Report No. C00-2232-17Ex

Conf-7209104--1

MUON TRIDENT DIRECT PAIR PRODUCTION AND OTHER PROBES OF
THE MUON-MUON INTERACTION*

Michael J. Tannenbaum

The Rockefeller University New York, New York 10021

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

(Presented at the Seminar on The μ -e Problem,
Moscow, U.S.S.R., 19-21 September, 1972)

* Work supported in part by U.S. Atomic Energy Commission,
Contract AT(11-1)-2232.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

MUON TRIDENT DIRECT PAIR PRODUCTION AND OTHER PROBES OF
THE MUON-MUON INTERACTION

Michael J. Tannenbaum
The Rockefeller University New York, New York 10021

Presented to: Seminar on The μ -e Problem
Moscow, U.S.S.R.
September 1972

My interest in Muon Physics stems primarily from the question of why muons exist at all. What would happen in a world without muons?

Obviously, pions would live much longer. This would seriously alter the composition of Cosmic Rays at the surface of the earth, but would have little other effect except, of course, that without muons, we wouldn't have had this lovely conference.

To put this in perspective, just consider a world with muons but without electrons. Atoms would shrink in size by a factor of 206.8. Nuclear beta decay would be completely changed, etc. We would not recognize the world.

This drastic situation predisposes us to the position that electrons must be more fundamental than muons, but I believe that this is simply prejudice. For all we know, the muon could actually be the fundamental entity, with the lower mass of the electron (which sets the scale of the world as we know it) being merely an accident! It's an interesting thought but the real question before us is what we can do to throw some light on this puzzle of the muon's existence. In fact, one early approach to the subject consisted of doing precisely that experiment (i.e., shining light on materials to try to produce muons).

Consider what can happen when a high energy photon of energy $\nu \geq 1 \text{ GeV}$ strikes a heavy nucleus. It most often makes electron pairs with cross section. (Fig. 1)

$$\sigma_{\text{pair}}(\nu) = \frac{4Z^2\alpha^3}{m_e^2} \frac{7}{9} \ln \frac{\nu}{6.7m_e}$$

However, it can also make muon pairs with a cross section found by substituting m_μ for m_e in the above expression, i.e., with a probability of about 40,000 times smaller.

Photoproduction of muon pairs was first definitively observed by Masek and Panofsky¹ in 1955 using 600 MeV gamma rays on aluminum. Their result was that

$$\sigma_{\text{exp}}/\sigma_{\text{theory}} = 1.42 \pm 0.34$$

which was significantly different from zero but could not rule out the possibility that the cross section was, for instance, exactly twice the value predicted from quantum electrodynamics. Nevertheless, this result plus the Mu-mesic X-ray studies of Fitch and Rainwater² in 1953 proved pretty conclusively that, to quote Masek and Panofsky, "The mu-meson appears to behave, as far as its scantily known experimental aspects are concerned, exactly analogously to an electron, with the single exception of their different masses." To put it more concisely, they concluded that the muon was simply a heavy electron.

This conclusion has prevailed unchanged right up to the present time....a period of seventeen years....in spite of the fact that during this interval a great deal of new and beautiful data have been accumulated. The only possible breakthrough on the subject was the discovery that neutrinos can tell the difference between muons and electrons,³ but this fact has merely served to make the puzzle even more intriguing.

Although surely the solution to the μ -e problem will involve higher mathematics, many of the experimental difficulties can be understood in terms of simple arithmetic. Electrons are obtainable in macroscopic quantities by simply plugging into any electric outlet; but in order to get muons in appreciable quantity, it takes a tertiary beam from an accelerator. Typical muon beam intensities are $\sim 10^6$ per pulse; with the

repetition rate varying from 1,8 seconds at Brookhaven to 5 seconds at NAL and 7 seconds at Serpukov. Thus the difference between muons/pulse and muons/second can be quite appreciable.

A typical target for a muon scattering experiment is 1 meter of liquid hydrogen or perhaps 1 meter of carbon; and the duration of an average experiment is 10^5 to 10^6 pulses. The detection system in a secondary beam experiment rarely has a geometrical efficiency of less than 1% and rarely approaches the maximum value of 100%, so we take 10% as typical.

