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METHOD OF RAPID DETERMINATION OF MHD EQUILIBRIUM PROPERTIES
WITH THE MODIFIED VERSION OF THE SURFAS CODE

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Method of Rapid Determination of MHD Equilibrium Properties with the Modified Version of the SURFAS Code

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Rapid determination of MHD equilibrium properties of tokamak plasmas is carried out by means of an approximation method based on the use of database files. These are computed a priori from MHD equilibrium solutions obtained by performing reconstruction to match experimental measurements, which include motional Stark effect (MSE) data. The procedure carries out a single iteration of Newton's method to determine the poloidal variation of the toroidal plasma current density in the equilibrium form $j_\phi = -2\pi(\mu_0 R p' + F F'/R)$ by representing $p'(\psi)$ and $F(\psi)F'(\psi)$ in series expansions of Chebyshev polynomials. The polynomial expansion coefficients are obtained through a least-squares data fitting process similar to that used in the equilibrium reconstruction. Knowing the current density j_ϕ allows the determination of the internal q -profile from the MSE data. This important stability parameter is generally unavailable from a current filament model. Numerical results calculated in this approach are compared with those determined from an accurate solution of the Grad-Shafranov equation, subject to a similar set of magnetic and pressure measurement constraints.

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I. Introduction

Computation of the magnetohydrodynamic (MHD) equilibrium properties of a tokamak plasma plays an important role in various fields of magnetic fusion research. The basic equation describing the axisymmetric toroidal equilibrium is a second-order partial differential equation, called the Grad-Shafranov equation. A wide variety of computational methods and codes have been developed to solve the Grad-Shafranov equation. Of particular interest are those that perform, through an iterative process, a simultaneous reconstruction to match electromagnetic experimental measurements, which include the toroidal plasma current, poloidal magnetic fluxes, motional Stark effect (MSE) data, and the diamagnetic toroidal flux. Unfortunately, such codes are not capable of yielding numerical results fast enough for real-time applications, such as on-line data analysis between successive discharges and plasma feedback control. A method developed to overcome this difficulty, the Swain-Neilson model [1], approximates the plasma toroidal current distribution by a finite number of filamentary currents within the plasma region. Several similar types of approximation methods have also been developed recently for fast determination of plasma equilibrium parameters [2, 3]. In general, they are quite useful for circular and slightly elongated plasmas. However, for highly indented plasmas, which can occur in the PBX-M device, these methods do not always converge well, nor do they yield sufficiently accurate results. Furthermore, if the experimental measurements include local quantities such as MSE data that must be matched at points within the plasma, filament models are not expected to be suitable for equilibrium reconstruction, unless the number of filaments is increased substantially.

In this report, we present a new method of rapid determination of MHD equilibrium parameters of tokamak plasmas based on the use of database files and a single iteration of Newton's method. The database files are computed from equilibrium solutions obtained through reconstruction of experimental measurements. The toroidal plasma current density $j_\phi(\psi, \theta)$ is determined in the usual equilibrium form in terms of $p'(\psi)$ and $F(\psi)F'(\psi)$ by representing them in rapidly converging series expansions, whose coefficients are computed in a least-squares data-fitting process. The design matrix elements are evaluated from averages of those in "closest" database files. The reconstruction of the current density

$j_\phi(\psi, \theta)$, although approximate, allows the determination of the internal q -profile from the MSE data, which cannot be accurately obtained using a current filament model. In addition, the procedure is rather insensitive to plasma shapes and thus equally useful for all shapes of plasmas, including highly indented ones. The computer code SURFAS has been developed to carry out the rapid equilibrium reconstruction for tokamak plasmas. The present version of SURFAS is considerably different from the original version written several years ago. In the original version, the poloidal dependence of $j_\phi(\psi, \theta)$ was represented in a Fourier series and the database files were generated by the code SURFIN, which evaluated matrix elements using a simple approximate reconstruction scheme of the equilibrium rather than from a solution of the Grad-Shafranov equation. Procedures involved in SURFIN and the original version of SURFAS are described in Appendix A.

II. Basic Formulation

The equation describing the ideal MHD equilibrium of an axisymmetric plasma is the Grad-Shafranov equation for the poloidal flux function $\psi(R, Z)$:

$$\Delta^* \psi \equiv R^2 \nabla \cdot \left(\frac{\nabla \psi}{R^2} \right) = 2\pi \mu_0 R j_\phi, \quad (1)$$

where

$$j_\phi = -2\pi \left(R p'(\psi) + \frac{F(\psi) F'(\psi)}{\mu_0 R} \right) \quad (2)$$

is the toroidal plasma current density, $p(\psi)$ is the pressure function,

$$F(\psi) = R B_\phi \quad (3)$$

is the toroidal field function, and the primes on $p'(\psi)$ and $F'(\psi)$ denote derivatives with respect to ψ . Note that in general, j_ϕ is a nonlinear function of ψ and R .

The numerical solution of the Grad-Shafranov equation is often obtained by expressing $p'(\psi)$ and $F(\psi) F'(\psi)$ in the series expansions

$$p'(\psi) = \sum_{n=1}^{N_p} \alpha_n p_n(\psi) , \quad (4)$$

$$F(\psi) F'(\psi) = \sum_{n=1}^{N_f} \beta_n f_n(\psi) , \quad (5)$$

and j_ϕ becomes a series expansion of the form

$$j_\phi(\psi, \theta) = \sum_{n=1}^N c_n J_n(\psi, \theta) , \quad (6)$$

where

$$N = N_p + N_f , \quad (7)$$

$$c_n = \begin{cases} \alpha_n, & 1 \leq n \leq N_p \\ \beta_{n-N_p}, & N_p < n \leq N \end{cases} , \quad (8)$$

$$J_n(\psi, \theta) = \begin{cases} -2\pi R(\psi, \theta) p_n(\psi) , & 1 \leq n \leq N_p \\ -\frac{2\pi f_n(\psi)}{\mu_0 R(\psi, \theta)} , & N_p < n \leq N \end{cases} . \quad (9)$$

Let y_m denote the m th data item representing an equilibrium quantity observed experimentally. The data include the total toroidal plasma current, the poloidal fluxes or external B-fields at observation points (or their differences), the internal Z-components of the magnetic fields (MSE data), and the diamagnetic toroidal flux. Because the safety factor at the magnetic axis q_0 is not determined experimentally unless an MSE measurement is made, it is useful to include a reasonable guess for q_0 in the experimental data set of $\{y_m\}$ if MSE data is not available. (Knowledge of the sawtooth inversion radius may be used to estimate q_0 .)

It is convenient to define the array $y^* = (y_m^*)$ which is that portion of the measurement due to the plasma current. Hence, y_m^* is obtained by subtracting the vacuum contribution due to external coil currents from the experimental measurement y_m . Therefore, y_m^* is a linear functional in $j_\phi(\psi, \theta)$ and can be represented in terms of c_n as

$$y_m^* = \sum_{n=1}^N A_{mn} c_n, \quad m = 1, 2, \dots, M, \quad (10)$$

where M is the number of experimental data and A_{mn} is the design matrix element, which generally involves a two-dimensional integration over the plasma cross-sectional domain.

In general, $M > N$, and the design matrix, $A = (A_{mn})$. Hence, c_n cannot be determined to satisfy Eq. (10) exactly for all m 's. The method of least-squares data fitting can be employed to evaluate c_n by minimizing the sum of squares of weighted residuals:

$$\epsilon^2 \equiv \sum_{m=1}^M W_m^2 \left(\sum_{n=1}^N A_{mn} c_n - y_m^* \right)^2, \quad (11)$$

where W_m are the weights.

Various numerical methods and codes [4–7] are known for determining the MHD equilibrium solution while matching the magnetic measurements. Throughout this paper, it is assumed that such a solution, obtained with $j_\phi(\psi, \theta)$ given by Eq. (2), is available. The equations needed to calculate the design matrix elements A_{mn} are presented in the remainder of this section.

