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THE EFFECT OF STRUCTURAL NONLINEARITIES ON THE
DYNAMIC RESPONSE OF AIRCRAFT TO IMPULSIVE LOADING

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ABSTRACT

Equations are developed for the symmetrical response to dynamic loads of an airplane consisting of a wing and fuselage in which the wing may have nonlinear load-deflection characteristics.

Solutions of the equations in which only one bending mode of the wing is considered indicate that the loads required to produce a specified tip deflection are greater for the nonlinear structure than for the corresponding linear structure for rectangular load impulses and loadings with finite rise times. The amount of load increase varies over a wide range depending on the type of loading and amount of nonlinearity. In general, the load increases are greater for the shorter duration impulses.

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INTRODUCTION

The prediction of damage to an aircraft caused by very high velocity gust conditions depends upon the knowledge of at least three major factors, (1) the loads imposed on the structure, (2) the response of the structure to the loads, (3) the conditions which cause structural failure. The present study is concerned with one aspect of the second of these factors, namely, the effect of structural nonlinearity on the response of the structure.

The behavior of a complicated structure such as an aircraft caused by time-dependent loads is very complex. The motions produced by a suddenly applied load may be made up of rigid body motions such as translation, pitching, rolling, and yaw, as well as structural deformations such as lifting surface bending and torsion. The problem may be further complicated by aeroelastic effects in which the displacements of the structure affect the aerodynamic forces present at any time. Other factors that affect the response of the structure are (1) structural damping, (2) behavior of the material under dynamic conditions, which we may call strain rate effects, and (3) the presence of nonlinearities. Calculation of the response of a structure is relatively straight forward although tedious, when the loads are known and when damping, strain rates, and nonlinearities are neglected. Such an analysis is usually expressed in terms of modes of motion in which as many modes are considered as the accuracy warrants.

The object of the present study is to make an estimate of the effect of the structural nonlinearity which is usually neglected in the conventional analysis. The types of nonlinearity considered are based on those observed in static tests of aircraft components such as wings and tails. In general, the deflections of the structure are not proportional to the loads, particularly at the higher stress levels. The deviation from linearity of load and deflection in many cases is very small, while for some cases it is appreciable. The amount of nonlinearity is expressed in terms of a coefficient, β , which is a measure of the deflection at the tip of the nonlinear wing as compared with the deflection of a corresponding linear wing at the same load. The value of β is arbitrarily called the per cent of nonlinearity.

The investigation is limited to the response of the airplane in a symmetrical plane; that is, the change in loading on the wing caused by gust will be symmetrical about the longitudinal axis of the airplane so that no roll or antisymmetric modes of the wing are excited. The analysis is further simplified by assuming that the system consists of a point mass, the fuselage, attached to a nonuniform beam, the wing. A one-mode response of the wing, assuming it to be fixed at the root, is determined for rectangular load impulses of various durations and for a ramp function loading of various rise times. The effect of the nonlinearity is expressed by means of a comparison of the magnitude of the dynamic load required to produce a given tip deflection with the magnitude of the static load required to cause the same tip deflection, for different amounts of nonlinearity. The ratio of these loads is called the load coefficient. The greater the load coefficient the greater the dynamic load must be to cause a given response. The results of the analysis for the case of the wing fixed at the root are extended to take into account the rigid body translation of the wing-fuselage combination.

CONCLUSIONS

The effect of the nonlinearity, in general, is to increase the load required to cause a specified response. The amount of increase depends upon the loading conditions and the degree of nonlinearity. For example, for a rectangular pulse of long duration, the load required is 10% greater for the structure with 20% nonlinearity and 34% greater for the structure with 40% nonlinearity as compared with the linear structure. These effects become more pronounced for the shorter duration impulse. For an impulse with a duration of 30% of the natural period of the structure, the corresponding increases in load are 20% and 49%, respectively.

