1 of 1

WSRC-TR-91-106

NRTSC
**NUCLEAR REACTOR TECHNOLOGY
AND SCIENTIFIC COMPUTATIONS**

KEYWORDS: ERROR
CURVE FIT
M&TE EVALUATION
QUALITY ASSURANCE
STATISTICS

RETENTION: PERMANENT

UNCERTAINTY AND CALIBRATION ANALYSIS (U)

D. Allan Coutts

Savannah River Laboratory

D. Allan Coutts

ISSUED: MARCH, 1991

Derivative Classifier *J.P. Faraci* 5/13/91

SRL SAVANNAH RIVER LABORATORY, AIKEN, SC 29808
Westinghouse Savannah River Corporation
Prepared for the U.S. Department of Energy under
Contract DE-AC09-76SR00001 *82 818035*

MASTER

DOCUMENT: WSRC-TR-91-106

TITLE: UNCERTAINTY AND CALIBRATION ANALYSIS (U)

APPROVALS

John C. Whitehouse

5/14/91

TECHNICAL REVIEWER

DATE:

D. R. Muhlbaier

7/11/91

D. R. MUHLBAIER, RESEARCH SUPV.

DATE:

A. J. Garrett

7/12/91

A. J. GARRETT, RESEARCH MANAGER

DATE:

CONTENTS

1.0	SUMMARY	1
2.0	INTRODUCTION	1
	2.1 Purpose	1
	2.2 Methodology	1
3.0	METHOD OF ANALYSIS	2
	3.1 Uncertainty Calculation Methods	2
	3.1.1 Simplified Method	2
	3.1.2 ANSI/ASME PTC 19.1 Method	2
	3.1.3 Hybrid Method	2
	3.1.4 Commentary	3
	3.2 Assumptions	3
	3.3 Terms/Definitions	3
	3.3.1 Definitions	3
	3.3.2 Nomenclature	5
	3.3.3 Equations - General	6
	3.3.4 Equations - Large Samples	7
	3.3.5 Equations - Small Samples	7
4.0	DISCUSSION	7
	4.1 Measurement Errors	7
	4.2 Confidence Level	9
	4.3 Instrument Tolerances	10
	4.4 Combining Uncertainties	10
	4.5 Calibration Efforts	11
	4.5.1 General	11
	4.5.2 Calibration of Comparative Devices	12
	4.5.3 Calibration of Measurement Instruments	12
	4.6 Uncertainty of Data and Calculations	14
	4.6.1 Uncertainty Estimate Methods for Measurements	14
	4.6.2 Sample Size	15
	4.6.3 Measured Data	15
	4.6.4 Adjusted Data	16
	4.6.5 Calculated Data	16
5.0	REFERENCES	16

APPENDIX A	-	Developmental and Sample Calculations (4)
APPENDIX B	-	Calibration Evaluation Method for Comparative Devices (2)
APPENDIX C	-	Calibration Evaluation Method by Development of a Curve Fit Equation (5)
APPENDIX D	-	Development of the Equations Used in Appendix C (5)
APPENDIX E	-	Uncertainty Example, Vessel Volume Calculation (4)
APPENDIX F	-	Pressure Gage Calibration (8)
APPENDIX G	-	Student-t Values (2)

LIST OF FIGURES

Figure D1	Inverse Tolerance Calculation	D-3
Figure D2	Slope Adjustment Calculation	D-3
Figure D3	Discernable Increment Adjustment Calculation	D-4
Figure F1	Bourdon Gage Calibration Curve	F-4
Figure F2	Bourdon Gage Correction Plot	F-5
Figure F3	Bourdon Tube Adjusted Data Uncertainty	F-6

LIST OF TABLES

Table 1	Highlights from QAP 3-1, 3-2 and 3-3	8
Table 2	References to QAP 3	9
Table 3	Highlights from QAP 11-1	11
Table A1	Significant Digits for M&TE Calibration	A-2
Table A2	M&TE Calibration Results	A-4
Table E1	Measurement Data	E-2
Table E2	Uncertainty Comparison	E-4
Table F1	Calibration Test Data	F-1
Table F2	Pressure Gage Data	F-1
Table F3	Statistical Data	F-2
Table F4	Tolerance Data	F-4
Table F5	Adjusted Test Data for Bourdon Tube Example	F-7

1.0 SUMMARY

- 1.1 All measurements contain some deviation from the true value which is being measured. In the common vernacular this deviation between the true value and the measured value is called an inaccuracy, an error or a mistake. Since all measurements contain errors, it is necessary to accept that there is a limit to how accurate a measurement can be. The uncertainty interval combined with the confidence level, is one measure of the accuracy for a measurement or value. Without a statement of uncertainty (or a similar parameter) it is not possible to evaluate if the accuracy of the measurement, or data is appropriate.
- 1.2 The preparation of technical reports, calibration evaluations and design calculations should consider the accuracy of measurements and data being used. There are many methods to accomplish this. This report provides a consistent method for the handling of measurement tolerances, calibration evaluations and uncertainty calculations.

2.0 INTRODUCTION

2.1 Purpose

- 2.1.1 The SRS Quality Assurance (QA) Program requires that the uncertainty of technical data and instrument calibrations be acknowledged and estimated. The QA Program makes some specific technical requirements related to the subject but does not provide a philosophy or method on how uncertainty should be estimated. This report was prepared to provide a technical basis to support the calculation of uncertainties and the calibration of M&TE for any activity within the Experimental Thermal-Hydraulics (ETH) Group.
- 2.1.2 The methods proposed in this report provide a graded approach for estimating the uncertainty of measurements, data and calibrations. The method is based on the national consensus standard, ANSI/ASME PTC 19.1.

2.2 Methodology

- 2.2.1 There are three uncertainty estimating methods discussed in this report: ANSI/ASME PTC 19.1, a simplified method and a hybrid of the two methods. The method used to estimate uncertainty should be selected by the task leader or test engineer depending on the magnitude of the uncertainty estimate, the available instrument information, the available manpower, and the significance of the data to other SRS activities. All three methods will provide a defensible estimate of the uncertainty but the ANSI/ASME method will provide the best estimate. It is also the most costly method in time and manpower to apply. All three methods will achieve the objectives of the QA Program.
- 2.2.2 One of the primary features of this report is the separation of M&TE tolerance from data uncertainty. While the uncertainty of an M&TE item can be used to

estimate the item's tolerance and the M&TE item's tolerance can be taken as the uncertainty of the measurement, the terms are not interchangeable. Tolerance is the acceptable measurement error for an M&TE item during calibration, uncertainty is the acknowledged limit of how accurate data is. If a correction or calibration curve is available for an M&TE item, the tolerance of the item cannot be reduced but the uncertainty of the measurements made with the item can be reduced. This report provides examples of how this estimation process works.

3.0 METHOD OF ANALYSIS

3.1 Uncertainty Calculation Methods

There are three basic methods described in this standard. They are all based on summing the uncertainties using the root-sum-square (RSS) rule:

$$\omega_R = \sqrt{\sum_{i=1}^j (\omega_i)^2} \quad (3.1)$$

3.1.1 *Simplified Method*

This method combines the known errors of each parameter of a resultant without distinguishing bias errors from precision errors using the RSS rule. (See equations 4.1 and 4.2.)

3.1.2 *ANSI/ASME PTC 19.1 Method*

This method combines the bias and precision errors of all parameters of a resultant separately, using the RSS rule. The total bias and precision errors are then combined:¹

$$\omega_R = \sqrt{B^2 + (tS)^2} \quad (3.2)$$

This method is very systematic; each elemental index is evaluated and categorized. Categories include: calibration, data acquisition and data reduction errors. Examples in the data acquisition category are thermal sensitivity shift, thermal sensitivity stability, vibration and shock. An example in the data reduction category is the error of a curve fit.

3.1.3 *Hybrid Method*

The simplified method and the ANSI/ASME PTC 19.1 methods are combined. The uncertainty of key instruments are calculated using the ANSI/ASME method while the total resultant uncertainty for a task is determined using the simplified method.

¹ Equation 2.26, ANSI/ASME 19.1.

3.1.4 *Commentary*

For most applications the simplified method is usually appropriate. If a specific instrument has been evaluated using the ANSI/ASME method then that uncertainty should be used. In some instances it may be more appropriate to use the hybrid method since the simplified method can sometimes inflate the uncertainty estimate slightly² from the ANSI/ASME Method.

3.2 **Assumptions**

The presentations in this report are based on the following assumptions.

- 3.2.1 The uncertainty of any measurement can be simulated using a Gaussian distribution (normal distribution).
- 3.2.2 The relative magnitudes of the bias error and precision error are not normally known.
- 3.2.3 The confidence level of all data at SRS is 95% unless otherwise stated. This is also referred to as the 2σ level of confidence.
- 3.2.4 The uncertainties are symmetrical (i.e. $x - \delta \leq x \leq x + \delta$). If the uncertainties are not symmetrical (i.e. $x - \alpha \leq x \leq x + \beta$) they should be handled as discussed in ANSI/ASME PTC 19.1, Section 2.6.2, page 12.

