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M.J. Syphers, L. Ahrens, W. van Asselt, J.M. Brennan

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The AGS γ_t -jump System

M. J. Syphers, L. Ahrens, W. van Asselt, J. M. Brennan

In an attempt to generate a lossless crossing of an accelerator's transition energy, one procedure is to alter the transition energy of the accelerator quickly as the beam passes through this energy region by changing the optics of the lattice – a so-called “transition jump,” or “ γ_t -jump” scheme. Such a system was first implemented at CERN[1] and later adopted at other accelerator laboratories. A scheme for the AGS was developed in 1986. A description of the AGS γ_t -jump system, and recent results from its commissioning are presented in this report.

1 First Principles

The transition energy, $E_t = \gamma_t mc^2$, of a synchrotron is the energy at which particles of various momenta have the same revolution period. At this energy, the slip factor,

$$\eta \equiv \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} = \frac{\Delta\tau/\tau}{\Delta p/p} \quad (1)$$

becomes zero. Changing the transition energy in the AGS amounts to changing the average value of the dispersion function, D_x , in the bending magnets of the accelerator. The value of γ_t for a synchrotron with sector bending magnets is given by

$$\frac{1}{\gamma_t^2} \equiv \frac{\Delta C/C}{\Delta p/p} = \left\langle \frac{D_x}{\rho} \right\rangle, \quad (2)$$

where the average is taken over the circumference of the accelerator. The value of ρ is path length dependent, and has infinite value in straight sections. Hence, it is the value of the dispersion function in the bending magnets which one is after in this exercise of altering γ_t .

By introducing quadrupoles in the normal AGS lattice, one can perturb the dispersion function and hence, hopefully, alter γ_t in a constructive way. The basic AGS lattice has 12-fold symmetry. With a base tune on the order of 9, two equivalent

Figure 1: The dispersion function of the AGS for the nominal lattice, and for the γ_t -jump (for $\Delta\gamma_t = 2$). The points plotted are located in the middle of the main bending magnets.

points in adjacent superperiods will be roughly 90° (or, actually, 270°) apart in beta-tron-phase. By placing two quadrupoles of opposite polarity two superperiods (180°) apart, the result is an amplification in the effect on the dispersion function, while the disturbances on the amplitude function will be localized and the tune will remain approximately constant. By choosing an appropriate configuration of quadrupoles such that the average dispersion in the bending magnets is lowered, γ_t can be raised accordingly. The choice of quadrupole locations for this purpose in the AGS was chosen after a study in 1986[2],[3].

Of course, the tune of the accelerator is not exactly 9, and hence the phases are only approximately those quoted above. The system employed in the AGS alters the dispersion function as shown in Figure 1. By varying the strength of the six quadrupoles, one can tune the accelerator to various values of γ_t . The perturbations to the lattice functions generated by the system can be predicted as shown in the subsections below.

1.1 Dispersion Function Perturbations

A small quadrupole perturbation $q \equiv \Delta B'\ell/(B\rho)$, at a location where the dispersion function has an unperturbed value D_0 , will cause a change in the slope of D at that point by an amount $\Delta D' = -D_0 q$. Since the dispersion function describes a closed

orbit in the accelerator, perturbations due to gradient errors can be easily estimated as follows. For six equally spaced quadrupoles of alternating polarity around the AGS, the change in the dispersion function at one of the "positive" quadrupoles will be, to lowest order,

$$\Delta D_0 = \frac{\sqrt{\beta_0}}{2 \sin \pi \mu} \sum_{i=1}^2 \Delta D'_i \sqrt{\beta_i} \cos(\psi_i - \pi \mu) \quad (3)$$

$$= \frac{\beta_0(-D_0 q)}{2 \sin \pi \mu} [\cos \pi \mu - \cos(\chi - \pi \mu)] \quad (4)$$

$$= -\frac{D_0(\beta_0 q)}{2 \sin \pi \mu} [\cos \pi \mu - 1] \quad (5)$$

$$= \frac{D_0(\beta_0 q)}{2} \tan\left(\frac{\pi \mu}{2}\right) \quad (6)$$

and the slope of the dispersion function, just after the quadrupole, changes by

$$\Delta D'_0 = -\frac{D_0 q}{2}. \quad (7)$$

In the above, $\mu = \nu/3$ is the tune across one-third of the ring (the new superperiod), and $\chi = 2\pi\nu/6$ is the phase advance between adjacent quadrupoles of the jump system.

