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**Compilation of Interim
Technical Research Memoranda
Volume I**

W. R. Shanahan

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those dynamical considerations not within the purview of the virial theorem but of importance to plasmoid propagation may be investigated. Plans are presently going forward to include such simulations in our continuing study of plasmoid propagation.

REFERENCES

1. S. Chandrasekhar and E. Fermi, "Problems of Gravitational Stability in the Presence of a Gravitational Field," Ap. J. 118, 116 (1953).

COMPILATION OF INTERIM TECHNICAL RESEARCH MEMORANDA
Volume I

by

W. R. Shanahan

ABSTRACT

Four interim technical research memoranda are presented that describe the results of numerical simulations designed to investigate the dynamics of energetic plasma beams propagating across magnetic fields.

INTRODUCTION

In this Compilation of Interim Technical Research Memoranda, we describe the results of numerical simulations designed to investigate the dynamics of energetic plasma beams propagating across magnetic fields. Questions concerning bulk plasma motion across magnetic fields were among the first to be examined in plasma physics, having first arisen in theoretical studies of the origin of magnetic storms^{1,2} and subsequently acquiring increased importance in the context of the quest to control thermonuclear reactions.^{3,4} These investigations were concerned primarily with parameter regimes for which the plasmoid kinetic-energy density is small compared to the energy density of the ambient field. For such cases both theory³ and experiment⁴ seem to have converged on a fairly consistent picture in which the external magnetic field is but slightly perturbed by currents induced in the beam and propagation occurs across the field by macroscopic charge polarization of the beam followed by $\underline{E} \times \underline{B}$ drifting of the plasma particles. In contrast, much of the present interest in cross-field plasma motion involves situations in which the beam kinetic-energy density is large compared to the magnetic-field energy density. Consequently, the induced currents can significantly perturb the surrounding field. Further, these currents do not flow lamina-ly, but rather excite waves which lead to significant plasma heating. Detailed theoretical investigation of these issues encounters the usual problems presented by the study of realistic plasma systems, including the effects of spatial and temporal inhomogeneity and of strong nonlinearity. These factors render an ab initio analytic theory difficult. The numerical simulations described in this report serve to identify the essential aspects of cross-field propagation which must be incorporated into a complete theoretical description.

All of the simulations described in this report were performed either with the two-dimensional, relativistic, fully electromagnetic particle-in-cell simulation code CCUBE⁵ or with its three-dimensional analog, IVORY.⁶ In the case of two-dimensional simulations, different aspects of cross-field propagation are revealed by placing the initial ambient field either within or perpendicular to the plane of computation. In the former case new components of the distorted magnetic field

can appear self-consistently and interact with the induced currents which generate them. The first memorandum of this Compilation, Plasmoid Transverse Constriction, of December 22, 1982, describes the central features of this interaction. Essentially, the currents induced in the plasma beam by its motion across the magnetic field produce a longitudinal magnetic-field component which then interacts with the current via the Lorentz force to yield a transverse plasma beam constriction. A substantial portion of the first memorandum is concerned with the identification of this phenomena and its differentiation from other possible constricting mechanisms. Other features of the simulations, concerned with acceleration and deceleration of various parts of the beam revealed with this choice of initial field direction, are also discussed there.

With the magnetic field placed perpendicular to the plane of computation, the induced currents no longer flow in the ignorable direction, and well-known charge polarization effects rapidly appear. If the particle Larmor radii were small compared to the dimensions of the plasma beam, such polarization would quickly lead to the usual $\underline{E} \times \underline{B}$ mode of propagation. In this more energetic situation, however, the relevant phenomena are apparently more complex. In the second memorandum, Plasmoid Heating, of August 19, 1983, these phenomena are described and the directions to be followed in the development of a comprehensive theory for their explanation are indicated. Evidence is culled from the simulations to demonstrate that the observed heating and accompanying beam dispersal are due to wave excitation by the induced motion of the beam electrons. Such excitation produces significant ion correlations and concomitant strong electron heating. The basic result of the research described in this memorandum is that even an initially cold neutralized plasma beam will quickly be heated, and therefore dispersed, if it begins cross-field propagation with the field pervading it. In this memorandum I suggest that less heating may be expected if the plasma beam is introduced adiabatically to the magnetic field. Subsequent simulations in which the field is turned on over a time of ~ 500 plasma periods indeed show less heating.

The results reported in the first two memoranda are obviously strongly conditioned by the low dimensionality of the plasma simulation code with which they were obtained. For example, the magnetic field