3.3 **Terms/Definitions**

3.3.1 *Definitions*

Accuracy^{*3} - The closeness of agreement between a measured value and a true value.

Average - The arithmetic mean of a set of data.

Bias Error* - The difference between the average of the total population and the true value. The constant component of repeatable error.

Calibration, Adjustment - The adjustment of an M&TE item to minimize the item's measurement error.

Calibration, Evaluation - The evaluation of an M&TE item to identify the item's measurement uncertainty to confirm the item is within its measurement tolerance.

² The bias error is multiplied by the Student-t value in the Simplified Approach. See Appendix A for a further explanation.

³ The symbol * indicates that the definition is excerpted from ANSI/ASME 19.1.

Confidence Level* - The relative frequency that the calculated statistic is correct.

Confidence Level, Significant - A confidence level of 95 percent.⁴

Confidence Level, Very Significant - A confidence level of 99 percent.⁵

Coverage* - The frequency that an interval estimate of a parameter may be expected to contain a true value.

Data, Adjusted - Field measurements which have been adjusted by the use of calibration curves, logic circuits, or computational routines.

Data, Calculated - Numerical values which have been derived from adjusted and/or recorded data.

Data, Recorded - Measurements recorded directly as indicated by field equipment. This can be both manually recorded or automatically recorded.

Degrees of Freedom* - A sample of N values is said to have N degrees of freedom, and the statistic calculated from it also is said to have N degrees of freedom. The degrees of freedom are reduced one for each previously calculated statistic used to calculate a new statistic.

M&TE - Measuring and Test Equipment.

M&TE System - A collection or assembly of individual M&TE items used to measure one or more parameters.

M&TE Tolerance - The allowable measurement error for an M&TE item which shall be considered still accurate based on a specified level of 95 percent. (See the definition for uncertainty.)

Mean - The averaged value for a population.

Measurement Error* - The difference between the true value and the measured value. It includes both bias and precision errors.

MSE - Error Mean Square or Residual Mean Square.⁶

Number of Samples, Large - Thirty or more data points.

Number of Samples, Small - Less than thirty data points.

Population - The set of all possible observations for a given statistical study. Eberhardt suggests that population "is used broadly to designate a group to

4 Lindeburg, page I-27.

5 Lindeburg, page I-27.

6 Neter, page 47.

be studied."⁷ A population may be finite or infinite. An example of a finite population is the employees of SRS. An example of an infinite population is the number of measurements made with a piece of M&TE equipment.

Precision Error* - The closeness of agreement between repeated measurements.

SSE - Error Sum of Squares, Residual Sum of Squares⁸ or Sum of the Square of the Residuals⁹.

Tolerance - The range of measurement error for a given instrument which will be exceeded for no more than 5 percent of time if used in a service with no significant field error effects.

Theoretical Tolerance - The instrument's tolerance based solely on the most recent calibration data or calibration data sets.

True Value* - The actual value of the parameter being measured.

Uncertainty - The possible range of measurement or calculation error for a stated parameter (equation or value) which is still accurate based on a specified confidence level. (Uncertainty is related to data, tolerance is related to equipment. It is often the case that the uncertainty of data is the tolerance of the equipment used to obtain the data. To avoid confusion the terms should not be interchanged.)

3.3.2 *Nomenclature*

$n \rightarrow$ number of samples

$N \rightarrow$ population size

$t \rightarrow$ Student-t value from Table G1

$T(n, \bar{X}, \mu, S) \rightarrow$ Student t Distribution Function

$x_i \rightarrow$ sample value

$\bar{X} \rightarrow$ sample average

$Z(n, \bar{X}, \mu, \sigma) \rightarrow$ Gaussian (Normal) Distribution Function

$\alpha \rightarrow$ confidence level

⁷ Wadsworth, page 9.1

⁸ Neter, page 47.

⁹ Walpole, page 282.

$\sigma \rightarrow$ standard deviation of a population

$\sigma_m \rightarrow$ standard deviation from the population mean

$\mu \rightarrow$ mean value of a population

$v \rightarrow$ degrees of freedom

$\omega \rightarrow$ magnitude of error

3.3.3 Equations - General

Population Mean¹⁰:

$$\mu = \frac{\sum_{i=1}^N x_i}{N} \quad (3.1)$$

Population Variance¹¹:

$$\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N} \quad (3.2)$$

Population Standard Deviation of the Mean¹²:

$$\sigma_m = \frac{\sigma}{\sqrt{N}} \quad (3.3)$$

Sample Mean:

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n} \quad (3.4)$$

Standard Normal Distribution Transform¹³:

$$Z = \frac{\bar{X} - \mu}{\sigma_m} \quad (3.5)$$

¹⁰ Walpole, page 81, Holman, page 51.

¹¹ Walpole, page 81, Holman, page 51.

¹² Holman, page 78.

¹³ Walpole, page 161.

Student t distribution transform¹⁴:

$$T = \frac{(\bar{X} - \mu) \sqrt{n}}{\sigma} \quad (3.6)$$

3.3.4 Equations - Large Samples

Sample Variance¹⁵:

$$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n} \quad (3.7)$$

Sample Standard Deviation of the Mean¹⁶:

$$S_m = \frac{S}{\sqrt{n}} \quad (3.8)$$

3.3.5 Equations - Small Samples

Sample variance¹⁷:

$$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n - 1} \quad (3.9)$$

4.0 DISCUSSION

4.1 Measurement Errors

- 4.1.1 Uncertainties are the measure of accuracy for a measurement based on a specified confidence level. For almost all measurements there is more than one uncertainty. A simple example is the use of a tape measure; the uncertainty terms would include:

inaccuracy during manufacture,
deformation under tension,
thermal expansion,
sag,
twist,
deformation because of prior abuse,
smallest indication of measurement, and
ability of the user to read the tape measure.

Depending on the required accuracy for the measurement some of these terms may be significant. To judge their significance an uncertainty analysis

14 Walpole, page 168.

15 Walpole, page 81, Holman, page 51.

16 Holman, page 78.

17 Walpole, page 158, Holman, page 64.

must be performed. To judge the accuracy of the measurement the significant uncertainty terms must then be combined.

- 4.1.2 Uncertainty terms can be categorized in many ways: By error type: bias errors (fixed) or precision errors (random or scatter). By source¹⁸: calibration errors, data acquisition errors, or data reduction errors. The categorizations for the tape measure example are shown in Table 1.

TABLE 1		
UNCERTAINTY CATEGORIZATIONS FOR A TAPE MEASURE		
Uncertainty Term	Error Type	Error Source
Inaccuracy during manufacture	Bias	Calibration
Deformation under tension	Bias	Data Acquisition
Thermal expansion	Bias	Data Acquisition
Sag	Bias	Data Acquisition
Twist	Bias	Data Acquisition
Deformation because of prior abuse	Bias	Calibration/Data Acquisition
Smallest indication of measurement	Scatter	Data Acquisition
Ability of the user to read the tape measure	Scatter	Data Acquisition

- 4.1.3 The evaluation of uncertainties does not lend itself to any tabular method. Each type of instrument has specific uncertainties inherent to its design. Its installation and mode of use can contribute additional uncertainties. A total immersion thermometer is designed to be read by totally immersing the entire thermometer length in the fluid whose temperature is being measured. If it is not possible to see the mercury thread while the thermometer is immersed, the thermometer must be withdrawn to be read. This introduces an uncertainty; how much does the thermometer cool before being read? If the thermometer is not fully immersed, a second uncertainty term is introduced since the mercury in the exposed portion will not be at the same temperature as the fluid being measured and will expand less (if the fluid temperature is higher than the ambient temperature). These two uncertainties are data acquisition errors. The calibration of the thermometer can not eliminate these uncertainties. It is possible to generate a set of calibration curves to estimate how fast the thermometer cools and the emergent stem error. When these two corrections are used the total uncertainty of the temperature reading will be reduced, however the uncertainty estimate must now include two curve fit (data reduction) error terms.

- 4.1.4 Uncertainties represent estimates of error. They may be based on engineering calculations, special calibration curves, measurement of a known parameter or historical data. An example of an engineering calculation would be the estimated thermal expansion of a tape measure when it is heated using

¹⁸

ANSI/ASME 19.1, page 7.

the expansion coefficient for steel. The method used to estimate most error terms must be based on judgement of the phenomena involved. Seldom is it feasible or cost effective to require that only one uncertainty estimate method be used. The one exception to this discretionary approach is the evaluation of instrument calibration data. For calibration evaluations it is as important to know if the uncertainty is changing, in addition to what the uncertainty is. This allows historic trending of instrument behavior. A systematic evaluation method is required if historic trending is to be preformed. This report documents several methods which may be used to evaluate a wide variety of calibration data.

- 4.1.5 Normally measurement errors are considered two-tailed. There is an equal chance that a given value is too high or too low and the stated uncertainty is symmetric. (See Assumption 3.2.4.) In general if no explanation is given measurement errors are assumed to be symmetric. If a measurement error is preceded by the symbol "±" then this implies that some consideration has been made relative to the assumption being true. The consideration does not need to be exhaustive, a previous assumption in the report or calculation may be the limit of the consideration. The use of the "±" symbol when discussing errors provides a notation that the error is considered symmetrically two-tailed rather than one-tailed or non-symmetric.