Note that with quadrupoles ≈ 1.5 betatron wavelengths apart, the periodic solution for the dispersion function perturbation will have a structure similar to that shown in Figure 2.

Having found ΔD_0 and $\Delta D'_0$ above, the maximum perturbation between the quadrupoles will be approximately ($\hat{\beta} \approx \beta_0$, and $\alpha_0 \approx 0$):

$$\Delta \hat{D}_0 = \sqrt{\Delta D_0^2 + (\beta_0 \Delta D'_0)^2} \quad (8)$$

$$= |\Delta D_0| \sqrt{1 + \cot^2\left(\frac{\pi \mu}{2}\right)} \quad (9)$$

$$= \frac{D_0(\beta_0 q)}{2} \sqrt{1 + \tan^2\left(\frac{\pi \mu}{2}\right)}. \quad (10)$$

As an example, take $q = 1/(50 \text{ m})$ (approximate value of γ_t quads running at 600 A near the AGS transition energy). If $\nu = 8.75$, then

$$\Delta \hat{D}_0 \approx \frac{(2m)(22m)(1/50m)}{2} \sqrt{1 + \tan^2(8.75\pi/6)} = 3.4m. \quad (11)$$

Figure 3 shows the result of a more detailed calculation using the code SYNCH[4] with the γ_t -jump system with $q = 1/(50 \text{ m})$.

Figure 2: Schematic of dispersion wave generated by the γ_t -jump system.

Figure 3: Dispersion function perturbation generated by the γ_t -jump system. Here, $q = 1/50\text{m}$.

1.2 Amplitude Function Perturbations

The quadrupole perturbations generated by the γ_t -jump system also alter the amplitude function of the AGS. Since the $\Delta\beta/\beta$ distortions travel at twice the betatron tune, and since the distortions are linear to first order, then for "small" quadrupole perturbations the treatment is similar to that above, namely

$$\left(\frac{\Delta\beta}{\beta}\right)_0 = -\frac{1}{2\sin 2\pi\mu} \sum_{i=1}^2 \beta_i q_i \cos(2\psi_i - 2\pi\mu) \quad (12)$$

$$= -\frac{\beta_0 q}{2\sin 2\pi\mu} [\cos(2\pi\mu) - 1], \quad (13)$$

or,

$$\left(\frac{\Delta\beta}{\beta}\right)_0 = \frac{\beta_0 q}{2} \tan \pi\mu, \quad (14)$$

and

$$\Delta\alpha_0 = \frac{\beta_0 q}{2}. \quad (15)$$

Here, $(\Delta\beta/\beta)_0$ and $\Delta\alpha_0$ are the changes in these quantities at one of the "positive" perturbing quads in the system. The maximum value of $\Delta\beta/\beta$ between the quadrupoles is given approximately by[5]

$$\left(\frac{\Delta\beta}{\beta}\right)_\pm = \pm \sqrt{\left(\frac{\Delta\beta}{\beta}\right)_0^2 + \Delta\alpha_0^2} + \frac{1}{2} |\det \Delta J| \quad (16)$$

where

$$|\det \Delta J| = \left| \det \begin{pmatrix} \Delta\alpha & \Delta\beta \\ -\Delta\gamma & -\Delta\alpha \end{pmatrix} \right| = \frac{\left(\frac{\Delta\beta}{\beta}\right)^2 + \left(\alpha \frac{\Delta\beta}{\beta} - \Delta\alpha\right)^2}{1 + \frac{\Delta\beta}{\beta}}. \quad (17)$$

In our example, with $q = 1/50m$, then

$$\left(\frac{\Delta\beta}{\beta}\right)_0 = \frac{(22m)(1/50m)}{2} \tan(8.75\pi/3) = -0.059, \quad (18)$$

$$\Delta\alpha_0 = \frac{(22m)(1/50m)}{2} = 0.22, \quad (19)$$

$$|\det \Delta J| = \frac{(-0.059)^2 + (0.22)^2}{1 - 0.059} = 0.055, \quad (20)$$

assuming $\alpha = 0$ at the quadrupole (a good approximation). Thus, we should see an amplitude function perturbation between the quadrupoles with extrema

$$\frac{\hat{\Delta\beta}}{\beta} = \frac{0.055}{2} + \sqrt{(0.059)^2 + (0.22)^2} = 0.26, \quad (21)$$

$$\frac{\check{\Delta\beta}}{\beta} = \frac{0.055}{2} - \sqrt{(0.059)^2 + (0.22)^2} = -0.20. \quad (22)$$

Figure 4: Amplitude function perturbation generated by the γ_t -jump system. Here, $q = 1/50\text{m}$.

