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Optimized minimal inductance transmission line configuration for Z-Pinch experiments (UCRL-JP-200381)

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Successful dynamic Z-pinch experiments generally require good current delivery to the target load. Power flow losses through highly inductive transmission line configurations reduce the current available to the load. In this Brief Report, a variational calculus technique is used to determine the transmission line configuration that produces the least possible inductance and therefore the best possible current delivery for Z-pinch experiments.

In recent years, Z-pinchs have experienced a renaissance. A number of z-pinch driven intense x-ray sources for inertial confinement fusion (ICF) capsules have been studied [1–5]. Experiments on the Sandia National Laboratory Z facility have also been performed to study the a number of basic physics issues of high energy density plasmas [6,7]. The Z facility delivers up to 3 MJ of electrical energy at 50 TW to four separate vacuum insulator stacks. Currents of up to 20 MA are fed through four magnetically insulated vacuum transmission lines (MITLs) to a vacuum convolute that combines the current as the power flow gets close to the target load [8].

Typically, power is supplied to the target load (the Z-pinch) through the anode-cathode (AK) gap at the base of the pinch. The AK gap is feed by a transmission line configuration coming from the vacuum convolute which, ideally has a low inductance maximizing the current delivery (see Fig. 1). On Z, target loads are generally raised above the convolute for the purpose of providing suitable line-of sight diagnostic access. However, raising the load incurs a cost in inductance that can have a non-negligible impact on the load performance. While a great deal of work has gone into designing target loads, the design of the transmission line configuration has apparently been somewhat neglected. In this Brief Report, an elementary variational calculus technique is applied to the problem of the design of a minimal inductance transmission line configuration.

Consider the situation illustrated in Figure 1. Power coming from the vacuum convolute enters the axisymmetric transmission line at (R_{out}, z_{out}) . The AK gap at the load is located at (R_{in}, z_{in}) . The desired power feed must connect these two points while costing as little as possible in inductance, L . By definition the current, I , and magnetic field, B , are related to the inductance through the expression

$$L = \frac{1}{\mu_0 I^2} \int B^2 dV \quad (1)$$

where μ_0 is the vacuum permeability and the integral is over the volume containing the magnetic field. From Eq. [1] it is clear that the magnetic field and volume conspire to produce inductance. For example, the (a) feed shown in Fig. 1 has a small volume, but because of the inverse radius dependence of the magnetic field ($B = \mu_0 I / 2\pi R$) the B^2 term is large over a significant portion of the volume. The feed shown in (c), keeps the magnetic field small by keeping it at large radius for most of the volume, but the volume is relatively large. The minimal inductance configuration must be a compromise between these two extremes. A conical feed like that shown in (b) approaches a minimal inductance configuration in certain limits, but as will be shown below, is not the absolute minimal inductance solution.

FIG. 1. Of all of the transmission lines shown (a) has the smallest volume, but since the magnetic field is brought to a small radius over a significant fraction of the volume the cost in inductance is the highest of all of the feeds shown in the figure. Feed (c) holds the magnetic field at large radius over most of the volume, but the volume is the largest of any shown again incurring a large cost in inductance—although not as large as (a). The conical configuration (b) appears to be a compromise between either (a) or (c), but it is not the minimum inductance configuration.

Rewriting Eq. [1] using the above expression for magnetic field and relating the volume to the area (A) normal

to the magnetic field between the conductors gives and alternate relation for the inductance

$$L = \frac{\mu_0}{2\pi} \int_{R_{in}}^{R_{out}} \frac{dA}{R} \quad (2)$$

Taking the normal width between the conductors of the transmission line to be w and the element of arc length along the conductors to be ds , then $dA = wds = w\sqrt{1+(z')^2}dR$, where $z' = dz/dR$ is the derivative of the curve $z(R)$ that describes the shape of the power feed (see Fig. 2). The minimum size of the gap w between the conductors of the feed is set by a break-down condition (physics that need not be considered here). If w is taken to be constant then the integral that needs to be minimized is

$$\int_{R_{in}}^{R_{out}} \sqrt{1+(z')^2} \frac{dR}{R}. \quad (3)$$

It is interesting to note that the above integrand is very similar to that obtained in the classic brachistochrone problem [9] with the exception of the denominator (in the brachistochrone problem the denominator is a $\sqrt{z}$ instead of R).

FIG. 2. The feed must connect the endpoints at (R_{in}, z_{in}) to those at (R_{out}, z_{out}) . The arc length along the feed is s and the width of the channel is w . The shape of the power feed, $z(R)$ that minimizes the inductance is determined by minimizing the integral shown in Eq. (2).

From the Euler-Lagrange equation, one finds that

$$\frac{\partial}{\partial z'} \left(\frac{\sqrt{1+(z')^2}}{R} \right) = \text{const.} \quad (4)$$

Solving the above equation, one obtains an expression for a circular arc

$$\begin{aligned} & \left(z - \bar{z} - \bar{R} \frac{\Delta R}{\Delta z} \right)^2 + R^2 \\ &= \bar{R}^2 \left[1 + \left(\frac{\Delta R}{\Delta z} \right)^2 \right] + \frac{1}{4} [(\Delta z)^2 + (\Delta R)^2] \end{aligned} \quad (5)$$

where we have used the position of the fixed endpoints of the configuration to determine the two constants of integration. In Eq. [5], $\bar{R} = \frac{1}{2}(R_{out} + R_{in})$, $\bar{z} = \frac{1}{2}(z_{out} + z_{in})$, $\Delta R = R_{out} - R_{in}$, and $\Delta z = z_{out} - z_{in}$. Thus, the minimal inductance feed is not a cone as one would obtain by connecting the endpoints by a straight-line in the R - z plane, but a dome shape as shown in Figure 3 when $\Delta z \bar{z} < -\Delta R \bar{R}$. When the change in vertical height is very small ($|\Delta z| \ll |\Delta R|$) Eq. [5] approaches a nearly horizontal straight-line. Curiously, when the change in height is large ($\Delta z \bar{z} > -\Delta R \bar{R}$), Eq. [5] indicates that the transmission line goes *outward* in radius from R_{out} along a circular trajectory before going inward in radius to connect to R_{in} . Going outward in radius takes advantage of the inverse radius dependence of the magnetic field, apparently compensating for the extra volume.

FIG. 3. The minimal inductance feed for a constant width anode-cathode gap has a dome shape instead of a conical one.

For typical Z dimensions ($R_{out} \approx 4 - 5$ cm, $R_{in} \approx 1$ cm, and $\Delta z \approx 2.2$ cm) the dome shaped transmission line offers a 3-7 % reduction in inductance as compared to a conical transmission line and a 60 % reduction in inductance as compared to the “L” shaped feed shown in Fig. 1. The distinction, while arrived at through elementary means, can have a significant practical impact as even small reductions in transmission line inductance allows more energy to be delivered to the load. Generalization of the above technique to include variable gap width (as is required for the outer MITL on Z) or other physical constraints is straightforward.

It should be clear from Eq. [1], that by minimizing the inductance we have minimized the magnetic energy contained in the volume bounded by the two conductors carrying the current. This energy is itself equal to the work required to drive the current through the conductors in the first place. It is interesting to note that the self-limited impedance of magnetically insulated transmission lines can be obtained by minimizing the total electromagnetic field and electron kinetic energies [10].

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