It is now fashionable to compare experiments in terms of the luminosity $\mathcal{L}$ which relates the number of events E obtained in the number of pulses (or seconds) τ , to the cross section σ and the acceptance of the detector A :

$$\text{Events} = \mathcal{L} \sigma A \tau$$

For a solid target, the luminosity equals the number of atoms/cm² times the average flux so that

$$\mathcal{L} = 0.42 \times 10^{31} \text{ cm}^{-2} / \text{sec}$$

for a flux of 10^6 /sec on 1 meter of liquid hydrogen. For 10^6 /sec on 1 meter of graphite (density 2.5)

$$\mathcal{L} = 1.5 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1} / \text{nucleon}$$

for incoherent scattering from the nucleons or

$$\mathcal{L} = 1.25 \times 10^{31} \text{ cm}^{-2} \text{ sec}^{-1} / \text{nucleus}$$

for coherent scattering from the nucleus as a whole,

The accessible cross section can be related to the luminosity in a background free situation by requiring the observation of 10 clean events for an experiment of 10^6 pulses. (This is clearly a radical assumption to give us the most optimistic cross section limit.) Then:

$$10 \text{ events} = [\mathcal{L}\sigma A] \ 10^6 \text{ pulses}$$

OR

$$\sigma \sim \frac{1}{10^4 \mathcal{L}}$$

Thus, for example, the present "state of the art" in μ -proton experiments is a 10^{12} muon total exposure ($10^6 \mu$ /pulse $\times 10^6$ pulses) which gives the minimum observable cross section in any bin as

$$\sigma = \frac{1}{10^4 \times 0.42 \times 10^{31}} = 2.4 \times 10^{-35} \text{ cm}^2$$

Historically, one series of attempts to find a μ -e difference has centered on trying to find a difference in the electromagnetic structure of these particles at very small distances.⁴ The logic is quite simple. Since electromagnetism is the strongest force available to the leptons, a difference in electromagnetic structure could give rise to a difference in self energies of these particles which would presumably explain the difference in rest masses.

Of course, the existence of this structure would violate quantum electrodynamics. It would indicate a deviation from the $\frac{1}{r}$ potential at small distances. To make matters worse, the structure would also violate relativity unless it was due to a space-time effect caused, e.g., by the exchange of the quantum of a new force.

The obvious reaction in which to probe for structure in the leptons is the scattering of high energy leptons from each other, or from other targets, at large four-momentum transfers. Since position and momentum are conjugate coordinates, large four-momentum transfers imply small distances probed. (In current jargon this would be called getting close to the "light cone").

The kinematics for scattering are shown in Fig. 2. The incident particle of mass m and four momentum p , scatters from the target particle of mass M at rest in the laboratory system. The lepton scatters through angle θ and comes out with four momentum p' in the final state. In terms of vector momenta $\underline{P}$, $\underline{P}'$ and energies E , E'

$$p \equiv (\underline{P}, iE)$$

$$p' \equiv (\underline{P}', iE')$$

The four-momentum of the virtual photon is

$$q = p - p' = (\underline{P} - \underline{P}', i[E - E'])$$

Defining ν as the lepton energy loss in the laboratory system

$$\nu \equiv E - E'$$

and θ as the laboratory scattering angle, then the invariant four-momentum transfer is given by

$$q^2 = 4E(E-\nu) \sin^2 \theta/2 + m^2 \nu^2 / E(E-\nu)$$

We define elastic scattering as the case when neither the target nor projectile break up, and no other particles are produced; and inelastic scattering as any other process in which the scattered projectile can be identified in the final state. Then we can write the cross-section in a form which applies to both cases.

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4} \left(1 - \frac{\nu}{E}\right) [W_2(q^2, \nu) \cos^2 \theta/2 + 2W_1(q^2, \nu) \sin^2 \theta/2]$$

$W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ are generalized form factors.

