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PREDICTING EXCESS REACTIVITY IN THE ETR
AS A FUNCTION OF POWER-TIME

J. W. Henscheid and E. E. Burdick

PHILLIPS
PETROLEUM
COMPANY

ATOMIC ENERGY DIVISION

NATIONAL REACTOR TESTING STATION
US ATOMIC ENERGY COMMISSION

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J. W. Henscheid and E. E. Burdick

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ABSTRACT

A semiempirical equation is used to predict excess reactivity in the ETR as a function of power-time in terms of reactivity parameters which are routinely measured in a mock-up of each ETR core in the ETR Critical Facility. Constants in the equation were evaluated by fitting the equation to plots of excess reactivity as a function of power-time obtained for several cores in the ETR. The equation and reactivity measurements fulfill the need for frequent and reliable predictions of core charge life as well as control rod positions during reactor operation.

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PREDICTING EXCESS REACTIVITY IN THE ETR AS A FUNCTION OF POWER-TIME

I. INTRODUCTION

1. ENGINEERING TEST REACTOR

The Engineering Test Reactor (ETR) [1, 2], a water cooled and moderated reactor fueled with highly enriched uranium, has the primary purpose of providing high-intensity neutron flux to a number of experimental facilities located within the fuel regions of the reactor. The experiments in most of these facilities are contained in steel tubes which penetrate the reactor pressure vessel above and below the reactor core.

Refueling shutdowns in the ETR occur at six-week intervals during which time the composition or location of many of the experiments is changed and, occasionally, one or more in-pile tubes are removed or installed. As would be expected, these changes may significantly alter the excess reactivity and neutron flux distribution. In order to compensate these effects or to provide particular irradiation environments for certain experiments, major changes in the in-pile experiment facilities are usually accompanied by adjustments in the number and configuration of the fuel assemblies comprising the core. In some cases, revisions also are made in the control rod withdrawal sequence.

2. NEED FOR EXCESS REACTIVITY AND CHARGE LIFE PREDICTIONS

Conducting neutron irradiation programs as complex as those in the ETR and coordinating operation with the Materials Testing Reactor (MTR) require efficient and precise scheduling of reactor operating and shutdown time. Consequently, it is important that the charge life of each new loading be reasonably well known or, conversely, that the reactor be loaded to give a charge life greater than a certain specified minimum. Occasionally the ETR will be shutdown prior to its scheduled shutdown date. If there is not sufficient excess reactivity to override xenon, the reactor will usually be refueled rather than waiting for xenon to decay. In this instance it is convenient as well as economical to refuel the ETR with a previously discharged core that was only partially spent. This is practical only if a reliable estimate can be made of the residual charge life of the core.

The operating conditions of most of the in-pile experiments are influenced by the withdrawal of control rods during the operating cycle. Consequently, it is important to be able to predict the control rod positions as a function of time. This is possible if the reactivity worths of the control rods are known and if the excess reactivity can be predicted as a function of time.

3. CHARACTERISTIC CURVE OF EXCESS REACTIVITY

Figure 1 shows three curves of excess reactivity as a function of power-time (in MWd), curve A being typical for a core loading in the ETR. The general shape of curve A is characteristic of a high-flux thermal reactor using a burnable poison (elemental natural boron in the ETR). The shape and amplitude of the curve depend primarily on the amount and distribution of the poison in the core. Without burnable poison the curve would be considerably different. For example, if the burnable poison were removed, the increased reactivity would have to be compensated either by reducing the amount of fuel in the core or by adding more control rods. In the first case (reducing fuel) the excess reactivity would change with time, as shown by curve B in Figure 1, and would result in a much shorter charge life. In the latter case (adding more control rods) the reactivity change would follow curve C in Figure 1 with only a small

Fig. 1 Excess reactivity as a function of power-time in the ETR.

increase in charge life. The initial rapid loss in reactivity in all three cases is primarily due to fission product poisoning.

4. PREDICTING EXCESS REACTIVITY AND CHARGE LIFE

Computation methods based on perturbation theory have been reasonably successful in predicting the charge life of a reactor core. McMurry, Grimaud, and Hanson [3] derived an expression for the charge life of a compact, single-region reactor (such as the MTR) in which the neutron flux distribution is assumed to be independent of time. It accounts for nonuniform fuel burnup and fission product buildup. In order to apply this equation to the ETR, it was subsequently extended to include the effects of a burnable poison [4]. McMurry further modified the methods used above to derive an equation for the variation of reactivity with time in the ETR [5].

Perturbation methods also were used by Wolfe to predict the variation of excess reactivity with time in the ETR [6]. In Wolfe's formulation, as contrasted to that of McMurry et al, the neutron flux is time as well as space dependent.

From an operational viewpoint, perturbation methods, such as those referred to above, have one serious drawback--they generally are too time consuming. These methods require machine computations which, however rapid, must be preceded by lengthy and laborious tabulations of nuclear constants. In the MTR, which contains no in-pile experiments, the initial flux distributions are nearly the same for each loading, and a perturbation technique for predicting the time dependence of the excess reactivity or the charge life of the core is particularly applicable and probably feasible. However, for the ETR, more often than not, the flux distribution changes significantly between loadings due to changes in the in-pile experiments. Using perturbation methods, the nuclear constants of each new core would have to be calculated and tabulated, the initial spatial distributions of flux and adjoint flux in two energy groups would then be obtained from machine computations, and finally, the excess reactivity would be calculated for a series of time increments. The reliability of this technique largely depends on the ability to calculate flux distributions from the nuclear constants. On the few occasions where calculated and measured thermal flux distributions for the ETR have been compared, the agreement was not particularly good.

Recognizing these problems, McMurry and Healy [7] developed a simplified computational method which, though still based on perturbation theory, utilized experimental data as input data for the computer. The required data--thermal flux distribution and weighting factors for fuel and poison--were to be obtained from the ETR Critical Facility (ETRC), a full scale nuclear mock-up of the ETR. However, this method still required an excessive amount of time to perform the measurements in the ETRC prior to each change in the ETR core loading.

A much simpler method has now been developed which will reliably predict excess reactivity as a function of power-time for each core loading in the ETR. Operational data from the ETR and experimental data from the ETRC have been used to obtain a semiempirical equation describing the time dependence of excess reactivity held down by the control rods. The equation contains two reactivity parameters which vary from one loading to the next and can be easily measured in the ETRC prior to each loading in the ETR. Computations to obtain predictions of excess reactivity as a function of power-time and the charge life of the core can be easily made on a desk calculator. The manner in which the semiempirical equation was obtained, and the measurements, experimental data, and computations required for its application are described in the following sections.

II. ETR CRITICAL FACILITY

The ETR Critical Facility [8, 9] is a low-power nuclear duplicate of the ETR. Prior to each new loading in the ETR, measurements are performed in a nuclear mock-up of the proposed ETR loading in the ETRC to determine, among other things, the thermal neutron flux and power density distributions, the reactivity worths of the control rods, the shutdown reactivity, and the excess reactivity held down by control rods and burnable poison.

A cross-sectional diagram of the ETRC is depicted in Figure 2 showing the locations of the in-pile experimental facilities, fuel assemblies, and control rods. The reactor "core" generally includes the entire 10 x 10 array of three-in.-square grid cells inside the beryllium reflector with a vertical height of 36 in. The core arrangement shown is only one example of several different core configurations which have operated in the ETR.

