AIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

Inches

MANUFACTURED TO AIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

DOE/BC/14899-15
Distribution Category UC-122

SCALING OF BUBBLE GROWTH IN A POROUS MEDIUM

Topical Report

by
C. Satik
X. Li
Y.C. Yortsos

September 1994

Work Performed Under Contract No. DE-FG22-93BC14899

Prepared for
U.S. Department of Energy
Assistant Secretary for Fossil Energy

Thomas B. Reid, Project Manager
Bartlesville Project Office
P.O. Box 1398
Bartlesville, OK 74005

Prepared by
University of Southern California
Department of Chemical Engineering
Los Angeles, CA 90089-1211

MASTER

ds
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Table of Contents

	<u>Page No.</u>
Abstract.....	1
Acknowledgment	15
References.....	15

List of Figures

<u>Figure No.</u>	<u>Page No.</u>
1 Gas cluster growth from carbonated water in glass micromodel.....	3
2 Gas cluster growth from carbonated water in a Hele-Shaw cell.....	5
3 Total net flux to a percolation cluster under quasi-static diffusion. Solid line is the theoretical slope.	5
4 Percolation and viscous fingering boundaries from simulations in a 50 x 50 square lattice. The solid line is a guide to the eye.....	11
5 Typical sequence of gas cluster growth under conditions of (a) capillary control ($Ca = 0.00001$) and (b) viscous control ($Ca = 0.1$).....	12

ABSTRACT

The growth of a single gas cluster in a porous medium driven by a constant supersaturation is examined. Patterns and rates of growth are derived. It is shown that the growth pattern is not compact and changes from pure percolation to pure Diffusion-Limited-Aggregation (DLA) as the size of the cluster increases. The scaling of the cluster sizes that delineate these patterns, with supersaturation and diffusivity is presented for the case of quasi-static diffusion. In 3-D the diffusive growth law is found to be $R_g \sim t^{\frac{2}{3}}$, which is different than the classical $R_g \sim t^{\frac{1}{2}}$.

Processes involving liquid-to-gas phase change in porous media are routinely encountered. Growth of a gas phase by solute diffusion in the liquid is typical of the "solution gas-drive" process for the recovery of oil¹. Growth of a vapor phase by heat conduction (boiling in porous media) is encountered in diverse applications, such as geothermal processes, thermal methods for oil recovery, nuclear waste disposal or enhanced heat transfer². Either process involves diffusion (and convection) in the pore space, driven by an imposed supersaturation in pressure or temperature, respectively.

Phase change proceeds by the two mechanisms of nucleation and phase growth. Heterogeneous nucleation, where microbubbles constrained in surface cavities grow at a cavity-specific supersaturation, dominates the onset of the phase change in porous media (at least at the observable stage³). Depending on pore surface roughness, a number of nucleation centers exist, thus phase growth occurs from a multitude of separate clusters. Contrary to growth in the bulk⁴ or in a Hele-Shaw cell⁵, however, bubble growth patterns in porous media are disordered and not compact. Surprisingly, this distinction was made only recently⁶. As in related immiscible displacement processes⁷, these patterns reflect the underlying pore microstructure. In single-bubble growth in random media we expect percolation-like regimes at small supersaturations and a transition to non-local regimes when percolation conditions are exceeded. The competition between multiple clusters is also different from the bulk. For example, clusters may grow in a diffusion field controlled by the Laplace equation (for the case of quasi-static diffusion), while percolation rules apply locally. Novel growth patterns are expected from this competition⁶. Furthermore, because clusters in porous media have the curvature of the pore geometry at the cluster perimeter instead of that of the equivalent, spherical cluster, Ostwald ripening⁸ is expected to be much less important than in the bulk.

While multiple cluster growth is important, an understanding of the simpler problem of single-bubble growth is presently lacking. Early studies were restricted to effects of nucleation⁹ or to theoretical models based on effective compact bubbles¹⁰. Recently¹¹, we have considered porous media effects by using a percolation approach for multiple cluster growth from distributed nucleation sites. The theory aimed at the understanding of nucleation effects at conditions of very small supersaturations, where capillarity predominates, and transport effects were not considered.

(a) Large scale sequence

(b) Pore scale sequence

Figure 1. Gas cluster growth from carbonated water in a glass micromodel.

In fact, that problem can be considered as a prototypical example of ordinary percolation, extended to account for growth from multiple centers¹².