Figure 4 shows the result from the SYNCH calculation.

Naturally, these first order estimates break down at higher currents in the system quadrupoles, when $(\beta_0 q)$ gets too large. But for most of the system's range, especially in the range of typical operation, the approximations provide reasonable estimates.

1.3 Estimation of Tune Shift

According to the well-known tune shift formula, $\Delta\nu = \beta q / 4\pi$, there should be no tune shift from six regularly spaced, alternating gradient quadrupoles in the AGS. However, the strength of these quadrupoles can be large enough that second order effects come into play. To second order, the tune shift from a distribution of gradient errors is[6]

$$\Delta\nu = \frac{1}{4\pi} \int k\beta_0 ds + \frac{1}{8\pi} \int k\beta_0 \left(\frac{\Delta\beta}{\beta} \right) ds \quad (23)$$

where β_0 is the unperturbed value of the amplitude function, and $\Delta\beta/\beta$ is the perturbation. For the AGS γ_t -jump system the above reduces to

$$\begin{aligned} \Delta\nu &= \frac{1}{4\pi} \sum \beta_i q_i + \frac{1}{8\pi} \sum \beta_i q_i \left(\frac{\Delta\beta}{\beta} \right)_i \\ &= 0 + \frac{1}{8\pi} \left[3\beta_0 q \left(\frac{\Delta\beta}{\beta} \right)_0 + 3\beta_0 (-q) \left(\frac{\Delta\beta}{\beta} \right)_1 \right] \end{aligned} \quad (24)$$

$$\begin{aligned}
&= \frac{3(\beta_0 q)}{8\pi} \left(\frac{\Delta\beta}{\beta} \right)_0 \left[1 - \frac{(\Delta\beta/\beta)_1}{(\Delta\beta/\beta)_0} \right] \\
&= \frac{3(\beta_0 q)^2}{16\pi} \tan \pi\mu \left[1 - \frac{(\Delta\beta/\beta)_1}{(\Delta\beta/\beta)_0} \right]
\end{aligned} \tag{25}$$

where the "0" and "1" subscripts refer to positive and negative quad locations respectively. But, from symmetry, $(\Delta\beta/\beta)_1 = -(\Delta\beta/\beta)_0$ and so the total tune shift will be

$$\Delta\nu \approx \frac{3 \tan \pi\mu}{8\pi} (\beta_0 q)^2. \tag{26}$$

As an example, for the AGS with a tune of 8.75, and for $q = 1/25\text{m}$, we get $\Delta\nu_x = -0.03$, and $\Delta\nu_y = -0.007$. Notice that both tune shifts are of the same sign, since they depend upon the square of the quadrupole strength.

1.4 Estimation of Shift in Transition Energy

To obtain an estimate of how much one can change γ_t with the AGS system, we can use Eq. 2 to see that

$$\Delta\gamma_t \approx \frac{-\gamma_{t0}^3}{2\rho} \Delta\langle D \rangle \tag{27}$$

for small variations in the lattice parameters. For the AGS system, consider the change in the dispersion function, ΔD , across one new superperiod (one third of the ring) as indicated in Figure 5. We can use Eq. 6, the change in the dispersion function at a "positive" quad, and propagate this change through two standard superperiods of the AGS. By summing up the changes ΔD in the bending magnets between two consecutive "positive" quads, say, and taking their average, one can employ Eq. 27 above to estimate $\Delta\gamma_t$.

We readily note that the change in the transition energy is a second-order effect. We know that the change, ΔD_1 , at a "negative" quad will be, to first order, opposite the change at a "positive" quad due to symmetry. (i.e., $\Delta D_1 = -\Delta D_0$, to first order.) Now if M_i is the transfer matrix from a γ_t -quad to the i -th bending magnet downstream, where

$$M = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix}, \tag{28}$$

then the contributions to $\Delta\langle D \rangle$ from the i -th dipole downstream of a "+" γ_t -quad and the i -th dipole downstream of a "-" γ_t -quad will be:

$$\begin{aligned}
\Delta\mathcal{D}_i &\equiv a_i \Delta D_0 + b_i \Delta D'_0 + a_i \Delta D_1 + b_i \Delta D'_1 \\
&= a_i \Delta D_0 + b_i \Delta D'_0 - a_i \Delta D_0 - b_i \Delta D'_0 \\
&= 0.
\end{aligned}$$

Figure 5: Sketch of dispersion function perturbation generated by the γ_t -jump system through four normal AGS superperiods.