Each of the 16 control rods in the core is composed of a poison section with a fuel section follower, the integral unit being contained inside a control rod guide tube. The poison sections on four of the 16 control rods (1 through 4) contain either cadmium or boron. These four control rods, called safety rods, are always completely withdrawn when the reactor is operating, (ie, the poison section is above the core and the fuel section is centered in the core). The twelve remaining control rods, called shim rods, contain nickel poison sections, usually referred to as gray sections.

In the ETR the 12 shim rods are gradually withdrawn during the operating cycle, some individually, others in ganged sets. Consequently, when a proposed ETR core is mocked-up in the ETRC, some of the shim rods are fully inserted, some are partially withdrawn, and some are completely withdrawn. Hence, aside from the in-pile experiments, the operational core is composed of fuel assemblies, control rod fuel sections, and nickel poison sections, all identical and interchangeable with their counterparts in the ETR.

The fuel assemblies and control rod fuel sections are composed of plates containing highly enriched uranium alloyed and clad with aluminum. Elemental natural boron is more or less uniformly dispersed in the alloy.

Although several of the in-pile experiments in the ETRC are completely interchangeable with corresponding experiments in the ETR, most of them in deference to cost are merely nuclear mock-ups. In general, however, each component part of an ETRC experiment mock-up bears a close resemblance to its ETR counterpart.

Outside the reactor core the resemblance of the ETRC to the ETR diverges as is readily seen by comparing the cross-sectional diagrams of the two reactors in Figures 2 and 3, respectively. Actually these apparent differences result in only a very small disparity between the neutron flux distributions within the cores of the two reactors, small enough that the additional cost of constructing an ETRC reflector identical to that in the ETR was not justified. (The beryllium pieces in the ETRC reflector were originally used in the MTR where they were subsequently replaced by experiments.)

Appendix A contains detailed information on standard core components of the ETRC. It includes calculated atom densities and macroscopic thermal absorption cross sections for fuel assemblies and control rods.

Fig. 2 Cross section of a typical core in the ETRC.

Fig. 3 Tank cross section of the ETR.

III. DEVELOPMENT OF TIME-DEPENDENT EXCESS REACTIVITY EQUATION

1. EXCESS REACTIVITY

The length of time that a reactor will operate with a given fuel loading, commonly called the charge life, is determined primarily by the amount of excess fuel initially loaded in the reactor. In order to extend the charge life without the encumbrance of many additional control rods, a distributed burnable poison may be included in the fabrication of the fuel assemblies. By properly proportioning the amount of fuel and burnable poison, the effect of the poison depletion can be made about equal to the effect of the fuel depletion during part of the cycle. In the ETR, boron is used as the burnable poison.

A measure of the excess fuel added to a reactor is the total excess reactivity $\rho_{\text{total}}^{[*]}$. This is the relative amount by which the effective multiplication factor is increased as a result of the additional fuel. Initially, when the reactor is just critical before the power is raised, the total excess reactivity is held down by the control rods and the boron in the fuel assemblies. Hence, the initial excess reactivity held down by the control rods is simply

$$\rho(0)_{\text{rods}} = \rho(0)_{\text{total}} - \rho(0)_{\text{boron}} \quad (1)$$

2. TIME DEPENDENCE OF EXCESS REACTIVITY

The change in excess reactivity with time depends on several factors. In the ETR, when the reactor power is raised, there is an initial loss in reactivity due primarily to the decrease in the density of the water moderator as the temperature of the reactor core increases. As the reactor continues to operate, the total excess reactivity decreases gradually due to fuel burnup while the excess reactivity held down by boron poison decreases due to boron burnup. An additional reactivity loss occurs due to the accumulation of fission product

[*] Reactivity is defined as $\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$

poisons. If the ETR is operated at a constant power, the excess reactivity held down by the control rods in a given core loading is given by

$$\rho(T)_{\text{rods}} = \rho(T)_{\text{total}} - \rho(T)_{\text{boron}} - \rho(T)_{\text{fp}} - \rho_{\text{temp}} \quad (2)$$

where all terms with the exception of ρ_{temp} have a functional dependence on the product of time and power or power-time T in units of MWd. With the exception of occasional power reductions, the ETR operates at a constant power and temperature and, hence, the term ρ_{temp} is approximately constant.

It remains to obtain analytical expressions for each of the three functions on the right side of Equation 2 which will describe their time behavior in terms of certain parameters or initial conditions. These conditions are fixed for a given core loading and can be determined from measurements in the ETRC. For a particular core loading, hence, a particular set of parameters, the curve of the equation obtained for ρ_{rods} should then describe the curve which would be observed in the ETR.

The rate of change of the total excess reactivity, ie, the reactivity loss per MWd due to fuel burnup, gradually increases with time. However, Wolfe has shown that for fuel burnup not exceeding 50%, the reactivity change with time is approximately constant [6]. Using this approximation the time dependence of the total excess reactivity will be given by

$$\rho(T)_{\text{total}} = \rho(0)_{\text{total}} - RT \quad (3)$$

where $\rho(0)_{\text{total}}$ is determined in the ETRC mock-up of the particular ETR loading, and R , the reactivity loss per MWd due to fuel burnup, is a constant.

The reactivity held down by the boron in the fuel assemblies is approximately proportional to the total amount of boron in the core. The boron burnup rate will vary throughout the core but, at any point, it will be proportional to the neutron flux and amount of boron at that point. Although the flux distribution varies with time due to control rod motion, fuel burnup, etc., these complex effects are neglected in order to obtain a simplified expression for the reactivity change due to boron burnup. Consequently, the approximation will be made that the reactivity held down by the boron in the core will decrease exponentially with time or

$$\rho(T)_{\text{boron}} = \rho(0)_{\text{boron}} e^{-AT} \quad (4)$$

where $\rho(0)_{\text{boron}}$ is determined for a given loading from excess reactivity measurements in the ETRC and A is a constant with dimensions of reciprocal MWd.

The reactivity held down by fission product poisons which accumulate as the fuel is burned can be separated into two components, one due to xenon poisoning and the other due to fission products other than xenon. The reactivity held down by xenon builds up very rapidly at first, reaching a maximum value after roughly 48 hours of reactor operation and then diminishing gradually throughout the rest of the operating cycle as the macroscopic fission cross section of the core decreases. The reactivity loss due to other fission products (primarily samarium-149) also increases rapidly at first and then continues to increase gradually with fuel burnup [10]. However, Wolfe has shown that the sum of these two effects is nearly constant after the first 48 hr [6]. Consequently, the time dependence for the excess reactivity held down by all fission product poisons will be approximated by the equation,

$$\rho(T)_{\text{fp}} = \rho_{\text{eq}} [1 - e^{-BT}] \quad (5)$$

where ρ_{eq} is the equilibrium value of reactivity held down by fission products. The constant B has dimensions of reciprocal MWd.

Substituting Equations 3, 4, and 5 into Equation 2 gives the desired equation for the excess reactivity in the control rods as a function of power-time,

$$\rho_{\text{rods}} = \rho(0)_{\text{total}} - RT - \rho(0)_{\text{boron}} e^{-AT} - \rho_{\text{eq}} [1 - e^{-BT}] - \rho_{\text{temp}} \quad (6)$$

Figure 4 is a graphical presentation of the time relation of each of the functions in Equation 6, ie, the linear decrease in total excess reactivity, the exponential decrease of the reactivity held down by the boron, the exponential buildup of the reactivity held down by fission product poisons, the reactivity loss due to the increased temperature and, finally, the algebraic sum of all the functions which is the excess reactivity in the control rods. For this illustration, values for the two reactivity parameters $\rho(0)_{\text{total}}$ and $\rho(0)_{\text{boron}}$ were taken for a typical ETR core loading. Although these parameters will vary from one core loading to the next, they are fixed for a particular loading and are obtained from measurements in the ETRC mock-up of that loading

Fig. 4 Time relation of functions in excess reactivity equation.

as shown on the following pages. Values used for the constant ρ_{temp} and the constant coefficients R , A , B , and ρ_{eq} were obtained from ETR excess reactivity plots as described on the following pages.

