

TABLE II.1.3. Final Equations for Solution in the STUFF Model

Neutrally Stable Stratification:

(mean-flow continuity)

$$\frac{\partial}{\partial x_i}(U_i) = 0,$$

(mean-flow momentum)

$$\frac{\partial}{\partial t}(U_i) + \frac{\partial}{\partial x_j}(U_i U_j) = \frac{\partial}{\partial x_j} \left[(\nu + \alpha \lambda \sqrt{2E}) \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] + S - \frac{\partial P}{\partial x_i},$$

and

(turbulent energy)

$$\frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x_j}(U_j E) = \alpha \lambda \sqrt{2E} \Omega^2 - \beta (2E)^{7/6} J^{1/3} + \frac{\partial}{\partial x_j} \left(\alpha \gamma \lambda \sqrt{2E} \frac{\partial E}{\partial x_j} \right).$$

Density Stratification:

(mean-flow continuity)

$$\frac{\partial}{\partial x_i}(U_i) = 0,$$

(mean-flow momentum)

$$\frac{\partial}{\partial t}(U_i) + \frac{\partial}{\partial x_j}(U_i U_j) = \frac{\partial}{\partial x_j} \left[(\nu + \alpha \lambda \sqrt{2E}) \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] - \frac{\partial P}{\partial x_i} + Q g_i + S,$$

(heat energy)

$$\frac{\partial}{\partial t}(Q) + \frac{\partial}{\partial x_j}(U_j Q) = \frac{\partial}{\partial x_j} \left[(\alpha \gamma \lambda \sqrt{2E} + D) \frac{\partial Q}{\partial x_j} \right] + \pi Q,$$

and

(turbulent energy)

$$\begin{aligned} \frac{\partial}{\partial t}(E) + \frac{\partial}{\partial x_j}(U_j E) &= \alpha \lambda \sqrt{2E} \Omega^2 - \beta (2E)^{7/6} J^{1/3} + \frac{\partial}{\partial x_j} \left(\alpha \gamma \lambda \sqrt{2E} \frac{\partial E}{\partial x_j} \right) \\ &\quad - \alpha \gamma \lambda \sqrt{2E} g_j \frac{\partial Q}{\partial x_j}. \end{aligned}$$

two-dimensional channel flow and axisymmetric flow problems. Consequently, a truly objective comparison of the heuristic turbulence model to data did not occur. After the publication of Ref. 3, Pritchett⁷ applied the turbulence model to the flat plate problem under laminar flow. He assumed that the transition point to turbulent flow coincided with the leading edge of the plate. Using experimental velocity profiles, the model was used to predict energy dissipation. Pritchett⁷ states that the calculations compared satisfactorily to the data. He also notes that this is not surprising because the heuristic turbulence model could handle the channel flow problem well.

For a clearer perspective, however, the limitations of the Gawain-Pritchett turbulence model are listed below.

a. Numerous aspects of the model are grossly oversimplified and incorporate several approximations in the equation set to effect closure. The applicability of the model to a general class of turbulent flow problems in its present formulation raises serious questions. The relative sensitivity of the solution to the choice of parameters intrinsic to the approximation is important to know. This would help to locate possible inaccuracies inherent in the approximation and to encourage improvements. Further study should be applied to:³

(1) The relationship between the local length scale λ of the mean flow and the correlation length λ^* of the turbulence.

(2) The generalized three-dimensional stress-strain relations that actually exist in regions of strong anisotropy, such as in the flow near a wall.

(3) The influence of various important parameters in describing the rate of dissipation of turbulent energy to heat.

b. The turbulence model as developed by Gawain and Pritchett does not handle stratification or density differences; the appendage to the turbulence model by the STUFF authors to treat this is unverified. Pritchett adds⁷ that the correction to the turbulent energy equation appearing in Table II.I.3 attempts to take into account the effects of stratification on suppressing the turbulent energy or augmenting it if the stratification is unstable. Pritchett notes that this additional term in the turbulent energy equation (see Table II.I.3) does follow directly from the original unaveraged Navier-Stokes equations if the same assumptions are made and once the Boussinesq term is added to the turbulent energy equation at the start of the derivation. That additional stratification correction term to the turbulent energy equation is thus the result of an identical derivation; it may be considered as a source/sink-type term. Pritchett asserts⁷ that the additional term is entirely self-consistent with the derivation. However, he notes that with the correction included, the turbulence model remains basically an isotropic one; the model does not treat anisotropic mixing.

c. In principle, the heuristic model can be used in an unsteady flow problem, allowing the calculation to proceed until a steady state is reached. The validity of the approximations made in the model for developing flows is unknown, since no adequate experimental data are available. Furthermore, the unsteady use of the turbulence model is expensive in terms of computation time.

Pritchett states⁷ that he expects the use of the steady turbulence model in the unsteady state not to be a problem in thermal-plume applications because the response time of the turbulence has to be fast compared to the mean flow. The turbulence "sees" a steady flow, certainly on the time scale of the problem.

d. No verification of this model for thermal-plume problems exists to prove that the Gawain-Pritchett formulation is superior to other alternative schemes, even mixing-length theory. More work should be done in evaluating diffusivity theories with reference to their application to numerical plume models.

4. Numerical Scheme/Boundary and Initial Conditions

The details on the numerical technique are sparse due to the lack of published material on the STUFF model. This reflects the proprietary nature of the code. Based upon Refs. 1-5 and discussions with J. Taft⁶ and J. Pritchett,⁷ the following steps summarize the method of solution of the equations.

a. Flow-field Calculation

(1) A corrector potential iteration scheme is initiated in this step by a first calculation of the velocities u , v , and w including only the diffusion and source terms of the momentum equations but ignoring the pressure gradient terms. The diffusion terms, i.e., the components of the strain-rate tensor, are calculated here from values at the previous time step. A second-order finite difference operator is used to do this with the resulting terms placed in temporary storage in a velocity array. Source terms in the momentum equations, if any, are then calculated and added to the array. Solving for the new velocities $\tilde{u}^{n+1}$, $\tilde{v}^{n+1}$, and $\tilde{w}^{n+1}$ in this first step is done by the Crowley scheme.¹⁰ The differencing of the advection terms is second order in space and carried out by forward differencing the velocity gradient terms for each cell and then adding a correction term that is diffusion-like in nature to help correct for the truncation error that would otherwise cause numerical diffusion. The STUFF scheme allows for a second iteration on $\tilde{u}^{n+1}$, $\tilde{v}^{n+1}$, and $\tilde{w}^{n+1}$ using the average of the values just calculated and those at time step n to initiate this calculation. This second iteration is carried out, just as above, to refine the values obtained in this step, and it makes the approximation second order in time as well as space.

(2) This step accounts for the pressure gradient terms that were ignored in the first approximation, step (1). The velocity field derived in step (1) is divergent; a vector field, with zero curl and divergence of opposite sign, derived from the pressure field must be added to get the correct velocity field at time $n + 1$. Since a pressure field alone cannot impart rotation to the fluid, the true new velocities cannot differ from those "intermediate" values by more than a vector field that is the gradient of a scalar function Ψ . Thus a property of a pressure field is that it cannot introduce any more vorticity into the flow. A scalar "velocity corrector potential" is defined to satisfy

$$u^{n+1} = \tilde{u}^{n+1} + \frac{\partial \Psi}{\partial x}, \quad (10)$$

$$v^{n+1} = \tilde{v}^{n+1} + \frac{\partial \Psi}{\partial y}, \quad (11)$$

and

$$w^{n+1} = \tilde{w}^{n+1} + \frac{\partial \Psi}{\partial z}. \quad (12)$$

The true new velocities u, v, w , above, then differ from the intermediate velocities $\tilde{u}^{n+1}, \tilde{v}^{n+1}, \tilde{w}^{n+1}$ only by a vector field defined by Ψ . This field is such that $\nabla \times \vec{u}^{n+1} = \nabla \times \vec{\tilde{u}}^{n+1}$ (since $\nabla \times \nabla \Psi = 0$), as it should be at all times.

The final new velocities must satisfy the continuity equation

$$\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y} + \frac{\partial w^{n+1}}{\partial z} = 0. \quad (13)$$

Combining Eqs. 10-13, we have

$$\begin{aligned} \frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y} + \frac{\partial w^{n+1}}{\partial z} &= \frac{\partial \tilde{u}^{n+1}}{\partial x} + \frac{\partial \tilde{v}^{n+1}}{\partial y} + \frac{\partial \tilde{w}^{n+1}}{\partial z} \\ &\quad + \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}. \end{aligned} \quad (14)$$

Since $\nabla \cdot \vec{u}^{n+1} = 0$ must be satisfied for the new velocity field at time $n + 1$, the following Poisson's equation for Ψ results:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = - \frac{\partial \tilde{u}^{n+1}}{\partial x} - \frac{\partial \tilde{v}^{n+1}}{\partial y} - \frac{\partial \tilde{w}^{n+1}}{\partial z}.$$

The boundary conditions on Ψ are that the normal derivative is assumed to be zero on all outer boundaries of the grid. The Poisson equation for Ψ is solved

using an over-relaxed Gauss-Seidel iteration procedure. If a conservative time step is utilized, 10-20 iterations are generally necessary for the solution to the Poisson equation to converge. A more liberal time step might require 100 or more iterations.

Concerning the treatment of the free surface boundary condition, the STUFF model makes the rigid-lid assumption. Consequently, there is essentially a rigid plate at the top of the (rigid) grid structure preventing vertical motion above the top of that grid. A free slip boundary condition (normal velocity is zero) is maintained at the top of the grid to divert flow horizontally. The Chorin method entirely avoids the calculation of pressures, and consequently neither the pressure field at the top of the grid nor the free surface heights are calculated in the computer code. However, the pressure field itself could be calculated easily enough from the velocity data output in terms of an integration of Poisson's equation (not discussed). The rigid lid condition enters the problem precisely in the surface boundary condition on the velocity corrector potential Ψ . Its boundary condition of zero normal derivative is reflected in the solution for Ψ .

The first half of the flow calculation (step 1) allows the choice of boundary conditions of two types: (1) the periodic form in which no flow leaves the top and sides of the grid, essentially all flow being directed along a duct, and (2) the type that requires the velocities themselves to be specified for all time on the external boundary. The latter option was chosen for the Point Beach and Pilgrim runs (Secs. 5 and 6 below). In these cases the upcurrent and downcurrent boundary values for velocity were chosen to be the ambient current velocity. The authors of the model hope this does not introduce too many errors on the downcurrent side, which should be the case if the downcurrent boundary is sufficiently far from the influence of the plume. A portion of the plume at Pilgrim did extend beyond the grid boundaries. The authors of the model do not estimate the region of influence of the plume to be very significant to the right of the final breakwater. The plume was probably not perturbing the downstream velocities significantly. The offshore velocities, however, might have caused more difficulties at Pilgrim since the plume did impinge on the offshore wall. The offshore grid velocities were chosen to allow a fixed amount of momentum through per unit time. This quantity was specified a priori and set equal to the initial component of offshore momentum of the jet. As the plume tended to transfer more than that fixed amount of momentum outside the offshore grid boundary, a reflection of momentum off that grid boundary occurred with the perturbation propagating back into the plume. The authors of the model hope that the effect is not felt more than three or four cells inward from the boundary. A much larger grid in the horizontal offshore direction would have been useful in removing that effect, but this was not possible to accomplish with the computer core size available for the computer run. Even boundary conditions more commonly used, such as a second derivative equal to zero, would still give the perturbative reflection. In all cases it is safest, though usually more expensive, to keep boundaries far from the influence of the plume.

Specifying actual velocities as boundary conditions is the only way to get a unique solution to the Navier-Stokes equations for the interior mesh points. Numerical solution of other fluid flow problems seem to imply that if the open boundaries are sufficiently far from the discharge, specifying velocities, gradients, or second derivatives as zero would yield the same results. Yet, for a situation like Pilgrim, where boundaries, especially the offshore one, are close to the flow and interact with it, more perturbations result from specifying fixed velocities than if second derivative boundary conditions were used. Hence, for close-in open boundaries, the type of boundary condition employed is important in the determination of the solution.

The solution of the flow equations as set up in STUFF involves mass and momentum conservation over the whole grid, i.e., as much mass and momentum flux enters the grid through the outfall and upcurrent flow as leaves the grid. A standard output in all STUFF runs is the sum of momentum flux over all cells in the grid. This is printed out at each time step so that the sum total momentum in the grid can be determined. The code passed through 1500 time steps in a past simulation with essentially no loss (under 1%) of mass or momentum in the finite difference scheme.

b. Calculation of Scalar Variables

The solutions for the turbulent-energy equation, passive contaminant equation, and heat-energy equation are carried out with the same procedure, since each is a scalar field, and all their equations are essentially the same. In this step an array of tracer particles is employed. Initially each cell is assumed to contain, typically, eight such tracers; each particle moves from the cell with the flow field, and diffuses according to local values of diffusivity. Each particle carries with it values of each field, whether it be turbulent energy E , Boussinesq parameter Q , or passive contaminant concentration. To find the values for these variables for a computational cell, a census of its particles is taken and the mean particle-based value is used for the cell as a whole. Solution to each of the scalar equations proceeds as follows: From the cell-based concentrations, the scalar diffusion terms are evaluated. Next, updating the particles is undertaken assuming each particle diffuses with the cell-based diffusion rates. At this point in the calculation, the cells have their old concentrations but the particles now have new values updated by Δt times the diffusion rate. The momentum equations are then solved to yield the u , v , and w velocity field. To evaluate the scalar advection term, the particles are moved into new cells under the local u , v , w velocities of that cell at time step n . The time-centered value (and spatially (linear) interpolated value) of velocity is used in the particle advection process. To calculate the local value of velocity requires a spatial interpolation, because the particles usually do not fall on the grid cell faces as they are defined. Finally the census procedure is again implemented to get cell-based values at the next time step. To calculate the eddy viscosities from the turbulent energy equation requires only the local velocity field, thus allowing the turbulent-energy solution to follow

the scalar-equation solution format. Time centering is done in the turbulent energy numerical solution as is done in solution of all scalar equations; the average of old and new values of velocity is used. The calculation of eddy viscosities and diffusivities uses values of velocity at time step n only; no time-centered velocity values are used.

Zero particles occur only very infrequently in a cell at a given time step. The cell-based concentration of a void cell is determined from the average of surrounding cells, the particles being actually considered as finite volumes rather than individual points when a census takes place.

The treatment of the particles as they interact with solid boundary surfaces, whether the surface is the bottom or shoreline, is an important consideration. Particles advected or diffused outside the grid are disregarded; they are advected out by the velocities specified at the boundary grid cells, which are described later. The question of whether mass and momentum are conserved in such a treatment is of importance. Two treatments are permitted in the STUFF code. The first is the destruction of the particles that interact with a boundary, clearly violating conservation of mass. However, for a grid of 10,000 particles, a plume problem like Pilgrim may involve the loss of 50 particles per time step but 20 is more typical. Since there are always new particles being generated, by the inflow of ambient current to the grid or else from the discharge channel, a serious reduction in particles is not likely to occur and the important cell averages are not significantly affected.

The second and more involved method is to force the local velocity of particles near wall boundaries to deflect those particles that are directed toward the solid boundaries by the following procedure. When a particle is directed toward an obstacle and enters the "last" grid cell before that boundary, the time step is usually small enough to prevent it from hitting the obstacle in that one time step. The time step in the diffusion solution in STUFF is never more than one-fourth the time necessary to pass through a cell; the STUFF model uses the Courant condition in its time iterations. For the particle located within the cell, a linear interpolation to derive the local velocity of that particle from the four nearest velocities involves the zero normal velocity at the boundary face. The interpolated value of these velocities generally involves a larger transportive velocity along the cell face and a smaller velocity toward the obstacle. Observing the path of the particle as time increases, with the sequence of linear interpolations at each time step continuing, the particle generally gets more and more deflected as it gets closer and closer to the obstacle face as the velocity component towards the obstacle decreases. In this way the component of velocity toward the obstacle becomes essentially zero. Thus for conservative time steps, the deflection of the particle is not through the obstacle but sideways. The normal velocity component of a particle is seldom large enough for it to travel across the boundary face in one or two time steps. To implement the second approach takes an additional few percent in computation time,

but based upon previous applications of the model with both approaches tested, only a very little difference occurs in the results. The first method technically involves a loss of mass because attributes of the particles are being destroyed. Yet, Taft states⁶ that though the particle concentrations are not of interest, the cell-based concentrations are most important; these latter concentrations are derived from an averaging procedure of typically eight particles per cell. Losing one particle out of eight does not, as found in the STUFF model, alter the average concentration significantly because the particles that are lumped together tend to have similar attributes. The usual mode of operation in the STUFF code is to destroy all particles hitting obstacles; this was done in the Pilgrim and Point Beach runs. An alternative procedure of reflecting a particle off of an obstacle back to its old position is not strictly correct, because of the implication that the derivative of the normal velocity is infinite at the obstacle. This "reflective fluid flow" procedure often proposed, implies that the stresses and acceleration of the fluid at the boundary point are infinite, which violates the fundamental continuum assumption of fluid mechanics.

Despite the confidence of the authors of the model in the destroyed-particle scheme, we are suspicious of its applicability to Pilgrim where the flow must pass around two breakwaters; other boundaries of concern are the free surface and the solid shoreline and bottom boundaries. An evaluation of both methods and a study of alternative procedures for particle-boundary interaction should be done. We have no way of passing judgment on the schemes used in STUFF without additional computer runs and a concerted effort in studying that problem.

The other boundary conditions are now summarized. A zero temperature gradient is assumed at all solid boundaries. Only normal velocities are specified along solid boundaries and these are assumed zero. Tangential velocities are not specified; hence, free-slip conditions are assumed for all boundaries. No viscous stress effects are simulated at the bottom. Any or all of the outer walls of the grid may be set to simulate rigid, impermeable "free slip" walls or the x direction may be chosen as periodic. This again allows the simulation of a series of sources and sinks in the horizontal direction and forces the total input flow rate to be constant in time.

Topography can be simulated through various mockup techniques. A series of continuous blocks can be used for the bottom and an arbitrary number of cells within the grid used to represent bottom boundaries. Depending upon the resolution, these bottom contours can be accurate or inaccurate representations. Obstacle cells with impenetrable walls can be set up as internal boundaries in any arrangement and number throughout the grid. Outer walls may also be set up as planes of symmetry to focus attention on that portion of the problem that need be modeled.

The model uses a set of artificial cells, two in each direction, one on either side of the computational grid cells in order to make the finite-difference routine more compact. These cells have fixed attributes that are

known a priori. The cells are used to set velocities at the boundaries, i.e., the velocity can be set at the face of an artificial cell for all time. The cells can be used as sources of momentum and turbulent energy. Their presence avoids numerous "if" tests in the computer code to check whether a cell is next to a boundary. These cells are part of the code's "intake/exhaust system" of grid cells that may be specified for a particular application. They also may be placed at arbitrary positions within the grid. The total set acts as sources and sinks of mass, momentum, and energy; it must be set up to insure conservation principles. This system of cells is used to set up an ambient cross-current at the upcurrent side of the grid and to maintain it with a judicious choice of exhaust cells on the opposite side of the grid; conservation of mass and momentum must be satisfied. Source and sink specifications for the scalar variables can also be made once the cell-cell block location and the start-stop time of the source or sink is specified.

The model has a flexible crosscurrent scheme in which velocities on the upcurrent and downcurrent boundaries may essentially be specified arbitrarily. The only requirement is that the flow must be conservative i.e., nondivergent. Flow into the grid structure must equal the flow out of the grid. The velocity field specified can vary arbitrarily in both the horizontal and vertical directions. The code has a potential flow solver to calculate a potential flow field for the boundary flow if one chooses potential flow as an initial condition for the entire grid. In that case, the inputted boundary flow must be uniform with height, with no allowance for stratification. Other methods of initiating the flow field exist in the code, however, that allow an essentially arbitrary boundary flow to be specified. The crosscurrent scheme is flexible in that one is not forced to align the grid with the mean flow. This latter advantage is useful when sloping boundaries are present and the mean flow is not along a principal axis. Ocean currents, which often have much spatial curvature, can be inputted as boundary conditions. To our knowledge, the STUFF model has not been run with time-varying input flow quantities.

As stated above, the time step for the STUFF model follows the Courant condition for stability. The model actually has three time-step criteria: a diffusion-limited time step, an advection-limited time step, and an internal-wave-limited time step. The latter prevents the propagation of waves in the model from causing instabilities. The minimum of the three time steps is used, the one defined by the diffusion is typical for plume-type problems with significant turbulence. A conservative time step for STUFF is usually taken to be one-fourth of the maximum allowable.

The initial conditions must be specified for the flow field and each of the scalar fields solved for in STUFF. The flow field can be specified arbitrarily but must be nondivergent. An example is the specification of uniform flow in the x direction. Such a flow is derivable from potential flow and the given boundary flow. Specification of cell blocks as having given flow characteristics is permitted. There is also the option for specifying combinations of these blocks with the flow derived from a potential function to obtain a resultant field.

Scalar fields may be initialized in any combination of the following ways:

- (1) Specification of a series of rectangular blocks at a given concentration,
- (2) Specification of a vertical profile of arbitrary functional form,
- (3) Specification of a series of spheres within the grid with arbitrary centroidal positions and arbitrary radii (here segments of spheres lying outside the grid are ignored), and
- (4) Specification of a series of Gaussian distributions throughout the grid.

One advertised¹ feature of the code is the technique of "buffering" the LaGrangian particles to mass storage devices. Groups of 500 are retrieved from these devices, calculations are made on them in core, and they are then stored away again. This allows the specification of an unlimited number of particles. The computing time associated with buffering the particles, censusing the particles, updating the particles, etc., runs into a cost of $\approx 25\%$ of the total computing time. Doubling the number of particles in the simulation adds another $\approx 25\%$ to the computer time. This is not insignificant and is really a fairly large fraction of the total computer time but the authors of the model feel that buffering of the particles is worth the expense in terms of accuracy gained. This option is available at present only for the three-dimensional version of the code, STUFF 3.

Another feature of the code advertised¹ is the "restart" capability. Under this feature the calculation can stop at any point, at which time a dump to tape of all current values of the variables is executed. The calculation can then be restarted at a later time by reading the tape to initialize the code variables.

Concerning output of intermediate calculations, the code produces plots and listings of the field arrays at specified intervals. The code plots contours from the cell-centered values of the field array in terms of vertical slices; i.e., values in an x-y vertical slice are plotted through the midpoint of each cell in the x direction.

5. Application to Point Beach (Unit 1)

The first warm-water-plume application of the STUFF model was made for the Point Beach Nuclear Generating Station located on the western shore of Lake Michigan. This case was run and presented in Ref. 1 to indicate the ability of the STUFF model to treat thermal-plume problems. The date chosen was May 20, 1971, for which field data had been acquired¹¹ during two time periods: 1438-1550 (plume A) and 1740-1850 (plume B). The computer run

was carried out for the latter set of measurements. Table II.I.4 summarizes the plant and meteorological data for plume B as well as the initial conditions used in the STUFF model run for that date and time. Figures II.I.1-3 show the plume measurements at the surface and at 3- and 6-ft depths for plume B. Superimposed on those drawings is the STUFF grid that was used in the plume calculations for the plume from Unit 1. Figure II.I.4 illustrates the temperature measurements at various cross sections of plume B.

The rectangular STUFF grid is oriented at a 60° angle to shore in keeping with the direction of the Unit 1 discharge. In this way the momentum of the jet is directed along the main axis of the grid. The ambient current used in the model was simulated in the upcurrent boundary cells by resolving the 0.3 fps (125°) measurement into two components, each parallel to a grid axis.

Results of the calculated temperature profiles are given in Figs. II.I.5-7. Only predictions of temperature in the horizontal plane at 3-, 9-, and 15-ft depths are presented. Comparisons of the model to data appear in Figs. II.I.8-10 in terms of surface centerline temperature decay, surface temperature half-widths, and surface isotherm areas. Also included are data from plume A (May 20, 1971; 1438-1550) and a third plume C (October 28, 1971; 1422-1539), because the outfall and environmental conditions measured for these plumes were very similar to those of plume B. When the predicted results are compared to a number of measured plumes under like conditions we expect a more statistically accurate comparison.

TABLE II.I.4. Plant and Meteorological Data
and Initial Conditions Used in STUFF Run
for Point Beach Unit 1 Plume,
May 20, 1971

Plant and Meteorological Data

Plant name: Point Beach Nuclear Plant (Unit 1)
Date and time of Survey: 20 May 1971 (1740-1850)
Plant load: 474 MWe
Discharge flow rate: 885 cfs
Outfall temperature: 17.5°C
Intake temperature: 8.9°C
Ambient water temperature: $8.2\text{-}8.5^\circ\text{C}$
Dry-bulb temperature: $12.2\text{-}13.9^\circ\text{C}$
Wind speed and direction: 10-15 mph, 270°
Ambient current speed and direction: 0.3 fps, 125°
Lake level: 579.5 ft (IGLD)
Lake surface conditions: 0-1-ft waves

Initial Conditions Used in STUFF Run

Discharge from two stacked cells in
upper two layers

Top cell	420 cfs
Mid cell	336 cfs
Outfall temperature	17.5°C
Ambient water temperature	8.3°C
Ambient current (z direction)	0.1 fps

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)
 PLANT LOAD (MW): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTFALL TEMPERATURE: 17.5°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT WAVES

Fig. II.I.1. Thermal Plume at Surface for Point Beach Power
 Plant (Unit 1): May 20, 1971, 1740-1850 Hours¹¹

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)
 PLANT LOAD (MW_e): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTFALL TEMPERATURE: 17.5°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT WAVES

Fig. II.I.2. Thermal Plume at 3-ft Depth for Point Beach Power
 Plant (Unit 1): May 20, 1971, 1740-1850 Hours¹¹

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)
 PLANT LOAD (MW): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTFALL TEMPERATURE: 17.6°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT. WAVES

Fig. II.I.3. Thermal Plume at 9-ft Depth for Point Beach Power
 Plant (Unit 1): May 20, 1971, 1740-1850 Hours

Fig. II.I.4. Vertical Cross Sections at Point Beach Power Plant (Unit 1): May 20, 1971, 1740-1850 Hours¹¹

Fig. II.I.5

Contour Plot of Predicted Temperature at 3-ft Depth;
Time = 460 sec, STUFF Model: Point Beach (Unit 1),
May 20, 1971. (Adapted from England and Taft.¹)

Fig. II.I.6
Contour Plot of Predicted Temperature at 9-ft Depth;
Time = 460 sec, STUFF Model: Point Beach (Unit 1),
May 20, 1971. (Adapted from England and Taft.¹)

Fig. II.I.7

Contour Plot of Predicted Temperature at 15-ft Depth;
Time = 460 sec, STUFF Model: Point Beach (Unit 1),
May 20, 1971. (Adapted from England and Taft.¹)

Fig. II.I.8. Comparison of STUFF Predictions to Point Beach (Unit 1) Data for Three Plumes of Similar Environmental and Outfall Conditions (including May 20, 1971, 1740-1850 hours): Surface Centerline Temperature Decay

Fig. II.I.9. Comparison of STUFF Predictions to Point Beach (Unit 1) Data for Three Plumes of Similar Environmental and Outfall Conditions (including May 20, 1971, 1740-1850 hours): Surface Temperature Half-widths

Fig. II.I.10. Comparison of STUFF Predictions to Point Beach (Unit 1) Data for Three Plumes of Similar Environmental and Outfall Conditions (including May 20, 1971, 1740-1850 hours): Surface Isotherm Areas

Several aspects of the model simulation make the predictions tenuous. First, an inadvertent error was made by not changing the volumetric expansion coefficient. The value for air was used⁶ in previous runs of STUFF and this was not changed to the appropriate value for water. The value of the resulting predictions is thus doubtful and no strong implications from the model-data comparisons can therefore be made. Second, the model was run with a plant flow rate of 771 cfs while in actuality the discharge flow was 885 cfs. A note in the Addendum of Ref. 11 was apparently not noticed. Third, the STUFF model was run for only 460 seconds of prototype time and not to steady state. The concave-down appearance of the centerline temperature and area predictions is an indication that the plume has not completed its dispersion. This concave-down trend, in steady state, should persist only near the outfall, followed by a concave-up trend for the rest of the plume. Fourth, the grid chosen is too small to contain the plume (see Figs. II.I.1-3) and hence the boundary conditions are inaccurate. These boundary conditions were set assuming the plume to be contained entirely within the grid as an approximation. Note from Fig. II.I.5 that some of the higher-temperature isotherms can be closed only outside the grid (for negative z values).

The predictions presented in Figs. II.I.5-7 indicate that the plume, as calculated, is not very buoyant. The plume extends to depths of 9 and 15 ft. This nonbuoyant nature may be due to the erroneous value of the volumetric expansion coefficient employed in the calculations. Note that the 3-ft calculations indicate a warm area on the lee side of the outfall. This does occur to some degree in the field as a result of a slowly recirculating eddy of partially-mixed plume water. In all, this Point Beach comparison is not a fair test of the model due to the inadequacies in the model simulation for this case.

6. Application to Pilgrim Plant

The Boston Edison Company has sponsored S³ to run the STUFF model in order to validate it with data acquired at the Pilgrim Station located on Cape Cod Bay. The date chosen was December 29, 1972, in which aerial infrared measurements of the plume surface temperature were available from a study done by the Coastal Research Corporation¹² for Boston Edison. Vast, Inc.,¹³ also carried out thermal surveys and dye tracer studies by boat on that date for Boston Edison. The Vast, Inc. study furnished ground truth and the ambient temperature for its own study as well as the aerial infrared study. In addition, information on the vertical structure of the plume was determined by the Vast study for the Pilgrim plume. Figure II.I.11 represents the variation of some of the major outfall and environmental parameters as a function of time, as measured by Vast, Inc., for the period of the December 29 survey. Note that the power plant intake temperature remained between 40.5 and 41.8°F; temperatures taken from the condenser outfall were between 68.9 and 70.1°F. The boat study was carried out over 18 days from December 12 to 29, but the aerial infrared study was executed only for December 29.

Fig. II.I.11. Variation of Major Parameters at Pilgrim for the December 29, 1972, Study. (Adapted from Vast, Inc.¹³)

The aerial study was designed to give information over a full tidal cycle. Four flight missions were flown to coincide with the four peak tidal states: seven flight passes from 0918 to 0950 to cover ebb tide; 17 flight passes at low tide from 1204 to 1340; four flight passes from 1535 to 1555 for flood tide; and 13 passes from 1724 through high tide at 1837. Eleven of the total 41 passes were used for analysis in graphically representing the four tidal cycles. For the model-data comparison, the thermal imagery at 1302 hours was used in which the plume was under low tide conditions. The wind was from the northwest at 15-20 mph. For December 29, although the sky was clear, a strong, steady wind generated 2 to 3-ft waves that continued in a southerly long-shore direction, accounting for the similarity and orientation of all plumes analyzed for that date.

A few comments about the aerial and boat data are useful here. The data showed that the wind-driven currents dominated the tidal currents during all phases of the tidal cycle on December 29. During the 29th, the wind was from the northwest at 9 to 19 mph during the morning and from 6 to 15 mph during the afternoon. The wind-driven current clearly dominated the tidal currents so that no shift in the plume direction was apparent. Due to wind dominance, the major part of the plume periodically moved in a direction opposite to the expected tidal current. As indicated in preliminary surveys and current meter data taken prior to the Vast Inc. study, tidal currents in the region of the plant are weak and surface currents are dominated by the wind. Recirculation was observed as the isotherms of the plume extended past the intake structure due to a north wind.

The amount of recirculation determined from the dye studies on December 29, was about 3% at low tide and approximately 11% at high tide. Also notable were the rapid changes in area enclosed by the isotherms; this was the most striking characteristic of the flights at 1302, 1319, and 1331. As discussed in more detail in the Pilgrim study, Sec. D of Ch. IV, this is a major problem in using aerial infrared data. Rapid changes in centerline temperatures and surface areas over a short period of time are unrealistic and make the data of questionable validity. (Refer to Sec. IV.D for details.) The Coastal Research Corporation quotes the accuracy of the published isotherms to ± 2 F°.