For the plasma current density represented by Eq. (6), the design matrix elements A_{mn} appearing in Eq. (10) are evaluated by carrying out the two-dimensional integration over the plasma domain

$$A_{mn} = \int \int G(R_m, Z_m; R(\psi, \theta), Z(\psi, \theta)) J_n(\psi, \theta) R^{-1}(\psi, \theta) \sqrt{g(\psi, \theta)} d\psi d\theta, \quad (12)$$

where

$$\sqrt{g(\psi, \theta)} = R \left(\frac{\partial R}{\partial \psi} \frac{\partial Z}{\partial \theta} - \frac{\partial R}{\partial \theta} \frac{\partial Z}{\partial \psi} \right) \quad (13)$$

is the Jacobian of the transformation from (R, Z) to (ψ, θ) , and $G(R_m, Z_m; R, Z)$ is the Green's function corresponding to the measured data y_m^* , which represents the contribution at (R_m, Z_m) due to a horizontal circular unit current of radius R , and centered at $(R = 0, Z)$. We now give the Green's functions, or explicit expressions of A_{mn} , for the various quantities which are to be matched to experimental data.

Poloidal flux function [10]: ψ

$$\begin{aligned}
G_\psi(R_m, Z_m; R, Z) &= 2\pi \int_0^{R_m} R' B_Z(R', Z_m; R, Z) dR' \\
&= 2\pi R_m A_\phi(R_m, Z_m; R, Z) \\
&= 2\mu_0 \sqrt{RR_m} D(k) / k,
\end{aligned} \tag{14}$$

where

$$B_Z(R, Z) = \frac{1}{R} \frac{\partial}{\partial R} [R A_\phi(R, Z)], \tag{15}$$

$$D(k) = \left(1 - \frac{1}{2}k^2\right) K(k) - E(k), \tag{16}$$

$$k^2 = \frac{4RR_m}{(R + R_m)^2 + (Z - Z_m)^2}. \tag{17}$$

Here, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds [9], respectively. In Eqs. (14) and (15), B_Z and A_ϕ are the Z -component of the magnetic field and ϕ -component of the vector potential, respectively, due to a unit filamentary current source. Equation (14) is the solution of

$$\Delta^* G(R_m, Z_m; R, Z) = 2\pi\mu_0 R \delta(R - R_m) \delta(Z - Z_m). \tag{18}$$

R-component of the field [10]: B_R

$$\begin{aligned}
G_{B_R}(R_m, Z_m; R, Z) &= \frac{\mu_0}{2\pi} \frac{Z}{R [(R + R_m)^2 + (Z - Z_m)^2]^{1/2}} \\
&\times \left[\frac{R^2 + R_m^2 + (Z - Z_m)^2}{(R - R_m^2) + (Z - Z_m)^2} E(k) - K(k) \right]
\end{aligned} \tag{19}$$

Z-component of the field [10]: B_z

$$G_{B_z}(R_m, Z_m; R, Z) = \frac{\mu_0}{2\pi} \frac{1}{\left[(R + R_m)^2 + (Z - Z_m)^2\right]^{1/2}} \times \left[\frac{R^2 - R_m^2 - (Z - Z_m)^2}{(R - R_m)^2 + (Z - Z_m)^2} E(k) + K(k) \right] \quad (20)$$

Total toroidal plasma current: I_p

$$G_{I_p}(R_m, Z_m; R, Z) = 1 \quad (21)$$

Diamagnetic toroidal flux: $\Delta\Phi$

Integration of Eq. (5) yields

$$F^2(\psi) = F_b^2 + 2 \sum_n \beta_n h_n(\psi), \quad (22)$$

where

$$h_n(\psi) = \int_{\psi_b}^{\psi} f_n(\psi') d\psi', \quad (23)$$

$$F_b \equiv F(\psi_b), \quad (24)$$

and ψ_b is the boundary value of ψ . The vacuum toroidal field (due to external coil currents) is given by

$$B_{\phi 0} = \frac{F_b}{R}. \quad (25)$$

In general,

$$F_b^2 \gg 2 \left| \sum_n \beta_n h_n(\psi) \right|, \quad (26)$$

and the linearized approximation for $F(\psi)$ becomes

$$F(\psi) \simeq F_b + F_b^{-1} \sum_n \beta_n h_n(\psi). \quad (27)$$

The diamagnetic toroidal flux (due to the plasma poloidal current) is

$$\Delta\Phi = \int (B_\phi - B_{\phi 0}) d^2x. \quad (28)$$

This can therefore be approximated by

$$\Delta\Phi \simeq \frac{1}{F_b} \sum_n \beta_n \int \frac{h_n(\psi)}{R} d^2x, \quad (29)$$

and the design matrix elements for $\Delta\Phi$ are

$$A_{mn} = \begin{cases} 0, & 1 \leq n \leq N_p, \\ \frac{1}{F_b} \int \int \frac{h_{n-N_p}(\psi) \sqrt{g(\psi, \theta)}}{R^2} d\psi d\theta, & N_p < n \leq N. \end{cases} \quad (30)$$

Safety factor at the magnetic axis: q_0

By integrating the toroidal component of Ampere's law, $\mu_0 \mathbf{j} \cdot \nabla \phi = \nabla \cdot (\mathbf{B} \times \nabla \phi)$, in a volume enclosing the magnetic axis, one obtains the safety factor at the magnetic axis:

$$q_0 = \frac{(1 + \kappa_0^2) F(\psi_0)}{\mu_0 \kappa_0 R^2(\psi_0) j_\phi(\psi_0)}. \quad (31)$$

Here, ψ_0 and κ_0 are ψ and the ellipticity at the axis, respectively. Substituting Eqs. (6) and (27) at ψ_0 into Eq. (31) yields

$$(1 + \kappa_0^2) F_b = \mu_0 q_0 \kappa_0 R^2(\psi_0) \sum_{n=1}^N c_n J_n(\psi_0) - (1 + \kappa_0^2) F_b^{-1} \sum_{n=1}^{N_f} \beta_n h_n(\psi_0), \quad (32)$$

which is of the form of Eq. (10), with

$$y_m^* = (1 + \kappa_0^2) F_b, \quad (33)$$

$$A_{mn} = \begin{cases} \mu_0 q_0 \kappa_0 R^2(\psi_0) J_n(\psi_0), & 1 \leq n \leq N_p, \\ \mu_0 q_0 \kappa_0 R^2(\psi_0) J_n(\psi_0) - (1 + \kappa_0^2) F_b^{-1} h_{n-N_p}(\psi_0), & N_p < n \leq N. \end{cases} \quad (34)$$

III. Equilibrium Quantities

The SURFAS code determines a wide variety of quantities characterizing a tokamak equilibrium. In this section, we give definitions of these quantities and some useful identities. Many of them are related to volume integrals representing measures of the plasma energy content, including

$$W_p = \int p d^3x, \quad (35)$$

$$W_I = \int \frac{B_p^2}{2\mu_0} d^3x, \quad (36)$$

$$W_\phi = \int \frac{B_\phi^2}{2\mu_0} d^3x, \quad (37)$$

$$W_{\phi 0} = \int \frac{B_{\phi 0}^2}{2\mu_0} d^3x, \quad (38)$$

where

$$B_p = (B_R^2 + B_Z^2)^{1/2} \quad (39)$$

is the poloidal component of the field and the domain of integration is the plasma volume.

The dimensionless characteristic parameters are defined by [11]

$$\beta_I = \frac{4}{\mu_0 I_p^2 R_m} W_p, \quad (40)$$

$$l_i = \frac{4}{\mu_0 I_p^2 R_m} W_I, \quad (41)$$

$$\mu_I = \frac{-4}{\mu_0 I_p^2 R_m} (W_\phi - W_{\phi 0}), \quad (42)$$

$$S_2 = \frac{2}{\mu_0 I_p^2} \int \frac{1}{R} \left(p + \frac{B_p^2 - B_\phi^2 + B_{\phi 0}^2}{2\mu_0} \right) d^3x, \quad (43)$$

$$S_3 = \frac{2}{\mu_0 I_p^2 R_m} (3W_p + W_I + W_\phi - W_{\phi 0}) , \quad (44)$$

$$S_1 = S_3 - S_2 , \quad (45)$$

where R_m is an arbitrary scale length representing a mean major radius. Straightforward algebra shows that these parameters satisfy [11]

$$3\beta_I + l_i - \mu_I = 2(S_1 + S_2) , \quad (46)$$

$$\beta_I + l_i + \mu_I = 2S_2(1 - \delta) , \quad (47)$$

which also imply

$$\beta_I = \mu_I + S_1 + S_2\delta , \quad (48)$$

$$l_i = 2S_2 \left(1 - \frac{3}{2}\delta\right) - S_1 - 2\mu_I , \quad (49)$$

$$\beta_I + \frac{1}{2}l_i = \frac{1}{2}S_1 + S_2 \left(1 - \frac{1}{2}\delta\right) , \quad (50)$$

where

$$\delta = 1 - R_T/R_{r1} , \quad (51)$$

and R_T is the Shafranov major radius

$$R_T = \frac{\int (2\mu_0 p + B_p^2 + B_{\phi 0}^2 - B_\phi^2) d^3x}{\int R^{-1} (2\mu_0 p + B_p^2 + B_{\phi 0}^2 - B_\phi^2) d^3x} . \quad (52)$$