IV. EXPERIMENTAL DETERMINATION OF REACTIVITY PARAMETERS

For a given ETR core loading the two reactivity parameters $\rho(0)_{\text{total}}$ and $\rho(0)_{\text{boron}}$ in Equation 6 can be determined from measurements in an ETRC mock-up of the loading. Since neutron flux and reactivity data are obtained from the ETRC on a routine basis prior to each new loading in the ETR, it is convenient to obtain measurements of these parameters at the same time. These latter measurements require, at most, an additional eight hours of ETRC time and about one man-hour of data reduction.

1. TOTAL EXCESS REACTIVITY

In determining the initial total excess reactivity $\rho(0)_{\text{total}}$ in a particular core loading in the ETRC, the fueled regions of the core are uniformly poisoned with boron-polyethylene tapes until the reactor is critical with all control rods withdrawn to their upper limits. The borated tapes are inserted in the water channels between the fuel plates in the fuel assemblies and control rod fuel sections. With the core in this "fully poisoned" condition, a core-averaged macroscopic absorption cross section is defined by

$$\Sigma_{\text{core}} = \frac{n_{\text{fa}} \Sigma_{\text{fa}} + n_{\text{fs}} \Sigma_{\text{fs}}}{n_{\text{fa}} + n_{\text{fs}}} \quad (7)$$

where n_{fa} and n_{fs} are the numbers of fuel assemblies and control rod fuel sections in the core, and Σ_{fa} and Σ_{fs} are the total macroscopic absorption cross sections of a fuel assembly and a control rod fuel section, respectively. It should be noted that the core-averaged cross section defined in this manner includes none of the in-pile experiment facilities.

The total excess reactivity of the core given by the relation,

$$\rho(0)_{\text{total}} = \frac{\Delta \Sigma}{\Sigma_{\text{core}} - 1/2 \Delta \Sigma} \quad (8)$$

where $\Delta \Sigma$ is the average change in Σ_{core} which would result if all of the boron in the core were removed. This includes the boron in the polyethylene tapes as well as the boron in the fuel assemblies and fuel sections. The derivation of this equation and a discussion of its application in the ETRC are given in Appendix C.

The average change in the core cross section $\Delta\Sigma$ is calculated from the relation,

$$\Delta\Sigma = \frac{\sigma_a \cdot N_O}{A} \cdot \frac{M}{V n} \quad (9)$$

where σ_a is the microscopic absorption cross section of natural boron in cm^2 , N_O is Avagadro's number, M is the total mass in grams of boron in the core, A is the atomic weight of natural boron, V is the volume of a standard grid cell in cm^3 , and n is the number of grid positions containing fuel assemblies and control rod fuel sections.

Due to metallurgical problems encountered in the fabrication of plate-type fuel assemblies containing elemental boron, the boron content of ETR fuel assemblies varies as much as 30 percent from the specified content. The assemblies with the highest boron contents are loaded in the center of the core, those with the lowest boron contents, around the periphery of the core. This arrangement tends to flatten the overall power distribution and also results in a better utilization of the burnable poison.

Since the neutron flux is usually highest in the center of the core, the effect on reactivity of removing boron from the center of the core is greater than the effect of removing the same amount from the outer extremities. Consequently, the effect of removing a nonuniform distribution of boron would be different than removing the same amount if it were uniformly distributed. The ratio of the effects which equivalent amounts of boron have in any two core positions is approximately equal to the ratio of the squares of the thermal neutron flux in the two positions [11].

The thermal neutron flux is routinely measured at the midpoint of each fuel assembly during the ETRC mock-up measurements. Thus, it is convenient to calculate the total "effective" content of the boron in the fuel assemblies from the relation,

$$M_{\text{eff}} = \frac{n \sum m_i (\phi_i)^2}{\sum (\phi_i)^2} \quad (10)$$

In this equation n is the number of fuel assemblies, m_i is the mass in grams of the boron in the i th fuel assembly, and ϕ_i is the thermal neutron flux at the midpoint of the i th fuel assembly. Both summations are over all n fuel assemblies. Σ_{fa} , Σ_{core} , and $\Delta\Sigma$ are calculated using this "effective" boron content.

2. EXCESS REACTIVITY HELD DOWN BY CONTROL RODS AND BORON

If the entire uniform distribution of boron in the form of tapes is removed, a number of the control rods must be inserted to keep the reactor just critical. This is commonly referred to as the "cold, clean, critical" condition. The excess reactivity held down by the inserted control rods is just equivalent to the excess reactivity held down by the uniform distribution of boron which was just removed, so that from Equation 8 ,

$$\rho(0)_{\text{rods}} = \frac{\Delta \Sigma'}{\Sigma_{\text{core}} - 1/2 \Delta \Sigma'} \quad (11)$$

where Σ_{core} is the same as in Equation 8 and $\Delta \Sigma'$ now corresponds to the amount of boron.

The remaining excess reactivity held down by the boron in the fuel assemblies and control rod fuel sections is then simply

$$\rho(0)_{\text{boron}} = \rho(0)_{\text{total}} - \rho(0)_{\text{rods}} \quad (12)$$

3. SHORTENED PROCEDURE FOR SMALL CORE CHANGES

Occasionally a new ETR loading will differ from a prior loading by, at most, a few minor in-pile experiments. If the total reactivity effect of these differences in experiments is small ($<0.01 \Delta k/k$) and is known or can be easily determined, it is not necessary to perform excess reactivity measurements on an ETRC mock-up of the new loading. $\rho(0)_{\text{total}}$ and $\rho(0)_{\text{rods}}$ will differ from the prior loading by the amount of the reactivity effect due to the experiment differences $\rho(0)_{\text{boron}}$ remains the same.

On the other hand, the only difference between a new ETR loading and a prior loading may be the boron content of the fuel assemblies. In this case, although $\rho(0)_{\text{total}}$ would remain the same, $\rho(0)_{\text{rods}}$ and $\rho(0)_{\text{boron}}$ would change by equal amounts--one decreasing and the other increasing, depending upon whether the boron content of the fuel assemblies is greater or less than the prior loading. The new values for $\rho(0)_{\text{rods}}$ and $\rho(0)_{\text{boron}}$ can be evaluated by repeating the calculations performed following the excess reactivity measurements on the ETRC mock-up of the prior ETR loading, but using the new fuel assembly boron contents. The value obtained for $\Delta \Sigma'$ (Equation 11) in this new

loading corresponds to the amount of boron which would have been added in the form of tapes if an excess reactivity measurement actually had been made with this new ETR loading.

For the case where a new ETR loading differs from a prior loading, both in experiments and fuel assembly boron content, the values for $\rho(0)_{\text{total}}$, $\rho(0)_{\text{rods}}$, and $\rho(0)_{\text{boron}}$ must all be adjusted as explained above.

It is to be noted that in all cases no adjustment is made in the value used for Σ_{core} in Equation 11. Since the procedure is restricted to those cases where $\rho(0)_{\text{total}}$ changes less than $0.01 \Delta k/k$, the resulting error in the values obtained for $\rho(0)_{\text{total}}$, $\rho(0)_{\text{rods}}$, and $\rho(0)_{\text{boron}}$ will be no more than $\pm 0.001 \Delta k/k$.