distortion which plays such an important role in the phenomena described in the first memorandum is doubtlessly exaggerated by the impossibility of the field being excluded from the path of a plasma beam infinitely extended in the third direction. Further, wave excitation cannot occur in the ignorable direction. These issues can be addressed only by analytic theory or by more sophisticated numerical tools. The factors which frustrate a purely analytic approach in two dimensions are naturally even more limiting in three. On the other hand, three-dimensional plasma simulation codes are still in a rather rudimentary state of development. We have addressed these issues, albeit in a very preliminary way, by using the quasi three-dimensional particle-in-cell code IVORY⁶ to simulate cross-field plasma beam propagation. The results of these simulations are described in the third memorandum, Three-Dimensional Rest Frame Plasmoid Simulations, of July 5, 1983. This memorandum also describes the implementation of a capability of simulating plasmoid propagation in the plasmoid frame. This feature permits one to overcome the twin problems of being able to simulate propagation only for periods equal to the beam transit time of the computational box and of eliminating spurious effects due to employing periodic boundary conditions in the longitudinal direction. These latter have been employed in the work of the first two memoranda to overcome the transit time limitations. Inclusion of three-dimensional effects leads to significant modification of previous results. In particular, the focusing effect described in the first memorandum has been largely eliminated. One cannot, however, immediately conclude that focusing is purely a fiction of low dimensionality since there is a tremendous difference in scale in the third direction, the two-dimensional case being naturally infinitely extended in that direction while the three-dimensional simulation plasmoid extends only $\pi(c/\omega_p)$.

In the first three memoranda of this Compilation, we have established that even an initially cold plasma beam is likely to be significantly heated during cross-field propagation. Effective propagation consequently depends on the discovery of means with which to control the resulting beam dispersal. In the laboratory this may perhaps be accomplished through the imposition of suitable additional fields, but in other environments this option may not be available. Interest naturally

turns toward the possibility of inducing suitable currents in the plasmoid to take advantage of pinch-like effects to retard dispersal. Suggestions in this direction usually encounter the assertion that all such efforts are doomed to failure by the virial theorem, a consequence of the fundamental equations of plasma dynamics.⁷ In the fourth memorandum, Virial Theorem Applied to Plasmoid Propagation, of April 7, 1983, we investigate the extent to which this result is true. We recover the well-known result that it is impossible to maintain a plasma in equilibrium by currents flowing within the plasma itself. The rather stronger statement, that the presence of a self-magnetic field always enhances dispersion, is apparently based on the neglect of certain surface terms which vanish in equilibrium but which do not do so if either the beam is not in equilibrium or if one redefines the system of interest to be but a part of the beam. These surface terms may be interpreted physically as the manifestation of pinching forces. Although the beam as a whole may well expand, there appears to be no fundamental prohibition against using such forces to confine at least part of the beam. Similar observations have been made by Alfven⁸ in an astrophysical context. The word "plasmoid" was originally coined to denote such a magneto-plasma entity,^{9, 10} although we use it in this Compilation to denote any relatively compact parcel of plasma. Hence, as used here it is synonymous with "neutralized plasma beam." Future progress in achieving effective plasmoid propagation seems to hinge on the degree to which the observations made in the fourth memorandum can be practically exploited toward this end.

REFERENCES

1. S. Chapman and V. C. A. Ferraro, "A New Theory of Magnetic Storms," Terr. Mag. (J. Geophys. Res.) 36, 77 (1931).
2. V. C. A. Ferraro, "On the Theory of the First Phase of a Geomagnetic Storm: A New Illustrative Calculation Based on an Idealized (Plane Not Cylindrical) Model Field Distribution," J. Geophys. Res. 57, 15 (1952).
3. G. Schmidt, "Plasma Motion Across Magnetic Fields," Phys. Fluids 3, 961 (1960).
4. D. A. Baker and J. E. Hammel, "Experimental Studies of the Penetration of a Plasma Stream in a Magnetic Field," Phys. Fluids 8, 713 (1965).
5. L. E. Thode, B. B. Godfrey, and W. R. Shanahan, "Vacuum Propagation of Solid Relativistic Electron Beams," Phys. Fluids 22, 747 (1979).
6. M. M. Campbell and B. B. Godfrey, "Ivory User's Manual," Mission Research Corporation report AMRC-N-221 (1983).
7. S. Chandrasekhar and E. Fermi, "Problems of Gravitational Stability in the Presence of Magnetic Fields," Ap. J. 118, 116 (1953).
8. H. Alfven, Cosmic Plasma (D. Reidel Publishing Company, Dordrecht, 1981).
9. W. Bostick, "Experimental Study of Ionized Matter Projected Across a Magnetic Field," Phys. Rev. 104, 292 (1956).
10. W. Bostick, "Experimental Study of Plasmoids," Phys. Rev. 106 (1957).

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PLASMOID TRANSVERSE CONSTRICTION

This is a PLASMOID research memo containing tentative results designed to enhance communication.

Initial simulation of an energetic ($\gamma = 2, n = 2.8 \times 10^{11}$) plasmoid propagating across a moderately strong ($B_0 = 714$ G) magnetic field revealed the presence of a pronounced focusing effect. Constriction in the transverse direction was observed, being most pronounced at the leading edge of the plasmoid. Originally, this effect was attributed simply to magnetic pressure. More precisely, focusing was thought to arise from the Lorentz force generated by the interaction between the diamagnetic current induced in the plasmoid and the self-consistently generated longitudinal component of the magnetic field. This component may appear computationally whenever the simulation is initialized with the ambient magnetic field in the plane of calculation. Questions concerning the isotropy of this effect can naturally be answered only to a limited extent by two-dimensional simulations. However, simulations initialized with the magnetic field perpendicular to the plane of calculation fail to exhibit constriction. This has been attributed to the absence in this configuration of a longitudinal component of the distorted field, thus strengthening our original interpretation.