To understand the effect of transport at larger supersaturations we focus here on the growth of a single bubble. The two relevant applications, growth by solute diffusion in a binary liquid, only one component of which is volatile, and by heat conduction in a single-component liquid, in the absence of solid conduction, will be uniformly treated. We consider a fixed far-field supersaturation (e.g. by lowering the pressure in a saturated solution or by raising the temperature in a superheated liquid, in the two applications, respectively). Other possibilities, are discussed elsewhere⁶. The particular emphasis is on developing a scaling theory for growth at conditions of quasi-static diffusion, guided by recent experimental observations.

Visualization of bubble growth in model porous media was recently conducted¹³ using 2-D etched-glass micromodels. Fig. 1a shows a typical CO_2 cluster evolving from carbonated water, initially saturated at 50 *psi*, the pressure of which was subsequently reduced to 14.7 *psi*. The non-compact nature of the cluster is clearly apparent. The cluster was found to grow in a manner similar to external drainage: gas-liquid interfaces were stable as they slowly advanced in the converging portion of a pore throat, but rapidly moved to occupy an adjacent pore body, in an event known as a "rheon"⁷, once the capillary pressure barrier of that throat was exceeded (Fig. 1b). During the early stages, the growth occurred "one-site-at-a-time", which is a mode of interface advance typical of invasion percolation. This motion was much slower than in typical external displacement experiments, however, being controlled here by mass transfer to the growing clusters. Similar events, although at a much faster speed, where inertia effects may also be relevant, were observed in related experiments¹⁴, where the boiling of ethanol in a glass micromodel was studied. By contrast, bubble growth under the same conditions in a Hele-Shaw cell showed compact growth patterns (Fig. 2).

Figure 2 Gas cluster growth from carbonated water in a Hele-Shaw cell

Figure 3. Total net flux to a percolation cluster under quasi-static diffusion. Solid line is the theoretical slope.

To model this process, we followed two approaches, an elaborate pore network simulation of multiple cluster growth, the details of which are given elsewhere¹³, and a scaling theory of single-bubble growth at low supersaturations, where mass transfer is diffusion-controlled (quasi-static limit), as described below. The particular problem pertains to the initial conditions $C = C_0$, $P_l = P_0$, where C denotes solute concentration and P_l liquid pressure, and the far-field conditions $C = C_0$, $P_l = P_\infty < P_0 \equiv KC_0$, where the solubility constant K was defined (see also below). Equivalently, the applied supersaturation can be expressed with the dimensionless Jacob number, $Ja = \frac{P_0 - P_\infty}{P_0}$. The concentration field in the liquid is quasi-static at low supersaturations ($Ja \ll 1$) (see also below) and satisfies the Laplace equation $D\nabla^2 C = 0$, where D is the solute diffusivity. It is this limit that is taken in this paper.

We consider a model of the porous medium in terms of an equivalent network of bonds (throats) and sites (pores) and we associate transport with bonds and storage (concentrations) with sites. The gas-liquid interface consists of menisci residing on the perimeter sites of the gas cluster. We take the gas to be inviscid and ideal, such that P_g is spatially uniform and $P_g V = nRT$, where n is the number of moles contained in the gas bubble (cluster) of volume V . For simplicity, we take linear phase equilibria, $P_g = KC_i$, in all perimeter sites i ($i = 1, N$). Bubble growth is dictated by the net mass transfer rate to all perimeter interfaces (whether moving or stationary) $\frac{dn}{dt} = \sum_{i,j} J_{ij}$, where the sum is over all perimeter sites i and all liquid-occupied sites j adjacent to them. The diffusive flux J_{ij} is $J_{ij} = DA_{ij} \frac{(C_i - C_j)}{l_{ij}}$, where the area A_{ij} and the length l_{ij} pertain to bond ij . The mass influx, $\frac{dn}{dt} > 0$, may result into either pressurization of the gas phase ($dP_g > 0$) or bubble growth ($dV > 0$). Pressurization is necessary to overcome the capillary pressure barrier of a perimeter bond and for the bubble to grow. This occurs when the capillary pressure across the bond is sufficiently large

$$P_g - P_{l,j} > \frac{2\gamma}{r_{p,ij}} \quad (1)$$

where $r_{p,ij}$ is the radius of the connecting bond and γ is the interfacial tension. Upon penetration, the interface advances and occupies site j . A measure of the diffusion driving force is the capillary number, here expressed as $Ca = \frac{l^* \mu_l Ja D}{\gamma k}$, where l^* is a characteristic lattice spacing, k is the permeability of the medium (which scales with the average pore

throat size¹⁵ r_p^* as $k \sim r_p^{*2}$) and we have defined a characteristic velocity based on diffusion $u^* = \frac{l^* J a D}{k}$. Note the difference with the expression for Ca in external displacements⁷. An additional number of relevance here is the solubility constant $\alpha = \frac{RT}{M_w K}$, where M_w is the solute molecular weight.