The usual definitions of poloidal, toroidal, and total betas are, respectively,

$$\beta_{\text{pol}} = \frac{W_p}{W_I} = \frac{\beta_I}{l_i} , \quad (53)$$

$$\beta_{\text{tor}} = W_p / W_\phi, \quad (54)$$

$$\beta_{\text{tot}} = \frac{W_p}{W_I + W_\phi} = \frac{1}{\beta_{\text{pol}}^{-1} + \beta_{\text{tor}}^{-1}}, \quad (55)$$

and the stored (thermal) energy is

$$W_s = \frac{3}{2} W_p. \quad (56)$$

For $|B_\phi - B_{\phi 0}| \ll |B_\phi|$, μ_I can be approximated in terms of $\Delta\Phi$ by

$$\mu_I \simeq -\Delta\Phi / \Delta\Phi_0, \quad (57)$$

$$\Delta\Phi_0 = \frac{\mu_0^2 I_p^2 R_T}{8\pi F_b}, \quad (58)$$

where $\Delta\Phi$ is defined in Eq. (28) and F_b in Eq. (24).

In SURFAS, the scale length R_m is taken to be R_T , or

$$\delta = 0. \quad (59)$$

The numerical calculation of S_2 and S_3 is carried out by expressing Eqs. (43) and (44) as contour integrals along the plasma boundary [11]:

$$S_2 = \frac{1}{2\pi\mu_0^2 I_p^2} \oint \frac{|\nabla\psi|^2}{R} dZ, \quad (60)$$

$$S_3 = \frac{1}{2\pi\mu_0^2 I_p^2 R_T} \oint \frac{|\nabla\psi|^2}{R} (n_R R + n_Z Z) ds, \quad (61)$$

where $ds = [(dR)^2 + (dZ)^2]^{1/2}$ and $\mathbf{n} = (n_R, n_Z)$ is the unit vector normal to the boundary. Derivations of Eqs. (60) and (61) are based on the use of the force balance equation $\nabla p = \mathbf{j} \times \mathbf{B}$ for ideal MHD equilibrium. In filament codes [1,2], Eqs. (60) and (61) are the only way of evaluating S_2 and S_3 . However, in SURFAS, which computes a distributed plasma

current, the volume integral definitions given by Eqs. (43) and (44) can also be used to obtain S_2 and S_3 .

Other parameters evaluated by SURFAS are

$$\begin{aligned} S_4 &= \frac{4}{\mu_0 I_p^2 R_m} W_{\phi 0} \\ &= \frac{4\pi F_b^2}{\mu_0^2 I_p^2 R_T} \oint \ln \left(\frac{R}{R_{\text{axis}}} \right) dZ, \end{aligned} \quad (62)$$

$$q_b = F_b \oint \frac{ds}{R |\nabla \psi|}, \quad (63)$$

$$\lambda = \frac{1}{2} l_i + \beta_I = \frac{1}{2} (S_2 + S_3), \quad (64)$$

where R_{axis} is the radial position of the magnetic axis and q_b is the safety factor at the boundary. In Eqs. (60)–(63), contour integrations are carried out along the plasma boundary.

A measure of the numerical accuracy of the computed equilibrium quantities can be given by the accuracy with which the Shafranov relations (46) and (47) are satisfied. Thus, SURFAS defines and computes two relative error functions

$$\varepsilon_1 = \left| \frac{2[(3\beta_I + l_i - \mu_I) - 2(S_1 + S_2)]}{(3\beta_I + l_i - \mu_I) + 2(S_1 + S_2)} \right|, \quad (65)$$

$$\varepsilon_2 = \left| \frac{2[(\beta_I + l_i + \mu_I) - 2S_2]}{(\beta_I + l_i + \mu_I) + 2S_2} \right|. \quad (66)$$

The plasma boundary is determined by values of ψ at scrape-off points and saddle points (separatrices). Let $\{(R_i, Z_i) : i = 1, 2, \dots\}$ be the set of the scrape-off and saddle points and $(R_{\text{lim}}, Z_{\text{lim}}) \in \{(R_i, Z_i)\}$ be the limiter point determined by

$$\psi_{\text{lim}} = \psi(R_{\text{lim}}, Z_{\text{lim}}) = \min\{\psi(R_i, Z_i) : i = 1, 2, \dots\}. \quad (67)$$

The value of ψ that determines the plasma boundary is defined by

$$\psi_b = \psi_0 + \alpha(\psi_{\text{lim}} - \psi_0) , \quad (68)$$

where α is the setback factor and usually, $\alpha = 0.95$.

IV. Generation of Database Files

A separate code, FS, has been developed to generate database files based on the MHD equilibrium form of $j_\phi(\psi, \theta)$ given by Eq. (2) and the flux coordinate system determined from a solution of the Grad-Shafranov equation. To obtain the solution, we use the Princeton Plasma Physics Laboratory (PPPL) FQ code [8], which is particularly convenient in the case of PBX-M, since it already has all of the necessary input structures for the system of external poloidal field coils, limiters, experimental observation system, etc. In both FQ and FS, $p'(\psi)$ and $F(\psi)F'(\psi)$ are expanded in the forms given by Eqs. (4) and (5), respectively, and their expansion coefficients, α_n and β_n , are determined by matching experimental observations through the least-squares data fitting process. In FS and SURFAS, the same functions $T_n^*(\hat{\psi})$ are used for both $p'(\psi)$ and $F(\psi)F'(\psi)$:

$$p_n(\psi) = f_n(\psi) = T_n^*(\hat{\psi}) - 1 \equiv u_n(\hat{\psi}) , \quad (69)$$

where

$$\hat{\psi} = \frac{\psi - \psi_0}{\psi_b - \psi_0} , \quad (70)$$

and $T_n^*(x)$ is the shifted Chebyshev polynomial of order n . (The FQ code uses a different, but equivalent, basis set.) The subtraction of 1 is introduced so that

$$u_n(1) = 0 , \quad (71)$$

which ensures the vanishing of $p'(\psi)$ and $F(\psi)F'(\psi)$ at the boundary, or

$$p'(\psi_b) = F'(\psi_b) = 0 . \quad (72)$$

This implies $j_\phi(\psi_b, \theta) = 0$. The number of terms for $p'(\psi)$ and $F(\psi)F'(\psi)$ needed to obtain reasonably accurate matching to the data are usually $N_p = 1$ or 2 and $N_f = 2$ or 3, respectively. While FQ carries out the numerical solution of the MHD equilibrium equation in the rectangular coordinate system (R, Z) , FS uses the flux coordinate system (ψ, θ) . The Jacobian of the transformation can be evaluated using Eq. (13) and the numerical results for $R(\psi_i, \theta_j)$ and $Z(\psi_i, \theta_j)$ obtained in FQ.

The vacuum poloidal field of PBX-M is generated by 42 individual external coils, but they consist of 9 independent coil groups, including the Helmholtz coil, which represents the buswork coils connecting the toroidal field coils. Since all coils in a group are connected in series, the vacuum contribution due to the external coil currents can be completely specified by the nine group coil currents. Let $\eta(\mathbf{x})$ denote each of the magnetic quantities $\psi, \partial\psi/\partial n, \mathbf{B}, I_p, \Delta\Phi$, etc., at $\mathbf{x} = (R, Z)$, generated by the plasma current distribution given by Eq. (6) and the external coil currents. Then, $\eta(\mathbf{x})$ can be expressed as

$$\eta(\mathbf{x}) = \sum_{n=1}^N c_n A_n(\mathbf{x}) + \sum_{k=1}^{N_g} I_k E_k(\mathbf{x}), \quad (73)$$

where $A_n(\mathbf{x}_m) = A_{mn}$ is defined by Eq. (12), N_g is the total number of the coil groups, and I_k is the k th group current. The external design matrix elements are given by

$$E_k(\mathbf{x}) = \sum_{l=1}^{N_k} T_{kl} G(R, Z; R_{kl}, Z_{kl}), \quad (74)$$

where N_k is the number of coils in the k th group; T_{kl} and (R_{kl}, Z_{kl}) are the number of windings and the location of the l th coil in the k th group, respectively; and $G(R, Z; R', Z')$ appearing in Eqs. (12) and (74) is the Green's function corresponding to $\eta(\mathbf{x})$.

