Lawrence Livermore Laboratory

ADVANCED-FUELED FUSION REACTORS SUITABLE FOR DIRECT ENERGY CONVERSION
PROJECT NOTE: TEMPERATURE-GRADIENT ENHANCEMENT OF ELECTRICAL FIELDS
IN INSULATORS

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ABSTRACT

Direct energy converters for use on controlled fusion reactors utilize electrodes operated at elevated voltages and temperatures. The insulating elements that position these electrodes must support large voltages and under some circumstances large thermal gradients. In this note, it is shown that even modest thermal gradients can cause major alterations of the electric-field distribution within the insulating element.

INTRODUCTION

Current designs for the "Venetian blind" direct converter require the suspension of several large planar arrays of carbon vane electrodes in the path of an ion flux escaping from a reactor. A method for supporting such arrays of vanes is shown in Fig. 1. Cooled, metal tubes were suggested as the support members for the vanes. The vanes are both at an elevated temperature and at (several different, e.g., V_1 and V_2) elevated voltages. To prevent the vane panels from shorting together, an insulator is required at the point of attachment of the vane array to the metal tube. The insulator such as the one in the illustration is subject to a temperature gradient. Using a very simplified model, studies have been made of the effect of the temperature gradient on the distribution of the electric field within the insulator. Two insulator geometries were considered: a uniform-height, annular cylinder and a tapered-height, annular cylinder. Because the calculations revealed some design pitfalls to be avoided, the results are discussed in this brief report. The key result is that the resistivity, which is very sensitive to temperature, dominates the field calculation; a significant distortion of the fields can occur.

Attention will be limited to the electrical gradients interior to the insulator. Any actual electrical failure is likely to be preceded either by the occurrence of a streamer discharge that develops within the vacuum

Fig. 1. A portion of the direct converter. Some insulators and the metallic, thermal-radiation shields have been omitted to achieve simplicity.

space surrounding the electrodes or by surface breakdown. Hence, a more comprehensive picture is required before the insulating strength can be predicted.

Figure 1 is not complete in several respects: additional insulators and metallic, thermal-radiation shields surround the support tube. These are deleted for the purpose of simplification.

PHYSICAL MODEL OF THE INSULATOR

There are two extremes of behavior which in varying degree are likely to be encountered in modeling the insulator material. In the first, the dielectric does not conduct, and the potential ϕ varies according to

$$\nabla \cdot \epsilon \nabla \phi = 0 ,$$

where ϵ is the dielectric susceptibility. In the presence of a temperature gradient, ϵ will vary with position, but since ϵ is generally a weak function of temperature, this reduces to

$$\nabla^2 \phi = 0 . \tag{1}$$

In the second extreme, which occurs at elevated temperatures, the material is conductive and the potential varies as

$$\nabla \cdot \frac{1}{\rho} \nabla \phi = 0 , \tag{2}$$

where ρ is the resistivity. In this case, ρ varies rapidly with temperature, and in the presence of a thermal gradient, Eq. (2) does not reduce to Eq. (1). Hence, the potential variation is different in the two cases.

To determine the regime of operation of a small volume element of the insulator, assume that its temperature is nearly uniform. The element may be modeled, as indicated in Fig. 2, by a capacitor in parallel with a resistor. If a voltage step is applied, the response is initially capacitive, but eventually becomes resistive. The time constant for this transition is $\tau = RC = \rho \epsilon$. In other words, the potential distribution initially corresponds to that of a perfect dielectric, but after several

Volume element

Dielectric susceptibility: ϵ
Resistivity: ρ

Equivalent circuit

$$C = \frac{\epsilon A}{l}$$

$$R = \rho \frac{l}{A}$$

$$\rho \epsilon = RC$$

Fig. 2. Model of a small volume element of an insulator.

$\rho\epsilon$'s it has changed into the distribution corresponding to a resistor. Since ϵ and ρ do not vary in the same manner with temperature (and hence position), these two distributions may be radically different. Table 1 tabulates ϵ , ρ , and τ for alumina over the temperature range of interest. From the computed τ values, one sees that the decay to the resistive solution takes place on a time scale which is short compared to the time during which the voltage is applied.