V. FITTING THE REACTIVITY EQUATION TO EXCESS REACTIVITY PLOTS FROM ETR

1. EXCESS REACTIVITY PLOTS FROM THE ETR

Figures 5 through 13 are plots of excess reactivity as a function of power-time for several different loadings in the ETR. The reactivity recorded at each point was obtained by converting ETR control rod positions to a reactivity value using control rod worths and calibration curves measured in the ETRC. By extrapolating the plots to zero excess reactivity, it is possible to obtain estimates of the potential charge life for each core. The differences between the plots are due to changes in the amount and configuration of fuel, boron, and in-pile experiments. The configuration of each core loading is shown in Appendix B along with other information pertinent to the loading.

The core loading to which each plot applies is designated in the figures by one or more "cycle" numbers. A cycle is merely a specific period of time (usually six weeks) during which the reactor is shut down and refueled, experiments are changed, and the reactor operated the remainder of that period. As a general rule, a new fuel loading is installed at the beginning of a new cycle, although this is not always the case. Occasionally, for operational reasons,

Fig. 5 Calculated and observed curves of excess reactivity for ETR Cycle 27.

Fig. 6 Calculated and observed curves of excess reactivity for ETR Cycle 28.

Fig. 7 Calculated and observed curves of excess reactivity for ETR Cycle 30.

Fig. 8 Calculated and observed curves of excess reactivity for ETR Cycle 32B.

Fig. 9 Calculated and observed curves of excess reactivity for ETR Cycle 33C.

Fig. 10 Calculated and observed curves of excess reactivity for ETR Cycle 41B.

Fig. 11 Calculated and observed curves of excess reactivity for ETR Cycle 44.

Fig. 12 Calculated and observed curves of excess reactivity for ETR Cycle 45.

Fig. 13 Calculated and observed curves of excess reactivity for ETR Cycle 46.

a particular fuel loading may be removed during one cycle and reloaded during a subsequent cycle. In other cases a loading may be operated for more than one cycle if the actual operating time were short during the initial cycle.

A change in the letter suffix on a cycle number signifies that the reactor was shut down during the cycle and a change made in the core or reflector. Generally, these mid-cycle changes are small and have little or no effect on the flux distribution or excess reactivity.

The sharp dips and peaks on the reactivity plots are a result of mid-cycle shutdowns as well as other occasional power reductions. When the reactor power is reduced, the concentration of xenon starts increasing (due to the iodine precursors) with a resulting loss in excess reactivity. Consequently, in returning to full power the control rods are withdrawn even further than they were prior to the power reduction. Once the power level is restored, however, the additional xenon which accumulated during the power reduction will initially burn at a faster rate than it is being produced via the newly created iodine precursors. This, of course, results in a temporary increase in excess reactivity until the xenon poison once again attains the equilibrium concentration it had before the power reduction.

The relative amplitudes of the dips and peaks in the reactivity plots are dependent upon the amount as well as the duration of the power reductions. A simultaneous plot of reactor power is shown with each of the reactivity plots in the figures. Since the reactor power is plotted against integrated power, the duration of a power reduction is not deducible from the plot.

In several of the figures there are slight but apparent discontinuities in the slopes of the plots, eg, the plot in Figure 5 at approximately 3700 MWd. In most cases these occur where one set of control rods has reached its upper limit and the next set of rods is being withdrawn. The discontinuity is attributed to the fact that the actual control rod worths are different than the measured worths which were used to obtain the reactivity points. In some of the core loadings not all of the control rod fuel sections were refueled along with the rest of the core (Appendix B). Since the rod worth measurements in the ETRC were made with new fuel sections, there were probably some differences in rod worths. Also, it is quite probable that the rod worths in the ETR change somewhat during the operating cycle due to burnup in the core.

The excess reactivity plots illustrated in Figures 5 through 13 were specially selected for this report for two primary reasons: (a) the total operating time for each of the fuel loadings whose reactivity plot is illustrated was sufficiently long to permit a reasonably accurate extrapolation to determine the potential charge life and (b) the plots were smoother due to fewer shutdowns, experiment changes, and power reductions.

No excess reactivity plots were selected from fuel loadings which operated in the ETR prior to Cycle 26. When the ETR began operating in 1957, none of the in-pile tubes were installed; the spaces for these tubes were temporarily filled with aluminum filler pieces and extra fuel assemblies. The gradual loss in excess reactivity as more and more in-pile tubes were installed and fuel elements were displaced made it necessary to increase the U-235 content of the fuel assemblies from the initial 255 to 320 g and finally to the current 400 g. With this last change, which occurred just prior to Cycle 26, the ETR achieved a comparatively stable mode of operation. Concurrent with this phase of ETR operations, the similarity between ETR experiments and their ETRC mock-ups was greatly improved. Techniques for nuclear measurements in the ETRC also had improved considerably by that time, which resulted in more complete and accurate data on subsequent ETR core loadings. Consequently, none of the excess reactivity plots from core loadings prior to Cycle 26 were included in this report.

2. EVALUATION OF CONSTANTS IN EXCESS REACTIVITY EQUATION

The excess reactivity plots from the ETR provided a means for evaluating the constant coefficients R , A , B , and ρ_{eq} in Equation 6. Using the measured values of $\rho(0)_{total}$ and $\rho(0)_{boron}$ in each case, different values for the constants were tried in Equation 6 to arrive at a set of values which would make the curve of the reactivity equation agree reasonably well with the observed plots in all cases.

Soon after the ETR and ETRC began operating, it was determined that the initial excess reactivity held down by the control rods was consistently less in the ETR than in the ETRC. This apparent difference (averaging about $0.005 \Delta k/k$) has been attributed primarily to differences between the reflectors of the two reactors [9]. Consequently, $0.005 \Delta k/k$ was subtracted from the right side of Equation 6.

In fitting the curve of the reactivity equation to the observed plots, a value of $0.003 \Delta k/k$ was assigned to the constant term ρ_{temp} . A reactivity effect of this order of magnitude would be predicted using either the calculated or the measured temperature coefficient of reactivity for the ETR [12].

The following values were finally selected for the remaining constants in the reactivity equation:

$$\begin{aligned} R &= 3.15 \times 10^{-5} \Delta k/k \text{ per MWd} \\ A &= 6.0 \times 10^{-4} \Delta k/k \text{ per MWd} \\ B &= 7.0 \times 10^{-3} \Delta k/k \text{ per MWd} \\ \rho_{eq} &= 0.05 \Delta k/k. \end{aligned}$$

3. THE FINAL EXCESS REACTIVITY EQUATION

On substituting the numerical values for the constants in Equation 6, the final form of the excess reactivity equation for the ETR is

$$\begin{aligned} \rho(T)_{rods} &= \rho(0)_{total} - 3.15 \times 10^{-5}T - \rho(0)_{boron} \exp(-6 \times 10^{-4}T) \\ &\quad + 0.05 \exp(7 \times 10^{-3}T) - 0.058. \end{aligned} \quad (13)$$

On inserting the values for $\rho(0)_{total}$ and $\rho(0)_{boron}$ obtained from the ETRC mock-up of a particular ETR loading, Equation 13 gives the excess reactivity in the control rods as a function of power-time T in MWd for that loading.