Similar expressions for W_p , W_ϕ , and $W_{\phi 0}$, given by Eqs. (35), (37), and (38), respectively, do not involve external coil currents:

$$W_p = \sum_{n=1}^{N_p} \alpha_n P_n, \quad (75)$$

$$W_\phi = W_{\phi 0} + \sum_{n=1}^{N_f} \beta_n H_n, \quad (76)$$

$$W_{\phi 0} = \frac{F_b^2}{2\mu_0} \int \frac{d^3 \mathbf{x}}{R^2}, \quad (77)$$

where

$$P_n = \int d^3 \mathbf{x} \int_{\psi_b}^{\psi} p_n(\psi') d\psi', \quad (78)$$

$$H_n = \frac{1}{\mu_0} \int \frac{h_n(\psi)}{R^2} d^3 \mathbf{x}. \quad (79)$$

W_I given by Eq. (36) can be expressed in terms of c_n and I_k . Writing the poloidal components of $\mathbf{B}$ in the form of Eq. (73),

$$B_R(\mathbf{x}) = \sum_{n=1}^N c_n A_n^{(R)}(\mathbf{x}) + \sum_{k=1}^{N_g} I_k E_k^{(R)}(\mathbf{x}), \quad (80)$$

$$B_Z(\mathbf{x}) = \sum_{n=1}^N c_n A_n^{(Z)}(\mathbf{x}) + \sum_{k=1}^{N_g} I_k E_k^{(Z)}(\mathbf{x}), \quad (81)$$

we obtain

$$W_I = \sum_{n,n'} c_n c_{n'} B_{nn'} + \sum_{n,k} c_n I_k D_{nk} + \sum_{k,k'} I_k I_{k'} G_{kk'}, \quad (82)$$

where

$$B_{nn'} = \frac{1}{2\mu_0} \int d^3 \mathbf{x} \left[A_n^{(R)}(\mathbf{x}) A_{n'}^{(R)}(\mathbf{x}) + A_n^{(Z)}(\mathbf{x}) A_{n'}^{(Z)}(\mathbf{x}) \right], \quad (83)$$

$$D_{nk} = \frac{1}{\mu_0} \int d^3 \mathbf{x} \left[A_n^{(R)}(\mathbf{x}) E_k^{(R)}(\mathbf{x}) + A_n^{(Z)}(\mathbf{x}) E_k^{(Z)}(\mathbf{x}) \right], \quad (84)$$

$$G_{kk'} = \frac{1}{2\mu_0} \int d^3 \mathbf{x} \left[E_k^{(R)}(\mathbf{x}) E_{k'}^{(R)}(\mathbf{x}) + E_k^{(Z)}(\mathbf{x}) E_{k'}^{(Z)}(\mathbf{x}) \right]. \quad (85)$$

We now outline the structure of a database file to be used by the SURFAS code by listing the matrix elements and other data written in the file:

- Characteristic parameter array, which consists of eight coil group currents I_k (excluding the Helmholtz coil current) and computed (or fitted) values of poloidal fluxes ψ , MSE data B_Z , and the diamagnetic toroidal flux $\Delta\Phi$. This array is used in selecting the “closest” database file(s). Note that the use of the computed values, the right-hand side of Eq. (10), is more appropriate for the selection of database file(s) than the use of the experimental data y_m^* , since the matrix elements in the database correspond to the computed values rather than to y_m^* .
- Miscellaneous parameters, which include shot number, zero shot number, time slice, plasma current I_p , Helmholtz coil current I_9 , toroidal field B_0 at $x_0 = 1.65$ m, q_0 , κ_0 , magnetic axis R_{axis} .
- Design matrix elements A_{mn} defined by Eq. (12) for I_p , ψ , B_Z , and $\Delta\Phi$, and by Eq. (34) for q_0 .
- External design matrix elements $E_{mk} = E_k(\mathbf{x}_m)$ defined by Eq. (74), where $\mathbf{x}_m$ represents the same experimental observation points as those in $A_{mn} = A_n(\mathbf{x}_m)$. These are used to subtract the vacuum contributions from the experimental data to obtain y_m^* appearing in Eq. (10).
- Midplane points $\mathbf{x}_m = (R_m, 0)$ and $A_n(\mathbf{x}_m)$ and $E_k(\mathbf{x}_m)$ for $\psi(\mathbf{x}_m)$.
- Plasma boundary points $\mathbf{x}_m = (R_m, Z_m)$ and $A_n(\mathbf{x}_m)$ and $E_k(\mathbf{x}_m)$ for $\psi(\mathbf{x}_m)$ and $\partial\psi/\partial n$ at $\mathbf{x}_m$.
- Limiter points $\mathbf{x}_m$, $A_n(\mathbf{x}_m)$, and $E_k(\mathbf{x}_m)$ for $\psi(\mathbf{x}_m)$.
- Separatrix vicinity points $\mathbf{x}_m$, $A_n(\mathbf{x}_m)$, and $E_k(\mathbf{x}_m)$ for $\psi(\mathbf{x}_m)$.
- P_n , H_n and $W_{\phi 0}$, defined by Eqs. (78), (79), and (77), respectively.
- Matrix elements for W_I : $B_{nn'}$, D_{nk} , and $G_{kk'}$ defined by Eqs. (83)–(85).
- Quantities defined in Section III ($W_p, W_I, W_\phi, \beta_I, l_i, \mu_I, S_1, S_2, S_3, R_T, \beta_{\text{pol}}, \beta_{\text{tor}}, \beta_{\text{tot}}, \Delta\Phi, \varepsilon_1, \varepsilon_2, \psi_b, \psi_0, \kappa_0$) and parameters related to the plasma shape (defined in Appendix B).

When the MHD equilibrium solution is available, numerical calculation of most quantities going into the database file is straightforward. However, the evaluation of the two matrix elements $B_{nn'}$ [Eq. (83)] and D_{nk} [Eq. (84)] is somewhat involved for two reasons. First, they are four-dimensional integrals (with toroidal symmetry considered), while all the others are two-dimensional integrals. For example, the first part of $B_{nn'}$ is

$$B_{nn'}^{(a)} = \frac{1}{2\mu_0} \int d^3x \int d^2x' G(\mathbf{x}; \mathbf{x}') J_n(\psi', \theta') \int d^2x'' G(\mathbf{x}; \mathbf{x}'') J_{n'}(\psi'', \theta'') , \quad (86)$$

where $G(\mathbf{x}; \mathbf{x}')$ is the Green's function of B_R given by Eq. (19). In a typical case, 94% of the computing time is used to evaluate $B_{nn'}$ and D_{nk} , while the computation of the rest of the data in the database file takes only 6%. The second problem is the singularity of $G(\mathbf{x}; \mathbf{x}')$ at $\mathbf{x} = \mathbf{x}'$. Although the singularity at $\mathbf{x} = \mathbf{x}'$ can be avoided by using alternating grid points for $\mathbf{x}$ and $\mathbf{x}'$, high numerical accuracy is difficult to achieve unless a special scheme of integration is employed near the singular points. We remark here that the evaluation of W_I itself can be done simply by using Eq. (36) with

$$B_p^2 = (2\pi R)^{-1} |\nabla\psi|^2 . \quad (87)$$

V. Numerical Procedure

The procedures carried out by the SURFAS code are outlined in this section. The names of subroutines involved in each step are given in parenthesis.

Step 1 (MATNIT): Read and normalize characteristic parameters.

