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CRITICAL PROPERTIES OF A SIMPLE SPIN GLASS MODEL

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1. INTRODUCTION

The spin glass state and the phase transitions into it have recently drawn much theoretical attention (1-9). In this phase the magnetic moments are supposed to be frozen but pointing in random directions, due to a competition between e.g. quenched ferromagnetic and antiferromagnetic interactions. In many systems of experimental interest (10-13) the magnetic ions are embedded in a metallic matrix and interacting with a RKKY coupling which becomes random in magnitude and sign due to the distribution of the interion spacing. However, in many theoretical models simpler short range random exchange interactions are assumed (2-9). At the present stage the theories appear not to be conclusive due to the use of mean field theory which neglects critical fluctuations and difficulties with the $n = 0$ (2,14) trick at low temperatures (7,8). The more recent renormalization group calculations (6) have used an expansion around $d=6$ (d is the number of space dimensions) which would not lead to accurate results at $d=3$. In view of those difficulties it seems worthwhile to have an exactly soluble model for the spin glass transition which although not realistic enough to correctly represent the systems under consideration may nevertheless give one some new insights on the problem. Such a model was recently suggested by Mattis (9), where by assuming a particular distribution of the exchange interactions, the spins can be redefined, leading to a uniform exchange model. Thus for a vanishing magnetic field all the properties of the Mattis model can be calculated in terms of those of the ordered model. A uniform magnetic field on the original model will become a random magnetic field (15,16) on the transformed spins. This leads to a number of predictions and conjectures (17) based on what is known on the random

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field model (16,18,19). The fact that the uniform magnetic field when applied on the spin glass has the same effect as a random field on an ordered uniform system is an exact result for the Mattis model. However, this type of result may be qualitatively valid also for more realistic spin glass systems. If indeed in the spin glass phase all spins are frozen and point in well-defined but random directions, an "up" magnetic field will appear to be of a random direction as viewed from the randomly varying "ordering" frame of reference of each spin. In fact, quite generally, the magnetic field tends to break up the spin glass ordering and turn it into a ferromagnetic type ordering. Our work highlights the importance of the magnetic field as a variable in the study of spin glasses. This has also motivated us to construct a scaling theory for the effects of the magnetic field whose range of validity is much broader than the original Mattis model.

In section 2 we describe the Mattis model as following from a particular quenched random solid solution picture, and discuss its zero-field properties. The random field model is reviewed in section 3. The application to the spin glass problem is made and the more general scaling theory presented in section 4. Section 5 contains concluding remarks, generalizations and a discussion of the limitations of the model.

2. THE MODEL AND ITS ZERO FIELD PROPERTIES

Consider (17) a quenched magnetic alloy of the composition $A_p B_{1-p}$, its Hamiltonian in a magnetic field B is:

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j - B \sum_i S_i^z \quad (1)$$

where $\vec{S}_i$ is an n component classical spin vector with a unit length (including the Ising case, $n=1$) and J_{ij} , the nearest neighbors exchange, is given by J_{AA} , J_{BB} or J_{AB} , depending on whether the pair of ions on sites i and j are AA , BB or AB , respectively. It is easily seen, by defining a variable ϵ_j to be 1 (-1) if the site j is occupied by a $A(B)$ ion, that in the case where $J_{AA} = J_{BB} = -J_{AB}$, which we shall henceforth assume, (1) becomes

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j \vec{S}_i \cdot \vec{S}_j - B \sum_i S_i^z \quad (2)$$

This is the model suggested by Mattis (9). In the classical case one may redefine the spins (9)

$$\vec{\tau}_i = \epsilon_i \vec{S}_i \quad , \quad (3)$$

(τ_i is again an n-dimensional unit vector) and obtain:

$$= -J \sum_{\langle i,j \rangle} \vec{\tau}_i \cdot \vec{\tau}_j - B \sum_i \epsilon_i \tau_i^z \quad (4)$$