Figure II.I.12 illustrates the region of Cape Cod Bay that was simulated in the STUFF model. An area 1000 m along the shore and 500 m offshore was modeled. The grid chosen has a longshore spacing $\Delta x = 20$ m, an offshore spacing $\Delta z = 20$ m, and a vertical spacing $\Delta y = 3$ m. This grid, mandated by computer limitations, is too small to include the entire plume and too coarse to resolve the fine detail of the plume. The core capacity of the UNIVAC 1108 allowed only two cells to be simulated in the vertical direction. Four or more cells would have been more suitable. The grid cells totaled 50 longshore, 25 offshore, and 2 in the vertical direction. Counting boundary cells, this becomes $52 \times 27 \times 4$.

Fig. II.I.12. Aerial Extent of Region Modeled by STUFF: Pilgrim Plant, December 29, 1972. (Adapted from Taft.²)

Figure II.I.13 illustrates the Eulerian mockup of the region. The dark cells illustrate areas that represent the shoreline; the gray cells indicate water of 3-m depth; the white cells indicate water of 6-m depth. This staircase simulation of bottom topography appears too coarse. The authors of the model state that their experience has shown that good results can be obtained using grids with even poorer resolution. They reason that the buoyant thermal plumes occupy only the first few feet of the water column and that their dynamic and energy interaction occurs predominantly in the upper layers. The authors state that the STUFF model is very sensitive to topographical mockup only when shallow regions occur due to submerged obstacles or hills. They conclude that a slight lack of resolution in bottom topography only slightly affects the solution. This argument is not very convincing. Furthermore, the Pilgrim plume stratifies in the intermediate and far fields, requiring good resolution in the vertical direction to define the plume adequately.

The intake and outfall are assumed, in the STUFF representation, to be in the upper of the two grid levels. The longshore side of the grid does not pass by the inlet port itself but some 80 m offshore. The authors assumed therefore, that the flow was mainly along the channel, the channel being five cells wide at the STUFF boundary, and hence the choice for the intake: five cells wide and one cell deep. For a 720 cfs flow into the intake, the intake velocity was determined to be

$$V_{\text{intake}} = \frac{720 \text{ ft}^3/\text{sec} \left(\frac{1}{3.281} \text{ m/ft} \right)^3}{(100 \text{ m})(3 \text{ m})} = 0.068 \text{ m/sec.}$$

Fig. II.I.13. STUFF Grid Mockup of Bottom and Shoreline Topography: Pilgrim Plant. (Adapted from Taft,²)

The outfall was simulated as one cell in the upper layer, hence 20 m wide and 3 m deep. The initial outfall velocity becomes

$$V_{\text{outfall}} = \frac{720 \text{ ft}^3/\text{sec} \left(\frac{1}{3.281} \text{ m/ft} \right)^3}{(20 \text{ m})(3 \text{ m})} = 0.34 \text{ m/sec.}$$

The ambient current present is simulated as the sum of the local tidal current and the wind-driven current. No ambient current was measured on December 29. Consequently, Boston Edison provided S³ with the Shureman formula¹⁴ to calculate the north-south and east-west components of the tidal current. The formula assumes that the tide can be decomposed into a discrete set of Fourier components whose amplitudes and phase angles depend on location and vary slowly during a given year. A table of amplitudes and phase angles for the important Fourier components in Cape Cod Bay near the Pilgrim Station were provided by Boston Edison. Utilizing the data, a calculation for 1302 on December 29, 1972, yielded a north-south component of -0.010 m/sec and an east-west component of 0.011 m/sec. (North and east are assumed to be directions of positive current flow.) The resultant tidal current was then calculated to be

$$[(0.011)^2 + (0.01)^2]^{1/2} = 0.015 \text{ m/sec},$$

with a direction almost exactly longshore, i.e., flowing towards the southeast.

The wind-driven component of the current was derived by S^3 by equating surface and bottom stresses under the assumption of a steady surface wind. The wind on December 29 was blowing almost exactly longshore toward the southeast at 15-20 mph. The 20-mph value was used in the wind-driven current calculation. Classical formulas from the literature were used to estimate the surface and bottom stresses. Equating those stresses yielded a velocity of 0.023 m/sec. This was assumed to be the mean current velocity over the depth of the fluid. The total current was summed to give $0.015 \text{ m/sec} + 0.023 \text{ m/sec} = 0.038 \text{ m/sec}$.

Fig. II.I.14. Ambient Longshore Current Profile: Pilgrim Plant Simulation of STUFF Model, December 29, 1972. (Adapted from Taft.²)

calculation is the ambient temperature, rather than the equilibrium temperature which is usually used in the literature. Surface heat transfer values varied for each surface grid cell. Surface heat loss was found to be insignificant in the calculation since the area covered by the STUFF grid was not large. The authors of the model estimate that with larger grids and longer residence times of the heat within the grid, surface heat loss will become more significant.

The aerial infrared measurements by CRC of surface temperature at 1302 hours are presented in Fig. II.I.15. The results of the STUFF run appear in Figs. II.I.16-19 with the horizontal isotherms and circulation patterns plotted for the upper and lower levels of the model. The flow lines indicate the general circulation pattern; the numbers next to the flow lines indicate the local magnitude of the flow velocity along the direction of the

Surface heat loss was calculated from formulas developed by Yotsukma.¹⁵ The advantage in the Yotsukma method is that the base temperature for the

Fig. II.I.16

Temperature Contours Predicted in Upper Level of STUFF Grid: Pilgrim Plant Simulation of STUFF Model, December 29, 1972; Undistorted Scale. (Adapted from Taft.²)

Fig. II.I.17. Temperature Contours Predicted in Lower Level of STUFF Grid: Pilgrim Plant Simulation of STUFF Model, December 29, 1972. (Adapted from Taft.²)

Fig. II.I.18. Flow in Upper Level of STUFF Grid: Pilgrim Plant Simulation of STUFF Model, December 29, 1972. (Adapted from Taft.²)

Fig. II.I.19. Flow in Lower Level of STUFF Grid: Pilgrim Plant Simulation of STUFF Model, December 29, 1972. (Adapted from Taft,²)

flow line. The intake and outfall flow has, in the STUFF simulation, created three eddies in the upper layer and one large eddy in the lower layer due to the near-breakwater between outfall and intake. The authors of the model state that these eddies immediately downcurrent of the outfall indicate the trapping of considerable amounts of heated effluent. The temperature rise in this region is more than 7°C above ambient. As the vertical velocities were negligible, they were not plotted.

Concerning possible recirculation through the plant, the temperature rise at the outer edge of the inlet breakwater is about $1.5\text{--}2.0^{\circ}\text{C}$ indicating some recirculation of heated effluent. However, the plume temperature has dropped to essentially ambient at the inlet cells themselves.

The aerial infrared data of 1302 hours were reduced and compared with the STUFF predictions. This work was done at Argonne; the results are presented in Figs. II.I.20-23 in terms of surface-centerline trajectory, surface-centerline temperature decay, surface-temperature half-widths, and surface

Fig. II.I.20. Comparison of STUFF Predictions with Pilgrim Plant Data, December 29, 1972, 1302 hours: Surface-centerline Trajectory

Fig. II.I.21. Comparison of STUFF Predictions with Pilgrim Plant Data, December 29, 1972, 1302 Hours: Surface-centerline Temperature Decay

Fig. II.I.22. Comparison of STUFF Predictions with Pilgrim Plant Data, December 29, 1972, 1302 Hours: Surface-temperature Half-widths

Fig. II.I.23. Comparison of STUFF Predictions with Pilgrim Plant Data, December 29, 1972, 1302 Hours: Surface Isotherm Areas

isotherm areas. The centerline temperature predictions are excellent. The half-widths and areas are larger than the data. The overall comparison is basically reasonable, based upon our experience with the quality of comparisons of other models to date. A critical discussion of the STUFF data comparison follows. Several points are discussed that tend to disqualify the comparison as a viable test of the model.

a. The aerial infrared data are of poor quality. Areas and centerline temperatures are often inconsistent among the 11 consecutive surveys taken over the single tidal cycle of December 29. For example, areas measured 17 min later at 1319 indicated a twofold-to-fivefold increase over those determined for 1302, which is physically improbable with the variation of environmental conditions that were present that day. This throws into doubt the accuracy and therefore utility of the data itself, which is discussed in greater detail in Ch. IV in the discussion of the Pilgrim site.

b. The calculation of ambient current is not correct. The tidal component is probably accurate, but the computation of the wind-induced current is without physical basis. Moreover, the value obtained for wind-induced current, namely 0.023 m/sec, is much too small for the wind-induced current resulting from a 20-mph wind. A rule-of-thumb, order-of-magnitude estimate of the current induced by the wind is 1 to 2% of the wind velocity. Wind-current studies done by Argonne National Laboratory on Lake Michigan during the summer and fall (1972-73) verified this order of magnitude estimate. From Fig. II.I.24 and the wind conditions of December 1972, we estimate the wind-induced current to be ~1% of the wind speed. Winter conditions might yield a

different correlation, however, but the graph of Fig. II.I.24 provides the best estimate that we feel is available. This 1% correlation would lead to a 0.09-m/sec current, a fourfold increase over the value used in the model calculation. So the wind-induced current, and hence the total current is probably much larger than simulated in the STUFF run. Recall that the tidal current was estimated at only 0.015 m/sec contributing only a small amount to the total current. In addition, what is not clear is that the tidal current should be added to the wind-induced current. It is conceivable that the tide and wind-induced currents might have been in opposing directions during the survey; in which case they should be subtracted with a predictable reversal of current direction occurring somewhere between surface and bottom to account for the tidally influenced bottom waters. In short, the STUFF run, with a smaller than expected ambient current simulated, should perhaps be compared more properly to a different set of low-tide data. A comparison of the STUFF model to areas and centerline temperatures for the entire low tide set of data appears in Figs. II.I.25 and 26. These kinds of additional comparisons would have been

Fig. II.I.24. Ratio of Current Speed to Wind Speed vs Direction (Current Meter Station A) Fall Season. (Adapted from O'Hagan.¹⁶)

Fig. II.I.25

Comparison of All Low-tide Data for Surface-centerline Temperatures (CRC Study) with STUFF Model Predictions

Fig. II.I.26. Comparison of All Low-tide Data for Surface Isotherm Areas (CRC Study) with STUFF Model Predictions

avoided had actual current measurements been made in the field at the time of the survey. A horizontal and vertical velocity profile of the ambient current would have removed the guesswork from determining ambient velocities for the plume simulation.

c. The simulation of outfall conditions by the STUFF grid also causes problems. The Pilgrim outfall, though trapezoidal (see Ch. IV), is simulated as rectangular, which is permissible, except that for low tide, a 3-m simulated depth of the outfall is too large. Using the STUFF input parameters

$$h_o = \text{outfall depth} = 3 \text{ m},$$

$$b_o = \text{outfall width} = 20 \text{ m},$$

$$u_o = \text{outfall velocity} = 0.34 \text{ m/sec},$$

$$T_o = \text{outfall temperature} = 21.1^\circ\text{C},$$

and

$$T_a = \text{ambient temperature} = 4.4^\circ\text{C},$$

the initial densimetric Froude number becomes 1.41, clearly a high-tide situation. Consequently, the STUFF simulation, based on the above schematization of the outfall, is a high-tide run; so the base data for comparison should be high-tide data. The comparison of the STUFF predictions to all the high-tide data taken on December 29, 1972, appears in Figs. II.I.27 and 28. The STUFF model

was run for the outfall and ambient temperatures measured at 1302 hours; these temperatures changed by only a few tenths of a degree from other data taken during that day. Fortunately, the wind conditions also did not change significantly between 1302 hours and high tide. Comparison of the model predictions for 1302 hours with all the data taken by CRC on December 29 appears in Figs. II.I.29 and 30.

Fig. II.I.27. Comparison of All High-tide Data for Surface-centerline Temperatures (CRC Study) with STUFF Model Predictions

Fig. II.I.28. Comparison of All High-tide Data for Surface Isotherm Areas (CRC Study) with STUFF Model Predictions

Fig. II.I.29. Replot of Surface-centerline Temperature Data from Coastal Research Study with STUFF Model Predictions, Emphasizing only Low-, Ebb-, Flood-, and High-tide Measurements

Fig. II.I.30. Replot of Surface Isotherm Area Data from Coastal Research Corporation Study with STUFF Model Predictions, Emphasizing only Low-, Ebb-, Flood-, and High-tide Measurements

d. The vertical and lateral resolution of the model and the simulation of bottom contours are quite coarse, as discussed above. Direct model-data comparison is difficult because the aerial infrared study measured temperatures in the top 0.1 mm of the water and the STUFF calculations presented one temperature for the top 3 m. Considering this, the STUFF temperatures should be quite a bit lower than the CRC measurements on a point-by-point basis.

e. The STUFF grid did not contain all the plume within its offshore boundary. The boundary conditions, then, are somewhat in error and the effect on the calculations cannot be known until a sensitivity analysis of the model is performed.

Some qualitative aspects of the predictions raise other questions. The eddies in the upper and lower grid levels as predicted by STUFF were not seen in the aerial infrared study or the MIT boat surveys (Sec. IV.D) of the Pilgrim plume. A more refined calculation horizontally and vertically needs to be done to see if they persist. Also, the zero recirculation directly into the intake was not observed in the field. As stated above, recirculation of ~3% for low tide and a recirculation of ~11% for high tide was estimated from the Vast, Inc., dye studies. Again, a more precise simulation of the field condition would be helpful in determining whether this is due to the model formulation or to the way this case was run.

Despite all these simulation problems, the STUFF model performed reasonably in terms of area, centerline temperature, and half-width predictions. An indirect cause of some of the difficulties with the predictions is the inability of the model to allow a fine grid to be used near and a coarser grid far from the outfall. More refinement in the vertical direction is probably needed. We believe that the model deserves further testing. Data should be used where uncertainties, such as current speed, are at a minimum.

7. Other Applications of STUFF Model

Although the STUFF model has been run for only two thermal plume problems, other runs of the model were carried out for other two- and three-dimensional flow situations. The other applications of the model are mentioned here along with references for further information.

a. The STUFF2 Highway Problem was a simulation⁴ with the 2-D version of STUFF. The problem was the determination of the local distribution of contaminant concentration from a highway surface, located below ground level, under the influence of a crosswind. The highway was simulated as a line source of pollution injecting contaminant continuously at the roadway surface, the contaminant representing automobile effluents. Two cases were simulated: one with, and one without, a building located upstream of the highway. The building had the effect of changing the flow pattern and developing

a large recirculating separated wake behind it that engulfed the entire highway. The presence of the building tended to trap the pollutant within the recirculation zone causing an increase in total solute concentration within the area of interest defined by the grid. However, the building presence caused more extensive turbulent dispersion of material within the wake, reducing peak and ground-level concentrations. For the simulation, a mesh spacing of $\Delta x = \Delta y = 3$ m was used with a total of 800 computational cells; four LaGrangian tracer particles were used per cell to carry the scalars.

The time development of the mean-flow field, the turbulent-energy distribution, the mean-flow kinetic energy, the contaminant isocon-tours, the total solute mass (in the computational region), the peak solute concentration, the height of the pollutant cloud, and, finally, the pollutant concentration at ground level were plotted. The calculation showed that a potential flow solution of the problem with no building simulated would probably provide reasonably accurate results. The second case, with the building simulated, required accurate mean flow and turbulence calculations in the region of separated flow for an accurate description of the pollutant dispersion; a potential flow solution is inadequate here.

No field data exist to verify the predictions of the model for these two cases.

b. The three-dimensional version of the model was used to solve a test problem simulating¹ the horizontal flow over a rectangular obstacle in a vertically stratified atmosphere. Stratification corresponding to a temperature lapse rate of one-half the dry adiabatic was used in the simulation; this was included in terms of the Boussinesq parameter Q . A linear upwind flow profile was assumed in the vertical direction; no crosswinds were specified. Since the wind profile is unidirectional, symmetry exists about a plane that bisects the obstacle in the direction of flow. A grid of eight cells in the horizontal and 5 in the vertical was used in the simulation taking advantage of the symmetry of the problem. After 50 time steps (non-steady state), the flow was found to accelerate around the obstacle more than over it. An eddy behind the obstacle was evident in the calculations. No information on time step size for the integration procedure was given; no data were presented for comparative purposes.

c. A two-dimensional viscous-wave problem was solved¹ to illustrate the lack of numerical diffusion in the STUFF solution for fluid density. The case considered was that of the oscillation of a viscous-damped internal wave in a channel of unit height acted upon by gravity. In the upper portion of the fluid, the density was 0.95, and in the lower portion, the density was 1.05. A stagnant situation was assumed initially. The Reynolds number of the flow was about 50. The thermal diffusivity was set equal to zero; consequently, the two fluids should not mix and the interface between them should remain sharply defined. The STUFF2 code and the purely Eulerian scheme of

Hirt and Cook¹⁷ were run to predict the fluid history. For both numerical solutions a 20 x 20 grid and a time step of 0.05 were used. The flow equations solved were identical in both cases. For the density-distribution solution, the Hirt-Cook method uses a pure Eulerian "donor-cell" first-order scheme;¹⁷ those calculations were compared to the LaGrangian/Eulerian scheme used in STUFF2.

The results showed a smearing of the interface of the two fluids in the Hirt-Cook density solution due to numerical diffusion. After $t = 25$, this smearing had spread over a region comparable to the initial wave height. The smeared flow affects the flow pattern due to the coupling of flow and density equations. Agreement between the Hirt-Cook scheme and STUFF2 was fairly good, however, until $t = 7$. Afterwards, the natural period of oscillation of the pure Eulerian solution began to increase with time. After a sufficiently long time, that frequency would eventually approach infinity with a uniformly mixed fluid field resulting, caused by numerical diffusion effects. The STUFF2 solution showed no smearing for the time interval covered in the calculation. This problem simulation was meant to prove that the numerical procedure in the STUFF code precludes numerical diffusion.

No data were presented for comparison, and no "exact" solution to the problem was presented to show that the STUFF2 solutions, at least those transient results presented, were accurate in themselves. Moreover, no other purely Eulerian schemes were tested and compared to the STUFF2 code.

8. Criticisms and Limitations

a. Limited Operational Experience in Application to Thermal Plumes

The Pilgrim calculations of the STUFF model indicated reasonable results. The model-data comparison is not a fair one because of the poor quality of the field data and the probably inaccuracy of the current velocity put into the model. It is not known what effect these factors had on the model-data discrepancies. Another problem is that the model was run with a poor choice of vertical step size simulating a high-tide plume rather than a low-tide one. Only a further sensitivity analysis of these model predictions to some key input parameters could determine how serious these deficiencies were in the Pilgrim comparisons.

The Point Beach comparison is not at all a fair test of the model since the run was carried out with an erroneous volumetric expansion coefficient and because the code was not run to steady state.

Our experience with the past computer runs of STUFF indicates that the model appears promising and that more work should be done in verifying its predictions.

b. The Gawain-Pritchett Turbulence Model

The turbulence model of Gawain and Pritchett is an important and integral part of the STUFF model, but has not been verified at all for the kinds of plume problems involving power plant discharges. The model was set up for turbulent incompressible flow without variable density effects; the slightly generalized version of the Gawain-Pritchett model, utilized when stratification effects are present, has not been verified for even simpler flow problems. The presentation of eddy viscosity and eddy diffusivity values generated in the plume by the STUFF model would be very useful. Analyzing the calculated values of viscosities and diffusivities in relation to our expectations based upon empirical and theoretical considerations would allow us to better assess the utility of the Gawain-Pritchett formulation. To date this has not been done in any of the published predictions presented for the model. A sensitivity analysis of the STUFF model to different viscosity and diffusivity laws would also be useful, even a sensitivity analysis to some of the parameters intrinsic to the Gawain-Pritchett model of turbulence.

The STUFF computer code has the turbulence scheme built into it in a modular fashion; in this way other turbulence schemes or turbulence laws may easily be used in the plume calculations.

c. Fixed Grid Structure

At present the STUFF model cannot treat variable grid sizes. Although the cell sizes may differ in the x, y, and z directions, the sizes must be equal for any one direction. This restriction in the model hampers its application when large increases in dimensions exist in a particular application. A fine grid structure near and a coarser grid far from the outfall would improve resolution and accuracy in the calculations, for a given amount of computer time. The authors of the model recognize this limitation in the code and propose to add variable grid spacings.

Pritchett notes⁷ that the reason the STUFF model was not developed with a variable grid structure is because of the nature of the particle scheme. Particle cells of different sizes can be very inefficient because a sufficiently high particle density in the whole grid is needed to properly represent the variables in the smallest cell; much computer time and storage is spent in treating those particles. The problem resides in any migration of the flow for a general application of the STUFF model. If a mass of fluid, represented by a fixed number of particles in a coarse zone, migrates over to a finely resolved region, then the fine grid system will likely not have enough particles per cell to properly represent the variables in the context of that fine grid. Some new algorithm needs to be added to the computer code to sense that condition and thereby increase the particle resolution for that mass. The opposite problem can occur. A fluid mass in a finely resolved

region may migrate to a coarsely resolved region. Here numerous particles may reside in a single cell, many more than are needed to determine the cell-based variables. The computer time required to track all of these superfluous particles could be costly. A remedy to these problems is an algorithm for particle creation and destruction. Pritchett notes⁷ that the inclusion of such a variable grid procedure in the STUFF model would take a fair amount of additional work but should proceed in a fairly straightforward manner.

The authors of the STUFF model envision a further improvement in the model, the addition of triangular cells. Presently, the boundaries are represented via rectangular cells at the outer edges of the grid structure. Internal and external boundaries are not always easily modeled in this way. They recommend the addition of obstacle cells that are triangular (wedge-shaped) in nature. In this way topographic shelving zones would be more easily modeled through the inclusion of appropriate rectangular/triangular cell systems. This would avoid some of the inaccuracies in the rectangular staircase type representation of sloping boundaries. Irregular bottom features can be more accurately modeled by judicious combinations of triangular and rectangular cells.

d. Free-slip Boundary Conditions

The STUFF model in its present form uses free-slip boundary conditions on all fluid-solid interfaces. These boundary conditions ignore the drag that is generated from flow over the bottom or against the shoreline boundaries. The Point Beach and Pilgrim calculations were carried out with these free-slip conditions. The use of the free-slip instead of the no-slip condition is an obvious omission, but we cannot assess its effect in the Point Beach and Pilgrim cases without a second run of the model with no-slip included. Taft notes⁶ that the no-slip condition would be implemented as a boundary condition in the intermediate or "tentative" step in the flow field calculations and not in the velocity corrector potential solution. Taft and Pritchett indicate^{6,7} that the requirement of no-slip at solid boundaries would be an easy change to implement in the code.

e. Inefficiency of Flow Calculations

A minor limitation is the inefficiency of the momentum equation solution. The differencing of the advection terms was improved three times in the Pilgrim simulation to allow faster computation. This was done during preliminary runs before a final Pilgrim prediction was computed.

The solution to the momentum equations is carried out in STUFF by the Gauss-Seidel iterative technique. A direct method such as fast Fourier transforms, odd-even reduction, nested bisection, etc., should provide a more efficient solution. This was not done in the development of the model because the solution of the momentum equations typically ran only 25-30% of

the total computer time; it was not deemed at the time to be worth the work involved. According to Taft,⁶ an efficient scheme for the momentum equations solution can be added to the STUFF code in a short time.

f. Computer Costs

The Pilgrim computer run required 4 hr of central processor time on a UNIVAC 1108 for steady state to be reached. Although the time of computation is not excessive in present numerical work, it greatly limits the worth of the model as a practical tool. The advent of faster computers might change that conclusion however.

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J. Till Numerical Model^{1,2}

1. Synopsis of Approach

Till has developed a computer model to treat the steady-state dispersion of thermal plumes in rivers. The model is an adaptation of COBRA,³ a code that has been used extensively to analyze heat transfer in the channels of a nuclear reactor core. A subchannel balancing technique is used in which rectangular parallelepiped subchannels are laid out in the direction of flow. At each location, the net flow of mass, momentum, and heat into each subchannel is balanced against the rate of change of the corresponding variable within the subchannel. Modeling plume dispersion becomes a matter of solving for the appropriate convective and turbulent transport variables between adjacent subchannels.

The approach yields a coupled set of ordinary differential equations that can be integrated numerically by a Euler predictor-corrector scheme. The set of equations is solved as an initial-value problem with the integration carried out in the direction of flow. For each conserved quantity, one ordinary differential equation is generated for each subchannel and that equation is coupled to the equations of adjacent subchannels through the exchange velocities. The independent variable in these equations is the downstream distance from the point of discharge.

A number of important assumptions made in the model are not explicitly stated but are implicit in the development. A better understanding of the subtle assumptions made and their effects on model predictions were obtained from analyzing the three prototype applications of the model. The model has been applied to two thermal discharge problems in rivers: a submerged discharge in the Mississippi River and a lateral surface discharge on the Ohio River. The predictions were not qualitatively correct, yet the precise reason for the discrepancies was difficult to ascertain. A third application² to heated discharges into Lake Michigan (see App. B) provided a clearer picture of the model and revealed that the model had serious problems in transporting momentum laterally and vertically, i.e., in the directions normal to the axis of the main flow. For this reason and others discussed in this review, the model is not recommended for either river or lake applications without further significant improvements in its formulation. Consequently, the formulation and applications of the model are reviewed below with emphasis on the areas of needed improvement.

2. List of Symbols

A	Cross-sectional area of the river
A_x	Longitudinal cross-sectional area
A_y	Lateral cross-sectional area

A_z	Vertical cross-sectional area
c_p	Specific heat at constant pressure
D	Hydraulic diameter of the river, $4A/P$; diameter of circular submerged discharge
d	Depth of river
E	Equilibrium temperature
F_g	Gravitational force
F_o	Initial densimetric Froude number
F_s	Shear force
f	Darcy friction factor
g_c	Gravitational constant
H	Depth of water
h	Enthalpy
K	Surface-heat-transfer coefficient
k	von Karman constant
M	Maximum number of subchannels adjacent to the subchannel of interest
$\dot{m}$	Mass flow rate
N	Maximum number of subchannels in the river cross section modeled
P	Wetted perimeter of river cross section
p	Pressure
$\dot{Q}$	Heat rate addition or loss per unit length due to sources and sinks
Re	Reynolds number based on D
Re_H	Reynolds number based on R_h
R_h	Hydraulic radius; A/P
s	Half-width of river
T	Temperature
T_s	Temperature of water surface
u, u_x	Longitudinal velocity
u_*	Friction velocity
$\bar{u}$	Average river velocity

$u_{\max}$	Maximum velocity of the river
v, u_y	Lateral velocity
w, u_z	Vertical velocity
w	Wind speed
w	Diversion crossflows between adjacent subchannels
w'	Turbulent crossflows between adjacent subchannels
x	Longitudinal space variable
y	Lateral space variable
z	Vertical space variable
z_s	Depth of shallow submerged circular discharge
Δh	Enthalpy lost by surface heat transfer
$\Delta T_{\max}/\Delta T_o$	Ratio of maximum, surface excess temperature (above ambient) to outfall excess temperature (above ambient)
$\Delta x, dx$	Grid spacing in longitudinal direction
$\Delta y, dy$	Grid spacing in lateral direction
$\Delta z, dz$	Grid spacing in vertical direction
ϵ_h	Eddy diffusivity of heat
ϵ_{hz}	Vertical eddy diffusivity of heat
ϵ_m	Eddy diffusivity of momentum
ϵ_{mz}	Vertical eddy diffusivity of momentum
θ	Slope of river
ν	Kinematic viscosity of water
ξ_y	$\sqrt{1 - (y/s)}$ (y measured from river bank)
ξ_z	$\sqrt{1 - (z/d)}$ (z measured from river bed)
ρ	Density of water (in units of lb_f/ft^3)
τ_x	Longitudinal shear stress
τ_y	Lateral shear stress
τ_z	Vertical shear stress

3. Derivation of Equations

The original set of equations for the Till model, derived in Till's doctoral dissertation, contained two errors. The bottom friction term had a

gravitational constant inadvertently placed in the denominator, and the Reynolds number used to define the river (and plume) longitudinal velocity was four times too large. The Rush Island and Philip Sporn cases were run before these inadvertent errors were noted. Till expects the second error to be much less significant than the reduction in bottom friction in the model. For the Point Beach application, the errors were corrected and, in addition, some terms thought to be important for a lake plume application were added. The river equations are derived in this section with the corrections presented. The additional terms added by Till for lake applications are also noted. A brief discussion of the revised equations appears in App. B.

The Till model is developed by conceptually dividing the river into longitudinal subchannels as shown in Fig. II.J.1. Mass, momentum, and energy balances are written for each subchannel. The term "diversion crossflow" is used in the Till model to represent convective transport from one subchannel to another. Turbulent crossflow is also allowed between adjacent pairs of cells. These turbulent and advective velocities between subchannels are the means by which mass, momentum, and energy are exchanged between adjacent subchannels. These are the only two forms of mixing, other than through pressure differences simulated in the basic equations. Figure II.J.2 illustrates a typical subchannel with turbulent and diversion crossflows shown.

Fig. II.J.1

Nodalization Procedure of the River. (Adapted from Till.¹)

The derivation of the model equations is summarized in Tables II.J.1-5 with Figs. II.J.3-7 showing the corresponding subchannel balancing. The term in the dashed box (see Table II.J.3) represents the term that was in error in the original Till river model and computer code. Its corrections are noted.

Table II.J.6 traces the terms in Till's river plume and lake plume model to terms in the basic Navier-Stokes equations of fluid flow. Table II.J.7 lists the unknown crossflows that enter each of the conservation equations. Further details concerning the development of the equations appear in Refs. 1 and 2 and App. B. The latest version of the model, the one used for the lake plume applications, may also be used for river plume applications. Verification of the two versions of the model is described below in Secs. 6-8.

Fig. II.J.2. Subchannels under Mass, Energy, and Momentum Exchange by Diversion and Turbulent Crossflows. (Adapted from Till.¹)

TABLE II.J.1. Continuity Equation: Till's River and Lake Plume Model

Performing a mass balance on subchannel i illustrated in Fig. II.J.3 yields

$$\dot{m}_i + w_{ji} dx + w_{li} dx + w_{ki} dx + w_{mi} dx + \dot{m}_i + d\dot{m}_i + w_{ik} dx + w_{lk} dx + w_{ij} dx + w_{im} dx + w_{il} dx + w_{im} dx, \quad (A)$$

where

$\dot{m}_i$ = mass flow rate in the longitudinal direction at subchannel i ,

w_{ij} = diversion crossflow from subchannel i into subchannel j (the other crossflows are obtained by permuting indices),

and

w'_{ij} = turbulent crossflow from subchannel i into subchannel j (the other crossflows are obtained by permuting indices).

Since the time average of the turbulent crossflows is zero,

$$w'_{ij} = w'_{ji}, \quad w'_{il} = w'_{li}, \quad w'_{ik} = w'_{ki}, \quad w'_{im} = w'_{mi}, \quad (B)$$

causing the turbulent terms to drop out of the mass conservation equation. Hence,

$$\frac{d\dot{m}_i}{dx} = w_{ji} + w_{li} - w_{ik} - w_{im} \quad (C)$$

or, equivalently,

$$\frac{d\dot{m}_i}{dx} = - \sum_{j=1}^M w_{ij} \quad i = 1, 2, 3, \dots, N, \quad (D)$$

where

N = maximum number of subchannels in the river cross section modeled,

and

M = maximum number of subchannels adjacent to the subchannel of interest; i.e., 1, 2, 3, or 4.