Despite this qualitative evidence in favor of a focusing view of transverse constriction, questions soon arose concerning the possible presence of other contributing factors. Of particular interest in this regard was the possible contribution of pinching forces due to return currents. No strong evidence excluding such forces could be extracted from the initial simulations. During the past two months we have

effected several simulations designed to address this issue. In the remainder of this memorandum, we describe the principal results of these simulations. Our basic conclusion is that focusing, rather than pinching, is the dominant mechanism of transverse plasmoid constriction.

The basic strategy behind this series of simulations is that of studying configurations in which the pinching return current is altered or eliminated while the focusing effects remain largely unaffected. Essentially two different configurations have been examined, both motivated by considerations concerning return current formation. In the first such configuration, the plasmoid is propagated across a magnetic field through a neutral background plasma. Here, the background plasma serves to at least partially short the electrostatic forces ultimately responsible for the return currents of interest. In the second variety of simulation effected, the advantage afforded by numerical simulation to examine unphysical parameter regimes is exploited to study the dynamics of a plasmoid with equal mass electrons and ions. The rationale here is simply that the species separation necessary to return current formation depends on the mass difference of the species, while the diamagnetic effects do not. It is this latter variety of simulation, when compared to the initial simulations with 50:1 mass ratio, which has hitherto been most useful in disentangling focusing-pinching issues. The problem of plasmoid propagation through a background plasma undoubtedly deserves further attention in its own right.

Some care is required when using plasmoid propagation through a background plasma to differentiate various aspects of the dynamics. A plasma which is too tenuous will naturally not significantly affect the phenomena being scrutinized, while an over-dense plasma will disrupt plasmoid integrity. Two extreme cases have thus far been examined. In the first the background plasma density is taken to be ten times that of the plasmoid. This is intended to be primarily a bench mark run to see if the dynamics is essentially what is to be expected in this

extreme case. Indeed, the basically unmagnetized ions propagate largely as before, their electrostatic repulsion being shorted by the background, while the separated electrons simply execute Larmor motion. The electric field by which the ions transfer momentum to the electrons in the vacuum case is shorted by the background plasma. Although this simulation gives us some confidence in our understanding of very basic plasmoid dynamics, it naturally is not very illuminating with regard to constricting mechanisms. A simulation with a background density one-tenth that of the plasmoid has also been performed. Such a low density is thought to be not inappropriate, since not only is it roughly the opposite of the first case, but should be sufficient to materially affect the presumably small charge separation attendant on current formation. The gross dynamics appears substantially the same as in the vacuum case. No firm conclusions can be drawn, however, since examination of the magnetic field and current profiles reveals no dramatic differences from the case without a background plasma. Currently, a simulation is being performed with roughly equal densities for the beam and plasma densities.

The above remarks make evident the need for a more robust approach to discriminating between focusing and pinching effects. Such an approach has apparently been found in very unphysical simulations employing equal mass electrons and ions. As already noted, the basic reasoning here is that mass differences are essential to the formation of electrostatic fields and resultant currents, while the diamagnetic currents and longitudinal field components that enter into focusing proper are less affected. Accordingly, a simulation with mass ratio 1:1 has been performed. This simulation is successful, in the sense that longitudinal currents and electric fields are effectively suppressed, but the moderately strong magnetic field combined with the small mass of the particles produces a very fast time scale for the dynamics. Concomitantly, the reduced kinetic energy density entails a rapid deceleration of the plasmoid.

In order to obtain a reasonable time scale, and simultaneously have a basis for comparison with previous simulations, we have performed a simulation with mass ratio 50:50. The essential result of this simulation is summarized in Fig. 1, which shows the focusing of the plasmoid in configuration space together with the axial profiles of the current-field components which enter into the focusing. Notice that the axial variation of the focusing reflects that of the profiles. At this time in the simulation, the current-field components which would produce pinching are basically zero. Axial currents and the corresponding magnetic-field component are produced in the 50:1 simulation by differential deflection in the magnetic field, an inductive effect that leads to current concentration in the center of the plasmoid, and return currents. Each of these is dependent on a mass difference, and each is effectively suppressed here. We therefore conclude that focusing, at least for these times, rather than pinching, is the dominant constricting mechanism.

The longer time scale afforded by the 50:50 simulation also helps to clarify other aspects of the plasmoid dynamics. This is an important point, since the late-time evolution of the two cases is by no means identical. Movies of the 50:1 simulation early gave the strong impression that the trailing edge of the plasmoid is being accelerated in the direction of plasmoid motion rather than simply colliding with the leading part of the plasmoid retarded by the snowplowed field. This effect is perhaps most dramatically evident in the electron phase plots, which show strong high-energy tails localized at the rear of the plasmoid. Inspection of the ion phase space shows that this species too is being accelerated in the direction of plasmoid motion. Obviously, this acceleration cannot be a purely electrostatic effect since species of opposite charge are being accelerated in the same direction. In addition to the longer time scale, the 50:50 simulation aids in the explication of this phenomena by eliminating any possible electrostatic contribution. Initiation of acceleration at the trailing edge is coincident with the appearance of a reversed magnetic field there. Basically, the combination of magnetic and inductive electric-field

effects which act to slow the particles at the leading edge are reversed at the trailing edge to provide acceleration. The inductively generated electric field in the ignorable direction acts to change the gamma of the particles, while the magnetic field converts the motion into the longitudinal direction. Since the two species are accelerated oppositely in the ignorable direction, both are effectively accelerated the same longitudinally.