This is a uniform exchange model, but the magnetic field, while remaining in the z direction, is random in both sign and magnitude. For the case which we shall consider, where $p = 1/2$ and $\bar{\epsilon} = 0$, the average $\overline{B\epsilon_i} = 0$ (a bar over the quantity denotes an average over the disorder configurations). Clearly, all the zero field properties of this model are expressible in terms of those for the uniform exchange (ordered) model. In particular, all the thermodynamics, including the value of T_c and the specific heat singularity of the model (2) are exactly the same (3) as those of the ordered model. The thermal average of a particular spin, in a given configuration of the ϵ 's is:

$$M_i = \langle S_i^z \rangle = \epsilon_i \langle \tau_i^z \rangle = \epsilon_i M_n(T) \propto \epsilon_i |t|_i^{\beta_n}, \quad (5)$$

for $T < T_c$, where M_n is the magnetization of the pure n component system and $t \equiv (T - T_c)/T_c$. z is assumed to be the direction along which the τ -system is ordered (20) and the subscript n refers to the critical exponents of the pure n-component system. Thus, clearly (for $\bar{\epsilon} = 0$):

$$\bar{M}_i = \bar{\epsilon} M_n(T) = 0 \quad (6)$$

The average magnetization is zero, while each spin, in a given configuration of the ϵ 's, points along $\epsilon_i \hat{z}$, which is the reason why the uniform field B appears as a random field to the spins as viewed around their random local average directions.

One may now define the usual spin-glass order parameter

$$Q \equiv \overline{|\langle S_i \rangle|^2} = \overline{|\langle \tau_i \rangle|^2} \propto \tau^{2\beta} \quad (7)$$

This is not unique. One may also consider other order parameters for example

$$Q \equiv \overline{|\langle S_i \rangle|} \propto |t|^\beta. \quad (8)$$

We shall have more to say about this arbitrariness in choosing the order parameter later.

The zero field spin-spin correlation function is given by

$$\Gamma^{\alpha\beta}(r_{ij}) \equiv \overline{\langle S_i^\alpha S_j^\beta \rangle} - \overline{\langle S_i^\alpha \rangle} \overline{\langle S_j^\beta \rangle} = \overline{\epsilon_i \epsilon_j} \Gamma_n^{\alpha\beta}(r_{ij}) \quad (9)$$

where $\Gamma_n(r_{ij})$ is the correlation function of the pure model. We can assume no long range correlations in the atomic positions: $\overline{\epsilon_i \epsilon_j} \rightarrow \bar{\epsilon}^2$ as $r_{ij} \rightarrow \infty$. Thus $\Gamma_n^{\alpha\beta}(r_{ij})$ will tend to a finite limit at long distances only if $\bar{\epsilon} \neq 0$ ($P \neq 1/2$) - which will then imply ferromagnetic long range ordering. In the case of interest to us, $\bar{\epsilon} = 0$ and we can assume that $\epsilon_i \epsilon_j$ either vanishes for $i \neq j$ or decays to zero as a function of r_{ij} on some finite microscopic scale, on the order of a lattice constant. The correlation (8) is thus short ranged, its singular behavior is given by $t^{1-\alpha(n)}$ for $T > T_c$ and by $|t|^{2\beta(n)}$ for $T < T_c$. The latter behavior is due to the subtracted $\langle S_i \rangle \langle S_j \rangle$ part in the definition (8).

The zero field magnetic susceptibility is given by

$$\chi = \frac{\partial M}{\partial B} = \frac{1}{TN} \sum_{i,j} \sum_{\alpha} \Gamma_n^{\alpha\alpha}(r_{ij}) = \frac{1}{T} \left\{ \sum_{\alpha} \Gamma_n^{\alpha\alpha}(0) + \frac{1}{N} \sum_{i \neq j} \overline{\epsilon_i \epsilon_j} \Gamma_n^{\alpha\alpha}(r_{ij}) \right\} \quad (10)$$

For $p \neq 1/2$ this will diverge like $|t|^{-\gamma_n}$ as $T \rightarrow T_c$. However, for the pure-spin-glass ordering case, $p = 1/2$, χ has an asymmetric cusp at T_c , behaving as $\frac{1}{T} + O(t^{1-\alpha})$ for $T > T_c$ and like $-O(|t|^{2\beta}) + O(|t|^{1-\alpha})$ for $T < T_c$. The $|t|^{1-\alpha}$ corrections follow when there are short range correlations in the ϵ 's.