TABLE II.J.2. Energy Equation: Till's River and Lake Plume Model

Performing an energy balance on subchannel i illustrated in Fig. II.J.4 yields

$$\begin{aligned} & \dot{m}_i h_i + w_{ji} h_j dx + w'_{ji} h_j dx + w'_{ki} h_k dx + w_{\ell i} h_{\ell} dx + w'_{\ell i} h_{\ell} dx + w'_{mi} h_m dx \\ & = \dot{m}_i h_i + d(\dot{m}_i h_i) + w_{ik} h_i dx + w'_{ik} h_i dx + w'_{ij} h_i dx + w_{im} h_i dx \\ & + w'_{im} h_i dx + w'_{i\ell} h_i dx + \dot{Q}, \end{aligned}$$

where

h_i = enthalpy of subchannel i

and

$\dot{Q}$ = heat rate addition or loss per unit length of subchannel.

Using Eq. B of Table II.J.1 on the turbulent crossflows and the continuity condition of Eq. C of Table II.J.1, the energy equation becomes

$$\begin{aligned} \dot{m}_i \frac{dh_i}{dx} &= -\dot{Q}_i + w_{ji}(h_j - h_i) + w_{\ell i}(h_{\ell} - h_i) + w'_{ij}(h_j - h_i) + w'_{ik}(h_k - h_i) \\ &+ w'_{i\ell}(h_{\ell} - h_i) + w'_{im}(h_m - h_i). \end{aligned}$$

Distinguishing the diversion and turbulence crossflows, this becomes

$$\dot{m}_i \frac{dh_i}{dx} = -\dot{Q}_i + \sum_{J=1}^M w'_{iJ}(h_J - h_i) + \sum_{J=1}^M \begin{cases} 0 & \text{if } w_{iJ} \geq 0, \\ w_{iJ}(h_i - h_J) & \text{if } w_{iJ} < 0; \end{cases}$$

$i = 1, 2, 3, \dots, N.$

TABLE II.J.3. The x-Momentum Equation: Till's River and Lake Plume Model

The rate of change of x-momentum in subchannel i is related to the external forces on that subchannel (see Fig. II.J.5) by

$$\begin{aligned} A_x p_i - A_x p_i - d(A_x p_i) - F_s + F_g = & w'_{ij} u_{xi} dx + w'_{ik} u_{xi} dx + w'_{ik} u_{xi} dx + w'_{il} u_{xi} dx \\ & + w'_{im} u_{xi} dx - w'_{ji} u_{xj} dx - w'_{ki} u_{xk} dx + w'_{im} u_{xi} dx - w'_{ji} u_{xj} dx - w'_{li} u_{xl} dx \\ & - w'_{mi} u_{xm} dx - w'_{li} u_{xl} dx + \dot{m}_i u_{xi} + d(\dot{m}_i u_{xi}) - \dot{m}_i u_{xi}. \end{aligned}$$

Using Eq. B of Table II.J.1 on turbulent crossflows, along with

$$(a) \quad \frac{d(\dot{m}_i u_{xi})}{dx} = \frac{d(\rho_i A_x u_{xi}^2)}{dx} = 2\dot{m}_i \frac{du_{xi}}{dx} + A_x u_{xi}^2 \frac{d\rho_i}{dx},$$

and the expression for gravitational force F_g

$$(b) \quad \frac{F_g}{\Delta x} = \rho_i A_x \cos \theta,$$

and the shearing force F_s

$$(c) \quad \frac{F_s}{\Delta x} = \tau_x = \rho_i \epsilon_{my} \frac{du_{xi}}{dy} + \rho_i \epsilon_{mz} \frac{du_{xi}}{dz},$$

yields

$$\begin{aligned} \frac{dp_i}{dx} = \rho_i \cos \theta \left[\frac{\tau_x \Delta x}{A_x g_c} \right] - \frac{2\dot{m}_i}{A_x g_c} \frac{du_{xi}}{dx} - \frac{u_{xi}^2}{g_c} \frac{d\rho_i}{dx} + \frac{1}{A_x g_c} \sum_{j=1}^M w'_{ij} (u_{xj} - u_{xi}) \\ - \frac{1}{A_x g_c} \sum_{j=1}^M \begin{cases} w_{ij} u_{xi} & \text{if } w_{ij} \geq 0, \\ w_{ij} u_{xj} & \text{if } w_{ij} < 0; \end{cases} + \frac{\tau_x \Delta x}{A_x} + \boxed{\frac{\tau_{wx} \Delta x}{A_x}}. \end{aligned}$$

$i = 1, 2, 3, \dots, N,$

The dashed box indicates a term in error in the Till river model and its associated computer code. The arrow shows the correct term. The error in the shear term equation reduced the effectiveness of the bottom friction in the model for the Rush Island and Philip Sporn applications. The term in the dashed box was corrected in the lake-plume application only. The wind stress term at the surface located in the solid box was added for the lake application to the right-hand side of the above equation.

For the lake-plume application, the river-bed-slope angle θ was taken as 90° .

TABLE II.J.4. The y-Momentum Equation: Till's River and Lake Plume Model

The rate of change of y-momentum in subchannel i is related to the external forces on the subchannel (see Fig. II.J.6) by

$$A_y p_i - A_y p_i - d(A_y p_i) = \dot{m}_{yi} u_{yi} + d(\dot{m}_{yi} u_{yi}) - \dot{m}_{yi} u_{yi} + w'_{i\ell} u_{yi} dy + w'_{im} u_{yi} dy \\ + w_{im} u_{yi} dy - w'_{\ell i} u_{y\ell} dy - w'_{mi} u_{ym} dy - w_{\ell i} u_{y\ell} dy.$$

Using the derivative of the acceleration term

$$\frac{d(\dot{m}_{yi} u_{yi})}{dy} = \frac{d(\rho_i A_y u_{yi}^2)}{dy} = 2\dot{m}_{yi} \frac{du_{yi}}{dy} + A_y u_{yi}^2 \frac{d\rho_i}{dy},$$

and Eq. B of Table II.J.1 yields

$$\frac{dp_i}{dy} = + \frac{F_s}{A_y \Delta y} - \frac{2\dot{m}_{yi}}{A_y g_c} \frac{du_{yi}}{dy} - \frac{u_{yi}^2}{g_c} \frac{d\rho_i}{dy} + \frac{1}{A_y g_c} \sum_{J=\ell, m} w'_{iJ} (u_{yJ} - u_{yi}) \\ - \frac{1}{A_y g_c} \sum_{K=\ell, m} \begin{cases} w_{iK} u_{yi} & \text{if } w_{iK} \geq 0, \\ w_{iK} u_{yK} & \text{if } w_{iK} < 0; \end{cases} \\ i = 1, 2, 3, \dots, N.$$

The lateral velocity and mass flow rates are easily expressed in terms of the lateral diversion crossflows by

$$u_{yi} = \frac{w_{iJ} \Delta x}{\rho_i A_y}$$

and

$$\dot{m}_{yi} = w_{iJ} \Delta x.$$

For the river model adaptation, Till neglects the turbulent terms with the assumption that the lateral momentum equation will itself be important only near the outfall. Till also drops the shear term for the river and lake model due to lack of information on the velocity distribution for a fluid in a crossflow. This term may, however, be important. The final form of the y-momentum equation becomes

$$\frac{dp_i}{dy} = - \frac{2w_{iJ}(\Delta x)^2}{\rho_i A_y g_c} \frac{dw_{iJ}}{dy} - \frac{w_{iJ}^2(\Delta x)^2}{\rho_i A_y g_c} \frac{d\rho_i}{dy} - \frac{\Delta x}{A_y g_c} \sum_{K=\ell, m} \begin{cases} w_{iK} \frac{w_{iJ}}{\rho_i} & \text{if } w_{iK} \geq 0, \\ w_{iK} \frac{w_{KJ}}{\rho_K} & \text{if } w_{iK} < 0; \end{cases} \\ i = 1, 2, 3, \dots, N, \\ + \boxed{\sum_{J=\ell, m} \frac{w'_{iJ}}{A_y g_c} (u_{yJ} - u_{yi})} + \boxed{\frac{\tau_{wy} \Delta y}{A_y}} + \boxed{\frac{u_x(\Delta x)}{g_c A_y} \frac{dw_{iJ}}{dx}}.$$

In the above equation, the Kth subchannel is a vertically connected adjacent subchannel and the Jth subchannel is a laterally connected subchannel. The terms in the boxes were added to the lake plume model. The first box represents the turbulent diffusion terms; the second box, the wind stress term; and the third box, the convective term $(-\rho_i u_x / g_c)(\partial u_y / \partial x)$.

TABLE II.J.5. The z-Momentum Equation: Till's River and Lake Plume Model

The rate of change of z-momentum in subchannel i is related to the external forces on the subchannel (see Fig. II.J.7) by

$$A_z p_i - A_z p_i - d(A_z p_i) - F_s + F_g = \dot{m}_{zi} u_{zi} + d(\dot{m}_{zi} u_{zi}) - \dot{m}_{zi} u_{zi} + w_{ik} u_{zi} dz + w'_{ik} u_{zi} dz \\ + w'_{ij} u_{zi} dz - w_{ji} u_{zj} dz - w'_{ji} u_{zj} dz - w'_{ki} u_{zk} dz.$$

Here only turbulent and diversion crossflows were considered in the lateral directions because the river convection is assumed to dominate the longitudinal transport.

Using the following identity for the derivative of the acceleration term

$$\frac{d(\dot{m}_{zi} u_{zi})}{dz} = \frac{d(\rho_i A_z u_{zi}^2)}{dz} = 2\dot{m}_{zi} \frac{du_{zi}}{dz} + A_z u_{zi}^2 \frac{d\rho_i}{dz},$$

along with Eq. B of Table II.J.1, yields,

$$\frac{dp_i}{dz} = \frac{F_g}{A_z} - \frac{F_s}{A_z} - \frac{2\dot{m}_{zi}}{A_z g_c} \frac{du_{zi}}{dz} - \frac{u_{zi}^2}{g_c} \frac{d\rho_i}{dz} + \frac{1}{A_z g_c} \sum_{J=j,k} w'_{iJ} (u_{zJ} - u_{zi}) \\ - \frac{1}{A_z g_c} \sum_{J=j,k} \begin{cases} w_{iJ} u_{zi} & \text{if } w_{iJ} \geq 0, \\ w_{iJ} u_{zJ} & \text{if } w_{iJ} < 0; \end{cases} \\ i = 1, 2, 3, \dots, N.$$

The vertical velocity and mass flow rates are easily expressed in terms of the vertical diversion crossflows by

$$u_{zi} = \frac{w_{iK} \Delta x}{\rho_i A_z}$$

and

$$\dot{m}_{zi} = w_{iK} \Delta x.$$

The Till river model neglects the turbulent terms because he expects the vertical momentum equation to be important only near the outfall. They are retained, however, for the lake plume application. The shear stress is also neglected (river and lake versions) due to lack of information on plumes in crosscurrents. This latter assumption is probably not a significant one. The final form for the z-momentum equation becomes

$$\frac{dp_i}{dz} = \rho_i \sin \theta - \frac{2w_{iK}(\Delta x)^2}{\rho_i A_z^2 g_c} \frac{dw_{iK}}{dz} - \frac{w_{iK}^2(\Delta x)^2}{\rho_i A_z^2 g_c} \frac{d\rho_i}{dz} - \frac{\Delta x}{A_z^2 g_c} \sum_{J=j,k} \begin{cases} w_{iJ} \frac{w_{iK}}{\rho_i} & \text{if } w_{iJ} \geq 0, \\ w_{iJ} \frac{w_{JK}}{\rho_J} & \text{if } w_{iJ} < 0; \end{cases} \\ i = 1, 2, 3, \dots, N, \\ + \left[\sum_{J=j,k} \frac{w'_{iJ}}{A_z g_c} (u_{zJ} - u_{zi}) \right] + \left[\frac{u_x(\Delta x)}{g_c A_z} \frac{dw_{iK}}{dx} \right],$$

where the Kth subchannel is a vertically connected adjacent subchannel, and the Jth subchannel is a laterally connected adjacent subchannel. Added in the lake plume version model were the turbulence terms and a convection term, $(\rho_i u_x / g_c) (\partial u_z / \partial x)$, where the river bed slope angle was taken as 90° .

Upper Left: Top View: Lateral Crossflows

Lower Left: Front View: Vertical and Lateral Crossflows

Lower Right: Side View: Vertical Crossflows

Fig. II.J.3. Diagram of Mass Balance with Crossflows. (Adapted from Till.¹)

Upper Left: Top View: Lateral Crossflows

Lower Left: Front View: Vertical and Lateral Crossflows

Lower Right: Side View: Vertical Crossflows

Fig. II.J.4. Diagram of Energy Balance with Crossflows. (Adapted from Till.¹)

Upper Left: Top View: Lateral Crossflows

Lower Left: Front View: Vertical and Lateral Crossflows

Lower Right: Side View: Vertical Crossflows

Fig. II.J.5. Diagram of x-Momentum Balance with Crossflows. (Adapted from Till¹)

Fig. II.J.6. Diagram of y-Momentum Balance with Vertical Crossflows. (Adapted from Till,¹)

Fig. II.J.7. Diagram of z -Momentum Balance with Lateral Crossflows. (Adapted from Till.¹)

TABLE II.J.6. Till Model Equations in Differential Form for the River and Lake Plume Model

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Energy equation:

$$\rho c_p u \left(\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left(\epsilon_h \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\epsilon_h \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\epsilon_{hz} \frac{\partial T}{\partial z} \right)$$

x-momentum equation:

$$\rho \left(\boxed{u \frac{\partial u}{\partial x}} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \rho g_c \cos \theta + \frac{\partial}{\partial y} \left(\epsilon_m \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\epsilon_{mz} \frac{\partial u}{\partial z} \right)$$

y-momentum equation:

$$\rho \left(\boxed{u \frac{\partial v}{\partial x}} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left(\epsilon_{mz} \frac{\partial v}{\partial z} \right)$$

z-momentum equation:

$$\rho \left(\boxed{u \frac{\partial w}{\partial x}} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_c \sin \theta + \frac{\partial}{\partial y} \left(\epsilon_m \frac{\partial w}{\partial y} \right)$$

Terms within solid boxes were added for the lake plume application. In the above equations, the river-bed-slope angle θ was taken as 90° in the lake plume applications. Surface pressure elevations are not considered in the model.

TABLE II.J.7. List of Unknowns in the Equations for the Till River and Lake Plume Model

<u>Diversion Crossflows</u>	<u>Turbulent Crossflows</u>
w_{ji}	w'_{ji}
w_{li}	w'_{li}
w_{ik}	w'_{ik}
w_{im}	w'_{im}

Continuity equation: $w_{ji}, w_{li}, w_{ik}, w_{im}$

Energy equation: $w_{ji}, w_{li}, w_{ik}, w_{im}; w'_{ji}, w'_{li}, w'_{ik}, w'_{im}$

x-momentum equation: $w_{ji}, w_{li}, w_{ik}, w_{im}; w'_{ji}, w'_{li}, w'_{ik}, w'_{im}$

y-momentum equation: $w_{il}, w_{im}; \boxed{w'_{li}, w'_{im}}$

z-momentum equation: $w_{ji}, w_{ik}; \boxed{w'_{ji}, w'_{ik}}$

Substitutions are made for the following:

$$\left. \begin{aligned}
 u_{xi}, \quad \dot{m}_{xi} &= u_{xi}\Delta x, \\
 u_{yi} &= \left(\frac{w_{ij}\Delta x}{\rho_i A_y} \right), \quad \dot{m}_{yi} = w_{ij}\Delta x, \\
 \text{and} \\
 u_{zi} &= \left(\frac{w_{ik}\Delta x}{\rho_i A_z} \right), \quad \dot{m}_{zi} = w_{ik}\Delta x.
 \end{aligned} \right\}$$

The terms in the solid boxes above were added for the lake plume model and did not appear for previous river plume applications.

4. Assumptions on the Longitudinal Velocity, Eddy Viscosities, Eddy Thermal Diffusivities, and Surface Heat Loss

Additional assumptions on the longitudinal velocity profile, eddy viscosity, and eddy thermal diffusivity coefficients are required before a solution can be obtained. The assumptions for the river plume model are discussed first, followed by a discussion of the adaptation of the model for lake applications. The river plume is considered by Till to be a steady flow with nearly all transport in the downstream direction. Lateral and vertical transport are both considered to be of secondary importance.[†] The river velocity profile is assumed to be given by

$$u_x = u_{\max} + \frac{u_*}{2k} \left[\xi_y + \ln(1 - \xi_y) + \xi_z + \ln(1 - \xi_z) \right], \quad (1)$$

where

u_x = downstream velocity,

$u_{\max}$ = maximum river velocity,

u_* = friction velocity,

k = von Karman constant,

$\xi_y = \sqrt{1 - (y/s)}$ (y measured from river bank),

$\xi_z = \sqrt{1 - (z/d)}$ (z measured from river bed),

s = half-width of the river (a function of depth),

and

d = depth of the river (a function of lateral position).

This form comes from arithmetically averaging the two one-dimensional von Karman logarithmic velocity distributions⁴ for open channel flow

$$u_x = u_{\max} + \frac{u_*}{k} \left[\sqrt{1 - (z/d)} + \ln(1 - \sqrt{1 - (z/d)}) \right] \quad (2)$$

and

$$u_x = u_{\max} + \frac{u_*}{k} \left[\sqrt{1 - (y/s)} + \ln(1 - \sqrt{1 - (y/s)}) \right]. \quad (3)$$

The attempt of Till to introduce two-dimensionality into the essentially one-dimensional velocity distribution by arithmetically averaging

[†]This is not explicitly stated as a model assumption, but work with the model indicates that it is true. This secondary importance of lateral and vertical transport represents a shortcoming of the model.

the components is questionable. The effect of this assumption on plume characteristics is probably profound, but as no sensitivity study has been made its precise effect is not known. A more realistic profile of the river velocity should be used. Recall the assumption in the Till river model that this distribution for longitudinal velocity is valid not only upstream of the discharge but in the plume itself and downstream of the plume. Applications of the code have indicated that the plume must indeed be a second order effect superimposed on the above velocity distribution before realistic predictions can be expected. For river plumes, this implies a large current with only little perturbation by the momentum of the discharge itself. For lake plume applications, Till used the component of ambient velocity in the x direction to define u_x . For the stagnant lake application of the Till model, u_x is set to zero. In the latter two cases, no ambient dominating flow existed and this very likely contributed to the poor predictions.

The parameters in Eq. 1 are determined as follows for the river plume model. The friction velocity in Eq. 1 is assumed to be given by

$$u_* = \bar{u}\sqrt{f/8}, \quad (4)$$

where f is a friction factor obtained by solving the equation⁵

$$\frac{1}{\sqrt{f}} = 2 \log_{10}(\text{Re}\sqrt{f}) + 0.4. \quad (5)$$

The Reynolds number Re in the above equation is defined by

$$\text{Re} = \frac{D\bar{u}}{\nu} \quad (6)$$

where

D = hydraulic diameter of the river = $4A/P$,

$\bar{u}$ = average river velocity,

ν = kinematic viscosity of river water,

A = area of flow,

and

P = wetted perimeter.

The maximum river velocity is assumed to be given by

$$u_{\max} = \sqrt{\frac{4gD \cos \theta}{f}}, \quad (7)$$

where g is the acceleration due to gravity.

Assumptions are also made for the turbulent exchange of momentum and energy between adjacent subchannels that Till represents by means of turbulent crossflow velocities. The turbulent crossflow of momentum is given by

$$w'_m = \rho \epsilon_m, \quad (8)$$

where ϵ_m is an eddy diffusion coefficient of momentum assumed to have the form⁶

$$\epsilon_{my} = 2ku_*(s - y)(1 - \xi_y) \quad (9)$$

in the lateral direction, and

$$\epsilon_{mz} = 2ku_*(d - z)(1 - \xi_z) \quad (10)$$

in the vertical direction. Till averages these two values to get a two-dimensional effect for a particular subchannel. Averaging lateral and vertical viscosities and using only those averaged values in both directions is definitely wrong, because the diffusivities are not usually of similar magnitude. Also, it is not clear that the viscosities chosen above to represent river flow are compatible with the velocity profile assumed in Eq. 1.

Till assumes that the spread of heat is greater than the spread of momentum. The turbulent crossflow of heat is given by⁷

$$w'_h = \rho \epsilon_h, \quad (11)$$

where

$$\epsilon_h = 1.85\epsilon_m. \quad (12)$$

The turbulent crossflows are shown in Fig. II.J.3 for an individual subchannel.

A major difficulty with the river plume model is in the choice of these eddy coefficients. The above form for the eddy diffusion coefficients indicates a zero value at the river shoreline and at midriver, with a peak reached somewhere in between. Holley⁸ indicates that the lateral eddy diffusion in rivers is greatest near the shore and least at midriver, varying smoothly over the river width. The particular form of the eddy diffusion coefficients must be substantiated before the model can be used with confidence in rivers.

Further difficulties exist with the definitions of the velocity profile, hydraulic radius, and friction factor. The velocity profile assumed for the river, as stated above, is based on von Karman's velocity-distribution law for turbulent pipe flow. Schlichting⁹ derives this distribution from von Karman's similarity hypothesis, indicating that the friction velocity is given by

$$u_* = \bar{u} \sqrt{f/8}, \quad (13)$$

where f is related to the Reynolds number based on the hydraulic diameter of the pipe

$$D = \frac{4A}{P}, \quad (14)$$

through the equation

$$\frac{1}{\sqrt{f}} = 2 \log_{10}(\text{Re} \sqrt{f}) - 0.8. \quad (15)$$

In open channel flow, the Reynolds number is customarily based on the hydraulic radius of the open channel given by

$$R_h = \frac{A}{P} = \frac{D}{4}. \quad (16)$$

Thus, the Reynolds number commonly used in the literature of open channel flow⁵ is only a quarter of that used for turbulent pipe flow. If the Reynolds number based on the hydraulic radius of Eq. 16 is used, Eq. 15 above becomes

$$\frac{1}{\sqrt{f}} = 2 \log_{10}(\text{Re}_H \sqrt{f}) + 0.4 \quad (17)$$

(since $\log_{10} 4 \approx 0.6$). Till uses Eq. 17 but defines the Reynolds number in terms of the hydraulic diameter, which is an inconsistent use of the equation, an error that was present in the Rush Island and Philip Sporn calculations. This river profile was not needed in the Point Beach applications.

In calculating the maximum river velocity, Till uses the following formula:

$$\begin{aligned} u_{\max} &= \sqrt{\frac{16gA \cos \theta}{fP}} \\ &= \sqrt{2} \bar{u}, \end{aligned} \quad (18)$$

where $\bar{u}$ is the average river flow velocity. This formula appears to us to be completely unjustified in the model development.

A further assumption in the Till river model leads to difficulties. The river is idealized as a uniformly sloping open channel making angle θ with the vertical direction. This sloping bottom provides the driving force for downstream river flow. As a result, the method of inputting the data to the computer code is awkward. Values must be guessed for the river velocity;

outfall velocity, and river slope from which the code calculates the actual initial conditions. The results then have to be compared with available data and new guesses made when the comparison is inadequate, an altogether inconvenient and untrustworthy procedure. From the viewpoint of the model user, such a method for inputting data is very troublesome. This problem was corrected in the lake plume version of the model.

The simulation of heat exchange with the atmosphere at the water surface is accounted for using the formula

$$\Delta h = -K(T_s - E), \quad (19)$$

where

Δh = the enthalpy transferred,

K = an empirical surface-heat-transfer coefficient,

T_s = surface temperature of the river,

and

E = equilibrium temperature of the river.

Heat loss is not expected to be significant in the near and intermediate fields where model predictions are most important. Because the model is numerical, the use of a linearized form for ΔH is not necessary; inclusion of the entire form for the various components for ΔH on a subchannel by subchannel basis would have been more accurate. More information concerning ΔH is given in App. B.

Keep in mind while reading the Rush Island and Philip Sporn applications in Secs. 6 and 7 that the runs were made with the following errors in the code:

- a. The bottom shear term in the x-momentum equation should not contain the gravitational acceleration constant in the denominator.
- b. The Reynolds number was incorrectly defined in the calculation of the longitudinal velocity for the river and plume, Eq. 17.

Those runs were also made with the untrustworthy input procedure. Each of these problems was corrected in the lake plume cases, but other, more significant, difficulties with the model exist as we further indicate.

5. Numerical Solution of Governing Equations

The correct set of equations for the Till model (river and lake applications) appears in Table II.J.8. The solution procedure, outlined in App. B, is conceptually the same for the river and lake plumes. Only a brief outline of the numerical technique is given here. More details appear in App. B and Ref. 1.

TABLE II.J.8. Final (Correct) Equations of the Till Model

Continuity Equation:

$$\frac{dm_i}{dx} = - \sum_{J=1}^M w_{iJ}$$

Energy Equation:

$$\dot{m}_i \frac{dh_i}{dx} = -\dot{Q}_i + \sum_{J=1}^M w'_{iJ}(h_J - h_i) + \sum_{J=1}^M \begin{cases} 0 & \text{if } w_{iJ} \geq 0, \\ w_{iJ}(h_1 - h_J) & \text{if } w_{iJ} < 0; \end{cases}$$

$i = 1, 2, 3, \dots, N.$

x Equation of Motion: ($\theta = 90^\circ$)

$$\frac{dp_i}{dx} = \frac{\tau_x \Delta x}{A_x} + \frac{\tau_{wx} \Delta x}{A_x} - \frac{2\dot{m}_i}{A_x g_c} \frac{du_{xi}}{dx} - \frac{u_{xi}^2}{g_c} \frac{d\rho_i}{dx} + \frac{1}{A_x g_c} \sum_{J=1}^M w'_{iJ}(u_{xJ} - u_{xi})$$

$$- \frac{1}{A_x g_c} \sum_{J=1}^M \begin{cases} w_{iJ} u_{xi} & \text{if } w_{iJ} \geq 0, \\ w_{iJ} u_{xJ} & \text{if } w_{iJ} < 0; \end{cases}$$

$i = 1, 2, 3, \dots, N.$

y Equation of Motion: ($\theta = 90^\circ$)

$$\frac{dp_i}{dy} = \frac{\tau_{wy} \Delta y}{A_y} - \frac{2w_{iJ}(\Delta x)^2}{\rho_i A_y^2 g_c} \frac{dw_{iJ}}{dy} - \frac{w_{iJ}^2(\Delta x)^2}{\rho_i^2 A_y g_c} \frac{d\rho_i}{dy} - \frac{u_x(\Delta x)}{A_y g_c} \frac{dw_{iJ}}{dx}$$

$$- \frac{\Delta x}{A_y^2 g_c} \sum_{K=\ell, m} \begin{cases} w_{iK} \frac{w_{iJ}}{\rho_i} & \text{if } w_{iK} \geq 0 \\ w_{iK} \frac{w_{KJ}}{\rho_K} & \text{if } w_{iK} < 0 \end{cases} + \frac{1}{A_y g_c} \sum_{J=\ell, m} w'_{iJ}(u_{yJ} - u_{yi});$$

$i = 1, 2, 3, \dots, N.$

z Equation of Motion: ($\theta = 90^\circ$)

$$\frac{dp_i}{dz} = \rho_i - \frac{2w_{iK}(\Delta x)^2}{\rho_i A_z^2 g_c} \frac{dw_{iK}}{dz} - \frac{w_{iK}(\Delta x)^2}{\rho_i^2 A_z g_c} \frac{d\rho_i}{dz} - \frac{u_x(\Delta x)}{A_z g_c} \frac{dw_{iK}}{dx}$$

$$- \frac{\Delta x}{A_z^2 g_c} \sum_{J=j, k} \begin{cases} w_{iJ} \frac{w_{iK}}{\rho_i} & \text{if } w_{iJ} \geq 0 \\ w_{iJ} \frac{w_{JK}}{\rho_J} & \text{if } w_{iJ} < 0 \end{cases} + \frac{1}{A_z g_c} \sum_{J=j, k} w'_{iJ}(u_{zJ} - u_{zi});$$

$i = 1, 2, 3, \dots, N.$

The solution consists of two steps. In the first step, the diversion crossflows are determined by solving a linear algebraic system using the Gauss-Seidel iterative technique. In the second step, using these diversion crossflows, the mass, energy, and momentum equations are integrated along the direction of river flow employing the Euler predictor-corrector method. In order to obtain the algebraic system, remove the pressure gradient terms from the equations of Table II.J.8 by taking pressure differences between the longitudinal momentum equation written for two adjacent subchannels. The algebraic system that results is very sparse in the case of the river model version. The matrix reduction is somewhat more complicated in the case of the lake model version because of the additional terms included.

6. Rush Island Plant Application

The first application of the Till model¹ was for the single-port submerged discharge at the Rush Island Power Plant on the Mississippi River near Crystal City, Missouri. Unfortunately, no experimental data (tank or field) is available for model comparison. The model predictions were, however, compared to the Union Electric Company's predictions for the submerged plume that appeared in the draft environmental statement¹⁰ for the power plant. This is not as satisfactory as a direct comparison to experimental data. The comparison of the Till and Union Electric Company predictions showed differences that were also seen in later comparisons between the Till model and field data (see Secs. 7 and 8).

The plant location is given in Fig. II.J.8. The basic parameters for the plant and environmental conditions appear in Table II.J.9. The actual discharge is through a single-port outlet of 11-ft diameter located about 40 ft from midchannel. The discharge is oriented down current (coflow), but at an angle of 20° from the river bottom. The velocity of discharge is about six times the average river velocity.

Fig. II.J.8

Rush Island Power Plant on Mississippi River. (Adapted from Till.¹)

TABLE II.J.9. Mean Atmospheric and River Conditions for Month of August, and Maximum Discharge Situation for Rush Island Plant

<u>Atmosphere</u>	
Temperature	79°F
Percent Relative Humidity	76
Wind Speed (ground level)	6 mph
<u>River</u>	
Depth at Discharge	24 ft
Width at Discharge (main channel)	1000 ft
Temperature	86.5°F
Flow Rate	60.7×10^3 cfs
Velocity	2.25 ft/sec
<u>Heated Effluent under Maximum Discharge</u>	
Plant Power Level	106.5°F
Flow Rate	1.34×10^3 cfs
Velocity	14.1 ft/sec

The system of subchannels was set up differently in the near- and far-field regions of the plume. The near field had a rectangular array of eleven subchannels laterally and four vertically. Each subchannel was 16 ft wide and 6 ft high. In this way, four subchannels piled vertically would cover the surface-to-bottom region of the river. The extent of the lateral subchannels did not cover the entire river width, as the plume was not expected to spread significantly in the lateral direction in the near field. The circular port is approximated by a single subchannel. The port area is 95.0 ft², and the subchannel area is 96.0 ft². The area of discharge is essentially the same, but the rectangular subchannel is, unfortunately, a distortion of the actual circular discharge. This distortion is discussed in more detail below. The actual configuration of subchannels and the location of the discharge (subchannel 39) is shown in Fig. II.J.9. The discharge is directed from subchannel 39

Fig. II.J.9. Configuration of Subchannel and Crossflow Arrangement for Near Region (0 to 500 ft) of Submerged Discharge. (Adapted from Till.1)

to subchannel 28 by specification of a diversion crossflow, forcing the transport at the first downriver step to account for the 20° upward angle of discharge from the horizontal plane.

Figures II.J.10 and 11 compare the Till model predictions to the Union Electric Company predictions in terms of isotherms predicted for the longitudinal section passing through the outfall. The Till predictions indicated a plume that did not rise as rapidly as the Union Electric predictions.

Fig. II.J.10. Calculated Values for Isotherms of Subsurface Discharge--Rush Island Plant, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.11. Side View of Subsurface Isotherms Near Single-port Discharge, Rush Island Plant, Union Electric Company Predictions. (Adapted from Till.¹)

A significant difference in surface centerline temperatures also exists. The Till model predicts a $1.0\text{ }^{\circ}\text{F}$ rise 200 ft down current from the discharge, a $2.0\text{ }^{\circ}\text{F}$ rise at 350 ft, and a $2.5\text{ }^{\circ}\text{F}$ rise at 500 ft. The Union Electric predictions show these temperatures occurring within 40 ft of the discharge. Till suspects that the longer distance needed to transport the heated effluent to the surface is due partly to the way the model simulates the discharge through the diversion crossflow mechanism. A second reason suggested by Till that might be responsible for the lack of significant vertical spreading in the near field is the choice of a rectangular subchannel ($16 \times 6\text{ ft}$) to represent a circular discharge. The lack of a variable subchannel structure in the Till model prevented the use of a square subchannel for the outfall because then the river depth would not be evenly divisible by the vertical dimension of the subchannel. Bear in mind that the Till predictions are being compared to Union Electric's prediction; there is no guarantee that the latter predictions are correct.

