Z’ generation with PYTHIA

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Abstract

This document is intended as a guide for getting started with the Z’ generation with PYTHIA[1]. Several different conventions used in literature are discussed, and the conversion among these is given. The Z’ couplings to fermions are given for the sequential Z’, the Z’ model-lines of Ref. [2], and the popular E6 Z’ models.

1 Notations and Conventions

The interaction of the Z’ boson to Standard Model (SM) fermions f can be generally written as:

\[ \mathcal{L} = \frac{g_{Z'}}{2} \bar{f} \gamma^\mu (z_{f_L} P_L + z_{f_R} P_R) f Z'_\mu \]  

(1)

where \( g_{Z'} \) is the \( U(1)_{Z'} \) gauge coupling, \( z_{f_L} \) and \( z_{f_R} \) are the left and right handed fermion charges, and \( P_L = (1 - \gamma^5)/2 \) and \( P_R = (1 + \gamma^5)/2 \) are the projectors for left and right-
handed chiral fields. In the above equation, it is customary to separate the chiral operator \( \gamma^5 \) term; we list below the main conventions used in literature:

\[
g z' \bar{f} \gamma^\mu (z_{fL} P_L + z_{fR} P_R) f Z'_\mu
\]

\[
= g z' \bar{f} \gamma^\mu \left( \frac{z_{fL} + z_{fR}}{2} - \frac{z_{fL} - z_{fR}}{2} \gamma^5 \right) f Z'_\mu
\]

\[
= g z' \bar{f} \gamma^\mu (C_{fV} - C_{fA} \gamma^5) f Z'_\mu
\]

\[
\equiv C_{fV} \bar{f} \gamma^\mu (C_{fV} - C_{fA} \gamma^5) f Z'_\mu
\]

\[
\equiv \frac{g}{4 \cos \theta_W} \bar{f} \gamma^\mu (C_V - C_A \gamma^5) f Z'_\mu
\]

where \( g = 0.626 \) is the SU(2)\(_L\) gauge coupling, and \( \theta_W \) is the Weinberg angle. Note the slightly different notations used to identify the four conventions. The axial and vector couplings defined above can be expressed as:

\[
C_{fV} = (z_{fL} + z_{fR})/2 \quad C_{fA} = (z_{fL} - z_{fR})/2 \quad \text{CDFT [2]}
\]

\[
C_{fV} = z_{fL} + z_{fR} \quad C_{fA} = z_{fL} - z_{fR} \quad \text{Halzen+Martin [3]}
\]

\[
C_{V^{A'}} = -g z'(z_{fL} + z_{fR})/2 \quad C_{A^{A'}} = g z'(z_{fL} - z_{fR})/2 \quad \text{Rosner87 [4]}
\]

\[
C_V = 2 \cos \theta_W (z_{fL} + z_{fR}) g z'/g \quad C_A = 2 \cos \theta_W (z_{fL} - z_{fR}) g z'/g \quad \text{PYTHIA [1]}
\]

From these conversion relations, it is now easy to express the input couplings to PYTHIA needed to implement various \( Z' \) models. As seen in Eqns. (4) and (5) (or Eqns. (8) and (9)), for the Rosner paper and PYTHIA cases the \( g z' \) is included in the \( V \) and \( A \) couplings.
<table>
<thead>
<tr>
<th>$f$</th>
<th>$Q_f$</th>
<th>$T^3_f = C_{fa}$</th>
<th>$C_{fy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e, \nu_{\mu, \tau}$</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$e^-, \mu^-$</td>
<td>-1</td>
<td>-1/2</td>
<td>-1/2 + 2\sin^2 \theta_W</td>
</tr>
<tr>
<td>$u, c$</td>
<td>2/3</td>
<td>1/2</td>
<td>1/2 - 4\sin^2 \theta_W/3</td>
</tr>
<tr>
<td>$d, s$</td>
<td>-1/3</td>
<td>-1/2</td>
<td>-1/2 + 2\sin^2 \theta_W/3</td>
</tr>
</tbody>
</table>

Table 1: SM couplings, from page 301 of Halzen+Martin [3].

<table>
<thead>
<tr>
<th>$V$</th>
<th>$d$</th>
<th>$A$</th>
<th>$V$</th>
<th>$u$</th>
<th>$A$</th>
<th>$V$</th>
<th>$c$</th>
<th>$A$</th>
<th>$V$</th>
<th>$\nu_e$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARU(121)</td>
<td>PARU(122)</td>
<td>PARU(123)</td>
<td>PARU(124)</td>
<td>PARU(125)</td>
<td>PARU(126)</td>
<td>PARU(127)</td>
<td>PARU(128)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1 + \frac{1}{3} \sin^2 \theta_W</td>
<td>-1</td>
<td>1 - \frac{3}{3} \sin^2 \theta_W</td>
<td>1</td>
<td>-1 + 3 \sin^2 \theta_W</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.693</td>
<td>-1</td>
<td>0.387</td>
<td>1</td>
<td>-0.08</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: SM couplings - **Pythia** implementation.