In Fig. 2 we present successive phase-space plots which exhibit the slowing of the leading edge of the plasmoid, the acceleration of the trailing, and the consequent overtaking of the former by the latter. It is in the stages subsequent to this first overtaking that the 50:1 and 50:50 simulations differ somewhat. The accelerated plasma naturally encounters the snowplowed field in both cases. In the former the light species readily interacts with the associated inductive field to yield a rapid deceleration. For the 50:50 case there is again deceleration, but a well-defined core oscillates with respect to a low energy halo. In both cases there appears a well-confined central plasmoid. It is at this point that pinching forces cannot be entirely ruled out for the 50:1 case since both current and field radial profiles are consistent with the existence of such forces. However, the smaller magnitude of the roughly calculated forces combined with the presence of confinement in the equal mass case indicates that focusing is here likewise the dominant effect.

Fig. 1. Transverse plasmoid constriction is due to magnetic focusing: (a) axial profile of transversely integrated induced current, (b) axial component of plasmoid-distorted magnetic field, and (c) Lorentz force generated by these current-field components reflected in focusing of plasmoid. Initial plasmoid velocity in x-direction, undisturbed ambient field in y-direction.

Fig. 2 Phase-space plots exhibit plasma deceleration at leading and acceleration at trailing plasmoid edge: (a) $\omega_p t = 320$, (b) $\omega_p t = 560$, (c) $\omega_p t = 660$.

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W. R. Shanahan, X-8

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PLASMOID HEATING

This is a PLASMOID research memo containing tentative results designed to enhance communication.

In this memorandum I summarize the clues from relevant simulations indicating the direction a theory should take to explain the intense heating and concomitant dispersal of the plasmoid, resulting from energetic cross-field plasmoid propagation numerical simulations.

When the ambient magnetic field is placed perpendicular to the plane of computation, this heating and dispersal is the principal phenomena to be observed. Although the expected charge polarization of the beam is apparent at very early times in the simulation, it is later largely obscured by wave generation and plasma-heating phenomena. The clues are, briefly, surface-initiated Langmuir waves weakly heat electrons until the time at which ion acoustic waves can be supported by the plasma. Thereafter, the ion fluctuations are apparently driven unstable by residual electron oscillations, producing significant ion correlations and related strong electron heating. Suprathermal ion fluctuations are, of course, a well-known component of collectively-induced "anomalous" plasma heating.¹ Full understanding of plasmoid heating is likely to have significant programmatic consequences since heating affects the conductivity, and hence the magnetic diffusivity time, of the plasmoid.

The simulation parameters used in the present heating studies are similar to those employed previously,^{2,3} namely, a slab plasmoid of relativistic energy factor $\gamma = 2$ and density $n = 2.8 \times 10^{11}$ is propagated perpendicular to a magnetic field with initial strength such that $\Omega/\omega_p = 0.004$, with Ω the nonrelativistic electron cyclotron

frequency and ω_p the usual Langmuir plasma frequency. For the plasmoid density used here, this corresponds to a field of approximately 7 G. The field is thus weaker by a factor of a hundred than that used in simulations described earlier. Such a weak field allows the wave generation and heating phenomena to be resolved on a reasonable time scale. We must also emphasize that the ambient field is in the direction perpendicular to the plane of computation. In consequence, behavior associated with dynamic field deformation, such as the plasmoid constriction described previously,² will not appear. In return, induced currents no longer flow unphysically along the ignorable direction, and charge polarization and wave generation effects can appear self-consistently.

In Fig. 1 we exhibit the ion configuration space for $\omega_p t \cong 500$. Significant plasmoid dispersal has already occurred, although for this plasmoid density this corresponds only to a propagation time and distance of some 17 ns and approximately 4.0 m, respectively. Of course, in interpreting these results allowance for the unrealistically small electron-to-ion mass ratio of 50:1 must be made. This ratio is chosen to exhibit ion dynamics within a computationally realistic time, but it naturally abets dispersal. Nevertheless, Fig. 1 demonstrates that heating and dispersal are important aspects of cross-field propagation.

Some initial understanding of the heating process can be obtained from Fig. 2, which is a transverse profile of the charge density. Charge has begun to accumulate on the sides as a consequence of the Lorentz deflection of the plasmoid particles, primarily of the lighter species. This graph, taken from a time early in the simulation, is thus representative of a stage of the beam dynamics assumed to have already occurred in the usual drift theory discussions⁴ of plasma bulk motion across magnetic fields. Note, however, that there is a dipole-like structure apparent in Fig. 2 which is inconsistent with "capacitor" models of plasmoid motion. This is actually the initial stage of a charge-density wave which ultimately extends throughout the bulk of the plasmoid. One also discerns in Fig. 2 a smaller charge

modulation in the interior of the plasmoid in regions which could not possibly be reached from the surface at this early time ($\omega_p t = 10$). This is probably due to a beam-like instability driven by the differential deflection of electrons and ions in the external magnetic field. Such an instability has already been detected in experiments⁵ involving low-energy plasma beams with resultant heating and dispersal of the beam. In the present parameter regime, however, the surface-initiated disturbance appears to ultimately dominate this process.