Let Y_{ji} represent the j th element of the characteristic parameter array, $I_p(i)$ the toroidal plasma current, and $B_0(i)$ the toroidal field at $x_0 = 1.65$ m, all in the i th database file. Each array contains N_{PR} elements, and there are N_{DB} database files. To normalize the array elements, we introduce (for $j = 1, 2, \dots, N_{PR}$)

$$A_j = \max\{\tilde{Y}_{ji} : i = 1, 2, \dots, N_{DB}\} , \quad (88)$$

$$A'_j = \tilde{Y}_{ji'}, \quad |\tilde{Y}_{ji'}| = \max\{|\tilde{Y}_{ji}| : i = 1, 2, \dots, N_{\text{DB}}\}, \quad (89)$$

$$B_j = \min\{\tilde{Y}_{ji} : i = 1, 2, \dots, N_{\text{DB}}\}, \quad (90)$$

and define the normalized element by

$$y_{ji} = \frac{\tilde{Y}_{ji} - \tilde{B}_j}{\tilde{A}_j - \tilde{B}_j}, \quad (91)$$

where $\tilde{Y}_{ji}$, $\tilde{A}_j$, and $\tilde{B}_j$ are given in Table I, which shows that four different normalization options are available (IPARM is the option input parameter). In the table, $K_{ji} = I_p(i)$ if j refers to a coil group current or a poloidal flux, and $K_{ji} = B_0(i)$ otherwise.

TABLE I. Definitions of $\tilde{Y}_{ji}$, $\tilde{A}_j$, and $\tilde{B}_j$

IPARM	0	1	2	3
$\tilde{Y}_{ji}$	Y_{ji}/K_{ji}	Y_{ji}/K_{ji}	Y_{ji}	Y_{ji}
$\tilde{A}_j$	A_j	A'_j	A_j	A'_j
$\tilde{B}_j$	B_j	0	B_j	0

Step 2 (SHOTRD): Read the shot data at all time slices t_k , $k = 1, 2, \dots, N_t$.

The data at each time slice includes the plasma current, the toroidal field coil current, and all quantities included in the characteristic parameter array.

Repeat Steps 3–12 for time slices t_k , $k = 1, 2, \dots, N_t$.

Step 3 (MOSSET): At a given time t_k , construct data array ($y_m^* : m = 1, 2, \dots, M$), the left-hand members in Eq. (10), and the characteristic parameter array ($X_j : j = 1, 2, \dots, N_{\text{PR}}$).

Step 4 (MATIN): Determine the “closest” database file and read data from this file ($N_A = 1$), or determine the closest N_A database files and obtain the data through weighted averaging of those read from these files ($N_A > 1$).

First, the characteristic parameter array elements X_j are normalized in the same way as is done for Y_{ji} in Step 1:

$$x_j = \frac{\tilde{X}_j - \tilde{B}_j}{\tilde{A}_j - \tilde{B}_j}, \quad (92)$$

where $\tilde{A}_j$ and $\tilde{B}_j$ are defined in Step 1, and

$$\tilde{X}_j = \begin{cases} X_j/K_j, & \text{IPARM} = 0 \text{ or } 1, \\ X_j, & \text{IPARM} = 2 \text{ or } 3. \end{cases} \quad (93)$$

Here, $K_j = I_p$ if j refers to a coil group current or a poloidal flux, and $K_j = B_0$ otherwise.

A measure of closeness of the shot configuration at a given time to the i th database file configuration can be given by the “distance” in the characteristic parameter space defined by

$$D_i = \left[\sum_{j=1}^{N_{\text{PR}}} w_j (x_j - y_{ji})^2 / \sum_{j=1}^{N_{\text{PR}}} w_j \right]^{1/2}, \quad i = 1, 2, \dots, N_{\text{DB}}, \quad (94)$$

where (w_j) is the parameter weight array.

Let the least element correspond to the I th database file:

$$D_I = \min\{D_i : i = 1, 2, \dots, N_{\text{DB}}\}. \quad (95)$$

There are two options for obtaining the data used in the rapid determination of the plasma equilibrium properties.

(A) Single database-file option ($N_A = 1$). We define $I_0 = I_0(k)$ such that the I_0 th database file is the actually chosen one representing the “closest” at time t_k . In general, $I_0 = I$. However, some exceptions to this general rule are useful when $D_I > 0$. Let D_J and D_K be the second and the third least members of $\{D_i\}$, respectively, ($D_I \leq D_J \leq D_K$), and

$$d_L = (D_L - D_I) / D_I, \quad (96)$$

$$L' = |L - I_0(k-1)|, \quad (97)$$

where $L = I, J$, or K . The exception rules follow:

[1] If $t_k - t_{k-1} > \text{DELTMN}$ or $I' = 0$, then $I_0 = I$ (no exception).

[2] If $I' = 1, J' = 0, d_J < \Delta_1$, then $I_0 = J$.

[3] If $I' = 1, K' = 0, d_K < \Delta_1$, then $I_0 = K$.

[4] If $I' > 1, J' \leq 1, d_J < \Delta_2$, then $I_0 = J$.

[5] If $I' > 1, K' \leq 1, d_K < \Delta_2$, then $I_0 = K$.

Here, DELTMN , $\Delta_1 \equiv \text{DELYD1}$, and $\Delta_2 = \text{DELYD2}$ are input parameters. Rules 4 and 5 should not be applied (let $\Delta_2 = 0$) if the database files are generated from multiple shots. The main purpose of these rules is to reduce the fluctuation or “jumping” of $I_0(k)$ as a function of time t_k . Matrix elements and other data are simply read from the I_0 th database file. It is to be noted that the chosen I_0 th database file is “closest” in the parameter space and not necessarily in time.

(B) Multiple database-file option ($N_A > 1$). Let $\{D_{I_n} : n = 1, 2, \dots, N_A\}$ be the subset containing the N_A least elements of $\{D_i : i = 1, 2, \dots, N_{\text{DB}}\}$, and define the normalized weights to be inversely proportional to the “distance” by

$$W_{I_n} = C/D_{I_n}, \quad (98)$$

where

$$C = \left[\sum_{n=1}^{N_A} 1/D_{I_n} \right]^{-1}. \quad (99)$$

The matrix elements and other data to be used in the rapid determination of the equilibrium are obtained by taking the weighted averages of those read from the N_A database files:

$$\gamma = \sum_{n=1}^{N_A} W_{I_n} \gamma_{I_n}, \quad (100)$$

where γ_{I_n} represents a data element in the I_n th file.

Step 5 (JPHI): Compute the coefficient array (c_n).

The data array (y_m^*) obtained in Step 3 and the design matrix (A_{mn}) obtained in Step 4 are used to solve Eq. (10) with a least-squares data fitting method. As in FQ, the singular value decomposition program SVD [12] is employed to compute the expansion coefficient array (c_n). It should be remarked here that while the data array elements y_m^* are exact (in the sense that they are the experimental measurements at the real time t_k), the design matrix elements A_{mn} are only approximate since they are obtained from the “closest” database file(s), which do not correspond to the real time t_k for the shot analyzed (except for special cases of $D_I = 0$).

Step 6 (MIDPSI): Calculate the magnetic axis R_{axis} .

Equation (73) is used to calculate the poloidal fluxes $\psi(\mathbf{x}_m)$ at midplane points $\mathbf{x}_m = (R_m, 0)$. The matrix elements $A_n(\mathbf{x}_m)$ and $E_k(\mathbf{x}_m)$ and points $\mathbf{x}_m$ are determined from the “closest” database file(s) in Step 4, expansion coefficients c_n are obtained in Step 5, and the external group coil currents $I_k = X_k$ ($k = 1, 2, \dots, N_g$) are the first N_g elements of the characteristic parameter array given in Step 3. The radial position of the magnetic axis R_{axis} is determined by the quadratic interpolation based on the $\psi(\mathbf{x}_m)$, which also determines $\psi_0 = \psi(R_{\text{axis}}, 0)$.

Step 7 (SEPRAT, SEARCH): Find saddle points.

The poloidal fluxes $\psi(\mathbf{x}_m)$ at the separatrix vicinity points $\mathbf{x}_m$ are computed using Eq. (73) as is done in Step 6. Then the saddle points are determined by the 9-point Lagrange interpolation based on the calculated $\psi(\mathbf{x}_m)$.

Step 8 (LIMIT): Calculate $\psi(\mathbf{x}_m)$ at scrape-off points $\mathbf{x}_m$.

Step 9 (SICRAP): Calculate ψ_b .