Figures II.J.12 and 13 compare cross-sectional predictions at a distance of 200 ft down current. The Till predictions do not show as great a lateral spread as do the Union Electric predictions. The rectangular simulation of the circular discharge is considered by Till to be the probable cause. We suspect, however, that simulating an 11-ft circular pipe by a slot 16 ft wide would result in greater lateral spreading for the Till model. Moreover, the far-field predictions maintain the same trend as do the near-field predictions.

The far field is treated by defining the new set of subchannels that appear in Fig. II.J.14 for downstream distances from 500 to 3000 ft. The subchannels are 50 ft long, 50 ft in lateral extent, and 6 ft deep. Smaller variations in velocities and temperatures in this region make this coarser grid permissible to save computer time. The total lateral extent of this new grid is much greater and nearly represents the full profile of the river. The total

Fig. II.J.12. Cross-sectional View of Calculated Isotherms at 200 ft Downstream from Rush Island Plant Submerged Discharge, Till Model Predictions. (Adapted from Till.1)

Fig. II.J.13. Cross-sectional View of Subsurface Isotherms at 200 ft Downstream from Single-port Submerged Discharge for Rush Island Plant, Union Electric Company Predictions. (Adapted from Till.¹)

Fig. II.J.14. Configuration of Subchannel and Crossflow Arrangement for Far Region (500 to 3000 ft) of Submerged Discharge. (Adapted from Till.¹)

grid section is 16,800 ft² and the river cross section is 18,000 ft². Till links the near- and far-field subchannel structure by adjusting the values calculated at the end of the near field. The precise method is not discussed; the use of conservation laws is presumed.

Predictions of the Till and Union Electric models are given in Figs. II.J.15-16 and 17-18 for distances of 600 and 1000 ft, respectively, from the outfall. These cross-sectional views indicate an approximately fully-mixed plume in the Union Electric case. The predictions of the Till model appear qualitatively incorrect. Figures II.J.15 and 17 indicate cooler water lying above warmer water! This seemingly unstable temperature distribution could occur in the near field as the jet is still rising due to buoyancy and inertial effects, but not in the far field where momentum forces are generally dissipated. The Union Electric isotherms appear qualitatively consistent in terms of a stable temperature field. Here again the Union Electric predictions show greater vertical and lateral spreading as in the near field. Recall that the

Fig. II.J.15. Cross-sectional View of Calculated Isotherms at 600 ft Downstream from Rush Island Plant Submerged Discharge, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.16. Cross-sectional View of Subsurface Isotherms at 600 ft Downstream from Single-port Submerged Discharge for Rush Island Plant, Union Electric Company Predictions. (Adapted from Till.¹)

Fig. II.J.17. Cross-sectional View of Isotherms at 1000 ft Downstream from Rush Island Plant Submerged Discharge, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.18. Cross-sectional View of Subsurface Isotherms at 1000 ft Downstream from Single-port Submerged Discharge for Rush Island Plant, Union Electric Predictions. (Adapted from Till.¹)

Union Electric predictions are not experimental data and the Till model cannot be considered as giving wrong predictions due to any quantitative disagreement between the two predictions.

The only experimental data we are aware of that is available for comparison with the Till predictions is the study of Partheniades et al.¹¹ Based upon their hydraulic model study, Partheniades et al. present a design curve (see Fig. II.J.19) to allow predictions of minimum surface dilution (maximum surface temperature rise) for a horizontal circular discharge in shallow water. For the Rush Island discharge with a densimetric Froude number of 12.0 and the ratio of water depth to pipe diameter of about 3.2, the dilution factor predicted is ≈ 0.27 and the maximum surface temperature should then be approximately 5.4°F above ambient or about 91.9°F . The Partheniades data and diagram are for a horizontal discharge; the Rush Island discharge is at 20° angle to the river bottom. One would therefore expect a higher maximum surface temperature for the Rush Island case due to the shorter trajectory to the surface and consequently the reduced entrainment. Table II.J.10 shows the temperatures by subchannel, predicted by the Till model, for the near field at 0, 100, 300, and 500 ft from the outfall. The highest surface temperature (examining subchannels 1-11) is 89.02°F for subchannel 7, at a distance of 500 ft from the discharge. Clearly, the surface boil and the maximum surface temperature occurs in the Till model beyond 300 ft from the discharge. From Fig. II.J.17, showing cross-sectional temperatures at 1000 ft downstream, the boil has yet to occur! In that figure, the heated water is mixing but the isotherms appear to be incorrectly receding toward the bottom. Till notes that the plume does not lift off the bottom in the region of calculation. Furthermore, the trend in the predictions is for the plume to return into the river bed from whence it came.

Fig. II.J.19

Maximum Surface Temperature Rise $\Delta T_{\max}/\Delta T_0$ in Terms of Relative Submergence and Initial Densimetric Froude Number of a Horizontally Discharged Circular Jet in Shallow Water. (Adapted from Partheniades et al.¹¹)

TABLE II.J.10. Subchannel Temperatures as a Function of
Distance from Discharge Point in Rush Island Plume
Simulation: Till Model (from Till¹)

Subchannel	0'	100'	300'	500'	Temperature Change 0 to 500 ft
	°F	°F	°F	°F	°F
1	86.50	86.50	86.54	86.73	.23
2	86.50	86.50	86.67	87.07	.57
3	86.50	86.52	86.96	87.62	1.12
4	86.50	86.60	87.45	88.35	1.85
5	86.50	86.77	88.01	89.02	2.52
6	86.50	86.75	87.83	88.95	2.52
7	86.50	86.77	88.01	89.02	2.52
8	86.50	86.60	87.45	88.35	1.85
9	86.50	86.52	86.96	87.62	1.12
10	86.50	86.50	86.67	87.07	.57
11	86.50	86.50	86.54	86.73	.23
12	86.50	86.50	86.56	86.78	.28
13	86.50	86.51	86.73	87.15	.65
14	86.50	86.56	87.11	87.80	1.30
15	86.50	86.76	87.86	88.77	2.27
16	86.50	87.40	89.00	89.97	3.47
17	86.50	88.53	90.04	90.77	4.27
18	86.50	87.40	89.00	89.97	3.47
19	86.50	86.76	87.86	88.77	2.27
20	86.50	86.56	87.11	87.80	1.30
21	86.50	86.51	86.73	87.15	.65
22	86.50	86.50	86.56	86.78	.28
23	86.50	86.50	86.58	86.82	.32
24	86.50	86.52	86.79	87.25	.75
25	86.50	86.62	87.33	88.06	1.56
26	86.50	87.10	88.52	89.44	2.94
27	86.50	89.00	90.86	91.62	5.12
28	86.50	94.27	94.21	94.80	8.30
29	86.50	89.00	90.86	91.62	5.12
30	86.50	87.10	88.52	89.44	2.94
31	86.50	86.62	87.33	88.06	1.56
32	86.50	86.52	86.79	87.25	.70
33	86.50	86.50	86.58	86.82	.32
34	86.50	86.50	86.59	86.83	.33
35	86.50	86.52	86.84	87.32	.82
36	86.50	86.67	87.52	88.28	1.78
37	86.50	87.46	89.21	90.07	3.57
38	86.50	91.05	93.17	93.78	7.28
39	106.50	104.54	101.50	98.21	-8.29
40	86.50	91.05	93.17	93.78	7.28
41	86.50	87.46	89.21	90.07	3.57
42	86.50	86.67	87.52	88.28	1.78
43	86.50	86.52	86.84	87.32	.82
44	86.50	86.50	86.59	86.83	.33

The fully-mixed condition predicted by Union Electric seems reasonable. Surface impingement should occur near the outfall due to the large momentum of discharge, the shallow water, and the 20° angle of discharge. If the outfall were a long slot of depth 11 ft, discharging horizontally, the surface impingement should occur¹² at approximately 2.5 times the depth of the discharge or 60 ft from the outfall. For a 20° angle discharge it should occur sooner. The estimate so obtained is still probably approximately correct for the circular discharge being simulated. Our best estimate, based upon the literature, indicates that the Till model greatly underestimates buoyancy and inertial effects in the vertical direction. The simulation of the thermal discharge as a second order effect imposed upon the current appears inadequate even in this high-velocity coflow problem.

It appears that the Till plume never fully rips the surface but continues to travel submerged along the x (downcurrent) axis. This anomaly may be related to a problem seen in the Point Beach application of the model. Because the model is unable to transport momentum in the lateral and vertical directions, lateral momentum there was quickly transferred to longitudinal momentum for no well-defined reason. This probably happens in the Rush Island case, momentum directed vertically is being transferred quickly to longitudinal momentum. Consequently surface impingement would be impossible or at least long delayed. The model therefore appears to be incapable of treating jet problems involving bending trajectories.

Till ran the Rush Island case a second time, assuming the discharge temperature to be 66.5°F , to test the sensitivity of the model to buoyancy. Figures II.J.20 and 21 show the results in terms of a comparison of cross-sectional isotherms for the positively buoyant and negatively buoyant cases. The comparison shows a reasonable configuration of the isotherms for the negatively buoyant case. Figures II.J.20 and 21 are baffling, however, for one would expect the positively buoyant plume to spread laterally more than the negatively buoyant plume due to the relative densities in the dispersion terms for each case.

Fig. II.J.20. Calculated Isotherms for Heated Discharge at 200 ft Downstream, Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.21. Calculated Isotherms for Cooled Discharge at 200 ft Downstream, Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

Till also carried out an interesting sensitivity analysis of the subchannel size. Runs were made with the lateral and vertical subchannel dimensions decreased to one-half and increased to twice the values used for the Rush Island case. Figures II.J.22 and 23 show the Rush Island predictions at 200 ft downcurrent with two different grid sizes: 10 x 8 x 6 ft and 10 x 32 x 6 ft. Decreasing the lateral dimension of the subchannel from 32 to 8 ft caused greater vertical spreading and less lateral spreading. The predictions look more physically reasonable with the small subchannel width. Till attributes the improvement to the use of two discharge subchannels and two vertical diversion crossflows, instead of one, to model the discharge. Consequently, Till recommends the use of multiple subchannels and diversion crossflows for large momentum jets.

Figures II.J.24 and 25 show the effect of changing subchannel height. Here the Rush Island results at 200 ft downstream are compared for

Fig. II.J.22. Calculated Isotherms for Subchannel Dimensions 10 x 8 x 6 ft at 200 ft Downstream; Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.23. Calculated Isotherms for Subchannel Dimensions 10 x 32 x 6 ft at 200 ft Downstream; Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.24. Calculated Isotherms for Subchannel Dimensions 10 x 16 x 3 ft at 200 ft Downstream; Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

Fig. II.J.25. Calculated Isotherms for Subchannel Dimensions 10 x 16 x 12 ft at 200 ft Downstream; Rush Island Case, Till Model Predictions. (Adapted from Till.¹)

grid configurations of 10 x 16 x 3 ft and 10 x 16 x 12 ft. The decrease in vertical dimension decreased vertical spreading and increased lateral spreading. The results unfortunately indicate a more unstably stratified situation for the smaller vertical grid spacing. The increased vertical spreading using the larger 12 ft vertical dimension occurs because the surface subchannel receives a portion of the thermal discharge directly.

Till found (not shown) that the model was basically insensitive to changes in size of the longitudinal dimension of the subchannels. Values in the neighborhood of 20 ft were tested.

Till next tried a subchannel optimization investigation to ascertain whether better results could be obtained with different subchannel dimensions. Since the model is relatively insensitive to the longitudinal dimension, only variation in the lateral and vertical dimensions were considered. The model predictions were found to be very sensitive to the cross-sectional area of the subchannels in the region tested. Subchannel cross-sectional areas varying from 50-200 ft² were tested by varying either the lateral or vertical dimension only. The Till model predictions appeared less sensitive to dimension variations in large subchannels than smaller ones.

Fig. II.J.26. Philip Sporn Power Plant on Ohio River with Field Data Locations. (Adapted from Till.¹)

7. Philip Sporn Surface Discharge Application

The second application of the Till model¹ was to the Philip Sporn Power Plant on the Ohio River. This plant is fossil-fired with a generating capacity of 1050 MWe. The heated effluent is discharged at a 45° angle to the current from a discharge channel 80 ft wide by 20 ft deep. The river is about 1000 ft wide at the discharge with a midchannel depth of 27 ft. Figure II.J.26 sketches the site and Table II.J.11 lists the river, outfall, and environmental conditions on August 23, 1967. On that date, field data were taken¹³ by the Department of Environmental Engineering at the Johns Hopkins University for the American Electric Power Service Company. This field data was acquired by making boat transects at cross sections A, B, C, D, and E located as shown in Fig. II.J.26. The ambient current was larger than the discharge

velocity implying that the plume was dominated by the advection of the river. A plume that hugs the near shore and slowly disperses laterally is the type expected and indeed appeared in the field data.

TABLE II.J.11. Atmospheric and River Conditions for August 23, 1967, and Maximum Discharge Situation for Philip Sporn Plant

Atmospheric Conditions

Average Temperature	67.2°F
Average Relative Humidity	91%
Average Wind Speed (ground level)	5 mph

River Conditions

Ambient Temperature	78.3°F
Flow Rate of River	19,000 cfs
Average Current Speed (maximum)	1.13 ft/sec

Heated Effluent under Maximum Discharge

Outfall Temperature	89.6°F
Plant Flow Rate	1,400 cfs
Discharge Velocity	0.87 ft/sec

Nondimensional Parameters of Interest

Initial Densimetric Froude Number	0.73
Outfall Aspect Ratio (ratio of width to depth)	4.0
Ratio of Outfall Velocity to Ambient Current	0.85

The Till simulation was carried out using the subchannel configuration given in Fig. II.J.27. The lateral subchannel size was chosen as 80 ft and the vertical subchannel size as 5 ft. The longitudinal size was chosen as 80 ft to coincide with the outfall width. The total grid system encompasses 53 subchannels and nearly covers the entire river cross section, i.e., the grid covers 21,200 ft² and the actual average river cross section is 24,000 ft². Till simulates the discharge as coming from the left-hand cells (1, 14, 27, 38) in the grid (see Fig. II.J.27).

Fig. II.J.27. Configuration of Subchannel and Crossflow Arrangement for Oblique Surface Discharge, Philip Sporn Plant. (Adapted from Till.¹)

The Till model is compared to the field data in Figs. II.J.28-30 for cross sections C, D, and E located 1000, 3000, and 4400 ft, respectively, downcurrent from the discharge. Our comments about the comparison follow.

a. The model grossly underpredicts the intrusion of the plume into the river. The predictions appear qualitatively incorrect as the predicted plume is nearly fully mixed (vertical isotherms), whereas the measured plume is more buoyant (lens-shaped isotherms) and spreads laterally more quickly.

Fig. II.J.28. Isotherms from Field Data and Calculated Values for Philip Sporn Power Plant at Position C, Approximately 1000 ft Downstream from Discharge Point. (Adapted from Till.¹)

Fig. II.J.29. Isotherms from Field Data and Calculated Values for Philip Sporn Power Plant at Position D, Approximately 3000 ft Downstream from Discharge Point. (Adapted from Till.¹)

Fig. II.J.30. Isotherms from Field Data and Calculated Values for Philip Sporn Power Plant at Position E, Approximately 4400 ft Downstream from Discharge Point. (Adapted from Till.¹)

The isotherm temperatures predicted at each cross section are the correct ones, but their location relative to the data within the cross section is quite different.

b. It appears that buoyancy is only weakly operative in the model. This is important and might be due to neglect of surface pressure variations in the model. The neglect of jet entrainment indicates that the model cannot simulate the near field, which might be the most significant reason for the small lateral movement of the heated water and the lack of a reasonably bending trajectory. Till suggests that the model-data discrepancy is partially due to the neglect of a 5-mph wind that was present on that date and directed across the river from the discharge side to the far side of the river. We are doubtful that a relatively small wind of this nature operating with a small fetch could be a significant factor in inducing such lateral mixing.

c. The reduced lateral spreading in the predictions might also be due (in part) to the simulation of the discharge. Markowsky¹⁴ of American Electric Power has experimented with the model and determined that the intrusion of the plume into the river can be more accurately modeled if a non-uniform initial flow profile is used. Realizing that the initial densimetric Froude number is a mere 0.73, the plume should be partially stratified as it enters the river. Also since the initial densimetric Froude number is less than unity, a cold-water wedge should enter the discharge channel. As this occurs, more heat will be discharged in the upper layers than the lower portion of the discharge canal. Instead of discharging 25% of the outflow into each of the four vertical levels as simulated by Till, Markowsky set up the discharge with 50% into the surface level, 35% into the second level, 15% into the third level, and 0% into the bottom level. This modification produced significant improvement in the model predictions.

d. As discussed in a previous section, the eddy diffusivity is probably incorrectly modeled. Till assumes that the eddy diffusivity near the shore is small. Because the discharge is essentially confined to only the four shoremost subchannels, the plume may be artificially forced to hug the shoreline. Once the plume is near the shore, it cannot spread as fast due to the small diffusivity near the shore.

e. The terms containing derivatives of diversion crossflows shown in the y- and z-momentum equations of Table II.J.8 were not included in the version of the model used to obtain the predictions given in Figs. II.J.28-30. These terms were added in a later version of the model (for the Point Beach application) and may result in improved predictions of plume intrusion and spreading in the river.

Till, again, carried out some interesting sensitivity tests.¹ The first involved the sensitivity of the model to buoyancy. An additional run was made with the Philip Sporn discharge at a cooled temperature of 67.0°F. This temperature corresponds to an excess temperature of equal magnitude and opposite sign compared to the actual discharge to the river simulated above. Figures II.J.31 and 32 make the comparison of heated and cooled jets at 1000 ft downcurrent. Like the heated jet, the cooled jet does not spread very rapidly laterally and appears to be almost fully mixed. The sinking nature of the plume is not observed. A plume fully submerged due to the long downstream distance from the discharge is to be expected. Whether buoyancy has been properly simulated in the model is questionable.

The next sensitivity test carried out was for subchannel dimensions. Varying the length of the subchannel from 40 to 120 ft changed the temperature distribution at 1000 ft downcurrent only slightly. An attempt to use a channel length of 160 ft resulted in too large a step size and failed. Figures II.J.33 and 34 indicate the effect of varying the lateral subchannel dimension from 40 to 160 ft. Greater lateral spreading occurred with the larger lateral dimension. The qualitative nature of the isotherms (i.e., showing an almost fully mixed condition) is unchanged. Figures II.J.35 and 36 show the effect of changing the vertical dimension of the subchannel from 2.5 to 10 ft. The larger dimension again produced greater mixing in that direction; i.e., an even more fully mixed condition for the 10-ft vertical dimension. The Philip Sporn subchannel sensitivity analysis indicated that greater spreading in a particular direction (lateral or vertical) is achieved by making subchannel dimensions larger in that direction. This spreading is due to the smearing effect a coarse spacing tends to have, an effect that is by no means physical. Use of a square cross-sectional area did not improve predictions as it did in the Rush Island case.

In an attempt to optimize the subchannel dimensions, Till determined the changes in the maximum lateral and vertical distances of isotherms from the point of discharge as the lateral and vertical dimensions of the subchannels were changed. Again, this was done by analyzing the predictions 1000 ft downcurrent of the discharge. The results are summarized in Figs. II.J.37 and 38. The lateral and vertical extent of the isotherms are

Fig. II.J.31. Calculated Isotherms for Heated Discharge at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.32. Calculated Isotherms for Cooled Discharge at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.33. Calculated Isotherms for Subchannel Dimensions 80 x 40 x 5 ft at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.34. Calculated Isotherms for Subchannel Dimensions 80 x 160 x 5 ft at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.35. Calculated Isotherms for Subchannel Dimensions 80 x 80 x 2.5 ft at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.36. Calculated Isotherms for Subchannel Dimensions 80 x 80 x 10 ft at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.37. Average Lateral Isotherm Deviations at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

Fig. II.J.38. Average Vertical Isotherm Deviations at 1000 ft Downstream; Philip Sporn Case. (Adapted from Till.¹)

very sensitive to the choice of subchannel size. This is disturbing since it is not clear, a priori, what dimensions for the subchannels should be chosen. No convergence is apparent as the spacing gets smaller and smaller. From these optimization runs, Till found that subchannels with larger lateral dimensions and smaller vertical dimensions yielded better results. The most

optimum dimensions of those tested in the Philip Sporn case were 160 ft in width and 2.5 ft in depth. The basically incorrect vertically-mixed nature of the predictions was not changed however. Till produced the same kind of graphs for this case as appear in Figs. II.J.27 and 28 for the Rush Island case. They are not as informative, however, since upon averaging the effects of lateral and vertical changes in subchannel size, the variation in terms of lateral and vertical movement of the isotherms in the resulting sum was negligible, although individual variations were usually significant. Similar variations for the Rush Island case led to the same kinds of trends in isotherm deviation, but there, a simple arithmetic average was more meaningful.

8. Applications to Point Beach Power Plant Unit 1^{1,2}

Argonne National Laboratory requested Till to run his model for two cases in which data were available for the Unit 1 discharge of the Point Beach Power Plant. The plant is located on the western shore of Lake Michigan. For the first date, May 18, 1972, the plume was under zero current and wind conditions. The ability of the model to handle stagnant ambient conditions was to be tested. For the second date, May 20, 1971, a small current and wind were present. Only the outfall and environmental parameters on these dates (see Tables B.1 and B.3 of App. B) and the bottom contours in the vicinity of the discharge were communicated to Till. These dates were the same ones as those for which Argonne sponsored the Paul-Lick computer runs. The Paul-Lick predictions for those two dates also appear in App. B with a direct comparison to data appearing in Sec. II.K. The reader is referred to App. B for an expanded discussion of the Till model runs for Point Beach. Briefly, Till employed the most recent version of the model, correcting the errors in the equations discussed earlier as well as adding some terms neglected in the river applications (see Table II.J.8). Our comments about the comparison are:

a. The Till model has a fundamental flaw in that, for stagnant ambient applications, the plume will be directed along the x axis wherever it is chosen. This fact was brought to light when Till ran his model for the stagnant case at Point Beach with the x axis taken directly offshore. The simulated plume quickly directed itself in this direction; i.e., offshore. At best, this means that the model cannot simulate any jet bending. Till and Schade² had to direct the x axis along the extended 60° line from the outfall to obtain any reasonable results. The lack of proper jet analysis, the neglect of jet entrainment and surface elevation changes in the near field, is probably the cause.

For applications with ambient currents present (e.g., Point Beach) and the x axis directed along the current, the plume very quickly turns in that direction independent of the relative magnitudes of outfall and ambient velocities. These directional dependence problems may be, in part, due to the single-pass solution technique of marching from the outfall along the x axis to obtain the solution.

b. The Till predictions for both cases look much like river plumes and not lake plumes. Related to this is another major fault of the model (Secs. 2.d and 2.g of App. B), its inability to sustain lateral and vertical transport for an extended longitudinal distance. Any large lateral transport, for instance, which occurs for a lake plume under small or zero currents, gets immediately (within a few subchannels from the discharge) transferred into longitudinal momentum. A study of the model development shows that the assumptions in the model derivation implicitly require large longitudinal transport and second-order lateral and vertical transport. This problem becomes more obvious upon application of the model to a few problems. Terms that were assumed insignificant when the model was applied to river plumes became very significant when applied to the lake plumes. The proper simulation of lake plumes under small or moderate currents becomes nearly impossible without addition of terms to the lateral and vertical momentum equations. The addition of terms was done to some degree by Till and Schade² but the results indicated only a small improvement.

The predictions in Figs. B.5 to B.12 of App. B are quantitatively and qualitatively incorrect. The total lateral spreading on either side of the outfall was about 300 ft, which is small compared to the data.

c. The model cannot handle upstream transport and has trouble handling low flows. This problem came to the fore in these Point Beach simulations. Trouble occurred with the turbulent and convective transport terms of the energy equations because the (low) flow velocity appears in the denominator and the turbulent mixing or eddy diffusivity and the convective crossflow terms both appear in the numerator. For large lateral movement of heated water (compared to the longitudinal flow), large energy differentials ensue and the code fails. Till dropped the troublesome terms when they occurred; clearly this is not adequate and reduces nonlongitudinal spreading.

d. The expected interaction between the terms in the equations of motion do not appear correct in the lake plume simulations, as discussed in Sec. 7 of App. B. In fact, pressure usually increased when flow was out of the subchannel and decreased when the flow was inward. The opposite trend should occur. When the correct trend occurred, it did so in a lethargic manner. A check of the code simulation of the equations and a study of the relative magnitudes of terms in relation to subchannel pressures needs to be done.

e. The model requires the longitudinal velocity u_x for each subchannel as input. Till and Schade used a uniformly zero velocity profile for the stagnant lake case and the ambient current velocity for the small current case. Clearly these small or zero values for u_x do not dominate the plume velocities as was implicitly assumed in the derivation of the model. The changes in the model made by Till and Schade do not alter this fact.

In all, the lake plume predictions were disappointing. The changes made to the model in Ref. 2 did not correct the fundamental inadequacies of the model. As the model stands, its only useful application might be for the prediction of the far field of a plume discharged into a river with a strong current. Even this would require significant code modifications to develop an effective computer model.

9. Criticisms and Limitations

a. Poor Results in Data Comparisons

The comparison of the Till model to the Point Beach data brought to light a number of problems that were present but not obvious in the Rush Island and Philip Sporn runs. The Till model predictions (river or lake application) have definite directional dependence. The plumes always quickly direct themselves along the direction of primary flow (typically chosen as the x axis) or along the x axis, wherever it is chosen, for discharges into a stagnant ambient. The model apparently cannot treat near-field interactions between the jet and ambient currents. This deficiency is most probably due to the neglect of certain important near-field phenomena in the model; notably, jet entrainment and surface elevations. We believe that the model approach may be used only, with any expectation of accuracy, for the far-field of plumes in a large current where the plume is a superimposed second-order effect. The present computer code would require major alteration to accomplish this however. Till expects that his model would work better for far-field applications and suspects that the model could be improved in that region if the diversion crossflows are removed from the model, with just the turbulent transport remaining. This remains to be shown as well as any improvement achieved over existing far field models.

b. No Upstream Transport

Since the model assumes that the principal direction of flow is downstream, it cannot handle situations where upstream transport is expected. The changes in the code from the river plume simulation to the lake plume simulation were meant to handle this problem, but were not successful. This limitation is important as upstream transport is possible for highly buoyant plumes that float and gradually work their way upstream due to buoyant forces. Again, the model cannot, in its present form, treat such situations successfully.

c. Errors in the Model (and Computer Code)

The bottom shear term in the governing equations as given in Table II.J.3 (river plume model) was dimensionally incorrect due to misuse of the gravitational constant g_c . The calculation of the frictional parameter f was also incorrect due to an inconsistent use of hydraulic radius. These errors were present in the Rush Island and Philip Sporn applications; however, they were corrected for the Point Beach applications. The effect of such errors on the predicted results is not known because the corrected code was not run for the previous cases. The effect of bottom shear was certainly reduced by the first error in the Rush Island and Philip Sporn cases. The effect of the incorrect calculation of f is probably less important.

d. Incorrect Eddy Viscosity and Eddy Diffusivity Assumptions

The Till model simulates flow in a river plume as if it were flow through an open channel. No change in diffusivities is made to account for the presence of a heated effluent. More important, the diffusivities employed do not match the trend in experimental data. Holley's experiments⁸ indicate that diffusivities reach their maximum values at the shorelines and their minimum values in the center of the river. The Till diffusivities follow the reverse trend.

e. Inconsistencies in Pressure Calculations

The pressure terms are extremely critical to proper model performance. In the model, the pressure differences are calculated at each forward step in x . For the model to run correctly, the diversion crossflows should then negate the pressure differences and bring them all to zero. The next step forward in x requires new conditions to be met, upsetting the pressures. Again, new calculations for the diversion crossflows should bring the pressure differences to zero. What actually happens in the model, is that the pressure differences do not even tend toward zero; instead, they have a tendency to keep growing. Till suspects that if this erroneous trend in the calculation of pressure can be corrected and with the input of good coefficients, a significant improvement in the model can be attained. We feel that the addition of a near-field simulation (jet entrainment, surface elevations, etc.), if it can be done within the framework of the Till model, should provide a much needed further improvement.

f. Need to Define Longitudinal Velocity

The distribution of the longitudinal velocities of the plume is required as input for the model. If the plume has any residual momentum (i.e., in the near and intermediate fields), the longitudinal velocity is actually an unknown quantity. Substituting the ambient current velocity for this longitudinal velocity implicitly assumes that the plume is riding with the current and has a negligible effect on the hydrodynamics. The Rush Island, Philip Sporn, and Point Beach applications each used an ambient current profile to represent the longitudinal velocity. The need for this velocity assumption actually makes the Till model development a combination of numerical and integral approaches. The Till model, then, cannot solve for the longitudinal velocity distribution of the plume.

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K. Paul-Lick Numerical Model¹⁻⁴

1. Synopsis of Approach

The three-dimensional differential-numerical model proposed by Paul and Lick is similar in many respects to the other differential-numerical models reviewed in this chapter, but there are some important distinctions in both the model equations and the method of solution. Two versions of the model are discussed in this section. First, an early version, which is detailed in Paul's doctoral dissertation,¹ assumed a uniform flat bottom and no crosscurrent in the receiving body. A later version⁴ removed these restrictions and also made some changes in the finite difference equations and boundary conditions. At this writing, the authors of the model are developing a user's manual for this later version of the computer code.

Paul and Lick begin with the three-dimensional equations of mass, momentum, and energy conservation for a turbulent incompressible fluid. The equation of vertical momentum conservation is reduced to just the body force and pressure gradient terms by means of the hydrostatic approximation. Transport quantities, such as the generalized stress tensor and the heat transport vector, are related to local gradients as is done in viscous, heat-conducting flows. Boundary conditions at the extremities of the grid system include the rigid-lid condition that requires no vertical motion at the free surface. This rigid-lid condition eliminates the direct calculation of the free surface elevation. In its place a surface pressure distribution is introduced and calculated. This surface pressure is shown to satisfy a Poisson-type equation by taking the divergence of the Navier-Stokes equations and then integrating the result vertically over the flow domain.

The model equations consist of: (1) the continuity equation; (2) the x-momentum equation; (3) the y-momentum equation; (4) the z-momentum equation (hydrostatic part only); (5) the energy equation; (6) the two-dimensional equation for the surface pressure; and (7) an equation of state. These seven equations, which form the basis of the model, are integrated numerically using an explicit finite-difference scheme following the five-step algorithm shown below.

- 1) The two-dimensional, Poisson-type equation is used to calculate the surface pressure from existing values of the nodal velocities. The alternating-direction-implicit method is used for this purpose.

- 2) Using existing nodal velocities, the nodal temperatures are computed using an explicit finite difference form of the time-dependent energy equation.

- 3) Densities are calculated from the equation of state and then starting with the surface pressures from Step 1 above, the pressure is calculated by integrating the density field vertically downward.

4) With updated values of the pressure and present nodal velocities, the time-dependent x and y-momentum equations are solved for the u and v velocities, respectively, by an explicit finite-difference expression.

5) The vertical velocities are computed by integrating the continuity equation.