Summing up, we would have no net change in γ_t . So, we must look at second-order terms in the perturbation of the dispersion function. To find the next term in the expansion of ΔD_0 , we generalize Eq. 6 to write

$$\frac{dD_0}{dq} = \frac{D_0(q)\beta(q)}{2} \tan \frac{\pi\mu}{2} \quad (29)$$

and, assuming the tune shift is negligible,

$$\ln D_0 = \frac{\tan \frac{\pi\mu}{2}}{2} \int \beta_0 \left(1 + \frac{\Delta\beta}{\beta}(q) \right) dq \quad (30)$$

$$= \frac{\tan \frac{\pi\mu}{2}}{2} \left(\beta_0 q + \beta_0 \int \frac{\beta_0 q}{2} \tan \pi\mu dq \right) \quad (31)$$

$$= \frac{\beta_0 q}{2} \tan \frac{\pi\mu}{2} + \frac{(\beta_0 q)^2}{8} \tan \pi\mu \tan \frac{\pi\mu}{2} + \text{const.} \quad (32)$$

$$\Rightarrow D_0^{\text{new}} = D_0 \exp \left\{ \frac{\beta_0 q}{2} \tan \frac{\pi\mu}{2} + \frac{(\beta_0 q)^2}{8} \tan \pi\mu \tan \frac{\pi\mu}{2} \right\}. \quad (33)$$

So, to 2nd order in $(\beta_0 q)$,

$$\Delta D_0 = \frac{D_0 \beta_0 q}{2} \tan \frac{\pi\mu}{2} + \frac{D_0 (\beta_0 q)^2}{8} \tan \frac{\pi\mu}{2} (\tan \frac{\pi\mu}{2} + \tan \pi\mu), \quad (34)$$

$$\Delta D'_0 = -\frac{Dq}{2} = -\frac{D_0 q}{2} - \frac{D_0 \beta_0 q^2}{4} \tan \frac{\pi\mu}{2}. \quad (35)$$

Adding the terms again for $\Delta\mathcal{D}$:

$$\Delta\mathcal{D}_i \equiv a_i\Delta D_0 + b_i\Delta D'_0 + a_i\Delta D_1 + b_i\Delta D'_1 \quad (36)$$

$$= a_i \left[2 \cdot \frac{D_0(\beta_0 q)^2}{8} \tan \frac{\pi\mu}{2} \left(\tan \frac{\pi\mu}{2} + \tan \pi\mu \right) \right] \\ + b_i \left[2 \cdot \left(-\frac{D_0\beta_0 q^2}{4} \tan \frac{\pi\mu}{2} \right) \right] \quad (37)$$

$$= -\frac{D_0(\beta_0 q)^2}{8} \left(\frac{\tan \pi\mu}{\cos^3 \frac{\pi\mu}{2}} \right) \mathcal{A}_i, \quad (38)$$

where

$$\mathcal{A}_i \equiv \sqrt{\frac{\beta_i}{\beta_0}} \left[\sin(\psi_i - \frac{3\pi\mu}{2}) + \left(\cos \frac{\pi\mu}{2} - \alpha_0 \sin \frac{3\pi\mu}{2} \right) \sin \psi_i \right]. \quad (39)$$

In the last expression, we have written a_i and b_i in terms of the unperturbed lattice functions:

$$a_i = \sqrt{\frac{\beta_i}{\beta_0}} [\cos \psi_i + \alpha_0 \sin \psi_i], \quad (40)$$

$$b_i = \sqrt{\beta_i \beta_0} \sin \psi_i. \quad (41)$$

Now, $\Delta\mathcal{D}_i$ above is for 2 bending magnets, the i -th magnet after a "+" quad, and the i -th magnet after a "-" quad. These two bending magnets are at equivalent points of the normal superperiods of the unperturbed AGS lattice. Thus,

$$\Delta\langle D \rangle = \frac{1}{2} \left(\frac{1}{40} \sum_{i=1}^{40} \Delta\mathcal{D}_i \right) \quad (42)$$

where the sum is over the 40 bending magnets in two consecutive normal AGS superperiods. Our estimate for $\Delta\langle D \rangle$ then becomes

$$\Delta\langle D \rangle = -\frac{D_0}{16} \left(\frac{\tan \pi\mu}{\cos^3 \frac{\pi\mu}{2}} \right) \langle \mathcal{A} \rangle_{\text{bends}/6} (\beta_0 q)^2 \quad (43)$$