The boundary flux value ψ_b is determined by Eq. (68), with ψ_{lim} defined by Eq. (67).

Step 10: Compute the new plasma boundary.

The poloidal fluxes $\psi(\mathbf{x}_m)$ and the normal derivatives $\partial\psi/\partial n$ at the boundary points $\mathbf{x}_m$ are determined from the database file(s) in Step 4. The linear corrections to $\mathbf{x}_m$ given by

$$\delta\mathbf{x}_m = \frac{\psi_b - \psi(\mathbf{x}_m)}{\partial\psi/\partial n} \mathbf{n} \quad (101)$$

are computed and the new (corrected) boundary points

$$\mathbf{x}_m^{(b)} = \mathbf{x}_m + \delta\mathbf{x}_m \quad (102)$$

are obtained. In Eq. (101), $\mathbf{n}$ is the unit vector normal to the boundary $\{\mathbf{x}_m\}$.

Step 11 (GEOMET): Calculate geometrical parameters, which are related to the plasma boundary shape and defined in Appendix B, using $\{\mathbf{x}_m^{(b)}\}$.

Step 12 (S1S2): Evaluate equilibrium parameters β_I , l_i , μ_I , S_1 , S_2 , S_3 , β_{pol} , β_{tor} , β_{tot} , and other equilibrium quantities defined in Section III.

Numerical evaluation of μ_I , l_i , and β_I is carried out in two different ways:

- (A) (IDPHIM = 1) If the experimental data of $\Delta\Phi$ is available, this data is used to compute μ_I from Eq. (57), and β_I and l_i are determined using Eqs. (48) and (49), respectively, with $\delta = 0$.
- (B) (IDPHIM = 0) If the experimental data of $\Delta\Phi$ is not available, l_i is evaluated with Eqs. (41) and (82) and β_I with Eq. (50). μ_I is obtained by rewriting Eq. (49) as

$$\mu_I = \frac{1}{2} (2S_2 - S_1 - l_i) . \quad (103)$$

Step 13 (SURFAS): Output results in printout files and in plots.

There are more than 60 plots available, which are listed in Appendix C, together with many plot option parameters.

A flow diagram describing the overall procedure involved in the SURFAS code is shown in Fig. 1.

VI. Numerical Results

As test cases, we have chosen PBX-M Shots 298601 and 298602 with Zero Shot 298448. The full equilibrium solutions were obtained for Shot 298601, using the FQ code at 63 sample times, from 80 msec to 700 msec with the interval of 10 msec. The FS code was used to generate a database file at each sample time, and thus the database consists of 63 files ($N_{DB} = 63$) from Shot 298601. A subset database with 13 files ($N_{DB} = 13$), corresponding to sample times from 100 msec to 700 msec with a 50-msec interval, was also used to examine the effect of N_{DB} on the rapid reconstruction. The SURFAS code was run at 301 time slices ($N_t = 301$) from 100 msec to 700 msec with a 2-msec interval for Shots 298601 and 298602, using the two databases ($N_{DB} = 63$ and $N_{DB} = 13$), with $N_A = 1$ and $N_A = 3$. (N_A represents the number of closest database files used for averaging).

Typically, the Cray 2/128 CPU time required for FQ run at one time slice ranges from 50 sec (16 iterations) to 80 sec (30 iterations), and the CPU time required to evaluate matrix elements for a database file with the FS code is about 80 sec. The VAX CPU time needed to complete a SURFAS run for one time slice is usually about 0.1 sec. The numerical results for Shot 298601 obtained from the SURFAS code with $N_{DB} = 63$ and $N_A = 3$ are plotted in Figs. 2–13 for R_{axis} , ellipticity K_b , plasma volume, β_I , l_i , μ_I , S_1 , S_2 , ε_1 , ε_2 , β_{pol} , and β_{tor} , respectively. In these figures, open circles represent the results obtained with the FQ equilibrium code, and solid curves are those determined with the SURFAS code.

An obvious way of determining the accuracy of the SURFAS results is to compare them with the corresponding FQ results. A less rigorous but useful measure of accuracy is provided by the errors ε_1 and ε_2 defined by Eqs. (65) and (66), respectively. Table II lists time averages of ε_1 , ε_2 , and the average distance

$$D = \sum_{n=1}^{N_A} W_{I_n} D_{I_n} = N_A C, \quad (104)$$

which follows from Eqs. (98) and (100). In the table, ε_1 and ε_2 were obtained using μ_I determined from Eqs. (57) and (29), l_i from Eqs. (41) and (82), and β_I from Eqs. (40) and (35). The time averages of the errors were taken from 150 msec to 700 msec, whereas $\langle D \rangle$ was calculated from 100 msec to 700 msec. Results corresponding to $N_t = N_{DB} = 61$ were

obtained by taking 61 time slices from 100 msec to 700 msec with a 10-msec interval, so that they coincide with the 61 database file times. Results of the time averages for D , ε_1 , and ε_2 in this case may be regarded as the largest lower bounds of the corresponding quantities in applications of the SURFAS code for the database used.

TABLE II. Time averages of the distance D and errors ε_1 and ε_2

Shot	N_t	N_{DB}	N_A	$\langle D \rangle$	$\langle \varepsilon_1 \rangle$	$\langle \varepsilon_2 \rangle$
298601	61	61	1	0.0440	0.0098	0.0182
298601	61	61	3	0.0564	0.0154	0.0374
298601	301	63	1	0.0536	0.0194	0.0488
298601	301	63	3	0.0624	0.0170	0.0440
298601	301	13	1	0.0870	0.0352	0.0918
298601	301	13	3	0.1150	0.0325	0.0860
298602	301	63	1	0.0714	0.0333	0.0794
298602	301	63	3	0.0774	0.0298	0.0718
298602	301	13	1	0.1023	0.0336	0.0850
298602	301	13	3	0.1305	0.0325	0.0812

Fig. 1. Flow diagram of the SURFAS code.

Fig. 2. Radial position of the magnetic axis.

Fig. 3. Ellipticity defined by Eq. (B3).

Fig. 4. Plasma volume.

Fig. 5. β_I defined by Eq. (40).

Fig. 6. l_i defined by Eqs. (41) and (82).

Fig. 7. μ_I defined by Eq. (42).

Fig. 8. S_1 defined by Eq. (45).

Fig. 9. S_2 defined by Eq. (43).

Fig. 10. ϵ_1 defined by Eq. (65).

Fig. 11. ϵ_2 defined by Eq. (66).

Fig. 12. Poloidal beta defined by Eq. (53).

Fig. 13. Toroidal beta defined by Eq. (54).

Appendix A. Original Versions of SURFIN and SURFAS

The original versions of SURFIN and SURFAS are based on the following two fundamental assumptions. First, the toroidal plasma current density is given by

$$j_{\phi}(\psi, \theta) = c_0 (1 - \hat{\psi})^{\gamma_0} + c_1 \hat{\psi} (1 - \hat{\psi})^{\gamma_1} + \sum_{n=2} c_n \hat{\psi}^{n-1} (1 - \hat{\psi})^{\gamma_n} \cos(n-1)\theta, \quad (\text{A1})$$

where γ_n is a predetermined constant and c_n is a coefficient determined by matching experimental measurements.

The second assumption is that the flux coordinate system is given by

$$R(\psi, \theta) = \sum_m R_m \hat{\psi}^m \cos m\theta, \quad (\text{A2a})$$

$$Z(\psi, \theta) = \sum_m Z_m \hat{\psi}^m \sin m\theta, \quad (\text{A2b})$$

where (R_m, Z_m) are Fourier components of the plasma boundary. Thus, internal coordinates are determined uniquely and solely by the boundary.

The SURFIN code has three different modes of run:

- A. Slow (normal) mode: a least-squares data fitting is carried out to evaluate the expansion coefficients c_n of $j_{\phi}(\psi, \theta)$ [Eq. (A1)] by matching the total plasma current, safety factor at the axis (q_0), and poloidal fluxes measured at input observer positions. Iteration continues until the plasma boundary converges and a database file is generated for use in the fast mode of run or by the SURFAS code.
- B. Setup mode: the matrices necessary to do one Newton's step are generated and stored.
- C. Fast mode: database files generated in the slow mode are used for rapid determination of the MHD equilibrium properties.