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The same trend is maintained for loadings with finite rise times. For a loading with a rise time of 20% of one period and a total duration of 50% of the period, the load required is 19% greater for the structure with 20% nonlinearity and 51% greater for the structure with 40% nonlinearity, as compared with the linear structure. For a rise time of 40% of the period, the corresponding values are 18% and 43%. These last values should be compared with the previous ones with caution, because the total impulse is different for the two cases.

It should be emphasized that the analysis takes no account of the tendency for a material to behave in a more linear manner when loaded dynamically. The assumption has been made that the structure responds dynamically in the same nonlinear way that it responds statically.

The effect of the rigid body translation may vary over a wide range depending on the ratio of wing weight and distribution to total aircraft weight. For an airplane with a wing having a uniformly distributed weight of 10% of the total weight, the loads required are about 10% greater to produce the same tip deflection.

On the basis of very few data, it is believed that typical aircraft components may have nonlinearities ranging from zero to 30%. The lower end of this range is more likely for so-called "stiff structures".

ANALYSIS OF A NONLINEAR STRUCTURE

The response of a structure subjected to dynamic loading in which stress is proportional to strain and other linearities are assumed, is often expressed in terms of the superposition of principal modes. The superposition is permissible because the systems are linear. In many cases a response of the structure, such as a displacement, can be closely approximated by using only one mode, neglecting the contributions of additional higher modes.

The presence of nonlinearities prevent the application of the usually modal analysis because the principal of superposition does not apply. However, some estimate can be made of the response of a nonlinear structure if it is idealized so that there is only one degree of freedom corresponding to a one-mode response. This is the viewpoint taken in the analysis of the nonlinear aircraft component such as a wing or tail considered herein.

Figure 1 indicates a wing or similar structure carrying a spanwise distributed load which is also a function of time. This load produces deflections which vary with time. If the structure is linear, restoring forces in the structure will be developed that are proportional to the deflections. That is, the restoring forces are twice as great for twice the deflection. We could represent this condition by a straight line variation between restoring force and deflection, $f = k_1 y_1$, as indicated in Figure 2. A nonlinear relation between restoring force and deflection is illustrated by the dashed line in Figure 2. A convenient representation of this type of variation is the so-called "soft" spring,

$$f = k_1 y - k_2 y^3. \quad (1)$$

A great variety of curves can be obtained by adjusting the values of k_1 and k_2 . It is assumed that the response of the wing can be expressed as

$$y(x, t) = q(t) \phi(x) \delta \quad (2)$$

where

$q(t)$ = a generalized coordinate
 $\phi(x)$ = shape of the deflection curve; $\phi(l) = 1$
 δ = tip deflection.

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FIGURE 1
WING DEFLECTION

FIGURE 2
RESTORING FORCE DEFLECTION CURVES

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The Lagrange equation of motion for this system is

$$\ddot{q} + \omega^2 q - \frac{\delta^2 \int_0^l k_2 \phi^4 dx}{\int_0^l m \phi^2 dx} q^3 = \frac{1}{\delta} \frac{\int_0^l w \phi dx}{\int_0^l m \phi^2 dx} \quad (3)$$

where

$$\omega^2 = \frac{\int_0^l k_1 \phi^2 dx}{\int_0^l m \phi^2 dx} \quad = \text{natural frequency squared of the system if it were linear.}$$

m = mass per unit span

For the case of a static load, w_s , which produces a static tip deflection δ , $\ddot{q} = 0$ and $q = 1$ in Eq. (3), and

$$\delta^2 \int_0^l k_2 \phi^4 dx = \omega^2 \int_0^l m \phi^2 dx - \frac{1}{\delta} \int_0^l w_s \phi dx. \quad (4)$$

If further we consider that the dynamic load has the same spanwise distribution as the static load, we may write

$$\frac{\int_0^l w_d(x, t) \phi dx}{\int_0^l w_s(x) \phi dx} = n F(t). \quad (5)$$

where

$$n = \frac{w_d}{w_s} = \frac{\text{maximum intensity of load applied dynamically}}{\text{intensity of load applied statically}}$$

$F(t)$ = time variation of dynamic load. $F(t)_{\max} = 1$.