The course of electron heating is summarized in Fig. 3, which suggests that such heating occurs in two clearly distinct phases. The first phase, which lasts until $\omega_p t \cong 140$, is characterized by relatively weak heating, while the second, which takes the electron temperature to ~ 1 keV by the end of the simulation, exhibits much stronger heating. Such an enhancement of electron temperature is frequently associated with the appearance of significant ion fluctuations through the well-known Dawson-Oberman formulae for high-frequency plasma conductivity.¹ Thus, we are motivated to examine the ion phase space. In Fig. 4 we show a transverse example of such a phase space for times which roughly bracket that for the onset of strong electron heating. For the earliest time shown there is but slight modulation of the ions apparent. In only ~ 6 ion plasma periods, however, there have appeared significant ion fluctuations. The appearance of these fluctuations is therefore well correlated with the beginning of intense electron heating.

There emerges, therefore, the following qualitative picture of electron heating in energetic cross-field plasma beam propagation. Surface-initiated charge-density fluctuations impart an oscillatory velocity to the initially cold electrons. The electrons are thereby weakly heated until such time that the plasma can support ion-acoustic waves. Thereafter, the ion-acoustic waves are driven unstable by the residual oscillatory motions of the electrons, thus producing the significant ion fluctuations. These enhanced ion fluctuations then contribute to the strong heating phase of Fig. 3. This scenario is, of course, quite reminiscent of what was once the conventional view of

electron heating in laser-plasma interactions. Parametric instabilities play an important role there, and we might expect something similar here. The details remain to be supplied.

For the time of initial strong electron heating there is little indication in ion configuration space of the fluctuations so clearly evident in the ion phase space. At later times, however, corrugations are clearly evident in the configuration space and serve to further support the relation between significant electron heating and the presence of ion fluctuations. In Fig. 5a we show the ion configuration space and in Fig. 5b an axial plot of the electron transverse velocity. One quickly perceives that both significant ion fluctuations and strong electron heating are localized toward the rear of the plasmoid, the presence of the one related clearly to that of the other. Details of the boundary of discernible ion modulation remain to be elucidated, but it appears to coincide roughly with Mach-like lines related to possible shock phenomena occurring at the head of the beam. Again, further theoretical work is required to elucidate this geometrically complex situation.

There are at least two features of the simulations described here which probably exaggerate the extent of wave generation and beam heating. Firstly, since the principal wave motion is surface-initiated, the sharp profile used for the initial plasmoid enhances wave excitation. A physically producible plasmoid would probably have diffuse edges and thus ameliorate disruption. Secondly, the plasmoid in the simulation is started in the full magnetic field, thus effectively being infinitely accelerated in the presence of the field. In reality, the plasmoid will be formed and accelerated at a finite rate and may be gradually introduced to the magnetic field.

Simulations in which the field is turned on over several hundred plasma periods in fact show reduced heating. These simulations will be described in a future memorandum.

Fig. 1. Significant beam dispersal is evident in ion configuration space plots at $w_p t \approx 500$.

Fig. 2. Transverse profile of longitudinally integrated charge density at early time $w_p t = 10$ shows beam polarization, but with evidence of wave generation.

Fig. 3. History of averaged electron temperature shows periods of weak and strong heating.

Fig. 4. Ion fluctuations are small at (a) $\omega_{pt} = 140$, noticeable at (b) $\omega_{pt} = 160$, and significant at (c) $\omega_{pt} = 180$, appearing roughly coincidentally with the onset of strong electron heating.

Fig. 5. Region of noticeable fluctuations in ion configuration space (a) coincides approximately with that of strong electron heating (b).

REFERENCES

1. J. Dawson and C. Oberman, "Effect of Ion Correlations on High-Frequency Plasma Conductivity," *Phys. Fluids* 6, 394 (1963).
2. W. Shanahan, "Transverse Plasmoid Constriction," Los Alamos National Laboratory memorandum X-8(12-82)13 (December 13, 1982).
3. W. Shanahan and C. Snell, "Three-Dimensional Rest Frame Plasmoid Simulations," Los Alamos National Laboratory memorandum X-8(7-83)5 (July 5, 1983).
4. G. Schmidt, "Plasma Motion Across Magnetic Fields," *Phys. Fluids* 3, 961 (1960).
5. V. Birko and G. Kirichenko, "Stability of a Neutralized Ion Beam in a Transverse Magnetic Field," *Sov. Phys. Tech. Phys.* 22, 202 (1978).

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THREE-DIMENSIONAL REST FRAME PLASMOID SIMULATIONS

This is a PLASMOID research memo that contains tentative results intended to enhance communication.

