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**Basic Probability
Characteristics of Wind-
Direction Fluctuations**

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David C. Powell

May 1982

**Prepared for the U.S. Department of Energy
under Contract DE-AC06-76RLO 1830**

**Pacific Northwest Laboratory
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BASIC PROBABILITY CHARACTERISTICS
OF WIND-DIRECTION FLUCTUATIONS

David C. Powell

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EXECUTIVE SUMMARY

The subject of this report is probability description of wind direction fluctuations within the height range above the surface of wind energy conversion systems (WECS). The essential question the report addresses may be stated as follows. Given that a WECS machine is oriented into the mean wind direction, prevailing for a period of 15 min or so, during what fraction of the time will the instantaneous wind direction deviate from that mean direction by an angle equal to or exceeding a given value. If this can be modeled, then as a corollary, the fraction of time that the deviation angle is within a certain sector can also be modeled. Or, the first question can be stated inversely. Given a fraction of time, what is the magnitude of deviation from the mean wind direction that will be exceeded over that fraction? More precisely, in either case this report deals only with the projection in the horizontal plane of the total wind direction fluctuations.

In one sentence, this report shows that a probabilistic model of wind direction fluctuations must take into consideration coincidence of high wind conditions with unstable conditions as well as with neutral conditions. This is established by comparing exceedance probabilities for given angles calculated using a model of the rms of wind direction fluctuations σ_θ constructed on the assumption of neutral conditions and using one incorporating the probability of σ_θ found from a histogram of values representing a series of 15 min test periods conducted in Kansas in 1968. Also magnitudes, i.e., the angle that will be exceeded over a given fraction of the time, were calculated first using the neutral model, then using a model incorporating the histogram; and results were compared. In this case it was found that the exceedance angle calculated to correspond to a given small fraction of time from the model including the histogram was about 1.5 to 2 times the value obtained from the neutral model.

The data analyzed to make this determination were all taken at a "reference" height of 11.3 m, and where the surface roughness was that of wheat stubble over a level plain. When these results were extrapolated to a typical WECS height of 50 m and to greater surface roughnesses, it was found that the model

incorporating the histogram yielded values of the exceedance angle that were typically 1.5 to 2.5 times as great as those calculated using the neutral model of σ_θ . Meanwhile when the original problem is considered, that of calculating an exceedance probability for a given deviation angle, the error is of orders of magnitude when the angle is about three times σ_θ .

Therefore in the Conclusions section of this report a new probabilistic model of wind direction fluctuations is formulated that uses meteorological equations for neutral and unstable conditions as well as a probability function for σ_θ taken from analysis of the Kansas data. The user may substitute any other probability function that he has reason to consider more representative than that of the Kansas data. The model is furthermore based on use of a slight modification of Gaussian distribution of the deviation angle θ when a mean wind direction can be defined.

The report also shows that the difference between σ_θ for neutral conditions and for unstable conditions is a function of spectral frequency such that the complications of the unstable model and the histogram need not be considered if fluctuations of frequencies below 0.01 Hz are not of interest.

Although the Conclusions section is given in terms of technical terms, the full understanding of which is not presumed of nonmeteorologists, appropriate numerical values are suggested such that the results are immediately translatable into a computer program.

This computer program is given in Appendix C. It may be run according to two options. At the first option the program computes only exceedance probability as described in this report and modeled in the Conclusions section. This is done for two angles. The difference between the two exceedance probabilities is the probability that the wind direction will be in the sector between the two angles.

At the second option, machine response characteristics are simulated and the output parameters are increased to include those especially pertinent to operational strategy. The probability modeling now applies to σ_θ values that

have been reduced to correspond to machine responsiveness within a user-specified bandpass filter. Furthermore frequency response characteristics are added such that ultimately the average time duration of a wind direction fluctuation exceeding a specified angular value may be calculated. Thus the exceedance probabilities and associated frequency characteristics of wind direction fluctuations, as seen through a bandpass filter, will be useful as inputs for yaw control system designs and as a factor in the determination of operating strategies in machine control algorithms.

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ABSTRACT

This report develops a probability model for the projection of wind direction fluctuations in the horizontal plane, applicable at heights of concern in Wind Energy Conversion Systems (WECS) operations. The model is given as exceedance probability. The textual development shows that such a model must include a model for the unstable boundary layer, otherwise the angular deviations from mean direction applying over a period such as 15 min and applying at a given exceedance probability may be underestimated by a factor of 2.

The model includes a probability function for σ_θ taken from a series of 44 tests conducted by the Air Force Cambridge Laboratory in Kansas in 1968. Tests with wind speeds below typical WECS cut-in speed were excluded from this sample. The model also includes a slight adjustment for small departure of the probability of θ from Gaussian.

The model is expressed in a code which may be run according to either of two options. At either option the program reads in two angles for which exceedance probability is computed. Additionally, the probability that the wind direction will be between these two angles is given by the difference between the two exceedance probabilities. At the first option the program produces the exceedance probabilities and sector probability just described. At the second option the program goes beyond the stated atmospheric model in the following respects. Firstly, a user-specified bandpass filter is invoked that corresponds to the frequency range of machine sensitivity to wind fluctuation frequencies. The values of σ_θ in the probability function from the Kansas data are reduced accordingly using lateral component turbulence spectra for neutral and unstable conditions. The output exceedance probabilities calculated correspond to the reduced values.

Secondly, the program produces frequency of outward slope level-crossings at the two angles specified and the average time interval between such a crossing and the return of wind direction to the same level (angle). This information may be used in yaw and operating strategies.

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1.0 INTRODUCTION

The introduction to a report of this kind need only relate the material to follow to previously published related reports and then provide a map indicating where the reader may go from here depending on his interest.

The only document published by the PNL Wind Characteristics Program Element (WCPE) dealing primarily with wind direction fluctuations is by Cliff (1979) titled "Wind Direction Change Criteria for Wind Turbine Design." Insofar as this model treats the atmosphere, it is a model of wind direction change criteria at neutral stability. The Cliff report goes beyond the present report in that such machine related topics as the upper frequency limit of machine responsiveness are modeled along with the use of a differencing or "gust rise" time. The results of the present report can be easily incorporated into a velocity difference model or "gust rise" since the essential Gaussian assumption of the present model was found from analysis of local data to hold for differenced data provided that the differencing interval is not less than 1 second.

As stated elsewhere in the report, statistics of the wind direction angle may be taken from statistics of the lateral component of turbulence v with minimal error. Therefore the reader who has read such reports as Frost et al. (1978), Powell and Connell (1980) or Doran and Powell (1980) would expect that the present report would be some sort of analog of these materials for wind direction fluctuations. However, none of these reports treat the coincidence of high winds and unstable conditions. Therefore, the reader is not made aware of the error made with respect to wind direction fluctuations when the unstable conditions are not modeled. The omission of modeling the unstable case is more serious with respect to wind direction fluctuations than with respect to wind speed fluctuations, which these documents are principally concerned with, as well as such gust models as those of Cliff and Fichtl (1978) and Huang and Fichtl (1979).

Therefore this report concentrates on the probabilistic structure of wind direction fluctuation modeling that is a necessary component of any of the

more complicated concepts presented in the aforementioned documents such as frequency of occurrence of positive-slope level crossings or a $GUST_0$ model.

The reader who is interested only in final recommendations may go directly from this section to the Conclusions section where the final probabilistic model is given. However, since this model is conveyed in terms of meteorological definitions that may not be familiar to nonmeteorologists, and since definition of mathematical and meteorological terms occupies different sections of the report, the meteorological definitions are repeated here.

The value of the rms of wind direction fluctuations σ_θ that is expected to apply at a given point in the atmospheric boundary layer depends on three dimensionless ratios, z/z_0 , z/L and z_i/L . The first two elements of these ratios are z -height of the point above the immediate surface z_0 -surface roughness length. Representative values are z_0 (see Counihan 1975):

smooth (mud, snow)	0.04 to 0.1 cm
moderately rough (short grass, crops, rural)	0.1 to 20 cm
rough (woods, suburbs)	100 to 150 cm
very rough (urban)	300 to 400 cm.

The third parameter z_i is the depth of the convective layer during unstable conditions. On a typical sunny day it increases from a few hundred meters in the morning to about 1500 m in the afternoon.

The remaining parameter, the Monin-Obukhov stability length L is more difficult to explain. The reader who is interested only in numerical values will find these in the Conclusions section. A cursory description is given in Section 4. A more technical description is given in Appendix B.

The reader with limited interest may therefore go to the Conclusion section from this point.

Section 2 gives the essential definitions of wind direction descriptor, its variance and spectrum, and the probability descriptor to be entertained. Section 3 outlines the probability problem. Section 4 is a detailed traverse of the working out of the probability problem, beginning with essential meteorological definitions. Section 5 treats these results from a spectral point of

view. Section 6 deals with the essential Gaussian assumption of turbulence modeling with respect to wind direction fluctuations. The Hanford data analyzed is described briefly in Appendix A. Appendix B has further technical definitions of meteorological stability parameters.

Appendix C relates the probability model to machine responsiveness. A bandpass filter, simulating the frequency response characteristics of a machine, is introduced; and the computed probability characteristics apply to this frequency band. Also frequency of level crossing calculations are included so that the average time duration of a wind direction excursion beyond an arbitrary angular value may be computed. The material in Appendix C should greatly increase the practical impact of the report.

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2.0 DEFINITIONS AND ILLUSTRATIONS

2.1 WIND DIRECTION DATA

Let us assume that over a period of time, T , which is typically 10 to 60 min, the horizontal projection of the instantaneous wind vector takes on directions that are confined to a certain sector of a circle in the x,y plane such that the direction and speed of a mean wind vector may be defined. This is illustrated by the arrow in Figure 2.1. At a point (x_h, y_h, z_h) , which we take to be the location of the hub of a WECS, we define the x -axis as extending horizontally in the downwind direction, the y - and z -axes being the other horizontal direction and the vertical direction, respectively, of a right-handed coordinate Cartesian system. The three wind components at (x_h, y_h, z_h) are $U(t)$, $V(t)$, and $W(t)$, the components in the x , y , and z direction, respectively. The wind direction data analyzed in this report are defined by

$$\theta(t) = \frac{180}{\pi} \tan^{-1} \frac{V(t)}{U(t)} \quad \text{.. (degrees)} \quad (2.1)$$

where t is time. These definitions are illustrated in Figure 2.1. In Equation (2.1) we will assume that the mean wind direction is perpendicular to the plane of blade rotation.

2.2 VARIANCES AND SPECTRA

The variance of these data is given by

$$\sigma_{\theta}^2 = \frac{1}{T} \int_{t_0}^{t_0 + T} \theta^2(t) dt, \quad (2.2)$$

The rms quantity referred to throughout this report is the square root of the variance. The variance is subject analytically to a spectral decomposition over an infinite frequency range. The decomposition function is called the power spectrum, or simply the spectrum, and is related to the variance by

$$\sigma_{\theta}^2 = \int_0^{\infty} \phi_{\theta}(n) \, dn, \quad (2.3)$$

where n is circular frequency and $\phi(n)$ is the power spectral density.

FIGURE 2.1. Geometry of Definitions in x-y Plane

2.3 EXCEEDANCE PROBABILITY

We use the term $P_{\theta}(x)$ to represent that probability that a given value of angular fluctuation θ will exceed an arbitrary angle x . This is exceedance probability. To represent probability density, we use the small p . Probability density has the property of integrating to a value of unity:

$$\int_{-\pi}^{\pi} p(\theta) \, d\theta = 1 \quad (\text{theta in radians}) \quad (2.4)$$

where the limits $-\pi$ to $+\pi$ are imposed rather than the usual infinity because these are the limits of angular measure.

We must also define the conditional probability density, the probability density of θ given a particular value of the rms. The notation for conditional probability uses a vertical line to separate the two arguments: $p(\theta|\sigma_\theta)$. Conditional probability integrates to unity as follows:

$$\int_0^\pi p(\sigma_\theta) \int_{-\pi}^\pi p(\theta|\sigma_\theta) d\theta d\sigma_\theta = 1 \quad (2.5)$$

Then the final expression for exceedance probability, the probability that a given angular fluctuation value will exceed x is

$$P_\theta(x) = \int_0^\pi p(\sigma_\theta) \int_x^\pi p(\theta|\sigma_\theta) d\theta d\sigma_\theta, \quad x << \pi. \quad (2.6)$$

This formula is for one-sided angular exceedance. The corresponding expression for exceedance of an absolute value of x can be obtained by multiplying the above by 2 if the probability density $p(\theta)$ is symmetrical. If not, another integration over from $-\pi$ to $-x$ must be added.

2.4 "WIND TURBULENCE CRITERIA"

Throughout this report there are references to Wind Turbulence Criteria. This phrase references a memorandum titled "Wind Gust Analysis for Wind Turbine Design" by David A. Spera and Timothy R. Richards of the NASA-Lewis Research Center. It has been published as Appendix A of Powell and Connell (1980). This memorandum has modeling elements that pertain to both mean wind and turbulence within the WECS layer.

