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INTRODUCTION

Standard surface impedance approximations are invalid at lower frequencies approaching DC since the cross sections of conductors are smaller than the skin depth. Hence, a volumetric formulation is typically used at these low frequencies for broadband simulation as necessitated in digital or ultra-wideband systems since the skin effect can be modeled explicitly. This modeling requires fine and frequency dependent volume meshing. However, an approach using higher-order elements and/or bases may alleviate these requirements. The intent of this paper is to present a tightly coupled circuit and hybrid boundary element (or integral equation)/finite element based electromagnetic simulation that has been coded in EIGER [1].

HYBRID FORMULATION

The approach used for the hybrid formulation has been presented in [2]. The body of interest is shown in Fig. 1. The surface of the body is used to create a boundary to separate the problem into interior and exterior regions. Finite elements are used in the interior volume V . A boundary element formulation is applied on the body surface S to model the exterior. The interior volume of the body is modeled with finite elements using the electric field Helmholtz equation, represented as

$$\nabla \times (\bar{\mu}_r^{-1} \cdot \nabla \times \mathbf{E}) - k_0^2 \bar{\epsilon}_r \cdot \mathbf{E} = -j\omega\mu_0 \mathbf{J}, \quad (1)$$

where $\mathbf{E}$ is the electric field, k_0 is the freespace wave number, ω is the radian frequency, and $\bar{\epsilon}_r$ and $\bar{\mu}_r$ are the relative permittivity and permeability dyads, respectively. The body exterior is modeled using boundary elements on the body surface, in this case using the electric field integral equation (EFIE), which is given by

$$\frac{\mathbf{M}}{2} - \hat{\mathbf{n}} \times \left(j\omega \mathbf{A} + \nabla \Phi + \frac{1}{\epsilon} \nabla \times \mathbf{F} \right) = -\hat{\mathbf{n}} \times \mathbf{E}^i, \quad \mathbf{r} \uparrow S, \quad (2)$$

where $\hat{\mathbf{n}}$ is the normal to the surface at the observation point, $\mathbf{E}^i$ is the impressed electric field, and $\mathbf{A}$, Φ , and $\mathbf{F}$ are the magnetic vector, electric scalar, and the electric vector potentials, respectively. Also, the jump discontinuity of the curl term in (2) has been removed and expressed explicitly in terms of the magnetic current $\mathbf{M}$. The fields and currents are expanded using the basis functions described in [3].

CIRCUIT-CONNECTION ALGORITHM

The contact-connection algorithm [4,5] is employed to couple the circuit to the electromagnetic surface. A circuit is attached to a spatially localized surface S_c by enforcing at this contact a modified current-continuity equation, a KCL connection, and a KVL connection from the contact to the circuit node. This is shown in Fig. 2. These three conditions are fundamental to the coupled hybrid-circuit formulation. On a contact surface S_c , the continuity equation is changed to account for injecting branch current from the circuit. This current introduces an additional source term in the continuity equation and thus affects the distribution of both the electromagnetic surface currents and surface charges. Hence, the continuity equation is modified to become

$$\nabla \cdot \mathbf{J} + j\omega\rho = \begin{cases} I_c, & r \in S_c, \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

where I_c is the contact current. This contact current provides a virtual extension from the distributive electromagnetic surface to the circuit node. The coupling between the electromagnetic formulation and the circuit is done by enforcing (2) at the boundary and coupling them to the circuit by including the contact current from (3). The localized circuit source attached to the contact produces an additional source or sink of charge that alters the scalar potential and the resulting electric field. Because of this additional current, the scalar potentials must be tied to the circuit node voltage V_n . A KVL expression sets the scalar potentials at the equipotential circuit voltage V_n . Finally, the contact current is connected to the circuit by including an addition term I_c to the KCL based circuit equation associated at circuit node n .

The hybrid-circuit formulation, including the connecting KCL and equipotential KVL equations, may be summarized as the following block-matrix equation:

$$\begin{bmatrix} FEM & C_A & 0 \\ C_B & BEM & X \\ 0 & -X^T & MNA \end{bmatrix} \begin{bmatrix} V_{EM} \\ I_{EM} \\ I_{ckt} \end{bmatrix} = 0, \quad (4)$$

where the FEM block matrix represents the finite elements, BEM represents the boundary elements and contacts, and the C blocks represent the cross-coupling between the finite elements and boundary elements. The X and X^T block matrices are the connection matrices between circuit nodes and contacts, and the MNA block matrix represents the modified nodal circuit analysis.

RESULTS

A rectangular connector (1 mm $\times$ 1 mm $\times$ 4 mm) is modeled to determine the convergence of the resistance curve around the first resonance of the structure. The results shown in Fig. 3 include the hybrid formulation (164 triangles and 216 tetrahedrons) and the boundary element method formulation (48 rectangles) from [5]. The complete basis order is denoted by p . The various results agree well except when $p=0$ for the boundary element method since convergence had not been obtained.

SUMMARY

A hybrid boundary element/finite element formulation is coupled with a technique for representing circuit-to-EM connections. Higher-order bases functions and elements

may be used in the analysis. Results were presented to validate the hybrid formulation compared to the boundary element formulation.

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Figure 1. A body is in space where the volume V is surrounded by the surface S .

Figure 2. This schematic shows the approach for creating the contact-circuit connection.

Figure 3. This is a plot of the resistance versus frequency for a rectangular connector. The connector was modeled using a boundary element method (BEM) and hybrid BEM/FEM formulation (FEM). The complete basis order p for each case is shown in the legend.