To characterize bubble growth in a porous medium requires that the two aspects of cluster pattern and its rate of growth to be determined. We recall that growth in the bulk or in an effective porous medium is compact and obeys the similarity scaling^{4,5} $R \sim t^{1/2}$. By contrast, in a random medium we expect a percolation pattern at sufficiently small cluster sizes and a departure towards a viscous fingering pattern at larger sizes.

1. Growth Patterns

• Percolation

The cluster will follow a percolation pattern, the characteristics of which are well known⁷, if perimeter bonds are invaded "one-at-a-time", such that the largest available bond is always invaded first. These rules are the same with invasion percolation in drainage, except that here invasion occurs from an internal, rather than an external source. Of key interest to our study is the delineation of the pattern limits. The following conditions must apply for a pure percolation pattern: 1. Immediately preceding and during the pressurization step, all interfaces reside in converging pore geometries, where they are stable. 2. During the filling step, only "one-site-at-a-time" is invaded, the simultaneous penetration of another throat not being possible.

The first condition is always satisfied during pressurization, since both liquid and gas pressures are spatially uniform (no flow). The validity of the second condition depends on the viscous pressure drop. During the filling of a partly occupied site l , the capillary pressure in the site is small, $P_g \sim P_{l,n}$. Simultaneous penetration of a perimeter bond at another location m would not be possible only if $P_g - P_{l,m} \leq \frac{2\gamma}{r_{p,im}}$, namely only if the difference between the pressures of the two sites is sufficiently small. Delineation of the percolation regime requires an estimate of the viscous pressure drop, which in this problem is directly related to mass transfer. We shall denote by R^p the radius of gyration of the cluster when this limit is reached and proceed for its estimation as follows.

We, first, note that during percolation growth occurs from one site only (say m) (although

mass transfer to the bubble is from all perimeter sites), hence the pressure field is set by the velocity of that site. The latter can be estimated from mass balance considerations

$$v_m \sim \frac{RT}{bP_\infty} \sum_{i,j} J_{ij} \quad (2)$$

where we introduced the geometrical length b , such that $b = 2\pi h r_s^*$ in 2-D, where h is the thickness of the system and r_s^* is a typical site radius (which was assigned to site m), and $b = 4\pi r_s^{*2}$ in 3-D. To calculate the mass flux, we must solve a problem of quasi-static diffusion in a Euclidean space, bounded internally by a fractal interface. Following Ref[6], the overall flux can be obtained from the mean-field solution, where the fractal cluster is replaced by a compact cluster of equivalent radius, R^p . We have

$$\sum_{i,j} J_{ij} \sim 2\pi h D R^p \left. \frac{\partial C}{\partial r} \right|_{r=R^p} = \frac{2\pi h D (C_0 - C_i)}{\ln \frac{R_s^*}{R^p}} \quad \text{or} \quad \sum_{i,j} J_{ij} \sim 4\pi D (C_0 - C_i) R^p \quad (3)$$

in the two geometries, respectively, where R_s denotes the outer boundary in 2-D. The scaling of the velocity follows by substitution of the Jacob number to yield

$$v_m \sim \left(\frac{\alpha J a D}{r_s^*} \right) \frac{1}{\ln \frac{R_s^*}{R^p}} \quad \text{or} \quad v_m \sim \left(\frac{\alpha J a D}{r_s^*} \right) \left(\frac{R^p}{r_s^*} \right) \quad (4)$$

in the two geometries, respectively. For a conservative estimate of the percolation limit we calculate the viscous pressure drop across a distance that scales with the cluster size, R^p . We take Poiseuille's law to describe viscous flow resistance across each pore, which is equivalent to discretizing Darcy's law in the liquid-occupied region. The fluid flow problem involves solving the Laplace equation subject to flow at the growth site m given from (4). Because the cluster is fractal, while flow occurs over a Euclidean space, we will use a mean field approach, as before. A similar approach was also followed by Lenormand¹⁷ in the external displacement problem. In 2-D geometries, the pressure obeys a logarithmic profile, from which the viscous pressure drop can be obtained

$$\Delta P_v \sim \frac{v_m \mu l r_s^*}{k} \ln \frac{R^p}{r_s^*} \quad (5)$$

In 3-D geometries, where spherical flow prevails, the profile is algebraic and we obtain

$$\Delta P_v \sim \frac{v_m \mu l r_s^{*2}}{k} \left(\frac{1}{r_s^*} - \frac{1}{R^p} \right) \sim \frac{v_m \mu l r_s^*}{k} \quad (6)$$