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3.0 DIFFERENCE IN THE PROBABILITY PROBLEM FOR ANGLE AS OPPOSED TO SPEED

The equation for exceedance of a wind speed corresponding to Equation (2.6) may be written

$$P_u(x) = \int_0^{\infty} p(\sigma_u) \int_x^{\infty} p(u|\sigma_u) du d\sigma_u \quad (3.1)$$

where it is assumed that values of u-component turbulence, the along-wind component, are effectively those of wind speed. In most measurements the difference is only a few percent. Assuming that turbulence is Gaussian, the second probability becomes the familiar expression

$$p(u|\sigma_u) = \frac{1}{\sqrt{2\pi} \sigma_u} e^{-\frac{1}{2} \left(\frac{u-U}{\sigma_u} \right)^2} \quad (3.2)$$

where U is the mean wind speed. The first probability density is handled by assuming, from the neutral model that σ_u is a constant multiple of mean wind speed. For example from the Wind Turbulence Criteria we have

$$[\sigma_u]_{\text{ref}} = \frac{1.04 U_{\text{ref}}}{\ln(1 + z_{\text{ref}}/z_0)} \quad (3.3)$$

where z_{ref} refers to a reference height of 9.1 m. Actually it turns out that neutral boundary layer theory represents σ_u as a constant throughout the entire boundary layer, which is generally treated in WECS literature as co-extensive with the WECS layer. But to relate to the point being developed here, it is advantageous to focus on the first equality of the following equation:

$$\sigma_u = C_z U(z) = \text{constant} \quad (3.4)$$

which says that σ_u is related to $U(z)$ by a height dependent constant.

The point is that we can now write

$$p(\sigma_u)d\sigma_u = p(U)dU, \quad (3.5)$$

and finally

$$P_u(x) = \int_0^{\infty} p(U) \int_x^{\infty} p[u|\sigma_u(U)] du dU \quad (3.6)$$

This construction is implicitly involved in all models dealing with probability of turbulence events and using the probability density of mean wind speed, which is available in place of probability density of turbulence rms, σ_u . Such a document is the model by Cliff and Fichtl (1978), where the Rayleigh density is chosen as the probability density of mean wind speed. In the Wind Turbulence Criteria, the Weibull distribution is chosen.

To make the main point of this section we first return to Equation (2.6) where the bothersome term $p(\sigma_\theta)d\sigma_\theta$ occurs. It is bothersome because, to the knowledge of the author, information on $p(\sigma_\theta)$ is lacking. Moreover we cannot solve the problem by substituting $p(U)dU$, for $p(\sigma_\theta)d\sigma_\theta$ since, particularly in the neutral boundary layer, σ_θ is theoretically not a function of U , as is shown in Section 4. Therefore, the most we can do to make Equation (2.6) more specific at this point is to assume that wind direction data are Gaussian and symmetrical and write for the absolute value of,

$$P_{|\theta|}(x) = \int_0^{\infty} p(\sigma_\theta) \int_x^{\pi} \frac{1}{\sqrt{\pi/2} \sigma_\theta} e^{-\frac{1}{2} \left(\frac{\theta}{\sigma_\theta} \right)^2} d\theta \cdot d\sigma_\theta \quad (3.7)$$

leaving $p(\sigma_\theta)$ for the moment unspecified (see Equation 7.1)..

This is the end of the portion of this report dealing strictly with mathematical, as opposed to meteorological, formation. The basic mathematical equation upon which all the positive results of this report are based is Equation (2.6). The more practical version, which incorporates the Gaussian

assumption and accordingly modifies the problem to deal with the absolute value of θ is Equation in (3.7). Since the upper limit of integration of the Normal Probability integral in Equation (3.7) is π rather than infinity, it follows that σ_θ must be small enough that the value of the integrand at $\theta = \pi$ is effectively that for θ equal infinity.

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4.0 ON THE MODELING OF THE PROBABILITY DENSITY OF σ_θ

This section is divided into three parts where in the topics are 1) correspondence of the rms of wind direction fluctuation σ_θ to the quotient of the rms of the lateral component of turbulence by the mean wind, σ_v/U ; 2) relations for σ_v/U , and consequently for σ_θ found in the literature; and 3) comparison of these values with certain observed values. All of this is related to the need for information about the probability density of σ_θ , $p(\sigma_\theta)$.

In the last two topics relations for σ_θ are given for both neutral and unstable conditions. This is done because a quasi-discontinuous increase of σ_θ occurs as atmospheric stability changes from neutral to very slightly unstable. Since the high wind conditions that are important to wind energy production include both conditions, accounting of both is needed for an adequate model. The neutral model will be considered adequate for such slightly stable conditions as sometimes prevail during high winds.

4.1 CORRESPONDENCE OF THE RMS OF WIND DIRECTION FLUCTUATIONS TO THE RMS OF THE LATERAL COMPONENT OF TURBULENCE

Our analysis indicates that the rms of wind direction fluctuations may be calculated directly from the rms of the lateral component of turbulence and the mean wind. Accordingly when wind angle is expressed in degrees, as in Equation (2.1) and when its variance is defined as in Equation (2.2), we should expect the relation between the rms of wind angle and that of the lateral component turbulence to be

$$\sigma_\theta(z) = \frac{180}{\pi U(z)} \sigma_v \quad (\text{degrees}) \quad (4.1)$$

Examination of the Hanford vertical plane array data analyzed for this document, see Appendix A, shows that σ_θ is underestimated by 2% to 3% by Equation (4.1) for all four tests. Since these differences are much less than many uncertainties that may emerge from our analysis, we will consider Equation (4.1) to be a good relation.

4.2 RELATIONS FOR TURBULENCE INTENSITY σ_v/U FOUND IN THE LITERATURE

4.2.1 Neutral Stability Model

For wind engineering purposes this model will apply for neutral conditions and for the slightly stable conditions that are pertinent to wind energy production. The Wind Turbulence Criteria, published as Appendix A of the report by Powell and Connell (1980) give the following equation for the intensity of the lateral component of turbulence:

$$\frac{\sigma_v}{U_r} = \frac{0.83}{\ln(1 + z_r/z_0)} , \quad (4.2)$$

where U_r is the mean wind speed at a reference level z_r , and where z_0 is the roughness length, for which typical values are given by Cliff and Fichtl (1978). The Wind Turbulence Criteria set the reference level at 9.1 m. For purposes of the present report, which applies to winds over level homogenous terrain, where a so-called constant flux layer is recognized by meteorologists to exist up to a height of about 30 m, we will assume that the reference level can be at any height from 9 to 30 m, preferably closer to the former.

For neutral stability the denominators of the two sides of Equation (4.2) are proportional for different z_r . Therefore it follows that

- σ_v is a constant through the constant flux layer and is proportional to U_r , the mean wind speed at a reference level;
- by contrast σ_θ is not a function of mean wind speed, but is a decreasing function of height

$$\sigma_\theta(z) = \frac{180}{\pi} \frac{0.83}{\ln(z/z_0)} \approx \frac{150}{\pi \ln(z/z_0)} . \quad (\text{degrees}) \quad (4.3)$$

According to Panofsky (1973) Equation (4.2) may be applied throughout the entire WECS layer. We similarly recommend Equation (4.3). The number 1 has been dropped from the denominator in going from Equation (4.2) to Equation (4.3) because the logarithmic parameter should not be used unless z/z_0 is large enough, (say 30 or 40) so that adding one more does not make the logarithm significantly different.

4.2.2 Unstable Conditions

For unstable conditions the analogous equations are

$$\frac{\sigma_v}{U_r} = \frac{0.4(12 + 0.5|z_i/L|)^{1/3}}{\ln(z_r/z_0) - \psi(x_r)} \quad (4.4)$$

$$\sigma_\theta(z) = \frac{72(12 + 0.5|z_i/L|)^{1/3}}{\pi[\ln(z/z_0) - \psi(x)]} \quad (\text{degrees}) \quad (4.5)$$

where

$$\psi(x) = 2 \ln \left[\frac{1+x}{2} \right] + \ln \left[\frac{1+x^2}{2} \right] - 2 \tan^{-1} x + \pi/2,$$

$$x = (1 + 16|z/L|)^{1/4},$$

see Paulson (1970).

In these equations L is the Monin-Obukhov stability parameter with dimensions of length (see Appendix B). For a full understanding of this parameter, the reader is referred to Lumley and Panofsky (1964) or Businger et al. (1971). For the engineer who is interested only in application, suffice it to say that L is negative for unstable conditions and becomes of smaller absolute value as instability increases. A suitable range of values for unstable conditions for high winds is -30 m to -500 m, where use of the former yields larger estimates of turbulence intensity.

The z_i is the depth of the convective layer under unstable conditions. A good approximation for a summer afternoon under sunny skies is 1500 to 2000 m. A series of maps by Holzworth (1972) for mornings and afternoons at various times of the year show spring and summer values over southwestern states attaining as much as 3000 m.

4.3 COMPARISON OF PREDICTED AND OBSERVED VALUES

The spread of values of σ_v/U_r at a single site is shown in Figure 4.1 see Izumi (1971). This is a histogram of values for forty-four 15-min periods

FIGURE 4.1. Observed σ_θ as a Function of Stability During Moderately High Winds

which are lumped together in 12 test periods most of which lasted for 1 hr, while some were 30 or 45 min. The values of σ_v/U_r are shown on the horizontal axis. The reference height z_r at which U_r is measured is 11.3 m. Since the data on this graph are only for σ_v observed at the same height, σ_v/U_r is also σ_θ . Only those tests were considered where the wind within the height of the measuring array (23 m) exceeded 7.75 m sec^{-1} during at least one 15-min period.

These data are plotted as a function of stability, which is represented on the horizontal axis by the Richardson number Ri . Ri is a function of the

shear of the mean wind $dU(z)/dz$, the lapse rate of potential temperature, (see Appendix B), the absolute temperature Θ , and the gravitational acceleration g . The relation is

$$Ri = \frac{g \, d\Theta/dz}{\Theta(dU/dz)^2} . \quad (4.6)$$

Positive values of Ri represent stable conditions, the zero value represents neutral conditions, and negative values represent unstable conditions. Ri is a one-to-one function of z/L , as is shown in Appendix B. In this graph we presume the neutral model of σ_v/U_r , Equation (4.2), to extend over stable conditions. This is indicated by the straight line at the value of $\sigma_v/U_r = 0.135$. The predicted values of σ_v/U_r for unstable conditions depend on the additional parameter, z_i , which is not specified in the source document. The two diagonal lines drawn on the right side (the unstable side) indicate the range of σ_v/U_r if z_i is presumed to vary from 500 m to 3000 m. Since many of the points on the unstable side fall outside the plausible range, there is much unparameterized variation of σ_v/U_r , i.e., these values depend on physical conditions that are not included in the parameters, z_r/z_0 , z_r/L , and z_i/L .

4.4 CHARACTERIZING $p(\sigma_\theta)$ FROM THE ONE SET OF 44 TEST PERIODS

Since U_r and σ_v were measured at the same height we may equate σ_v/U_r to σ_θ . From the 44 data plotted in Figure 4.1, we find

$$\begin{aligned} \text{mean } \sigma_\theta &= 0.152 && \text{radians} \\ \text{variance } \sigma_\theta &= 0.37 && \text{radians squared} \\ \text{skewness } \sigma_\theta &= 1.04 \\ \text{kurtosis } \sigma_\theta &= 3.79. \end{aligned}$$

We note that the mean value is 13% greater than that obtained using the neutral model, Equation (4.3), which predicts a mean value of 0.135 for σ_θ . Also we note that the distribution is far too skewed to be considered Gaussian. This is because the values for neutral to stable conditions cluster while those for unstable conditions spread.

A probability density function for σ_θ may be constructed from the following histogram of values from the 44 values.

Since the experimental function of $p(\sigma_\theta)$ that could be constructed from this table comes from only 44 tests rather than hundreds, as would be desired, there is a question about how representative the derived pdf is. But it will have to do for the present since the author is aware of no other tabulations of similar character and magnitude. There are most certainly aspects of the unevenness that are not representative. Anyone using the data in Table 4.1 would want to smooth the sample at the value 0.18 radians among the neighboring values. Also the highest value 0.35 radians would probably have a lower probability from a larger data sample.

TABLE 4.1. Histogram of σ_θ Values From Figure 4.1

<u>σ_θ (rad) cases</u>	<u>σ_θ (rad) cases</u>
0.09 7	0.20 3
0.10 8	0.21 2
0.11 3	
0.12 2	0.23 2
0.13 1	0.24 1
0.14 2	0.26 1
0.15 3	0.28 1
0.16 1	0.35 <u>1</u>
0.17 1	44
0.18 4	
0.19 1	

4.5 USING THE DERIVED PDF OF σ_θ

Obviously the probability that a wind direction fluctuation will exceed a certain magnitude will be underestimated if the assumed rms value is less than that which actually applied. It is perhaps less obvious, but equally true that if the correct average value of σ_θ is used rather than the probability function $p(\sigma_\theta)$ that makes up that average, the exceedance probability for

large magnitudes will be further underestimated. In other words the probability that a direction fluctuation will exceed a large magnitude calculated from

$$P_{|\theta|}(x) = \frac{1}{\sqrt{\pi/2} \bar{\sigma}_{\theta}} \int_x^{\pi} e^{-\frac{1}{2} \left(\frac{\theta}{\bar{\sigma}_{\theta}} \right)^2} d\theta, \quad \bar{\sigma}_{\theta} = \int_0^{\pi} \sigma_{\theta} p(\sigma_{\theta}) d\sigma_{\theta} \quad (4.7)$$

is smaller than that calculated from the more correct Equation (3.7).