Below T_c , χ is a sum of $(1-M_n^2)/T$ and a term proportional to E_n/T and to the possible short range correlation in the ϵ 's. Thus, as $T \rightarrow 0$ χ (eq. 9) vanishes exponentially for the Ising model and as a power of T for $n \geq 2$. It should be emphasized that those predictions are specific for the Mattis model and may well change when the exchange distribution allows for weakly coupled spins and clusters.

The singular behavior of the resistivity, ρ , is a quantity which is relatively easy to measure. If there are short range correlations in the ϵ 's, one finds here a cusp-like behavior of ρ characterized both above and below T_c by the exponent $1-\alpha$ (21).

It is interesting to note that although the model considered here should not be expected to describe real systems, the cusps in χ and ρ appear to agree qualitatively with experiments (10-12). The behavior of the specific heat does not agree with the experiments of ref. 13a where no anomalies in the specific heat were found. However, some anomalies were found for different materials in ref. 13b. What is perhaps more serious is that the low temperature behavior of the specific heat in this model is identical to that of the pure case. The special $O(T)$ (or $O(T^2)$)⁸ term in the specific heat at low temperatures (1,7) does not appear, probably due to the lack of "loosely bound clusters" in this model.

As noted by Mattis (9), the nonlinear magnetic susceptibility diverges more strongly than the linear one as $T \rightarrow T_c$. By symmetry $M''(B) = 0$, $M'''(B = 0)$ is proportional to the fourth cumulant average of M

$$M'''(0) = -\frac{6}{T^3 N} \sum_{i,j} \Gamma_n^{zz}(r_{ij}) \{ \Gamma_n^{zz}(r_{ij}) - 2 M_n(T)^2 \} t^{-\gamma_{3,B}} \quad (11)$$

Assuming scaling for the pure correlation function Γ_n^{zz} , one finds that

$$\gamma_{3,B} = 2\gamma(n) - 2 + \alpha(n) \quad (12)$$

$\gamma_{3,B}$ is thus positive and M''' diverges for $d < 4$. It is interesting to note (17), that the divergence is related to the correlation function of the order parameter Q , $\Gamma_Q^{\alpha\beta}(r_{ij}) = \{ \Gamma_n^{\alpha\beta}(r_{ij}) \}^2$, so that $\gamma_{3,B} = \gamma_Q$. Note that since $\nu_Q = \nu_n$, $\beta_Q = 2\beta_n$, $\eta_Q = d - 2 - 2\eta_n$, the critical exponents associated with the order parameter Q satisfy all the usual scaling relations.

On the other hand, one can also easily evaluate the correlation function of the order parameter $\tilde{Q}$ (eq. 7). It is easily seen that $\Gamma_{\tilde{Q}}^{\alpha\beta}(r_{ij}) = \Gamma_n^{\alpha\beta}(r_{ij})$, thus all exponents of $\tilde{Q}$ are identical to those of the pure system - $\nu_{\tilde{Q}} = \nu_n$, $\eta_{\tilde{Q}} = \eta_n$, $\gamma_{\tilde{Q}} = \gamma_n$, $\beta_{\tilde{Q}} = \beta_n$. These exponents again satisfy all the usual scaling relations. It is interesting to note that $\gamma_{\tilde{Q}} > \gamma_Q$; in fact, above $d=4$ $\gamma_{\tilde{Q}}=1$ while γ_Q does not diverge. Thus the instability of $\tilde{Q}$ is more pronounced than that of Q , which suggests that for this particular model $\tilde{Q}$ may be a more appropriate choice of the "main" order parameter than Q . While this question clearly needs further study, we remark that the order parameter $\tilde{Q}$ (which is equal to $\langle \tau \rangle$) appears in fact to be analogous to the one suggested by Kosterlitz, Thouless and Jones (8b) in another exactly solvable model. There the exchange matrix is diagonalized by a linear transformation on the spins and the most unstable mode, corresponding to the maximum eigenvalue, is suggested as the natural order parameter. In our case this mode is trivially associated with the variable τ .

