A COMPARISON OF SINGLE AND
DOUBLE NULL POLOIDAL
FIELD DIVERTOR CONFIGURATIONS

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A COMPARISON OF SINGLE AND DOUBLE NULL POLOIDAL FIELD DIVERTOR CONFIGURATIONS

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ABSTRACT

A comparison is made of single and double null poloidal field divertor configurations for Tokamak plasmas. Geometrical, magnetic field, MHD equilibrium and stability, and economic characteristics are considered. We find the single null to be preferable for small minor radii plasmas such as might be used in a fusion-fission hybrid reactor. However, the more efficient use of the available toroidal field by a double null configuration results in an increasing economic superiority that becomes the dominant consideration for pure fusion size plasmas.

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I. INTRODUCTION

We describe a study of the relative merits of two tokamak plasma configurations with poloidal magnetic field divertors. These are the single null separatrix configuration used in the Princeton Reference Design,¹ and the double null separatrix used in many conceptual designs such as UWMAK-I.² The comparison is made with regard to the intrinsic geometrical, magnetic field, MHD equilibrium and stability, and economic characteristics of the two configurations.

This investigation is an outgrowth of a conceptual design for a Fusion-Fission Tokamak Hybrid Reactor,³ in which a configuration with a divertor was deemed necessary to a long burn cycle. The plasma is fueled and thus requires an exhaust system to maintain a steady state reaction. Pumping on the scrapeoff plasma near the separatrix surface in a shielded region of the divertor channel provides such a system, and also impedes the passage of impurities into the reacting plasma.

The results described below are most pertinent to a hybrid plasma, which has a much smaller volume than a pure fusion plasma because of the large energy contribution of its fission style blanket. We do however, discuss the changes that may be expected in larger pure fusion devices where these are relevant.

II. GEOMETRIC PROPERTIES

Typical single and double null configurations are shown in Figs. 1 and 2. These figures illustrate our belief that the separatrix surface must be drawn far from the plasma to provide a large well-shielded area for pumping. Our designs incorporate a minimum of 1 m of pure shielding for coils and structure in direct line with the plasma, and upwards of 0.5 m for regions subject to neutron streaming. Finally, we have attempted to provide 0.5 m clearance for the scrapeoff plasma.

In the case of the single null, these criteria impose no constraints on the design regardless of plasma size. As is described in the next section, we can create the necessary equilibrium vacuum poloidal field with coils placed far from the plasma, and thus can always use the blanket to provide structural shielding even with an 0.5 m scrapeoff. The path of the separatrix passes naturally through a well-shielded region where pumping can take place, without approaching any of the field coils too closely.

In contrast, the double null configuration must be driven by large positive current coils that are comparatively close to the plasma. This creates difficulties in shielding these coils. It is also difficult to pull the separatrix away from the plasma to a pumping region whilst still shielding the coils that perform this function, and providing adequate width in the divertor channel. These problems are particularly acute in smaller machines. However, since shielding distances and

scrapeoff width do not scale with plasma size, these difficulties will be greatly reduced in pure fusion devices. Relaxation of our criteria, that are perhaps conservative, together with a concerted design effort might yield a satisfactory solution for the smaller plasmas.

A further geometric advantage of the single null is that it utilizes a larger solid angle of blanket coverage. This is because the divertor channel opening is narrower, having two arms as opposed to the four of the double null, and is at a smaller major radius than the corresponding double null opening. This major advantage is however, more than outweighed by the decreased toroidal field that results at the single null magnetic axis from a given toroidal field coil. The horizontally elongated single null has a larger major radius than its corresponding vertically elongated double null. The $1/R$ dependence of the toroidal field results in lower achievable densities in single null plasmas. This well-recognized behavior is quantified in the section on economics.

III. MAGNETIC FIELD PROPERTIES

It has previously been reported⁴ that a single null equilibrium configuration is sensitive to radial perturbations. This results from motion of the 'X' point enclosing differing plasma volumes. We have eliminated this problem by strongly curving the vacuum poloidal field in the vicinity of the 'X' point, whilst maintaining a nearly uniform field throughout most of the plasma as shown in Fig. 3. Such a field localizes

the position of the ' χ ' point for a fairly wide range of plasma parameters. This particular vacuum field has been designed for a decay index $n = -\partial(\ln B_V)/\partial(\ln R)$ appropriate to providing a plasma equilibrium that is stable to radial and vertical rigid body displacements.⁵ The coil set has zero net turns which may facilitate disassembly, and was designed with a simultaneous minimization of the total turns.⁶ Further, the coils are situated far from the plasma to minimize shielding problems.

