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QUARK LINE DIAGRAMS, RULES AND SOME RECENT DATA

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There has been a great deal of interest recently in the so-called OZI rule,¹⁻³ also referred to as the Quark-Line Rule.⁴ This note will refer to various aspects of this rule and point out that the Quark Line Rule alone has some serious limitations. However, by an appropriate expansion into a set of quark line diagram rules based on observations and other dynamical considerations it may be possible to develop Quark Line Diagram techniques that are applicable to hadron dynamics in a qualitative and perhaps even semi-quantitative sense analogous to the way Feynman graphs and their associated rules are used in both a qualitative and semi-quantitative sense in many instances of hadron dynamics. We shall concern ourselves in this note primarily with the u, d, and s quarks.

Okubo has recently written a survey of the Quark Line Rule (UR641, Department of Physics and Astronomy, Rochester, New York) in which he has reviewed the Quark Line Rule (or OZI rule) both theoretically and experimentally.

The algebraic version of the QLR was first proposed by Okubo¹ and referred to as the moment Ansatz. He proposed that the singlet member of the octet, ω_8 and the singlet ω_0 following from the SU(3) symmetry for mesons be considered one unified nonet which is represented by a non-traceless tensor $G_{\nu}^{\mu}(\mu, \nu = 1, 2, 3)$. The nonet Ansatz was to demand that the
$$\text{Tr } G = \sum_{\lambda=1}^3 G_{\lambda}^{\lambda}$$
 should not enter into any physical expression at least

within the first order approximation. Under this assumption he showed that $M(\phi \rightarrow \rho\pi) = 0$, $M(\phi \rightarrow \pi^+\pi^-\pi^0) = 0$. The physical ϕ and ω are allowed to be coherent mixtures of ω_8 and ω_0 and when the mixing angle $\theta = \tan^{-1}(1/\sqrt{2})$ the above matrix element relations are true and this mixing angle is known as ideal mixing. This ideal mixing gave rise to nonet mass formulae in very good agreement with experiment for the 1^- nonet.

Zweig in his papers² noted that the nonet Ansatz can easily be reinterpreted in terms of the quark model.

Let q_1, q_2, q_3 be u, d and s quarks respectively and represent the non-traceless tensor of G_V^μ as

$$G_V^\mu = q_\nu \bar{q}_\mu$$

where each element represents a bound state of a quark anti-quark.

Then the ideal mixing angle for the nonet results in the ϕ -meson being regarded as a bound state of $s\bar{s}$ whereas the ω -meson consists of $u\bar{u}$ and $d\bar{d}$ but not $s\bar{s}$ components.

Then, if we draw the following quark diagrams following Zweig's approach (see Figs. 1 and 2), we see that the $\omega \rightarrow \pi^+\pi^0\pi^-$ can be drawn as a graph containing only connected quark lines whereas for $\phi \rightarrow \pi^+\pi^0\pi^-$ the quark lines are disjointed and we have the so-called hairpin diagram which is considered forbidden (at least to a substantial suppression factor). The nonet Ansatz by Okubo and also found independently by Iizuko et al. would make the same predictions.

However, the quark line diagram rule is of course more useful in visualizing processes and in fact as one gets to diagrams of greater complexity such as in hadron interactions which lead to productions of other hadrons the Quark Line diagrammatic approach clearly can be a great aid to one's thinking provided it told the whole story.

As far as ϕ decay goes, the value⁵ of $\Gamma(\phi \rightarrow \pi^+ \pi^- \pi^0) / \Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0) \simeq 0.074$. The matrix element for these decays $M(\phi \rightarrow \pi^+ \pi^- \pi^0) / M(\omega \rightarrow \pi^+ \pi^- \pi^0) \simeq 0.10$. $(g_{\phi\rho\pi} / g_{\omega\rho\pi})^2$ is estimated⁴ to be $\simeq 0.007$. Thus the QLR seems to work (strong suppression of a hairpin diagram seems to be quite good, i.e. suppression by a factor ~ 100).