Previous simulations of energetic cross-field plasmoid propagation¹ suffer from several limitations, the most serious of which are the restriction to two dimensions and the use of periodic boundary conditions in the direction of propagation. The low dimensionality is a consequence of intrinsic limitations of the computer hardware and simulation software available at the time these studies were initiated, while the use of periodic boundary conditions arises from the desire to examine propagation dynamics for times longer than that required for the plasmoid to transit the computational box. There are, however, several features of the cross-field propagation problem which are fully three-dimensional and require the examination of the effects of macroscopic field gradients. The magnetic-field perturbations due to induced currents flowing in the plasmoid are expected to be modified in a fully three-dimensional treatment. Use of periodic boundary conditions naturally precludes study of gradient effects. During the past three months, we have sought to address these issues by using a quasi three-dimensional particle-in-cell plasma simulation code to perform cross-field plasmoid propagation studies in the rest frame of the plasmoid. The new code permits us to address at least qualitatively the effects of higher dimensionality, while transformation to the plasmoid rest frame allows extended study of gradient effects by exchanging spatial inhomogeneities of the external magnetic field in the "lab" frame for space-time inhomogeneity in the plasmoid frame. Significant differences emerge between two and three dimensions.

In Fig. 1 we exhibit the basic initial geometry used in both the two- and three-dimensional simulations. A plasmoid, initially at rest, is placed in the center of the computational box. A rising magnetic field is then convected into the box according to a hyperbolic tangent prescription. In the absence of the plasmoid, the initially inhomogeneous external magnetic field evolves into one that is spatially uniform, as shown in Fig. 2. Naturally, since we are simulating, as viewed from the lab frame, a plasma beam propagating across a static magnetic field, the rest frame dynamics must include an external electric field pointing in the direction perpendicular to both that of plasmoid motion and of the magnetic field. Note that, although the initial magnetic-field strength in the vicinity of the plasmoid and the rise time of the field may be made as small and as long, respectively, as one desires, in this case we have not striven particularly to so control these parameters. Our objective here, rather, is to examine the effects of higher dimensionality and of gradients with as little computer time as possible. Simulations employing gentler initial conditions are presently being performed.

The three-dimensional PIC code, IVORY, used in the present study, is a product of Mission Research Corporation,² and is essentially an extension of the two-dimensional code used previously.¹ IVORY models variation in the third direction by effecting a Fourier transform in that direction of all field and current quantities and then retaining a finite number of modes. This code is thus most appropriate for problems that have a clearly defined uniform analog, such as the study of beam instabilities exhibiting azimuthal variation. Its applicability to an intrinsically three-dimensional problem, such as the present one, must remain a question for study. Nevertheless, one expects to obtain at least qualitative insight concerning the effects of higher dimensionality from the results of running this code. In the following, we have retained the uniform ($k_z = 0$) mode and the first two nontrivial modes, with three particles/mode in the third direction. Variations in these parameters yielded some differences in the fine details of the simulations, but did not significantly affect the gross features reported here.

In Fig. 3 we display the basic results of this series of simulations. Figure 3a is the plasmoid configuration space plot taken from a two-dimensional simulation of an initially stationary plasmoid subjected to the rising magnetic field of Fig. 2. Here, we have employed a final vacuum magnetic-field strength, as measured in the laboratory frame, of approximately 700 G and a corresponding laboratory relativistic factor $\gamma = 2$. Focusing is apparent, although there are some differences from the periodic simulations described previously.² These differences may be traced to the effect of the spatial inhomogeneity of the external magnetic field along the length of the plasmoid with resultant variation in the diamagnetic current structure. Figure 3b exhibits the results of a similar three-dimensional simulation. No remnant of focusing is evident here. Rather, substantial disruption of the plasmoid has taken place. Comparison of magnetic-field profiles for the two-dimensional case and of the uniform mode profiles for the three-dimensional case exhibit little difference. Inspection of current profiles is more revealing. Figure 4 is a comparison of the two- and three-dimensional cases for the axial profile of the transversely integrated induced current density. Figure 4a represents a typical current profile quite similar to that found in earlier periodic simulations. In contrast, Fig. 4b reflects the more complex inductive field structure revealed by the inclusion of variation in the third direction. Examination of axial and transverse profiles of higher modes shows qualitatively expected behavior. For example, while the uniform mode of the magnetic field continues to exhibit snowplowing, significant corrections are evident in higher modes. The transverse profile of the longitudinal magnetic field, which in two dimensions exhibits a characteristic focusing form, shows in one higher mode a qualitatively opposite form with a quantitative correction of 30-40%. Obviously, all these factors affect the Lorentz force previously found to be responsible for transverse plasmoid constriction.

One cannot, however, immediately conclude that focusing is a spurious effect of low dimensionality. In the two-dimensional simulation, the plasmoid extends infinitely in the third direction, whereas

for the three-dimensional case the width has been restricted to the small value of $\pi c/\omega_p$. Thus, the effect evident in Fig. 3 may well be merely a reflection of inadequate scale. This last observation emphasizes the importance of a quantitative theory of diamagnetic field response.

Fig. 1. Initial configuration in both two and three dimensions consists of stationary plasmoid placed in center of computational box.