To assess the practical importance of modeling the severity of wind direction fluctuations after the $p(\sigma_{\theta})$ derived from the Kansas data, as opposed to ignoring it, Figures 4.2 and 4.3 have been added.

The four lines on these two figures are all ratios that require careful definition. The ratios plotted in Figure 4.2 correspond to direction solution Equation (3.7) (given a magnitude of wind direction deviation, x , calculate the exceedance probability) while those plotted in Figure 4.3 correspond to the indirect solution (given an exceedance probability calculate the corresponding x .) The ratios plotted are as follows:

FIGURE 4.2 - solid line: Ratio of the exceedance probability obtained by using $p(\sigma_{\theta})$ from analysis of the Kansas data to that obtained by using its average value. This is the ratio of the left side of Equation (3.7) to that of Equation (4.7) as a function of x/σ_{θ} .

dashed line: Ratio of the exceedance probability obtained by using $p(\sigma_{\theta})$ from analysis of the Kansas data to that obtained by substituting the value from the neutral model (0.135 radians) into Equation (4.7) for the average value of the Kansas data (0.152 radians).

FIGURE 4.3 - solid line: Given the exceedance probability on the horizontal-axis, the ratio is that of x from Equation (3.7) corresponding to this probability to x from Equation (4.7) corresponding to the same probability.

dashed line: The same ratio multiplied by 1.13 (0.152/0.135) since the x in the denominator is that from the neutral model.

FIGURE 4.2. Ratios of Exceedance Probability of σ_θ Using a) Observed Distribution of σ_θ , and b) Neutral Model of σ_θ for $(z, z_0) = (11.3, 0.025 \text{ m})$.

FIGURE 4.3. Ratios of $σ_θ$ at Given Exceedance Probability Using a) Observed Distribution of $σ_θ$ and b) Neutral Model of $σ_θ$ for Same (z, z_0)

It is easily seen from Figure 4.2 that exceedance probabilities can be underestimated by orders of magnitude when $p(σ_θ)$ is ignored. Also, the figure illustrates how much more serious the error is when from the neutral model is used rather than that which is the average of the applying to the data analyzed.

From Figure 4.3 we see that the error in exceedance magnitude associated with a given exceedance probability is generally less than a factor of 2.0. For example, if you calculate using the neutral model, without $p(σ_θ)$, of course, the wind directional deviation exceeded 0.001 of the time, the materials analyzed in this report indicate that the angle value you get should be multiplied by about 1.7. On the other hand if the deviation value was calculated using the correct average value but not $p(σ_θ)$, the deviation value obtained should be multiplied by about 1.5.

All the arguments presented thus far are concerned with the magnitude of the rms and its probability distribution. Nothing has been said about any additional corrections needed to account for the fact that the probability

function of θ itself may or may not be Gaussian, may or may not meet the Gaussian assumption expressed within both Equation (3.7) and (4.7). Examination of this question is postponed until Section 6.

4.6 EXTRAPOLATION OF PREVIOUS CHARACTERIZATION TO WECS HEIGHT AND TO OTHER SURFACE ROUGHNESS

In Sections 4.4 and 4.5 evidence was given showing that for a data set of 44 samples, the probability density of $p(\sigma_\theta)$ is such that the angle values responding to small exceedance probabilities should be 1.5 to 2 times greater than the angles predicted by assuming that the neutral value. However, the data analyzed were all taken at a "reference" height of 11.3 m and over a field of wheat stubble, with a roughness length z_0 of 0.025 m.

The neutral model of σ_θ from Equation (4.3) is much simpler than a model including the behavior of during unstable conditions as shown in Equation (4.5) and the necessary probability distribution, for which we must use the example in Table 4.1. Therefore, it would be convenient if the extrapolation ratios shown in Figure 4.3 were found applicable at other heights and surface roughness. There is no immediate obvious reason why this should or should not turn out to be accurate enough.

Formally this amounts to defining the following ratio, which is represented in Figure 4.3 for the limited case where $z = 11.3$ m and z_0 is 0.025 m. More generally this ratio is

$$R_p = \frac{x \text{ of } \left[P_{|\theta|}(x) \text{ of } p(\sigma_\theta)_{z, z_0, z_i, L} \right]}{x \text{ of } \left[P_{|\theta|}(x) \text{ of } \sigma_\theta(z, z_0) \right]} \quad (4.10)$$

where z is now a WECS height, z_0 varies over a wide range, and z_i and L are representative values, or a set of them yet to be determined. $P_{|\theta|}(x)$ is defined by Equation (3.7). The result is that the extrapolation cannot be clearly formulated or exhibited. Therefore as a model of σ_θ , we have no choice but to recommend one including all the elements mentioned above. For clarity this is done in Section 7, where all the necessary equations and tables are repeated.

It is, however, of interest to see how this ratio turns out when z is 50 m, and when z_0 varies from 0.025 m to 1.0 m. This is shown in Figure 4.4, where the lowest curve is R_p when z is 11.3 m and z_0 is 0.025 m, and is identical with the dashed line on Figure 4.3. The other curves indicate that R_p is generally greater at 50 m than at 11.3 m, and that it increases with increasing surface roughness. However, in view of the spread between the lines, the final recommended model will be constructed from all the elements considered thus far rather than an extrapolation of the neutral model.

FIGURE 4.4. Ratios of σ_θ at Given Exceedance Probability Using a) Observed Distribution of σ_θ and b) Neutral Model of σ_θ for Several (z, z_0)

Another representation that should be of interest at this point is one showing how σ_θ varies theoretically as a function of $(z, z_0, z_i, -L)$ during unstable conditions. This is shown in Figure 4.5.

In this figure the vertical axis is height z . The horizontal axis is σ_θ in degrees; however, it is in three discontinuous sections, one for each of three roughness values, $z_0 = 0.01, 0.1, \text{ or } 1.0$ m. Each set has a set of four

FIGURE 4.5. σ_θ as a Function of Height and Roughness for Different Convective Layer Depths

distinct lines and a shaded area. The left most distinct line is σ_θ at neutral stability according to Equation (4.3). The second, third and fourth distinct lines are all for σ_θ at unstable conditions, according to Equation (4.5). They are for z_i equal 500 m, 1500 m, and 2500 m, respectively. These three lines are for $-L = 50$ m, which is the reciprocal of the average value of L^{-1} from the high wind, unstable tests of the Kansas data.

The shaded area shows the range of σ_θ for $-L = 200$ m, which is a representative value for higher winds. This shaded range is equivalent to the entire spread of the three lines for $-L = 50$ m.

For even higher winds, say $-L = 500$ m, the shaded area collapses into little more than a stripe that is located at the left end of the shaded area for $-L = 200$ m. As is expected, the ratio between σ_θ for unstable conditions

and that for neutral conditions decreases with the higher wind conditions. Therefore the exceedance ratios R_p shown on Figure 4.4 would be reduced somewhat for higher wind cases.

Having made these points we now turn to the spectral distribution of σ_θ^2 to see how the contributions to σ_θ^2 vary with frequency.

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5.0 POWER SPECTRAL MODEL FOR WIND DIRECTION FLUCTUATIONS

In Section 4 it was shown that the rms of wind direction fluctuations σ_θ has a probability distribution of its own that must be considered in predicting the exceedance probabilities of large directional fluctuations. The variance of wind direction fluctuations σ_θ^2 is, of course, the integral of the power spectrum of these fluctuations, see Equation (2.3); and must, in WECS application, be considered as a function of frequency, which is the variable of integration. That is, the sensitivity of a machine to the spread of turbulence rms discussed in Section 4 depends on the frequency range of sensitivity of the machine. In Section 5 we show that the spread of σ_θ as indicated in Section 4 is chiefly due to the difference of power spectral contributions below 0.01 Hz when the comparison is made between neutral and slightly unstable conditions.

Section 5 is divided into these subsections. In Section 5.1 comparison is made between a sample power spectrum for $v(t)$ and one of $\theta(t)$. The comparison here shows that the statement made in Section 4.1 of the approximate equivalence of σ_v and σ_θ can be strengthened to include the spectral decomposition of the corresponding variances. The two spectra follow closely at all banded frequency estimates.

In Section 5.2, formulae and graphs are given comparing neutral and stable conditions, showing the frequency range over which the difference becomes important.

In Section 5.3 the important point of the entire Section 5 is briefly stated.

5.1 EQUIVALENCE OF POWER SPECTRA FOR $v(t)$ AND $\theta(t)$

To compare the power spectra of $v(t)$ and $\theta(t)$ it is necessary first to define a form of one that has the physical dimensions of the other. Accordingly, for use in Section 5.1 only, we define

$$\tilde{\theta}(t) = U \tan^{-1}[V(t)/U(t)] \quad (5.1)$$

which is related to wind angle with dimension of degrees by

$$\theta(t) = \frac{180}{\pi U} \gamma(t) . \quad (5.2)$$

Next we introduce the form of the power spectrum that is used by meteorologists and which appears in such wind energy oriented publications as Frost et al. (1978). This is $n\phi(n)$, which is logarithmically integrated to yield the variance according to

$$\sigma^2 = \int_0^{\infty} n\phi(n) d[\ln(n)] . \quad (5.3)$$

The correspondence between the v spectrum $n\phi_v(n)$ and the θ spectrum $n\phi_{\theta}(n)$ is shown in Figure 5.1 for one of the unstable samples. Since the differences between these two spectra are much smaller than the differences between sample v spectra and models of the same, we shall model the v spectra and assume that the corresponding θ spectra are identical for all practical purposes.

In the remainder of this section we shall first set forth model v spectra for neutral and unstable conditions and then compare with spectra from experimental data.

5.2 MODELS OF v SPECTRA FOR NEUTRAL AND UNSTABLE CONDITIONS

5.2.1 Neutral Conditions

Perhaps the most familiar model of the v spectrum for WECS design engineers is that given by Frost et al. (1978). In this form the power spectrum, $n\phi_v(n)$ is normalized by the variance σ_v^2 . The equation is

$$\frac{n\phi_v(n)}{\sigma_v^2} = \frac{0.164(f/f_{ov})}{1 + 0.164(f/f_{ov})^{5/3}} \equiv \psi(n), \quad (5.4)$$

FIGURE 5.1. Correspondence of Power Spectra for v and $\tilde{\theta}$

where

$$f = nz/U,$$

and

$$f_{ov} = 0.0265.$$

Use of the dimensionless frequency $f = nz/U$ involved the following assumptions:

- a turbulence length scale U/n is preserved at different wind speeds, U
- this length scale is proportional to the height, z
- the eddies, or fluctuations, are advected at the speed of the mean wind--Taylor's hypothesis.

Of these assumptions the most questionable is the assumption that turbulence lengths are proportional to height throughout the entire WECS range which is up to, say, 150 m. However, at the present state of the arts, we recommend following the statements in Frost et al. (1978) which suggested this usage while acknowledging that the model was developed from measurements taken in the lowest 23 m.

Here it should also be stated that the form of Equation (5.4) was first given by Kaimal (1973), where the intent was to apply it to stable conditions and is a function of Richardson number, see Equation (4.6). As Kaimal originally wrote it, we have

$$f_{ov} = 1.5 Ri \quad 0.05 \leq Ri \leq 0.25 . \quad (5.5)$$

Frost et al. (1978) have adapted the equation to neutral conditions by supplying the value of f_{ov} given in the third member of Equation set (5.4).

5.2.2 Unstable Conditions

The Kaimal v spectrum for unstable conditions, see Kaimal (1978), require three different equations for three different frequency ranges modeling the following physical situations.

- A high frequency range in which the spectrum depends on two dimensionless functions of height z . These are nz/U and z/L .
- A low frequency range in which the spectrum depends on two dimensionless functions of the height of the lowest inversion aloft, z_i . These are nz_i/U and z_i/L .
- A transition region between these two.

The equations are

$$\frac{n\phi_v(n)}{\sigma_v^2} = \frac{0.4f^{-2/3}}{A} \quad \frac{nz}{U} \geq 1 \quad (5.6a)$$

$$\frac{n\phi_v(n)}{\sigma_v^2} = \frac{0.4f^{-q}}{A} \quad \frac{nz}{U} < 1, \frac{nz_i}{U} > 4 \quad (5.6b)$$

$$\frac{n\phi_v(n)}{\sigma_v^2} = \frac{f_i}{1 + 3.1f_i^{5/3}} \quad \frac{nz_i}{U} \leq 4 \quad (5.6c)$$

where

$$A = \frac{(12 + 0.5|z_i/L|)^{2/3}}{1 + 0.75|z/L|^{2/3}}$$

$$f_i = nz_i/U$$

$$q = \frac{\ln(0.30A)}{\ln(0.25z_i/z)}$$

Comparison of the Kaimal-Frost spectrum for neutral conditions with the Kaimal spectrum for unstable conditions for several values of z_i is shown in Figures 5.2, 5.3, and 5.4 for the data described in Appendix A at heights of 12.2, 36.6, and 61 m. In all cases the Monin-Obukhov L was set at -50 m, which is representative of the Kansas data as explained in Section 4.6. All of the graphs show considerably more low frequency energy for z_i equal 1500 m or 2500 m, which is typical of sunny afternoon conditions in spring or summer. At the lower height, there is considerable difference even when z_i is as small as 500 m. Typical values of z_i for different times of the year and at different locations are given by Holzworth (1972).