For inelastic scattering, the target breaks up into a collection of particles with invariant mass w . The energy

loss ν is given by

$$\nu = \frac{q^2 + w^2 - M^2}{2M}$$

In the case of elastic scattering, the target does not break up but absorbs all the four momentum of the virtual photon itself and recoils with kinetic energy T . This constrains ν to its lowest value for a given q^2

$$\nu = \frac{q^2}{2M} = T$$

The generalized form factors are then related to the squares of $G_E(q^2)$ and $G_M(q^2)$, the charge and magnetic form factors of the target.

$$W_2(q^2, \nu) = \frac{G_E^2(q^2) + \frac{q^2}{4M^2} G_M^2(q^2)}{1 + \frac{q^2}{4M^2}} \delta\left(\nu - \frac{q^2}{2M}\right)$$

$$W_1(q^2, \nu) = \frac{q^2}{4M^2} G_M^2(q^2) \delta\left(\nu - \frac{q^2}{2M}\right)$$

If there is some structure in the reaction, represented by the exchange of a quantum of mass Λ , then the cross-section is modified from the QED value.⁵ (Fig. 3)

$$\sigma \rightarrow \frac{\sigma_{\text{QED}}}{\left(1 + \frac{q^2}{\Lambda^2}\right)^2}$$

Although this is not rigorous, it serves as a convenient way to compare experiments. We define

$$\frac{\delta\sigma}{\sigma} \equiv \frac{\sigma_{\text{meas}} - \sigma_{\text{QED}}}{\sigma_{\text{QED}}} \equiv \frac{1}{\left(1 + \frac{q^2}{\Lambda^2}\right)^2} - 1$$

so,
$$\frac{\delta\sigma}{\sigma} \approx \frac{-2q^2}{\Lambda^2}$$

i.e.,
$$\frac{1}{\Lambda^2} = \frac{-1}{2q^2} \frac{\delta\sigma}{\sigma}$$

Hence, Λ^{-2} is the cross-section deviation from QED weighted by $\frac{1}{q^2}$. The smaller the value of Λ^{-2} , the better the agreement with QED.

Because scattering muons from atomic electrons is akin to scattering automobiles from target ping-pong balls, (i.e., the projectile doesn't lose much momentum), searches for lepton structure via scattering have been limited to comparing the scattering of muons and electrons from the same target,

typically protons. These searches have been plagued by irreconcilable absolute normalization differences between the muon and electron experiments, which are each beset by completely independent experimental problems. (Fig. 4)

The conclusion of these experiments,⁶ essentially unchanged since the pioneering work of the Columbia-Rochester group at Brookhaven in 1965, is that there is a normalization difference of $\sim 8\%$ between μ and e scattering, the muon cross sections being lower; but if you ignore that then the precision on Λ^{-2} is $\sim \pm 0.02$ to 0.03 GeV^{-2} . On the other hand, a real effect with Λ^{-2} on the order of 0.03 to 0.10 GeV^{-2} can not be ruled out. The results are summarized in Table I.

These experiments are still going on and have become quite elegant of late. Figure 5a shows the Columbia, Rochester, Rockefeller, NAL, NYU, Harvard experiment recently completed at Brookhaven. The muon beam, incident from the left side, is pitched upwards and then straightened out by a pair of 18D36 bending magnets. Six planes of scintillation hodoscopes containing a total of ~ 250 counters with 5 nanosecond resolving time, surround the second bending magnet and are used to momentum analyze each individual beam particle to a precision of $\pm 1\%$. At a mean momentum of $7.5 \text{ GeV}/c$, the momentum spread in the beam was 15% FWHM, and the intensity on a 5 inch x 5 inch

area was 10^6 useable muons, per 10^{12} protons incident on the target. The beam halo had an exponential form with fall-off parameter between 2 and 4 inches.

The muon beam was incident on a liquid hydrogen target, 48 inches in length. Considerable data were also taken with liquid deuterium in the target and with solid targets of carbon, aluminum, copper, and lead. The detector for the experiment was a pair of wide aperture magnetic spectrometers using magnetostrictive wire spark chambers, triggered by scintillation counter hodoscopes. Forward going muons which had scattered between 8° and 24° were detected in the forward spectrometer based on a 48D48 magnet with 24-inch aperture. This spectrometer included a 6 foot thick iron absorber to separate out the scattered muon from any pions which might be produced. Particles emerging from the target between 30° and 60° were detected in the recoil spectrometer based on a 72D18 magnet with 18-inch gap. This spectrometer had time of flight measurements to separate recoil pions from protons. Triggers were obtained independently from each of the forward and recoil spectrometers so that the two reactions:

$$\mu + p \rightarrow \mu + X$$

(elastic and inelastic muon scattering)

and

$$\mu + p \rightarrow p + \mu^*$$

(study of excited muons by the missing mass method)

were measured simultaneously. In conjunction with the muon scattering trigger, a large solid angle recoil detection system involving the front recoil spark chambers and time of flight counters in front of the magnet was used to study the angular distribution and multiplicity of particles produced in deeply inelastic scattering.