Table 1. Variation in a volume element of the insulator of dielectric susceptibility ϵ , resistivity ρ , and time constant τ for alumina.

Temperature (°C)	ϵ^a (10^{-14} F/cm)	ρ^b ($\Omega \cdot \text{cm}$)	$\tau = \rho\epsilon$ (s)
25	9.8×8.854	$>10^{15}$	$>8.7 \times 10^2$
300	9.8×8.854	2×10^{13}	17.4
500	9.8×8.854	5×10^{10}	4.3×10^{-2}
700	9.8×8.854	5×10^8	4.3×10^{-4}
1000	9.8×8.854	5×10^6	4.3×10^{-6}

^aValues for AD-999 alumina manufactured by Coors Porcelain Company. Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Energy Research & Development Administration to the exclusion of others that may be suitable.

^bValues taken from W. H. Kohl, *Handbook of Materials and Techniques for Vacuum Devices* (Reinhold Publishing Co., New York, 1967) p. 71. It should be noted that the resistivity of Coors AD-999 is about two orders of magnitude higher than the values of alumina resistivity found in Kohl's book.

The resistivity may be approximated by

$$\rho = \exp \left[- \left(\frac{T - 200}{C_0} \right)^{1/2} + 16.9 \ln (10) \right], \quad (3)$$

where

$$C_0 = 1000 / [5.4 \ln (10) - 16.9 \ln (10)]^2.$$

This representation of ρ will be used in some of the following calculations. It is accurate to within $\pm 0, -36\%$ for temperatures exceeding 300°C . This accuracy is adequate to illustrate the magnitude of the problem.

To compute the distribution of electric fields within the insulator for times exceeding a few τ , the resistivity and hence the temperature distribution must be known.

To compute the temperature distribution, the thermal conductivity must be known as a function of temperature. Table 2 illustrates the accuracy with which the experimental data are fit by the expression $13.5/T$, where T is the temperature in $^{\circ}\text{C}$. Above 300°C , the approximation is quite good.

Table 2. Experimental values of thermal conductivity and values derived from $13.5/T$. The figures indicate that $13.5/T$ is a useful expression for the range $300\text{--}800^{\circ}\text{C}$.

Temperature ($^{\circ}\text{C}$)	Thermal Conductivity ($\text{cal}\cdot\text{s}\cdot\text{cm}^{-2}/^{\circ}\text{C}$)	
	Experimental ^a	$13.5/T$
25	0.088	0.54
300	0.044	0.045
500	0.027	0.027
800	0.018	0.017

^aValues from American Lava Corporation, Chart No. 711, ALSIMAG 753.

CASE I: UNIFORM-HEIGHT, ANNULAR-CYLINDER INSULATOR

The uniform-height, annular cylinder (Fig. 3) represents the simplest insulator geometry. If cylindrical symmetry is assumed, the heat flow at a radius r is given by

$$-2\pi r K(T) \frac{dT}{dr} = H_T, \text{ a constant thermal power,}$$

where $K(T)$ is the thermal conductivity. If $K(T)$ is approximated by

$$K(T) \approx \frac{D}{T},$$

Fig. 3. Case I: The uniform-height, annular cylinder insulator.

where D is a constant (equal to 13.7), and T is the temperature in °C, then

$$-dT K(T) = \frac{H_T}{2\pi} \frac{dr}{r},$$

$$-D \frac{dT}{T} = \frac{H_T}{2\pi} \frac{dr}{r}.$$

The preceding equation has a solution of the form:

$$\frac{T}{T_0} = \left(\frac{r}{r_0}\right)^{-H_T/2\pi D}.$$

Since H_T is unknown, the exponent must be fixed by examining the boundary conditions, which are noted in Fig. 3. The result is

$$T = T_0 \left(\frac{r}{r_0}\right)^{\ln(T_1/T_0)/\ln(r_1/r_0)}. \quad (4)$$

For the calculation of the field E and voltage V, cylindrical symmetry was again assumed. For the voltage one can write

$$-2\pi r \frac{1}{\rho(T)} \frac{dV}{dr} = J,$$

where ρ is the temperature-dependent electrical resistivity, and J is the current per unit of axial length of the insulator. J is independent of r.