4. THE CHARGE LIFE EQUATION

The charge life of the loading can be obtained directly without plotting the curve of Equation 13. It is simply the value of T when $\rho(T)_{rods}$ is equal to zero. Although Equation 13 cannot be solved for T explicitly, by rearranging and simplifying, it can be written in a form amenable to solution by iteration:

$$T_c = \frac{\rho(0)_{\text{total}} - \rho(0)_{\text{boron}} \exp(-6 \times 10^{-4} T_c) - 0.058}{3.15 \times 10^{-5}} \quad (14)$$

The fourth term on the right side of Equation 13 was neglected since it becomes negligible for T greater than 700 MWd. Since the exponential factor in Equation 14 is very small for charge lives of the order of 4000 MWd or more, T_c can be easily calculated with no more than two iterations.

5. COMPARISON OF CALCULATED AND OBSERVED CURVES OF EXCESS REACTIVITY

Comparison of the excess reactivity curves obtained using Equation 13 with the actual plots of excess reactivity obtained from the ETR are shown in Figures 5 through 13. In the earlier loadings the calculated curves tend to be slightly higher than the observed plots and, hence, overestimate charge lives by as much as 300 MWd. Since more accurate and complete information was obtained in the ETRC mock-ups of the more recent loadings, a greater emphasis was placed on these loadings in evaluating the constants in the reactivity equation. As a result, the calculated reactivity curves are in closer agreement with the excess reactivity plots for the latter loadings, and predict the actual charge life within 200 MWd or approximately five percent.

VI. DISCUSSION

In nearly all of the excess reactivity plots between 400 and 1500 MWd there is little or no apparent change in the reactivity held down by the control rods. In contrast, the curve of Equation 13 has no such flat region and after the initial reactivity loss due to fission product poisons, the curve tends to increase slightly for a period of time before falling off again. With the reactivity equation in the form developed in this report, it is impossible to make the curve of the equation flat in this region without adversely affecting the curve for larger values of T . A good fit of the reactivity curve to the experimental points in this latter region is necessary in order to obtain an accurate prediction of charge life.

The coefficient of T in the exponent of the fission product poisoning term in Equation 6 largely determines the rate of the initial rapid decrease in the curve of the reactivity equation. It also affects, to some extent, the difference between the amplitudes of the trough and crest of the curve. The value selected for this constant ($B = 7 \times 10^{-3} \Delta k/k$ per MWd) is a compromise. A lesser value would result in a smaller difference between the trough and crest and, hence, a flattened curve; but the amplitude at the crest would increase and the initial reactivity decrease would occur more slowly than in the observed plot. If a greater value were selected for the constant the effects would be reversed.

If it is assumed that the fission product poisoning was in equilibrium during the flat region of the observed reactivity plots, there would be but one explanation for the fact that there was little or no change in the reactivity held down by the control rods -- the opposing reactivity effects due to uranium and boron burnup must have been nearly equal in this region. In the reactivity equation this phenomenon could be described only if the functional forms for the time dependence of both ρ_{total} and ρ_{boron} were nearly alike for T less than about 1500 MWd. One possibility is that the boron in the fuel assemblies is not completely dispersed and some of the boron is self-shielded. This being the case, the simple exponential function used to describe the time dependence of the reactivity held down by the boron in the fuel assemblies (Equation 4) would not be valid. With a self-shielded burnable poison, the slope of the curve of a modified function would initially be less than in the simple exponential function and, in fact, would tend to be nearly linear for awhile. If the slope in this linear region equaled

the slope of the total excess reactivity curve in the same region, then the reactivity effects due to uranium and boron burnup would indeed be completely compensating. However, it is unlikely that such a perfect match would exist for the length of time indicated by the observed reactivity plots.

In the foregoing attempt to explain the flat region of the excess reactivity plots, it was assumed that the reactivity held down by fission product poisons reached a constant equilibrium value soon after start-up. If on the other hand, no equilibrium value exists and the reactivity change cannot, in fact, be described by a simple exponential function (as in Equation 5), then it may be possible that the flat region could be attributed to the accumulation of fission product poisons.

VII. REFERENCES

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APPENDIX A

DETAILS OF ETRC CORE COMPONENTS

Table I is a list of physical constants associated with the three basic core components in the ETRC: fuel assembly, control rod fuel section, and control rod gray section. Dimensional drawings of these components are shown in Figures 14, 15, and 16. All values in Table I are based on cross-sectional areas at the reactor midplane and a core height of 36 in. Values for the control rod fuel and poison sections include the control rod guide tube. All atom densities and macroscopic absorption cross sections were calculated assuming the constituent materials of the core components to be homogenized over the volume of one grid cell (3.04 x 3.04 x 36 in.). The atom density of a particular element is given by

$$N_i = \frac{\rho_i V_i}{V_T} \cdot \frac{N_O}{A_i} = \frac{M_i}{V_T} \cdot \frac{N_O}{A_i}$$

where ρ_i is the density of element i in g/cm³ at STP, V_T is the volume of an ETRC grid cell (5452 cm³), V_i is the volume occupied by element i in cm³, N_O is Avogadro's number (0.6023×10^{24}), A_i is the atomic weight in grams of element i , and M_i is the mass in grams of element i in the volume V_T . The atom density of hydrogen (in water) is calculated by first calculating the molecular density of water. Twice this number is the atom density of hydrogen.

The macroscopic thermal absorption cross sections given in Table I were calculated from the relation

$$\Sigma_a = N\sigma_a$$

where N is the atom density of the particular element in atom/cm³ and σ_a is the microscopic thermal absorption cross section in cm². The values given in the table for $\Sigma(235)$ also include a non- $\frac{1}{v}$ factor of 0.98.

TABLE I

PHYSICAL CONSTANTS FOR ETRC CORE

	Fuel Assembly	Control Rod Fuel Section (14 plates)	Control Rod Fuel Section (16 plates)	Control Rod Gray Section
<u>Nominal Contents</u>				
U-235	400 g	160 g	183 g	-----
Boron	2.3 g	0.92 g	1.05 g	-----
<u>Material Volumes</u>				
Grid Cell	5452 cm ³	5452 cm ³	5452 cm ³	5452 cm ³
Metal	2136 cm ³	3034 cm ³	2901 cm ³	-----
Water	3316 cm ³	2418 cm ³	2551 cm ³	2922 cm ³
Metal/Water ratio	0.6441	1.255	1.137	-----
Aluminum	-----	-----	-----	1340 cm ³
Nickel	-----	-----	-----	1190 cm ³
<u>Atom Densities</u>				
U-235	0.0188×10^{22} atoms/cm ³	0.00752×10^{22} atoms/cm ³	0.00361×10^{22} atoms/cm ³	-----
U-238	0.0013×10^{22} "	0.00053×10^{22} "	0.00061×10^{22} "	-----
Boron	0.00235×10^{22} "	0.00094×10^{22} "	0.00107×10^{22} "	-----
Aluminum	2.36×10^{22} atoms/cm ³	3.35×10^{22} atoms/cm ³	3.21×10^{22} atoms/cm ³	1.481×10^{22} atoms/cm ³
Water	2.03×10^{22} "	1.49×10^{22} "	1.57×10^{22} "	1.794×10^{22} "
Hydrogen	4.07×10^{22} "	2.97×10^{22} "	3.13×10^{22} "	3.588×10^{22} "
Nickel	-----	-----	-----	1.994×10^{22} "
Cobalt	-----	-----	-----	0.004×10^{22} "
<u>Macroscopic Absorption</u>				
<u>Cross Sections (2200 m/sec)</u>				
Σ (235)	0.1279 cm^{-1}	0.0511 cm^{-1}	0.0586 cm^{-1}	-----
Σ (B)	0.0179 cm^{-1}	0.0071 cm^{-1}	0.0081 cm^{-1}	-----
Σ (Al)	0.0054 cm^{-1}	0.0077 cm^{-1}	0.0074 cm^{-1}	0.003 cm^{-1}
Σ (H)	0.0135 cm^{-1}	0.0099 cm^{-1}	0.0104 cm^{-1}	0.012 cm^{-1}
Σ (H ₁)	-----	-----	-----	0.091 cm^{-1}
Σ (Co)	-----	-----	-----	0.001 cm^{-1}
Σ (Total)	0.1647 cm^{-1}	0.0758 cm^{-1}	0.0845 cm^{-1}	0.107 cm^{-1}
$\Delta\Sigma$ (235)	$0.00032 \text{ cm}^{-1}/\text{g}$	$0.00032 \text{ cm}^{-1}/\text{g}$	$0.00032 \text{ cm}^{-1}/\text{g}$	-----
$\Delta\Sigma$ (B)	$0.00778 \text{ cm}^{-1}/\text{g}$	$0.00778 \text{ cm}^{-1}/\text{g}$	$0.00778 \text{ cm}^{-1}/\text{g}$	-----

Fig. 15 Control rod fuel section and guide tube.