Fig. 2. (a) Vacuum magnetic field is initially spatially inhomogeneous. (b) External field is smoothly convected into computational region. (c) In absence of beam, field eventually reaches steady uniform value.

Fig. 3. Results of rest frame simulations differ dramatically between two (a) and three (b) dimensions.

Fig. 4. (a) Two-dimensional transversely integrated axial profile shows focusing character and (b) significant differences apparent in uniform mode.

REFERENCES

1. W. R. Shanahan, "Plasmoid Transverse Constriction," Los Alamos National Laboratory memorandum X-8(12-82)13 (December 22, 1982).
2. Mark M. Campbell and Brendan B. Godfrey, "Ivory User's Manual," Mission Research Corporation report AMRC-N-221 (1983).

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W. R. Shanahan, X-8,

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VIRIAL THEOREM APPLIED TO PLASMOID PROPAGATION

This is a PLASMOID research memo that contains tentative results intended to enhance communication.

There is a "folk" result to the effect that plasmoid propagation cannot be enhanced by currents flowing within the plasmoid itself. Usually this assertion is coupled with a reference to the virial theorem. This virial theorem is thus exalted to a fundamental law of nature that essentially precludes certain lines of investigation, and reference to a contrary possibility is greeted as if one had suggested that it is possible to pick oneself up by his bootstraps. In this memorandum I investigate the extent to which the above bit of conventional wisdom is true and suggest a computational program to investigate the exceptions revealed. Such a program is necessary since the only thing that is demonstrated here is that the virial theorem is consistent with the presence of pinching forces, these forces being manifested through usually neglected surface terms. Fully self-consistent numerical simulations are required to determine if this observations has any meaningful role to play in enhancement of plasmoid propagation.

The conventional derivation¹ of the virial theorem begins with the usual equation of motion for ideal magnetohydrodynamics:

$$\rho \frac{du_i}{dt} = - \frac{\partial}{\partial x_i} \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \frac{\partial}{\partial x_j} (B_i B_j) \quad (1)$$

with ρ the plasma mass density, $\underline{x}$ the position of a fluid element, $\underline{u}$ its velocity, and $\underline{B}$ the magnetic field. Further, the summation convention, according to which repeated indices are to be summed over their range, is in force. Multiply both sides by x_i , again apply the summation convention, and integrate over the volume of the plasma "system." The word "system" is emphasized in the preceding sentence since consideration of precisely which portion of the plasma is included is important for arriving at unambiguous results. Various integrations by parts will now be performed to reduce Eq. (1) to a standard form. The left-hand side may be transformed as:

$$\begin{aligned} \int \rho x_i \frac{du_i}{dt} dV &= \int \frac{1}{2} \frac{d^2}{dt^2} (\rho x^2) dV - \int \rho u^2 dV \\ &= \frac{1}{2} \frac{d^2 I}{dt^2} - 2T, \end{aligned} \quad (2)$$

where I is the moment of inertia of the plasmoid, T is simply the contribution of the bulk kinetic energy, and where all volume integrals must be understood to be taken over a fixed mass of plasma. Equation (1) consequently becomes:

$$\frac{1}{2} \frac{d^2 I}{dt^2} - 2T = - \int x_i \frac{\partial}{\partial x_i} \left(p + \frac{B^2}{8\pi} \right) dV + \frac{1}{4\pi} \int x_i \frac{\partial}{\partial x_j} (B_i B_j) dV \quad (3)$$

Note that we have left unspecified which volume is being integrated over. In particular, we have left open the option of including only part of the plasmoid. The first and second volume integrals, I_1 and I_2 , appearing on the right-hand side of Eq.(3) may now be transformed by integrations by parts to yield:

$$I_1 = \int (p + \frac{B^2}{8\pi}) \underline{x} \cdot d\underline{S} - 3 \int (p + \frac{B^2}{8\pi}) dV \quad (4)$$

and

$$I_2 = \frac{1}{4\pi} \int [(\underline{x} \cdot \underline{B}) \underline{B} \cdot d\underline{S} - B^2 dV] \quad (5)$$

Therefore, Eq.(2) becomes

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2T + \int (3p + \frac{B^2}{8\pi}) dV + \frac{1}{8\pi} \int [2(\underline{x} \cdot \underline{B}) \underline{B} - (8\pi p + B^2) \underline{x}] \cdot d\underline{S} \quad (6)$$

The volume integrals in Eq. (6) may be given simple physical interpretations. Clearly,

$$M = \int \frac{B^2}{8\pi} dV \quad (7)$$

the total magnetic energy of the considered system, while the material pressure volume integral may be expressed in terms of the total internal energy, U , and the ratio of specific heats γ :

$$\int 3p dV = 3(\gamma - 1)U \quad (8)$$

Equation (6) then becomes:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2T + 3(\gamma - 1)U + M + \frac{1}{8\pi} \int [2(\underline{x} \cdot \underline{B}) \underline{B} - (8\pi p + B^2) \underline{x}] \cdot d\underline{S} \quad (9)$$