4.2 Confidence Level

- 4.2.1 The confidence level is the relative frequency that the calculated statistic or measured value is within the stated uncertainty. A calculated statistic or measured value which is within the stated uncertainty would be considered true or correct.
- 4.2.2 There are several "standard" confidence levels which may be used. Table 2 provides information on several possible confidence levels.

TABLE 2			
CONFIDENCE LEVEL AND COVERAGE INFORMATION			
Description ¹⁹	Confidence Level	Uncertainty Interval	Chance of results falling within specified deviation ²⁰
		0.6745 σ	1:1
		σ	2.15:1
Significant	95%	1.96 σ	19:1
		2 σ	21:1
Highly Significant	99%	2.576 σ	99:1
		3 σ	369:1

¹⁹ Lindeburg, page 1-26.
²⁰ Holman, page 64.

4.2.3 The 95 percent confidence level is the one proposed by ANSI/ASME PTC 19.1. Since it is a widely accepted level and provides a simple method for combining uncertainties, (See section 4.4 Combining Uncertainties) the confidence level used in determining the uncertainty interval at SRS should be 95 percent. The Student-t for the standard deviation term will be taken as 2.00 for large samples. (See section 4.6.2, Sample Size.)

4.2.4 The value of the uncertainty interval is very dependant on the confidence level used. Because of this, if the confidence level is not 95 percent (or higher) then the confidence level must be explicitly stated.

4.3 Instrument Tolerances

4.3.1 The theoretical tolerance is the uncertainty for an instrument based solely on the calibration data. Often this theoretical tolerance does not represent the known uncertainty of an instrument in use. To account for this the M&TE tolerance should be the greater of:

1. The manufacturer's stated tolerance for the item,
2. The calculated uncertainty of the calibration based on equations 4.1 and 4.2 (See also Appendix A),
3. The calculated uncertainty of the instrument system (Note: If not calibrated as a system the uncertainty is the RSS of the individual M&TE item tolerances.).

4.3.2 The tolerance for any M&TE item should normally not be tighter than (i.e. less than) that specified by the manufacturer. The uncertainty related to the data obtained can be reduced by the use of calibration curves and/or application of special test procedures.

4.3.3 The calculated uncertainty of a calibration should be determined by methods stated in Sections 4.4 and 4.5 to ensure that it is possible to evaluate variations in the uncertainty (historical trending) in addition to the theoretical uncertainty.

4.4 Combining Uncertainties

4.4.1 The uncertainty, ω_R , for a resultant R where:

$$R = R(x_1, x_2, x_3, \dots, x_n) \quad (4.1)$$

should be based on²¹:

$$\omega_R = \left[\sum_{i=1}^n \left(\frac{\partial R}{\partial x_i} \omega_i \right)^2 \right]^{1/2} \quad (4.2)$$

- 4.4.2 The uncertainty may be calculated by partial derivatives of the resultant R or by using a Sequentially Perturbing Data-Reduction Program²²:

$$\frac{\partial R(x_i)}{\partial x_i} = \frac{R_{x_i + \delta x_i} - R_{x_i}}{\delta x_i} \quad (4.3)$$

where the equation is evaluated for each x_i parameter.

4.5 Calibration Efforts

4.5.1 General

The method used to calibrate an instrument must be selected based on instrument's accuracy, precision and use. The first factor which must be determined is if the instrument is a comparative style or a measurement style.

- **Comparative Gages and Templates:** Comparative gages and templates are M&TE devices which can only be used to judge if an item is not at a specific dimension. Examples of comparative devices are: gage blocks, standard weights (for use with a balance), gage pins, feeler gages, and radius templates. (Go-not-go gages are an example of two comparative gages which are connected together. The first gage indicates that the part is large enough, the second indicates that it is not too large.)
- **Measurement Instruments:** Measurement instruments are M&TE devices which indicate a dimension (or multiple dimensions) for a given input or set of inputs. Examples of measurement instruments are rulers, micrometers, voltmeters, pressure transducers and flowmeters. Measurement instruments can be further subdivided into:
 1. Adjustable and fixed settings,
 2. Direct and indirect reading and
 3. Transforming and non-transforming.

TABLE 3			
Examples of Instrument Types			
Instrument	Setting	Reading	Transform
ruler	fixed	direct	non-transforming
caliper	adjustable	indirect	non-transforming
rotometer	fixed	direct	transforming
mass flowmeter	adjustable	direct	transforming

- **Significant Figures for M&TE Calibrations:** The M&TE tolerance (ω_T) should not contain significant digits which are 1/10 of the minimum

discernable measurement. (See Appendix A for an explanation of this requirement.)

Roundoff of Figures for M&TE Calibrations: The standard method of rounding the last significant figure up or down is based on the rule: Round down if next digit is 5 or less, round up if more than 5. It is clear that this would reduce some stated tolerances such that the confidence level is less than the required 95 percent. Roundoff of M&TE calibration uncertainties shall be such that the maximum round down of any calculated tolerance ω_T will be 5 percent. (See Appendix A.)

4.5.2 Calibration of Comparative Devices

By its nature a comparative device has a very narrow range of use. This type of device is primarily used to judge one or more of the following:

1. Does the part being evaluated have the same dimension (e.g. weight, height, roughness, etc.) as the comparative device?
2. Is the part being evaluated smaller than the comparative device?
3. Is the part being evaluated larger than the comparative device?

The calibration of a comparative device is then directed to verifying that the comparative device is capable of fulfilling its defined task (i.e. The dimension (e.g. weight, height, roughness, etc.) is within an acceptable variation.)

Calibration Requirements - The calibration evaluation method for comparative devices is described in Appendix B. Significant parameters which must be reported on the calibration evaluation sheet are:

- μ_o The specified operating dimension for the comparative device.
- $\bar{X}$ The operating dimension calculated from the calibration data for the comparative device.
- ω_C The combined uncertainty of the calibration equipment used to evaluate the comparative device.
- ω_E The uncertainty of the calibration which is a result of calibration data scatter.
- ω_T The specified tolerance of the comparative device.

4.5.3 Calibration of Measurement Instruments

Measurement instruments normally have one or more outputs which vary based on one or more inputs. This procedure only addresses the calibration of measurement instruments which have one input. Multiple outputs can be evaluated using this procedure if each is handled separately. These methods are limited to M&TE equipment which exhibit a linear behavior.

Assume that the M&TE item which must be calibrated has one dependant variable and only one independent variable (e.g. An applied pressure produces a voltage output for a pressure transducer or a dial deflection for a

bourdon tube pressure gage.) and the two variables are linearly related such that:

$$\mu_{y|x} = \alpha + \beta x \quad (4.4)$$

Equation 4.4 represents the true relationship between x and y . This equation may be estimated from test data using a linear regression by the method of least squares.

$$y = a + b x \quad (4.5)$$

An estimated equation should also be developed from the manufacturer's operating data for the instrument. (e.g. For a pressure transducer with a range of 0 to 25 psia, with an output of 1 to 6 volts, two operating points have been established and a calibration curve can be estimated.) Although this equation may not be used in the reduction of data, the equation is important in evaluating if the instrument is operating properly.

Data Collection Planning - Calibration measurements should be made at a minimum of three levels (i.e. three different values of x) with at least three measurements made at each level²³. If the instrument to be calibrated might exhibit a hysteresis effect the number of measurements at each level should be even (e.g. pressure instrumentation). This will only result in 9 or 12 data point sets and the calculated tolerance might be unnecessarily high because of the Student t Value which must be used.

If the Student t Value must be reduced, additional measurements should be taken. In general the preferred calibration data set is 5 levels with 4 data points at each level.

Calibration Requirements - The calibration evaluation method for measurement devices is described in Appendix C. Significant parameters which must be reported on the calibration evaluation sheet are:

- a The first parameter from the calibration curve.
- b The second parameter from the calibration curve.
- x_{low} The minimum value for the instrument's range.
- x_{high} The maximum value for the instrument's range.
- ω_u The minimum discernible increment which may be made with the instrument. (This is normally the smallest increment on an instrument's readout device. For analog outputs this value is normally not applicable.)

23

This is based on the following information from Neter, page 176: "Use three levels whenever a description of the response curve by its slope and curvature is likely to be adequate; this should cover most cases. Use four levels if further examination of the shape of the response curve is important."

- ω_C The combined uncertainty of the calibration equipment used to evaluate the instrument. This is independent of the device being calibrated.
- ω_E The uncertainty of the calibration which is a result of calibration data scatter. (This is related to how well the calibration curve represents the calibration data.)
- ω_F The uncertainty of the calibration if the calibration curve is not used to reduce measurements made with the instrument. (For some instruments such as pressure transducers this value may not seem to be applicable since a curve fit must be used to convert voltage readings into pressure measurements. For this and similar cases ω_F represents the deviation from the equation represented by the vendor's nominal operating curve data or the previous calibration curve fit.)
- ω_T The specified tolerance of the instrument.