Substitution of the expression for the velocity gives the final results

$$\Delta P_v \sim \frac{\alpha \mu_l J a D}{k} \frac{\ln \frac{R^p}{r_*^p}}{\ln \frac{R_a}{R^p}} \quad \text{or} \quad \Delta P_v \sim \frac{\alpha \mu_l J a D}{k} \frac{R^p}{r_*^p} \quad (7)$$

in the two geometries, respectively.

Having obtained expressions for the viscous pressure drop, we next define the limit of the percolation pattern by requesting that variations in the gas saturation, S , are negligible¹⁷, ($\frac{\Delta S}{S} = \epsilon$), where ϵ is a small number. Using percolation theory this is equivalently expressed as $\frac{\Delta S}{S} \sim (p - p_c)^{-1} \Delta p$, where p is the percolation probability and p_c the percolation threshold. In capillary-controlled displacements, p is related to the pore throat (bond) size distribution $\alpha_p(r_p)$ via $p = \int_{r_p}^{\infty} \alpha_p(r) dr$, hence $\Delta p \sim \alpha_p(r_p) \Delta r_p \sim \frac{\Delta r_p}{\sigma r_p}$, where σ is a dimensionless measure of the standard deviation of $\alpha_p(r)$. To relate Δr_p to ΔP_v , we first note that because the gas pressure is spatially uniform, variations in liquid pressure equal those in capillary pressure, $\Delta P_v \sim \Delta P_c$. The latter can be related, in turn, to variations in the occupied pore sizes, $\Delta P_c = \frac{2\gamma \Delta r_p}{r_p^2}$, where we have used the equilibrium condition (1). Next, we identify the cluster extent with the correlation length ($\xi \sim \frac{R^p}{l^*}$), thus obtaining $\frac{R^p}{l^*} \sim (p - p_c)^{-\nu}$, where ν is the correlation length exponent, equal to $\frac{4}{3}$ in 2-D and 0.88 in 3-D. Finally, we express $p - p_c$ in terms of R^p , replace variations in p by variations in r_p , and substitute the relation with P_v , to yield

$$\left(\frac{R^p}{l^*}\right)^{\frac{1}{\nu}} \left(\frac{\ln \frac{R^p}{r_*^p}}{\ln \frac{R_a}{R^p}}\right) \frac{\alpha r_p^* C a}{\sigma l^*} \sim \epsilon \quad \text{or} \quad \left(\frac{R^p}{l^*}\right)^{\frac{1}{\nu}+1} \frac{\alpha r_p^* C a}{\sigma r_*^p} \sim \epsilon \quad (8)$$

in 2-D or 3-D geometries, respectively. Equation (8b) is similar to the analogous expression that delineates the limit of the percolation pattern in external displacements¹⁷. It is apparent from (8) that the radius at the percolation limit decreases as Ca increases, namely as Ja or D increase.

• Viscous Fingering

As the cluster size increases, for a fixed Ca , the pattern departs from percolation and tends towards a non-local pattern. To infer its characteristics, a linear stability analysis for the equivalent problem in an effective porous medium (or a Hele-Shaw cell) was performed⁵. It was shown that in the absence of capillarity, the growth possesses the Mullins-Sekerka instability of solidification¹⁸, which here also happens to coincide exactly with the viscous fingering instability of external displacement. As a result, we expect a limiting pattern of the

DLA type at sufficiently large sizes. This was indeed confirmed with numerical simulations (see below).