3. PROPERTIES OF THE RANDOM FIELD MODEL

Eq. (4) with $B \neq 0$ contains a quenched random magnetic field. The general problem where the field conjugate to the order parameter is random and quenched was considered for the ideal Bose gas ($n \rightarrow \infty$) in ref. 15, and for general n in ref. 16. Such models appear to be physically realizable at present for spin glasses, as discussed here, and for displacive and charge-density-wave transitions; their critical properties are sufficiently different from

the usual ones to warrant a careful study. Two main general results were obtained in ref. 16. Heuristic arguments were given to show that in systems with a continuous symmetry ($n \geq 2$) an arbitrarily weak random field should eliminate long range ordering for $d \geq 4$. This can be seen as due both to the diverging fluctuations perpendicular to the assumed direction of ordering and to the instability of the ordered state towards domain formation following the statistical fluctuations of the random field. The latter argument indicates that for $n=1$ the random field will break the long range order for $d \geq 2$. Also, the dimensionality at which the critical behavior becomes mean-field-like was found to be $d=6$ (independent of n) and expansions for the critical exponents were found to order $\epsilon = 6-d$. These expansions are identical to those of the pure (no random field) model around $d=4$. This latter identity has been proven to $O(\epsilon^3)$ by Grinstein (22) who also found that the hyperscaling law $\nu d = 2-\alpha$ is replaced by $\nu(d-2) = 2-\alpha$ and conjectured that this reduction of d by two might hold more generally. This conjecture was proven to be valid to all orders in perturbation theory in ref. 19 where heuristic arguments were also given to interpret it physically. It appears that the lowering of the effective dimensionality by two can be obtained approximately by assuming (19) that the dominant cause for disorder close enough to T_c is not thermal fluctuations but the disordering energy due to statistical fluctuations in the random field. A separate heuristic argument was given (19) for $n=1$ using Thompson's (23) picture. For $n=1$ the reduction of d by two is not valid for lower dimensionalities (the critical dimension below which long range order is lost decreases from $d=2$ with the random field to $d=1$ with no random field). This particular qualitative argument yields $\nu = d+2/4(d-2)$, i.e. $\nu=5/4$ at $d=3$ and $\nu \rightarrow \infty$ when $d \rightarrow 2$. Although one should not trust this prediction numerically, it is suggested that the critical behavior with the random field at $d=3$ and $n=1$ will be extremely interesting and very different from the usual one. Experiments on Ising like spin glasses in a magnetic field might probe this interesting new critical behavior, while the usual Heisenberg case should verify the ideas about the elimination of the long range order in a random field.

4. APPLICATION TO THE SPIN GLASS IN A FIELD

In the first part of this section we shall apply the results of section 4 to the Mattis model in a field. Although the results are strictly valid only for this particular case, our above arguments indicate that they may have a wider range of validity. In the second part of this section we present a more general scaling theory for the effect of a magnetic field on a spin glass.

The fact that a random field disrupts the long range ordering in continuous symmetry systems suggests that the sharp phase transition will also be eliminated. This latter statement is not obviously valid however, due to the fact that a sharp transition to another spin-glass-like phase cannot be ruled out, although one would not expect a sharp transition with a nonzero field present which is conjugate to the ordering mode. Assuming that the random field (in the τ system) does indeed smear the transition-one would predict the cusps in ρ and χ at T_c to be smeared out in the presence of the field. Such smearing does indeed occur experimentally, for systems that approximate Heisenberg spin-glasses. A real test for our theory would be if the measurements of $\rho(T)$ and $\chi(T)$ with and without the field would be made for an Ising-like spin glass. Our prediction in this case would be that the cusps in ρ and χ should not be smeared in a $d=3$ system, because for Ising-like systems the random field disrupts ordering only below $d=2$! Another important prediction would be the unusual values of the critical exponents for an Ising-like spin glass in a field at $d=3$.