4.6 Uncertainty of Data and Calculations

4.6.1 *Uncertainty Estimate Methods for Measurements*

There are two methods which may be used to estimate the uncertainty of a measurement:

1. Use the specified tolerance of the M&TE item which was used to make the measurement.
2. Use the estimated uncertainty of the instrument when the measurement was made.

The principle concern is that the correct uncertainty is used based on the pre-test and post-test calibrations. The primary difference between these two methods is who is responsible for identifying an out of calibration condition.

For method 1, the M&TE out-of-calibration system as described in the WSRC 1Q Manual, Procedure QAP 12-1, Section 4.10 will be used to notify all users of the out of calibration item. If the M&TE item is ever found to be out of calibration all users (since the last calibration) will be notified of the out-of-calibration condition. The user can then evaluate the effect of the out of calibration condition.

For method 2, the calibration may show the instrument to be within its tolerance. However, the user of the measurement data is responsible for identification of a condition where the estimated uncertainty based on pre-test calibration data is still valid. Use of this method should be described in the test plan or procedure. If the post-test calibration data indicates that the original uncertainty estimate is correct, then the original estimated uncertainty is considered correct. If the post-test calibration data indicates a higher uncertainty then the original estimate, then the uncertainty estimate is revised.

4.6.2 *Sample Size*

To establish an uncertainty estimate the size of the sample set must be known. Larger sample sets normally better represent the population as a whole. For small sample sets an adjustment must be made to account for the uncertainty that the population is correctly represented.

For routine tasks (e.g. operation, maintenance and construction activities) the sample size will normally be assumed large and no special review of the sample size will be made. Because of this, the uncertainty must be discussed on a project or activity basis. (Justification: Since the confidence level of operation, maintenance and construction data will be based on a project or activity basis, the number of samples related to a given project or activity will greatly exceed 30.)

For M&TE calibration, research activities and design basis calculation testing, the stated uncertainty of a value should be calculated using the methods described here or the method used in ANSI/ASME PTC 19.1. The uncertainty of data should be considered as:

Large Samples²⁴: $\omega_E = |t \cdot S_m|$ where $t = 2.0$ (4.6)

Small Samples:

$$\omega_E = \left| \frac{t \cdot S}{\sqrt{n}} \right| \text{ where } t = T(v) \text{ and } v = n - 1 \quad (4.7)$$

The value of t is listed in table 2.5 of ANSI/ASME PTC 19.1 and Appendix G.

4.6.3 *Measured Data*

To avoid the confusion related to the uncertainty of measured data, the uncertainty of all measured data should be the greater of:

1. The instrument tolerance
2. The calculated uncertainty of the measurement
3. The known uncertainty.

When the uncertainty of measured data has been reduced using correction factors or curve fits as described in method 2 in section 4.6.1, the data shall be considered adjusted data.

4.6.4 Adjusted Data

The uncertainty related to measured values (data) may be reduced by the application of calibration curves or correction factors. When such methods are used they should be handled as calculations.

4.6.5 Calculated Data

The uncertainty related to calculated data shall be calculated from the uncertainty of the calculation input data.

5.0 REFERENCES

Abernethy, R. B., R. P. Benedict, and R. B. Dowdell, *ASME Measurement Uncertainty*, Journal of Fluids Engineering, June 1985, vol 107, page 161.

ANSI/ASME MTC-2M, *Measurement Uncertainty for Fluid Flow in Closed Conduits*, The American Society of Mechanical Engineers, New York, 1983.

ANSI/ASME PTC 19.1, *Measurement Uncertainty, Part 1*, The American Society of Mechanical Engineers, New York, 1985.

Control of Measuring and Test Equipment, WSRC Manual 1Q, Procedure QAP 12-1, Revision 0, 4/1/90.

Holman, J. P. and W.J. Gajda, Jr. *Experimental Methods for Engineers*, 3rd Ed, McGraw-Hill Book Co, New York, 1978.

Lindeburg, Michael R. *Mechanical Engineering Review Manual*, 7th Ed, Professional Publications, San Carlos, CA, 1985.

Moffat, R. J, *Using Uncertainty Analysis in the Planning of an Experiment*, Journal of Fluids Engineering, June 1985, vol 107, page 173.

Neter, John, William Wasserman, and Michael H. Kutner, *Applied Linear Regression Models*, Irwin, Homewood, IL, 1983.

Sweeney, R. J, *Measurement Techniques in Mechanical Engineering*, John Wiley & Sons, Inc., New York, 1953.

Sydenham, P. H, *Mechanical Design of Instruments*, Instrument Society of America and The Institute of Measurement and Control, Research Triangle Park, NC, 1986.

Test Control, WSRC Manual 1Q, Procedure QAP 11-1, Revision 0, 4/1/90.

Wadsworth, Harrison M., *Statistical Methods for Engineers and Scientists*, McGraw-Hill Publishing Co., New York, 1990.

Walpole, Ronald E. and Raymond H. Myers, *Probability and Statistics for Engineers and Scientists*, Macmillan Publishing Co, Inc, New York, 1972.

APPENDIX A

DEVELOPMENTAL AND SAMPLE CALCULATIONS

Selection of Student t Value for Large Samples:

Uncertainty for Large Samples ($n \geq 30$) using equation 4.6 for a Gaussian Normal Distribution:

$$\alpha = \frac{1 - 0.95}{2} = 0.025 \rightarrow z = 1.96 \approx 2.0 \quad (\text{A-1})$$

$$\text{uncertainty} = \omega = |\bar{X} - \mu| = t \cdot \sigma_m = 2.0 \sigma_m \quad (\text{A-2})$$

Uncertainty for Small Samples ($n < 30$) using equation 4.7 for a two-tailed Student t Distribution:

$$\alpha = \frac{1 - 0.95}{2} = 0.025 \text{ where } t \text{ will vary from } 1.960 \text{ to } 12.706 \quad (\text{A-3})$$

$$\text{uncertainty} = \omega = |\bar{X} - \mu| = t \cdot \frac{S}{\sqrt{n}} \quad (\text{A-4})$$

Differences between the Simplified and ANSI/ASME PTC 19.1 Methods:

The uncertainty of a result using the simplified method is:

$$\omega_R = t \sqrt{B^2 + S^2} \quad (\text{A-5})$$

The uncertainty of a result using the ANSI/ASME PTC 19.1 method is:

$$\omega_R = \sqrt{B^2 + (t S)^2} \quad (\text{A-6})$$

The difference between the uncertainties is:

$$\Delta \omega = |t \sqrt{B^2 + S^2} - \sqrt{B^2 + (t S)^2}| \quad (\text{A-7})$$

The largest variation is when S is zero:

$$\Delta \omega = |\sqrt{(t B)^2} - \sqrt{B^2}| = |tB - B| = (t - 1) B \quad (\text{A-8})$$

For large samples t will approach 2:

$$\Delta\omega = (2 - 1) B = B \quad (A-9)$$

If there is no precision error the greatest difference between the methods for large samples is the bias error.

Justification for the Four to One Rule for M&TE Calibrations

The Quality Assurance Program requires that all calibration standards have an uncertainty no greater than one-fourth of the tolerance for the equipment under calibration. This is referred to as the 4 to 1 rule. It is justified by:

$$\omega_{\text{total}} = \left[(\omega_{\text{specified}})^2 + \left(\frac{\omega_{\text{specified}}}{4} \right)^2 \right]^{1/2} = 1.03 \omega_{\text{specified}} \quad (A-10)$$

Equation A-10 indicates that if the 4 to 1 rule is met, the calibration standard will result in an uncertainty increase of less than 3%. This is normally considered negligible.

In many instances it is not feasible to have calibration standards to this accuracy. An example is for flow meters where there can be a considerable scatter of the flow standard data. The methods in this document can be used to provide the justification for not complying with the 4 to 1 rule.

Roundoff and Significant Figures for M&TE Calibrations

For most instruments there is a minimum discernable measurement. Normally this is the smallest division on an instrument's scale. (Note: For many instruments such as thermometers and pressure gages it is routine practice to record data to 1/2 of the increment. For calculation of the uncertainty the minimum discernable value will be considered 1 division.)

Digital gages also have a minimum discernable measurement. This may not be the number of digits displayed. Often the last digit will change in increments of 2 or 5 units. If this is the case the minimum discernable measurement is 2 and 5 units respectively. (Note: This minimum discernable measurement is sometimes referred to as counts.)

The M&TE tolerance (ω_T) should not contain significant digits which are 1/10 of the minimum discernable measurement.

Example A1: Let X be considered significant digits for measurements.

TABLE A1			
SIGNIFICANT DIGITS FOR M&TE CALIBRATIONS			
Description of Measurement		Comment	Reported Measurement units
M&TE Range		Low	0
		High	XXXX00
Minimum Discernable Measurement			X00
Tolerance:	Case 1	Preferred	XX00
	Case 2	Acceptable	X000
	Case 3	Not Acceptable	X0
	Case 4	Not Acceptable	XX0

The tolerance stated for Case 3 is not acceptable since it indicates that the tolerance is less than the minimum discernable measurement for the item. Case 4 is not acceptable since the tolerance should represent the number of significant digits for the instrument.