## 2 PYTHIA Generation of Different $Z'$ Models

### 2.1 Sequential $Z'$

The parameters available\(^1\) in **Pythia** to implement the couplings for a given $Z'$model are PARU(121)-PARU(128) for the 1\textsuperscript{st} generation quarks and leptons, PARJ(180)-PARJ(187) for the 2\textsuperscript{nd} generation, and PARJ(188)-PARJ(195) for the 3\textsuperscript{rd} generation, respectively. The default values are those of a sequential $Z'$, which are the same as for the Standard Model $Z$ boson. In Halzen+Martin, these couplings are expressed as: $C_{f\nu} = T^3_f - 2Q_f \sin^2 \theta_W$ and $C_{fa} = T^3_f$ (see Table 1).

We can use Eqns. (7) and (9), plugging in the Standard Model $g_{Z'} = g/\cos \theta_W$, to obtain the **Pythia** coefficients: $C_V = 2C_{f\nu}$ and $C_A = 2C_{fa}$, respectively. The exact values of the vector ($V$) and axial ($A$) couplings are given in the fourth row of Table 2. If one uses the **Pythia** value $\sin^2 \theta_W = 0.23$, one obtains the values listed in the fifth (last) row, which coincide with the defaults listed in the **Pythia** manual. The couplings to the 2\textsuperscript{nd} and 3\textsuperscript{rd} families must be set to the same values, i.e. PARU(121)=PARJ(180)=PARJ(188), etc.

### 2.2 CDDT model-lines

In the CDDT paper [2], four general classes of $Z'$ models (or model-lines) are discussed. Table 3 presents the fermion charges for these model-lines. These charges are plugged in

\(^1\)In addition to these, one can also set the $Z'$ mass via PMAS(32) parameter, etc.
$$gZ' = \gamma Z' \cdot \frac{g}{\cos \theta_W} \quad \text{with} \quad \gamma Z' \equiv \frac{gZ'}{g} \cos \theta_W$$

(10)

For example, for a $q + xu$ family $Z'$ with $gZ' = 0.1$ and $x = 1.0$, we have $\text{PARU(121)} = \text{PARJ(180)} = \text{PARJ(188)} = -2(1 - 3) \cdot 1.402 \cdot 0.1/3 = 0.187$, etc.

2.3 E6 $Z'$s

In the E6 models, the SM structure is extended to include the $U(1)_\psi \times U(1)_\chi$ groups\cite{5}. The gauge fields corresponding to these groups $Z'_\psi$ and $Z'_\chi$ can be massive, and are not
true mass eigenstates since these states can mix. Let us define the mass eigenstates by:

\[ Z'(\theta) = Z'_\psi \cos \theta + Z'_X \sin \theta, \quad Z''(\theta) = Z'_\psi \sin \theta - Z'_X \cos \theta \] (11)

with \( \theta \) being a parameter dependent on the Higgs vev’s and the gauge couplings \( g_\psi \) and \( g_X \) corresponding to \( U(1)_\psi \) and \( U(1)_X \). Different definitions of \( \theta \) exist in the literature; however, this is irrelevant if one restricts the discussion only to \( Z'_\psi, Z'_X, Z'_\eta \), and \( Z'_f \) as it is customarily the case in experimental searches. In what follows we assume the \( \theta \) definition from Ref. [4] (Rosner). As we will show, other definitions [2, 5] lead to the same results. To simplify the problem, we assume that \( Z''(\theta) \) is heavy enough to decouple from the \( Z \) and \( Z'(\theta) \), and will not be discussed further. The \( Z'(\theta) \) is light enough to mix with the standard model \( Z \). According to Eqn. (3) of Ref. [4], we can write:

\[ g_\theta = \sqrt{\frac{5}{3}} g_Z \sin \theta_W = \sqrt{\frac{5}{3}} g \tan \theta_W \equiv \sqrt{\frac{5}{3}} \gamma \] (12)

where \( g = 0.626 \) is the \( SU(2)_L \) gauge coupling. In this notation, and using Table I of Ref. [4], we can calculate the fermion couplings for the \( Z'(\theta) \), listed in Table 5. For this Table, to get the neutrino couplings we used the \( e \) couplings and Eqn. (8) to calculate the \( z_{eL} \) charge \( (z_{eL} = z_{eL}, z_{eR} = 0) \). To get the \textsc{pythia} parameters, we used the relations from Eqns. (8) and (9).