By substituting Equations (4) and (5) into (3), it can be shown that

$$\ddot{q} + \omega^2 (q - \beta q^3) = n \omega^2 (1 - \beta) F(t) \quad (6)$$

where

$$\beta = 1 - \frac{1}{\delta \omega^2} \frac{\int_0^l w_s \phi dx}{\int_0^l m \phi^2 dx}. \quad (7)$$

The value of n which satisfies Equation (6) and makes $q = 1$, is the ratio of the dynamic load to the static load that is required to make the tip deflection produced dynamically equal to the static tip deflection.

The coefficient of nonlinearity, β , is a measure of the amount of nonlinearity in the system. The relation between β and the restoring force variation will be discussed in the next section. However, a simple way to estimate β for a given structure will be given here.

Let us consider a fictitious system in which we assume there is no nonlinearity, so that $k_2 = 0$. We apply a static load w_s so as to produce a shape ϕ with an end deflection Δ . Then

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from Equation (3) ($\ddot{q} = 0$, $q = 1$) we have

$$\frac{1}{\Delta} \frac{\int_0^L w_s \varphi dx}{\int_0^L m \varphi^2 dx} = \omega^2 \quad (8)$$

Substituting this relation into the value for β given by Equation (7) results in

$$\beta = 1 - \frac{\Delta}{\delta} \quad (9)$$

where

Δ = deflection produced by static load if structure were linear
 δ = deflection produced by static load.

The results are summarized as follows

$$\ddot{q} + \omega^2(q - \beta q^3) = n \omega^2(1 - \beta) F(t) \quad (10)$$

where

q = time dependent coordinate with maximum value of unity
 ω = natural frequency of linear structure. Frequency observed for small amplitudes. (rad sec⁻¹)
 n = ratio of maximum intensity of load applied dynamically to intensity of load applied statically to produce same tip deflection
 β = coefficient of nonlinearity = $1 - \Delta/\delta$
 Δ = deflection caused by static load assuming linearity
 δ = actual deflection produced by static load
 $F(t)$ = time variation of application of dynamic load.

Equation (10) is to be used as follows: Suppose we have a wing or other structural component for which we have such information as the load-deflection curves for increments of static load up to some load of probable interest, such as the ultimate load. We may plot the deflection at the tip versus the load and obtain a curve which in general will be nonlinear. Corresponding to the load which produces the maximum deflection, δ , we can determine a deflection, Δ , which would be produced if the structure were linear by extending the linear portion of the curve up to the final load. In this way we determine the coefficient of nonlinearity, β . The frequency, ω , is obtained from vibration tests or calculations based on a linear structure or small amplitudes of motion. In general, this frequency corresponds to the first principal frequency of the structure. Next we consider the manner in which the dynamic load varies with time, $F(t)$. This is established by the loading conditions and may be a unit step function, a sinusoid pulse of unit amplitude, or any other variation of interest. These factors are introduced into Equation (10) and the equation is solved for the factor n which will make the maximum value of q equal to unity.

The value of n thus determined is the ratio of the magnitude of the dynamic load to the magnitude of the static load with the same spanwise distribution which is required to produce the same tip deflection.

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STRUCTURAL NONLINEARITIES

The types of nonlinearity that are considered in the dynamic response analyses are those having a cubic parabola variation as indicated in Equation (1). Different degrees of "softness" can be obtained by adjusting the values of the constants associated with the linear and the cubic terms. Figure 3 indicates the relation between restoring force and displacement for different amounts of nonlinearity. The coefficient of nonlinearity, β , is indicated for each curve. It is expected that for most aircraft structures β will range in value from zero to approximately 0.3.

It is of considerable interest to know the type of nonlinearities encountered in actual aircraft structures. These nonlinearities are determined from static load-deflection measurements. Unfortunately, there is not much data available on the static response of aircraft components up to complete failure. Since deviations from linearity are not expected until the stresses become high, the lack of failure data makes it difficult to estimate this factor. The curve shown in Figure 4 gives the relation between the load and tip deflection of a wing of the C-54 aircraft. It should be noted that the maximum value of the load is 100% design ultimate which does not mean that the structure actually failed. According to these data, which are considered to be only approximate, the value of β is 0.32. A similar curve is shown in Figure 5 for the vertical fin of the B-47B aircraft. For this surface, β is equal to 0.22.