Reiterating, β , β_0 , α_0 , and D_0 are unperturbed values of the lattice functions (β_0 , α_0 and D_0 are at a γ_t -quad), $\Delta\psi$ is the phase advance from the nearest upstream γ_t -quad, $\mu = \nu/3$, and the average taken in the above expression is over the bending magnets in one sixth of the AGS circumference.

So, we finally put this result into Eq. 27 to obtain

$$\Delta\gamma_t \approx \frac{\gamma_{t0}^3 D_0}{32\rho} \left(\frac{\tan \pi\mu}{\cos^3 \frac{\pi\mu}{2}} \right) \langle \mathcal{A} \rangle_{\text{bends}/6} (\beta_0 q)^2. \quad (44)$$

For the AGS, with $\nu = 8.77$,

$$\langle \mathcal{A} \rangle_{\text{bends}/6} \approx 0.025 \quad (45)$$

Figure 6: Change in transition energy (γ_t) in the AGS. The solid line is the equation in the text, while the dashed line is the result of calculations using the computer code SYNCH.

and taking $\nu = 8.77$, $\gamma_{t0} = 8.5$, $\rho = 85.38$ m, and $D_0 = 2.1$ m, we get

$$\Delta\gamma_t \approx 1.7 (\beta_0 q)^2. \quad (46)$$

Displayed in Figure 6 are the results for the AGS as computed by the above formula, and by use of the SYNCH code. Note that this formula breaks down for $\beta_0 q \gg 1$. For reference, $(\beta_0 q) \approx 1$ for protons at 9 GeV, and $I = 1.5$ kA in the jump-quadrupoles. Quick estimates of system parameters can be made using the following summary:

$$\Delta\gamma_t = 1.7 (\beta_0 q)^2 \quad (47)$$

$$\hat{D} = \hat{D}_0(1 + 3.2\sqrt{\Delta\gamma_t}) \quad (48)$$

$$\hat{\beta} = \hat{\beta}_0(1 + 0.4\sqrt{\Delta\gamma_t}) \quad (49)$$

$$\Delta\nu_x = -0.02\Delta\gamma_t, \quad \Delta\nu_y = -0.005\Delta\gamma_t \quad (50)$$

It should also be pointed out that the value of $\Delta\gamma_t$ will depend upon the tune of the AGS. While the tune (i.e., μ) is seen explicitly in Equation 44, the value $\langle \mathcal{A} \rangle$ also depends upon the base tunes of the accelerator. One expects this, since changing the tune (both horizontal and vertical) will produce different phase advances between the various bending magnets and hence the dispersion function perturbation will proceed

Figure 7: Transition energy (γ_t) as a function of horizontal and vertical base tunes. Here, $\beta_{0q} = 1$.

in a different manner. Near an integer tune, the dispersion function perturbations will be greatly amplified. A plot of γ_t versus tune with the jump on is shown in Figure 7. The tunes are the “base” tunes before the jump is implemented. A value of $\beta_{0q} = 1$ was used. For reference, the uncorrected tunes of the AGS are approximately $\nu_x = 8.77$, $\nu_y = 8.67$.

2 Description of the System

The AGS γ_t -jump system consists of six quadrupoles located at ring coordinates A17, C17, E17, G17, I17, and K17. The quadrupoles in A, E, and I sectors are focusing in the horizontal plane, while those in C, G, and K sectors are defocusing. Table 1 lists the parameters of the quadrupole magnets and their power systems.