The process for the slow-mode run of SURFIN (for given shots and zero shots at a given time slice) can be summarized as follows:

1. Estimate an initial plasma boundary $[R_b(\theta), Z_b(\theta)]$.

2. Fourier-analyze the boundary in the form

$$\begin{cases} R_b(\theta) &= \sum_m R_m \cos m\theta \\ Z_b(\theta) &= \sum_m Z_m \sin m\theta \end{cases} \quad (\text{A3})$$

3. Define internal grid points $[R(\psi_j, \theta_i), Z(\psi_j, \theta_i)]$ by using Eqs. (A2).

4. Evaluate design matrix elements A_{mn} in this flux coordinate system.

5. Using the given (input) data set $\{y_m^*\}$ and weights $\{W_m\}$, determine coefficients c_n by minimizing ϵ^2 defined by Eq. (11).

6. Perform a postprocess computation of matrix elements for ψ and $\nabla\psi$ at the boundary points, ψ at midplane points, ψ at scrape-off points, and ψ near separatrices.

7. Determine the new boundary value of the flux function ψ .

8. Obtain the new plasma boundary $[R_b(\theta), Z_b(\theta)]$ by adding the linear corrections given by Eq. (101) to the old boundary.

9. Compare the new boundary with the old one. If the new one is close enough to the old one, go to 10. Otherwise, go to 2.

10. Write the database file.

The SURFAS code is a small subset of SURFIN that corresponds to the portion of the fast mode of run and therefore carries out rapid approximate determination of MHD equilibrium properties by linear interpolation based on the database files generated by SURFIN in the slow mode. The major difference between the original and current versions of the SURFAS code lies in the method of calculating the MHD equilibrium quantities (and hence the database file), and also in the approximation method of rapid determination of the equilibrium.

Appendix B. Geometrical Parameters

To give expressions for parameters characterizing the plasma boundary shape, let $\{R_b(\theta), Z_b(\theta)\}$ be the set of the boundary points and assume the symmetry

$$R_b(\theta) = R_b(-\theta) , \quad (\text{B1a})$$

$$Z_b(\theta) = -Z_b(-\theta) , \quad (\text{B1b})$$

and assume that the boundary is indented at $\theta = \pi$. In terms of

$$R_{\max} = \max\{R_b(\theta)\} , \quad (\text{B2a})$$

$$R_{\min} = \min\{R_b(\theta)\} , \quad (\text{B2b})$$

$$Z_{\max} = \max\{Z_b(\theta)\} \equiv Z_b(\theta^*) , \quad (\text{B2c})$$

$$Z_{\min} = \min\{Z_b(\theta)\} = Z_b(-\theta^*) , \quad (\text{B2d})$$

the geometrical parameters are defined by

$$\text{Ellipticity:} \quad K_b = \frac{Z_{\max} - Z_{\min}}{R_{\max} - R_{\min}} \quad (\text{B3})$$

$$\text{Indentation:} \quad d = \frac{R_b(\pi) - R_{\min}}{R_{\max} - R_{\min}} \quad (\text{B4})$$

$$\text{Minor radius:} \quad a = \frac{1}{2} [R_{\max} - R_b(\pi)] \quad (\text{B5})$$

$$\text{Geometric center:} \quad R_{\text{geo}} = \frac{1}{2} [R_{\max} + R_b(\pi)] \quad (\text{B6})$$

$$\text{Triangularity:} \quad \Delta = \frac{R_{\text{geo}} - R_b(\theta^*)}{R_{\max} - R_{\min}} \quad (\text{B7})$$

$$\text{Shift:} \quad \Delta R = \frac{R_{\text{axis}} - R_{\text{geo}}}{R_{\max} - R_{\min}} . \quad (\text{B8})$$

Appendix C. Description of the Input File and Input Parameters

SURFAS.INI: Input file

First line: Comment. Appears at bottom of each plot page.

Second line: \$NOMINE Namelist label in program.

Lines following up to the \$ sign are namelist input variables.

Default values, shown here in square brackets [ ], are given in file DEFAULT.FOR.

After the \$ sign are two sets of channel information; the first one is for the SHUNT case and the second for ROG-IBW case.

INPUT PARAMETERS

The following four parameters are the diamagnetic loop corrections (henries) due to different coil groups. They are used only if IDLCOR = 1

AMDLOH OH coils

AMDLIF IF coils

AMDLEF EF coils

AMDLTF TF coils

APLASM Atomic weight of plasma [1.0]

BEDYSC Eddy field scale factor [1.0]

DBASE Database array

Database file extensions are 001, 002, . . . , NDBFIL(I)

**DELTIM Difference between time samples (sec) [0.002]

DELTMN Used only if NDBA = 1 for database file selection [0.009]

DELYD1 Used only if NDBA = 1 for database file selection [0.080]

DELYD2 Used only if NDBA = 1 for database file selection [0.190]

DLMOV Maximum movement for surface extrapolation (meters) [0.05]

DSCRAP Distance from scape-off point (meters) [0.025]

DTBND Time between boundary plots (>0 and IPLOT(7) = 1) [0]

FSCALE	Array of scale factors for archive data	
IBPLOT	Beta(perp), beta(par) condensed plots. (0 = no, 1 = yes)	[0]
IDLCOR	Diamagnetic loop correction (0 = no [default], 1 = yes)	
	See parameters AMDLOH, . . . above	
IDPHIM	Mode option for computing μ_i , l_i , and β_i . (See Step 12 on page 22)	
IDSCUT	Disruption cutoff of data	[1]
	0 = use only plasma current cutoff at each end (PLCUT)	
	1 = use plasma current at beginning and plasma disruption at the end	
	Time of disruption used for IPLOT(40) and for database dump	
	2 = same as 1 except that time for plasma shape given at the console is used for IPLOT(40) and database dump	
IDSHOT	Shot ranges for databases	
IEDCON	Line average density: = 0, from shot data [default]	
	= 1, constant value from terminal	
IEDDY	Eddy currents included? (0 = no [default], 1 = yes)	
IFILTP	Array of archive channel indicators: 1 = MOAS, 2 = MDAS	
IFIXBD	Fix boundary (XB,ZB), PSIMIN, PSILIM to those of FQ?	
	0: No (recompute to update)	
	1: Yes (use those from FQ without updating for debugging)	
IGDS	Graphics output (0 = terminal screen [default], 1 = GDS)	
IGXMIN	Window limit for boundary plot	[300]
IGXMAX	Window limit for boundary plot	[900]
IGZMIN	Window limit for boundary plot	[100]
IGZMAX	Window limit for boundary plot	[700]
INDFLX	Flux indices to be matched	[4, 3, 2, 1]
	Must be (4, 3, 2, 1) or (14, 15, 16, 17, 18)	
INOISE	Noise removal from TF coil data (0 = no [default], 1 = yes)	

IPARM	Option for normalization mode for characteristic parameters	[1]
	[See Eqs. (88)-(91) and Table I.]	
	BPARM(J)	Divide by I_p or B_0 ?
0	Minimum	yes
1	0	yes
2	Minimum	no
3	0	no
IPAUSE	SG graphics pause (0 = no [default], 1 = yes)	
IPLFIX	Option to set ranges for time plots from terminal	[0]
	0: the code sets the ranges	
	1: user inputs the ranges from terminal	
IPLLOT	Array of plot selectors (See description and table on pages)	
IPRINT	Print switch	
= 0:	No print out	
= 1:	Print out in file SURFAS.OUT (small amount)	
= 2:	Print out in file SURFAS.OUT (large amount)	
= -1:	Debug-dump characteristic parameters in file MATDMP.OUT	
= 4,5:	Print out computed matrix elements to compare with those in the database file; sets IFIXBD = 1, NDBA = 1	
ISHOT	Shot number	
ISHOT0	Baseline shot number	
ISUDM	Archive PK:[PBX.SU.DATA]SUDMxxx.xxx for diamagnetic calculation (0 = no [default], 1 = yes)	
ITAVER	Number of points averaged for archive data smoothing	[15]
ITDUMP	Option to dump database (TDMPxx.xxx) at time given at terminal for plasma shape or at disruption cutoff. For latter see IDSCUT, above (0 = no [default], 1 = yes)	