The total exposure obtained was $\sim 10^{12}$ muons distributed over assorted targets. Preliminary analysis of a liquid hydrogen run with 0.3×10^{12} muons gave roughly

250 elastic muon scatterings
7500 inelastic muon scattering events
and 10000 events of the type

$$\mu + p \rightarrow p + \text{anything}$$

OR

$$\mu + p \rightarrow \pi^+ + \text{anything}$$

The preliminary results for elastic scattering are shown in Figure 5b. Note that the q^2 scale now goes from 0.7 to 3.1 (GeV/c)². The solid line gives the fit to the electron data. It is clear that there is no gross discrepancy between the muon and electron data even at q^2 up to 3 GeV² but at this stage of the analysis it is too early to tell whether the 8% normalization difference is still observed.

Another aspect of muon scattering for which we have good data, but the interpretation is unclear, is the process

$$\mu + p \rightarrow p + \text{anything}$$

We measure the angle and energy of the proton recoiling from the target and can compute the invariant mass m of the "anything." Hopefully this method would let us observe excited μ^* resonances

$$\mu + p \rightarrow p + \mu^*$$

Our acceptance goes from μ^* mass squared between

$$1.3 \leq m^2 \leq 3.4 \text{ GeV}^2$$

and in this region we can measure the momentum transfer dependence of the cross section versus m^2 , i.e.,

$$\frac{d^2\sigma}{dt dm^2}$$

In Figure 5c we show this t distribution integrated over the entire missing mass range. The cross section is very large since most of the protons recoiling from the target are produced by "deeply inelastic scattering"

$$\begin{array}{c} \mu + p \rightarrow \mu + X \\ \quad \quad \quad \downarrow \\ \quad \quad \quad p + Y \end{array}$$

It is therefore very interesting to note that the t distribution of these protons from inelastic scattering is very similar to the t distribution from elastic scattering (solid line).

Just for amusement, we can also integrate our cross section over the observed t range, $0.4 \leq t \leq 2.0 \text{ GeV}^2$, to obtain the μ^* mass spectrum

$$\frac{d\sigma}{dm^{*2}}$$

This is shown in Figure 5d. No strong signal of μ^* is observed. The sensitivity of the experiment is limited by the large background from inelastic scattering. We are presently working on ways to improve this situation.

An important breakthrough on lepton scattering at high momentum transfer has been provided by storage rings.⁷ At ORSAY in France, FRASCATI in Italy, NOVOSIBIRSK in the U.S.S.R. and soon SPEAR in the U.S., the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$ can be studied with colliding beams of electrons and positrons of energy $\sim 1 \text{ GeV}$ each. (Fig. 6)

At FRASCATI⁸, by measuring the reactions

$$\bar{e} e \rightarrow \bar{\mu} \mu$$

$$\bar{e} e \rightarrow \bar{e} e$$

in the same apparatus at the same time, they can find the ratio of these two reactions, at a fixed q^2 of about 3 GeV^2 (timelike), independently of any unknown normalization. The

measured value of the ratio agrees with that predicted by QED to 2% with an 8% standard deviation, or

$$\begin{aligned}\Lambda^2 &= 0.003 \pm 0.013 \text{ GeV}^2 \\ &= 1.2 \pm 5.1 \times (10^{-15} \text{ cm})^2\end{aligned}$$

ruling out any possibility of muon-electron relative structure at this level. It is my opinion that this resolves the apparent μ -e discrepancy observed in the elastic and inelastic scattering experiments, proving that it was a systematic effect having to do with normalization difficulties.⁹