Fig. 16 Control rod gray section and guide tube.

The following microscopic absorption cross sections were used in computing the cross sections in Table I [*].

$$\sigma_a(235) = 694.0 \text{ b}$$

$$\sigma_a(238) = 2.710 \text{ b}$$

$$\sigma_a(\text{Al}) = 0.230 \text{ b}$$

$$\sigma_a(\text{H}) = 0.332 \text{ b}$$

$$\sigma_a(\text{B}) = 755.0 \text{ b}$$

$$\sigma_a(\text{N}_i) = 4.8 \text{ b}$$

$$\sigma_a(\text{Co}) = 37.0 \text{ b}$$

All other elements or impurities (eg, O, Mn, Mg, etc) have been neglected since their cross sections and/or masses are negligible when calculating Σ_a .

The value given for $\Sigma(\text{N}_i)$ is an effective cross section having been reduced five percent from the calculated homogenized value to account for the more heterogeneous geometry of the gray section.

The factors $\Delta\Sigma(235)$ and $\Delta\Sigma(\text{B})$ have been included as an aid in adjusting $\Sigma(235)$ and $\Sigma(\text{B})$, respectively, when the U-235 and boron contents differ from the nominal contents given.

The 14-plate 160-g control-rod fuel sections were used in the ETR through Cycle 43. Subsequent cycles have all used the 16-plate 183-g control-rod fuel sections.

[*] D. J. Hughes, R. B. Schwartz, Neutron Cross Sections, Second Edition BNL-325 (1958).

APPENDIX B

ETR CORE CONFIGURATIONS

The following pages contain diagrams of the ETR core configurations referred to in this report. The legend for the core diagram in Figure 2 also applies to the core diagrams shown in this appendix. Supplementary information on each core also is included.

ETR Cycle 27 Core Loading (Figure 17)

The Cycle 27 core loading contained fifty-one 400-g fuel assemblies. The natural boron content of these assemblies ranged from 2.00 to 2.46 g with a flux-weighted average of 2.17 g.

The control rod fuel sections contained 160-g U-235 with the natural boron content ranging from 0.70 to 0.96 g.

The control rod withdrawal sequence used during Cycle 27 as well as pertinent control rod reactivity worths measured in the ETRC were as follows:

Fig. 17 ETR Cycle 27 core loading.

<u>Control rod withdrawal step</u>	<u>Control rods withdrawn</u>	<u>Reactivity worth</u>
1	1, 2, 3, 4	
2	10	
3	13	
4	12	
5	5, 11, 15	0.042 $\Delta k/k$
6	7, 9, 14	0.054 $\Delta k/k$
7	6, 8, 16	0.020 $\Delta k/k$

The following additional reactivity data were obtained from the ETRC for the Cycle 27 loading:

$$\rho(0)_{\text{total}} = 0.219 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.104 \Delta k/k$$

ETR Cycle 28 Core Loading (Figure 18)

The Cycle 28 core loading contained forty-six 400-g fuel assemblies and five 320-g fuel assemblies, the latter being denoted in the diagram by asterisks (*). The natural boron content of these assemblies ranged from 1.80 to 2.16 g with a flux-weighted average of 2.06 g.

Control rods 6, 8, and 16 were not refueled since they had accumulated little or no burnup during the preceding cycle. All control rod fuel sections initially contained 160-g U-235 with the natural boron content ranging from 0.73 to 0.93 g.

The control rod withdrawal sequence used during Cycle 28 as well as pertinent control rod reactivity worths measured in the ETRC were the same as in Cycle 27.

The following additional reactivity data were obtained from the ETRC for the Cycle 28 loading:

$$\rho(0)_{\text{total}} = 0.213 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.098 \Delta k/k$$

ETR Cycle 30 Core Loading (Figure 19)

The Cycle 30 core loading contained fifty 400-g fuel assemblies. The natural boron content of these assemblies ranged from 2.03 to 2.51 g with a flux-weighted average of 2.40 g.

Control rods 6, 8, 9, 12, 15, and 16 were not refueled since they had accumulated only a small amount of burnup in the preceding cycle. All control

Fig. 18 ETR Cycle 28 core loading.

Fig. 19 ETR Cycle 30 core loading.

ETR Cycle 32B Core Loading (Figure 20)

The Cycle 32B core loading contained forty-nine 400-g fuel assemblies and one special 400-g assembly without boron in the position denoted with an asterisk (*) on the diagram. The natural boron content of the other fuel assemblies ranged from 1.91 to 2.13 g with a flux-weighted average of 2.05 g.

All control rod fuel sections contained 160-g U-235 with the natural boron content ranging from 0.73 to 0.91 g.

The control rod withdrawal sequence used during Cycle 32B as well as pertinent control rod reactivity worths measured in the ETRC were as follows:

<u>Control rod withdrawal step</u>	<u>Control rods withdrawn</u>	<u>Reactivity worth</u>
1	1, 2, 3, 4	
2	10	
3	13	

rod fuel sections initially contained 160-g U-235 with the natural boron content ranging from 0.73 to 0.84 g.

The control rod withdrawal sequence used during Cycle 30 was the same as in Cycle 27. The control rod reactivity worths also were estimated to be the same.

The following additional reactivity data were obtained from the ETRC for the Cycle 30 loading:

$$\rho(0)_{\text{total}} = 0.206 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.111 \Delta k/k$$

Fig. 20 ETR Cycle 32B core loading.

<u>Control rod withdrawal step</u>	<u>Control rods withdrawn</u>	<u>Reactivity worth</u>
4	12	0.004 $\Delta k/k$
5	7, 11, 15	0.047 $\Delta k/k$
6	5, 9, 14	0.035 $\Delta k/k$
7	6, 8, 16	0.026 $\Delta k/k$

The following additional reactivity data were obtained from the ETRC for the Cycle 32B loading:

$$\rho(0)_{\text{total}} = 0.208 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.100 \Delta k/k$$

ETR Cycle 33A, B, and C Core Loading (Figure 21)

The Cycle 33A, B, and C core loading contained fifty 400-g fuel assemblies. The natural boron content of these assemblies ranged from 2.18 to 2.60 g with a flux-weighted average of 2.45 g.

Control rods 6, 8, and 16 were not refueled since they had accumulated little or no burnup during the preceding cycle. All control rod fuel sections initially contained 160-g U-235 with the natural boron content ranging from 0.91 to 0.94 g.

The control rod withdrawal sequence and reactivity worths were the same as in Cycle 32B.