To delineate the pattern boundary, we proceed in a fashion analogous to the previous. Let us denote by R^{vj} the size of the cluster at this limit. For a conservative estimate we again follow Lenormand¹⁷ and request that variations in the capillary pressure are small compared to the viscous pressure drop between adjacent liquid sites, $\Delta P_c \ll \Delta P_v$. For the latter, we shall take

$$\Delta P_v \sim \frac{v_m \mu_l r_s^*}{k} \ln 2 \quad \text{or} \quad \Delta P_v \sim \frac{v_m \mu_l r_s^*}{2k} \quad (9)$$

in the two geometries, respectively. After substitution of v_m and some manipulation, we obtain the results

$$\left(\frac{1}{\ln \frac{R_s}{R^{vj}}} \right) \left(\frac{\alpha Ca}{\sigma} \right) \sim \epsilon^{-1} \quad \text{or} \quad \left(\frac{R^{vj}}{l^*} \right) \left(\frac{\alpha Cal^*}{\sigma r_s^*} \right) \sim \epsilon^{-1} \quad (10)$$

in the two geometries, respectively. Large values of Ca result in promoting a pure DLA growth pattern at smaller cluster sizes. We shall point out that in the above derivations snap-off phenomena, where an advancing gas-liquid meniscus becomes disconnected from the main cluster due to a capillary instability, were not considered. Also implicit was the assumption of sufficiently large cluster and system sizes for statistical theories, such as percolation, to be meaningful. The latter condition is very important in numerical simulations as discussed below.

We used the numerical simulator in 2-D geometries^{6,13} to test the validity of the two limiting cluster sizes. Simulations were performed in square lattices of variable sizes, but not larger than 50x50. First, the mean-field results for mass transfer were tested. Fig. 3 shows a plot of the mass flux to a percolation cluster as a function of the cluster size. Good agreement with the theoretical scaling is found provided that the computational domain is large (here equal to ..x...) and the cluster size sufficiently large, but not too large for the boundary to affect the radial symmetry. In these simulations, quasi-static diffusion to a percolation cluster was calculated. The full growth problem was solved in the subsequent simulations.

Figure 4. Percolation and viscous fingering boundaries from simulations in a 50x50 square lattice. The solid line is a guide to the eye.

(a) Capillary Control

(b) Viscous Control

Figure 5. Typical sequence of gas cluster growth under conditions of
 (a) capillary control ($Ca = 0.00001$) and
 (b) Viscous control ($Ca = 0.1$).

Plotted in Fig. 4 are numerical results for R^p and R^{vf} , where only quasi-static diffusion outside the cluster was considered (although the simulator can also account for convection and transients^{13,14}). While the trend of the numerical results is consistent with the theory, a quantitative agreement is difficult to be ascertained. This is due to the small size of our computational domain, which gives rise to substantial finite-size effects. As pointed out above, the cluster must be large enough for meaningful percolation statistics, yet small enough in comparison to the outer boundary for the mean-field transport theories to be valid. The transition from percolation to DLA patterns is illustrated in the sequence of snapshots of Fig. 5, which illustrates cluster growth under conditions of percolation (Fig. 5a) or viscous (Fig. 5b) control.

2. Rates of Growth

From the above theory, rates of bubble growth can be calculated. If N_g is the number of sites occupied by the gas cluster and V_s the average site volume we have

$$V_s \frac{dN_g}{dt} = v_m b \quad (11)$$

where N_g is related to the radius R_g via $N_g \sim \left(\frac{R_g}{l^*}\right)^{D_f}$, where D_f is the fractal dimension of the cluster. Substitution from (4) yields the results

$$\left(\frac{R_g}{l^*}\right)^{D_f} \left[1 + D_f \ln \frac{R_g}{R_g}\right] \sim \frac{\alpha b Ja D D_f}{V_s r_s^*} t \quad \text{or} \quad \left(\frac{R_g}{r}\right)^{D_f-1} \sim \frac{(D_f-1) \alpha b l^* Ja D}{D_f V_s r_s^{*2}} t \quad (12)$$

in the two geometries, respectively. The two scalings should be contrasted to the scaling for growth in the bulk, $R_g^2 \sim t$. As a result of its ramified structure, the cluster grows faster than in its effective medium analogue (for example, $R_g \sim t^{\frac{2}{3}}$, for the 3-D case for either the percolation or the viscous fingering limit). We conclude that during single bubble growth in a porous medium the following regimes develop in succession: a short duration early-time regime, where finite size effects dominate, the growth is still compact and the effective medium scaling applies; a percolation regime ($R_g \leq R^p$); a transition to a viscous fingering regime ($R^p \leq R_g \leq R^{vf}$); and a DLA regime ($R^{vf} \leq R_g$).