A smearing of the cusp in χ at a finite field has also been predicted by the previous theories (1-8) of the spin glass transition. We remark however that a $|t|^{1-\alpha}$ singularity is an extremely delicate critical effect which can be hardly expected to be accurately obtained from a mean field theory. Thus, further work is needed in order to ascertain the validity of the cusp in χ as obtained in mean field theory, as well as its smearing in a magnetic field. It should also be noted that in the mean field theory, similar results would be obtained for Heisenberg and Ising systems. The smearing is thus predicted in these theories to occur also in the Ising case. As mentioned above, experiments on the latter systems would be invaluable for clarifying this question.

Since the magnetic field is an important variable for spin glasses, it is natural to write down a scaling theory for the singular part of the free energy as a function of t and B . This has the form (choosing B^2 as the variable, due to symmetry reasons)

$$F(t, B^2) \propto |t|^{2-\alpha} f(B^2 |t|^{-\phi}) \quad (13)$$

where α is the specific heat index of the system without a field and ϕ is the cross-over exponent. For the particular case of the Mattis model $\alpha=\alpha_n$ and (16,17) $\phi=\gamma_n$. However, eq. 13 should be generally valid even for realistic systems, albeit with different values of α and ϕ . From a scaling relation like (13) one can easily obtain the critical singularities in the higher order nonlinear susceptibilities (24). The k^{th} magnetic field derivative of M diverges as $|t|^{-\gamma_{k,B}}$ where

$$\gamma_{k,B} = \alpha - 2 + (k+1)\phi/2 \quad (14)$$

Eq. (12) for the divergence of M''' in the Mattis model is a special case of this with $k=3$, $\alpha=\alpha_n$, $\phi=\gamma_n$. In the general case measurements of χ and M''' (which should be much more strongly divergent!) should yield the correct exponents α and γ . Similar scaling assumptions can be made for the correlation functions.

5. CONCLUDING REMARKS

It should be emphasized that the particular model embodied in Eq. (2) and obtained for the AB mixture when $J_{AA} = J_{BB} = -J_{AB}$ and $p=1/2$, is indeed very specialized. If the above restrictions are relaxed, one can easily show that the additional terms are relevant in the renormalization group sense. Thus, the critical behavior of real spin glasses at $B=0$ can be expected to be different from that of the pure system. However, the difference between our results and those of Ref. 6 may not be totally due to the difference in the models considered. For the particular case of the Mattis model one can see that the usual truncation of the expansion of the effective ($n \rightarrow 0$) Hamiltonian in the cumulants of the exchange distribution does not yield the correct results at any finite order. The possible necessity of summing the cumulants to all orders could also affect the result of a renormalization group calculation (25), although it might well be an artifact of the Mattis model.

Having a solvable model allowed us to obtain some insights on the importance of the magnetic field and the identification of the "main" order parameter. A more general scaling theory for the former effect was given. Experiments on real spin glasses with magnetic fields should be invaluable to clarify the phase diagram, the nonlinear properties in the field and the question of the smearing of the cusps. Finally, the critical behavior of an Ising-like spin glass in a small field is suggested as an outstanding problem both experimentally and theoretically.

The model considered here can also be used in studies of the time dependence of the magnetic properties and of the transformation between quenched and annealed behaviors (26).

We conclude by mentioning that random single site anisotropies which are of interest can be handled by similar models. A particular model might be given by

$$= -J \sum_{\langle ij \rangle} (\hat{x}_i \cdot \hat{S}_i)(\hat{x}_j \cdot \hat{S}_j) - B \sum_i S_i^z \quad (15)$$

where $\hat{x}_i$ is a randomly oriented unit vector. Denoting $\tau_i = \hat{x}_i \cdot \vec{S}_i$ yields an Ising like model with random field which should be similar to the model of Ref. 27 (28). Also, random anisotropy fields might play a role similar to that of the random magnetic field in breaking long range order for $n \geq 2$.