2. List of Symbols

A_H	horizontal eddy coefficient for momentum
A_V	vertical eddy coefficient for momentum
b_o	jet inlet channel width
B_H	horizontal eddy coefficient for temperature
B_V	vertical eddy coefficient for temperature
C_p	specific heat of water at constant pressure
Fr	Froude number; $u_o/\sqrt{gh_o}$
Fr^*	densimetric Froude number; $u_o/\sqrt{gh_o\Delta\rho}$
$f(\Delta T)$	equation of state
g	acceleration due to gravity
g_i	specified velocity and temperature inlet profiles
h	depth of receiving water basin
h_o	jet inlet channel depth (same as basin depth for constant depth basin)
i, j, k	unit vectors in x, y, z directions
k	Coriolis parameter
K	index corresponding to z direction; dimensional surface-heat-transfer coefficient
K_1	K index referring to surface
K_2	K index referring to bottom
M	index corresponding to y direction
M_1	M index referring to jet centerline
M_2	M index referring to jet inlet corner
M_3	M index referring to dividing line between constant- and variable-spacing
M_4	M index referring to outer y boundary
N	index corresponding to x direction
N_1	N index referring to jet inlet

N_3	N index referring to dividing line between constant and variable spacing
N_4	N index referring to outer x boundary
P	pressure
Pr	turbulent Prandtl number; A_H/B_H
P_s	surface pressure
q_h	heat-transport vector
q_s	surface-heat transfer at the air-water interface
q_T	heat-transport vector
Re	turbulent Reynolds number; $u_o b_o / A_H$
R_o	nondimensional Coriolis parameter; kb_o^2 / A_H
s	centerline distance
S	dimensionless centerline distance; s/b_o
T	temperature
T_E	equilibrium temperature
$\bar{\bar{T}}_s$	generalized stress tensor
t	time
U	vertically-integrated u velocity
u	velocity in x direction
u_o	outfall velocity
v	velocity in y direction
$\vec{v}$	vector velocity
V	vertically-integrated v velocity
w	velocity in z direction
x	coordinate along jet centerline
x_L	outer x boundary
$\vec{X}$	body force
y	horizontal coordinate perpendicular to jet centerline
y_L	outer y boundary
z	vertical coordinate
α_1	ratio of adjacent y grid spacings for variable-spacing region
α_2	ratio of adjacent x grid spacings for variable-spacing region

β	B_v/B_H
γ	A_v/A_H
ΔT	nondimensional temperature difference $(T - T_E)/T_E$
Δx	nondimensional x-grid spacing
Δy	nondimensional y-grid spacing
Δz	nondimensional z-grid spacing
$\Delta \rho$	nondimensional, density difference $(\rho - \rho_E)/\rho_E$
κ	nondimensional, surface-heat-transfer coefficient
ρ	density of fluid
ρ_E	density evaluated at the equilibrium temperature
ρ_o	density of discharge
Σ	right-hand side of surface pressure equation
σ	transformed z coordinate

3. Basic Assumptions and Derivation of Model Equations

The equations of the Paul-Lick model are derived using the coordinate system shown in Fig. II.K.1. The origin of the rectangular Cartesian system is taken as the surface and midpoint of the discharge channel. The offshore direction is denoted as x in space and u in velocity, the shore-parallel direction as y in space and v in velocity, and the vertical direction as z in space and w in velocity.

Fig. II.K.1. Jet Structure as Visualized by Paul-Lick Model. (Adapted from Paul and Lick.¹⁻⁴)
ANL Neg. No. 190-1165.

The basic equations are developed from the time-dependent, three-dimensional equations of motion for a fluid. In accordance with the Boussinesq approximation, density variations are ignored

in all but the gravity term. In this way, the fluid is treated as incompressible, yet the coupling of flow and energy equations (through the density) is retained. The work and dissipation terms are removed from the energy equation by assuming that velocities and density variations are small. These generalized equations are given in Table II.K.1.

One major simplification made in the equations of Table II.K.1 is the reduction of the vertical momentum equation to hydrostatic form. The hydrostatic approximation has been extensively used in the modeling of oceanic

basins and lakes.⁵⁻¹⁰ This simplification results in a considerable savings of computational effort due to the many terms dropped from z-momentum equation. The density itself is assumed to be a function of temperature only, thus the vertical pressure gradient can be directly related to the temperature field.

TABLE II.K.1. Time Averaged Equations of Flow

1. Continuity	where $\vec{v}$ = vector velocity
$\nabla \cdot \vec{v} = 0$	ρ = density
2. Conservation of Linear Momentum	P = pressure
$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \nabla \cdot \bar{\bar{T}}_s + \vec{X}$	$\bar{\bar{T}}_s$ = generalized stress tensor
3. Conservation of Thermal Energy	T = temperature
$\rho C_p \frac{DT}{Dt} = \nabla \cdot \vec{q}_T$	$\vec{X}$ = body force
	$\vec{q}_T$ = heat transport vector
	$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$
	t = time

A second major assumption concerns the local stresses and the local heat transport vector. Following the traditional line of thinking, the authors assume that the combined turbulent and molecular transport of momentum and heat can be formulated as a diffusion process as follows:

$$\begin{aligned}
 \frac{1}{\rho} (\nabla \cdot \bar{\bar{T}}_s) \cdot i &= A_H \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(A_v \frac{\partial u}{\partial z} \right) \\
 \frac{1}{\rho} (\nabla \cdot \bar{\bar{T}}_s) \cdot j &= A_H \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(A_v \frac{\partial v}{\partial z} \right) \\
 \frac{1}{\rho C_p} (\nabla \cdot \vec{q}_h) \cdot k &= B_H \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(B_v \frac{\partial T}{\partial z} \right)
 \end{aligned} \tag{1}$$

where the A_H and A_v represent, respectively, horizontal and vertical eddy diffusion coefficients for momentum and B_H and B_v represent, respectively, horizontal and vertical eddy diffusion coefficients for heat. There is little justification for this treatment except that it is commonly used. In the literature,⁵⁻¹⁰ the horizontal diffusion coefficients are often taken to be equal in both directions and constant. To account for the influence of buoyancy on vertical transport, which has been noted by some investigators,¹¹⁻¹² the vertical coefficients are taken to depend on local vertical density gradients. Under the foregoing assumptions, the governing equations become those presented in Table II.K.2.

One additional (and important) equation becomes necessary because of (1) the way the numerical scheme is developed and (2) the use of the rigid-lid condition. The authors of the model force the free surface to coincide with the top of the grid system. This has the effect of damping out certain oscillatory modes of solution, resulting in a more rapid approach to steady state. From a practical point of view, the rigid-lid condition allows large

TABLE II.K.2. Equations of the Paul-Lick Model

Conservation of Mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of Longitudinal Momentum

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} (u^2) + \frac{\partial}{\partial y} (vu) + \frac{\partial}{\partial z} (wu) = - \frac{1}{\rho_E} \frac{\partial P}{\partial x} + A_H \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right)$$

Conservation of Lateral Momentum

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} (uv) + \frac{\partial}{\partial y} (v^2) + \frac{\partial}{\partial z} (wv) = - \frac{1}{\rho_E} \frac{\partial P}{\partial y} + A_H \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right)$$

Hydrostatic Approximation

$$\frac{\partial P}{\partial z} = \rho g$$

Conservation of Heat

$$\frac{\partial \Delta T}{\partial t} + \frac{\partial}{\partial x} (u \Delta T) + \frac{\partial}{\partial y} (v \Delta T) + \frac{\partial}{\partial z} (w \Delta T) = B_H \left(\frac{\partial^2 \Delta T}{\partial x^2} + \frac{\partial^2 \Delta T}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(B_V \frac{\partial \Delta T}{\partial z} \right)$$

Equation of State

$$\rho = f(\Delta T)$$

where

u, v, w = components of velocity

P = pressure

ΔT = temperature excess over reference temperature

ρ = density of fluid

ρ_E = reference density

A_H = constant horizontal eddy viscosity

A_V = variable vertical eddy viscosity

B_H = constant horizontal eddy diffusivity

B_V = variable vertical eddy diffusivity

g = acceleration due to gravity

time steps to be taken in the integration of the transient flow problem. They speculate that computer time is reduced by about two orders of magnitude by imposing the rigid-lid condition.

As a result of the rigid-lid condition, the authors introduce a surface pressure distribution, P_s . This distribution is easily translated into free surface elevations using the assumed hydrostatic approximation. While these surface elevations are immeasurably small in the field, their spatial gradients are quite important to the mathematical modeling of the flow. The above surface pressure, P_s , is shown to satisfy a two-dimensional Poisson-type equation by taking the divergence of the Navier-Stokes equations and then integrating vertically over the flow field as outlined in Table II.K.3. As readily can be seen from that table, the surface pressure, P_s , arises as a constant of integration and represents the pressure at the top of the grid structure, which is due to the slight elevation of the free surface above the grid.

The equations of Tables II.K.2 and 3 can be nondimensionalized by introducing the following nondimensional variables:

$$u \rightarrow \frac{U}{u_o},$$

$$v \rightarrow \frac{v}{u_o},$$

$$w \rightarrow \frac{b_o w}{h_o u_o},$$

$$x \rightarrow \frac{x}{b_o},$$

$$y \rightarrow \frac{y}{b_o},$$

$$z \rightarrow \frac{z}{h_o},$$

$$t \rightarrow \frac{A_H t}{b_o^2},$$

$$P \rightarrow \frac{P \text{ Re}}{\rho_o g h_o F r^2},$$

where

$$\text{Re} = \frac{u_o b_o}{A_H} \text{ and } F r = \frac{u_o}{\sqrt{g h_o}}$$

TABLE II.K.3. Derivation of Poisson-type Equation for Surface Pressure

- I. Integrate the two horizontal momentum equations over the vertical coordinate. Due to rigid lid condition, integration is from $z = 0$ to $z = h_0$.

$$\frac{\partial U}{\partial t} = \int_0^{h_0} \left[- \left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) + A_H \frac{\partial^2 u}{\partial x^2} + A_H \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right) \right] dz - \frac{1}{\rho_0} \int_0^{h_0} \frac{\partial p}{\partial x} dz$$

$$\frac{\partial V}{\partial t} = \int_0^{h_0} \left[- \left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} \right) + A_H \frac{\partial^2 v}{\partial x^2} + A_H \frac{\partial^2 v}{\partial y^2} + \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right) \right] dz - \frac{1}{\rho_0} \int_0^{h_0} \frac{\partial p}{\partial y} dz$$

where

$$U = \int_0^{h_0} u dz$$

$$V = \int_0^{h_0} v dz$$

- II. Integrate the hydrostatic equation over the vertical coordinate twice:

$$\int_0^{h_0} p dz = \int_0^{h_0} \left(\int_0^z \rho g dz' \right) dz + h_0 P_s(x, y)$$

The integration constant, P_s , is a function of both x and y and is the pressure that exists at the surface $z = 0$.

- III. The pressure gradient terms in (I) can then be written

$$\int_0^{h_0} \frac{\partial p}{\partial x} dz = \int_0^{h_0} \left(\int_0^z g \frac{\partial \rho}{\partial x} dz' \right) dz + h_0 \frac{\partial P_s}{\partial x}$$

$$\int_0^{h_0} \frac{\partial p}{\partial y} dz = \int_0^{h_0} \left(\int_0^z g \frac{\partial \rho}{\partial y} dz' \right) dz + h_0 \frac{\partial P_s}{\partial y}$$

- IV. The two vertically integrated momentum equations now have the form

$$\frac{\partial U}{\partial t} = - I_x + A_x - \frac{1}{\rho_0} h_0 \frac{\partial P_s}{\partial x} - \frac{1}{\rho_0} R_x$$

$$\frac{\partial V}{\partial t} = - I_y + A_y - \frac{1}{\rho_0} h_0 \frac{\partial P_s}{\partial y} - \frac{1}{\rho_0} R_y$$

where

$$I_x = \int_0^{h_0} \left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) dz$$

$$A_x = \int_0^{h_0} \left[A_H \frac{\partial^2 u}{\partial x^2} + A_H \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right) \right] dz$$

$$R_x = \int_0^{h_0} \int_0^z \left(g \frac{\partial \rho}{\partial x} dz' \right) dz$$

$$I_y = \int_0^{h_0} \left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} \right) dz$$

$$A_y = \int_0^{h_0} \left[A_H \frac{\partial^2 v}{\partial x^2} + A_H \frac{\partial^2 v}{\partial y^2} + \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right) \right] dz$$

$$R_y = \int_0^{h_0} \left(\int_0^z g \frac{\partial \rho}{\partial y} dz' \right) dz$$

- V. The integrated continuity equation, with the use of the zero flow conditions at $z = h_0$, is

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = w(z=0)$$

- VI. Taking the divergence of the two vertically integrated horizontal momentum equations results in

$$\begin{aligned} \frac{h_0}{\rho_0} \left(\frac{\partial^2 P_s}{\partial x^2} + \frac{\partial^2 P_s}{\partial y^2} \right) &= \frac{\partial}{\partial x} \left(- I_x + A_x - \frac{1}{\rho_0} R_x \right) \\ &\quad + \frac{\partial}{\partial y} \left(- I_y + A_y - \frac{1}{\rho_0} R_y \right) \\ &\quad - \frac{\partial}{\partial t} (w(z=0)) \end{aligned}$$

- VII. Final Equation

$$\frac{\partial^2 P_s}{\partial x^2} + \frac{\partial^2 P_s}{\partial y^2} = \Sigma(x, y)$$

$$\begin{aligned} \Sigma(x, y) &= \frac{\partial}{\partial x} \int_0^1 \int_0^1 Re \left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \\ &\quad + \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right) dz + \frac{\partial}{\partial y} \int_0^1 \left[- Re \left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} \right) \right. \\ &\quad \left. + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right) \right] dz - \frac{\partial}{\partial t} (w(z=0)) \\ &\quad - \frac{Re}{Fr^2} \left[\int_0^1 \frac{\partial}{\partial x} \left(\int_0^z \frac{\partial \rho}{\partial x} dz' \right) dz + \int_0^1 \frac{\partial}{\partial y} \left(\int_0^z \frac{\partial \rho}{\partial y} dz' \right) dz \right] \end{aligned}$$

and

$$Pr = \frac{A_H}{B_H}, \quad (2)$$

where

h_0 = depth of basin,

b_0 = full width of discharge channel,

u_0 = velocity at origin of coordinate system (center and top of discharge channel).

When these transformations are used the equations reduce to the form given in Table II.K.4.

TABLE II.K.4. Nondimensionalized Form of Equations of Table II.K.2.

Conservation of Mass	Density/Temperature Relationship
$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$	$\Delta \rho = f(\Delta T)$
Conservation of Longitudinal Momentum	The Poisson Equation for Surface Pressure
$\frac{\partial u}{\partial t} + Re \left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) = - \frac{\partial p_s}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ $+ \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right) - \frac{Re}{Fr^2} \int_0^z \frac{\partial \Delta \rho}{\partial x} dz,$	$\frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2} = \frac{\partial}{\partial x} \int_0^1 \left[- Re \left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \right) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right.$ $+ \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial u}{\partial z} \right) \Big] dz + \frac{\partial}{\partial y} \int_0^1 \left[Re \left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} \right) \right.$ $+ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right) \Big] dz - \frac{\partial}{\partial t} [w(z=0)]$ $- \frac{Re}{Fr^2} \left[\int_0^1 \frac{\partial}{\partial x} \left(\int_0^z \frac{\partial \Delta \rho}{\partial x} dz' \right) dz + \int_0^1 \frac{\partial}{\partial y} \left(\int_0^z \frac{\partial \Delta \rho}{\partial y} dz' \right) dz \right].$
Conservation of Lateral Momentum	
$\frac{\partial v}{\partial t} + Re \left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} \right) = - \frac{\partial p_s}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ $+ \left(\frac{b_0}{h_0} \right)^2 \frac{1}{A_H} \frac{\partial}{\partial z} \left(A_V \frac{\partial v}{\partial z} \right) - \frac{Re}{Fr^2} \int_0^z \frac{\partial \Delta \rho}{\partial y} dz,$	
Conservation of Heat Energy	
$Pr \left[\frac{\partial \Delta T}{\partial t} + Re \left(\frac{\partial u \Delta T}{\partial x} + \frac{\partial v \Delta T}{\partial y} + \frac{\partial w \Delta T}{\partial z} \right) \right] = \frac{\partial^2 \Delta T}{\partial x^2}$ $+ \frac{\partial^2 \Delta T}{\partial y^2} + \left(\frac{b_0}{h_0} \right)^2 \frac{1}{B_H} \frac{\partial}{\partial z} \left(B_V \frac{\partial \Delta T}{\partial z} \right),$	

The seven equations presented in Tables II.K.2 and 3 are formulated in general geometry, but are used by Paul and Lick only in the case of a uniform flat bottom receiving body. They find it more convenient to treat irregular bottom topography through the use of the following transformation of the vertical coordinate,

$$\sigma = z/h(x, y) \quad (3)$$

TABLE II.K.5. Paul-Lick Equations for Variable Bottom

Conservation of Mass	Equation of State
$\frac{1}{h} \frac{\partial(hu)}{\partial x} + \frac{1}{h} \frac{\partial(hv)}{\partial y} + \frac{\partial \Omega}{\partial \sigma} = 0$	$\Delta p = f(\Delta T)$
Conservation of Longitudinal Momentum	Equation of Surface Pressure
$\frac{\partial u}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(hu^2)}{\partial x} + \frac{1}{h} \frac{\partial(huv)}{\partial y} + \frac{\partial(\Omega u)}{\partial \sigma} \right] - R_0 v = -\frac{\partial P}{\partial x}$ $- \frac{\text{Re}}{\text{Fr}^2} \left[h \int_0^\sigma \frac{\partial \Delta p}{\partial x} d\sigma + \frac{\partial h}{\partial x} \left(\int_0^\sigma \Delta p d\sigma - \sigma \Delta p \right) \right] + \frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial u}{\partial y} \right) \right]$ $+ \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial u}{\partial \sigma} \right)$	$\frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2} = -\frac{\partial}{\partial x} \int_0^1 A d\sigma - \frac{\partial}{\partial y} \int_0^1 B d\sigma - \frac{\partial}{\partial t} (w(\sigma=0))$ $+ \frac{1}{h} \frac{\partial h}{\partial x} \int_0^1 A' d\sigma + \frac{1}{h} \frac{\partial h}{\partial y} \int_0^1 B' d\sigma,$
Conservation of Lateral Momentum	where
$\frac{\partial v}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(huv)}{\partial x} + \frac{1}{h} \frac{\partial(hv^2)}{\partial y} + \frac{\partial(\Omega v)}{\partial \sigma} \right] + R_0 u = -\frac{\partial P}{\partial y}$ $- \frac{\text{Re}}{\text{Fr}^2} \left[h \int_0^\sigma \frac{\partial \Delta p}{\partial y} d\sigma + \frac{\partial h}{\partial y} \left(\int_0^\sigma \Delta p d\sigma - \sigma \Delta p \right) \right] + \frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial v}{\partial y} \right) \right]$ $+ \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial v}{\partial \sigma} \right)$	$\sigma = z/h(x, y)$ $\Omega = \frac{1}{h} \left[w - \sigma \left(u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} \right) \right] = \frac{d\sigma}{dt}$ $A = \text{Re} \left[\frac{1}{h} \frac{\partial(hu^2)}{\partial x} + \frac{1}{h} \frac{\partial(huv)}{\partial y} + \frac{\partial(\Omega u)}{\partial \sigma} \right] + R_0 v + \frac{\text{Re}}{\text{Fr}^2} \left[h \int_0^\sigma \frac{\partial \Delta p}{\partial x} d\sigma \right.$ $\left. + \frac{\partial h}{\partial x} \left(\int_0^\sigma \Delta p d\sigma - \sigma \Delta p \right) \right] - \frac{1}{h} \frac{\partial}{\partial x} \left(h \frac{\partial u}{\partial x} \right) - \frac{1}{h} \frac{\partial}{\partial y} \left(h \frac{\partial u}{\partial y} \right) - \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial u}{\partial \sigma} \right)$ $B = \text{Re} \left[\frac{1}{h} \frac{\partial(huv)}{\partial x} + \frac{1}{h} \frac{\partial(hv^2)}{\partial y} + \frac{\partial(\Omega v)}{\partial \sigma} \right] - R_0 u + \frac{\text{Re}}{\text{Fr}^2} \left[h_0 \int_0^\sigma \frac{\partial \Delta p}{\partial y} d\sigma \right.$ $\left. + \frac{\partial h}{\partial y} \left(\int_0^\sigma \Delta p d\sigma - \sigma \Delta p \right) \right] - \frac{1}{h} \frac{\partial}{\partial x} \left(h \frac{\partial v}{\partial x} \right) - \frac{1}{h} \frac{\partial}{\partial y} \left(h \frac{\partial v}{\partial y} \right) - \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\gamma \frac{\partial v}{\partial \sigma} \right)$
Conservation of Heat Energy	$A' = -A - \frac{\partial P_s}{\partial x}$
$\text{Pr} \left[\frac{\partial \Delta T}{\partial t} + \text{Re} \left[\frac{1}{h} \frac{\partial(hu \Delta T)}{\partial x} + \frac{1}{h} \frac{\partial(hv \Delta T)}{\partial y} + \frac{\partial(\Omega \Delta T)}{\partial \sigma} \right] \right] =$ $\frac{1}{h} \left[\frac{\partial}{\partial x} \left(h \frac{\partial \Delta T}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \Delta T}{\partial y} \right) \right] + \left(\frac{b_0}{h_0} \right)^2 \frac{1}{h^2} \frac{\partial}{\partial \sigma} \left(\beta \frac{\partial \Delta T}{\partial \sigma} \right)$	$B' = -B - \frac{\partial P_s}{\partial y}$
Hydrostatic Approximation	
$\frac{1}{h} \frac{\partial P}{\partial \sigma} = \frac{\text{Re}}{\text{Fr}^2} (1 + \Delta \rho)$	

The velocity and temperature profiles are specified over the channel outlet. At other points along the shore, the "no-slip" condition is used on tangential velocities, the impermeability condition imposed on normal velocities, and a zero heat flux enforced on the temperature field. These conditions can be stated mathematically as follows:

$$\begin{aligned}
 x = 0, \quad 0 \leq y \leq \frac{1}{2}, \quad 0 < z < 1 & \left\{ \begin{array}{l} u = g_1(y, z) \\ v = g_2(y, z) \\ \Delta T = g_3(y, z) \end{array} \right. \\
 x = 0, \quad \frac{1}{2} < y < y_L, \quad 0 < z < 1 & \left\{ \begin{array}{l} u = 0 \\ v = 0 \\ \frac{\partial \Delta T}{\partial x} = 0 \end{array} \right. \quad (4)
 \end{aligned}$$

As shown in Fig. II.K.2, in the case of no crossflow and a discharge angle of 90° , Paul and Lick use symmetry to reduce the problem in half. At the plane of symmetry, the following restrictions on velocity and temperature are made:

$$0 < x \leq x_L, \quad y = 0, \quad 0 \leq z < 1 \left\{ \begin{array}{l} \frac{\partial u}{\partial y} = 0 \\ v = 0 \\ \frac{\partial \Delta T}{\partial y} = 0 \end{array} \right. \quad (5)$$

When symmetry does not exist, downstream boundary conditions must be used.

Along the bottom, as at the shoreline, conditions of no-slip and impermeability are imposed on velocity and an insulated surface condition is imposed on temperature as follows:

$$0 < x \leq x_L, \quad 0 < y \leq y_L, \quad z = 1 \left\{ \begin{array}{l} u = 0 \\ v = 0 \\ w = 0 \\ \frac{\partial \Delta T}{\partial z} = 0 \end{array} \right. \quad (6)$$

Momentum and energy transport is allowed across the free surface to account for wind stress and surface heat loss. The vertical velocity is, however, set to zero in what is called the rigid-lid condition. These boundary conditions can be stated as follows:

$$0 \leq x \leq x_L, \quad -y_L \leq y \leq y_L, \quad z = 0 \quad \left\{ \begin{array}{l} w = 0 \\ \frac{\partial u}{\partial z} = \frac{1}{\rho A_V} \tau_{wx} \\ \frac{\partial v}{\partial z} = \frac{1}{\rho A_V} \tau_{wy} \\ \frac{\partial \Delta T}{\partial z} = \frac{1}{\rho B_V} \kappa \Delta T. \end{array} \right. \quad (7)$$

According to Paul and Lick, the specification of boundary conditions at the open water extremities of the grid are "somewhat up to the whim of the investigator." They cite a discussion by Roache¹³ in which he concludes that far-boundary conditions are not too important as long as they do not severely restrict the flow. We feel this to be a point that needs investigation, particularly when the grid chosen for plume problems has not, to date, included "all" of the plume.

In the early form of the model, Paul and Lick arbitrarily required velocities and temperatures to vary linearly at the open-water boundaries. The justification is that such a requirement maintains a smooth variation of the physical variables. This condition translates mathematically to:

$$0 < x \leq x_L, \quad y = y_L, \quad 0 < z < 1 \quad \left\{ \begin{array}{l} \frac{\partial^2 u}{\partial y^2} = 0 \\ \frac{\partial^2 v}{\partial y^2} = 0 \\ \frac{\partial^2 \Delta T}{\partial y^2} = 0, \end{array} \right. \quad (8)$$

and

$$x = x_L, \quad 0 < y < y_L, \quad 0 < z < 1 \quad \left\{ \begin{array}{l} \frac{\partial^2 u}{\partial x^2} = 0 \\ \frac{\partial^2 v}{\partial x^2} = 0 \\ \frac{\partial^2 \Delta T}{\partial x^2} = 0. \end{array} \right. \quad (9)$$

In a later version of the model, Paul and Lick decided that setting first-order derivatives to zero was more appropriate, except in those cases where the cross-boundary flow is explicitly known. In the exceptions, the velocity is specified. Neither of these methods appears totally justified to us. The assertion that such boundary conditions do not matter has not been sufficiently demonstrated by the authors.

The boundary conditions on the surface pressure are somewhat difficult to define. They evaluate the x and y partial derivatives of the surface pressure at the boundaries by vertically integrating the x and y momentum equations, respectively. However, the specification of the derivatives at the boundary does not result in a unique solution for the surface pressure, and the pressure must be specified at one point. Since the gradients and not the pressures themselves are needed in the model calculations, this uniqueness question of the pressure solution is sidestepped. The effect of different pressure boundary conditions on the solution is an area that deserves further investigation.

5. Numerical Technique

Two remarks need to be made concerning the ways the governing equations of Tables II.K.4 and 5 are finite differenced. First, it should be noted that the advection terms are written in what is often called the conservation form. The conservation form is the one in which the velocity is brought inside the divergence operator thusly;

$$\vec{u} \cdot \nabla \Psi = \nabla \cdot (\vec{u} \Psi), \quad (10)$$

where Ψ is a scalar such as u , v , w or T . Equation 10 is valid by continuity. The term conservative is attached to the right-hand side of Eq. 10 because the various terms of its finite difference expression can be readily identified with mass fluxes into the grid cell from adjacent cells. A more detailed explanation of this point is given by Roache.¹³ The other numerical convenience, which deserves noting, is the casting of the heat equation into a form that uses temperature excesses, an approach that tends to minimize the accumulation of truncation and roundoff error.

Fig. II.K.3. One Cell in the Grid Network.
(Adapted from Paul and Lick,¹)
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The finite difference grid used by Paul and Lick in the early stages of model development is shown in Figs. II.K.3-6. The indices N , M , and K correspond to the x , y , and z axes, respectively. The authors elect to define the velocity components u and v at the nodal points and define the remaining variables, w , T , ρ , and P , on the horizontal faces of the grid cell. This arrangement, illustrated in Figs. II.K.4 and 5, leads to a convenient

finite difference expression of the basic equations. The definition points for individual variables are summarized in Table II.K.6. Spatial variations of the dependent variables are described in terms of central differences. For first derivatives, the three neighboring points are half-nodal spacings apart; for second derivatives, the spacing between the points are at full nodes. If a variable is needed at a location and it is not defined, a simple average of the two neighboring points is made. At boundaries where the normal derivative is required, a one-sided difference formulation is used. No fictitious grid points outside the region are employed.

Fig. II.K.4. Horizontal Position of Variables. (Adapted from Paul and Lick.¹)

Fig. II.K.5. Vertical Position of Variables. (Adapted from Paul and Lick.¹)

Fig. II.K.6. Horizontal and Vertical Grid Networks. (Adapted from Paul and Lick.¹)

TABLE II.K.6

Position of Definition
of Dependent Variables

	x	y	z
u	N	M	K
v	N	M	K
w	$N + \frac{1}{2}$	$M + \frac{1}{2}$	K
P	$N + \frac{1}{2}$	$M + \frac{1}{2}$	—
T	$N + \frac{1}{2}^*$	$M + \frac{1}{2}^*$	K
ρ	$N + \frac{1}{2}^*$	$M + \frac{1}{2}^*$	K

*ALSO ALONG $N=N_1$ AND $M=M_1$

dependent variables are the greatest, and a coarser grid where changes are expected to be less rapid.

In a later use of the model, Paul and Lick have slightly altered the choice of grid system as is shown in Figs. II.K.7 and 8. The modified grid structure lends itself to a convenient form for the finite difference expressions.

In the form of the model originally described in Paul's doctoral dissertation,¹ a combination of both simple-forward and three-level temporal differencing is used. The latter method of differencing was dropped in the modified model because it was found to be less stable than simple-forward differencing.

The sequence of steps in the integration algorithm has been given previously (see Synopsis of Approach). Several important comments are

Fig. II.K.7

Arrangement of Variables in
Grid Sections. (Adapted from
Paul and Lick.⁴)

- - u, v
- Δ - T, ρ, P_s
- $\blacktriangle$ - T, ρ
- $\square$ - w

VERTICAL PLANE AT HALF-NODAL SECTION

Fig. II.K.8
Typical Nodal Cells for Grid System

needed to fully explain that algorithm. First, the most time-consuming part of the calculation is the solution of the second-order equation for the surface pressure in spite of Paul's use of the accelerated alternating-direction-implicit method. This surface pressure is an essential part of the model since it controls the overall pressure field and thus the driving force for the flow. A considerable savings in computational effort could be achieved if this solution could be hastened. Furthermore, a numerical trick devised by Hirt and Harlow¹⁴ is used by Paul and Lick in the integration of the auxiliary surface-pressure equation to correct the nonzero surface velocity that can result from vertical integration of the continuity equation. Such a nonzero vertical velocity is inconsistent with the rigid-lid condition and would, if left uncorrected, result in the net loss of mass through the surface. The corrective procedure is to use the time derivative, $\partial w / \partial t$, as a simple backward difference with the present value of w set to zero. In this way, one is continually forcing the surface velocity to zero and, in essence, conserving mass.

To stabilize an unstable density profile (heavy fluid above lighter fluid) that may arise, Paul and Lick use a final numerical device of artificially mixing to a uniform density the vertical column of nodal values. The effects of this arbitrary "fix-up" are presently unknown.

6. Application to Cuyahoga River Discharge into Lake Erie

Seven simulations of the Cuyahoga River discharge into Lake Erie have been made. These calculations include three cases where the river is warmer than the lake:

- 1) low flow rate of the river typical of later summer,
- 2) high flow rate of the river typical of spring,
- 3) low flow rate with a simulated variable bottom.

Two simulations were prepared for cases where the river is colder than the Lake Erie waters to indicate a sinking plume:

- 4) low flow rate of the river, cooled jet,
- 5) high flow rate of the river, cooled jet.

In addition to these five runs, two cases were considered where the lake and river were taken to be the same temperature.

Table II.K.7 summarizes the grid pattern for the Cuyahoga simulations. A constant grid spacing is used near the outfall (horizontal mesh) with a variable grid spacing used further from the discharge. Table II.K.8 summarizes the physical conditions and physical parameters for the low-flow simulation. Table II.K.9 gives the same information for the high-flow simulation. The inlet conditions for the low- and high-flow cases are given in Table II.K.10. The velocity distribution of the inlet was derived from a simple closed-form simulation of three-dimensional, constant temperature, rectangular channel flow.

The solution procedure followed for all Cuyahoga simulations was:

(a) The constant temperature cases were run first. A small number of horizontal points was used initially with more and more grid points added as time increased until the grid for the variable temperature case was obtained. The growth of the grid looked like a series of basins increasing in size.

(b) Results for the variable temperature case were obtained by using the above results as initial conditions and then slowly changing the temperature at the inlet until the specified inlet temperatures were reached. Too abrupt an inlet temperature change at the start of the code was found to induce large vertical velocities causing instabilities.