On the other hand, the diagram $\phi \rightarrow K^+ K^-$ (see Fig. 3) is allowed by the QLR and about half the ϕ 's decay by this diagram and another $\approx 35\%$ decay by $K_L K_S$ which is also an allowed diagram.

A point which Okubo⁴ made is that if one considers the QLR allowed diagram (see Fig. 4) for the process $K^- K^+ \rightarrow \pi^- + \rho^+$ and then considers a two step process each of which is allowed by the QLR, $\phi \rightarrow K \bar{K}$ and $K \bar{K} \rightarrow \pi^- \rho^+$. The combined diagram, Fig. 5a, is allowed. This diagram he states is topologically equivalent to the QLR forbidden diagram, Fig. 5b. He therefore finds this curious and attempts to explain this inconsistency by some mathematical arguments involving cancellations in the intermediate states.

I would like to point out that if in Fig. 5a one pulled the part of the quark line diagram containing the ϕ far enough to the left, one would again obtain the disjointed diagram in Fig. 5b which would be QLR forbidden.

Therefore, it is obvious that a second requirement must be placed on drawing quark line diagrams, namely that quark lines which start in the initial state can only be joined together in the initial state before they interact with quark lines in the final state. Otherwise one could always turn a disjointed diagram of the hairpin type into a connected diagram. We shall call this QLR Number 2.

This QLR #2 implies that quark line diagrams cannot contain final state interactions. Let us now consider quark line diagrams for the ϕ meson as produced in hadron-hadron collisions. We will be especially

interested in non-strange hadron initial systems as these will involve disconnected diagrams (QLR forbidden) for ϕ production where there are no kaons in the final state and will be QLR allowed when one or more $K\bar{K}$ pairs are produced in the final state. Fig. 6 shows the Quark Line Diagram for $\pi^- p \rightarrow \phi + n$ with the expected hairpin diagram for the ϕ . Of course, one hopes that Quantum Chromodynamics will eventually be able to explain hadron dynamics. In the connected diagrams according to quantum chromodynamics pairs of quarks in a hadron particle are bound together by single gluon exchanges which for simplicity we have not shown. In disconnected diagrams such as a hairpin, at least two gluon exchanges are required for scalar and three for vector particles. Therefore the single ϕ hairpin is connected via a triple gluon exchange.

Let us now consider the reaction $\pi^- p \rightarrow \phi\phi n$. This can be represented by the diagram in Fig. 7, where the $\phi\phi$ system requires two gluon exchange (2 dotted lines) if it is a scalar, and three gluon exchange (2 dotted lines and dash-dot line) if it is a vector.

One might naively think that if ϕ production requires one hairpin, perhaps $\phi\phi$ production requires two. This obviously is not correct since one can always combine the two ϕ hairpins into one disconnected part of the graph as shown for $\phi\phi$.

In fact, $\phi\phi$ production is no more forbidden by the simple Quark Line Rule than single ϕ production.⁺ From the point of view of QCD, $\phi\phi$ production in a scalar state requires only two gluon exchange and is thus less forbidden than ϕ production which requires three gluon exchange.*

+ This ignores the possibility that one may pay an additional penalty factor for creating a second $s\bar{s}$ quark pair. The subject of penalty for $s\bar{s}$ creations will be discussed later.

* We are assuming so-called precocious asymptotic freedom in which the gluon coupling constant is small even at these moderate energies.