All the scalings above rely on the assumption of quasi-static mass transfer. For growth in the bulk this is equivalent to the condition $Ja \ll 1$. In the general case, the equivalent condition can be shown to be $Ja t^{\frac{3-D_f}{D_f-1}} \ll 1$. Contrary to the compact cluster case

($D_f = 3$), the validity of the quasi-static approximation is time-dependent for fractal patterns ($D_f < 3$). To determine whether it holds when the limiting radii are reached, as assumed above, we substitute the equations for growth (12) to (8) and (11) to obtain the conditions $Ja^{1-\frac{\nu}{\nu+1}(3-D_f)} \ll 1$ for the percolation boundary, and $Ja^{1-\frac{D_f(3-D_f)}{D_f-1}} \ll 1$ for the viscous fingering boundary, respectively. Since $D_f \sim 2.5$ for both patterns, the two conditions are found to be still equivalent to the bulk condition $Ja \ll 1$.

ACKNOWLEDGEMENT

This work was supported in part by DOE Contract No. DE-FG22-90BC14600 and by a grant from Chevron Petroleum Technology Corporation.

REFERENCES

1. Hunt, E.B.Jr. and Berry, V.M.R., *AICHEJ*, 560 (1956); Dumore, J.M., *SPEJ*, 211 (1970); De Swaan, A., *J.Pet. Tech.*, 907 (1981).
2. Cornwell, K., Nair, B.G., and Patten, T.D., *Int. J. Heat Mass Transfer* **19**, 236 (1976); Sondergeld, C.H. and Turcotte, L., *J. Geophys. Res.* **82**, 2045 (1977); Thome, J.R., *Enhanced Boiling Heat Transfer*, Hemisphere Publishing Co. (1990).
3. Yousfi, E., Zarcone, C., Bories, S. and Lenormand, R., *C. R. Acad. Sci. Paris* **313**, Serie II, 1093 (1991).
4. Scriven, L.E., *Chem. Eng. Sci.* **10**, 1 (1959); Szekely, J. and Martins, G.P., *Chem. Eng. Sci.* **26**, 147 (1971); Plesset, M.S., and Prosperetti, A., *Ann. Rev. Fluid Mech.* **9**, 145 (1977).
5. Li, X. and Yortsos, Y., C., *Phys. Fluids A*, submitted (1993).
6. Li, X. and Yortsos, Y., C., *Chem. Eng. Sci.*, submitted (1993).
7. Wilkinson, D. and Willemsen, J.F., *J. Phys. A: Math. Gen.* **16**, 3365 (1983); Lenormand, R., *J. Phys.: Condens. Matter* **2**, SA79 (1990); Dullien, F.A.L, *Porous Media: Fluid Flow and Pore Structure*, Academic Press (1992).
8. Lifshitz, I.M. and Slyozov, V.V., *J. Phys. Chem. Solids* **19**, 35 (1961); Marqusee, J.A. and Ross, J., *J. Chem. Phys.* **79**, 373 (1983).
9. Kennedy, H.T. and Olson, C.R., *Trans. AIME* **195**, 271 (1952); Wieland, D.R. and Kennedy, H.T., *Trans. AIME* **210**, 122 (1957); Chatenever, A., Indra, M.K. and Kyte, J.R., *J. Pet. Tech.*, 13 (1959)

10. Epstein, D., *J. Geophys. Res.* **72**, 3701 (1967); Abgrall, E. and Iffly, R., *Rev IFP*, 667 (1973); Kashchiev, D. and Firoozabadi, A., *J. Chem. Phys.* **98**, 1 (1993).
11. Yortsos, Y.C. and Parlar, M., paper SPE 19697, presented at the 1989 Fall Meeting of the Society of Petroleum Engineers, (Oct. 8-9, 1989).
12. Parlar, M. and Yortsos, Y.C., *J. Coll. Intf. Sci.* **124**, 162 (1988); Parlar, M. and Yortsos, Y.C., *J. Coll. Intf. Sci.* **132**, 425 (1989).
13. Li, X. and Yortsos, Y.C., paper SPE 22589 presented at the 1991 Fall Meeting of the Society of Petroleum Engineers, Dallas, TX (Oct. 6-9, 1991).
14. Satik, C., *PhD Dissertation*, University of Southern California (1993).
15. Katz, A.J. and Thompson, A.H., *Phys. Rev. B* **34**, 8175 (1987).
16. Witten...
17. Lenormand, R., *Proc. Roy. Soc.*
18. Langer, J.S., *Rev. Mod. Phys* **52**, 1 (1980); Pelce, P. (ed), *The Dynamics of Curved Fronts*, Academic Press (1988).

**DATE
FILMED**

10/21/94

END