A completely independent and potentially more exciting way to look for structure inside muons is to make precision measurements of their static properties. Muons, by virtue of their electric charge, are continuously emitting and reabsorbing virtual photons. This process causes infinities with respect to charge and mass renormalization (Fig. 7a), but has a finite effect in the magnetic moment giving rise to the so-called anomalous magnetic moment,¹⁰ $a = \frac{g-2}{2}$ (Fig. 7b), where the g factor relates the magnetic moment $\underline{\mu}$ of a particle to its spin angular momentum $\underline{s}$, its charge e and mass m :

$$\underline{\mu} = g \frac{e}{2mc} \underline{s}$$

The presence of a special quantum in this reaction has

the same effect as in elastic scattering, (Fig. 8) It modifies the anomaly by a factor of

$$\left(\frac{1}{1 + \bar{q}^2/\Lambda^2} \right)^2$$

so that¹⁰

$$\frac{\delta a}{a} = \frac{-2\bar{q}^2}{\Lambda^2} = -\frac{2}{3} \frac{m_\mu^2}{\Lambda^2}$$

which means that the average of q^2 in the reaction is

$$\bar{q}^2 \approx \frac{1}{3} m_\mu^2$$

Note that this is an extremely small effect, changing the anomaly by about 10^{-4} for $\Lambda^{-2} = 0.01 \text{ GeV}^{-2}$.

The reason I said that this was exciting is that the quantum in this reaction is coupled only to muons. No other particles are involved. Suppose, for instance, that there were a specifically muonic force coupled only to muons and their relatives. It would show up in $g-2$ but in no other reaction I have mentioned so far. That this could have an enormous effect is obvious when you think about pions.

Pions have roughly the same mass as muons when compared to the scale of the electron mass; and if you studied only pion-electron scattering, nothing special would be apparent. It is only when you start mixing pions with their strongly interacting relatives that the sparks begin to fly.

Thus if you scale by their mass splitting with respect to electrons, there could in principle be a muon-muon force with nearly the same strength as the pion-pion force. Now, I call that really exciting!

The fact that $g \neq 2$ for a muon, means that as it moves through a magnetic field in a circular orbit, its spin gets out of phase with its orbital motion resulting in a precession of the spin in the rest frame of the muon. For a magnetic field $\underline{B}$, the cyclotron frequency $\underline{\omega}_c$ of the orbit is

$$\underline{\omega}_c = \frac{e}{mc} \underline{B}$$

while the precession frequency $\underline{\omega}_p$ of the muon spin is

$$\underline{\omega}_p = g \frac{e}{2mc} \underline{B}$$

$$\underline{\omega}_p - \underline{\omega}_c = \left(\frac{g-2}{2}\right) \frac{e}{mc} \underline{B} \equiv \underline{\omega}_a$$

It is interesting to note that for motion in a plane perpendicular to the magnetic field $\underline{B}$, the anomalous precession frequency, ω_a , of the muon spin in its rest system when measured in laboratory time is given exactly by the above non-relativistic value.

This effect can be detected because the parity violating decay

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

causes electrons to be preferentially emitted opposite to the direction of the muon spin. Consequently, as the spin precesses with respect to the direction of motion, the energy spectrum of decay electrons in the laboratory system is modulated at a frequency ω_a .

This has been observed in a beautiful series of experiments at CERN, starting in the late fifties and culminating with the magnificent experiment of the muon storage ring group, published in final form this year.¹¹ Their result for the time distribution of $\mu \rightarrow e \nu \bar{\nu}$ decay, modulated by ω_a , over four decades^{on} log paper is shown in Fig. 9. The precision of the data is staggering; they have hundreds of 1% bins. Their conclusion is once again that QED is valid -- this time, of course, for a purely muonic reaction

$$\frac{\delta a}{a} = (2.5 \pm 2.7) \times 10^{-4}$$

$$\Lambda_{\mu}^{-2} = -0.034 \pm 0.037 \text{ GeV}^{-2}$$

Of course, the drawback of this method for studying muon-muon reactions is that you are fixed in q^2 , on the order of m_{μ}^2 , so that you have to resort to higher and higher precision to see any structure. Furthermore, the experiments are already so precise that the effect of strong interactions is beginning to limit the precision of the quantum electrodynamics calculations.¹² (Fig. 10) To make any headway, it is clear that you have to go to higher q^2 . In the absence of muon targets, or muon-muon colliding beams, the only way to attain this is to go up one order in QED.