Let us now consider the process $\pi^- p \rightarrow K^- K^+ \phi N$ where N is a system with no strange particles with a baryon number = 1. For simplicity we take N = neutron however all the conclusions are true for the general system N and this applies to previous diagrams where we have used n (neutron) - these can be replaced by N without changing the conclusions in regard to the simple Quark Line Rule. Fig. 8 shows that $\pi^- p \rightarrow K^+ K^- \phi n$ is a connected quark line diagram* and allowed by the simple OZI rule. In this case if we were to join together in the final state the $\bar{u}$ line from the π^- with u line from the proton we would obtain the process $\pi^- p \rightarrow \phi \phi n$ which is forbidden by the OZI rule, thus again illustrating the necessity of QLR #2 which forbids this procedure. Fig. 9 shows that the process $\pi^- p \rightarrow K^+ K^- \left(\begin{smallmatrix} N \\ n \end{smallmatrix} \right)$ is allowed by the simple OZI rule. If in the final state we joined the $\bar{u}$ line from the π^- with the u line from the K^+ , we would have $\pi^- p \rightarrow \phi n$ as an allowed process. This again illustrates the necessity of QLR #2 which forbids this procedure.

Fig. 10 shows that the process $\pi^- p \rightarrow K^+ K^- K^+ K^- n$ is also a connected quark line diagram and thus OZI allowed. In all the foregoing we have implicitly assumed that as long as we have a quark line connected diagram it does not cost us any penalty to create a $q\bar{q}$ pair except perhaps the change in the phase space factor.

As long as we restrict ourselves to $u\bar{u}$ and $d\bar{d}$ pairs this is probably not too bad as a crude approximation. However, when we create an $s\bar{s}$ pair, experiments suggest that we pay a penalty of ~ 10 -100 (i.e. ~ 10 at Fermilab energy $\sim$ several hundred GeV) 30-100 at AGS and CERN energies (~ 20 GeV).

* The crossing over of the two quark lines from top to bottom is a quark rearrangement which should not affect the connected status of the diagram. It can be considered equivalent to a single pion exchange.

We shall explore this point in further detail shortly but for the time being let us postulate QLR #3 that creating a quark pair costs us a penalty of ~ 10 -100 for the first $s\bar{s}$ pair when there were originally no strange quarks previously. Vice versa, if we start with strange quarks only and go to ordinary quarks, we may also pay a penalty, but probably a much smaller one.

If we also create a second $s\bar{s}$ pair we may pay a much smaller penalty than for the first since it may be easier for the first $s\bar{s}$ pair to help create a second pair of the same type then it was for u and d and their antiquarks.

Taking account of the QLR one could argue that the overall suppression factor (OSF) for a hairpin diagram such as occurs in $\pi^- p \rightarrow \phi + n$ is due to the product of two factors, the first being the suppression factor for creating a pair of strange quarks (SFSQ) and the second suppression factor being due to the disconnected hairpin (HSF) requiring multi-gluon exchange.

Thus, for example, if $\pi^- p \rightarrow \phi n$ has a typical suppression factor of ~ 100 , if we assume SFSQ ~ 10 -30, then the HSF (hairpin suppression factor may only be ~ 3 -10) - much less than previously thought.

Furthermore, if we compare the processes $\pi^- p \rightarrow \pi^+ \pi^- \phi n$ and $\pi^- p \rightarrow K^+ K^- \phi n$ with the above stated suppression factors, the two cross sections may turn out to be comparable even though $\pi^- p \rightarrow K^+ K^- \phi n$ is allowed by the simple QLR whereas $\pi^- p \rightarrow \pi^+ \pi^- \phi n$ is forbidden by the simple QLR. As we shall see shortly the experimental data indicate the two cross sections are probably comparable. Thus QLR #3 may indeed be required.

Incidentally the penalty we have to pay for creating an additional pair of charmed quarks may be much larger and that may explain why although the simple QLR seems to work much better for charmed quarks in the ψ -decay,

yet $\psi\bar{D}\bar{D}$ is not comparable in the production process to ψ + ordinary hadrons even though the former is allowed by the QLR and the latter is not. It should be noted that the concept of penalty factors downgrades both the simplicity and credibility of quark line rules.

Let us look at some of the recent CERN data⁵ based on observation of 45 ϕ 's produced by 19 GeV/c π^- incident on proton. They conclude:

$$\frac{\sigma(\phi\pi^+\pi^-\pi^-p)}{\omega\pi^+\pi^-\pi^-p} \frac{0.7 \mu b}{130 \mu b} = 0.005^{+0.005}_{-0.002}$$

which gives the typically expected simple OZI rule suppression of ϕ .