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REFERENCES

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1. C. Held and M.W. Klein, Phys. Rev. Lett. 35, 1783 (1975) contains many previous references; see also M.W. Klein, unpublished.
2. S.F. Edwards and P.W. Anderson, J. Phys. F 5, 965 (1975).
3. D. Sherrington and B.W. Southern, J. Phys. F 5, L49 (1975).
4. K.H. Fischer, Phys. Rev. Lett. 34, 1438 (1975) and Solid State Comm. 18, 1515 (1976).
5. D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1792 (1975).
6. A.B. Harris, T.C. Lubensky and J.H. Chen, Phys. Rev. Lett. 36, 415 (1976).
7. For a recent review, see D. Sherrington, AIP Conf. Proc. (in press).
- 8a. D.J. Thouless, P.W. Anderson, E. Lieb and R.G. Palmer, unpublished.
- 8b. J.M. Kosterlitz, D.J. Thouless and R.C. Jones, Phys. Rev. Lett. 36, 1217 (1976).
9. D.C. Mattis, Phys. Lett. 56A, 421 (1976).
10. Many experiments are reviewed in J.A. Mydosh, AIP Conf. Proc. 24, 131 (1974); T.A. Fiory, AIP Con. Proc. (in press).

11. C.N. Guy, J. Phys. F5, L242 (1975).
12. D.E. Murnick, A.T. Fiory and W.J. Kossler, Phys. Rev. Lett. 36, 100 (1976).
- 13a. L.E. Wenger and P.H. Keesom [Phys. Rev. B11, 3497 (1975)].
- 13b. B.V.B. Sarkissian and B.R. Coles, Comm. on Physics 1, 17 (1976) and references therein.
14. V.J. Emery, Phys. Rev. B11, 239 (1975).
15. P. Lacour-Gayet and G. Toulouse, J. Phys. (Paris) 35, 425 (1974).
16. Y. Imry and S. Ma, Phys. Rev. Lett. 35, 1399 (1975). In this paper the spin definition is also mentioned as possibly leading to a random magnetic field.
17. A. Aharony and Y. Imry, unpublished.
18. G. Grinstein, unpublished.
19. A. Aharony, S. Ma and Y. Imry, unpublished.
20. It may be argued that $\langle \tau_i^z \rangle = 0$, on the grounds that the field conjugate to the order parameter $\langle \tau \rangle$ is strictly zero, while an infinitesimal symmetry breaking field is needed, which is conventionally sent to zero after the thermodynamic limit is taken. While this remark is correct in principle, it does not appear to be of much physical relevance. Although the infinite time average of τ_i is zero, the average τ should point, within any reasonable macroscopic time, in a well defined direction which we choose as the z-axis.
21. M.E. Fisher and J.S. Langer, Phys. Rev. Lett. 20, 665 (1968). Note that in the Born approximation, the leading singularity in the resistivity below T is $|t|^{1-\alpha(n)}$ and not $|t|^{2\beta(n)}$ due to the fact that ρ is determined by the average $\langle SS \rangle$ correlations without the subtraction in Eq. 8. We thank M.E. Fisher and S. Alexander for discussions on this point. See also, e.g., S. Alexander, J.S. Helman and I. Balberg, Phys. Rev. B13, 304 (1976).
22. G. Grinstein, unpublished.
23. C. Thompson, J. Phys. A9, L25 (1976).
24. Y. Imry, Phys. Rev. B13, 3018 (1976).

25. See, in this connection, A. Aharony, Y. Imry and S. Ma, Phys. Rev. B13, 466 (1976).
26. The annealed case of the present model was considered by E. Domany, Phys. Lett. 49A, 339 (1974). See also, A. Aharony and B.A. Huberman, unpublished.
27. R. Harris, M. Plischke and M.J. Zuckermann, Phys. Rev. Lett. 31, 160 (1973).
28. Lubensky (private communication) found an Ising spin glass phase for this system, using the methods of Ref. 6.