We note that for z_i equal 500 m or more, the model spectra do not converge sufficiently at the low frequency end that an integral scale could be meaningfully specified.

5.3 CONCLUSION PERTAINING TO SPECTRAL DISTRIBUTIONS

In general the difference between the Kaimal-Frost spectrum (the neutral model) and the spectra for unstable conditions is significant principally at frequencies lower than 0.01 Hz. Therefore, the difference between the neutral model and the unstable model for turbulence intensity, Equations (4.3) and (4.5) may be important principally for responsiveness below 0.01 Hz.

FIGURE 5.2. Model Power Spectra for v for Neutral and Unstable Conditions, $z = 12.2 \text{ m}$

FIGURE 5.3. Model Power Spectra for v for Neutral and Unstable Conditions, $z = 36 \text{ m}$

FIGURE 5.4. Model Power Spectra for v for Neutral and Unstable Conditions, $z = 61 \text{ m}$

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6.0 EXCEEDANCE PROBABILITY OF WIND DIRECTION FLUCTUATIONS DURING STEADY MEAN WIND CONDITIONS

The usual model of the probability density of any turbulence component during a period of time when mean wind conditions are steady is the Gaussian model. Applying this reasoning to the probability density of wind direction fluctuations, we hypothesize

$$p(\theta|\sigma_\theta) = \frac{1}{\sqrt{2\pi} \sigma_\theta} e^{-\frac{1}{2} \left(\frac{\theta}{\sigma_\theta} \right)^2}, \quad -\pi < \theta \leq \pi \quad (6.1)$$

(It is well for the reader to note that this is the second probability in the integral of Equation (2.6). Whereas the sections prior to this point have concentrated on the first probability in this integral, this section concentrates on the second one).

In view of the constraints upon the orthogonal components of turbulence expressed by the governing equations of turbulent motion, the Navier-Stokes equations [see Hinze (1959)], it is expected that the probability density of turbulence components will not be truly Gaussian, but close enough that the kurtosis of the data is a useful indication of the degree of departure from Gaussian distribution. For wind direction fluctuations, we can write this as

$$K_\theta = \overline{\theta^4} / \left(\overline{\theta^2} \right)^2 \quad (6.2)$$

The kurtosis of any sample of Gaussian distribution is 3. A kurtosis greater than 3 indicates that the probability of encountering extreme values is greater than in Gaussian distribution of the same standard deviation. An example is exponential distribution, for which the kurtosis is 6. Generally, the analysis of turbulence data reveals it to have slightly higher than Gaussian kurtosis.

This turns out to be true for the four data samples of data (see Appendix A) from the Hanford vertical plane array that were used to generate the power spectra in Section 5. The kurtosis K_θ for the four tests were:

TABLE 6.1. K_{θ} of Four Hanford Vertical Plane Array Samples

<u>Test Name</u>	<u>Kurtosis</u>
NA102	3.45
NA106	3.4
NA111	3.4
NA412	3.5

The point is further illustrated by the exceedance probability functions for the same four tests plotted in Figures 6.1 and 6.2. Horizontal-axis values are for

$$P_{\theta}(x) = \frac{1}{\sqrt{2\pi} \sigma_{\theta}} \int_x^{\pi} e^{-\frac{1}{2} \left(\frac{\theta}{\sigma_{\theta}} \right)^2} \dots$$

These plots are on Gaussian paper such that departure from Gaussian probability in the data sample is reflected by departure from a straight line in the graphical representation of the sample probability. Two sample functions are displayed on each figure, and for the sake of greater clarity, the numbering on each axis is reversed for plotting the analysis of the second function. Thus one axis numbering at the bottom and at the left applies to the filled in circle function, and the axis numbering at the top and at the right applies to the open circle function. The slope of the straight lines is proportional to the rms of the data. If the data are of Gaussian distribution, the function plotted on this paper will be a straight line.

These lines are not straight, but with a curvature showing that the probability of encountering large values is greater than would be predicted by assuming Gaussian distribution. Just how serious this is depends on whether the engineer's problem requires consideration of the error in probability or the error in exceedance value. Graphically the first error is represented by a horizontal line drawn from the sample value to the closest diagonal line, while the second error is represented by a vertical line from the sample values to the closest diagonal line. Thus where the sample values are on the

FIGURE 6.1. Probability Distribution of θ for Tests NA102, NA111

order of $3\sigma_{\theta}$ or more, the error in probability may exceed an order of magnitude, while the error in magnitudes are generally within 20%.

In Sections 4 and 5 it was shown or stated that the rms and spectra of θ can be taken directly from that of the lateral turbulence component v with negligible error. Presumably the same can be done with probability distribution. Therefore, a short discussion of kurtosis values for v , K_v is added to the report at this point.

Kurtosis values are given by Izumi (1971) for hourly data runs including some, but not all of the hourly runs that were broken up into 15-min segments for the analysis given in Section 4 of the present report. In fact values of K_v are given for only 6 of the 12 hourly segments of high wind speed cases

FIGURE 6.2. Probability Distribution of θ for Tests NA106, NA412

that were used to produce the forty-four 15-min segments analyzed in Section 4. The values at the 11.31 m height from which the Section 4 data were taken were 2.77, 2.81, 3.22, 3.31, 3.15, and 3.81, which yield an average value of 3.18. Further analysis including all 25 tests represented in Izumi's table, whether high wind speed or not, yielded an average K_v values of 3.12.

A larger sample of K_v values is available from data analyzed by the author part of which was published by Elderkin et al. (1971), Elderkin et al. (1972), and Powell (1974), and part of which remain unpublished. These data were taken either by sonic or Gill anemometers exposed at heights varying from 7.5 m to 60 m above homogenous desert terrain. The sampling interval varied from 0.1 to 0.4 sec, and the length of the samples from 14 to 55 min, each having 2^{13} data points. Two modes of analysis are reported on here are

1. data unfiltered except by the sampling interval and total record length
2. data additionally filtered by a highpass filter with a half power point at about 8 min.

Graphs of K_v values for each category are given in Figure 6.3 below.

FIGURE 6.3. Distribution of Kurtosis of v for Hanford Tests

These results all indicate that K_v is generally greater than the Gaussian value of 3.0; therefore, the probability of extreme fluctuations is greater than that predicted using Gaussian probability. The average K_v value for the highpass filtered Hanford data is greater than that for the unfiltered Hanford data because the highpass filter removes meandering character that reduces

kurtosis. This step is appropriate in this context because it is analogous to the effect of reorienting WECS machines into the mean wind direction.

On the whole it seems that the Gaussian model for steady-state turbulence is the appropriate basic model. However, some small adjustment remains to be made to reflect the expected kurtosis of the data.

Therefore, the chief problem in using the basic probability equation of this report Equation (2.6) lies with the first probability in the integrand, the probability density of σ_θ rather than the second probability, $p(\theta|\sigma_\theta)$, which has been treated in this section.

7.0 CONCLUSIONS--THE RECOMMENDED MODEL

The original motivation of this report was to formulate as a function of meteorological variables an exceedance probability model for θ which is the projection in the horizontal plane of the instantaneous angle between the wind and the mean azimuth, which is assumed to be that toward which a WECS machine is oriented. This model is given mathematically in Equation (3.7);

$$P_{|\theta|}(x) = \int_0^{\infty} p(\sigma_{\theta}) \int_x^{\pi} \frac{1}{\sqrt{\pi/2} \sigma_{\theta}} e^{-\frac{1}{2} \left(\frac{\theta}{\sigma_{\theta}} \right)^2} d\theta d\sigma_{\theta} \quad (7.1)$$

without meteorological detail other than the assumption that during the interval when a mean wind direction can be defined, the instantaneous angle deviations θ are of approximately Gaussian distribution.

The further specification of this model is divided into two parts that are obvious from the equation. First we discuss the probability model of σ_{θ} and then the Gaussian part of the model.

7.1 THE PROBABILITY MODEL OF σ_{θ}

It was found through analysis of parameterization published for the Kansas 1968 AFCRL that the greatest errors in using Equation (7.1) are those attending improper formulation of the probability function of $p(\sigma_{\theta})$, such as is done when the value of σ_{θ} that prevails under neutral conditions is assumed to be applicable with a probability of 1.

There are other types of models basic as the exceedance model for which this is also true. One is the probability that θ will be within a certain small range $x \leq \theta \leq x + \Delta \theta$. This is given by

$$\begin{aligned} P(x \leq \theta \leq x + \Delta \theta) &= P_{|\theta|}(x) - P_{|\theta|}(x + \Delta x) \\ &\approx \Delta \theta \int_0^{\infty} p(\sigma_{\theta}) \frac{1}{\sqrt{\pi/2} \sigma_{\theta}} e^{-\frac{1}{2} \left(\frac{x}{\sigma_{\theta}} \right)^2} d\sigma_{\theta} \end{aligned} \quad (7.2)$$

Again the greatest problem is that of formulating $p(\sigma_\theta)$. Therefore the principal meteorological offering of this report is an example of probabilistic model for σ_θ .

The model includes Equation 4.3 for neutral conditions, which is repeated here:

$$\sigma_\theta(z, z_0) = \frac{0.83}{\ln(z/z_0)} \cdot \quad (\text{radians}) \quad (7.3)$$

The model includes Equation 4.5 for unstable conditions, which is repeated here:

$$\sigma_\theta(z, z_0, z_i, -L) = \frac{0.4(12 + 0.5|z_i/L|)^{1/3}}{\ln(z/z_0) - \psi(x)} \quad (\text{radians})$$

$$\psi(x) = 2 \ln \frac{1+x}{2} + \ln \frac{1+x^2}{2} - 2 \tan^{-1} x + \pi/2 \quad (7.4)$$

$$x = (1 + 16|z/L|)^{1/4} \quad (\text{see Equation (4.5)}).$$

For average high wind conditions, recommended values of $-L$ and z_i are 50 m and 1000 m, respectively. For still higher wind conditions, a values of 200 m for $-L$ is recommended.

The probability function $p(\sigma_\theta)$ recommended is that of the 44 Kansas tests. This was given in Table 4.1. The version below is slightly smoothed. The argument values of σ_θ in Table 7.1 apply to the special case where the z and z_0 equal 11.3 and 0.025 m, respectively. To extrapolate this table to other heights and roughness, the argument values but not the probabilities are extrapolated. To make the extrapolation we assume that the values of σ_θ in the table from 0.09 to 0.13 radians represent neutral conditions and that those from 0.14 radians up represent unstable conditions. Therefore, the values of σ_θ are extrapolated as follows.

TABLE 7.1. Recommended Basic Table for Probability of σ_θ

σ_θ	Probability	σ_θ	Probability
0.09	0.159	0.18	0.046
0.10	0.182	0.19	0.045
0.11	0.068	0.20	0.068
0.12	0.045	0.21	0.023
0.13	0.023	0.22	0.023
0.14	0.045	0.23	0.045
0.15	0.068	0.24	0.023
0.16	0.023	0.26	0.023
0.17	0.045	0.28	0.023
		0.35	0.023

For $0.09 \leq \sigma_\theta \leq 0.13$ in Table 7.1

$$\sigma_\theta(z, z_0) = RN \sigma_\theta(11.3, 0.025) \quad (\text{in the table})$$

(7.5)

$$RN = \frac{\sigma_\theta(z, z_0) \text{ Equation (7.3)}}{\sigma_\theta(11.3, 0.025) \text{ Equation (7.3)}}$$

For $0.14 \leq \sigma_\theta$ in Table 7.1

$$\sigma_\theta(z, z_0) = RU \sigma_\theta(11.3, 0.025) \quad (\text{in the table})$$

(7.6)

$$RU = \frac{\sigma_\theta(z, z_0) \text{ Equation (7.4)}}{\sigma_\theta(11.3, 0.025) \text{ Equation (7.4)}}$$

Use of Equation (7.4) requires entering values for $-L$ and for z_i . Recommended values for high wind conditions generally the coincidence of high wind conditions and unstable conditions generally are $z_i = 1000$ m and $-L = 50$ m. For still higher wind conditions a value of 200 m is recommended for $-L$.

For complex terrain sites the best policy may be to rely on measurements. The most basic principle involved here is that as the wind approaches a point over complex terrain, it "remembers" the low frequency portion of the spectral content as it was upwind, but not so the high frequency portion, which is influenced more by local conditions (see Panofsky et al. 1982). Of course, this is all relative, and there is even more reason for uncertainty about the low frequency content which has been remembered over terrain that may be difficult to describe in terms of one roughness length.

Also from this reference one would expect σ_θ to be larger at sites where the terrain features tend in any way to enclose the point of interest such as in a valley or canyon. Accordingly, σ_θ would be smaller on an exposed summit. However, since this seems opposite of what one would intuitively expect for wind direction fluctuations, again measurement is suggested.