DETERMINATION OF LOAD COEFFICIENT, n

The value of the load coefficient n , which is the ratio of dynamic load to static load required to produce a given tip deflection, is determined for several loading conditions by means of Equation (10). Specifically, the load considered is one that is distributed spanwise and applied either as a rectangular impulse of duration τ , or applied as a ramp function with rise time t_0 , as indicated in Figure 6. These time-dependent loadings are arbitrarily selected in order to study the effect of pulse duration and rise time on the response of a nonlinear system. The spanwise distribution of the load on the structure is unspecified, but it is presumed that it would be similar to the distribution used in the static tests from which the deformed shape ϕ , and the static tip deflection δ , are determined.

Equation (10) is solved for each load condition so as to make the maximum value of the generalized coordinate, q , equal to unity. The value of the load coefficient thus determined is a measure of the effectiveness of a load applied dynamically to produce a specified tip deflection. Since the equations involved are nonlinear, no general solutions were sought. Particular solutions were obtained graphically by a method discussed in Reference 1, and several check points were obtained by means of the REAC analog computer at the Sandia Corporation.

The results of the analysis are indicated in Figures 7, 8, and 9. In these figures the time has been taken as a dimensionless ratio to the period of vibration of the linear structure, T . This period is related to the frequency, ω , that appears in Equation (10) by $T = 2\pi/\omega$.

The duration of a rectangular impulsive load required to produce a given tip deflection as compared with a static load is given in Figure 7. It is apparent from the curves that a duration time of about $1/2$ the period is as effective as a much longer duration. For these long durations, only half as much load is required dynamically to produce the same deflection as a static load for the linear structure. This factor increases to 55% for a structure with a nonlinearity coefficient of 0.2, and is 67% for a nonlinearity coefficient of 0.4. Hence, 34% more dynamic load is required for the case of $\beta = 0.4$ than for the linear case to produce the same deflection. All of these effects become more pronounced for the shorter durations. For example, for a duration of $0.3 T$ the load coefficient is 0.61, 0.73, and 0.91 for $\beta = 0, 0.2$, and 0.4 , respectively. Thus for $\beta = 0.4$ it is necessary to apply 91% of the static load dynamically to produce the same deflection. This is 49% more load than is necessary for the linear structure.

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FIGURE 6
TYPES OF APPLIED LOADS

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An indication of the effect of rise time of the impulse is given in Figures 8 and 9. The dashed curve for zero rise time and for the linear structure is shown for comparison in these figures. For a rise time of $t_0 = 0.2 T$, (Fig. 8), it is evident that for long durations the load coefficient approaches the same values as for the rectangular impulse of Figure 7. The total impulse time (i.e., the sum of rise time and duration with constant amplitude) to reach these values is, however, greater than before. An estimate of the effect of only the rise time is obtained by comparing the curves for zero rise time and the curve for 0.2 T rise time for the linear structure. At a total impulse time of 0.3 T, the load coefficient, n , is 61% for $t_0 = 0$ and 87% for $t_0 = 0.2 T$. Thus, 42% more load is required for the longer rise time. The load coefficients for this same total impulse time are 103% and 130% for β of 0.2 and 0.4. Hence, 49% more load is required for the most nonlinear case than for the linear case for a rise time of 0.2 T, and 113% more load is required for the most nonlinear case with a rise time of 0.2 T than for the linear case with zero rise time.

All of the effects discussed for the rise time of 0.2 T are accentuated for a rise time of 0.4 T as shown in Figure 9.