The following additional reactivity data were obtained from the ETRC for the Cycle 33 loading:

$$\rho(0)_{\text{total}} = 0.198 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.118 \Delta k/k$$

Fig. 21 ETR Cycle 33A, B, and C core loading.

ETR Cycle 41B Core Loading
(Figure 22)

The Cycle 41B core loading contained fifty-one 400-g fuel assemblies. The natural boron content of these assemblies ranged from 2.01 to 2.60 g with a flux-weighted average of 2.45 g.

Thirteen of the control rods were not refueled since they had accumulated only a small amount of burnup. All control rod fuel sections initially contained 160-g U-235 with the natural boron content averaging about 0.95 g.

Fig. 22 ETR Cycle 41B core loading.

The control rod withdrawal sequence used during Cycle 41B as well as pertinent control rod reactivity worths measured in the ETRC were as follows:

<u>Control rod withdrawal step</u>	<u>Control rods withdrawn</u>	<u>Reactivity worth</u>
1	1, 2, 3, 4	
2	10	
3	13	
4	12	0.004 $\Delta k/k$
5	7, 11, 15	0.045 $\Delta k/k$
6	5, 9, 14	0.0345 $\Delta k/k$
7	6, 8, 16	0.0245 $\Delta k/k$

The following additional data were obtained from the ETRC for the Cycle 41B loading:

$$\rho(0)_{\text{total}} = 0.213 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.119 \Delta k/k$$

ETR Cycle 44 Core Loading (Figure 23)

The Cycle 44 core loading contained fifty-one 400-g fuel assemblies. The natural boron content of these assemblies ranged from 1.56 to 2.84 g with a flux-weighted average of 2.28 g.

Control rods 6, 7, 9, 14, and 16 were not refueled since they had accumulated only a small amount of burnup during the preceding cycle. The fuel sections of these control rods initially contained 160-g U-235 and an average of 0.85 g of natural boron. All other control rod fuel sections contained 183-g U-235 with the natural boron content ranging from 0.55 to 1.06 g.

The control rod withdrawal sequence used during Cycle 44 as well as pertinent control rod reactivity worths measured in the ETRC were as follows:

Fig. 23 ETR Cycle 44 core loading.

<u>Control rod withdrawal step</u>	<u>Control rods withdrawn</u>	<u>Reactivity worths</u>
1	1, 2, 3, 4	
2	10	
3	13	
4	12	0.005 $\Delta k/k$
5	7, 11, 15	0.047 $\Delta k/k$
6	5, 9, 14	0.036 $\Delta k/k$
7	6, 8, 16	0.026 $\Delta k/k$

The following additional data were obtained from the ETRC for the Cycle 44 loading:

$$\rho(0)_{\text{total}} = 0.225 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.109 \Delta k/k$$

ETR Cycle 45 Core Loading (Figure 24)

The Cycle 45 core loading contained fifty-one 400-g fuel assemblies. The natural boron content of these assemblies ranged from 1.43 to 3.20 g with a flux-weighted average of 2.57 g.

Control rods 6, 8, and 16 were not refueled since they had accumulated little or no burnup during the preceding cycle. The fuel sections on these three control rods contained 160-g U-235 with an average natural boron content of about 0.93 g. All other control rod fuel sections contained 183-g U-235 with the natural boron content ranging from 0.52 to 0.71 g.

The control rod withdrawal sequence and reactivity worths were the same as in Cycle 44.

The following additional reactivity data were obtained from the ETRC for the Cycle 45 loading:

$$\begin{aligned}\rho(0)_{\text{total}} &= 0.231 \Delta k/k \\ \rho(0)_{\text{boron}} &= 0.118 \Delta k/k\end{aligned}$$

ETR Cycle 46A and B Core Loading (Figure 25)

The Cycle 46A and B core loading contained fifty-one 400-g fuel assemblies. The natural boron content of these assemblies ranged from 1.10 to 2.90 g with a flux-weighted average of 2.51 g.

Control rods 5, 6, 8, 9, 14, and 16 were not refueled since they had accumulated only a small amount of burnup during the preceding cycle. With the exception of rod 5, the fuel sections of these control rods initially contained 160-g

U-235 and an average of 0.85 g of natural boron. All other control rod fuel sections initially contained 183-g U-235 with the natural boron content ranging from 0.50 to 0.77 g.

Fig. 24 ETR Cycle 45 core loading.

Fig. 25 ETR Cycle 46A and B core loading.

The control rod withdrawal sequence and reactivity worths were the same as in Cycle 45.

The following additional reactivity data were obtained from the ETRC for the Cycle 46 loading:

$$\rho(0)_{\text{total}} = 0.231 \Delta k/k$$

$$\rho(0)_{\text{boron}} = 0.116 \Delta k/k$$

An experiment change between Cycle 46A and 46B resulted in a loss in total excess reactivity of $0.004 \Delta k/k$.

APPENDIX C

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REACTIVITY DETERMINATIONS IN THE ETRC BY THE DISTRIBUTED POISON METHOD

(J. B. Willis)

Several different forms of the equation involved in the distributed poison method of determining reactivity have been used at this laboratory. It is the purpose of this discussion to show in part how some of these forms have been derived and to derive a continuous function which will more accurately predict large values of reactivity. Table C-1 shows the values of reactivity predicted by these various forms for a range of critical core cross sections and the respective poison cross sections commonly encountered in ETRC Critical Facility (ETRC) measurements.

TABLE C-1

RESULTS OF REACTIVITY CALCULATIONS FOR A TYPICAL ETRC CORE
BY VARIOUS DISTRIBUTED POISON FORMULAS

$\rho = \frac{\Sigma_p}{\Sigma_{core} - \Sigma_p}$ % $\Delta k/k$	$\rho = \frac{\Sigma_p}{(\Sigma_a + \Sigma_p) - \Sigma_{core}}$ % $\Delta k/k$	$\rho = \frac{\Sigma_p}{\Sigma_{core} - 0.5 \Sigma_p}$ % $\Delta k/k$	$\rho = \log_e \frac{\Sigma_{core}}{\Sigma_{core} - \Sigma_p}$ % $\Delta k/k$	Poison Cross Section $\Sigma_p, \text{ cm}^{-1}$	Critical Core Cross Section Including Poison $\Sigma_{core}, \text{ cm}^{-1}$
4.64	4.41	4.51	4.51	0.00671	0.15221
8.33	7.69	8.00	8.00	0.01201	0.15613
10.76	9.71	10.21	10.22	0.01536	0.15811

All excess reactivity measurements in the ETRC are made using the method of distributed poison. Measurements of large reactivities (≈ 10 percent $\Delta k/k$) are commonly encountered in conjunction with ETR core loadings. Therefore it is mandatory that any inherent limitations of the distributed poison method be evaluated which might lead to errors when the method is applied to large reactivity worths.

A change in the amount of poison in a reactor core will cause a change in the multiplication of neutrons. The four-factor formula describes the multiplication of neutrons in a bare reactor of infinite dimensions. It can be modified to describe the effective multiplication factor for the ETRC, a finite, reflected

reactor, by multiplying the infinite multiplication factor by a fast neutron nonleakage probability L_f and a thermal neutron non-leakage probability L_t , giving:

$$k_{\text{eff}} = \eta \epsilon p f L_t L_f. \quad (1)$$

The equation of the distributed poison method, $\rho = -\Delta\Sigma/\Sigma_c$, applicable to reactivity measurements in the ETRC, can be derived from the modified four-factor formula. In this equation Σ_c is the macroscopic absorption cross section of a critical core and ρ is the reactivity when a small amount of poison of macroscopic absorption cross section $\Delta\Sigma$ is either removed from or added to the critical core. The derivation of the distributed poison equation is based on the assumption that small uniform changes in the amount of poison in a critical core will affect only the thermal utilization factor.