7.2 THE GAUSSIAN PART OF THE MODEL

According to the figures in Section 6, the exceedance angle associated a given probability departs from the Gaussian starting when the exceedance probability $P_{|\theta|}(x)$ is about 0.02 such that x should be about 20% greater when P is 0.002 and possibly 40% greater when P is 0.0002. Mathematical expression of this requires that when P is less than 0.02, x be upwardly extrapolated to

$$x_{\text{Extrapolated}} = x_{\text{Gaussian}} \left[1 + 0.2 \log_{10}(0.02/P) \right] \quad (7.7)$$

$$0.0002 \leq P \leq 0.02$$

For higher values of P (smaller angle) the Gaussian part of the model is recommended without qualification.

7.3 CODING OF THE MODEL

Appendix C contains the model code which may be run under two options. At the first option the model produces exactly the information described in Section 7. This is the wind direction probability model that corresponds to

the title of this report. At the second option the model produces additional information that is more engineering oriented, and thus provides a base of information that should be useful in relation to yaw control system design and determination of operating strategies. This information is described verbally and in familiar engineering mathematics in Appendix C.

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GLOSSARY

Some parameters defined in the text are not defined in the glossary because the definition in context, usually by equation, is more meaningful.

f	Dimensionless circular frequency nz/U
f_{ov}	Characteristic value of dimensionless frequency of a power spectral model for lateral turbulence (dimensionless).
g	Gravitational acceleration (length time^{-2})
Hz	Hertz (sec^{-1})
K_θ	Kurtosis of θ , (dimensionless)
k	von Karman's constant, taken to be 0.40
L	Monin-Obukhov stability length (length)
n	Circular frequency
$n\phi_v(n)$	Logarithmic turbulence spectrum for v ($\text{length}^2\text{time}^{-2}$)
$n\phi_\theta(n)$	Logarithmic turbulence spectrum for θ (degrees^2 or radians^2)
$p_A(x), p(x)$	Probability density of the random variable A at the level x (dimensions are the inverse of the dimensions of A).
$P_A(x), P(x)$	Probability distribution of the random variable A at the level x --probability that A will be less than x (dimensionless).
Ri	Richardson number--a dimensionless measure of atmospheric stability
$\hat{T}$	Temperature (degrees)
t	Time
U	Average value of $U(t)$ (length time^{-1})
U_r	Average value of horizontal wind speed at a reference height r (length time^{-1})

$U(t)$	Total value of longitudinal wind component (length time ⁻¹)
u_*	Friction velocity (length time ⁻¹)
$u(t)$	Longitudinal component of turbulence (length time ⁻¹)
unfiltered data	Data which is filtered only by the sampling interval and the total time length of the sample
$V(t)$	Total value of lateral wind component (length time ⁻¹)
$v(t)$	Lateral component of turbulence (length time ⁻¹)
z	Height above the surface (length)
z_i	Depth of the convective layer, or, height above surface of lowest temperature inversion aloft (length)
z_0	Surface roughness length (length)
z_r	Reference height (length)
$\hat{\theta}$	Potential temperature (temperature degrees)
$\theta(t)$	Wind angle (or direction) turbulence value (degrees)

APPENDIX A

APPENDIX A

A.1 DATA SET

The four data runs analyzed in this report were taken from the several anemometers of the Hanford vertical plane array described by Doran and Powell (1980), see Appendix A. Pertinent description of these runs is given in Table A.1.

TABLE A.1. Data Runs Analyzed in This Report

<u>Run Name</u>	<u>NA102</u>	<u>NA106</u>	<u>NA111</u>	<u>NA412</u>
Date	5/8/79	5/8/79	5/8/79	12/10/79
Initial Time	2231	2231	2231	1048
Duration (hr)	5	5	5	5
z (m)	61	12.2	36	19.4
$\bar{U}$ (m sec ⁻¹)	13.3	9.6	11.8	11.7
$\bar{\theta}$ (degree)	-13.6°	-13.6°	-12.2°	-15.6°
σ_{θ} (degree)	4.0	6.2	4.9	6.25
σ_{θ} (radians)	0.069	0.108	0.085	0.109

In the run names the first number is a digit for the test and the last two digits indicate the anemometer number. The first three runs are from simultaneous records taken by a different anemometer number. The anemometers were u-v-w Gill anemometers.

A.2 ATMOSPHERIC STABILITY

Because of the high wind conditions prevailing during all these tests the stability was approximately neutral. However, using mean wind and temperature data from the nearby Hanford 125-m tower, it was possible to calculate Monin-Obukhov L values (see Equation B.2) of approximate validity. The calculated values were about 500 m for test period NA1 and about -500 m for test period NA4. This indicates slightly stable conditions during the first period and slightly unstable conditions during the second period.

APPENDIX B

APPENDIX B

In this appendix potential temperature and the dimensionless stability parameter z/L is technically explained. Also relations are given between the two dimensionless stability parameters z/L and Richardson number for stable and unstable conditions.

B.1 POTENTIAL TEMPERATURE

If a parcel of air is moved up and down, expanding as it goes up and contracting as it goes down due to change of atmospheric pressure but not adding nor subtracting heat due to any other physical circumstance, i.e., not adding or subtracting heat by conduction or due to radiation, the temperature property of the air parcel that remains the same is the potential temperature. The actual temperature of the parcel increases as the parcel moves down and decreases as the parcel moves up. However, each time the parcel revisits a given level, the temperature the parcel has at that level is the same as at the last visit, if no heat energy has been gained or lost.

The potential temperature is that actual temperature such as air parcel has when the pressure is 1000 millibars, which is approximately the mean pressure of the atmosphere at sea level. The relation between actual temperature $\hat{T}$ and potential temperature Θ at pressure p is

$$\Theta = \hat{T} \left(\frac{1000}{p} \right)^{0.286} \quad (B.1)$$

when the temperature is in absolute units.

B.2 THE DIMENSIONLESS STABILITY PARAMETER z/L

The denominator of the dimensionless stability parameter z/L is a parameter with the dimensions of length and is formulated as follows

$$L = - \frac{u_*^3}{k(g/\hat{T})(H/\rho_o c_p)} \quad (B.2)$$

where $(g/\hat{T})$ is the buoyancy parameter, $(H/\rho_0 c_p)$ is the heat flux correlation and k is von Karman's constant, equal to 0.4; and where $\hat{T}$ is the absolute temperature, u_* is called the friction velocity and is related to the stress τ_0 caused at the surface by wind and the surface air density ρ_0 by

$$u_* = \sqrt{\tau_0 / \rho_0} . \quad (B.3)$$

All three of these parameters explain essential interactions between the surface and the atmosphere. The buoyancy parameter expresses the interaction of gravity with gas that has a volume proportional to absolute temperature, and thus expresses the reason for an atmosphere enveloping the earth's surface. The square of the friction velocity is proportional to the surface momentum flux, the eating away of momentum from the atmosphere by the surface. This causes the wind shear that in turn causes the turbulence. The heat flux is the heat source or heat sink effect of the surface on turbulence. The heat flux correlation is positive for unstable conditions, zero at neutral conditions and negative for stable conditions. Therefore L is negative for unstable conditions, infinite at neutral conditions, and positive for stable conditions. Atmospheric stability is quantified by the dimensionless height z/L , which is negative for unstable conditions, zero for neutral conditions, and positive for stable conditions.

B.3 RELATIONS BETWEEN THE RICHARDSON NUMBER AND z/L

In Section 4 reference was made to both of the standard dimensionless meteorological stability parameters, the Richardson number Ri and z/L . Relations given by Businger et al. (1971) for these two parameters are for neutral to stable conditions:

$$Ri = \frac{\frac{z}{L} \left(0.74 + 4.7 \frac{z}{L} \right)}{\left(1 + 4.7 \frac{z}{L} \right)^2} , \quad (B.4)$$

for unstable conditions

$$Ri = z/L . \quad (B.5)$$

APPENDIX C

APPENDIX C

This appendix contains the code of the atmospheric probability model described in Section 7 and the engineering extension referred to at the end of the section. One of the required parameters to read in is the option parameter KOPT, according to

KOPT = 0 - Run only the probability model of Section 7
KOPT = 1 - Add engineering input and output information as described below

In either option the model reads in two angles for which exceedance probability is to be computed according to Equation (7.1). These exceedance probabilities are then computed, as well as the difference between them. The difference is the probability that the wind direction will be in the sector between the two specified exceedance angles. This input and output is in degrees. The only other required input for running the model in the first option are hub height z and surface roughness length z_0 , both in meters. Values of 50 m and 1000 m, respectively, are provided in the code for z_i and $-L$, respectively, via a data statement. The user may wish to modify this specification on occasions.

To run the model with KOPT = 0, only one data card is required. The variable names are THETA1, THETA2, Z, Z0, and KOPT. Since there is no format specification, the card looks like: 15., 20., 50., 0.1., 1..

Before discussing additional input for running the model with KOPT = 1, we first describe the additional capability.

C.1 BANDPASS FILTER OPTION

The probabilities computed with KOPT = 0 correspond to atmospheric wind direction fluctuation behavior within the band applying to the AFCRL data, which is roughly 0.001 Hz to 10.0 Hz. (The data samples analyzed are 15 min long wherein the sampling rate is 20 data per second). If machine responsiveness is such that the user is not interested in atmospheric data with more

high frequency information than that contained in 30-sec averages, the values of σ_θ used for neutral and unstable conditions should correspond to integration of the lateral turbulence spectra to a frequency no greater than the reciprocal of twice 30 sec, or 0.0167 Hz. Similarly, if the machine is small and reorients easily, there may be no interest in wind direction fluctuations at frequencies less than, say once in 5 min. Thus there may be no interest in integration at frequencies below 0.0033 Hz in this case. The program therefore reads in two frequencies defining a sharp cut-off band that is presumed to represent machine responsiveness.

The normalized spectra for turbulence under neutral conditions, Equation (5.4), and unstable conditions, Equation (5.6), are integrated within this band to find the fraction of the variance within the band. (Since the original normalized spectra are not perfectly normalized, it is necessary also to integrate again using the band 0.001 Hz to 10 Hz, and divide the two fractions, obtained using the first integration by those obtained from the second integration. These quotients are the true fractions of variance required). Since σ_θ is the square root of the variance, the correction of σ_θ values from Equations (7.3) and (7.4) respectively, are made by multiplying these values by the square roots of the fractions of variance computed from the two integrations of each power spectrum. This is all accomplished in SUBROUTINE TEILER.

Analytically this amounts to the following. It is easier to illustrate with just one stability; therefore we use neutral stability. The steps are as follows:

1. Specify two frequencies in Hz that define the frequency band of machine sensitivity. Call them EN1 and EN2.
2. Integrate the normalized spectrum for neutral stability, Equation (5.4) from EN1 to EN2 using the natural logarithm of frequency as the differential $d[\ln(n)]$. Call this fraction FRNEU.
3. Integrate the same spectrum from 0.001 Hz to 10 Hz. Call the resulting the number VANEU.

4. Compute the value of σ_θ to be used in the model for neutral conditions, replacing the value from Equation (7.3) with

$$\sigma_\theta \text{ (machine)} = \sigma_\theta \text{ Eq. (7.3)} (\text{FRNEU/VANEU})^{0.5} \quad (\text{C.1})$$

Then adjust σ_θ for unstable conditions, Equation (7.4), in the same way using the spectrum of Equation (5.6).

The output exceedance probabilities will now correspond to wind direction fluctuations as "seen" or "heard" by a machine responding to atmospheric fluctuations within this frequency band.

There is one important caveat, however. The assumption of Gaussian distribution of θ does not hold for arbitrarily narrow frequency bands because in the limit the signal must tend to the form of a sine wave for which the probability distribution is not Gaussian but bimodal, with a kurtosis of 1.5. But, for bands no narrower, or little narrower, than a frequency decade, the kurtosis stays up around the Gaussian value of 3; therefore it seems that the model should serve designers who see a band of about a frequency decade or larger as representing the frequency responsiveness of their machines.

All exceedance probabilities are for exceedance on both sides of the centerline. All sector probabilities are for the two sectors, one on each side of the centerline.

C.2 STATISTICS OF FREQUENCY OF OCCURRENCE AND OF DURATION

For a model to be useful in relation to control or operating strategies, or for fatigue calculations, spectral information must be brought into the calculations to determine duration and frequency information, in addition to using the spectra as an adjunct to dimensionless probability calculations as has been described above.

To this end we introduce the familiar concept of positive-slope level crossings, wherein the present context we mean crossing in the direction away from the centerline. For a machine responding between frequencies EN1 and EN2, the equations are

$$N_{\theta} = N_0 \int_0^{\pi} p(\sigma_{\theta}) e^{-\frac{1}{2} \left(\frac{\theta}{\sigma_{\theta}} \right)^2} d\sigma_{\theta} \quad (C.2)$$

$$N_0 = 2 \left[\frac{\int_{EN1}^{EN2} n^2 \psi(n) d[\ln(n)]}{\int_{EN1}^{EN2} \psi(n) d[\ln(n)]} \right]^{1/2} \quad (C.3)$$

where $\psi(n)$ is either the normalized spectrum of Equation (5.4) or that of Equation (5.6). Actually both spectra are so integrated, and the final result is obtained by weighing equally the results obtained using the neutral spectrum and those obtained by using the unstable spectrum. They are weighed equally because the 44 tests (see Section 4) divide about equally between unstable and neutral tests, as defined in Section 7. The program also displays for each stability separately.

The final answer, N_{θ} is the frequency of level crossings at the specified angle on either side of the centerline and with slope away from the centerline. N_0 is the frequency of crossing the centerline, with either slope.