TABLE II.K.7. Grid Parameters for the Cuyahoga River Application

number of points in x direction (N_4)	21
number of points in y direction (M_4)	21
number of points in z direction (K_2)	8
constant x/b_0 spacing (Δx)	0.1
constant y/b_0 spacing (Δy)	0.1
constant z/h_0 spacing (Δz)	0.143
last point in x direction with constant spacing (N_3)	3
last point in y direction with constant spacing (M_3)	8
ratio of grid spacings in x direction for variable spacing (α_2)	1.24
ratio of grid spacings in y direction for variable spacing (α_1)	1.28
maximum x value	20.7
maximum y value	9.42

TABLE II.K.8. Physical Conditions and Parameters for the Paul-Lick
Computer Run for the Cuyahoga River Discharge
(Summer Conditions)

time of year	August
river flow rate	120 ft ³ /sec (3.5 m ³ /sec)
river width	250 ft (76.2 m)
river depth	28 ft (8.54 m)
lake surface temperature	49° (9.4°C)
average wind speed	8.4 mph (3.76 m/sec)
average daily gross solar radiation	1720 BTU/ft ² -day (472 langleys)
average relative humidity	72%
average air temperature	70.4°F (21.3°C)
average dewpoint temperature	61°F (16.1°C)
equilibrium temperature (T _E)	72°F (22.2°C)
turbulent Prandtl number (Pr)	1.0
horizontal eddy coefficient (A _H)	0.028 ft ² /sec (26 cm ² /sec)
vertical eddy coefficient (A _V)	10 ⁻² ft ² /sec (9.3 cm ² /sec)
surface heat transfer coefficient (K)	156 BTU/ft ² -day-°F (37 watt/m ² -°C)
non-dimensional surface heat transfer coefficient (κ)	0.0814
temperature variation	.05°F (.028°C)
maximum velocity (u _o)	0.0334 ft/sec (1.0 cm/sec)
turbulent Reynolds number (Re)	300
Froude number (Fr)	1.11 x 10 ⁻³
densimetric Froude number (Fr*)	0.414
equation-of-state	$\Delta\rho = -1.04 \times 10^{-2} \Delta T$
jet aspect ratio (b _o /h _o)	8.93
ratio of vertical to horizontal eddy coefficient (A _V /A _H)	0.358

TABLE II.K.9. Physical Conditions and Parameters for the Paul-Lick
Computer Run for the Cuyahoga River Discharge
(Spring Conditions)

time of year	May
river flow rate	1200 ft ³ /sec (35 m ³ /sec)
river width	250 ft (76.2 m)
river depth	28 ft (8.54 m)
lake surface temperature	49°F (9.4°C)
average wind speed	10.5 mph (4.69 m/sec)
average daily gross solar radiation	1830 BTU/ft ² -day (503 langleys)
average relative humidity	66%
average air temperature	58°F (14.4°C)
average dewpoint temperature	46.7°F (8.2°C)
equilibrium temperature (T_E)	49°F (9.4°C)
turbulent Prandtl number (Pr)	1.0
horizontal eddy coefficient (A_H)	.103 ft ² /sec (95.7 cm ² /sec)
vertical eddy coefficient (A_V)	10 ⁻² ft ² /sec (9.3 cm ² /sec)
surface heat transfer coefficient (K)	110 BTU/ft ² -day-F° (26 watt/m ² -C°)
nondimensional surface heat transfer coefficient (κ)	.0572
temperature variation	5 F° (2.8 C°)
maximum velocity (u_o)	.362 ft/sec (11 cm/sec)
turbulent Reynolds number (Re)	875
Froude number (Fr)	1.2 x 10 ⁻²
densimetric Froude number (Fr*)	.806
equation of state	$\Delta\rho = -2.19 \times 10^{-3} \Delta T$
jet aspect ratio (b_o/h_o)	8.93
ratio of vertical to horizontal eddy coefficient (A_V/A_H)	.097

TABLE II.K.10. Inlet Conditions for the Summer and Spring Simulations of the Cuyahoga Discharge

Inlet Conditions for the Cuyahoga River in Summer							
y =	0.0	0.1	0.2	0.3	0.4	0.5	
z = 0.0	1.0	.988	.943	.832	.574	0.0	
.143	.980	.968	.924	.816	.564	0.0	
.286	.919	.908	.868	.768	.533	0.0	
.429	.817	.808	.773	.686	.481	0.0	
.571	.675	.668	.640	.570	.405	0.0	
.714	.492	.486	.467	.419	.302	0.0	
.857	.267	.264	.254	.229	.169	0.0	
1.0	0.0	0.0	0.0	0.0	0.0	0.0	
Nondimensional Velocities							
z = 0.0	.143	.286	.429	.571	.714	.857	1.0
6.94×10^{-4}	6.94	6.78	6.25	4.65	2.78	.694	0.0
Nondimensional Temperature for Heated Jet							
z = 0.0	.143	.286	.429	.571	.714	.857	1.0
6.94×10^{-4}	$-.694 \times 10^{-4}$	-2.78	-4.65	-6.25	-6.94	-6.94	-6.94
Nondimensional Temperature for Cooled Jet							

Inlet Conditions for the Cuyahoga River in Spring							
y =	0.0	0.1	0.2	0.3	0.4	0.4	
z = 0.0	1.0	.971	.880	.709	.430	0.0	
.143	.981	.953	.863	.696	.423	0.0	
.286	.923	.897	.815	.659	.402	0.0	
.429	.826	.804	.731	.595	.367	0.0	
.571	.688	.670	.611	.501	.313	0.0	
.714	.506	.494	.453	.375	.240	0.0	
.857	.278	.271	.250	.210	.139	0.0	
1.0	0.0	0.0	0.0	0.0	0.0	0.0	
Nondimensional Velocities							
z = 0.0	.143	.286	.429	.571	.714	.857	1.0
.102	.102	.0998	.0918	.0684	.0408	.0102	0.0
Nondimensional Temperature for Heated Jet							
z = 0.0	.143	.286	.429	.571	.714	.857	1.0
.102	-.0102	-.0408	-.0684	-.0918	-.102	-.102	-.102
Nondimensional Temperature for Cooled Jet							

Figures II.K.9-14 show the predictions of the high-flow, heated jet case. The surface isotherms and the surface-velocity distributions are given in Figs. II.K.9 and 10. Vertical cross sections of the jet indicating isotherms and jet velocities are shown in Figs. II.K.11 and 12. The longitudinal temperature and velocity profiles are given in Figs. II.K.13 and 14.

Fig. II.K.9. Surface Isotherms Predicted by Paul-Lick Model:
High-flow-rate Case. (Adapted from Paul and Lick.¹)
ANL Neg. No. 190-1192.

Fig. II.K.10. Surface Velocities Predicted by Paul-Lick Model:
High-flow-rate Case. (Adapted from Paul and Lick.¹)
ANL Neg. No. 190-1187.

Fig. II.K.11. Lateral-temperature Structure Predicted by Paul-Lick Model: High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1191.

Fig. II.K.12. Lateral-velocity Structure Predicted by Paul-Lick Model: High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1159.

Fig. II.K.13. Longitudinal-temperature Structure Predicted by Paul-Lick Model: High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1163.

Fig. II.K.14. Longitudinal-velocity Structure Predicted by Paul-Lick Model: High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1143.

Additionally, a prediction for a sinking plume is included. The parameters are the same as for the heated jet, except that the receiving body is taken to be 5°F warmer than the discharging fluid rather than the reverse situation previously calculated. Surface, lateral, and longitudinal profiles of the resulting plume are given in Figs. II.K.15-20, as determined by the authors. A detailed discussion of Paul-Lick results appears in Refs. 1-3, along with numerous other temperature and velocity profiles.

Fig. II.K.15. Horizontal Isotherms at $z/h_0 = 0.857$ Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1183.

Fig. II.K.16. Surface Velocities Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1145.

Fig. II.K.17. Longitudinal-temperature Structure Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1149.

Fig. II.K.18. Longitudinal-velocity Structure Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1153.

Fig. II.K.19. Lateral-temperature Structure Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.²) ANL Neg. No. 190-1179.

Fig. II.K.20. Lateral-velocity Structure Predicted by Paul-Lick Model (Cooled Jet): High-flow-rate Case. (Adapted from Paul and Lick.¹) ANL Neg. No. 190-1142.

We made a number of observations from these simulations. First, for very small initial densimetric Froude numbers, a cold water wedge may be present in the river channel upstream from the point of discharge. If such a wedge were indeed present then the validity of the simulations would be questionable since no wedge was considered in setting up the problem. Secondly, for the low-flow heated and cooled jets, the specification of a 0.05°F temperature excess between river discharge and ambient lake water precludes any kind of field verification since the resolution of present-day field measurements is not that refined. Also, the flow field resulting from the 1 cm/sec river discharge velocity is of too small a magnitude to be measured in the field with conventional techniques. Third, verification of model predictions on a laboratory scale would be equally difficult. For complete geometric similarity, a water depth of $0.1\text{--}0.2$ inches would result. At those water depths, surface tension and Froude number effects would become important. To avoid these difficulties in Kuhlman's hydraulic model¹⁵ for the Cuyahoga River discharge, the vertical dimensions of the channel were exaggerated roughly by a factor of four. Unfortunately, the site of the river entrance is not amenable to either field- or tank-verification. The authors acknowledge our comments above and reply that the major reason for the Cuyahoga calculations done to date was to test the model algorithm and only secondarily to provide physically correct predictions. In any case, the predictions presented for the five variable-density cases can be used only for intercomparison, as a check with physical intuition, or as a check of sensitivity of the model to grid configuration, outlet conditions, and diffusivities. This leads to an additional limitation of the model predictions because Paul and Lick used constant coefficients of eddy viscosity and eddy thermal diffusivity, a gross oversimplification for the vertical diffusivities should be dependent on the local velocity gradient and local stratification. Clearly, no universally accepted form exists for the vertical diffusivity, but many forms are more accurate and more reasonable than a constant value when the plume is buoyant.

Figures II.K.21-23 show the centerline temperature decay, half-widths, and areas obtained from the surface predictions of five Cuyahoga runs. Cases I and III are both based on low flow rates. In Case III the basin depth,

Fig. II.K.21. Comparison of Centerline Predictions for the Five Cuyahoga River Discharge Cases Simulated

Fig. II.K.22

Comparison of Temperature Half-width
Predictions for the Five Cuyahoga River
Discharge Cases Simulated

Fig. II.K.23. Comparison of Surface-Isotherm Area Predictions for
the Five Cuyahoga River Discharge Cases Simulated

instead of remaining constant, drops off to twice its initial value within two and one-half river widths and then maintains that value throughout the remainder of the simulation region. Also, the temperature difference for Case III was 0.025 F° (instead of 0.05 F° as used in Case I), since the densimetric Froude number of 0.414 ($\Delta T = 0.05$ F°) led to large buoyant vertical velocities and instabilities in the calculations. With the smaller temperature difference, the initial densimetric Froude number increased to 0.585, removing the numerical instabilities. Otherwise, the parameters and conditions for Cases I and III were identical.

One further remark concerning Case III is appropriate here. Using $\Delta T = 0.05 F^\circ$ resulted in large vertical velocities due to the reduced influence of friction as the bottom dropped off. The nondimensionalized equations indicate that the friction term has a factor of $1/h^2$ in front of it. So, for the same eddy viscosities, doubling the bottom depth reduces the relative importance of friction in the vertical momentum equation by a factor of roughly 4. This reduced friction coupled with large buoyancy forces produced the large vertical velocities and the corresponding instabilities in the solution.

Our observations and critical comments on the results presented in Figs. II.K.21-23 follow.

1) The centerline temperature decay for the low-flow, flat-bottom depth case is more rapid than that of the low-flow, variable-depth case. The same trend exists for the surface areas. This is surprising since more cold entrainment water is available for the variable-bottom case (since the lake is deeper) and the higher densimetric Froude number for Case III should add to the mixing. All other parameters including viscosities and diffusivities were assumed to be the same for both cases. At present no definite explanation exists for this discrepancy. The authors speculate¹⁶ that the discrepancy might be due to different finite difference forms of the constant-depth cases and variable-depth cases (one Cuyahoga case and also two later Point Beach cases). In formulating the variable depth equations, the locations of the dependent variables were redefined (see above discussion in this subsection and Appendix B) to keep better track of conserved quantities like momentum and energy. The difference equations were rewritten to assure that all the heat and momentum from one cell are transported into other cells. The goal was that the equations provide complete conservation of energy and momentum between cells. Second, the boundary conditions at the surface and bottom were also redefined. With these changes, there is negligible heat loss to the overlying air due to the numerical scheme, suggesting that this approach would give increased temperatures for all applications. As will be seen later, however, the heat loss from the surface is far less than the heat injected by the river. The only way to ascertain with some certainty whether model changes have caused this apparent discrepancy in model predictions is to compare both low-flow cases using the "improved" finite-difference equations.

2) The high-flow-rate discharge ($Fr^* = 0.806$) does not dilute as well as the low-flow-rate discharge ($Fr^* = 0.414$; compare Cases I and II, constant-depth bottom). The ability to induce entrainment by the higher densimetric Froude number jet is not indicated from the comparison of centerline temperature decay and isotherm areas. This might be explained in part by an improper choice of eddy diffusivities. In the low-flow case, A_H was taken to be $26 \text{ cm}^2/\text{sec}$, and A_V was taken to be $9.3 \text{ cm}^2/\text{sec}$; whereas for the high-flow case, A_H was taken to be $95.7 \text{ cm}^2/\text{sec}$, and A_V was taken to be $9.3 \text{ cm}^2/\text{sec}$. Paul and Lick chose the horizontal diffusivities from channel-flow equations expecting that near the source, the channel-flow diffusivities would be more indicative of the jet diffusion.

Vertical diffusivities were chosen based upon ambient lake data. The vertical diffusivity should, in fact, be smaller for the low Fr^* case (Case I); in fact, it is too large for both cases. The lower Fr^* indicates greater buoyant stratification and less vertical diffusion, hence, a smaller A_V . Ambient lake values for A_V measured without stratification will be too large. The use of the large A_V value leads to the nearly vertically mixed plume seen in the plots of cross-sectional temperatures. More proper values for A_V , if a constant value is used (and it should not), are probably on the order 1-3 cm²/sec. This number is based more on the region of stratified flow than the short initial region where large changes are taking place. The more vertically stratified plume that appears for the high-flow case might well be due to the larger horizontal diffusivity, i.e., the larger A_H/A_V value.

As stated above, the high-flow case ($Fr^* = 0.806$) should induce greater mixing and provide better dilution than the low-flow case ($Fr^* = 0.414$). To better understand why this does not occur in this Cuyahoga application of the model, Paul and Lick analyze the order of magnitude of terms in the equations of motion. Referring to Table 4, the relative effect of vertical diffusion to horizontal advection is seen by comparing the ratio of Re to $(b_o/h_o)^2 A_V/A_H$ for each case. For the low-flow simulation, this ratio is 10.5, and for the high-flow case, it is 113. This order of magnitude difference is the reason Paul and Lick give to explain why vertical diffusion is larger in the low-flow case than in the high-flow case. The densimetric Froude numbers in the two cases are only a factor of 2 apart, from which the authors indicate that the horizontal advection in both cases is nearly the same. It is important to note that this ratio is very dependent on the value chosen for A_V . As stated above, it is our opinion that the values chosen are inconsistent with data for buoyant jets; i.e., the ratio

$$Re / \left(\frac{b_o}{h_o} \right)^2 \frac{A_V}{A_H} = \frac{u_o h_o^2}{b_o A_V}$$

should be larger for the low-flow case. In this way, less vertical entrainment would occur, as expected, for a lower densimetric Froude number discharge.

We are not sure at this point whether the argument presented by the authors of the model is valid. The river excess temperatures, and hence density effects, are significantly different in the high-flow and low-flow cases. More importantly, we believe that their argument is oversimplified in terms of the complexity of the problem. In any case, the model predicts a trend that contradicts our intuition.

3) The plume half-width comparisons indicate a decrease in half-width for Cases I and III, at the lowest isotherms. We are unaware of any field data that support this kind of result. Our experience indicates that the lowest isotherms are spread out, being much larger in centerline distance and areas than indicated by the model. In contrast, the isotherms of the

Paul-Lick model appear almost equally spaced. Although we expect half-width to be a monotonically increasing function, the field data is not sufficiently refined at the lowest isotherms to demonstrate that this happens all the time.

4) Surface heat loss calculations reveal that not all the heat flow discharged is lost to the atmosphere within the region of computation. For the low-flow, constant-depth case, the heat discharged is 374.4 Btu/sec while within the 0.1 ΔT isotherm only 72.2 Btu/sec is transferred to the atmosphere. For the low-flow variable-depth case, 187.2 Btu/sec is discharged with 72.2 Btu/sec lost to the atmosphere. Finally, in the high-flow case, 374,400 Btu/sec is released with only 15,120 Btu/sec lost to the atmosphere. In this latter case, only isotherms down to $\Delta T = 0.5$ were closed in Ref. 1 and consequently only heat lost within that isotherm was calculated. In actuality isotherms down to 0.2 could have been closed in the computational grid net, although this was not indicated¹⁶ in Ref. 1. In the Paul-Lick calculations, especially the high-flow calculation, the computational grid chosen was too small, with the plumes clearly dispersing outside the grid. This is a most important observation since it implies that the boundary conditions at the grid boundaries are undoubtedly violated to some degree. The condition, $P_s = 0$, indeed allows a non-zero normal gradient for P_s but requires a zero tangential gradient. The condition $P_s = 0$, along the outer two boundaries, indicates a zero gradient. This is clearly violated if any kind of residual plume is passing through those boundaries; tangential pressure gradients cannot then be zero, indicating some error. Thus, the condition of linear decay of temperature and velocity is also violated, but probably to a lesser degree. Also, the return flow required underneath a plume as it spreads beyond the computational grid is not adequately defined with outer grid boundary conditions like

$$\frac{\partial^2 u}{\partial^2 x} = \frac{\partial^2 T}{\partial x^2} = \dots = 0.$$

Indeed, a sensitivity analysis of the boundary conditions would be quite valuable here to determine the precise region influenced by the outer boundary conditions and also how significant the boundary condition errors are from choosing too small a computational grid.

The temperature and velocity variation at the outer boundary locations in the model simulation most probably indicates a rather flat curve approaching ambient values. (The numerical results presented in Refs. 1-4 did not allow a precise plot of the velocity and temperature variation near the outer boundaries.)

To assess in some way the adequacy of the far boundary conditions on pressure, Paul compared the pressure gradients found at steady state in the low-flow, constant-depth Cuyahoga simulation to those derived by the simple closed-form solution obtained by considering the river outfall as a point source. The gradients from that calculated pressure distribution differed at most by 5% from those determined by the numerical model at the

outer-boundary locations. The precise meaning of this 5% difference in terms of its effect on the flow field can be determined only from an otherwise identical run with a larger grid size; the effect of even a 5% difference is not really known due to the complexity of the model.

7. Application to Point Beach Unit 1 Discharge into Lake Michigan

Parameters and physical conditions for two runs of the model for Point Beach Unit 1 are described in Appendix B. Also included are model predictions and a short discussion by the authors themselves.*

Figures II.K.24-33 illustrate the field data acquired at the plant on the two dates considered. Stagnant lake conditions prevailed on May 18, 1972, and moderate wind and current were present on May 20, 1971. To better facilitate a comparison, Figs. II.K.34-45 plot the model predictions and data in terms of jet trajectories, surface centerline temperature decay, surface half-widths of temperature, and surface isotherm areas. The comparisons are discussed below.

a. The centerline-temperature decay is quite good in the region for which numerical results exist corresponding to a centerline distance $S \lesssim 200$ m and a centerline excess temperature ratio $\theta_c/\theta_0 \gtrsim 0.6$. The case with zero wind and current (May 18, 1972) was initially run for only 1/2 hour CPU time (IBM 360); see Figs. II.K.34-37 for a comparison with data. Subsequently, the case was rerun to steady state (May 18, 1972) in 3 hours CPU time; see Figs. II.K.38-41 for an updated comparison to Point Beach data. The second case with a moderate wind and current (May 20, 1971) was run to steady state within 1½-2 hours CPU time (IBM 360); see Figs. II.K.42-45. In fact, the overall comparison is good for the areas, half-widths, and centerline temperatures. The only disappointment is that steady-state results could not be obtained beyond 200 m from the discharge because of internal gravity waves. It appears that a longer time of calculation yields a larger region of steady state. The difficulty in damping these waves is a present limitation of the model in that long run times are needed for only a small area near the discharge to reach steady state. The use of the Paul-Lick model for environmental impact as well as mixing zone calculations is thus limited. Future work with the model is necessary to change this situation.

These oscillations may be related to the thermal fronts described in Chapter III. There are several important distinctions; however, the most important being that the thermal fronts observed in the field show their greatest intensity near the outfall and tend to die out as one moves away from the discharge. In the numerical solution, the oscillations seem to disappear near the outfall first. The region free of oscillations then gradually grows

*Subsequent to the calculations carried out by Paul and Lick for Argonne, the first case was recomputed; this time calculations were allowed to reach steady state. Those results are presented in Appendix B. Also in Appendix B are newly available two-dimensional constant temperature predictions for Point Beach Unit 1, which were used to study the effects of the lake bottom on the plume trajectory.

Fig. II.K.24. Thermal Plume at Surface for Point Beach
 Nuclear Power Plant: May 18, 1972,
 1645-1751 Hours. ANL Neg. No. 190-983.

Fig. II.K.25. Thermal Plume at 0.5-, 1.0-, and 1.5-m Depths for Point Beach Nuclear Power Plant: May 18, 1972, 1645-1751 Hours. ANL Neg. No. 190-1043.

Fig. II.K.26. Thermal Plume at 2.0-, 2.5-, and 3.0-m Depths for Point Beach Nuclear Power Plant: May 18, 1972, 1645-1741 Hours

Fig. II.K.27. Vertical Cross Sections for Point Beach Nuclear Power Plant:
May 18, 1972, 1645-1751 Hours. ANL Neg. No. 190-1085.

Fig. II.K.27 (Contd.). ANL Neg. No. 190-1083.

THERMAL PLUME - SURFACE

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)
 PLANT LOAD (MW_e): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTFALL TEMPERATURE: 17.5°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT. WAVES

Fig. II.K.28. Thermal Plume at Surface for Point Beach Power Plant (Unit 1):
 May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-325.

Fig. II.K.29. Thermal Plume at 1-ft Depth for Point Beach Power Plant (Unit 1):
 May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-326.

Fig. II.K.30. Thermal Plume at 3-ft Depth for Point Beach Power Plant (Unit 1):
 May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-327.

THERMAL PLUME - 6 FT. DEPTH

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)
 PLANT LOAD (MW_g): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTLET TEMPERATURE: 17.5°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT. WAVES

Fig. II.K.31. Thermal Plume at 6-ft Depth for Point Beach Power Plant (Unit 1):
 May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-328.

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT & METEOROLOGICAL DATA

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)
 DATE & TIME OF SURVEY: 20 MAY 71 (1740-1050)
 PLANT LOAD (MW): 474
 DISCHARGE FLOW RATE: 771 CFS
 OUTFALL TEMPERATURE: 17.5°C
 INTAKE TEMPERATURE: 8.9°C
 AMBIENT WATER TEMPERATURE: 8.2-8.5°C
 DRY BULB TEMPERATURE: 12.2-13.9°C
 WET BULB TEMPERATURE: N. A.
 WIND SPEED & DIRECTION: 10-15 MPH - 270°
 AMBIENT CURRENT SPEED & DIRECTION: 0.3 FPS - 125°
 LAKE LEVEL: 579.5 FT. (IGLD)
 LAKE SURFACE CONDITIONS: 0-1 FT. WAVES

Fig. II.K.32. Thermal Plume at 9-ft Depth for Point Beach Power Plant (Unit 1):
 May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-329.

VERTICAL CROSS SECTIONS

ALL TEMPERATURES IN DEGREES CENTIGRADE

PLANT NAME: POINT BEACH NUCLEAR PLANT (UNIT 1)

DATE & TIME OF SURVEY: 20 MAY 71 (1740-1850)

Fig. II.K.33. Vertical Cross Sections at Point Beach Power Plant (Unit 1): May 20, 1971, 1740-1850 Hours. ANL Neg. No. 190-394.

Fig. II.K.34. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface-centerline Trajectory - Initial Run. (Nonsteady state.)

Fig. II.K.35. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface Centerline-temperature Decay - Initial Run. (Nonsteady state.)

Fig. II.K.36. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Temperature Half-widths - Initial Run. (Nonsteady state.)

Fig. II.K.37. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface-isotherm Areas - Initial Run. (Nonsteady state.)

Fig. II.K.38. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface-centerline Trajectory - Second Run. (Steady state.)

Fig. II.K.39. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface Centerline-temperature Decay - Second Run. (Steady state.)

Fig. II.K.40. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Temperature Half-widths - Second Run. (Steady state.)

Fig. II.K.41. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 18, 1972, Surface-isotherm Areas - Second Run. (Steady state.)

Fig. II.K.42. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 20, 1971, Surface-centerline Trajectory

Fig. II.K.43. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 20, 1971, Centerline-temperature Decay

Fig. II.K.44. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 20, 1971, Temperature Half-widths

Fig. II.K.45. Comparison of Paul-Lick Model and Field Data for Point Beach (Unit 1): May 20, 1971, Surface-isotherm Areas

outward from the discharge as more time steps are taken. These facts, coupled with the Paul-Lick observations that the wave structure depends on the initial flow and temperature conditions, suggest to us that these oscillations are attributable to the model alone and are not a physical reality.

b. The computational grid in both cases is too small to contain the thermal plume. An 800-m distance to the far grid boundary is not enough to insure plume containment. Again, the adequacy of the far boundary conditions comes into serious question. The effect of these erroneous boundary conditions (erroneous in terms of the small grid chosen) can only be determined by a sensitivity analysis, one of whose components would be sensitivity to the size of the grid system. Paul and Lick used a variable grid structure so that greater refinement is possible near the outfall, but this also means less accuracy at the grid points near the not-too-distant, far grid boundary.

The pressure boundary conditions were changed in the model for the Point Beach simulation. The outer-grid boundaries had normal-pressure gradients specified, which were determined from values of the variables obtained at the previous time iteration. In this way, the wall boundary conditions (see Fig. II.K.2) are extended to apply at the other three boundaries also. A reference pressure at some point had to be specified to produce a unique solution to the Poisson equation, since the boundary conditions on surface pressure are all prescribed in terms of normal derivatives. This should be an improvement over the surface-pressure boundary conditions used for the Cuyahoga River simulation since more natural boundary values are used. This is true provided that there are indeed small changes from one time step to another. Here again a sensitivity analysis on boundary conditions and computational grid size would be most useful in interpreting the results of the Point Beach runs.

c. The non-steady-state results of May 18, 1972 are quite interesting. The trend is for far too great a temperature decay at the lower isotherms. The temperature decay there is concaved down as though constrained by an outer boundary condition; however, this shape is more likely the result of the model not being run to steady state. The incompleteness is also seen in the isotherm area plot where larger areas are predicted near the outfall and areas below the field data are predicted for $\theta/\theta_0 < 0.7$. This is indicative to some degree of an undeveloped plume.

d. Half-width predictions for the model are quite large in comparison to the field data for the steady-state run of May 20, 1971. This may be due, in part, to the large horizontal eddy diffusivity, which causes increased lateral spreading. The general underprediction of surface centerline temperatures is also partly due to the extremely large vertical diffusivity.

The isotherms predicted for May 20, 1971 are not compacted on the windward side of the jet as seen in the data. Field observations show that moderate currents and winds usually cause a sharp edge to form on the upcurrent side. A more complete discussion of model predictions for Point Beach is given in Appendix B.

e. The trajectory plots indicate plume bending for the stagnant case of May 18, 1972. Since the constant temperature variable depth application of the model also reveals a plume bending, the authors of the model attribute the trajectory bending to the variable nonsymmetric bottom contour or to the way the specific values for the bottom contour $h(x, y)$ were chosen as input values for their model. We suspect that the difficulty lies in the specification of the shoreline as a solid wall or else in the specification of the nearshore and offshore boundary conditions. The simulation of the nearshore solid wall, in the presence of a 60° angle discharge, might allow plume water to pile up on the nearshore, causing adverse pressure gradients which in turn divert the plume slightly offshore. We suspect that the bending in the data for the May 18, 1972 case is probably due to the nonstagnant environmental conditions that were apparently present although unmeasurable. Instrumentation for wind and current measurements has a threshold below which readings of zero are given. The Bendix Q-15 current meter reads zero for current speeds under 2 cm/sec. Also, field data can never be taken under completely stagnant environmental conditions because such conditions never exist for the total duration of a field survey. The Paul-Lick model was run for precisely zero wind and current and consequently should give zero bending for symmetric bottom contours, or only very slight bending if nonsymmetric bottom contours are allowed.

f. As stated in Appendix B, the eddy coefficients were chosen based upon some previous data taken at Point Beach. The ratio of horizontal to vertical coefficient was chosen so that for the application of the two-dimensional variable-depth version of the model, the predictions matched the previously measured velocity decay for one Point Beach plume. The horizontal diffusivity was chosen using previous experience with other than Point Beach data. These facts obscure the complete objectivity of the comparisons. Some of the good agreement might indeed be due to the choice of diffusivities based upon similar field data.

g. Appendix B illustrates the application of the two-dimensional constant-temperature version (constant-depth lake bottom assumed) of the Paul-Lick model to Point Beach Unit 1. Paul and Lick solve the transient problem with a steady-state solution approached as time increases. The integrated velocities presented as time increases are noteworthy; the flow was initiated by assuming an impulsive start and a steady discharge thereafter. Vortices formed on either side of the discharge flow and were then swept out with the flow as time increased. A similar feature was evident in the three-dimensional variable-density case.

8. Criticisms and Limitations

a. General Verification

In spite of the applications of the model to the Cuyahoga River discharge and to the Point Beach Unit 1 discharge, the model remains unverified.

Predictions for the Cuyahoga case cannot be verified with data due to the extremely low velocities and low temperatures that result from the discharge. Poor judgment has been exercised in the choice of diffusivity coefficients in both applications as well as in the choice of the grid for the computations. Lack of a run to steady state has, in at least one case, hampered data comparison efforts. The Point Beach data comparisons are not completely objective, for a simpler version of the Paul-Lick model was used to help define diffusivities from velocity data measured at Point Beach. Internal gravity waves prevented a steady-state solution from being reached for a good part of the flow field in a number of cases, including both the Cuyahoga River and Point Beach applications.

In our opinion, however, the formulation of the model for heated surface discharges appears satisfactory provided that (1) the eddy coefficients and the grid configuration are wisely chosen and (2) the model is applied to cases for which the assumptions are valid (e.g., hydrostatic approximation). We feel that these basic assumptions were valid in the Cuyahoga River and Point Beach applications.

b. Eddy Diffusivities

The Cuyahoga River application was made using constant values for viscosities and eddy thermal diffusivities. The vertical diffusivity should have been reduced to account for buoyancy. In addition, the constant vertical diffusivities were chosen unrealistically large with inconsistent values of A_V and A_H used between the high and low densimetric Froude number cases. A slight improvement was made in one Point Beach application where vertical diffusivity decreasing with increasing stratification was used. Unfortunately, abnormally large coefficients were used in this functional form. Eddy coefficients more in line with the state of the art should have been employed.

c. Computational Grid

For the Cuyahoga River (high-flow case most significantly) and both Point Beach cases, the computational grid chosen was too small. Surface heat loss calculations indicated that most of the discharged heat was not lost to the atmosphere in the region of computation. The boundary conditions are very likely inaccurate due to the small computational grid and may provide unnecessary restrictions on the flow. A sensitivity analysis with respect to the size and spacing of the computational grid would be most useful in providing indication as to relative significance (if any) of the error incurred. The same conclusions apply to the Point Beach applications.

d. Transient Analysis

The Paul-Lick model can conceptually treat transient problems such as startup of a power plant, although the treatment can be expected to be no better than the corresponding results for steady state.

e. Hydrostatic Approximation

The hydrostatic approximation is valid when the pressure and gravity terms in the vertical momentum equation strongly dominate the convective and diffusive terms. Paul states, based on calculations with his model, that vertical velocities in the plume problems treated are one-to-two orders of magnitude smaller than necessary to invalidate the hydrostatic approximation. We feel that the way to test the validity of this approximation is to measure the pressure distribution and to determine the deviation from horizontal planes of constant pressure.

f. Rigid-lid Condition

The rigid-lid condition eliminates surface waves. When applied, it greatly reduces the computation time of the model. The approximation cannot be made when free surface variations are rapid or large (storm-surge problems). Surface waves, though physically real, are generally assumed unimportant in surface plume problems. The rigid-lid condition, however, does not eliminate the internal gravity waves that hopefully die out as the solution approaches steady state. The rigid-lid condition might, however, change the characteristics of these internal gravity waves by damping out the higher frequency oscillations.