The electrical field is given by

$$E = -\frac{dV}{dr} = +\frac{J}{2\pi r} \rho(T), \quad (5)$$

where T is an already calculated function of r [see Eq. (4)]. To find J, this equation is integrated to yield

$$J = -\frac{2\pi (V_1 - V_0)}{\int_{r_0}^{r_1} \frac{\rho(T)}{r} dr}.$$

With J known, one can use Eqs. (3), (4), and (5) to find E.

As an example of how E behaves, consider a uniform-height, annular-cylinder insulator in which the inner wall is assumed to be cooled. First, we calculate the electric field for the parameters shown in Fig. 4(a). Graphs of the temperature and electric field are shown in Fig. 4(b). The essential feature is the very large electric fields that occur in the cool, high-resistivity, 0.5-cm-thick, inner layer of the insulator. The electric field in the absence of a temperature gradient is also shown in Fig. 4(b). The enhanced field at the cooler, inner, insulator face is eight times greater than the peak field in the absence of a temperature gradient. A second calculation for a thicker wall insulator is given in Fig. 5(a + b). Again a factor of eight field enhancement occurs.

The field enhancement in both of these examples occurs near the negative electrode. Since breakdown is frequently initiated at the intersection of the insulator, negative electrode, and vacuum space (the so called "triple point"), the enhancement occurs where the system can least afford it. Although no corresponding graph is shown, we found that raising the overall temperature of the insulator decreased the peak field, but as expected the leakage current then tended to rise.

CASE II: THE TAPERED-HEIGHT, ANNULAR-CYLINDER INSULATOR

It was clear from the uniform-height insulator studies that the temperature gradient caused a resistivity variation that resulted in the bulk of the voltage drop occurring across a small portion of the insulator. The tapered design shown in Fig. 6 was introduced with the objective of increasing the width of the cool, high-resistivity region by limiting the heat flow through the outer portion of the insulator.

If we assume that the equal-temperature surfaces are cylinders, the heat flow H_{11} is given by

$$- 2\pi r h(r) K(T) \frac{dT}{dr} = H_T, \quad (6)$$

where H_T is the heat flow and is a constant independent of r , and $h(r)$ is the insulator height and does depend upon r . With

Inside wall	Outside wall
$r_0 = 6 \text{ cm}$	$r_1 = 8 \text{ cm}$
$T_0 = 300^\circ \text{C}$	$T_1 = 600^\circ \text{C}$
$V_0 = 0$	$V_1 = 10^5 \text{ V}$

(a)

(b)

Fig. 4. Distribution of the electric field within the 2-cm-thick, uniform-height, annular-cylinder insulator: (a) parameters used in calculation, and (b) graph showing that the field at the cooler face is eight times stronger than in the absence of any temperature gradient.

Inside wall

$$\begin{aligned} r_0 &= 6 \text{ cm} \\ T_0 &= 300^\circ\text{C} \\ V_0 &= 0 \end{aligned}$$

Outside wall

$$\begin{aligned} r_1 &= 10 \text{ cm} \\ T_1 &= 600^\circ\text{C} \\ V_1 &= 10^5 \text{ V} \end{aligned}$$

(a)

(b)

Fig. 5. Distribution of the electric field within the 4-cm-thick, uniform-height, annular cylinder insulator: (a) parameters used in calculation, and (b) graph showing that again the temperature gradient enhances the peak field at the cooler face by a factor of eight over the uniform-temperature case.

Fig. 6. Case II: The tapered-height, annular-cylinder insulator.