When performing calculations related to calibrations the number of significant digits used should be at least one greater than the number of significant digits for the tolerance.

The roundoff of the stated tolerance value should not be made in the same fashion as normal roundoff values (e.g. 54 is rounded to 50 and 56 is rounded to 60). Rounding down will result in a lower tolerance than should be reported and the required confidence level will not be bracketed. Rounding up will result in a higher tolerance which will bracket the required confidence level. While the rounding up of all tolerance numbers will always bracket the required confidence level it can also be excessively conservative. (See the roundoff requirement in Section 4.5.1.)

Example A2: The data in Table A2 represents the calibration of an M&TE item.

TABLE A2		
M&TE CALIBRATION RESULTS		
Description of Measurement	Comment	Reported Value units
M&TE Range	Low	0.00
	High	100.00
Previous M&TE Tolerance, ω_T		0.02
Minimum Discernable Measurement		0.02
Calibration Uncertainty, ω_C		0.008
Curve Fit Uncertainty, ω_E		0.02

The revised M&TE Tolerance would indicate:

$$\omega_T = [(0.020)^2 + (0.005)^2]^{1/2} = 0.0206 \quad (\text{A-11})$$

$$\frac{0.0206 - 0.020}{0.020} \times 100\% = 3\% \quad (\text{A-12})$$

It is clear that to revise the tolerance from 0.02 up to 0.03 units would be excessively conservative. It may be that the instrument consistently reads 0.02 units higher (or lower, but not both) than the applied standard. To account for this a roundoff error of 5% should be accepted. This acceptable roundoff error is based on engineering judgement and is in line with the 4-to-1 calibration rule.

The maximum acceptable round down of any calculated tolerance ω_T should be 5 percent. For the above example the maximum acceptable round down would be:

$$0.02 \text{ units} \times 5\% = 0.001 \text{ units} \quad (\text{A-13})$$

The previous M&TE tolerance (in Table A2) should then be considered correct and the instrument in calibration since equation A-11 will be rounded to 0.02 units.

APPENDIX B
CALIBRATION EVALUATION METHOD
FOR COMPARATIVE DEVICES

Step 1: Obtain the Calibration Data:

Obtain the present specified calibration information for the device:

- μ_0 The specified operating dimension (thickness, diameter, weight) for the comparative device.
- $\bar{X}$ The operating dimension calculated from the calibration data for the comparative device.
- ω_C The combined uncertainty of the calibration equipment used to evaluate the comparative device.
- ω_E The uncertainty of the calibration which is a result of calibration data scatter.
- ω_T The specified or theoretical tolerance of the comparative device.

Obtain comparative calibration data for the device which is to be calibrated. A minimum of three measurements should be made, however additional measurements should be considered.

Step 2: Calculate Statistical Data from the Calibration Data:

Calculate the parameters:

- $\bar{X}$ From Equation 3.4.
- S From Equation 3.9.

Step 3: Calculate Calibration Data Uncertainty (Precision Error):

Calculate the uncertainty of the calibration data, ω_E , based on the calibration data using equation 4.6 or 4.7.

Large Samples¹:

$$\omega_E = |t \cdot S_m| \quad \text{where: } t = 2.0 \quad (B-1)$$

Small Samples:

$$\omega_E = \left| \frac{t \cdot S}{\sqrt{n}} \right| \quad \text{where: } t = T(v) \text{ and } v = n - 1 \quad (B-2)$$

¹ Holman, page 78.

Step 4: Calculate the Fixed Instrument Error (Bias Error):

$$\omega_F = |\bar{X} - \mu_0| \quad (B-3)$$

Step 5: Calculate an M&TE Tolerance:

$$\omega_T = [(\omega_E)^2 + (\omega_F)^2 + (\omega_C)^2]^{1/2} \quad (B-4)$$

where:

- ω_C is the calculated uncertainty for the calibration process. This is independent of the equipment being calibrated.
- ω_E The uncertainty of the calibration data from Step 3.
- ω_F The uncertainty due to the difference between the specified operating value and the actual operating value calculated in Step 4.
- ω_T The theoretical tolerance of the M&TE item after calibrating.

Step 6: Compare the Calculated and Specified Tolerances:

If a tolerance has been previously specified (M&TE recalibration or vendor's stated tolerance) then the evaluation process must determine:

1. Is the item operating within the previously stated tolerance?
2. Should the stated tolerance be revised? (Note: The results of a single calibration are normally not adequate to justify the reduction of a stated M&TE tolerance.)

If the stated tolerance for the M&TE item is greater than, or equal to the theoretical tolerance calculated in Step 5 then the previously stated tolerance is considered correct.

APPENDIX C

CALIBRATION EVALUATION METHOD BY DEVELOPMENT OF A CURVE FIT EQUATION

Assumed correct relationship between x and y is:

$$\mu_{Y|X} = \alpha + \beta x \quad (C-1)$$

Estimated relationship from test data using a linear regression by the method of least squares:

$$y = a + b x \quad (C-2)$$

Step 1: Obtain the Calibration Data:

Obtain the present specified calibration information for the device:

a	The first parameter from the calibration curve.
b	The second parameter from the calibration curve.
ω_C	The combined uncertainty of the calibration equipment used to evaluate the instrument. This is independent of the device being calibrated.
ω_E	The uncertainty of the calibration which is a result of calibration data scatter.
ω_F	The uncertainty of the calibration if the calibration curve is not used to reduce measurements made with the instrument.
ω_T	The specified or theoretical tolerance of the instrument.
x	The applied standard input which is measured by the instrument being calibrated.
y	The output from the instrument being calibrated for an applied standard input.
$\omega_{y\text{low}}$	The uncertainty of the value y at the low end of the range for instrument being calibrated.
$\omega_{y\text{high}}$	The uncertainty of the value y at the high end of the range for the instrument being calibrated.
$t_{\alpha/2}$	The Student-t value for a confidence value of α .
x_{low}	The minimum value for the instrument's range.
x_{high}	The maximum value for the instrument's range.

Obtain calibration data for the device which is to be calibrated. A minimum of three measurements should be made, however additional measurements should be considered.

Step 2: Calculate the Curve Fit Parameters

For the method of least squares a and b may be estimated as¹:

$$b = \frac{n \sum_{i=1}^n x_i y_i - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right)}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2} \quad (C-3)$$

$$a = \bar{Y} - b \bar{X} \quad (C-4)$$

Significant parameters which should be calculated are:

Error Sum of Squares:

$$SEE = \sum_{i=1}^n (y_i - a - b x_i)^2 \quad (C-5)^2$$

Error Mean Square:

$$MSE = s^2 = \frac{SEE}{n - 2} \quad (C-6)$$

$$S_{xx} = \sum_{i=1}^n (x_i - \bar{X})^2 \quad (C-7)$$

Step 3: Verify a Linear Relationship³ Between x and y :

Assumptions:

1. Two-tailed Evaluation:
2. The sample was randomly selected.
3. The population is gaussian.
4. The uncertainty of the calibration process (ω_C) is small compared to instrument tolerance (i.e. ω_C is 1/4 of the specified uncertainty ω_T .)
(Note: If this is not true than the tolerance values determined by this method will be higher (more conservative) than what would be expected if the uncertainty of the calibration process was not assumed negligible.)

State the Null Hypothesis: The parameter y is independent of x .

$$H_0: \beta = 0 \quad (C-8)$$

1 Walpole, page 283 and Neter, page 38.
2 Walpole, page 282, and Neter, page 47.
3 Walpole, page 229.

State the Alternative Hypothesis: The parameter y is dependant on x .

$$H_1: \beta \neq 0 \quad (C-9)$$

Select the Critical Region:

$$|T| > t_{\alpha/2} \text{ where: } \frac{\alpha}{2} = \frac{1 - 0.95}{2} = 0.025 \text{ and } v = n - 2 \quad (C-10)$$

Compute the Test Statistic:

$$T = \frac{b}{\sqrt{MSE/S_{xx}}} \quad (C-11)$$

Conclusion: If the test statistic is in the critical region, consider that H_0 is not true (e.g. a linear association exists)⁴. If the test statistic is not in the critical region (H_0 is true) than the operation of the M&TE Item is not linear and further investigation is required.

Step 4: Compare a and b with Previous Calibration Data⁵:

The confidence intervals for the estimates of α and β based on a $(1 - \alpha)100\%$ confidence level are:⁶

$$b - \omega_b < \beta < b + \omega_b \quad (C-12)$$

$$a - \omega_a < \alpha < a + \omega_a \quad (C-13)$$

Step 5: Calculate the Revised Uncertainties of ω_a , and ω_b .

$$\omega_b = t_{\alpha/2} \left[\frac{MSE}{S_{xx}} \right]^{1/2} \quad (C-14)$$

$$\omega_a = t_{\alpha/2} \left[\frac{MSE \sum_{i=1}^n x_i^2}{n S_{xx}} \right]^{1/2} \quad (C-15)$$

⁴ Neter, page 174.

⁵ If available.

⁶ Walpole, page 287 and 289, Neter, 66 and 69.