As a function of current and beam momentum, the change in γ_t from the system can be expressed as

$$\Delta\gamma_t \approx 52 \times \left(\frac{I[\text{kA}]Q}{p[\text{GeV}/c]} \right)^2. \quad (51)$$

where I is the current in the γ_t quads measured in kA, Q is the charge of the individual particles (in units of e), and p is their momentum measured in GeV/c. In terms of

Effective length	0.422	m
$\int B'dl/I$	0.85	T/kA
Max. current	3.0	kA
Inductance	400	μH
Resistance (mag.& cable)	4-5	$\text{m}\Omega$
Discharge Resistor	0.75	Ω
Discharge time constant	0.53	msec
q (@ $I=1$ kA, $\gamma = 9$ for p):	0.03	m^{-1}
Lattice functions		
at the quadrupoles:		
β_x	22	m
α_x	-0.04	
β_y	11	m
α_y	-0.06	
D	2.1	m
D'	0.003	

Table 1: Parameters of γ_t -jump quadrupoles.

the relativistic kinematic factor $\beta\gamma$ and particle atomic weight A (in amu 's),

$$\Delta\gamma_t \approx 59 \times \left(\frac{I[\text{kA}]}{\beta\gamma} \right)^2 \left(\frac{Q}{A} \right)^2. \quad (52)$$

The time and magnitude of the γ_t perturbation can be varied manually through the control system, or via an applications program.[7] The program allows the user to specify a "jump size" ($\Delta\gamma_t^j$) and a "distance" $h \equiv \gamma_t - \gamma$, which is maintained prior to the actual jump time. (See Figure 8.) When the jump occurs, the current required

Figure 8: Parameters specified in the γ_t -jump system applications code.

in the quadrupoles is

$$I^j[\text{kA}] = \sqrt{\frac{\Delta\gamma_t}{52}} \left(\frac{p^j[\text{GeV}/c]}{Q} \right) \quad (53)$$

with

$$p^j c = mc^2 \sqrt{(\gamma_{t0} + \Delta\gamma_t - h)^2 - 1} \quad (54)$$

where m is the rest mass of the particles. Or, we can write

$$I^j[\text{kA}] = \sqrt{\frac{\Delta\gamma_t}{59}} \left(\frac{(\beta\gamma)^j A}{Q} \right) \quad (55)$$

with

$$(\beta\gamma)^j = \sqrt{(\gamma_{t0} + \Delta\gamma_t - h)^2 - 1}. \quad (56)$$

3 Effects of Orbit on System Operation

Commissioning of the final AGS γ_t -jump system began in March 1994 during the start-up period of the High Energy Physics run. Polarities of the six pulsed quadrupole magnets were checked by observing the tune shift induced by each one individually, and magnet power supply programs and discharge times were checked for simultaneity. Once the commissioning began in earnest, beam loss was noticed to occur during the time the quadrupoles were ramping to their final level. It was soon found that the equilibrium orbit in the AGS was shifting by 10 or more millimeters during this procedure with $\Delta\gamma_t \approx 1.5$. (See Figure 9.) By once again pulsing the quadrupoles individually, the centers of the quadrupoles with respect to the original equilibrium orbit could be deduced, and measures could be taken to attempt to reduce these offsets.

At that time, the equilibrium orbit of the AGS contained a large excursion through the E17 quadrupole which could account for much of the observed perturbation. This was soon rectified by physically moving nearby AGS main magnets. Afterward, another set of measurements were performed by pulsing individual quadrupoles. The next worst offender was the quadrupole at K17. A remotely tunable orbit bump through this region was installed by powering windings on the back legs of 8 AGS main magnets, connected in such a way that a $3\lambda/2$ local orbit bump is created. By adjusting the radial position in the AGS to center the beam in the other 5 γ_t quadrupoles, and then adjusting this local orbit bump, the distortions to the equilibrium orbit during the powering of the system was lowered to less than 2.5 mm.

It was also noticed that beam loss during the γ_t -jump procedure could be minimized by turning off the chromaticity sextupoles during this time. The change of the natural chromaticity of the AGS due to the γ_t -jump optics is only $\Delta\xi_x = -1.2$, and $\Delta\xi_y = -0.3$ for $\Delta\gamma_t = 1.5$. However, since the system alters both the dispersion function and the amplitude function at the locations of the sextupoles used for chromaticity correction, the actual chromaticity generated by these sextupoles during the

Figure 9: Perturbation of AGS equilibrium orbit during pulsing of γ_t -jump system with $\Delta\gamma_t \approx 1.4$. The vertical scale is in centimeters. These data were obtained April 24, 1994, prior to implementation of a local orbit bump at the K17 γ_t -jump quadrupole.

jump can be quite large. For instance, if the sextupole circuits are tuned to produce $\Delta\xi_x = \Delta\xi_y = 0$, and then the system is pulsed to generate $\Delta\gamma_t = 1.5$, the resulting chromaticity in the AGS will be $\Delta\xi_x = 14$ and $\Delta\xi_y = -4$. If necessary, new settings of the sextupole circuits could be implemented to create zero chromaticity during the jump, but this option has not been exercised to date.