With

$$A = \frac{h_1 r_0 - r_1 h_0}{r_0 - r_1}, \quad (7)$$

and

$$B = \frac{h_0 - h_1}{r_1 - r_0}, \quad (8)$$

one can write the height as a function of the radius in the form:

$$h(r) = A - Br. \quad (9)$$

As in Case I, we'll assume $K = \frac{D}{T}$, where D is a constant. Using Eqs. (7) through (9), Eq. (6) becomes

$$\frac{dT}{T} = - \frac{H_T}{2\pi DB} \frac{dr}{\left(\frac{A}{B} - r\right)r}. \quad (10)$$

By using a partial fraction expansion the integrand can be expanded as follows:

$$\frac{1}{\left(r - \frac{A}{B}\right)r} = \frac{B}{A} \left[\frac{1}{r - \frac{A}{B}} - \frac{1}{r} \right] \quad (11)$$

With this expansion, Eq. (10) can be integrated to yield

$$\ln \frac{T}{T_0} = \frac{H_T}{2\pi DB} \frac{B}{A} \ln \left(\frac{r - A/B}{r_0 - A/B} \cdot \frac{r_0}{r} \right). \quad (12)$$

By taking inverse logarithms, Eq. (12) becomes

$$\frac{T}{T_0} = \left(\frac{r}{r - A/B} \cdot \frac{r_0 - A/B}{r_0} \right)^{-\frac{H_T}{2\pi DA}}. \quad (13)$$

From the boundary condition at $r = r_1$ where $T = T_1$,

$$-\frac{H_T}{2\pi DA} = \ln (T_1/T_0) / \ln \left(\frac{r_1}{r_1 - A/B} \cdot \frac{r_0 - A/B}{r_0} \right).$$

Equation (13) describes the temperature distribution. The resistivity as a function of radius follows immediately from the empirical fit of resistivity to temperature given in Eq. (3).

The electrical field for the tapered-height, annular-cylinder insulator follows from a modification of the calculations for Case I. Here, we consider the total current through the insulator I rather than the current per unit length J ,

$$I = 2\pi r h(r) \frac{1}{\rho(r)} \left(-\frac{dV}{dr}\right). \quad (14)$$

The field is given by

$$\left(-\frac{dV}{dr}\right) = I \frac{\rho(r)}{2\pi r h(r)}.$$

The total current I can be evaluated by integrating Eq. 14 from r_0 to r_1 . This results in

$$I = -2\pi \frac{V_1 - V_0}{\int_{r_0}^{r_1} \frac{\rho(r)}{r h(r)} dr}.$$

The temperature and field distributions are plotted for two different parameter sets in Figs. 7 and 8. As expected, the thermal gradient near the hot, thin, outer portion of the insulator is increased and that near the inner portion is reduced (as compared with the uniform-height case). Consequently, the region of relatively high resistivity is widened and the peak field is decreased. Because of tapering of the insulator, the peak field in absence of any temperature gradient occurs in the thin, outer portion of the insulator. The presence of a temperature gradient changes the field distribution dramatically; there is a factor of two increase in peak field.

CONCLUSIONS

In addition to all the usual problems of high-voltage insulator design, the presence of a temperature gradient distorts the internal

Fig. 7. Distribution of the electric field within the 2-cm-thick, tapered-height, annular-cylinder insulator: (a) parameters used in calculation, and (b) graph showing that the insulator experiences a peak field that is a factor of two less than that of the untapered insulator (see Fig. 4).

Inside wall

$$\begin{aligned} r_0 &= 6 \text{ cm} \\ h_0 &= 10 \text{ cm} \\ T_0 &= 300^\circ\text{C} \\ V_0 &= 0 \end{aligned}$$

Outside wall

$$\begin{aligned} r_1 &= 10 \text{ cm} \\ h_1 &= 1 \text{ cm} \\ T_1 &= 600^\circ\text{C} \\ V_1 &= 10^5 \text{ V} \end{aligned}$$

(a)

(b)

Fig. 8. Distribution of the electric field within the 4-cm-thick, tapered-height annular-cylinder insulator: (a) parameters used in calculation, and (b) graph showing that the insulator experiences a peak field that is a factor of two less than that of the untapered insulator (see Fig. 5).

electrical field. Unfortunately, with the gradient present, the peak field shifts to the critical "triple point" where strong fields can least be tolerated. The effect of tapering has been to reduce the field at the inner radius by a factor of two. The extent to which one can take advantage of tapering is limited by the field increase in the thin end of the insulator when the gradient is absent. On occasion, this condition will be encountered. Further, it is doubtful that the simple theory employed here is accurate for extremely sharp tapers. More likely, one reaches a point of diminishing improvement near the factor of two peak field decrease which arose in the examples.

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