The values for the parameters η and ϵ of the modified four-factor formula are generally fixed by the character of the fuel in a given core loading; but, the remaining parameters may vary to a greater or lesser extent as the poison content of the core is perturbed. For this reason, the above assumption restricts the thermal utilization factor to small changes, or else the resonance escape probability and the nonleakage probabilities change.

The distributed poison formula also can be derived from a two group treatment of the differential neutron balance equations. In this manner, McMurry and Metcalf [1] have shown that the distributed poison formula has the form

$$\rho = \frac{\Sigma_p}{\Sigma_a + \Sigma_p} \quad (2)$$

where ρ is the reactivity after a core of macroscopic absorption cross section at criticality, $\Sigma_a + \Sigma_p$, has been perturbed by the removal of an amount of poison cross section Σ_p . It is easily seen that for the removal of a small amount of poison this relationship is equivalent to the distributed poison formula derived from the modified four-factor formula.

The distributed poison method, including the use of the poison formula, is demonstrated in the following hypothetical determination of the total worth of a ganged set of ETRC control rods in a given core configuration. Consider

core loading, which is critical with a set of control rods, whose worth is to be determined, fully inserted. The total worth of the set will be found by adding the incremental worths measured during the following stepwise withdrawal:

- (1) The core is uniformly poisoned with borated polyethylene tapes.
- (2) The ganged set of control rods is withdrawn until the reactor is critical.
- (3) The macroscopic absorption cross section Σ_{a1} of the core for the final critical state is computed. This cross section includes the cross sections of all the distributed boron tapes.
- (4) The macroscopic absorption cross section Σ_{p1} is computed for all the distributed poison in the core.
- (5) The reactivity associated with the withdrawn segment of the ganged set of control rods is calculated from the ratio of Σ_{p1}/Σ_{a1} .
- (6) Steps 1 through 5 are repeated with the exception that the macroscopic absorption cross section Σ_{p2} is used in the second ratio Σ_{p2}/Σ_{a2} where Σ_{p2} is the total of the cross sections due to the second distributed group of borated tapes.
- (7) The above procedure is repeated until the ganged set of control rods is calibrated over its entire length.
- (8) The total worth of the set of control rods is the sum of the worths of the segments,

$$\rho_T = \sum_{i=1}^N \rho_i \quad (3)$$

This procedure of calibrating a ganged set of control rods and adding up the resultant incremental reactivities is analogous to the method of calibrating the reactivity of a shim rod by positive period measurements, relating these periods to respective reactivities by the inhour equation, and adding up all the reactivities to get the total worth of the rod. Experimental evidence from the SPERT projects has indicated that reactivities are additive in measurements of this type [2].

The practice of adding reactivities can be formalized with a resultant mathematical refinement in the case of the distributed poison method. In the limit of the sum of the infinitesimal reactivity increments, the distributed poison formula takes the form:

$$\rho_T = \lim_{N \rightarrow \infty} \sum_{i=1}^N \rho_i = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{\Sigma_{p_i}}{\Sigma_{a_i} + \Sigma_{p_i}} = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{d\Sigma_i}{\Sigma_{core_i}} \quad (4)$$

It is not possible rigorously to pass from the limit of the sum to an integral because $d\Sigma_i$ is the distributed poison variable while Σ_{core_i} is not only a function of the varying distributed poison but also is a function of the fuel and other poison in the core, and these materials are varying because the control rods are being withdrawn. However, if the core is imagined to be distorted throughout the calibration, ie, if the variation in macroscopic absorption core cross section due to the moving control rods is neglected and the control rods are imagined to be fully inserted during the calibration, then it is possible to pass from the limit of the sum to an integral and obtain the following expression for the total worth of the ganged control rods:

$$\rho_T = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{d\Sigma_i}{\Sigma_i} = \int_{\Sigma_{core \text{ final}} - \Sigma_{poison \text{ total}}}^{\Sigma_{core \text{ final}}} \frac{d\Sigma}{\Sigma} \quad (5)$$

or

$$\rho_T = \log_e \frac{\Sigma_{core \text{ final}}}{\Sigma_{core \text{ final}} - \Sigma_{poison \text{ total}}} \quad (6)$$

Since the control rods are imagined to be fully inserted during the calibration, they are to be treated as fully inserted when the macroscopic absorption cross section of the core in its final critical state is computed. Alternatively, it could be imagined that the core is distorted throughout the calibration with the control rods fully withdrawn. If this approximation is used, the total worth of the ganged control rods is given by the same expression as above, but the macroscopic cross section of the core in the final critical state is computed with the control rods fully withdrawn. When a reactivity worth of ≈ 6 percent $\Delta k/k$ is calculated for the two cases of distortion, the results differ by ≈ 3 percent.

The steps involved in determining the total worth of a ganged set of control rods may in practice be greatly reduced if the logarithmic approximation is used since the reactivity has been shown above to be a function of the cross section of the core in its final critical state and the cross section of the distributed poison. Past calculations of reactivities in the ETRC have assumed a distorted core. The functional dependence of reactivity on macroscopic cross section was of the form

$$\rho = \frac{\Sigma_p}{\Sigma_{\text{core final}} - \frac{1}{2}\Sigma_p} \quad (7)$$

and had been derived from the sum of a finite number of incremental reactivities

$$\rho_T = \sum_{i=1}^N \frac{\Sigma_{p_i}}{\Sigma_{\text{core initial}}} \quad (8)$$

A graph of ρ vs Σ (Figure 26) can be drawn when the distortion principle is used in the calculation of reactivities. It illustrates why the previous dependence of reactivity on macroscopic absorption cross section closely approximates the logarithmic function. The first approximation to the logarithmic function over the interval Σ_p is the evaluation of the slope of the curve at the midpoint of the interval Σ_p , ie,

$$\rho = \frac{\Sigma_p}{\Sigma_M} \quad (9)$$

or

$$\rho = \frac{\Sigma_p}{\Sigma_{\text{core initial}} + \frac{1}{2}\Sigma_p} = \frac{\Sigma_p}{\Sigma_{\text{core final}} - \frac{1}{2}\Sigma_p} \quad (10)$$

Several calculations have shown that the slope $1/\Sigma$ of the logarithmic function is very insensitive to changes in Σ encountered in typical ETRC measurements. This first approximation differs from the logarithmic function by less than 0.5 percent.

Much of the ETRC operational work is accomplished through the use of distributed poisons. For this reason it is convenient to have a rule-of-thumb guide in relating distributed poison to reactivity. It has been determined that 12.3 g of natural boron uniformly distributed in the ETRC core is equivalent

Fig. 26 Comparison of the logarithmic function of the distributed poison method and the first approximation of the logarithmic.

to ≈ 1 percent $\Delta k/k$ for a wide range of core configurations made up of 400-g fuel elements and numerous poison sections. The general applicability to many core configurations of this operational guide stems from the insensitivity of the slope of the logarithmic function to changes in the total core cross section Σ . However, the graph shows that as more poison is added to a core, the effect of the poison tends to decrease.

The prime object of this work has been to develop a consistent procedure for measuring reactivities in which the limitations are clearly known and also to derive a continuous function that would relate cross section to reactivity. Further work is planned to determine the limiting amount by which the thermal utilization factor may be changed without appreciably affecting the resonance escape probability and the nonleakage probabilities of the effective multiplication factor.

2. REFERENCES

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