Consider the direct production of muon pairs by incident muons in the field of a heavy nucleus, or the muon trident.¹³ (Fig. 11) This is very similar to the photopair production process (Fig. 1) except now the incident photons have a large invariant four momentum, like in scattering, so that they can probe the structure of the muon. In addition, our friend the magic muon force could come into play, which is what we were hoping for in the first place.

However, if we tried to use the muon trident reaction to find structure in the muon merely by looking at some q^2 dependent modification of the cross section, e.g.,

$$\frac{\delta\sigma}{\sigma} \sim \frac{-2q^2}{\Lambda^2}$$

we would be in trouble for at least two reasons. First of all, the total trident cross section is extremely small, down by a factor of $\sim \frac{\alpha}{\pi}$ from photopair production; and secondly, the q^2 dependence of the cross section is very rapid, at least $\frac{1}{q^4}$ and perhaps even $\frac{1}{q^8}$.

The beauty of the trident configuration is that it is sensitive to the actual production and decay into muon pairs of this hypothetical quantum of the muon-muon force. The incident muon can radiate a real one of these quanta, which will then decay into a pair of muons and appear like a trident, with the exception that the invariant mass of the produced muon pair will always be equal to the mass of the quantum. (Fig. 12)

The price you pay for this possibility is that QED tridents can proceed through a similar mechanism, timelike virtual photons, which complicates the calculation considerably. (Fig. 13) As if that weren't enough, because there

are two identical muons in the final state, e.g., $2 \mu^-$ in the reaction

$$\mu^- + Z \rightarrow \mu^- + Z + \mu^- + \mu^+,$$

you have to anti-symmetrize the total production amplitude by subtracting from the above diagrams the same diagrams with the momenta of the identical particles exchanged. That leaves a total of 8 amplitudes to be summed and squared to obtain the seven-fold differential cross section. This explains why the full trident differential cross-section was not calculated until 1967.¹⁴ In fact, the higher order Born corrections and radiative corrections to tridents have still not been calculated.

Before leaving this subject, it is worthwhile to note that the trident cross section calculation contained an implicit assumption -- namely, that the spin 1/2 muons require an anti-symmetric wave function; i.e., they obey Fermi statistics and the Pauli exclusion principle just like electrons. In fact, this is not necessarily true. The spin-statistics theorem only requires that spin 1/2 particles not have symmetric wave functions; it doesn't require them to be antisymmetric.¹⁵ This still leaves open the possibility that the muon obeys a more complicated field theory, one in which wave functions have more options than fully symmetric or fully antisymmetric.

Could that be the solution to the puzzle??!!!! Well, you don't really know until it is experimentally proved or disproved. With the exception of some para-statistics theories,¹⁶ the only^{way} that the statistics of particles can be determined is to observe two or more identical particles in the final state. For electrons, nucleons, photons and pions, such final states are abundant in nature; but for muons the first possibility of observing one came with the muon trident reaction.

Although it is impossible to compute the trident cross section if you leave the statistics of the muon unspecified, it is possible to compute the cross section in two special cases: 1) Fermi statistics, and 2) The absence of statistics, i.e., produced muon pair distinguishable from incident muon. After the rigamarole of integrating the complicated differential cross section for both of these cases, the result is quite simple.¹⁷ The effect of Fermi statistics is to depress the trident cross section in the region of phase space where the two identical muons have similar energies and directions, i.e., small invariant mass.

The main problem with observing the muon trident experimentally, is that the cross section is so very small. For reference, the total cross sections for muon trident production in carbon at incident muon energies up to 500 GeV are given in Figure 13e. If we restrict the final state so that all three muons have energies greater than 1.5 GeV, to facilitate in separating the muons from

pions, then the total trident cross-section for 11 GeV muons incident on lead is 73 nanobarns per lead nucleus. This corresponds to a mean free path of about 5000 km of lead, roughly the distance from New York to San Francisco. The strong interaction mean free path by comparison is 15 cm. There are other technical problems in doing this experiment, but these are all exacerbated by the tiny cross-section and need not be dealt with in this report.