EFFECT OF RIGID BODY MOTION

The response to dynamic loading of a nonlinear beam-type structure has been discussed in the previous sections. In particular, the beams have been fixed at one end and the displacements have been specified relative to the fixed root section. The results, therefore, may be suitable for predicting the response of a wing panel that is rigidly restrained at one end. Actually, in normal operation, the wing is attached to the fuselage and any impulsive load applied to the aircraft while it is in flight will cause the whole airplane to translate as well as cause wing bending deflections relative to the fuselage. Since it is the relative or bending deflection of the wing which is likely to be of interest in any damage study, it is desirable to see how this is modified because of the rigid body motion of the complete aircraft.

The analysis of the rigid body effect will be limited to the case of symmetrical loading so that the aircraft will translate without rolling or pitching. The assumption is also made that all the mass of the aircraft other than the wing is concentrated at the midspan of the wing. The system considered is shown in Figure 10. The spanwise load distribution indicated is assumed to represent the load that is applied impulsively in addition to the steady-state flight loads.

The total displacement at any wing station is taken as

$$y(x, t) = y_0(t) + y_1(x, t) = y_0(t) + q_1(t) \varphi(x) \quad (11)$$

The nonlinear restoring forces are taken as in Equation (1) to be

$$F = k_1 y_1 - k_2 y_1^3 \quad (12)$$

The Lagrange equations for this system consist of two differential equations for the unknowns y_0 and q_1 . By eliminating the unknown y_0 and changing the form of the one remaining equation in a manner similar to that followed in the development of Equation (10), it can be shown that

$$\ddot{q}_1 + \omega_1^2(q_1 - \beta q_1^3) = n_1 \omega_1^2(1 - \beta) F(t) \quad (13)$$

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FIGURE 10

TRANSLATION MOTION OF AIRCRAFT

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where

q_1 = time-dependent coordinate with maximum value of unity.

$$\omega_1^2 = \omega^2 \left(\frac{1}{1 - \frac{m_1^2}{m_2 M}} \right) = \text{natural frequency of wing fixed at root as previously defined times a modification factor}$$

$$n_1 = n \frac{1}{\left(1 - \frac{m_1 \gamma}{M} \right)} = \text{load coefficient of wing fixed at root times a modification factor}$$

$F(t)^\beta$ = coefficient of nonlinearity
= time variation of application of dynamic load

$$m_1 = \int_{\text{wing}} m \phi \, dx = \text{generalized wing mass}$$

$$m_2 = \int_{\text{wing}} m \phi^2 \, dx = \text{second generalized mass of wing}$$

m = mass per unit span of wing

ϕ = shape of wing deflection, $\phi_{\text{tip}} = 1$

M = total mass of aircraft

$\gamma = w_o/w_1$ = ratio of generalized distributed loads

$$w_o = \int_{\text{wing}} w \, dx = \text{total applied load on wing (time-dependent)}$$

$$w_1 = \int_{\text{wing}} w \phi \, dx = \text{generalized load on wing.}$$

The form of Equation (13) is identical to Equation (10) from which the load coefficients for the wing fixed at the root were determined. The results obtained for the fixed wing can therefore be transformed to the case in which rigid body translation is considered. The relation between the load coefficient is

$$\frac{n_1}{n} = \frac{1}{\left(1 - \frac{m_1 \gamma}{M} \right)} \quad (14)$$

In general, this ratio is greater than unity indicating that more load is required to produce a given flexural response for the wing in a flight condition than for the wing fixed at the root. This ratio is a function of the relative mass of wing to the total mass of the airplane and to the load distribution, so that it is difficult to make general statements about its magnitude. However, an estimate can be made for a special case. The case considered is for a wing with uniform mass distribution, a uniform load distribution, and to a deflection shape the same as the first principal mode of vibration. Figure 11 shows the ratio of the load required to produce a given deflection of the wing in flight to the load required to produce the same deflection with the wing fixed at the root for different ratios of wing mass to aircraft mass. For a ratio of wing mass to aircraft mass of 0.1 the ratio of the load coefficient is about 1.10. This indicates that about 10% more load is required in this case to cause the given wing deflection if the whole system is allowed to move.

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