Another useful piece of information is the average time duration of the excursions beyond the specified angular deviation. This is easy because we already have the fraction of the time the wind exceeds this angle, Equation (7.1), and the frequency of exceedances, Equation (C.2). The result is that D , the average duration of an exceedance in seconds, is given by

$$D = \frac{P_{|\theta|}}{N_{\theta}} \quad (\text{seconds}) \quad (C.4)$$

C.3 ADDITIONAL INPUTS

Additional input requirements are three parameters that appear on one data card. They are EN1 and EN2, the band limiting frequencies in Hz and U, the average wind speed in m sec^{-1} presumed to apply at hub height. The unformatted input looks like

0.001, 0.05, 8.0

Since there is some correlation between wind speed and magnitude of the Monin-Obukov L, -L is set at 100 m if a lesser specification was previously made and U is 10 m sec^{-1} or more.

Obviously the values of ZI and ELM (convective layer depth and -L in meters) appearing in the program in data or FORTRAN statements are not hard numbers. Doubling the original ELM value to 100 m and the high wind values to 200 m is not out of line. Also ZI may be increased to a value of 1500 m if typical afternoon maximum conditions are to be modeled. For southwestern type climates (see Holzworth 1972) an even greater increase may be justifiable. Increase of ELM reduces turbulence, while increase of ZI increases turbulence.

The engineer will probably want to specify at least a few values of U that are within the operating range of his machine and assign a probability to each that is appropriate for his site. Then to finish the job he would have to run the model separately for each U and then appropriately average the results outside the program. The author welcomes correspondence regarding interpretation of results from this model.

C.4 PRINTOUT OF CODE AND SAMPLE CALCULATIONS

The printout of the code ends with three examples. For each example THETA1 and THETA2 are 15 and 20 degrees, respectively. The options are

1. KOPT = 0
2. KOPT = 1, EN1 = 0.001, EN2 = 0.05 Hz
3. KOPT = 1, EN1 = 0.001, EN2 = 0.01 Hz.

It is interesting to observe in the output for the filtered cases that the results depend on the unstable model and the fraction of the time unstable conditions are presumed to exist, rather than on the neutral model. That is the thesis of this report, where relatively large exceedance angles are considered.


```

C****
C THIS PROGRAM COMPUTES EXCEEDANCE PROBABILITIES FOR ABSOLUTE
C VALUES OF LATERAL ANGULAR DEVIATION OF THE WIND FROM ITS MEAN
C DIRECTION ACCORDING TO THE MODEL GIVEN IN EQUATION (7.1) OF
C THE REPORT "BASIC PROBABILITY CHARACTERISTICS OF WIND DIRECTION
C FLUCTUATIONS" BY D. C. POWELL. THE PROBABILITY DENSITY FUNCTION
C USED IS THAT OF TABLE 7.1. THE CORRESPONDING VALUES OF SIGMA THETA
C ARE ADJUSTED TO FIT USER-SUPPLIED VALUES OF HUB HEIGHT Z AND SURFACE
C ROUGHNESS LENGTH Z0 AS EXPLAINED IN SECTION 7.1.
C THE PROGRAM REQUIRES INPUT OF:
C 1. THETA1 AND THETA2 - TWO NON-NEGATIVE ANGLES FOR WHICH EXCEEDANCE
C PROBABILITY IS TO BE COMPUTED. THE DIFFERENCE BETWEEN THE TWO
C EXCEEDANCE PROBABILITIES, THE PROBABILITY THAT THE ANGULAR
C DEVIATION OF THE WIND WILL LIE IN THE SECTOR BETWEEN THE TWO
C ANGLES ON EITHER SIDE OF THE MEAN WIND DIRECTION, IS ALSO PRINTED.
C THETA1 AND THETA2 ARE ENTERED IN DEGREES AND CONVERTED TO RADIANS.
C 2. Z AND Z0. SEE PAGE 2 IN REPORT FOR SELECTION OF Z0 VALUE.
C ALL ANGLE VALUES ARE IN RADIANS UNLESS OTHERWISE STATED.
C ALL LENGTHS ARE IN METERS.
C THIS PROGRAM WAS WRITTEN AND CHECKED BY D. C. POWELL 3/27/82.
C
C ON 3/30/82 ADDITIONAL CAPABILITY WAS ADDED TO RESTRICT SIGMA THETA
C BY OPTION TO A FREQUENCY BAND CORRESPONDING TO MACHINE SENSITIVITY.
C THE FOLLOWING ADDITIONAL INPUTS ARE REQUIRED TO USE THIS OPTION.
C 3. KUPT = 0 - DO NOT USE OPTION.
C = 1 - USE OPTION AND READ IN:
C EN1 AND EN2, THE LIMITING FREQUENCIES IN HZ,
C U - THE MEAN WIND SPEED PRESUMED AT HEIGHT Z.
C AT THE END OF THE PROGRAM, THE ORIGINALLY SPECIFIED ANGLES
C THETA1, AND THETA2, ARE UPGRADED DUE TO NON-GAUSSIAN DISTRIBUTION
C OF THETA IF THE EXCEEDANCE PROBABILITY FOR EITHER IS LESS THAN
C 0.02. THIS IS EXPLAINED IN SECTION 7.2 OF THE REPORT. THIS
C FEATURE IS BYPASSED IF A FILTER IS USED SINCE THE DEPARTURE OF
C FILTERED THETA FROM GAUSSIAN DISTRIBUTION MAY NOT APPLY.
C
C ADDITIONAL LOGIC WAS ADDED TO CALCULATED LEVEL-CROSSING RATE AT
C THE SPECIFIED ANGLES AND AVERAGE DURATION OF EACH EXCURSION
C BEYOND THE SPECIFIED LEVEL.
C CHECK OF THESE FEATURES WAS FINISHED 4/2/82 BY D. C. POWELL.
C****
C****
C SST ARE ORIGINAL VALUES OF SIGMA THETA FROM TABLE 7.1.
C SIP ARE THE PROBABILITIES CORRESPONDING TO THESE VALUES.
C ST ARE ADJUSTED VALUES OF SIGMA THETA FROM THE TABLE FOR GIVEN (Z,Z0).
C****
0001 DIMENSION SST(20),SIP(20),ST(20)
C**
C THIS IS SIGMA THETA - NEUTRAL.
C**
0002 S(7,Z0) = 0.83/ALOG(7/70)
C**
C THIS IS SIGMA THETA - UNSTABLE.
C**
0003 SU(Z,Z0,Z1,ELN,X) = (0.4*(12.+0.5*71/ELN)**0.333)/(ALOG(Z/Z0)-X)
C**
C NEXT SEVEN LINES ARE A GAUSSIAN ERROR FUNCTION FOR TWO-SIDED EXCEEDANCE.
C**