The muon trident was finally pinned down in an experiment by a Harvard, University of Massachusetts, McGill group at Brookhaven,¹⁸ published in early 1971. A previous report by a Utah group had claimed to see tridents in a cosmic ray experiment,¹⁹ but it was pointed out by an English group²⁰ that the Utah result was five times higher than their measured upper limit as well as a factor of 10^4 higher than the theoretical value.

The layout of the BNL experiment is shown in Fig. 14. An incident muon beam of momentum 10.5 ± 1.5 GeV/c passed through a momentum analyzer (magnets D5, D6 and associated optical spark chambers) and struck a lead target which was 8" x 8" by 4" thick. The size of the beam was defined by scintillation counters to be 4" square at the target and contained a flux in this area of 12,000 μ per pulse.

Particles emerging from the target (Fig. 15) passed through four 1" thick scintillators (T counters), a front

spark chamber array, and then into a large aperture magnet 120D36 in which beamlike charged particles were deflected to the right and oppositely charged particles to the left. Following the magnet, the particles passed through the rear optical spark chamber and counter array which included a 1 meter thick iron absorber to identify penetrating particles.

A trident trigger was defined by at least two minimum-ionizing particles emerging from the target (as indicated by greater than twice minimum pulse heights in all four T counters) and by two penetrating particles, one of each charge. For each trigger the spark chambers were photographed, and these pictures were used to reconstruct the momenta of the incident and all three outgoing particles.

Since the lead nucleus doesn't absorb any appreciable kinetic energy for muon tridents, the difference between incident and total outgoing energies was calculated for all events in which one incident and three outgoing momenta were reconstructed, with at least two penetrating particles in the final state. The result is shown in Fig. 16. The clean peak shows that muon tridents were produced. The peak is shifted since on the average, two minimum ionizing muons pass through the entire 4 inch target. A total of 89 trident events were observed during our 200 hour run -- i.e., on the order of 1 event every 2 hours!

Table II shows the number of events detected compared with the yields predicted for Fermi statistics and no statistics. The experiment clearly favors Fermi statistics. The major experimental correction is due to the fact that we could require all three final state particles to penetrate the iron in only 2/3 of our data. From these events, we could measure the pion contamination in the sample of events where only two final state particles can be seen to penetrate the iron.

The pseudotrident correction is for the two step process of bremsstrahlung by the incident muon followed by muon pair production by the radiated photon. This reaction is completely indistinguishable from direct pair production but can be easily calculated.

It is interesting to note that the theoretical higher Born correction to the "events predicted" is as large as the experimental corrections. I believe that theoretical uncertainties will tend to discourage people from doing higher precision trident experiments.

The invariant mass distribution of the two identical muons of the trident (Fig. 17) is also consistent with Fermi statistics. A maximum likelihood fit to the data introducing a parameter α , which equals 1 if muons are Fermions and 0 if statistics are irrelevant, gives the result

$$\alpha = 1.15 \pm 0.25$$

which clearly shows that muons are Fermions.

Well, we're back to where we started -- muons are still heavy electrons. However, we have in some sense improved our thinking. If we really want to look for a muon-muon force, we should study reactions like muon tridents, which contain only muons, and look for anomalies. Alternatively, we can try to discover the quantum of the muon-muon force which will manifest itself by its strong decay into muon pairs.

This latter approach has been taken by Lederman and collaborators²¹ who tried to directly produce muon pairs by blasting the entire external proton beam into a target. The method has one obvious advantage over producing tridents with muons in that the primary proton beam has 10^6 times more flux than any muon beam. Without trying to belittle this factor of 10^6 improvement in rate, I must say that the real advantage of this method is that it greatly enhances the timelike production of muon pairs. Whether this will increase the signal or the background in trying to search for the quantum of the muon-muon force is not at all clear, but it is certainly a good method for studying timelike muon pair production. We can estimate the enhancement of the timelike processes by looking at Figures 13a and 13b. By replacing

the incident muon (p_1, p^*, p_2) by a proton, we allow the scattering from the nucleus to proceed via strong interactions instead of electromagnetic. This enhances the timelike production by roughly the ratio of the total strong interaction cross section to the total electromagnetic interaction cross section