The double null vacuum field is shown in Fig. 4. In order to generate the nulls of an 'inside D' configuration (nulls on the toroidal axis side of the magnetic axis) it is necessary to produce a vacuum field that is at some points concave with respect to the toroidal axis. Such field shaping can give rise to the rigid body vertical instabilities that are discussed below. Zero net turns can also be achieved, but the total poloidal field current is typically several times larger than for the single null, due to the conflict between the positive divertor coils and the negative equilibrium field coils. The single null coil set also aids in the establishment of the ohmic heating current, freeing the ohmic heating coils to offset resistive losses, thereby sustaining a longer burn time. The double null coil set can often inhibit the establishment of the ohmic heating current, resulting in much shorter burn times for the same ohmic heating core criteria.

A final single null advantage is illustrated in Fig. 5, one of a series of equilibria which shows the start up of a

single null plasma on a separatrix always lying in the same divertor channel. The initial pressure and currents are 1% of the final values, and the vacuum field shape is not changed. Procedures that appear satisfactory have been proposed for the double null, but the configuration cannot be initiated as a double null with a separatrix in the divertor channels.

IV. MHD EQUILIBRIUM AND STABILITY PROPERTIES

Free boundary solutions of the Grad-Shafranov equation

$$R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial \bar{\Psi}}{\partial R} + \frac{\partial^2 \bar{\Psi}}{\partial z^2} = -4\pi^2 \left(R^2 \mu_0 \frac{dp}{d\bar{\Psi}} + R_0^2 B_0^2 g \frac{dg}{d\bar{\Psi}} \right) = 2\pi R J_\phi \quad (1)$$

for the poloidal magnetic flux

$$\bar{\Psi} = \frac{1}{2\pi} \int \underline{B} \cdot \nabla \theta d\tau$$

have been obtained for both configurations using the Princeton Equilibrium Code.⁷ The coordinate system is illustrated in Fig. 6. The pressure and toroidal field function are prescribed as

$$P(\bar{\Psi}) = P_m \left(\frac{\bar{\Psi}_\ell - \bar{\Psi}}{\bar{\Psi}_\ell - \bar{\Psi}_m} \right)^\alpha$$

$$g(\bar{\Psi}) = \frac{R B_\phi}{R_0 B_0} = 1 + g_m \left(\frac{\bar{\Psi}_\ell - \bar{\Psi}}{\bar{\Psi}_\ell - \bar{\Psi}_m} \right)^\alpha \quad (2)$$

where the subscript ℓ refers to values at the bounding plasma surface, and the subscript m to values at the magnetic axis.

Figures 7 and 8 show two examples of these solutions. The ideal and resistive local interchange stability criteria⁸ can be evaluated from these equilibria. We find that our solutions for both configurations satisfy these criteria with both large and small plasmas.

The following stability results were obtained using the PEST package⁹, which numerically examines the ideal linear MHD stability of our equilibria.

We have previously discussed the effect of vacuum field shaping on rigid body modes. We find that our single null equilibria, whose vacuum fields were designed to exclude such modes, have no rigid body instabilities. However, the curvature of the double null vacuum field generates a vertical instability. The projection of this mode on a poloidal plane is shown in Fig. 9. The existence of this mode for 'inside D' configurations is well-known, and it has been postulated that sufficient passive stabilization will be generated by machine structure to allow for active feedback stabilization. This mode is stabilized by a perfectly conducting wall at the plasma edge, but we have not as yet investigated the realistic situation with the wall a finite distance from the plasma. The effects of plasma profiles, field tailoring and nonlinearity¹⁰ on the growth rate of the mode are not completely understood. However, we find that it is not sufficient that the magnetic axis be in a region of favorable curvature for stabilization of the mode. We believe that the limitations placed on the double null configuration by this mode will only finally be answered by an experiment such as PDX.¹¹

Recent studies indicate that ballooning modes may provide the limit on the achievable beta in reactors¹². Our use of 'conventional tokamak scaling' for the plasma parameters has probably not generated optimized reactor plasma equilibria, since we restricted ourselves to a narrow class of plasma profiles. The equilibria generated in this manner were all found to be stable to both kink and ballooning modes. A study to determine the effect of shape and plasma profiles on the critical beta for ballooning is currently in progress.

V. ECONOMIC PROPERTIES

We consider a highly simplified economic model, utilizing a semi-empirical plasma model, to study the relative economic advantages of the single and double null devices. In particular, we are concerned about the relatively inefficient use of a given toroidal field by the single null configuration.