We suspect that the rigid-lid approximation is valid in surface plume applications; however, a comparison with the equivalent free-surface model is the best test. Including a free surface in the model was estimated by Paul and Lick to result in a one-hundred-fold increase in computer time over the rigid-lid simulation. Including free surface oscillations means including free-surface waves, which reduces the time step necessary for stability and consequently increases computer time. Also, it should be kept in mind that the rigid-lid approximation does allow pressure variations at the surface. These surface pressure variations can be translated into equivalent water levels above that plane.

The authors expect that the rigid-lid and free-surface versions of their model would yield the same steady-state solution. The transient solutions would provide the same general flow field with differences from the rigid-lid model due solely to free-surface gravity waves (permitted in the free-surface model) and internal gravity waves. The rigid-lid model would totally eliminate free surface waves and provide some damping of the internal gravity waves. Paul and Lick argue that, based upon the analysis by Haq,¹⁷ their model is unaffected by the rigid-lid approximation. However, Haq's analysis consisted of a rigid-lid/free-surface model comparison for time-dependent, wind-driven flow in a constant-depth lake. No thermally-driven currents were present in Haq's analysis, so it really cannot be applied to the Paul-Lick simulation. The final test is a direct comparison between equivalent free-surface and rigid-lid models for thermal plumes.

g. Computer Time and Storage Requirements

Each run of the model requires 250K words of computer storage and processor time of as much as 3 hr. This is not untypical of three-dimensional numerical models; however, the attendant computer costs limit the practicability of the model.

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L. Waldrop-Farmer Numerical Model¹⁻⁴

1. Synopsis of Approach

Waldrop and Farmer have developed a three-dimensional, differential-numerical model to treat the dispersion of thermal plumes. Several versions have been tried in the course of development. Because of the generality of the model equations, it has also been applied to related problems. In fact, the model was originally formulated to treat the sedimentary transport instrumental in the Delta formation at the South Pass of the Mississippi River.¹ The reader is alerted that our discussion includes many of the different forms in which the model has been used.

The model equations are derived from the generalized conservation equations for momentum, energy, and salt to which the equation of continuity is added. The equation set is closed by relating the unknown transport quantities (surface stress tensor, heat flow vector, and mass flow vector) to the corresponding local gradients (velocity gradients, temperature gradients, and concentration gradients) in a manner similar to that used in laminar flow. In some versions of the model, the hydrostatic approximation is made. Also, certain compressibility effects are removed via the Boussinesq approximation.

The simplified equation set still contains time derivatives and as such is an initial-value problem in time and a boundary-value problem in space. The set can thus be solved as a transient flow problem to treat situations such as plant startup, although in most cases the time-dependent equations are merely a vehicle to obtain the steady-state solution. This approach is sometimes referred to as the "asymptotic time technique."

By dropping the vertical acceleration term from the vertical momentum equation, Waldrop and Farmer write the equation set as a sequence of parabolic equations, which allows the use of an explicit finite difference scheme to solve the equations. Finite difference means that temporal and spatial derivatives are approximated by the weighted algebraic sum of nodal variables. The scheme is explicit because all nodal variables used within each equation are known except one. In this way, the equations can be solved explicitly for the single unknown in terms of known values.

The difference equations are solved in the following order.

- a. The x-momentum equation is solved for the u-velocity component (x direction offshore, u velocity in direction of x axis).
- b. The y-momentum equation is solved for the v-velocity component (y direction shore parallel, v velocity in direction of y axis).
- c. The conservation of salt and energy equations are solved for the salt concentration and temperature at each node.

d. The continuity equation (time-independent) is solved from the bottom to the surface for the w-velocity component (w velocity in vertical or z direction).

e. The free-surface elevation is computed (adjusted) by performing a mass balance on the topmost layer of grid cells.

f. The z-momentum equation is solved in space (time-independent) for the pressure field. The integration is carried out vertically downward from the free surface. This integration and determination of pressure at each grid row does become simpler if the hydrostatic approximation is made.

It should be noted that only steps a-c involve time integrations. The remaining steps are spatial integrations.

2. List of Symbols

A	Aspect ratio of river outlet, width to depth
b	Half-width of the channel flow
C_1, C_2, C_3, C_4	Constants of stretching transformation
C_p	Specific heat of water at constant pressure
$\bar{D}$	Salt-diffusivity tensor
D_x, D_y, D_z	Diffusion coefficients for salt
F	Location of free surface as a function of time
F_0	Initial densimetric Froude number
g	Acceleration due to gravity
g_1, g_2, g_3, g_4	Convergence parameters
H	Depth of receiving-water body
i, j, k	Indexing variables
P	Pressure
q	Magnitude of horizontal velocity vector
$\vec{q}_s$	Salt-transport vector
$\vec{q}_T$	Heat-transport vector
R	Ratio of ambient current to discharge velocity
R_i	Local Richardson Number
S	Salt concentration
s	Centerline distance
T	Temperature
T_0	Reference temperature
$\bar{T}_s$	Generalized stress tensor
t	Time
u, v, w	Velocity components
U_{max}	Maximum channel velocity
$\vec{v}$	Vector velocity

W_T	Surface temperature half-widths
x, y, z	Spatial coordinates
$\vec{X}$	Body-force vector
X, Y	Transformed (stretched) coordinates
$X', X'' (Y', Y'')$	First and second derivatives, respectively, of stretching transformation with respect to x (y)
$\bar{\alpha}$	Heat-diffusivity tensor
α_H	Horizontal eddy diffusivity
$\alpha_x, \alpha_y, \alpha_z$	Eddy-diffusion coefficients for heat
β_1, β_2	Density constants
ΔH	Elevation of river surface above coastal current
$\bar{\epsilon}$	Eddy-diffusivity tensor
$\epsilon_x, \epsilon_y, \epsilon_z$	Eddy viscosities
ϵ_H	Horizontal eddy viscosity
η	Relaxation parameter
θ	Plume excess temperature above ambient
θ_c	Difference of surface centerline temperature and ambient temperature
θ_o	Difference of outfall temperature and ambient temperature
ξ_n	Convergence criterion
ρ	Density of water
ρ_o	Reference density

3. Derivation of the Model Equations

The governing equations are formulated in terms of the coordinate system shown in Fig. II.L.1. In that figure, the flow field is shown to be a

rectangular parallelepiped. For cases including bottom interaction, the idealization of Fig. II.L.1 must be altered slightly. This matter is treated later.

Fig. II.L.1. Discharge Situation as Idealized by the Waldrop-Farmer Model. (Adapted from Waldrop and Farmer.¹)

The derivation begins with the so-called "time-averaged" equations of flow as presented in Table II.L.1. Several comments are necessary. First, certain compressibility effects have already been removed from the equation set by implicit use of the Boussinesq approximation. Second, the pressure has been separated from the generalized stress tensor for convenience. Third, the work and dissipation terms have been omitted from the thermal energy equation.

TABLE II.L.1. Time-averaged Equations of Flow

1. Continuity	Here,
$\nabla \cdot \vec{v} = 0.$	$\vec{v}$ = vector velocity
	ρ = density
2. Conservation of Linear Momentum	P = pressure
$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \nabla \cdot \bar{\bar{T}}_S + \vec{X}.$	$\bar{\bar{T}}_S$ = generalized stress tensor
	T = temperature
3. Conservation of Thermal Energy	S = salt concentration
$\rho C_P \frac{DT}{Dt} = \nabla \cdot \vec{q}_T.$	$\vec{X}$ = body force
	$\vec{q}_S$ = salt-transport vector
	$\vec{q}_T$ = heat-transport vector
4. Conservation of Salt	$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$
$\rho \frac{DS}{Dt} = \nabla \cdot \vec{q}_S.$	t = time

Unfortunately, the equations of Table II.L.1 (six in number) contain a total of twenty-two dependent variables requiring that additional assumptions be made to close the system. First, the density is assumed to depend explicitly on S and T by an equation of state

$$\rho = \rho_0 + \beta_1 S + \beta_2 (T - T_0), \quad (1)$$

where ρ_0 and T_0 are reference values and β_1 and β_2 are empirically determined constant coefficients. This assumption reduces the number of unknowns by one.

Next, the generalized stress tensor is assumed to be the following tensorial product

$$\bar{\bar{T}}_S = \bar{\bar{\epsilon}} \cdot \nabla \vec{v}, \quad (2)$$

where $\bar{\bar{\epsilon}}$ is a known eddy diffusivity tensor. For the sake of convenience, the diffusivity tensor is assumed to have the form

$$\bar{\bar{\epsilon}} = \epsilon \bar{\bar{I}}, \quad (3)$$

where $\bar{\bar{I}}$ is the identity tensor in three dimensions and ϵ is a function of the local flow characteristics. In the sedimentary transport model, ϵ is taken to be constant. In later versions, the eddy diffusivity is assumed to vary as

$$\epsilon = 0.0256bU_{\max}[1.0 - 0.99 \exp(-|u/v|)], \quad (4)$$

where b and $U_{\max}$ are in some way characteristic of the jet flow. These reductions eliminate nine variables from the governing equations.

5

In a similar manner, the transport of salt and heat are related to local gradients by

$$\vec{q}_T = \bar{\alpha} \cdot \nabla T \quad (5)$$

and

$$\vec{q}_s = \bar{D} \cdot \nabla S,$$

where $\bar{\alpha}$ and $\bar{D}$ represent thermal and mass eddy-diffusion tensors, respectively. The off-diagonal components of $\bar{\alpha}$ and $\bar{D}$ are assumed to be zero, which reduces the number of diffusion coefficients to three for heat and three for mass. The components of $\bar{\alpha}$ corresponding to the x and y directions are both assumed equal to ϵ , and the component corresponding to the z direction is taken to be 0.2ϵ . The reduction in vertical eddy diffusivity is meant to account, in some way, for the suppression of vertical mixing by buoyant forces. There is little basis for this arbitrary reduction of the vertical diffusivity, particularly when no consideration of the local densimetric Froude number is made, i.e., local gradients of density. Constant eddy-mass-diffusion coefficients are used. These final two assumptions reduce the number of unknowns by six and thus close the system.

Waldrop and Farmer recognized that vertical velocities are typically small and that some simplification can be achieved through this knowledge. A somewhat limited assumption, which follows, is that vertical accelerations are also small, allowing the time derivative of the vertical velocity to be dropped from the z-momentum equation. This modification substitutes a spatial-integration step in the solution algorithm for a temporal-integration, but, more important, it allows pressure to be calculated explicitly. A bolder simplification is the hydrostatic approximation in which all velocity terms are dropped from the z-momentum equation leaving only the pressure gradient and body force terms. This hydrostatic assumption allows pressure to be explicitly calculated by a simple vertical integration of the density field, greatly reducing the computation effort of each time step. Unfortunately, there is no a priori criterion for judging the adequacy of these assumptions for individual plume problems. Order-of-magnitude estimation of the terms in the vertical momentum equation is the best method we know.

Note that the hydrostatic approximation does not require that vertical velocities necessarily be small. Reasonably large vertical velocities can occur provided that the acceleration and convective terms in the z-momentum equation are small in comparison to the pressure and gravity terms. As long as the effect of the "weight" of the plume far outbalances vertical convection, the hydrostatic approximation is still valid. For most heated surface discharges or freshwater discharges into salt water, this approximation is probably quite good. High densimetric-Froude-number discharges with large vertical spreading pose the greatest concern.

Using the weaker of the above approximations in the vertical momentum equation, Waldrop and Farmer give as the equations of the sedimentary transport model those presented in Table II.L.2. Note that the three momentum equations presented contain second-order derivatives of density that should not be there. The error arises from the form of the generalized conservation equations from which the equations of Table II.L.2 are derived. Waldrop and Farmer use $D(\rho\vec{v})/Dt$ in place of $\rho(D\vec{v}/Dt)$. This is incorrect, but either way makes no significant difference. In any case, the argument is as follows.

TABLE II.L.2. Basic Equations of the Waldrop-Farmer (Sedimentary-transport) Model

1. Volume Dilatation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

2. Approximate Equation of State

$$\rho = 1.0 + \beta_1 S.$$

3. Conservation of Species Equation

$$\frac{DS}{Dt} = D_x \frac{\partial^2 S}{\partial x^2} + D_y \frac{\partial^2 S}{\partial y^2} + D_z \frac{\partial^2 S}{\partial z^2}.$$

4. x-Momentum Equation

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \left[\epsilon_x \frac{\partial^2 u}{\partial x^2} + \epsilon_y \frac{\partial^2 u}{\partial y^2} + \epsilon_z \frac{\partial^2 u}{\partial z^2} \right] - u \left[D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right].$$

5. y-Momentum Equation

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \left[\epsilon_x \frac{\partial^2 v}{\partial x^2} + \epsilon_y \frac{\partial^2 v}{\partial y^2} + \epsilon_z \frac{\partial^2 v}{\partial z^2} \right] - v \left[D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right].$$

6. z-Momentum Equation

$$\rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} + \left[\epsilon_x \frac{\partial^2 w}{\partial x^2} + \epsilon_y \frac{\partial^2 w}{\partial y^2} + \epsilon_z \frac{\partial^2 w}{\partial z^2} \right] - w \left[D_x \frac{\partial^2 \rho}{\partial x^2} + D_y \frac{\partial^2 \rho}{\partial y^2} + D_z \frac{\partial^2 \rho}{\partial z^2} \right] - \rho g.$$

When the conservation of momentum equations are derived for an arbitrary compressible fluid, the advection and acceleration terms are shown to be (in tensorial notation)⁵

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j},$$

where u_i represents the i th component of the mass-averaged velocity. An equivalent form derived with the aid of continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$

is

$$\rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = \rho \frac{Du_i}{Dt}.$$

Either of these forms is acceptable; however, Waldrop and Farmer write instead

$$\frac{\partial \rho u_i}{\partial t} + u_j \frac{\partial \rho u_i}{\partial x_j},$$

which is only approximately correct.

As noted above, the error made is insignificant and can be ignored in plume problems. Waldrop and Farmer have carried out an order of magnitude analysis and examined the actual computer code calculations to determine the relative magnitude of terms in the differential equations for the Mississippi Delta problem. This confirmed that the added terms are indeed insignificant and may be neglected. Their calculations indicate that these additional density terms are three orders of magnitude smaller than the other terms in the equation.

When the full hydrostatic approximation is made, the equations of Table II.L.3 result. These equations form the basis of the thermal plume

TABLE II.L.3. Basic Equations of the
Waldrop-Farmer (Thermal-plume) Model

1. Continuity

$$\nabla \cdot \vec{v} = 0.$$

2. Conservation of Heat Equation

$$\frac{\partial T}{\partial t} = -\nabla \cdot (T\vec{v}) + \frac{\partial}{\partial x} \left(\alpha_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\alpha_z \frac{\partial T}{\partial z} \right).$$

3. x-Momentum Equation

$$\frac{\partial u}{\partial t} = -\nabla \cdot (u\vec{v}) - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(\epsilon_x \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\epsilon_y \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\epsilon_z \frac{\partial u}{\partial z} \right).$$

4. y-Momentum Equation

$$\frac{\partial v}{\partial t} = -\nabla \cdot (v\vec{v}) - \frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(\epsilon_x \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\epsilon_y \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(\epsilon_z \frac{\partial v}{\partial z} \right).$$

5. z-Momentum Equation

$$\frac{\partial P}{\partial z} = -\rho g.$$

model used by the authors. Omission of all velocity-containing terms from the z-momentum equation greatly simplifies the equation set and thus the numerical solution.

With the general form of the governing equations set forth, we proceed to the difficult problem of specifying boundary conditions.

4. Initial and Boundary Conditions

The explicit time algorithm used to integrate the model equations requires that all six variables (u , v , w , P , T , and S) be initialized at each nodal point before the time integration is begun. A crude guess suffices, but a close estimate of the steady-state solution is probably better. In the Mississippi Delta problem, Waldrop and Farmer used the "cold-start conditions" in which all grid points in a direct line with the river mouth are assumed to have the same velocity components as the corresponding point on the river mouth. Other grid points are assigned a uniform lateral-velocity profile. The authors of the model keep previously computed values for problems of similar geometry, yet different boundary conditions, on tape and use these steady-state values as initial values for their new problem. It is presently unclear what effect the initial guess has on the convergence rate or whether all initial guesses eventually lead to the same steady-state solution.

It is essentially the boundary conditions that distinguish one plume problem from another. Therefore, the specification of these conditions is an important matter. Unfortunately, Waldrop and Farmer do not go into detail as to the precise formulation of each boundary condition; we can, however, sketch the procedure they recommend.

Within the power plant outfall or river mouth, Waldrop and Farmer assume a one-dimensional flow in the direction of the channel. For the Mississippi Delta problem, the down-river velocity profile is assumed to be parabolic in both the lateral and vertical directions. Waldrop and Farmer prefer an experimental river velocity profile at some upstream plane, if such data are available. Model calculations would begin at this point and be carried out down river and eventually into the receiving water body. The surface height at the outlet is also needed, and for all but one computer run of the Delta problem, this height is taken to be the same as the initial height of the receiving body.

The near-shoreline boundaries are treated as vertical. Waldrop and Farmer state that the coarse spacing of the grid points in the offshore direction precludes the resolution required to enforce a no-slip condition at these boundaries. Consequently, reflection principles are used to enforce tangential flow.

Along the bottom, a no-slip condition is specified for the velocity. This creates a difficulty in the case of strong bottom interaction, since the

rectangular grid pattern does not match the irregular bottom contours. To compensate for the inadequacy, Waldrop and Farmer use nonlinear interpolation to approximate the value of the dependent variables at the first grid point above the bottom. The adequacy of this procedure is open to question.

A simplified form of the kinematic-boundary conditions was used at the surface in the original form of the model. To obtain a rigorous expression for the kinematic conditions and to see what has been omitted by Waldrop and Farmer, we must return to first principles. Suppose the free surface is specified by an equation of the form

$$F(x, y, z, t) = 0, \quad (7)$$

then F must satisfy

$$\left. \frac{DF}{Dt} \right|_{F=0} = 0. \quad (8)$$

If we further suppose that F is given by

$$F(x, y, z, t) = z - h_s(x, y, t) \quad (9)$$

where h_s represents the height of the free surface, then Eq. 8 becomes

$$w(x, y, h_s, t) = \frac{\partial h_s}{\partial t} + u(x, y, h_s, t) \frac{\partial h_s}{\partial x} + v(x, y, h_s, t) \frac{\partial h_s}{\partial y}. \quad (10)$$

In general, u , v , and w are not known at $z = h_s$; therefore, the authors make the following assumptions:

$$w(x, y, h_s, t) \approx w(x, y, h_0, t), \quad (11)$$

and

$$\frac{\partial h_s}{\partial x}, \frac{\partial h_s}{\partial y} \approx 0, \quad (12)$$

where h_0 is the nominal height of the free surface and the top of the grid system. Thus, Eq. 10 becomes

$$w(x, y, h_0, t) \approx \frac{\partial h_s}{\partial t}. \quad (13)$$

This simplification dismisses certain physical aspects of Eq. 10, but the amount of error thus produced is uncertain.

The simulation of the free-surface boundary condition has recently been upgraded by Waldrop and Farmer. A mass balance is now

performed for the topmost grid cells and the height of water above (or below) the grid system surface is adjusted accordingly. The modification does not replace the physical effects lost in going from Eq. 10 to Eq. 13, but it does at least assure an integral mass balance at the grid surface. As to the surface height itself, it is really the relative surface height or its slope that controls the pressure throughout the flow field. A problem that the Waldrop-Farmer model, as well as other numerical models, faces is the specification of surface elevation at the outfall above the receiving body. This important parameter is only about 1 mm and cannot be in any way measured in the field.

Waldrop and Farmer treat this problem by assigning to the outfall at each time step the height of a point somewhere off the discharge as computed from the previous time step. This procedure seems questionable to us, although we do not know its effect on the final solution. As is seen later, in one case the outfall was raised 15 cm above the ambient elevation as measured at the far boundary of the grid. This value seems much too large to us. A more realistic value would be a few millimeters. What effects this height has on model predictions only further testing can resolve.

The specification of boundary conditions at the remaining lateral extremities of the grid pattern is facilitated by the use of a stretching transformation on the horizontal spatial coordinates of the form

$$X = \frac{1}{C_1} \tan^{-1} \frac{x}{C_2} \quad (14)$$

and

$$Y = \frac{1}{C_3} \tan^{-1} \frac{y}{C_4} \quad (15)$$

A uniformly-spaced grid pattern in the transformed coordinates X and Y corresponds to a pattern in the true spatial coordinates that becomes coarser and coarser as one moves away from the outfall. This enables a larger lateral extent to be covered by the grid system, while at the same time retaining finer resolution near the discharge point. The philosophy here is that a fine grid should be used near the outfall while a somewhat coarser grid is permissible away from the outfall where variables are varying more slowly. Whether the grid spacing is always good is not known and each case should be considered individually. A tradeoff exists between the resolution that can be achieved and the computer time required to solve the finite difference equations.

The stretching transformation also assists in minimizing the difficulties caused by the finite grid system, but does not help in answering the difficult question of what variable specifications need to be made at the boundaries. For example, in the Mississippi Delta problem the upcurrent velocity profile was assumed to be uniform in the offshore direction and to vary in the manner of classical boundary layer flow in the vertical direction.

The effects of upstream and nearshore boundary conditions on the numerical solution are not presently clear, but there is substantial reason for caution. Because the equations are cast in the form of a sequence of parabolic integration steps, the effect of these boundary conditions may be significant. In fact, Waldrop and Farmer observed that upcurrent boundary effects are convected downstream in model calculations.

As the governing equations are stated, boundary conditions are also required on the downcurrent and offshore boundary planes. What is quite surprising is that in the numerical solution these conditions are not really enforced. Variables are left unconstrained at these boundaries. While this is desirable because it eliminates the need to impose arbitrary restrictions on the flow field, the question arises "Are we covering one error with yet another?" First, we distort the problem by introducing the finite grid structure and then we destroy the elliptical nature of the conservation equations by integrating them as a parabolic set.

We can gain more insight into this problem by examining the finite difference technique itself.

5. Numerical Technique

The governing equations are second order in spatial derivatives, yet Waldrop and Farmer managed to cast the problem into a sequence of first order integrations as shown in Table II.L.4. This approach means that implicit (matrix) solutions are never needed. The solution scheme is completely explicit in time, i.e., the solution at each time step is computed solely with the values from the previous time step. Since this is true, the order in which the values are calculated is rather unimportant. One important point is made, however. The calculations of u , v , and T are made using so-called "windward" differencing where, in the expression of the advection terms, no downstream values are ever used. As it happens, the code begins with the lowest values of y and proceeds to larger values of y , which is downstream progression in some cases and upstream in others.

Since no boundary conditions are enforced at the offshore and downstream vertical boundary planes, the model clearly cannot handle backwash effects but the question arises whether there is a more basic problem. In particular, we wonder whether the solution algorithm preserves in any way the spatially elliptical nature of the model equations. The validity of the transient predictions of the model hinge on this question. Waldrop and Farmer expect that their procedure gives the correct solution since they suspect that the elliptic nature is retained in the way pressure is handled. For pressure, central rather than windward differencing is used. A set of fictitious grid points is established using the assumption that the variation in pressure is linear at the boundary. Despite this explanation, no hard proof exists and the concerns we all have remain.

TABLE II.L.4. Numerical Solution Procedure of Waldrop-Farmer Model

1. The x-momentum equation given below is integrated explicitly in time for the u-velocity component:

$$\frac{\partial u}{\partial t} = -X' \left[\frac{\partial(u)^2}{\partial X} + \frac{1}{\rho} \frac{\partial P}{\partial X} \right] - Y' \cdot \frac{\partial(uv)}{\partial Y} - \frac{\partial(uw)}{\partial Z} + \epsilon_x \left[(X')^2 \frac{\partial^2 u}{\partial X^2} + X'' \frac{\partial u}{\partial X} \right] \\ + \epsilon_y \left[(Y')^2 \frac{\partial^2 u}{\partial Y^2} + Y'' \frac{\partial u}{\partial Y} \right] + \epsilon_z \left[\frac{\partial^2 u}{\partial Z^2} \right].$$

The corresponding finite-difference equation is

$$u(n+1; i, j, k) = u(n; i, j, k) - X'(i) \left[(u(n; i+1, j, k))^2 - (u(n; i-1, j, k))^2 \right] \\ + \frac{1}{\rho} [P(i+1, j, k) - P(i-1, j, k)] \frac{\Delta t}{2\Delta X} - Y'(j) (u(n, i, j+1, k) v(n, i, j+1, k) \\ - u(n; i, j-1, k) v(n; i, j-1, k)) \frac{\Delta t}{2\Delta Y} - (u(n; i, j, k+1) w(i, j, k+1) \\ - u(n; i, j, k-1) w(i, j, k-1)) \frac{\Delta t}{2\Delta Z} + \left(\epsilon_x \left[(X'(i))^2 (u(n; i+1, j, k) - 2u(n; i, j, k) \right. \right. \\ \left. \left. + u(n; i-1, j, k)) \frac{1}{(\Delta X)^2} + X''(i) (u(n; i+1, j, k) - u(n; i-1, j, k)) \frac{1}{2\Delta X} \right] \right. \\ \left. + \epsilon_y \left[(Y'(j))^2 (u(n; i, j+1, k) - 2u(n; i, j, k) \right. \right. \\ \left. \left. + u(n; i, j-1, k)) \frac{1}{(\Delta Y)^2} + Y''(j) (u(n; i, j+1, k) - u(n; i, j-1, k)) \frac{1}{2\Delta Y} \right] \right. \\ \left. + \epsilon_z \left[(u(n; i, j, k+1) - 2u(n; i, j, k) + u(n; i, j, k-1)) \frac{1}{(\Delta Z)^2} \right] \right) \frac{\Delta t}{\rho},$$

where

$$X'(i) = \frac{dX}{dx}(i) \quad Y'(j) = \frac{dY}{dy}(j)$$

and

$$X''(i) = \frac{d^2X}{dx^2}(i) \quad Y''(j) = \frac{d^2Y}{dy^2}(j).$$

2. In a similar manner, the y-momentum equation is integrated explicitly in time for the v-velocity component.
3. Depending on whether salt or temperature (or both) is the unknown of interest, one (or both) of the following steps is executed.
- The equation of salt conservation is integrated in time for the salt concentration. Current values of the velocity and past values of the salt concentration are used.
 - The equation of heat conservation is integrated in time for the temperature. Current values of the velocity and past values of the temperature are used. In addition, over-relaxation is used in some cases to speed convergence as follows:

$$T(n+1; i, j, k) = T(n; i, j, k) + \frac{\partial T}{\partial t}(n; i, j, k) n \Delta t,$$

where n is the relaxation parameter.

...At this point, u , v , S , and T have been computed for the current time step. Now two parabolic integrations in space are performed.

TABLE II.L.4. (Contd.)

-
4. The continuity equation is integrated vertically upward from the bottom for the w-velocity component. The corresponding finite-difference expression is:

$$\begin{aligned}
 w(i, j, k) = w(i, j, k - 1) - 0.25 \left\{ X'(i) [u(i + 1, j, k) - u(i - 1, j, k) \right. \\
 + u(i + 1, j, k - 1) - u(i - 1, j, k - 1)] \frac{1}{\Delta X} + Y'(j) [v(i, j + 1, k) \\
 \left. - v(i, j - 1, k) + v(i, j + 1, k - 1) - v(i, j - 1, k - 1)] \frac{1}{\Delta Y} \right\} \Delta z.
 \end{aligned}$$

5. The free-surface elevation is computed by performing a mass balance on the topmost layer of grid cells. The top surface of the grid pattern coincides with the nominal elevation of the free surface. When all of the velocity components are known from Steps 1 through 4, the net mass flow into each surface grid cell can be computed and the surface elevation $h_s(x, y, t)$ adjusted to conserve mass.
6. Pressure is determined by integrating the z-momentum equation (either with or without the velocity terms) vertically downward from the free surface assuming a zero pressure there. No boundary conditions are imposed on pressure at the bottom.

....This completes the integration step.

The steps of Table II.L.4 are carried out until the solution satisfies a criterion that shows it to be "sufficiently" close to steady state. The most stringent criterion that can be applied is a pointwise requirement on the difference between the dependent variables at two successive iteration steps. Waldrop and Farmer opt for a somewhat weaker criterion, which attempts to assess the global convergence of the solution. A convergence parameter g_1 is defined as

$$g_1 = \sum_{i,j,k} [|S(n + 1; i, j, k) - S(n; i, j, k)|]. \quad (16)$$

Parameters g_2 , g_3 , and g_4 are similarly defined for the dependent variables u , v , and T , respectively. The solution is considered to have reached steady state when

$$g_n < \xi_n, \quad n = 1, 2, 3, 4 \quad (17)$$

where ξ_n is a convergence criterion on the variable corresponding to n . Figure II.L.2 shows the convergence history of a typical case.

This type of global convergence criteria is certainly reasonable; however, we feel that a better test of the convergence to steady state (perhaps in addition to the authors' criterion) would be an integral check of the net mass, momentum, energy, and salt transport into the grid system. The net flow into the closed grid region can be computed from the calculated values of the dependent variables. Steady state would be assumed only after the net flows into the grid region had all become sufficiently close to zero. We believe this integral check would be a more physically valid test.

Fig. II.L.2a. Typical History of Convergence Process. (Adapted from Waldrop and Farmer.¹)

Fig. II.L.2b. Typical Convergence History of Velocity Components. (Adapted from Waldrop and Farmer.¹)

We now consider sample model calculations made by Waldrop and Farmer.

6. Mississippi Delta Problem

The first calculations with the Waldrop-Farmer model involved the simulation of flow and sediment deposition at the South Pass of the Mississippi River. The river mouth is approximately elliptical, with a surface width of 137 m and a mid-river depth of 10.5 m. The mean slope of the bottom was taken to be 0.0026 in the model calculation, a value typical of the bottom slope thought to exist in the initial stages of Delta formation. In the Delta problem, there was a negligible temperature differential between river and gulf water, but the density of seawater was 2.2% greater than that of the water flowing from the river mouth. The maximum velocity of the river was taken as 1 m/sec at mid-river on the surface of the river mouth. The tidal crosscurrent was assumed to have a maximum velocity of 1 m/sec at the surface and then to decrease parabolically with depth. The surface elevation within the river mouth was assumed to be equal to the elevation of the receiving body.

Results from this case appear in Figs. II.L.3-4 indicating the kind of predictions one can obtain with the model. For this run, a $19 \times 31 \times 10$ grid

was used with the offshore boundary 920 m from the nearshore and the up-current and downcurrent boundaries located 12,500 m from the centerline of the river. The stretching transformation was used to place the latter boundaries at these distances. In the computation, essentially all the high-speed core of the LSU IBM/360 digital computer was used.