```

```

0004      G(Z) = (1.0+D1*Z+D2*Z**2+D3*Z**3+D4*Z**4+D5*Z**5+D6*Z**6)**(-16)
0005      D1 = 0.043867347
0006      D2 = 0.0211410061
0007      D3 = 3.2776263E-3
0008      D4 = 3.80036 E-5
0009      D5 = 4.88906 E-5
0010      D6 = 5.383 E-6

C***
C      Z1 IS CONVECTIVE LAYER DEPTH.
C      ELM IS THE NEGATIVE OF THE MONIN-OBUKHOV LENGTH FOR UNSTABLE CONDITIONS.
C***
0011      DATA Z1, ELM/1000.,50./
0012      PI = 3.14159265

C**
C      ENTER SIGMA THETA VALUES FOR PROBABILITY TABLE.
C**
0013      DO 1 I = 1, 16
0014      FI = 1
0015      1 SST(I) = 0.01 * (FI + 8.)
0016      DATA(SST(I),I=17,19)/.26,.28,.35/

C**
C      ENTER HISTOGRAM VALUES FOR PROBABILITY TABLE.
C**
0017      DATA(STP(I),I=1,10)/7.,8.,3.,2.,1.,2.,3.,1.,2.,2./
0018      DATA (STP(I),I=11,19)/2.,3.,1.,1.,2.,1.,1.,1.,1.,1./

C**
C      CONVERT HISTOGRAM VALUES TO PROBABILITY BY DIVIDING BY NUMBER OF TESTS.
C**
0019      DO 2 I = 1, 19
0020      2 STP(I) = STP(I)/44.
0021      WRITE(6,2001)
0022      2001 FORMAT(1H,'EXCEEDANCE PROBABILITY CALCULATIONS FOR WIND DIRECTION
      * FLUCTUATIONS'/' ACCORDING TO EQUATION (7.1) IN THE REPORT'
      */' BASIC PROBABILITY CHARACTERISTICS OF WIND DIRECTION
      * FLUCTUATIONS'/' BY D. C. POWELL'/)

0023      RATIOU = 1.0
0024      RATIOB = 1.0

C***
C      READ 180 ANGLES IN DEGREES. THEN CONVERT THEM TO RADIAN.
C      ALSO READ IN HEIGHT AND SURFACE ROUGHNESS LENGTH IN METERS.
C      ALSO READ KDOT AS 0 OR 1 AS EXPLAINED IN INITIAL COMMENT.
C***
C*****
C
0025      READ(5,*)THETA1, THETA2, Z, 70, KDOT

C*****
C*****
0026      WRITE(6,2101)THETA1, THETA2
0027      2101 FORMAT(1H,'THETA1 AND THETA2 IN DEGREES',F16.1, F7.1)
0028      THETA1 = PI*THETA1/180.
0029      THETA2 = PI*THETA2/180.
0030      WRITE(6,2102)Z, 70
0031      2102 FORMAT(1H,'HEIGHT AND ROUGHNESS LENGTH IN METERS', F7.1,F7.3)
0032      WRITE(6,2103)Z1,ELM
0033      2103 FORMAT(1H,'CONVECTIVE LAYER DEPTH',F7.0,' METERS'/'
      * MONIN-OBUKHOV LENGTH FOR UNSTABLE CONDITIONS -',F4.0,' METERS')
0034      IF(KDOT.EQ.0)GO TO 25

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C**
C   READ LIMITING FREQUENCIES OF MACHINE SENSITIVITY EN1, EN2.
C   READ MEAN WIND SPEED U AT HEIGHT Z IN M/SEC.
C**
CREAD***
C   IF KOPT.NE.0 - - - - THEN
0035 READ(5,*)EN1, EN2, U
C
CREAD***
0036 WRITE(6,2201) EN1, EN2
0037 2201 FORMAT(1H0, 'FRACTION OF SIGMA THETA BETWEEN',F9.5,' AND'
      *,F9.5,' HZ USED')
0038 WRITE(6,2211)U
0039 2211 FORMAT(1H, 'MEAN WIND SPEED ASSUMED AT',F6.1,' M/SEC')
C**
C   CALCULATE FRACTION OF VARIANCE BETWEEN EN1 AND EN2.
C   CROSSEU IS ZERO CROSSING RATE FOR NEUTRAL CONDITIONS - HZ.
C   CROSUNS IS ZERO CROSSING RATE FOR UNSTABLE CONDITIONS.
C   BOTH LEVEL CROSSING RATES ARE WITH BOTH SLOPES.
C**
0040 CALL TEILER(0.001,10.,U, Z, Z1, ELM, VANEU, VAUNS,CROSSEU,CROSUNS)
0041 CALL TEILER(EN1, EN2, U, Z, Z1, ELM, FRNEU, FRUNS,CROSSEU,CROSUNS)
0042 RATNEU = SQRT(FRNEU/VANEU)
0043 RATUNS = SQRT(FRUNS/VAUNS)
C**
C   COMPUTE NEUTRAL AND UNSTABLE VALUES OF SIGMA THETA IN RADIANS.
C   IF(KOPT.NE.0)REDUCE THESE VALUES ACCORDING TO BAND (EN1,EN2).
C**
0044 25 A = S(11.3,0.025)
0045   B = S(7,70)
0046   IF(KOPT.NE.0) B = B * RATNEU
0047   RN = B/A
0048   CALL PHIOFX(11.3,ELM,PHIX1)
0049   CALL PHIOFX(7,ELM,PHIX2)
0050   AU = S(2, 70, Z1, ELM, PHIX1)
0051   BU = S(2, 70, Z1, ELM, PHIX2)
0052   IF(KOPT.NE.0) BU = BU * RATUNS
0053   RU = BU/AU
0054   BDEG = 180.*B/PI
0055   BUDEG = 180.*BU/PI
0056   WRITE(6,2611)BDEG, BUDEG
0057 2611 FORMAT(1H, 'SIGMA THETA (Z,Z0) NEUTRAL' F13.1, ' DEGREES'/
      *, 'SIGMA THETA (Z,Z0,Z1,-1) UNSTABLE', F6.1, ' DEGREES')
0058   IF(KOPT.NE.0) WRITE(6,2711)RATNEU, RATUNS
0059 2711 FORMAT(1H, 'FRACTION OF SIGMA THETA IN BAND FOR NEUTRAL AND
      * UNSTABLE CONDITIONS',2F8.3)
C**
C   REVISE SIGMA THETA VALUES IN STABILITY TABLE FOR CURRENT (Z,Z0).
C**
0060 DO 3 I = 1, 5
0061 3 ST(I) = SST(I)*RN
0062 DO 4 I = 6,17
0063 4 ST(I) = SSI(I)*RU
0064 RTHRTG = 4N/RATNEU
0065 RTHRTG = 4U/RATUNS
0066 WRITE(6,4001) RTHRTG, RTHRTG
0067 4001 FORMAT(1H, 'R', RU - EQUATIONS (7.5) AND (7.6)',2F7.3)

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C**
C      COMPUTE GAUSSIAN ERROR FUNCTION FOR THETA1 AND THETA 2.
C      SUM1, SUM2 ARE EXCEEDANCE PROBABILITY FOR THETA1, THETA2.
C      ESUM1, ESUM2 ARE THE PROBABILISTIC FACTOR IN A LEVEL-CROSSING
C      FREQUENCY EXPRESSION.
C**
0068      SUM1 = 0.0
0069      SUM2 = 0.0
0070      ESUM1 = 0.0
0071      ESUM2 = 0.0
0072      DO 5 I = 1, 19
0073      Y1 = THETA1/ST(I)
0074      E1 = G(Y1)
0075      Y2 = THETA2/ST(I)
0076      E2 = G(Y2)
0077      SUM1 = SUM1 + E1 * STP(I)
0078      SUM2 = SUM2 + E2 * STP(I)
0079      ESUM1 = ESUM1 + EXP(-0.5 * Y1 * Y1) * STP(I)
0080      ESUM2 = ESUM2 + EXP(-0.5 * Y2 * Y2) * STP(I)
0081      IF(I.NE.5) GO TO 45
C      THESE ARE PARTIAL SUMS FOR NEUTRAL STABILITY.
0082      SNSUM1 = SUM1
0083      SNSUM2 = SUM2
0084      ENSUM1 = ESUM1
0085      ENSUM2 = ESUM2
0086      45 CONTINUE
CWRITE      WRITE(6,4998)I, ST(I), STP(I)
0087      4998 FORMAT(1H,'I', ST(I), STP(I)',F10.4)
CWRITE      WRITE(6,4999)Y1,Y2,E1,E2,SUM1,SUM2,ESUM1,ESUM2
0088      4999 FORMAT(1H,'Y,E,SUM,ESUM',A11.3)
0089      5 CONTINUE
C      SECTOR PROBABILITY
0090      DIFF = ABS(SUM2 - SUM1)
0091      WRITE(6,6001)SUM1, SUM2, DIFF
0092      6001 FORMAT(1H0,'EXCEEDANCE PROBABILITIES FOR THETA1, THETA2, AND
* DIFFERENCE.'// DIFFERENCE IS THE SECTOR PROBABILITY.'//,3E12.3)
0093      WRITE(6,6101)
0094      6101 FORMAT(1H0,'PROBABILITIES INCLUDE EXCEEDANCE ON BOTH SIDES
* OF CENTER LINE.')
0095      IF(KOPT.EQ.0)GO TO 9
C      ZERO-CROSSING FREQUENCY - BOTH SLOPES
0096      CROSLV = 0.5 * (CROSNEU + CROSUNS)
0097      WRITE(6,6201)CROSLV
0098      6201 FORMAT(1H0,'ZERO CROSSING FREQUENCY',F10.4,' HZ')
CWRITE      WRITE(6,6211)SNSUM1, SNSUM2, ENSUM1, ENSUM2
0099      6211 FORMAT(1H,'SNSUM, ENSUM',4E12.4)
C      NEXT 4 VARIABLES ARE PARTIAL OUTWARD SLOPE LEVEL CROSSING RATES
C      FOR NEUTRAL OR UNSTABLE CONDITIONS - BOTH SIDES COMBINED.
0100      FN1 = CROSNEU * ENSUM1
0101      FU1 = CROSUNS * (ESUM1 - ENSUM1)
0102      FN2 = CROSNEU * ENSUM2
0103      FU2 = CROSUNS * (ESUM2 - ENSUM2)
CWRITE      WRITE(6,6231)FN1, FU1, FN2, FU2
0104      6231 FORMAT(1H,'FN1, FU1, FN2, FU2'/1H,8E12.4)
0105      F1 = (FN1 + FU1)
0106      F2 = (FN2 + FU2)
0107      D1 = 3.141/F1

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0108      D2 = SUM2/F2
0109      WRITE(6,6301)F1, F2
0110 6301 FORMAT(1H, 'OUTWARD SLOPE LEVEL CROSSING FREQUENCY AT EACH
      * ANGLE', 2E12.4, ' HZ')
0111      WRITE(6,6401)D1, D2
0112 6401 FORMAT(1H, 'AVERAGE DURATION OF EXCEEDANCE EXCURSIONS FOR EACH
      * ANGLE', 2F6.2, ' SEC')
0113      WRITE(6,6501)
0114 6501 FORMAT(1H0, 'SEPARATE ANALYSES FOR (1)NEUTRAL-STABLE AND (2)
      * UNSTABLE CONDITIONS')
0115      WRITE(6,6511)
0116 6511 FORMAT(1H, 'EXCEEDANCE PROBABILITY FOR THETA1')
0117      RNEU = 44./21.
0118      RUNS = RNEU/(RNEU - 1.0)
0119      A = RNEU * SWSUM1
0120      B = RUNS * (SUM1 - SWSUM1)
0121      WRITE(6,6521)A, B
0122 6521 FORMAT(1H, 2E11.3)
0123      WRITE(6,6531)
0124 6531 FORMAT(1H, 'EXCEEDANCE PROBABILITY FOR THETA2')
0125      A = RNEU * SWSUM2
0126      B = RUNS * (SUM2 - SWSUM2)
0127      WRITE(6,6521) A, B
0128      WRITE(6,6601)CRISVEU, CRISUNS
0129 6601 FORMAT(1H, 'ZERO CROSSING RATES', 2E11.3, ' HZ.')
0130      WRITE(6,6611)
0131 6611 FORMAT(1H, 'LEVEL CROSSING RATES FOR THETA1')
0132      A = RNEU * FN1
0133      B = RUNS * FU1
0134      WRITE(6,6612)A, B
0135 6612 FORMAT(1H, 2E11.3, ' HZ.')
0136      WRITE(6,6621)
0137 6621 FORMAT(1H, 'LEVEL CROSSING RATES FOR THETA2')
0138      A = RNEU * FN2
0139      B = RUNS * FU2
0140      WRITE(6,6612)A, B
0141      WRITE(6,6631)
0142 6631 FORMAT(1H, 'AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA1')
0143      A = SWSUM1/FN1
0144      B = (SUM1 - SWSUM1)/FU1
0145      WRITE(6,6641)A, B
0146 6641 FORMAT(1H, 2F6.1, ' SECONDS')
0147      WRITE(6,6651)
0148 6651 FORMAT(1H, 'AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA2')
0149      A = SWSUM2/FN2
0150      B = (SUM2 - SWSUM2)/FU2
0151      WRITE(6,6641)A, B

C****
C      IF THE FULL TURBULENCE SPECTRUM IS REFLECTED IN SIGMA THETA,
C      CORRECT THE ORIGINALLY SUBMITTED ANGLES FOR NON-GAUSSIAN
C      DISTRIBUTION OF THETA, SEE SECTION 7.2 OF REPORT.
C****

0152      IF(KOPT.NE.0)GO TO 9
0153      IF(SUM1.GE.0.02.AND.SUM2.GE.0.02)GO TO 9
0154      WRITE(6,7001)
0155 7001 FORMAT(1H0, 'THE ORIGINAL ANGLES, THETA1 AND THETA2, ARE UPGRADED
      * BECAUSE OF NON-GAUSSIAN DISTRIBUTION OF THETA.')

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0156      IF(SUM1.GE.0.02) GO TO 8
0157      H = SUM1
0158      IF(4.LT.0.0002)H = 0.0002
0159      THETA1 = THETA1 * (180./PI) * (1.0 + 0.2*ALOG10(0.02/H))
0160      WRITE(6,7501)THETA1
0161 7501 FORMAT(1H,'THETA1',F7.1,' DEGREES')
0162      IF(SUM2.GE.0.02) GO TO 9
0163      d = SUM2
0164      IF(4.LT.0.0002)H = 0.0002
0165      THETA2 = THETA2 * (180./PI) * (1.0 + 0.2*ALOG10(0.02/H))
0166      WRITE(6,8501)THETA2
0167 8501 FORMAT(1H,'THETA2',F7.1,' DEGREES')
0168      STOP
0169      END

```

PROGRAM SECTIONS

Name	Bytes	Attributes
0 SCODE	2478	PIC CON REL LCL SHR EXE RD NOVRT LONG
1 SPDATA	1670	PIC CON REL LCL SHR NOEXE RD NOVRT LONG
2 SLICAL	700	PIC CON REL LCL NOSHR NOEXE RD WRT LONG

ENTRY POINTS

Address	Type	Name
0-00000000		EPTHETASKAIN

STATEMENT FUNCTIONS

Address	Type	Name	Address	Type	Name	Address	Type	Name
0-00000923	R*4	G	0-00000ACD	R*4	S	0-000008E4	R*4	SU

VARIABLES

Address	Type	Name	Address	Type	Name	Address	Type	Name	Address	Type	Name
2-0000013C	R*4	A	2-00000170	R*4	AU	2-00000160	R*4	B	2-0000017C	R*4	BOEG
2-00000174	R*4	BU	2-00000180	R*4	BUNEG	2-000001C0	R*4	CRJSLV	2-0000014C	R*4	CROSNEU
2-00000150	R*4	CRJSLV	2-000000F0	R*4	D1	2-000000F4	R*4	D2	2-000000F8	R*4	D3
2-000000FC	R*4	D4	2-00000100	R*4	D5	2-00000104	R*4	D6	2-0000018C	R*4	DIFF
2-00000140	R*4	E1	2-000001A8	R*4	E2	2-0000010C	R*4	EL4	2-00000100	R*4	EL4
2-00000134	R*4	EN1	2-0000013C	R*4	EV2	2-00000184	R*4	ENSUM1	2-00000188	R*4	EN3U42
2-00000134	R*4	ESU41	2-00000198	R*4	ESUM2	2-00000104	R*4	F1	2-00000108	R*4	F2
2-00000118	R*4	FI	2-000001C4	R*4	FN1	2-000001CC	R*4	FN2	2-00000154	R*4	FRNEU
2-00000158	R*4	FRJMS	2-000001C8	R*4	FU1	2-000001D0	R*4	FU2	2-00000114	R*4	I
2-00000134	R*4	KOPT	2-00000168	R*4	PHIX1	2-0000016C	R*4	PHIX2	2-00000110	R*4	PI
2-0000011C	R*4	RATNEU	2-00000120	R*4	RATINS	2-00000164	R*4	R4	2-000001DC	R*4	RNEU
2-00000134	R*4	R4JITG	2-00000178	R*4	R4	2-000001E0	R*4	RUN3	2-00000188	R*4	RURITG

C****
C THIS PROGRAM INTEGRATES THE QUASI-NORMALIZED LATERAL COMPONENT
C TURBULENCE SPECTRUM OVER A FREQUENCY BAND BETWEEN FREQUENCIES
C EN1 AND EN2. SPECTRA FOR NEUTRAL AND UNSTABLE CONDITIONS ARE
C EQUATIONS (5.2) AND (5.6), RESPECTIVELY FROM THE PCKELL REPORT.
C OTHER REQUIRED INPUTS ARE HEIGHT Z, MEAN WIND SPEED U APPLYING
C AT HEIGHT Z, CONVECTIVE LAYER DEPTH Z1, AND NEGATIVE OF
C MONIN-OBUKHOV LENGTH ELM. THE OUTPUTS ARE THE FRACTIONS OF
C VARIANCE FRNEU AND FRUNS THAT APPLY FOR NEUTRAL AND UNSTABLE
C CONDITIONS, RESPECTIVELY, PLUS THE CORRESPONDING ZERO CROSSING
C FREQUENCIES OF BOTH SLOPES, CROSNEU AND CROSUNS.
C THE PROGRAM MUST BE RUN TWICE SINCE THE SPECTRA ARE NOT
C PERFECTLY NORMALIZED. FIRST THE CALLING PROGRAM ENTERS
C 0.001 AND 10.0 AS EN1 AND EN2; SECOND THE DESIRED FREQUENCIES
C ARE ENTERED. THE QUOTIENTS FORMED APPROPRIATELY FROM OUTPUT
C VALUES (FRNEU = 2ND RUN/FRNEU = 1ST RUN) ETC - OR RATHER
C THE SQUARE ROOTS OF THESE QUOTIENTS ARE THE FRACTIONS OF
C SIGMA THETA THAT APPLY IN THE BAND EN1 TO EN2 FOR NEUTRAL
C AND UNSTABLE CONDITIONS, RESPECTIVELY. SINCE THE VALUES
C OF CROSNEU AND CROSUNS FROM THE FIRST CALLING ARE NOT USED,
C THE CALLING IS IN THE ORDER SPECIFIED.
C THIS PROGRAM WAS WRITTEN AND CHECKED BY D. C. POWELL, 4/2/82.