We may now extract from Eq.(9) the bit of folklore mentioned in the first paragraph. Assume first that the plasmoid is in equilibrium. Under such conditions all time derivatives and surface terms vanish, the latter since they represent stress flux across the surface, which must vanish in equilibrium. If we consider a finite plasmoid, and extend all volume integrals over the entire system, Eq.(9) becomes:

$$3(\gamma - 1)U + M = 0 \quad (10)$$

However, each of the quantities appearing on the left-hand side of Eq.(10) is non-negative, so one is presented with a contradiction. This is nothing but the well-known result that a plasma may not be maintained in equilibrium only by currents flowing entirely within the plasma. Further, if one ignores the surface term contribution in Eq.(9), one likewise concludes that self-currents cannot even retard plasmoid expansion, thus arriving at the assertion motivating this memorandum. (In astrophysical applications, for which the extended virial theorem was first derived, a contribution from gravitational forces appears in Eq.(1), yielding a stabilizing negative term in Eq.(10). Of course, such a term is completely negligible here.)

There are two observations to be made concerning Eq.(9) with respect to its significance for plasmoid propagation. Firstly, there is no requirement that the plasmoid propagate in a state of equilibrium, but only that it not disperse to the extent that its energy flux is everywhere below the value considered to be useful. Equally important, one is not constrained to consider the entire body of the plasma as the system of ultimate interest, but rather may restrict attention to some portion of it. In this case, the surface terms appearing in Eq.(9) contribute and, at least from the vantage point of the virial theorem, there is no longer an absolute prohibition against the retardation of plasmoid expansion. A particularly simple illustration of this may be seen in a case in which only the material pressure term contributes to the surface term in Eq.(9). If, for example, material were being ablated from the surface of the plasmoid, then a portion of the plasmoid may be being compressed even though the plasmoid as a whole is experiencing disassembly. In a case in which magnetic forces are important, this point may be expressed somewhat more graphically. If from the argument following Eq.(10) one concludes that external "coils" of some sort are required to maintain equilibrium, then one may regard the expendable part of the beam as a species

of coil. Although it is unlikely that one portion of the beam may be configured to provide an exact equilibrium for another part, it is at least no longer obvious that retardation may not be effected.

Having noted that plasmoid dispersion retardation is not absolutely prohibited by the virial theorem, one is naturally interested in determining whether such partial confinement is possible in a practical sense. This is an extremely complicated question, touching almost all issues of plasmoid propagation. However, one would at least like to be assured that, in principle, beam currents may be initialized to permit the surface terms in Eq.(9) to dominate the volume contribution. To show that this is indeed the case, we specialize to the case of an infinitely long cylindrical beam with all quantities depending only on the radial variable. Further, we admit only currents flowing in the axial direction. Although this model obviously cannot reflect all properties of an axially bounded beam, it may be considered to be an idealization of a plasmoid in the form of a high aspect ratio torus. In the geometry of the model, Eq.(1) reduces to:

$$\rho \frac{du_r}{dt} = - \frac{\partial}{\partial r} \left(p + \frac{B_\theta^2}{8\pi} \right) - \frac{B_\theta^2}{4\pi r} \quad (11)$$

If Eq.(11) is multiplied by $2\pi r^2 dr$, integrated over r , and subjected to the same manipulations used to obtain Eq.(9) from Eq.(1), one obtains:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2T_r + 2(\gamma - 1)U - 2\pi r_p^2 \left(p + \frac{B_\theta^2}{8\pi} \right)_{r=r_p}, \quad (12)$$

where now the integrated energies have the significance of quantities per unit length, T_r is the radial kinetic energy, and r_p is the radius of the plasma system considered. Note that no magnetic volume contribution appears in Eq. (12). The high symmetry of our model has

conspired with the cylindrical geometry to exclude the explicit appearance of such a term. Only the surface contribution appears, providing a particularly striking illustration of the importance of including all terms in the virial theorem.

The only thing that has been demonstrated here is that the virial theorem by itself does not preclude retardation of dispersal. In fact, one can summarize the argument by saying that the presence of pinching forces is consistent with the virial theorem and that such forces manifest themselves through the usually neglected surface terms. Note also that this is perhaps as far as the virial theorem itself can take us. Examination of Eq.(12) would seem to indicate that any internal energy contribution can be compensated by establishing a sufficiently strong field, or, equivalently, a sufficiently large initial current. However, apart from the practical difficulty of actually establishing such a current, there is the consideration that for a plasmoid of given density this may require imparting a drift velocity of the electrons above that sufficient to trigger various instabilities. Such instabilities in turn might give rise to enhanced heating and plasmoid dispersal. This is an example of the sort of consideration necessary for the evaluation of the possibilities revealed here.

Naturally, one would like to have a method of testing these concepts and carrying them beyond the simple static arguments used above. The possibility of such a method is presented by the two-dimensional, electromagnetic, fully relativistic particle-in-cell plasma simulation code CCUBE. Recently, I have developed a simple way in which to initialize this code to represent a plasmoid with virtually any algebraically representable cross section propagating in the direction perpendicular to the plane of computation. Initial current profiles may be similarly structured. In particular, the idealized geometry used above for the example may be closely approximated. Histories of the moment of inertia of the beam inclusive of various initial radii may be established. Thus, the basic physical concepts discussed here may be examined in the context of numerical simulations. Further, some of