Step 6: Verify If Method Is Appropriate⁷:

$$\frac{t_{\alpha/2}^2 \cdot \text{MSE}}{b^2 S_{xx}} \leq 0.1 \quad \text{where: } v = n - 2 \quad (\text{C-16})$$

Step 7: Calculate the Inverse Prediction Uncertainty, ω_E

If the value of y is an analog type value (e.g. There is no minimum discernable increment for y. See Appendix D for an explanation.)

$$\omega_{y_{\text{low}}} = t_{\alpha/2} \sqrt{\text{MSE}} \left[1 + \frac{1}{n} + \frac{(x_{\text{low}} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (\text{C-17})$$

$$\omega_{y_{\text{high}}} = t_{\alpha/2} \sqrt{\text{MSE}} \left[1 + \frac{1}{n} + \frac{(x_{\text{high}} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (\text{C-18})$$

where ω_E is the larger of:

$$\omega_E = \begin{cases} \omega_{x_{\text{low}}} = \left| \frac{\omega_{y_{\text{low}}}}{b} \right| \\ \omega_{x_{\text{high}}} = \left| \frac{\omega_{y_{\text{high}}}}{b} \right| \end{cases} \quad (\text{C-19})$$

If the value of y has a minimum discernible increment (ω_u)

$$\omega_{y_{\text{low}}} = \frac{\omega_u}{2} + t_{\alpha/2} \sqrt{\text{MSE}} \left[1 + \frac{1}{n} + \frac{(x_{\text{low}} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (\text{C-20})$$

$$\omega_{y_{\text{high}}} = \frac{\omega_u}{2} + t_{\alpha/2} \sqrt{\text{MSE}} \left[1 + \frac{1}{n} + \frac{(x_{\text{high}} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (\text{C-21})$$

where ω_E is the larger of value determined from equation C-19.

Step 8: Calculate the Fixed Curve Error⁸:

Let the fixed curve equation be:

$$y = a_F + b_F x \quad (C-22)$$

ω_F is the larger of value of:

$$\omega_F = \left(\begin{array}{l} \left| x_{low} - \frac{a_F - a + b_F x_{low}}{b} \right| \\ \left| x_{high} - \frac{a_F - a + b_F x_{high}}{b} \right| \end{array} \right) \quad (C-23)$$

Step 9: Calculate an M&TE Theoretical Tolerance:

$$\omega_T = [(\omega_E)^2 + (\omega_F)^2 + (\omega_C)^2]^{1/2} \quad (C-24)$$

Step 10: Compare the Calculated and Specified Tolerances:

If a tolerance has been previously specified (M&TE recalibration or vendor's stated tolerance) then the evaluation process must determine:

1. Is the item operating within the previously stated tolerance?
2. Should the stated tolerance be revised? (Note: The results of a single calibration are normally not adequate to justify the reduction of a stated M&TE tolerance.)

If the previously stated tolerance for the M&TE item is larger than, or equal to the theoretical tolerance calculated in Step 9 then the previously stated tolerance is considered correct.

⁸ If applicable. Instruments which have a numeric or dial readout can be used to obtain data without the calibration curve. In such instances the fixed curve error must be considered as part of the instrument M&TE tolerance.

APPENDIX D

DEVELOPMENT OF THE EQUATIONS USED IN APPENDIX C

General:

Equation C-2 is written in the form:

$$\hat{y} = a + b x \quad (D-1)$$

When the symbol " $\hat{}$ " denotes a computed value (not a calibration data value.)

The confidence interval for the mean response estimate $\mu_Y|_{x_0}$ based on a $(1 - \alpha)100\%$ confidence level is¹:

$$\hat{y}_0 - t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}}} < \mu_Y|_{x_0} < \hat{y}_0 + t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}}} \quad (D-2)$$

where $t_{\alpha/2}$ is evaluated for $v = n - 2$, and x_0 is a variable.

The confidence interval for a single response y_0 based on a $(1 - \alpha)100\%$ confidence level is²:

$$\hat{y}_0 - t_{\alpha/2} s \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}} \right]^{1/2} < y_0 < \hat{y}_0 + t_{\alpha/2} s \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (D-3)$$

Uncertainty of x if y is Analog Information:

Equation D-3 is the prediction interval on a future observed response. It is a response of y for a given value of x . Instruments in normal use rely on the calibration curve (Equation D-1) to predict x for a recorded or indicated value of y . The additional uncertainty of this arrangement must be accounted for³.

The values of x_{high} and x_{low} are based on the operating range of the instrument which is being calibrated.

$$\omega_{y_{\text{low}}} = t_{\alpha/2} \sqrt{\text{MSE}} \left[1 + \frac{1}{n} + \frac{(x_{\text{low}} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (D-4)$$

¹ Walpole, page 290.

² Walpole, page 292.

³ Neter, page 173.

$$\omega_{y_{high}} = t_{\alpha/2} \sqrt{MSE} \left[1 + \frac{1}{n} + \frac{(x_{high} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (D-5)$$

If the slopes of the nominal y equation, the maximum y equation and the minimum y equation (See Figure D1) are considered equal, the relation between ω_x and ω_y can be simplified. From Figure D2:

$$\frac{\omega_y}{\omega_x} = b \quad (D-6)$$

The maximum uncertainty value of x (ω_E) for an indicated value of y would then be the larger of:

$$\omega_E = \begin{cases} \omega_{x_{low}} = \frac{|\omega_{y_{low}}|}{b} \\ \omega_{x_{high}} = \frac{|\omega_{y_{high}}|}{b} \end{cases} \quad (D-7)$$

Uncertainty of x If y has a Minimum Discernable Increment:

When the value of y represents a digital or dial output rather than an analog output the additional uncertainty of the indicated value of y must be accounted for. (For analog equipment this uncertainty is accounted for by combining the uncertainty of the M&TE item being calibrated and the instrument transforming the analog signal.)

From Figure D3 where ω_u represents the uncertainty associated with the indication of y. (Normally this is one unit or count for the gage. Example: If a weight scale has a digital readout with a range of 0 to 1000# where the digital readout is always in even values (e.g. 242#), the value of ω_u will be: 2#.)

$$\omega_{y_{low}} = \frac{\omega_u}{2} + t_{\alpha/2} \sqrt{MSE} \left[1 + \frac{1}{n} + \frac{(x_{low} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (D-8)$$

$$\omega_{y_{high}} = \frac{\omega_u}{2} + t_{\alpha/2} \sqrt{MSE} \left[1 + \frac{1}{n} + \frac{(x_{high} - \bar{X})^2}{S_{xx}} \right]^{1/2} \quad (D-9)$$

The values of ω_E can be calculated from Equation D-7.

Figure D1, Inverse Tolerance Calculation

Figure D2, Slope Adjustment Calculation

Figure D3, Discernable Increment Adjustment Calculation

Calculation of the Uncertainty If the Correction Curve Is Not Used.

For many instances the correction curve will not be used to obtain or reduce data. An example of this is when a bourdon tube pressure gage is calibrated. An uncertainty related to the curve fit equation (ω_E) can be generated but since in practice the curve fit equation is not used an additional uncertainty (ω_F) exists. (e.g. The dial of the gage is assumed correct. This is often a good assumption, the only penalty related to the assumption is that the stated uncertainty of the collected data is higher.)

Rewrite Equation D-1 to reflect that it is the correction equation parameters:

$$y = a_E + b_E x_E \quad (D-10)$$

If a different correction curve is selected a second set of correction equation parameters are generated:

$$y = a_F + b_F x_F \quad (D-11)$$

The uncertainty (error) between the use of these two different equations will be:

$$\omega_F = |x_F - x_E| = \left| \frac{y - a_F}{b_F} - \frac{y - a_E}{b_E} \right| \quad (D-12)$$

$$\omega_F = \left| \frac{(a_F + b_F x_F) - a_F}{b_F} - \frac{(a_F + b_F x_F) - a_E}{b_E} \right| \quad (D-13)$$

$$\omega_F = \left| x_F - \frac{a_F - a_E + b_F x_F}{b_E} \right| \quad (D-14)$$

The maximum uncertainty value ω_F must then be evaluated at the top and bottom of the instrument's operation range. The uncertainty is the larger of

$$\omega_F = \left(\begin{array}{c} \left| x_{low} - \frac{a_F - a_E + b_F x_{low}}{b_E} \right| \\ \left| x_{high} - \frac{a_F - a_E + b_F x_{high}}{b_E} \right| \end{array} \right) \quad (D-15)$$

APPENDIX E

UNCERTAINTY EXAMPLE. VESSEL VOLUME CALCULATION

The storage volume of a tank must be determined. The tank is 10' tall with a nominal 2' diameter. While the tank is round at the bottom it is elliptical at the top because it is open on the top.

Method 1: Measuring the tank size with a tape measure.