C****
C SUBROUTINE TEILER(EN1,EN2,U,Z,Z1,ELM,FRNEU,FRUNS,CROSNEU,CROSUNS)

0001 C
0002 C AN(ZI,ELM) = (12.0 + 0.5*(ZI/ELM))**0.666667
0003 C AD(Z,ELM) = 1.0 + 0.75*(Z/ELM)**0.666667
0004 C A(Z,ZI,ELM) = AN(ZI,ELM)/AD(Z,ELM)
0005 C F(EN,Z,U) = EN * Z/U
0006 C Q(Z,ZI,A) = ALOG(0.30*A)/ALOG(0.25*ZI/Z)
0007 C SNEU(F) = F/(1.0 + 69.6 * F**1.666667)
0008 C SLOW(F) = F/(1.0 + 3.1 * F**1.666667)
0009 C SMED(EN,Z,U,A,Q) = 0.4/(A * F(EN,Z,U)**Q)
0010 C SHIGH(EN,Z,U,A) = 0.4/(A * F(EN,Z,U)**0.666667)
0011 C
0012 C RATIO = 10.0**0.025
0013 C DELLNN = ALOG(RATIO)
0014 C FRNEU = 0.0
0015 C FRNEU2 = 0.0
0016 C AZ = A(Z,ZI,ELM)
0017 C QZ = Q(Z,ZI,AZ)
0018 C EN = EN1
0019 C INTEGRATE NEUTRAL SPECTRUM, EQUATION (5.2)
0020 C 1 FZ = F(EN,Z,U)
0021 C ANEU = SNEU(FZ)
0022 C FRNEU = FRAEU + ANEU
0023 C FRNEU2 = FRNEU2 + ANEU * EN * EN
0024 C PANEU = 6.19 * ANEU
C*RITE WRITE(6,1001)EN, PANEU, FRNEU, FRNEU2

```

0023      1001 FORMAT(1H,'EN, AVEU, FRNEU, FRNEU2',4E12.4)
0024      EN = EN * RATIO
0025      IF(EN.LE.EN2)GO TO 1
0026      CROSNEU = 2.0 * SQRT(FRNEU2/FRNEU)
CWRITE      WRITE(6,1501)CROSNEU
0027      1501 FORMAT(1H,'CROSNEU',F10.4)
0028      FRNEU = 6.19 * FRNEU * DELLN
C      C      INTEGRATE LOW FREQUENCY PART OF UNSTABLE SPECTRUM, EQU(5.6C)
0029      FRUNS = 0.
0030      FRUNS2 = 0.
0031      B = F(EN,2I,U)
0032      IF(B.GT.4.0)GO TO 3
0033      EN = EN1
0034      2 FZ1 = F(EN,ZI,U)
0035      ALLOW = SLOW(FZ1)
0036      FRUNS = FRUNS + ALLOW
0037      FRUNS2 = FRUNS2 + ALLOW * EN * EN
CWRITE      WRITE(6,2001)EN, ALLOW, FRUNS, FRUNS2
0038      2001 FORMAT(1H,'EN, ALLOW, FRUNS, FRUNS2',4E12.4)
0039      EN = EN * RATIO
0040      IF(EN.GT.EN2)GO TO 6
0041      B = F(EN,ZI,U)
0042      IF(B.LT.4.0)GO TO 2
C      C      INTEGRATE MIDDLE FREQUENCY PART OF UNSTABLE SPECTRUM, EQU(5.6B)
0043      3 B = F(EN,Z,U)
0044      IF(B.GT.1.0)GO TO 5
0045      4 AMED = SLOW(EN,Z,U,AZ,QZ)
0046      FRUNS = FRUNS + AMED
0047      FRUNS2 = FRUNS2 + AMED * EN * EN
CWRITE      WRITE(6,4001)EN, AMED, FRUNS, FRUNS2
0048      4001 FORMAT(1H,'EN, AMED, FRUNS, FRUNS2',4E12.4)
0049      EN = EN * RATIO
0050      IF(EN.GT.EN2)GO TO 6
0051      B = F(EN,Z,U)
0052      IF(B.LE.1.0)GO TO 4
C      C      INTEGRATE HIGH FREQUENCY PART OF UNSTABLE SPECTRUM, EQU.(5.6A)
0053      5 AHI = SHIGH(EN,Z,U,AZ)
0054      FRUNS = FRUNS + AHI
0055      FRUNS2 = FRUNS2 + AHI * EN * EN
CWRITE      WRITE(6,5001)EN, AHI, FRUNS, FRUNS2
0056      5001 FORMAT(1H,'EN, AHI, FRUNS, FRUNS2',4E12.4)
0057      EN = EN * RATIO
0058      IF(EN.LE.EN2)GO TO 5
0059      6 CROSUNS = 2.0 * SQRT(FRUNS2/FRUNS)
0060      FRUNS = FRUNS * DELLN
CWRITE      WRITE(6,6001)CROSNEU, CROSUNS
0061      6001 FORMAT(1H,'CROSNEU, CROSUNS',3E12.4)
0062      RETURN
0063      END

```


12-Apr-1982 09:04:04
27-Mar-1982 13:11:25

VAX-11 FORTRAN V2.2-40
_DHA1: (812013)PHIOFX.FOR14

Page 1

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C****
C   THIS SUBROUTINE CALCULATES THE STABILITY FUNCTION FOR
C   UNSTABLE ATMOSPHERES THAT IS CALLED PHI(X) AND IS THE SECOND
C   FUNCTION OF THE DENOMINATOR OF EQUATION (4.5) OF THE POWELL
C   REPORT ON WIND DIRECTION PROBABILITY. PROGRAM WAS WRITTEN
C   AND CHECKED BY D. C. POWELL 3/27/82.
C****
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```
0001 SUBROUTINE PHIOFX(Z,ELM,PHIX)
0002 ZOL = Z/ELM
0003 IF(ZOL.GT.1.0)ZOL = 1.0
0004 X = (1.0 + 16. * ZOL)**0.25
0005 A = 2.0 * ALOG (0.5 * (1.0 + X))
0006 B = ALOG (0.5 * (1.0 + X*X))
0007 C = 2.0 * ATAN(X)
0008 PHIX = A + B - C + 0.5 * 3.14159265
0009 RETURN
0010 END
```

PROGRAM SECTIONS

C-15

Name	Bytes	Attributes
0 SCODE	127	PIC CON REL LCL SHR EXE RD NMRT LONG
2 SLOCAL	20	PIC CON REL LCL NOSHR NOEXE RD WRT LONG

ENTRY POINTS

Address	Type	Name
0-00000000		PHIOFX

VARIABLES

Address	Type	Name	Address	Type	Name	Address	Type	Name	Address	Type	Name
2-00000000	R*4	A	2-0000000C	R*4	B	2-00000010	R*4	C	4P-00000000	R*4	ELM
4P-00000000	R*4	PHIX	2-00000004	R*4	X	4P-00000004	R*4	Z	2-00000000	R*4	ZOL

FUNCTIONS AND SUBROUTINES REFERENCED

4TMSALOG 4TMSATAN

Total Space Allocated = 147 Bytes

EXCEEDANCE PROBABILITY CALCULATIONS FOR WIND DIRECTION FLUCTUATIONS
ACCORDING TO EQUATION (7.1) IN THE REPORT
BASIC PROBABILITY CHARACTERISTICS OF WIND DIRECTION FLUCTUATIONS
BY D. C. POWELL

THETA1 AND THETA2 IN DEGREES 15.0 20.0
HEIGHT AND ROUGHNESS LENGTH IN METERS 50.0 0.100
CONVECTIVE LAYER DEPTH 1000. METERS
MINIMUM-BOUNDARY LENGTH FOR UNSTABLE CONDITIONS = 50. METERS
SIGMA THETA (Z,Z0) NEUTRAL 7.6 DEGREES
SIGMA THETA (Z,Z0,Z1,-L) UNSTABLE 12.6 DEGREES
RV, RU = EQUATIONS (7.5) AND (7.6) 0.984 1.121

EXCEEDANCE PROBABILITIES FOR THETA1, THETA2, AND DIFFERENCE.
DIFFERENCE IS THE SECTOR PROBABILITY.
0.127E+00 0.637E-01 0.637E-01

PROBABILITIES INCLUDE EXCEEDANCE ON BOTH SIDES OF CENTER LINE.

EXCEEDANCE PROBABILITY CALCULATIONS FOR WIND DIRECTION FLUCTUATIONS
 ACCORDING TO EQUATION (7.1) IN THE REPORT
 BASIC PROBABILITY CHARACTERISTICS OF WIND DIRECTION FLUCTUATIONS
 BY D. C. POWELL

THETA1 AND THETA2 IN DEGREES 15.0 20.0
 HEIGHT AND ROUGHNESS LENGTH IN METERS 50.0 0.100
 CONVECTIVE LAYER DEPTH 1000. METERS
 MONIN-ORUKHGV LENGTH FOR UNSTABLE CONDITIONS - 50. METERS

FRACTION OF SIGMA THETA BETWEEN 0.00100 AND 0.05000 HZ USED
 MEAN WIND SPEED ASSUMED AT 8.0 M/SEC
 SIGMA THETA (Z,Z0) NEUTRAL 8.4 DEGREES
 SIGMA THETA (Z,Z0,Z1,-L) UNSTABLE 10.9 DEGREES
 FRACTION OF SIGMA THETA IN BAND FOR NEUTRAL AND UNSTABLE CONDITIONS 0.035 0.060
 RN, RU - EQUATIONS (7.5) AND (7.6) 0.984 1.121

EXCEEDANCE PROBABILITIES FOR THETA1, THETA2, AND DIFFERENCE.
 DIFFERENCE IS THE SECTOR PROBABILITY.
 0.922E-01 0.415E-01 0.507E-01

PROBABILITIES INCLUDE EXCEEDANCE ON BOTH SIDES OF CENTER LINE.

ZERO CROSSING FREQUENCY 0.0344 HZ
 OUTWARD SLOPE LEVEL CROSSING FREQUENCY AT EACH ANGLE 0.6027E-02 0.3067E-02 HZ
 AVERAGE DURATION OF EXCEEDANCE EXCURSIONS FOR EACH ANGLE 15.30 13.53 SEC

SEPARATE ANALYSES FOR (1) NEUTRAL-STABLE AND (2) UNSTABLE CONDITIONS
 EXCEEDANCE PROBABILITY FOR THETA1
 0.260E-02 0.174E+00
 EXCEEDANCE PROBABILITY FOR THETA2
 0.111E-03 0.793E-01
 ZERO CROSSING RATES 0.394E-01 0.294E-01 HZ.
 LEVEL CROSSING RATES FOR THETA1
 0.400E-03 0.112E-01 HZ.
 LEVEL CROSSING RATES FOR THETA2
 0.208E-04 0.585E-02 HZ.
 AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA1
 6.5 15.6 SECONDS
 AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA2
 5.4 13.6 SECONDS

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EXCEEDANCE PROBABILITY CALCULATIONS FOR WIND DIRECTION FLUCTUATIONS
ACCORDING TO EQUATION (7.1) IN THE REPORT
BASIC PROBABILITY CHARACTERISTICS OF WIND DIRECTION FLUCTUATIONS
BY D. C. POWELL

THETA1 AND THETA2 IN DEGREES 15.0 20.0
HEIGHT AND ROUGHNESS LENGTH IN METERS 50.0 0.100
CONVECTIVE LAYER DEPTH 1000. METERS
MONIN-OBUKHCV LENGTH FOR UNSTABLE CONDITIONS - 50. METERS

FRACTION OF SIGMA THETA BETWEEN 0.00100 AND 0.01000 HZ USED
MEAN WIND SPEED ASSUMED AT 8.0 M/SEC
SIGMA THETA (Z,Z0) NEUTRAL 4.3 DEGREES
SIGMA THETA (Z,Z0,Z1,-L) UNSTABLE 8.9 DEGREES
FRACTION OF SIGMA THETA IN BAND FOR NEUTRAL AND UNSTABLE CONDITIONS 0.558 0.711
RN, RU - EQUATIONS (7.5) AND (7.6) 0.984 1.121

EXCEEDANCE PROBABILITIES FOR THETA1, THETA2, AND DIFFERENCE.
DIFFERENCE IS THE SECTOR PROBABILITY.
0.544E-01 0.200E-01 0.344E-01

PROBABILITIES INCLUDE EXCEEDANCE ON BOTH SIDES OF CENTER LINE.

ZERO CROSSING FREQUENCY 0.0109 HZ
OUTWARD SLOPE LEVEL CROSSING FREQUENCY AT EACH ANGLE 0.1314E-02 0.5528E-03 HZ
AVERAGE DURATION OF EXCEEDANCE EXCURSIONS FOR EACH ANGLE 41.42 36.20 SEC

SEPARATE ANALYSES FOR (1) NEUTRAL-STABLE AND (2) UNSTABLE CONDITIONS

EXCEEDANCE PROBABILITY FOR THETA1

0.206E-04 0.104E-00

EXCEEDANCE PROBABILITY FOR THETA2

0.599E-07 0.383E-01

ZERO CROSSING RATES 0.117E-01 0.100E-01 HZ.

LEVEL CROSSING RATES FOR THETA1

0.124E-05 0.251E-02 HZ.

LEVEL CROSSING RATES FOR THETA2

0.433E-08 0.106E-02 HZ.

AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA1

16.7 41.4 SECONDS

AVERAGE DURATION OF EXCEEDANCE EXCURSIONS - THETA2

13.8 36.2 SECONDS

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