We postulate a maximum field of 16 Tesla at the inner major radius of the toroidal field coil h , so that the field at the magnetic axis is given by

$$B_0 = \frac{16h}{R_0}$$

The geometric relationships shown in Figs. 1 and 2 are derived from a variety of numerical equilibrium solutions. We see that the height of the toroidal field coil H and the major radius R_0 depend on the average plasma minor radius defined in terms of the plasma volume V ,

$$a = \left(\frac{V}{2\pi^2 R_0} \right)^{1/2},$$

and h , and invariant parameters such as blanket size.

We have placed two requirements on the permissible equilibria to scale the pressure at the magnetic axis p_m , and the discharge current I . These are that the Kruskal-Shafranov limit is not violated, and we therefore chose the safety factor at the magnetic axis $q_m \sim 1.1$; and that we should not have reversed currents in the plasma.

We begin by calculating

$$q_m = \lim_{r \rightarrow 0} \left(\frac{r B_o}{R_o B_p} \right) = \frac{2 B_o}{R_o \mu_o J_o}$$

where the current density on axis can be found from (1) and (2) as

$$J_o \approx \frac{2 \pi R_o \alpha}{(\bar{\psi}_\ell - \bar{\psi}_m)} \left(p_m + \frac{q_m B_o^2}{\mu_o} \right)$$

Then following Greene et al.⁵

$$g_m = - f_a^{(1)2} C_1(\alpha) [1 - C_2(\alpha) \langle \beta_\theta \rangle]$$

with

$$(\bar{\psi}_\ell - \bar{\psi}_m) = B_o R_o a C_3(\alpha) f_a^{(1)}$$

$$\langle \beta_0 \rangle = C_4(\alpha) \frac{a^2 p_m}{I^2}$$

$$\langle \beta \rangle = C_5(\alpha) \frac{p_m}{B_o^2}$$

where the various functions of α are determined to match our equilibrium solutions. In this manner, we obtain for the single null

$$q_m = \frac{6 \cdot 1 \times 10^7 h a^2}{R^2 \alpha I} \left(1 + .6 \langle \beta_\theta \rangle \left(\frac{\alpha-1}{2\alpha-1} \right)^2 \right)^{-1} \quad (3)$$

By considering when $J_\phi = 0$ at $R = R_0 - a$, we can similarly obtain a maximum value $\langle \beta_\theta \rangle$, such that no reversed current appears.

$$\langle \beta_\theta \rangle_{\max} = \frac{(2\alpha-1)^2}{0.6 \left(4\alpha - (\alpha+1)^2 \left(1 - \frac{a}{R_0} \right)^2 \right)} \quad (4)$$

From (3) and (4) we find the discharge current and the maximum p_m as functions of h , a and α . For the single null

$$I = \frac{5.55 \times 10^7 h a^2}{R_0^2} \left(1 - \frac{(\alpha-1)^2}{\gamma} \right)^{-1}$$

$$p_m = 1.27 \times 10^{-8} I^2 \frac{(\alpha-1)^2 (2\alpha-1)^2}{a^2 \gamma}$$

where

$$\gamma = 4\alpha - (\alpha+1)^2 \left(1 - \frac{a}{R_0} \right)^2$$

$$\langle p \rangle = \frac{1.9 p_m}{(\alpha+1)^2}$$

$$\langle \beta \rangle = \frac{1.87 \times 10^{-8} p_m R_0^2}{h^2 (\alpha+1)^2}$$

$$\langle \beta_{\theta} \rangle_{\max} = \frac{(2\alpha-1)^2}{.6\gamma}$$

$$R_O = h + 2.5 + 1.95a$$

(5)

$$H = 5.5$$

and for the double null

$$I = \frac{6.1 \times 10^7 h a^2}{R_O^2 \alpha} \left(1 + \frac{(\alpha-1)^2}{\gamma} \right)^{-1}$$

$$p_m = 1.11 \times 10^{-8} \frac{I^2 (\alpha+1)^2 (2\alpha-1)^2}{a^2 \gamma}$$

where

$$\langle p \rangle = \frac{2.3 p_m}{(\alpha+1)^2}$$

$$\langle \beta \rangle = \frac{2.3 \times 10^{-8} p_m R_O^2}{h^2 (\alpha+1)^2}$$

$$\langle \beta_{\theta} \rangle_{\max} = \frac{(2\alpha-1)^2}{.6\gamma}$$

$$R_O = h + 2.4 + a$$

(6)

$$H = 5.5 + 1.5a$$

A comparison of these formulae with equilibrium data is given in Table 1

For the remainder of our economic relations we use the wall loading

$$P_w \propto \langle p \rangle^2 V/A_s \propto \frac{\langle p \rangle^2 a^2}{2a+1}$$

where the wall surface area $A_s = 4\pi^2 R_o (a+1/2)$.