Another effect due to closed orbit offsets through non-zero sextupoles is the introduction of quadrupole fields which will alter the optics. For example, in the AGS, the necessary sextupole strength to bring the natural chromaticity to zero is approximately

$$\frac{B''\ell}{(B\rho)} = 0.75 \text{ m}^{-2}. \quad (57)$$

The gradient of this sextupole field, at a closed orbit offset $x_{c.o.}$, will generate a quadrupole perturbation of strength $q = S \cdot x_{c.o.}$. Thus, the perturbation of the dispersion function caused by a 5 mm offset in this single sextupole will have an amplitude

$$\Delta D = \frac{\Delta D'_s \sqrt{\beta_s \beta}}{2 \sin \pi \nu} \quad (58)$$

$$= \frac{D_s q \sqrt{\beta_s \beta}}{2 \sin \pi \nu} = \frac{D_s S x_{c.o.} \sqrt{\beta_s \beta}}{2 \sin \pi \nu} \quad (59)$$

$$\approx \frac{(2m)(0.75/m^2)(0.005m)\sqrt{(22m)(16m)}}{2 \sin(8.77\pi)} = 0.1m. \quad (60)$$

Figure 10: Computed change in dispersion function due to measured orbit distortion in AGS. The sextupoles were adjusted to provide zero chromaticity for the normal, unperturbed lattice.

The effect of 20 sextupoles will depend upon the particular closed orbit and the phases between these magnets. The change in the dispersion function caused by the measured orbit distortion generated by the γ_t system is shown in Figure 10. This calculation was for $\Delta\gamma_t = 1.3$, as was the set-up during measurements taken April 24, 1994.

4 Measurement of Dispersion Function

The major effect of the γ_t -jump system which is readily measurable is the change in the AGS dispersion function. The AGS contains pick-up electrodes (PUEs) at 72 locations in the ring which can measure transverse positions of the beam, averaged over several milliseconds. By measuring orbits for various radial offsets (i.e., various momenta) and subtracting, one obtains signals which are directly proportional to the dispersion function at the PUE locations.

The method for adjusting the momentum is to adjust the radial position feedback loop of the AGS RF system. This system adjusts the frequency of the RF cavities to maintain a specified average position signal between two PUEs. The two PUEs in the AGS used for this purpose are located 180° apart in betatron phase and there are no γ_t -jump quadrupoles in between. Hence, the average value of the dispersion function

Figure 11: (a) Measurement of $\Delta p/p$ generated by two different settings of RF radial feedback loop with transition jump off. (b) Measurement repeated with transition jump on. The vertical scale is in centimeters.

for these two locations is unaffected by perturbations to the dispersion generated by the quadrupoles.

Measurements were made of the equilibrium orbit at the “jump time,” at various momenta (radial position loop settings) with and without the jump quadrupoles being pulsed. If one assumes the average dispersion to be its predicted value with the jump turned off (1.8 m for the 72 PUE locations), then the measured average value of the PUE positions will yield the momentum offset produced, namely $\Delta p/p = \langle x_{pue} \rangle / \langle D_{pue} \rangle$. The measurement is then repeated with the jump quadrupoles pulsing. Dividing these measurements by the calculated $\Delta p/p$ yields the dispersion function during the jump.

Figure 11(a) shows the difference of the PUE positions measured for two settings of the radial feedback loop. Here, the γ_t -jump was turned off. The same measurement, for the same loop settings, was repeated with the γ_t -jump turned on. The results are shown in Figure 11(b). Figure 12 shows the results for three such sets of measurements taken on the AGS for different radial position settings. It should be pointed out that the “SYNCH” curve shown in Figure 12 includes the effects of the closed orbit through the powered chromaticity sextupoles.

Figure 12: Measured AGS dispersion function during the γ_t -jump, assuming predicted dispersion function without the jump. The solid curve is the dispersion function predicted using SYNCH.