Measurement Data:

Diameter at base = 2' - 0"

Diameter at top = 2'-3"

Height at three places = 10'-0"

The tape measure is marked in 1/16" increments.

$$\text{Volume} = \pi \left(\frac{d}{2}\right)^2 \cdot H = \pi \left(\frac{2}{2}\right)^2 \cdot 10 = 31.4 \text{ ft}^3 \quad (\text{E-1})$$

$$\omega_V = \left[\left(\frac{\partial V}{\partial d} \omega_d \right)^2 + \left(\frac{\partial V}{\partial h} \omega_h \right)^2 \right]^{1/2} \quad (\text{E-2})$$

$$\omega_V = \left[\left(\pi \frac{d}{2} \omega_d \right)^2 + \left(\pi \left(\frac{d}{2}\right)^2 \omega_h \right)^2 \right]^{1/2} \quad (\text{E-3})$$

The uncertainty of the height will be about equal to the tape measure's increments. The uncertainty of the nominal tank diameter is not as easy to obtain.

$$\text{From Equation. 3.4:} \quad \bar{D} = \frac{2.0 + 2.25}{2} = 2.125 \text{ ft} \quad (\text{E-4})$$

$$\text{From Equation. 3.9:} \quad S = \left[\frac{(2.0 - 2.125)^2 + (2.25 - 2.125)^2}{2 - 1} \right]^{1/2} = 0.177 \text{ ft} \quad (\text{E-5})$$

$$\text{From Equation. 4.7:} \quad v = 2 - 1 = 1 \rightarrow t = 12.706 \quad (\text{E-6})$$

$$\omega_E = \left| \frac{12.706 \cdot 0.177}{\sqrt{2}} \right| = 1.59 \text{ ft} \quad (\text{E-7})$$

$$\omega_V = \left[\left(\frac{\pi(2)(10)}{2} 1.59 \right)^2 + \left(\pi \left(\frac{2}{2}\right)^2 (0.0052) \right)^2 \right]^{1/2} \quad (\text{E-8})$$

$$\omega_V = [2492.6 + 0.0003]^{1/2} = 2493 \text{ ft}^3 \quad (\text{E-9})$$

The total uncertainty for the volume calculation is unreasonable. A better evaluation of the vessel diameter is required. The diameter is measured at ten locations.

TABLE E1		
MEASUREMENT DATA		
Diameter feet -in	Diameter Feet	(Diameter-Average) ² square feet
2 - 0	2.000	0.00006
2 - 3	2.250	0.05856
1 - 8 9/16	1.714	0.08644
1 - 9	1.750	0.06656
1 - 10	1.833	0.03063
1 - 11 1/2	1.958	0.00250
2 - 1 1/16	2.088	0.00640
2 - 1 13/16	2.151	0.02045
2 - 1 15/16	2.161	0.02341
2 - 2 1/16	2.177	0.02856

$$\bar{X} = \frac{20.082}{10} = 2.008 \quad (E-10)$$

$$S = \left[\frac{0.32357}{10 - 1} \right]^{1/2} = 0.1896 \quad (E-11)$$

From Equation. 4.7 $v = 10 - 1 = 9 \rightarrow t = 2.262 \quad (E-12)$

$$\omega_{d_1} = \left| \frac{2.262 \cdot 0.1896}{\sqrt{10}} \right| = 0.1356 \text{ ft} \quad (E-13)$$

The measurement error for the diameter is still $\pm 1/16$ inch:

$$\omega_{d_{Total}} = \left[(0.1356)^2 + (0.005)^2 \right]^{1/2} = 0.1357 \text{ ft} \quad (E-14)$$

The volume error will then be:

$$\omega_V = \left[\left(\frac{\pi (2)(10)}{2} 0.1357 \right)^2 + \left(\pi \left(\frac{2}{2} \right)^2 0.0052 \right)^2 \right]^{1/2} \quad (E-15)$$

$$\omega_V = [18.2 + 0.0003]^{1/2} = 4.27 \text{ ft}^3 \quad (E-16)$$

This is a more acceptable uncertainty. It can be further reduced by taking additional measurements or by using a different volume measurement technique.

Method 2: The tank is weighed empty, then filled with water and weighed a second time. The difference in weight may be used to calculate the tank volume.

Tank weight empty = 745 # ±5 #

Tank weight full = 2697 # ±20 #

Water Density (18°C) = 998.60 kg/m³ (ASME/ANSI PTC 19.2, Table 2.3A)

1 kg/m³ = 0.06243 lb/ft³ (Crane Tech Paper 410, 1976, page D-10)

$$V = \frac{W_{\text{full}} - W_{\text{empty}}}{\rho} \quad (\text{E-17})$$

$$V = \frac{2697\# - 745\#}{\left(998.6 \frac{\text{kg}}{\text{m}^3}\right) \left(\frac{0.06243 \frac{\#}{\text{ft}^3}}{1 \frac{\text{kg}}{\text{m}^3}}\right)} = 31.31 \text{ft}^3 \quad (\text{E-18})$$

The errors in this measurement method are:

The tank empty weight	±5#
The tank full weight	±20#
The conversion factor	negligible
The water temperature	±2°C
The water density	998.21 kg/m ³ (20°C) 998.94 kg/m ³ (16°C)

$$\omega_{\text{p low}} = 998.94 - 998.60 = 0.34 \frac{\text{kg}}{\text{m}^3} \quad (\text{E-19})$$

$$\omega_{\text{p high}} = 998.60 - 998.21 = 0.39 \frac{\text{kg}}{\text{m}^3} \times \frac{0.06243 \frac{\#}{\text{ft}^3}}{\text{kg/m}^3} = 0.0243 \frac{\#}{\text{ft}^3} \quad (\text{E-20})$$

$$\frac{\partial V}{\partial W_{\text{full}}} = \frac{1}{\rho} = \frac{1}{998.6 \cdot 0.06243} = 0.016 \text{ft}^3/\# \quad (\text{E-21})$$

$$\frac{\partial V}{\partial W_{\text{empty}}} = \frac{-1}{\rho} = \frac{-1}{998.6 \cdot 0.06243} = -0.016 \text{ ft}^3/\# \quad (\text{E-22})$$

$$\frac{\partial V}{\partial \rho} = - \frac{(W_{\text{full}} - W_{\text{empty}})}{\rho^2} = - \frac{(2697 - 745)}{(998.6 \cdot 0.06243)^2} = -0.502 \text{ ft}^6/\# \quad (\text{E-23})$$

$$\omega_V = \left[\left(\frac{\partial V}{\partial W_{\text{empty}}} \omega_{\text{empty}} \right)^2 + \left(\frac{\partial V}{\partial W_{\text{full}}} \omega_{\text{full}} \right)^2 + \left(\frac{\partial V}{\partial \rho} \omega_{\text{density}} \right)^2 \right]^{1/2} \quad (\text{E-24})$$

$$\omega_V = \left[(0.016 \cdot 5)^2 + (0.016 \cdot 20)^2 + (0.502 \cdot 0.0243)^2 \right]^{1/2} \quad (\text{E-25})$$

$$\omega_V = [0.0064 + 0.1024 + 0.0001]^{1/2} = 0.330 \text{ ft}^3 \quad (\text{E-26})$$

The uncertainty for the volume measurement varies for the two methods. The difference is shown in Table E2.

TABLE E2				
UNCERTAINTY COMPARISON				
Method	Calculated Volume cu ft	Uncertainty cu ft	Minimum Volume cu ft	Maximum Volume cu ft
1	31.4	4.27	27.1	35.7
2	31.31	0.33	30.98	31.64

The water method provides a much lower uncertainty than the dimensional measurement method. If an even smaller uncertainty is required, a more accurate weight scale must be used since it contributes the most in equation E-26.

When the data is to be reported the best format would be: 31.3 ±0.4 cubic feet. The 0.33 value is rounded up since the change introduced by rounding down is greater than 5%.

$$\frac{0.3 - 0.33}{0.33} \times 100\% = 9.1\% \quad (\text{E-27})$$

APPENDIX F

PRESSURE GAGE CALIBRATION

Calibration Data: The calibration data shown in Table F1 was taken with a dead weight tester. The accuracy of this tester is ± 0.2 psi.

TABLE F1							
CALIBRATION TEST DATA							
Applied psig	Indicated psig	Applied psig	Indicated psig	Applied psig	Indicated psig	Applied psig	Indicated psig
0	0	180	183	20	21	170	173
10	10	190	192	30	31	160	163
20	20	180	182	40	40	150	152
30	31	170	172	50	54	140	142
40	41	160	162	60	60	130	131
40	41	150	152	70	72	120	122
50	51	140	142	80	79	110	112
60	61	130	132	90	94	100	101
70	71	120	122	100	102	90	91
80	81	110	112	110	112	80	80
90	92	100	102	120	120	70	70
100	102	90	92	130	132	60	60
110	112	80	81	140	142	50	51
120	122	70	71	150	152	40	38
130	132	60	61	160	162	30	30
140	142	50	51	170	172	20	18
150	152	40	41	180	182	10	10
160	162	30	31	190	192		
170	172	20	20	180	183		

Calculation of M&TE Tolerance: Following the methodology in Appendix C the tolerance for this gage may be calculated from the test data.

Step 1: Pressure Gage Data is shown in Table F2.