We then calculate the effective blanket area as A_s minus the divertor arm area

$$A_{w1} = A_s - 4\pi(h+2 \cdot 25)$$

$$A_{w2} = A_s - 8\pi(h+2 \cdot 4 + 0.75a)$$

with the power output

$$O \propto A_w P_w$$

for identical efficiencies, and the cost approximated as

$$C \propto \left(\frac{R+H}{2} \right)^{1.8}$$

The price of electricity is given by

$$P = C/O$$

This calculation of cost is considered adequate to examine comparative behavior. A more detailed calculation with regard to hybrids is in preparation.

The more efficient use of the available toroidal field by a double null is graphically illustrated by Figs. 10 and 11. The higher field permits the use of much higher plasma pressures. This translates into the substantial difference in the price of electricity for a given power output that is plotted in Fig. 12.

At the smaller plasma minor radii and thermal output powers of hybrid plasmas the difference in the price of electricity is not significant. This can be traced to both configurations having similar toroidal fields at the magnetic axis, since blanket width and the other non scaling lengths are comparable with the plasma minor radius. Also, in more detailed studies, a beam penetration requirement tends to limit the plasma pressures that can be used.

However, as the plasma minor radius is increased, the toroidal field effect results in greater power output at significantly lower cost for the double null. This configuration is thus clearly preferable for pure fusion devices.

VI. CONCLUSIONS

A single null poloidal divertor configuration is more easily shielded and requires less poloidal field energy than a corresponding double null configuration. Unlike the double null, it can be designed without rigid body instabilities, and is thus preferable for small hybrid plasmas where there is no

clear economic advantage. However, the larger output power capacity and lower price of electricity of the double null for larger pure fusion plasmas overwhelms all other single null advantages. Only if it were found impossible to control or suppress the double null vertical instability would it seem practical to consider a single null pure fusion reactor.

It should be noted that we have used so-called 'conventional tokamak scaling' in this paper. Were we to have employed the flux conserving tokamak concept¹³ or some other scaling model, then the economic results could change radically. The former admits much larger plasma betas, and results in smaller plasma volumes for a given neutron yield. Under these circumstances, the limiting plasma pressure may be determined by the onset of ballooning instabilities. The effect of plasma shape and profiles on these modes is under active study, and the use of such a scaling model was considered premature.

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TABLE 1. - COMPARISON OF EQUILIBRIUM DATA WITH SCALING FORMULAE

EQUILIBRIUM DATA											SCALING RESULTS		
Case	Nulls	P_0	I	α	h	a	R	$\langle \beta \rangle$	$\langle \beta_\theta \rangle$	q_0	$\langle \beta \rangle$	$\langle \beta_\theta \rangle$	q_0
1	1	6×10^5	2.5×10^6	2	3.65	1.7	9	.008	4.15	1.24	.0075	4.04	1.25
2	1	6.3×10^5	1.6×10^6	2	3	1.3	9	.0119	6.00	.84	.0118	6.06	.85
3	1	6×10^5	2×10^6	2	3	1.5	9	.0116	4.94	.94	.0113	4.86	.95
4	1	6×10^5	2.5×10^6	1.5	3	1.55	9	.0164	5.04	1.24	.0163	4.91	1.24
5	1	6×10^5	2.5×10^6	1.2	3	1.50	9	.0202	5.75	1.43	.0210	5.86	1.57
6	1	5×10^5	1×10^6	2	2.43	.86	6.5	.008	5.51	1.01	.0074	5.35	.95
7	2	8.95×10^5	3.4×10^6	2	3.25	1.37	6.8	.010	2.42	1.16	.010	2.42	1.12
8	2	7×10^5	1×10^6	2	2.41	.75	5.5	.008	6.38	1.07	.010	6.56	1.01

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Fig. 1. Single Null Configuration.

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Fig. 2. Double Null Configuration.

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Fig. 3. Magnetic flux contours for single null vacuum poloidal field.

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Fig. 4. Magnetic flux contours for double null vacuum poloidal field.

Fig. 5. Intermediate single null start-up configuration at 10% plasma pressure and discharge current.

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Fig. 6. Coordinate System.

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Fig. 7. Magnetic flux contours for single null equilibrium configuration.

774034

Fig. 8. Magnetic flux contours for double null equilibrium configuration.

774036

Fig. 9. Double null vertical instability.

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Fig. 10. Carpet of single null average plasma pressure (Nm^{-2}) with minor radius (a) and core size (h).

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Fig. 11. Carpet of double null average plasma pressure (Nm^{-2}) with minor radius (a) and core size (h).

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Fig. 12. Price of electricity against power output. Corresponding minor radii (m) are indicated