5 Longitudinal Phenomena

The above discussion has concentrated on the transverse particle dynamics associated with generating a different transition energy for the AGS. In fact, the major benefits of crossing transition more quickly appear in the longitudinal degree of freedom. Much discussion of the longitudinal dynamics of crossing transition can be found in the literature.[8],[9],[10] If the product $|eV \cos \phi_s|$ is held constant (where V is the peak accelerating voltage per turn in the RF cavities, and ϕ_s is the synchronous phase), and if we assume that $\frac{d}{dt}|\eta/\gamma|$ is also constant while crossing transition, then one can solve analytically for the momentum spread and bunch length at the time the transition energy is crossed. These conditions are approximately true in the AGS. One finds that the momentum spread at transition varies as $\dot{\gamma}^{-1/6}$, while the bunch length varies as $\dot{\gamma}^{1/6}$. In the event that γ_t is also changing, then $\dot{\gamma}$ can be replaced by $\dot{\gamma} - \dot{\gamma}_t$. In the AGS, running with protons,

$$\dot{\gamma} = \frac{epv}{mc^2} \dot{B} \approx 50/\text{sec} \quad (61)$$

in the vicinity of γ_t . Since the γ_t -jump system quadrupoles have their current discharged through a resistor, with time constant τ_0 , then the rate of change of γ_t which

is produced can be written as

$$\dot{\gamma}_t(t) = - \frac{2\Delta\gamma_t}{\tau_0} e^{-2t/\tau_0} \quad (62)$$

where here, $\Delta\gamma_t$ is the maximum value generated just before the jump. As an example, for $\Delta\gamma_t = 1.5$, $\tau_0 = 0.5$ msec, then after 0.5 msec one has $\dot{\gamma}_t = -800/\text{sec}$. Thus, implementation of the jump increases the bunch length at transition by a factor $(850/50)^{1/6} = 1.6$, and decreases the momentum spread at transition by a factor $(50/850)^{1/6} = 0.6$. However, one has to remember that the peak dispersion function is increased by a factor of 5 to do this.

The increase in the bunch length decreases the line density and hence increases the threshold of the microwave instability,[10] by about a factor of four in our example above. In addition, the mismatch of the bunch with the re-established RF bucket after transition crossing is reduced by this same factor.[10]

6 Recent Results and Comments

As of mid-June, 1994, the AGS transition-jump system has been operating routinely for several weeks. The transmission of beam across transition is routinely better than 99% at AGS record intensities in excess of 3.5×10^{13} protons per pulse. One γ_t -jump quadrupole power supply (at the E17 location) was found to have pulse-to-pulse jitter on the order of 10-20%, which has been fixed. To date, the chromaticity and tunes are left uncorrected during the jump. It has also been noticed that the lattice perturbations produced by the system conspire to make the largest beam size at the most critical aperture restrictions, namely at the injection and extraction points. By changing the polarities of all of the γ_t -jump quadrupole magnets, the locations of largest beam size can be moved, placing them in the presence of perhaps less restrictive apertures. This option may be exercised as the intensity of the AGS continues to be raised, but so far has not been necessary.

Figure 13 shows the horizontal beam profile measured at various times of the AGS cycle, showing the change occurring due to the transition-jump system. At the location of the monitor, the dispersion function changes from 2.1 m to about -5 m for $\Delta\gamma_t = 1.5$. In the figure, the beam width decreases until the dispersion is zero, and then increases until the maximum negative value of the dispersion is reached. The points are taken 1 msec apart. Notice that the total discharge time of the magnets is approximately 5 msec, and the beam size is seen to go through a minimum again during this time. This is an indication of how well the six quadrupoles discharge simultaneously.

Operationally, the optimum AGS performance was obtained by diluting the longitudinal emittance after the crossing of the transition energy. This dilution is required for optimizing slow resonant extraction, and was previously performed early in the AGS cycle using a high frequency (VHF) RF cavity. During this last HEP run, a

Figure 13: Horizontal beam size (mm, rms) vs. time during the AGS cycle near the transition jump.

"front porch" was added to the AGS magnet wave form, several milliseconds after transition crossing, at which time the VHF dilution cavity was used. Longitudinal emittance early in the cycle could still be controlled somewhat by injection RF phase mismatch. With the final dilution taking place after transition, the losses due to the large excursions of the dispersion function could be minimized.

Figure 14 shows the effect of the jump system on the longitudinal profile of the AGS beam during the jump. The total beam intensity was 2.5×10^{13} with the jump on, 1.5×10^{13} with the jump off. The change in the jump time (as expected from Figure 8 is easily visible.

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Figure 14: Longitudinal beam profiles vs. time. The “shift” occurs at transition. Time flows “up” with 1 msec between traces. (a) Jump system off. (b) Jump system on. Note the change in the transition crossing time when the system is on. The horizontal scale is in nanoseconds.

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