When they compare*

$$\frac{\phi K^+K^-\pi^-p}{\rho^0 K^+K^-\pi^-p} \approx 0.45^{+0.25}_{-0.15}$$

Since these are both OZI rule allowed processes, they again find reasonable agreement with the QLR.

However, when they compare*

$$\frac{\phi\pi^+\pi^-\pi^-p}{\phi K^+K^-\pi^-p} = \frac{44 \pm 10}{1.15 \times (22 \pm 6)} = 1.7^{+0.9}_{-0.5}$$

they find the numerator which is a OZI rule (or QLR) suppressed process is comparable to the denominator which is an OZI rule (or QLR) allowed process.

Thus the penalty factor in producing an additional pair of kaons at their energies appears to be comparable to or larger than the Zweig suppression. Lipkin⁶ has made this argument to explain QLR violations. Thus QLR #3 must always be kept in mind.

A BNL/CCNY group has recently submitted for publication⁷ (Phys. Rev. Letters, BNL Report #23701, dated December 14, 1977) the results of an experiment with the BNL Multiparticle Spectrometer which studied the reactions:

* Acceptance corrections have not been made in detail and these are to be considered crude estimates.

$$\pi^- p \rightarrow K^+ K^- K^+ K^- N \quad (1)$$

$$\rightarrow K^+ K^- \phi N \quad (2)$$

$$\rightarrow \phi \phi N \quad (3)$$

where N is mostly neutrons.

In this first publication, Reaction (3) was primarily discussed, since it was the first observation of this low cross section ~ 25 nb process. It should be noted that in the BNL/CCNY publication, their Fig. 2 (which is Fig. 11 of this paper) shows a scatter plot of the effective mass of one $K^+ K^-$ pair on the y-axis versus the effective mass of the second $K^+ K^-$ pair on the x-axis. Each point is plotted twice due to the double ambiguity in selecting a pair. Thus we clearly see a sizeable ϕ band along each axis. In fact, with a ϕ resolution of ~ 15 MeV there was an increase in the peak level of the $K^+ K^-$ effective mass plot in the ϕ band of about a factor of ~ 10 , and where the two ϕ bands overlapped a substantial darkening is seen which corresponds to an additional increase in peak level by about a factor of 10. If one had never heard of the OZI or Quark Line Rule, this result would be quite understandable on the following basis. The ϕ can be considered a fairly elastic resonance in the $K^+ K^-$ scattering. Thus when one produces a $K^+ K^- K^+ K^-$ state by any mechanism the matrix element for a particular correlation between the kaons becomes enhanced by a factor whenever the $K^+ K^-$ are in the ϕ mass band. When the two pairs of $K^+ K^-$ are both in the ϕ -mass band this final state interaction is enhanced by the enhancement factor squared. Our experimental resolution of ~ 15 MeV is about three times the ϕ width, therefore in this consideration the enhancement factor at the peak should be probably larger by about a factor of 3. Nevertheless, we can ignore the resolution effect when comparing single ϕ and double ϕ production since it will cancel out.

Thus the K^+K^- effective mass plot peak enhancement factor ≈ 10 in producing single ϕ would be expected to correspond to a peak enhancement factor ~ 100 in producing a $\phi\phi$ pair compared to $2K^+2K^-$ regions just outside the ϕ bands which is indeed highly consistent with the observations.

Remember however that the OZI-Quark Line Rule has been completely ignored in these considerations. In fact as the author showed earlier, final state interactions cannot be allowed in the QLR framework as they will defeat hairpin or disconnected diagrams.