**ITIMST	Call TIMSET? (0: no, 1: yes)	[1]
IVALID	Use which measurements for SVD data matching?	[7*1]
ITTF	Print total toroidal flux (TTFLUX)? (0: no, 1: yes)	[0]
JPRINT(1) = 1,	Generate FQ.P (file containing FQ results)	
= 2,	Read from FQ.P	
= 0,	None of the above	
JPRINT(2) = 1,	Write (in SURFAS.OUT) TIMES,DMIN1,DMIN2,DMIN3, DMINA,WCHMAT,JMIN1,JMIN2,JMIN3,ESR1,ESR2	[1]
JTAVER	Number of points averaged to smooth out time-plots Used only for XLSUB1, XLSUB2	[9]
JPLOT	Array of plot options (see the description at the end)	
KCHAN	Array of archive channel numbers	
NCHAN	Number of channels for reading archive data	[8]
NGDS	GDS screen designator (FSxx). Used when IGDS = 1	[0]
NGRUPS	Number of external coil groups	[8]
NDBA	N _A : Number of "closest" database files to be used for averaging; must be ≤ 3	[3]
NDBASE	Number of databases DBASE	
NDBFIL	Array for number of database files	
**NTIMES	Number of time samples (= number of shapes calculated)	[100]
PARWHT	Weight array (w_j) for selecting the closest database file; used in Eq. (94)	
PLCUT	Plasma current (amps) cutoff for time range of data Do not solve when $I_p < \text{PLCUT}$	[1.e+5]
PSIRAT	Setback factor α for scrape off	[0.95]
Q00	q_0 asymptotic decay value	[0.9]
QT0	q_0 at $t = 0$	[0.9]

RELERR	Relative error for least-squares fit (SVD)	[1.e-6]
T0ESR	Evaluate time averages of ϵ_1 and ϵ_2 from T0ESR to TIMES(NTIMES)	
TAUQ	Q decay constant	[0.0]
TBOUND	Time for Boundary plot in a summary plot	[0.5]
TIMES	Array of sample times	
TQ0	Time start for q_0 decay	[0.0]
**TZERO	Starting time for data sampling = TIMES(1)	[0.1]
WGHTX	Weight array for SVD data matching	[7*1.0]
XNOISE	Option to remove noise spikes from time plots	[2.0]
ZEFF	Z effective	[1.5]

**Note: These parameters are used to define TIMES(I).

PLOTTING UNDER CONTROL OF ARRAY “IPLOT” AND “JPLOT”

IPLOT(K) = 0, no plot [default]
 = 1, SURFAS plot, no FQ.P points
 = 2, SURFAS plot, plus FQ.P points

JPLOT(1) overrides IPLOT(K) [1]
 = 0, no plots [same as IPLOT(K) = 0 for all K's]
 = 1, SURFAS plots only
 = 2, SURFAS plots, plus FQ.P points [if IPLOT(K) = 2]

JPLOT(2) Symbol for FQ.P points [1]

JPLOT(3) Size for FQ.P points = 0.1 * JPLOT(3) [5]

* Plots 42–45, 49, 60–63 have more than one trace on a time axis.

** All plots are full-page plots vs time (sec) except those for K = 7, 30, 40, and 50.

K	Plot
** 7	Plasma boundary (see DTBND above)
18	λ ; Eq. (64)
19	Geometric center R_{geo} ; Eq. (B6)
20	β_I ; Eq. (40)
21	Average “distance” of N_A closest database files; Eq. (94)
22	l_i ; Eqs. (49) and/or [(41) and (82)]
23	μ_I ; Eq. (57) or (42)
24	Indentation; Eq. (B4)
25	Triangularity; Eq. (B7)
26	q_b ; Eq. (63)
27	Shift of magnetic axis; Eq. (B8)
28	Ellipticity; Eq. (B3)

- 29 WCHMAT, average index of database files chosen
- ** 30 Summary plots. Five pages, up to four per page.
  - 31 S_1 ; Eq. (45)
  - 32 S_2 ; Eq. (60)
  - 33 S_3 ; Eq. (61)
  - 34 X of scape-off point
  - 35 Z of scape-off point
  - 36 B_x (external)
  - 37 B_z (external)
  - 38 B_x (eddy)
  - 39 B_z (eddy)
- ** 40 Page of text similar to database dump
  - 41 $\Delta\Phi$ from Eq. (57); the same as $K = 90$ if IDPHIM = 1
- * 42 Minor radius and geometric center
- * 43 Plasma current, loop voltage, and surface voltage
- * 44 Energy stored, power injected, and power absorbed
- * 45 Plasma volume, (indentation current)/(plasma current)
  - 46 Total toroidal flux within the plasma boundary
  - 47 ψ_b (boundary); Eq. (68)
  - 48 $l_i \times$ (plasma current)
- * 49 $\Delta\Phi$ and $\Delta\Phi_0$
- ** 50 q_b (edge) vs $Ne \times R / B_t$
  - 51 Loop voltage
  - 52 Surface voltage
  - 53 Minor radius; Eq. (B5)
  - 54 Stored energy; Eq. (56)
  - 55 Injected power

56	Absorbed power
57	Volume
58	Area
59	Energy confinement time (τ_e)
* 60	Beta-toroidal, λ , beta-poloidal
* 61	Beta-toroidal, plasma current, power injected
* 62	I_p , I_p (smoothed), dI_p/dt
* 63	Beta-toroidal, power injected, beta-poloidal
64	Magnetic axis, R_{axis}
65	ψ_0 (axis)
71	ϵ_1 ; Eq. (65)
72	ϵ_2 ; Eq. (66)
73	Beta-poloidal; Eq. (53)
74	Beta-toroidal; Eq. (54)
75	Beta-total; Eq. (55)
80	Plasma current I_p
81	CGRUP(1): $I(EF)$
82	CGRUP(2): $I(IF)$
83	CGRUP(3): $I(DF)$
84	CGRUP(4): $I(T1)$
85	CGRUP(5): $I(T2)$
86	CGRUP(6): $I(T3)$
87	CGRUP(7): $I(T4)$
88	CGRUP(8): $I(OH)$
89	CGRUP(9): $I(HE)$, Helmholtz current
90	DPHIT = $\Delta\Phi$: Diamagnetic flux (due to plasma poloidal current)
91	DSIOBP(1): Plasma flux Ds(5-1) or FL1P

- 92 DSIOBP(2): Plasma flux Ds(4-1) or FL2P
- 93 DSIOBP(3): Plasma flux Ds(3-1) or FL3P
- 94 DSIOBP(4): Plasma flux Ds(2-1) or FL4P
- 95 DSIOBP(5): Plasma flux FL5P

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References

- [1] D. W. Swain and G. H. Neilson, Nucl. Fusion 22 (1982) 1015.
- [2] L. L. Lao, H. St. John, R. D. Stambaugh, and W. Pfeiffer, Nucl. Fusion 25 (1985) 1421.
- [3] B. J. Braams, W. Jilge, and K. Lackner, Nucl. Fusion 26 (1986) 699.
- [4] J. L. Luxon and B. B. Brown, Nucl. Fusion 22 (1982) 813.
- [5] J. Blum, J. Ch. Gilbert, and B. Thooris, Paramagnetic Identification of the Plasma Current Density from Magnetic Measurement and Pressure Profile, Report of JET Contract No. JT3/9008, Abingdon, Oxfordshire (1985).
- [6] L. L. Lao, H. St. John, R. D. Stambaugh, A. G. Kellman, and W. Pfeiffer, Nucl. Fusion 25 (1985) 1611.
- [7] Yu. K. Kuznetsov, V. N. Pyatov, and I. V. Yasin, Sov. J. Plasma Phys. 13 (1987) 75.
- [8] C. E. Kessel, F. Hofmann, S. C. Jardin, B. Kaita, F. Levinton, and M. Okabayashi, Bull. Am. Phys. Soc. 34 (1989) 2050.
- [9] M. Abramowitz and I. A. Stegun, eds., Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55 (Dover, New York, 1965) ch.17.
- [10] W. R. Smythe, Static and Dynamic Electricity, 3d ed. (McGraw-Hill, New York, 1968) p. 291.
- [11] V. D. Shafranov, Plasma Phys. 13 (1971) 757.
- [12] G. E. Forsythe, M. A. Malcolm, and C. B. Moler, Computer Methods for Mathematical Computations (Prentice-Hall, Englewood Cliffs, N.J., 1977), ch. 9.

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