TABLE F2		
PRESSURE GAGE DATA		
a	0 psi	No correction curve is used for this gage.
b	1	The gage is assumed correct.
ω_u	2 psi	The gage is marked in 2 psi increments. Some data was taken at 1/2 of this increment.
X_{low}	0 psig	
X_{high}	200 psig	

Step 2: The statistical data, based on the values in Table F1, is shown in Table F3.

TABLE F3	
STATISTICAL DATA	
a	0.05241 psi
b	1.0136
EE	53.04 (psi) ²
MSE	0.7367 (psi) ²
S _{xx}	208900 (psi) ²
Σ x _i ²	917,200 (psi) ²
$\bar{X}$	97.84 psi

Step 3: $v = n - 2 = 74 - 2 = 72$ (F-1)

Since the degrees of freedom is greater than 30; $t_{\alpha/2}$ will be taken as 2.0.

$$T = \frac{1.0136}{\sqrt{\frac{0.7367 \text{ (psi)}^2}{208900 \text{ (psi)}^2}}} = 540 > 2$$
 (F-2)

Since the test statistic, T, is in the critical region, H_0 is not true and a linear relationship exists.

Step 4: No previous data is available on this gage.

Step 5:

$$\omega_a = 2.0 \left[\frac{(0.7367)(917,200)}{(74)(208,900)} \right]^{1/2} = 0.418 \text{ psi}$$
 (F-3)

$$\omega_b = 2.0 \left[\frac{0.7367}{208900} \right]^{1/2} = 0.00376$$
 (F-4)

Step 6:

$$\frac{(2)^2 (0.7367)}{(1.0136)^2 (208900)} = 13.7 \times 10^{-6} \leq 0.1$$
 (F-5)

This confirms that the method is appropriate.

Step 7: (Eq. C-20 & C-21)

$$\omega_{y_{low}} = \frac{2.0}{2} + 2.0 \sqrt{0.7367} \left[1 + \frac{1}{74} + \frac{(0 - 97.84)^2}{208900} \right]^{1/2} = 2.767 \text{ psi} \quad (\text{F-6})$$

$$\omega_{y_{high}} = \frac{2.0}{2} + 2.0 \sqrt{0.7367} \left[1 + \frac{1}{74} + \frac{(200 - 97.84)^2}{208900} \right]^{1/2} = 2.770 \text{ psi} \quad (\text{F-7})$$

$$\omega_E = \frac{2.770 \text{ psi}}{1.0136} = 2.733 \text{ psi} \quad (\text{F-8})$$

Step 8:

$$\omega_F = \left\{ \begin{array}{l} \left| 0 - \frac{0 - 0.05241 + 1(0)}{1.0136} \right| \\ \left| 200 - \frac{0 - 0.05241 + 1(200)}{1.0136} \right| \end{array} \right\} = \left\{ \begin{array}{l} 0.052 \text{ psi} \\ 2.735 \text{ psi} \end{array} \right\} \quad (\text{F-9})$$

Step 9:

$$\omega_T = [(2.733)^2 + (2.735)^2 + (0.2)^2]^{1/2} = 3.87 \text{ psi} \quad (\text{F-10})$$

The gage tolerance, ω_T would be 4 psi since its dial is graduated in 2 psi increments.

Reduction of the Tolerance by Splitting the Operating Range

If this tolerance is higher than acceptable for this gage, it is possible to reduce the tolerance by establishing multiple operating ranges. Steps 1 through 6 are repeated.

Step 7:

$$\omega_{\text{low}} = \frac{2.0}{2} + 2.0 \sqrt{0.7367} \left[1 + \frac{1}{74} + \frac{(100 - 97.84)^2}{208900} \right]^{1/2} = 2.728 \text{ psi} \quad (\text{F-11})$$

from Eq. C-19

$$\omega_E = \frac{2.728 \text{ psi}}{1.0136} = 2.691 \text{ psi} \quad (\text{F-12})$$

Step 8:

$$\omega_F = 100 - \frac{0 - 0.05241 + 1(100)}{1.0136} = 1.393 \text{ psi} \quad (\text{F-13})$$

Step 9:

$$\omega_T = [(2.691)^2 + (1.393)^2 + (0.2)^2]^{1/2} = 3.037 \text{ psi} \quad (\text{F-14})$$

$$\frac{3.037 - 3}{3.037} \times 100\% = 1.2\% < 5\%$$

Since rounding down will result in less than a 5% change, ω_T would be 3 psi for the range 0 to 100 psig.

The tolerance for this gage could then be stated as:

TABLE F4	
TOLERANCE DATA	
Range psig	Tolerance psi
0 to 100	3
101 to 200	4

Figure F-1, Bourdon Gage Calibration Curve

Figure F-2, Bourdon Gage Correction Plot

Uncertainty Calculation with Correction Curve Use

If it is necessary to reduce the uncertainty of the data obtained with this pressure gage the calibration plot (Figure F-1) or the correction curve (Figure F-2) may be used. The measured data would be adjusted to provide adjusted data which is free of much of the bias error. The correction curve from the calibration plot is:

$$P_{\text{corrected}} = \frac{P_{\text{indicated}} - 0.05241}{1.0136} \quad (\text{F-15})$$

The adjusted (corrected) pressure is shown in Table F5. Also shown in Table F5 is the deviation between the applied pressure and the adjusted pressure. The uncertainty of this adjusted data would be:

$$u_R = [(0)^2 + (2.735)^2 + (0.2)^2]^{1/2} = 2.74 \text{ psi} \quad (\text{F-16})$$

The test data should be rounded to the nearest 1 psi and the uncertainty should be rounded to 3 psi when the final data is reported.

Figure F-3, Bourdon Tube Adjusted Data Uncertainty

TABLE F5**ADJUSTED TEST DATA FOR BOURDON TUBE EXAMPLE**

Applied psig	Indicated psig	Corrected psig	Error psig	Applied psig	Indicated psig	Corrected psig	Error psig
0	0	-0.052	0.052	20	20	19.681	0.319
10	10	9.815	0.185	20	21	20.667	-0.667
20	20	19.681	0.319	30	31	30.534	-0.534
30	31	30.534	-0.534	40	40	39.413	0.587
40	41	40.400	-0.400	50	54	53.226	-3.226
40	41	40.400	-0.400	60	60	59.146	0.854
50	51	50.266	-0.266	70	72	70.986	-0.986
60	61	60.133	-0.133	80	79	77.892	2.108
70	71	69.999	0.001	90	94	92.691	-2.691
80	81	79.865	0.135	100	102	100.584	-0.584
90	92	90.718	-0.718	110	112	110.451	-0.451
100	102	100.584	-0.584	120	120	118.344	1.656
110	112	110.451	-0.451	130	132	130.183	-0.183
120	122	120.317	-0.317	140	142	140.050	-0.050
130	132	130.183	-0.183	150	152	149.916	0.084
140	142	140.050	-0.050	160	162	159.782	0.218
150	152	149.916	0.084	170	172	169.648	0.352
160	162	159.782	0.218	180	182	179.515	0.485
170	172	169.648	0.352	190	192	189.381	0.619
180	183	180.501	-0.501	180	183	180.501	-0.501
190	192	189.381	0.619	170	173	170.635	-0.635
180	182	179.515	0.485	160	163	160.769	-0.769
170	172	169.648	0.352	150	152	149.916	0.084
160	162	159.782	0.218	140	142	140.050	-0.050
150	152	149.916	0.084	130	131	129.197	0.803
140	142	140.050	-0.050	120	122	120.317	-0.317
130	132	130.183	-0.183	110	112	110.451	-0.451
120	122	120.317	-0.317	100	101	99.598	0.402
110	112	110.451	-0.451	90	91	89.731	0.269
100	102	100.584	-0.584	80	80	78.879	1.121
90	92	90.718	-0.718	70	70	69.012	0.988
80	81	79.865	0.135	60	60	59.146	0.854
70	71	69.999	0.001	50	51	50.266	-0.266
60	61	60.133	-0.133	40	38	37.410	2.560
50	51	50.266	-0.266	30	30	29.517	0.453
40	41	40.400	-0.400	20	18	17.708	2.292
30	31	30.534	-0.534	10	10	9.815	0.185

APPENDIX G
STUDENT-t VALUES

Figure G1, Student-t Critical Values (one-tailed shown)

TABLE G1		
CRITICAL STUDENT-T VALUES ¹		
FOR 95% CONFIDENCE		
Degrees of Freedom	Two-tailed ($\alpha = 0.025$)	One-tailed ($\alpha = 0.050$)
1	12.706	6.314
2	4.303	2.920
3	3.182	2.353
4	2.776	2.132
5	2.571	2.015
6	2.447	1.943
7	2.365	1.895
8	2.306	1.860
9	2.262	1.833
10	2.228	1.812
11	2.201	1.796
12	2.179	1.782
13	2.160	1.771
14	2.145	1.761
15	2.131	1.753
16	2.120	1.746
17	2.110	1.740
18	2.101	1.734
19	2.093	1.729
20	2.086	1.725
21	2.080	1.721
22	2.074	1.717
23	2.069	1.714
24	2.064	1.711
25	2.060	1.708
26	2.056	1.706
27	2.052	1.703
28	2.048	1.701
29	2.045	1.699
Infinity	1.960	1.645