In the QLR framework $\pi^-p \rightarrow \phi K^+K^-N$ is an allowed reaction whereas $\phi\phi N$ is a forbidden reaction. Thus the author would expect that if the QLR were strictly operative where the two ϕ bands cross in the K^+K^- by K^+K^- effective mass scatter plot one would observe no enhancement except for a factor of two due to the crossing of the two ϕ bands since once a single ϕK^+K^- is produced (QLR allowed), there would be a prohibition or very strong suppression against producing a second ϕ . In particular one would hardly expect the same enhancement factor for producing the first ϕ to occur when producing the second.*

The author is well aware that previous arguments on the Quark Line Rule in production processes have restricted themselves to comparing total cross sections for various reactions and never have applied the differential comparison argument made here. However, I can think of no reason why this differential comparison argument is not correct since if one considers the differential cross section (i.e. a limited region of $K^+K^-K^+K^-$ phase space around the single ϕ bands) $K^+K^-\phi n$ can be created without a disconnected diagram (i.e. a series of one gluon exchanges) whereas $\phi\phi N$ requires a disconnected diagram (i.e. two or three gluon exchanges).

* There is the possibility that the observed $\phi\phi$ enhancement comes from decay of a resonance(s) (i.e. strangeonium) into $\phi\phi$ which is allowed by the QLR. Reference 7 has alluded to this possibility but there is not as yet any convincing evidence for this. A new investigation of much higher sensitivity is planned.

Since we are in the above always dealing with $K^+K^-K^+K^-n$ states, any penalties for the creating of $s\bar{s}$ pairs have already been paid in all cases, and are not a consideration.

Nevertheless since the QLR is conventionally discussed in terms of ratios of total cross sections let us also consider the ratios of

$$\pi^- p \rightarrow \phi K^+ K^- n \quad (A)$$

$$\rightarrow \phi\phi n \quad (B)$$

where (A) is QLR allowed and (B) is QLR forbidden.

In order to do this one must make some calculation about the behavior of the acceptances of the apparatus for the two processes. Making the simple assumption for process A, where $\phi K^+ K^-$ is considered a single body X, that Xn is two body phase space production and that X decays by phase space considerations to $\phi K^+ K^-$, and for process B, $\phi\phi$ is considered a single body X and the previous procedure is followed. We find⁸ that over the range 2-2.8 GeV where practically all of the data is observed the acceptances for A and B are essentially the same within a factor of two. Thus, it appears reasonable to use the ratio of the observed number of events to estimate the ratio of the total cross sections. Doing this we obtain,

$$\frac{\sigma(\pi^- p \rightarrow K^+ K^- \phi n)}{\sigma(\pi^- p \rightarrow \phi\phi n)} \sim 4-10 .$$

Thus it seems reasonable to conclude from the BNL/CCNY data and the above arguments that the quark line rule suppression is relatively small (i.e. $\sim 4-10$) when one would be led to expect a factor of ≈ 100 . Thus the BNL/CCNY experiment does not evidence the usual suppression factors expected for the Quark Line Rule and in fact strongly supports the mechanism of final state interaction (i.e. isobar production) of ϕ and $\phi\phi$. As noted earlier the Quark Line Rule

cannot allow final state interactions and maintain its consistency. Therefore one has reasonable evidence that the QLR is either not correct or badly violated in the above discussed reactions. Furthermore, as previously stated, the requirement of penalty factors to explain data downgrades both the simplicity and credibility of quark line rules.

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Figure 1 $\omega \rightarrow 3\pi$

Figure 2 $\phi \rightarrow 3\pi$

Figure 3 $\phi \rightarrow K^+ K^-$

Figure 4 $K^- K^+ \rightarrow \pi^- \rho^+$

Figure 5a $\phi \rightarrow \pi^- \rho^+$ via two steps

Figure 5b $\phi \rightarrow \pi^- + \rho^+$ (one step)

Figure 6 $\pi^- p \rightarrow \phi n$

Figure 7 $\pi^- p \rightarrow \phi \phi n$

Figure 8 $\pi^- p \rightarrow K^- K^+ \phi n$

Figure 9 $\pi^- p \rightarrow K^- K^+ n$

Figure 10 $\pi^- p \rightarrow K^+ K^- K^+ K^- n$

Figure 11