Fig. II.L.3a. Velocity Vectors in a Top View of the Surface.
(Adapted from Waldrop and Farmer.¹)

Fig. II.L.3b. Isoconcentration Contours in a Top View of the Surface. (Adapted from Waldrop and Farmer.¹)

Fig. II.L.4a. The y-z Plane View of Isoconcentration Contours: $x = 100m$.
(Adapted from Waldrop and Farmer.¹)

Fig. II.L.4b. The y-z Plane View of Velocity Vectors: $x = 18m$.
(Adapted from Waldrop and Farmer.¹)

Fig. II.L.4c. The y-z Plane View of Velocity Vectors: $x = 200m$.
(Adapted from Waldrop and Farmer.¹)

Fig. II.L.4d. Variation in Surface Elevation in the Horizontal
Direction: $x = 30m$. (Adapted from Waldrop and Farmer.¹)

Figure II.L.3 shows the surface velocity pattern predicted by the model indicating near stagnation flow on both sides of the jet near the outfall. Surface isoconcentration contours ($S = 0$, fresh water; $S = 1$, salt water) are also presented in Fig. II.L.3. The jet does not appear to bend until about 50 m from the outfall and then does so rather abruptly. Moreover, the isoconcentration contours are relatively compact and do not fan out at increasing distances as field data would most likely indicate. The cause of this problem is not known with certainty, but it might be that the model was not run to steady state.

The diagram of velocity vectors in the y-z plane at an 18-m distance from the river mouth is very interesting (see Fig. II.L.4b). It indicates two large symmetrical vortices formed about the centerline of the jet. Waldrop and Farmer attribute these vortices to pressure gradients in the y direction, caused by the density difference between the two fluids. The circulation resulting from these buoyant vortices convects salt water into the jet from below. The authors of the model state that these vortices increase the mixing of salt and fresh water by bringing the higher density bottom water to the surface. Interestingly, dye experiments⁶ carried out at South Pass indicate that such vortices do exist. Also, salinity measurements of Wright and Coleman⁷ indeed indicate that salinities within the plume are greater than those of the salt water at the periphery. Vertical mixing, resulting from turbulence, along with vertical convection velocities brought some of the warmer water of high salinity to the surface. Thermal imagery studies have further verified that vertical mixing is significant, indicating that warm gulf water emerges at the surface of the plume 2-3 channel widths from the river mouth. These qualitative observations by no means indicate that the model has been verified. It is unfortunate that no plume data were acquired by which to verify these Mississippi Delta predictions.

Waldrop and Farmer made eight computer runs for the South Pass problem to test the model sensitivity and to acquire predictions for different cases of interest. Table II.L.5 summarizes these eight runs (cases) in terms of the initial input conditions that were used.

TABLE II.L.5. Run Log for Analysis of Delta Flow Fields. (Adapted from Waldrop and Farmer.¹)

Title	Run	Maximum River Velocity (m/sec)	Minimum Coastal Current (m/sec)	σ_t	Elevation of River Surface (cm)	Eddy Visc./Diff. Coefficient
Baseline	1	2.5	0	22	0	Nominal
Buoyancy Analyzed	2	2.5	0	5	0	Nominal
Hydraulic Head Effects	3	2.5	0	22	15	Nominal
Turbulent Mixing Analyzed	4	2.5	0	22	0	(Nominal) $\times 10^{-1}$
Low-river Stage	5	1.0	0	22	0	Nominal
Flood Stage	6	4.0	0	22	0	Nominal
Low-river Stage Maximum Coastal Current	7	1.0	-1.0	22	0	Nominal
Flood Stage Maximum Coastal Current	8	4.0	-1.0	22	0	Nominal

The major purpose was to assess the influence on the mixing and flow patterns of turbulent mixing, hydraulic head, buoyancy, and inertia of the river and coastal currents. Details of the predictions for each case are presented by Waldrop and Farmer,¹ who attempt to support the results of these computer calculations by physical arguments. To better understand the implications of the predictions, we have compared subsets of those predictions with each other in Figs. II.L.5-8. Numerical results from plots like those of Fig. II.L.3 and 4

Fig. II.L.5a. Variation of Freshwater Concentration at the Surface with Centerline Distance: Cases I-IV

Fig. II.L.5b. Variation of Freshwater Concentration Half-width at the Surface with Centerline Distance: Cases I-IV

Fig. II.L.5c. Area of Freshwater Isocontours
 at the Surface: Cases I-IV

Fig. II.L.6a. Variation of Freshwater Concentration at the Surface
 with Centerline Distance: Cases I, V and VI

Fig. II.L.6b. Variation of Freshwater Concentration Half-width at the Surface with Centerline Distance: Cases I, V and VI

Fig. II.L.6c. Area of Freshwater Isocontours at the Surface: Cases I, V and VI

Fig. II.L.7a. Variation of Freshwater Concentration at the Surface with Centerline Distance: Cases I, V and VII

Fig. II.L.7b. Variation of Freshwater Concentration Half-width at the Surface with Centerline Distance: Cases I, V and VII

Fig. II.L.7c. Area of Freshwater Isocontours:
Cases I, V and VII

Fig. II.L.8a. Variation of Freshwater Concentration at the Surface
with Centerline Distance: Cases I, VI and VIII

Fig. II.L.8b. Variation of Freshwater Concentration Half-width at the Surface with Centerline Distance: Cases I, VI and VIII

Fig. II.L.8c. Area of Freshwater Isocontours: Cases I, VI and VIII

taken from Ref. 1 were reduced in terms of surface centerline decay of fresh-water concentration ($1 - S$), areas of surface isocontours, and half-widths of those surface contours. The information in Table II.L.5 was put in a more useful form by the calculation of the basic nondimensional parameters for those cases. Those parameters are presented in Table II.L.6. The results of our intercomparison study are given below.

TABLE II.L.6. Basic Parameters for Mississippi Delta Computer Runs

Run	F_0	Re	A	R	ΔH	Eddy Viscosity/Diffusivity Coefficient
1. Baseline	1.61	0.67	13.0	0	0	Nominal
2. Buoyancy Analyzed	2.99	0.67	13.0	0	0	Nominal
3. Hydraulic Head Effects	1.61	0.67	13.0	0	15 cm	Nominal
4. Turbulent Mixing Analyzed	1.61	6.67	13.0	0	0	Nominal $\times 10^{-1}$
5. Low-river Stage	0.644	0.27	13.0	0	0	Nominal
6. Flood Stage	2.58	1.07	13.0	0	0	Nominal
7. Low-river Stage Maximum Coastal Current	0.644	0.27	13.0	1.0	0	Nominal
8. Flood Stage Maximum Coastal Current	2.58	1.07	13.0	0.25	0	Nominal

Re = turbulent Reynolds number
 F_0 = initial densimetric Froude Number
A = aspect ratio (ratio of river width to depth)
R = ratio of ambient current to discharge velocity
 ΔH = elevation of river surface

a. When the buoyancy of the freshwater is reduced (Run 2 compared to Baseline Run), the model indicates that less mixing occurs. Waldrop and Farmer explain this to be due to the weaker buoyant vortices that occur in the Reduced Buoyancy Case; the buoyant vortices are weaker because the pressure gradients that drive these vortices are weaker. Reducing the strength of these vortices reduces the mixing. Figure II.L.5 illustrates this reduced mixing for Case 2. However, this is not physically correct. Tank studies have indicated that plumes having larger initial densimetric Froude numbers have smaller surface concentrations and areas. Consequently, Case 2 with $F_0 = 2.99$ should provide a more rapidly diluted plume than Case 1 with $F_0 = 1.61$. The cause of this discrepancy is of serious concern. Perhaps the eddy viscosities and diffusivities should not have been fixed for both cases. One would expect a larger vertical viscosity and diffusivity for Run 2, the case of a larger initial densimetric Froude number. A further sensitivity analysis in which the diffusivities are changed (consistently) from the baseline values would be of help. Hopefully, upon input of more realistic diffusivities in Cases 1 and 2, the proper trend would result. Of course, this must be tested before any faith

can be placed in the predictive capacity of the model. If the check described is not successful, implications are that the model may not have correct boundary conditions or, worse, that the model does not give a correct solution to the Navier-Stokes equations.

One should also consider the inadequacy of the eddy diffusivities for the Mississippi Delta cases. Prandtl's third eddy-viscosity model was used to calculate momentum diffusivities by

$$\epsilon = 0.0256bU_{\max}$$

The value $\epsilon = 5.12 \times 10^4 \text{ cm}^2/\text{sec}$ was used ($b = 100 \text{ m}$, $U_{\max} = 2 \text{ m/sec}$) as a constant value for all three directions throughout the calculation. Waldrop and Farmer recognize the tenuous nature of using an ϵ value, determined for a circular nonbuoyant jet into a quiescent medium, for the problem of an elliptical river discharge into and interacting with a tidally influenced sea. However, there are further inadequacies in their choice. First, taking b in the Prandtl equation as the channel width is arbitrary. Choosing the channel depth instead would have reduced the very large viscosities and hopefully the large vertical mixing that was observed in all eight runs (see for example Fig. II.L.4a). Second, for low densimetric Froude numbers (0.6-3.0), one would expect a vertical value smaller than the horizontal value, typically by an order of magnitude. The value $5.12 \times 10^4 \text{ cm}^2/\text{sec}$ is most unrealistic for a vertical diffusivity; this value should cause a large amount of vertical mixing that would not otherwise occur. Third, the viscosity, at least its vertical component, must be dependent upon local density and velocity gradients to provide a first-order approximation to reality.

b. Increasing the surface height of the river mouth (Run 3 compared to Baseline Run) 15 cm above the far boundary of the region of computation increased the jet surface height at the first grid point offshore typically 1 cm and never more than 4 cm. The increase in pressure at all depths was significant. Waldrop and Farmer state that this caused less mixing. Also, the jet in Run 3 maintained its initial character longer. (See Fig. II.L.5.) This result is hard to understand since the relative fluid heights should intuitively increase mixing, at least vertically. The increase in surface height at the mouth of the discharge should provide a greater pressure force longitudinally to push the fluid further out from the shore. The increase in head of the discharge water should thus provide greater longitudinal momentum and hence greater mixing.

c. In Run 4 all eddy coefficients were reduced by an order of magnitude; this produced the most significant change. The buoyant vortices became stronger; Waldrop and Farmer state that this increased the lateral entrainment of salt water near the bottom (see Fig. II.L.5). Mixing along the centerline increased near the bottom but decreased near the top.

Reduction of all diffusivities and viscosities should produce less dispersion and this was not observed. The increased entrainment near the bottom and increased vertical velocities should not occur.

d. Runs 5 and 6 indicate how increasing and decreasing the river velocity, compared to Run 1, influences the mixing process of the salt and fresh water. The higher river velocity was found to increase the effect of convection and delay mixing; buoyant vortices were less pronounced indicating more horizontal spreading of the fresh water. For the low-river-flow case, buoyancy was more pronounced with vortices that apparently induced more mixing near the bottom, and a smaller surface plume resulted.

The trends for Runs 1, 5, and 6 as seen in Fig. II.L.6 are not physically correct as evidenced by available tank data. Smaller densimetric Froude numbers yield less diluted plumes. The trend toward higher concentrations should be in the order 6-1-5, not 5-1-6. The fixing of viscosities and diffusivities for all three runs might be at fault. Again, it might be more serious. Fixing the river height at its baseline value in spite of the low- or high-flow nature of the river itself is also unrealistic in these three simulations.

e. The low-river case (1.0 m/sec) with the greatest coastal current (1.0 m/sec), simulated in Run 7, indicated a rather sharply deflected plume. A large-scale recirculation region on the lee side of the river plume was observed. This is a likely place for deposition of suspended material from the edge of the river. Predictions for the flood river (4.0 m/sec) with maximum observed coastal current (1.0 m/sec), as in Run 8, looked remarkably similar to those of the zero current case of Run 6. Apparently, for the model, an ambient current of 1.0 m/sec is small compared to a river velocity of 4.0 m/sec; deflection was observed to increase, however, at distances farther from the river mouth. The trends shown in Figs. II.L.7 and 8 are also physically incorrect. Plumes for Runs 7 and 8 should show a much more diluted plume than simulations 5 and 6 since much dilution water is available, provided no significant cutoff of inshore entrainment occurs. Model predictions showed no such cutoff. Moreover a current ratio, R , of 0.25 or 1.0 should provide a more strongly bent plume based upon available data for those low initial densimetric Froude numbers.

Some additional comments are useful here. First, Runs 5 and 7 are physically unrealistic since a wedge of salt water will intrude into the river and alter its flow characteristics from those assumed in the model runs. Second, plume widths are essentially constant with only slight lateral spreading. This is hard to understand considering the strong buoyant nature of all eight plumes ($F_0 = 0.6 - 3.0$). We speculate that this is due to the large vertical diffusivity and large vertical mixing seen in the numerical predictions. Third, centerline decay of freshwater concentration has a concave down appearance with a concave upward turn that is delayed to far downstream. The nonconcave upward appearance might be due to an incomplete run to steady state.

The above intercomparison of model predictions indicates that much work needs to be done to verify the Waldrop-Farmer model. The eight computer runs should be repeated with improvements in the model included, especially with a change in the surface-height boundary condition. If incorrect physical results persist (even after more appropriate dispersion coefficients are input for all runs), then further sensitivity studies and a study of the model boundary conditions and numerical technique will be needed. A resolution of these questions is necessary for any future confidence in the model for practical applications. All the effects investigated by the runs listed in Table II.L.5 are significant in their influence on the mixing and flow patterns. Field data are needed for a precise verification of the model as well as for a more accurate determination of boundary conditions, especially surface heights and river velocity profiles within the mouth.

7. Point Beach Plume Comparisons

The Point Beach application was made after the Mississippi Delta simulation and included the revision of the surface boundary condition described above. The computer run was made by Waldrop and Farmer and is presented in Ref. 3; the comparisons with data were done at Argonne.

The Waldrop-Farmer numerical model comparison to the Point Beach data⁸ of September 1, 1971 is presented in Table II.L.7 and Figs. II.L.9-14.

TABLE II.L.7. Plant and Meteorological Data for Point Beach
Cases To Be Compared to Waldrop-Farmer Calculations

	1 Sept 71	1 Sept 71	Used in Model Calculations
Symbol	A	B	
Time	0950-1130	1445-1727	
Plant Load (MWe)	480	480	
Outfall Temperature (°C)	27.1	26.5	27.0
Ambient Water Temperature (°C)	17.4	17.4	17.0
Ambient Current Speed (cm/sec)	Not Measured	Not Measured	0.0
Discharge Flow Rate (m ³ /sec)	25.1	25.1	
Width of Discharge (m)	10.7	10.7	10.9
Depth of Discharge (m)	4.1	4.1	
Average Outfall Velocity (m/sec)	0.57	0.57	
Average Wind Speed (m/sec)	4.9	4.9	
Air Temperature (°C)	19.8	18.9	
Initial Densimetric Froude Number	1.91	1.98

Fig. II.L.9

Geometry for Discharge at Point Beach Power Plant. (Adapted from Waldrop and Farmer.³)

Fig. II.L.10. Surface-temperature Measurements at Point Beach Power Plant (Unit 1):
 September 1, 1971, 0950-1130 Hours. ANL Neg. No. 190-368.

Fig. II.L.11. Surface-temperature Measurements at Point Beach Power Plant (Unit 1):
 September 1, 1971, 1445-1727 Hours. ANL Neg. No. 190-373.

Fig. II.L.12. Drogue Studies at Point Beach Power Plant (Unit 1): September 1, 1971, 1300-1337 Hours. ANL Neg. No. 190-409.

Fig. II.L.13. Velocity Vectors at the Surface (left diagram) and at a Depth of 2.4 m (right diagram), Predicted by the Waldrop-Farmer Model for Point Beach (Unit 1) on September 1, 1971. ANL Neg. No. 190-1397.

Fig. II.L.14

Surface Isotherms Predicted by the Waldrop-Farmer Model for Point Beach (Unit 1) on September 1, 1971. (Adapted from Waldrop and Farmer.³) ANL Neg. No. 190-1393.

Temperature data were taken in the morning (see Fig. II.L.10) and afternoon (see Fig. II.L.11); between these times drogues were released from the out-fall and tracked using transits (see Fig. II.L.12). The model was run for a zero ambient current, although a moderate but unmeasured north current apparently existed on that date. The Waldrop-Farmer velocity predictions for the surface and 2.4-m level appear in Fig. II.L.13. Surface isotherm predictions appear in Fig. II.L.14; the 22°C isotherms of the morning and afternoon surveys are superimposed on the diagram for qualitative comparison. Comparison of surface drogue trajectories appear in Fig. II.L.15. The model-data comparison is summarized in Figs. II.L.16 and 17 in which model predictions for the surface values of centerline trajectory, centerline temperature, isotherm areas, and temperature half-widths on both sides of the centerline are plotted with the data. The computer run took about 3 hours on the IBM/360 when a grid of 29 (shore-parallel) x 17 (offshore) x 15 (vertical) cells was employed.

Fig. II.L.15. Pathlines of Floating Particles Predicted by the Waldrop-Farmer Model for Point Beach (Unit 1) on September 1, 1971 (left diagram). Velocity vectors in a vertical cross section 27 m offshore predicted by the Waldrop-Farmer model for Point Beach (Unit 1) on September 1, 1971 (right diagram). (Adapted from Waldrop and Farmer.³) ANL Neg. No. 190-1392.

Fig. II.L.17. Comparison of Waldrop-Farmer Model Predictions to Two Sets of Temperature Data Taken at Point Beach (Unit 1) on September 1, 1971. ANL Neg. No. 190-1398.

The numerical model predicts a considerably smaller plume than is actually present. At $\theta/\theta_0 = 0.4$ for example, the predicted plume area is about 7 times smaller than the measured area. Also, for the stagnant ambient conditions used in the run, the centerline should not turn offshore as it does but should maintain a nearly straight line trajectory from the 60° outfall. A

slight deviation due to nonsymmetric bottom contours might be present. Data taken on dates of stagnant or near-stagnant ambient lake conditions at Point Beach prove this point. The bending that appears in the data taken on September 1, 1971, is probably due to the presence of an unmeasured ambient current and not to any natural bending caused by the irregular bottom topography. The bending in the numerical results is due to the predominance of the pressure-gradient terms over the inertial terms in the Waldrop-Farmer solution of the equations of motion. The right side of the numerical results (looking offshore) maintained a height of about 2 mm above the left side resulting in horizontal-pressure gradients that caused the deflection. The deflection was greater with depth (see Fig. II.L.13) since there is less inertia at the greater depths. We do not understand at this point whether the deflection is due to faulty model simulation, the numerical scheme used for solution, or to some error in the computer code.

A further discrepancy between the model and the data is the compactness of the lower isotherms predicted by the model (Fig. II.L.14), seen further in the top-left diagram of Fig. II.L.17. The centerline temperature of the numerical results appears to have a concave-downward decay instead of the expected gradual dieoff. As noted earlier, this abnormal behavior may be caused by stopping the run before steady state was achieved.

The effects of the boundary conditions (especially on pressure) on the numerical solution are still unknown. Waldrop and Farmer acknowledge the need for a larger grid in Point Beach simulation. They suspect that the disappointing comparison is due in part to the small computational grid. They recommend a grid large enough to contain the plume or large enough to cover a region in which the variables hardly change. Only a sensitivity analysis of the model to computational grid size could ascertain the precise effect of the grid. The authors of the model also suggest another simulation of the Point Beach plume with the effect of the intake included to ascertain whether it has any effect on plume motion.

Another problem with the Point Beach simulation is that the horizontal and vertical diffusivities are essentially unknown. The use of vertical diffusivities equal to 20% of the horizontal diffusivities throughout overemphasizes vertical mixing and might have partly contributed to the extremely rapid decay of temperature and velocity.

8. Startup of a Power Plant on a River

A third application of the Waldrop-Farmer model is the simulation of the unsteady discharge resulting from startup of an unidentified power plant on a river. The plant under consideration has an elliptical outfall configuration of a maximum depth of 5 m and a width at the surface of 25 m. In this simulation the ambient river temperature was taken as 15°C and the outfall temperature as 25°C. The plant discharge rate was 48 m³/sec; the river flow rate was taken as 150 m³/sec. The outfall and river bottom contour are shown in Fig. II.L.18.

Fig. II.L.18

Geometry for Discharge into a Flowing River. (Adapted from Waldrop and Farmer.³)

The simulation procedure was carried out as follows:

(1) A rectangular grid was set up along the river with the upcurrent boundary located 250 m from the centerline of the discharge canal. The maximum river velocity at this boundary is 0.40 m/sec. In this first step, no heated water was discharged. An initial guess of the river flow field was easily made. For the first $1\frac{1}{2}$ min of the computer run, the flow was allowed to adjust itself to account for the unevenness of the river bottom. After these 200 time steps the river flow appeared steady.

(2) The plant discharge was turned on. Velocities within the discharge canal were then linearly increased for $\frac{1}{2}$ min (67 time steps, $\Delta t = 0.45$ sec) at which time the full $48\text{-m}^3/\text{sec}$ flow was reached. The authors of the model noted that although only a small amount of water had entered the river, the velocity patterns were significantly changed downstream and as far as midstream of the river.

(3) Boundary conditions were then fixed at the canal mouth and upstream for the remainder of the computer run. Calculations were monitored every 2 min (267 time steps, $\Delta t = 0.45$ sec) from this point. Waldrop and Farmer noted that in the first 2 min, the velocity field essentially reached steady state. Surface velocity vectors are illustrated, 8 min of prototype time after full operation, in Fig. II.L.19. The numerical results presented are

Fig. II.L.19

Surface Velocity Vectors in a Flowing River. (Adapted from Waldrop and Farmer.³)

representative of all plan views of the flow field for times of 2 or more minutes after full operation of the discharge. The flow field appeared similar with depth but with velocity vectors of progressively smaller magnitude. The only noticeable exception to the constancy of the flow pattern with time was the vertical velocity component. This component apparently resulted from buoyancy effects and was on the order of mm/sec.

Fig. II.L.20. Pathlines of Floating Particles Discharged into a Flowing River. (Adapted from Waldrop and Farmer.³)

Figure II.L.20 illustrates the pathlines of particles released at the surface of the discharge channel. Note that the particles beginning at the centerline and on the upcurrent side of the jet have nearly parallel trajectories and penetrate to nearly half the river width, while the particle beginning on the lee side of the channel outlines a region of recirculation.

Figures II.L.21 and 22 show the surface and cross-sectional isotherms. The slow response of temperature to changes in the flow field was

Fig. II.L.21. Surface Isotherms of a Developing Flow. (Adapted from Waldrop and Farmer.³)

Fig. II.L.22. A Vertical View of Developing Isotherms 35 m Offshore of the Discharge Canal. (Adapted from Waldrop and Farmer.³)

noted. Temperature changes are dependent on both the flow field and on the rates of diffusion. Advection effects are felt quickly, but diffusion effects act very slowly. The slow response of the isotherms is seen in Figs. II.L.21 and 22 in which isotherms after two minutes, eight minutes, and at steady state are plotted.

Figure II.L.23 plots the basic plume characteristics at the surface as predicted by the Waldrop-Farmer model with a comparison to the predictions of the Shirazi-Davis model for the identical situation. Unfortunately, no experimental data are available for this case; however, the models do predict relatively consistent results. Again, the concave-downward axial decay of temperature as predicted by Waldrop-Farmer is notable (see Fig. II.L.23b). This indicates perhaps that the calculations, although run for about one hour of computer time, were not run to steady state. Waldrop and Farmer state that under ordinary circumstances it takes approximately 30 min of computer time for each two-minute interval of prototype time. Consequently, several hours of computer time are needed to attain steady state. Therefore, Waldrop and Farmer used a larger time step to integrate the energy equation. This was done to speed convergence to steady-state temperatures; in this way only one hour of computer time was needed.

Fig. II.L.23a. Comparison of Predicted Trajectories from Waldrop-Farmer Numerical Model (steady-state solution) and Shirazi-Davis Model: Power Plant Start-up Problem

Fig. II.L.23b

Comparison of Predicted Centerline Temperature Decay from Waldrop-Farmer Numerical Model (steady-state solution) and Shirazi-Davis Model: Power Plant Start-up Problem

Fig. II.L.23c

Comparison of Predicted Surface Temperature Half-widths from Waldrop-Farmer Numerical Model (steady-state solution) and Shirazi-Davis Model: Power Plant Start-up Problems

Fig. II.L.23d. Comparison of Predicted Surface Isotherm Areas from Waldrop-Farmer Numerical Model (steady-state solution) and Shirazi-Davis Model: Power Plant Start-up Problem

In the comparison of Fig. II.L.23, the centerline for the Waldrop-Farmer solution was chosen as that noted in Fig. II.L.23a. It appears from the isotherm maps that the temperature centerline returns to the shoreline after about 75 m downstream. Axial temperature decay and temperature half-widths were extracted using this centerline. It is disturbing and somewhat mystifying to us that the velocity centerline is more offshore than the temperature centerline. Again, the solution may not have fully reached steady state at the time the computer run was terminated. A disparity between the velocity and temperature centerlines was also seen in the Point Beach simulation. Waldrop and Farmer explain the disparity in the river simulation in terms of the large current that prevents entrainment water from reaching the lee side of the jet, producing hotter temperatures near the shore. This effect, along with the superposition of the large river velocity on the plume, leads the authors of the model to conclude that the centerlines will be different. We agree that the lateral temperature and velocity distributions will not be similar to one another, but a significant displacement of the maximum points of the lateral velocity and temperature distribution is hard for us to accept. Experimental data on the velocity and temperature profiles for a plume in a large current would settle our differences in physical interpretation.

9. Holston River Data Comparisons

The most recently published comparisons⁴ of the Waldrop-Farmer model are for TVA's John Sevier Steam Plant on the Holston River in eastern Tennessee. Two boat surveys were carried out, the first in March 1973 and the second in July 1973. The field measurements appear in Figs. II.L.24 and 25 on the left sides of the diagrams. A detention dam exists across the river separating the intake (not in the diagrams) from the outfall.

Fig. II.L.24. Field Measurements and Results of Waldrop-Farmer Calculations for the John Sevier Steam Plant, March 1973. (Adapted from Waldrop and Farmer.⁴)

Fig. II.L.25. Field Measurements and Results of Waldrop-Farmer Calculations for the John Sevier Steam Plant, July 1973. (Adapted from Waldrop and Farmer.⁴)

For the winter case the plant was discharging $11.7 \text{ m}^3/\text{sec}$ with an excess temperature (above ambient) of 7.8 C° . The river flow over the dam was approximately $28.3 \text{ m}^3/\text{sec}$. The river depth downstream of the dam was about 1-2 m deep. The thermal plume remained close to the nearshore with very slow horizontal dispersion.

For the summer case, the plant was discharging $31.3 \text{ m}^3/\text{sec}$ with an excess temperature rise (above ambient) of 7.8 C° . The river flow over the dam was $52.4 \text{ m}^3/\text{sec}$. The river depth near the outfall rose to 4-5 m. As opposed to the winter case, a cold-water wedge moved up the discharge channel, with the plume becoming significantly stratified and reaching the far bank. Most of the river water flowed under the plume.

Waldrop and Farmer ran their computer code (latest version discussed above) for horizontal diffusivities of 20, 40, 60, and $100 \text{ cm}^2/\text{sec}$ and with vertical viscosities and diffusivities defined,¹⁰ respectively, as

$$\epsilon_z = \epsilon_H(1 + 10R_i)^{-1/2}$$

and

$$\alpha_z = \alpha_H[1 + (10/3)R_i]^{-3/2},$$

where

$$R_i = -(\rho/\rho) \left(\frac{\partial p}{\partial z} \right) \left(\frac{\partial q}{\partial z} \right)^{-2}, \quad (q = \sqrt{u^2 + v^2})$$

is the gradient Richardson Number defined locally from the horizontal flow and vertical density gradient. In this way the vertical diffusivity is attenuated when stratification is present. The horizontal diffusivities were chosen from measured ambient values in the literature on shallow streams. Computer runs were made with the same viscosities and diffusivities for the winter and summer case.

The results of the four runs for each case were plotted with the data (not shown). For these cases the model was found not to be significantly sensitive to the value of horizontal diffusivity. The computer run predictions in Figs. II.L.24 and 25 are for the $60 \text{ cm}^2/\text{sec}$ value. The model-data comparison for the summer case is quite good. The trend of data and model are different here than for the winter plume: the data show a plume that disperses less than predicted by the model. The model-data comparison for the winter case is reasonable. The data show the plume traveling offshore some distance before bending downcurrent, but the model shows an extremely rapid bend and a strong tendency of the plume to hug the shore. The data show the plume more dispersed toward mid-river, but the predicted plume has its isotherms more compacted against the nearshore. Predictions for both cases

within 800 m ($\sim \frac{1}{2}$ mile) of the outfall were essentially insensitive to viscosity and diffusivity values. Only within 800-1600 m of the outfall did the model predictions exhibit sensitivity to diffusivity. We suspect that near the outfall, inertial and buoyancy forces predominated and consequently the model is insensitive to diffusivities, provided they are chosen to be approximately correct. Only in the far field did the value of diffusivity have a significant effect. The larger diffusivity caused a faster diffusion of heat toward the far bank. A variation of 20-100 cm^2/sec produced only approximately a doubling of the distance of an isotherm from the near bank. The effect of the Richardson number was probably insignificant in the winter case due to the large advection and the resulting fully-mixed condition. The summer case, on the other hand, had reduced inertial forces and stratification. In spite of the horizontal viscosity or diffusivity chosen, the plume isotherms had the same general behavior with lateral dispersion being the main difference. These comparisons are reasonable and encourage application of the model with a smaller current to determine whether the good model-data comparison will persist. Also, predictions beyond the relatively short down-current distance modeled here would be helpful for determining the far-field nature of the model predictions.

The Holston River simulations indicate that a reasonably satisfactory comparison to data can be obtained with proper choice of viscosities and diffusivities. Other simulations are necessary before this statement can be considered to be true in general. The proper choice of viscosities and diffusivities will probably always remain a major issue.

10. Criticisms and Limitations

Several criticisms have been given during the discussion of the governing equations and the numerical technique. The most important are reviewed below.

- a. The hydrostatic approximation may not be valid in all plume problems of interest.
- b. The values used to seed the solution algorithm may be unwisely chosen. Furthermore, there is little information available from which to conclude that the same steady-state solution (as measured by the Waldrop-Farmer convergence criterion) will be obtained no matter what initial guess is used.
- c. The handling of boundary conditions is too vague, and the seriousness of this matter is not adequately addressed in the model.
- d. Approximate kinematic surface conditions are used.
- e. The way in which the equations are integrated may change the elliptic nature of the equations.
- f. An integral check of the conservation equations should be performed as a test of convergence to steady state.

There are a few additional issues that need to be raised at this point.

g. The modeling of turbulent transport

The same general criticism applies to this model that applies to all numerical models (see Chapter III). This criticism is that the functional form of the eddy-diffusion coefficients is not known with any degree of certainty. Relating transport quantities to local gradients is an expedient assumption at best. An improved treatment may be possible by using more complex forms of the diffusion coefficients, but the ability to determine these forms even by fitting to data is doubtful.

As was noted earlier, Waldrop and Farmer arbitrarily reduce vertical eddy diffusion of heat to 20% of the horizontal values. First, this assumption is inconsistent with values in the literature (which are typically 1% or less) and, second, the omission of dependence on local density gradients is unwise.

In some cases, the authors attempt to match channel-flow and open-water diffusivities; the procedure is certainly quite arbitrary.

h. The omission of surface heat loss, wind, and Coriolis effects

These effects are normally small in the near-field region but may control the flow external to the grid structure and thus determine boundary conditions on the grid pattern. In fact, the only external flow consideration made is that of uniform current. These omissions cloud the applicability of the model to complete-field situations.

i. Limited operational experience in application to thermal plumes

Comparisons with prototype data have been limited due in part to the recent origin of the model and to the difficulty in applying a numerical model to a large number of cases. Until more verification work is completed, predictive use of the model is still risky.

j. Computer costs

The sample cases the authors have published required about three hours of central processor time on an IBM/360 to reach steady state. It is still questionable whether true steady state was reached in most of the runs to date. While the time of computation is not excessive in terms of present numerical work, it greatly limits the worth of the model as a practical tool.

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