Comparing to the \( 10 + 5 \) results from the last row of Table 4, the following conversion relations can be written down:

\[ \gamma_{Z'} = \frac{\sqrt{10} \cos \theta + \sqrt{6} \sin \theta}{4}, \quad \text{with} \quad s \equiv \sin \theta_W \] (13)

\[ x \gamma_{Z'} = \frac{\sqrt{10} \cos \theta - 3 \sqrt{6} \sin \theta}{4}, \quad \text{with} \quad s \equiv \sin \theta_W \] (14)
Table 5: The general E6 fermion couplings as function of $\theta$, following Rosner's definition of $\theta$. We used the shorthand notations $\gamma \equiv g \tan \theta_W$, and $s \equiv \sin \theta_W$, respectively. To fit all values in the Table, we omitted the $C_{V_2', u}$ column, as this parameter is zero; that is, PARU(123)=0 for all E6 models.

The next step is to particularize the general $Z'(\theta)$ to obtain the popular models. Table 6 lists the couplings for these models; the values were obtained simply by plugging each $\theta$ value in the expressions from Table 5. Given Eqns. (13)-(14), the same results from Table 6 can also be found if one particularizes the CDDT model-lines as follows [6]:

- $Z'_{\psi}$ is obtained in the $10 + x\bar{5}$ model-line (last row in Table 4), for $x = 1$ and $\gamma_{Z'} = \sqrt{\frac{5}{3}} \sin \theta_W$.

- $Z'_{\chi}$ is obtained in the $10 + x\bar{5}$ model-line (last row in Table 4), for $x = -3$ and $\gamma_{Z'} = \sqrt{\frac{5}{7}} \sin \theta_W$.

- $Z'_{\eta}$ is obtained in the $10 + x\bar{5}$ model-line (last row in Table 4), for $x = -1/2$ and $\gamma_{Z'} = \sin \theta_W$.

- $Z'_t$ is obtained$^2$ in the $d - xu$ model-line (eighth row in Table 4), for $x = 0$ and $\gamma_{Z'} = \sqrt{\frac{10}{2}} \sin \theta_W$.

- $Z'_{sq}$ is obtained in the $10 + x\bar{5}$ model-line (last row in Table 4), for $x = -8$ and $\gamma_{Z'} = \frac{1}{4} \sin \theta_W$.

$^2$This is also the special $10 + x\bar{5}$ case: $\gamma_{Z'} \to 0$, and $x\gamma_{Z'} = -\frac{\sqrt{10}}{4} \sin \theta_W$ (as $x \to \infty$).
Table 6: Popular E6 models - PYTHIA implementation. We used the notation: $s \equiv \sin \theta_W = \sqrt{0.23}$.

- $Z'_N$ is obtained in the $10 + x\bar{5}$ model-line (last row in Table 4), for $x = 2$ and $\gamma Z' = \sqrt{6} \sin \theta_W$.

We finally note that Ref. [5] uses a different definition of $\theta$: $Z' = Z_\psi \cos \theta - Z_\chi \sin \theta$. Working out the $V$ and $A$ couplings by using Eqn. (2,7) and Table 2 from [5], we obtain the same PYTHIA couplings as given in our Table 5. Using the definitions (i.e. $\theta$ values) of $Z'_\psi$, $Z'_\chi$, $Z'_\eta$, and $Z'_I$ given in Ref. [5], we find the couplings for $Z_\psi$ and $Z_\eta$ to be identical to the ones listed in our Table 6, while the couplings for the $Z_\chi$ and $Z_I$ are equal to the negative of the corresponding values from Table 6 (which is equivalent to the transformation $\theta \to -\theta$).

Figure 1 shows a cartoon with several definitions of the E6 angle $\theta$. 
3 Conclusions

In this document we briefly list several conventions used in the $Z'$ literature, and give the 
PYTHIA implementation for the sequential $Z'$, the model-lines of Ref. [2], and the popular 
E6 models. We hope this will serve as a useful first guide for the $Z'/\text{PYTHIA}$ beginner. 
We thank Marcella Carena, Bogdan Dobrescu, Tim Tait, and Muge Karagöz for useful 
discussions.

References

for PYTHIA versions 6.2xx-6.3xx.


[6] The $Z'_N$ and the secluded $Z'_sq$ E6 models are taken from: J. Kang and P. Langacker, 