$$\sigma_T \text{ strong} \approx 40 \times 10^{-27} \text{ cm}^2$$

while

$$\sigma_T \text{ electric} \approx \frac{4\pi\alpha^2}{(q_n^2)_{\min}}$$

The minimum momentum transfer to the nucleus required to make a muon pair of mass m is

$$(q_n^2)_{\min} \approx \left(\frac{m^2}{2E} \right)^2$$

where E is the energy of the incident proton. Writing this in terms of s , the total energy in the center of mass for a proton-proton collision

$$s \approx 2m_p E$$

then

$$(q_n^2)_{\min} \approx m_p^2 \left(\frac{m^2}{s} \right)^2$$

where m_p is the mass of the proton.

Thus

$$\begin{aligned} \text{enhancement} &\approx \frac{\sigma_{\tau \text{ strong}}}{\sigma_{\tau \text{ electric}}} = \left(\frac{m^2}{s}\right)^2 \frac{40 \times 10^{-27} \text{ cm}^2}{4\pi\alpha^2/m_p^2} \\ &\approx \left(\frac{m^2}{s}\right)^2 \times 1.35 \times 10^5 \end{aligned}$$

While this computation grossly oversimplifies the processes which must be occurring in the reaction, the order of magnitude of the result is probably correct.

The results of the experiment²¹ are well known (Fig. 18) and have been widely discussed at this conference so that I will not discuss them again. However, I will say that I think that Lederman must really like the experiment because he has already proposed it at four different laboratories: BNL, the ISR, NAL and ISABELLE. The layout of the ISR experiment²² is shown in Figure 19. The purpose of the experiment is to search for electron pairs produced at roughly 90° with respect to the two colliding proton beams. The electrons are detected by core-readout wire spark chambers and their energies are measured by lead glass total absorption Cerenkov counters. The experiment is still in progress and we would hope that eventually the experimenters would also try to detect muon pairs. The luminosity of the ISR is on the order of 10^{30} per sec cm^2 so that by my rule-of-thumb, their minimum observable cross section will be around 10^{-34} cm^2 .

Before leaving the present and discussing three possibilities for the future which really excite me, I should like to point out that if there really is a quantum coupled to "mu-ness" it may not be directly coupled to strongly interacting particles.²³

Thus, this hypothetical quantum may not be directly produced in proton-proton collisions. It is therefore imperative to keep searching for the quantum of "mu-ness" in purely leptonic processes such as muon or neutrino initiated tridents.

The exciting possibilities for the future, which I mentioned, all have to do with opening up the realm of high energy weak interactions to experimenters. The first facility, which is scheduled for completion in 1976, is a 100 to 200 GeV muon beam with intensity 2×10^8 per pulse to be built at super-CERN.²⁵ This could yield a luminosity of $10^{34}/\text{cm}^2\text{sec}$ at $s = 400 \text{ GeV}^2$ and should enable us to see high energy weak interactions initiated by muons. For instance, Bjorken and Llewellyn-Smith²⁴ in their review article on weak interactions and heavy leptons set limits of

$$10^{-37} \text{cm}^2 \leq \sigma \leq 10^{-35} \text{cm}^2$$

for the process

$$\begin{aligned} \mu^+ + n &\rightarrow M^0 + \text{hadrons} \\ &\quad \downarrow \\ &\quad \mu^+ + \mu^- + \bar{\nu}_\mu \end{aligned}$$

with 100 GeV incident muons if the mass of M^0 is < 4 GeV, Thus these experiments begin to border on the realm of feasibility.

The real step forward will come with the next generation of high energy machines now under active study in the U.S. Isabelle at Brookhaven²⁶ is a superconducting intersecting storage ring accelerator, with colliding proton beams of 200 GeV each. The design luminosity is $10^{34} \text{cm}^{-2}/\text{sec}$ at $s = 1.6 \times 10^5 \text{GeV}^2$. An alternative idea is PEP at SLAC²⁷ which is a 15 GeV electron beam colliding with 70 GeV protons. The design luminosity is $10^{32} \text{cm}^{-2}/\text{sec}$ at $s = 4.2 \times 10^3 \text{GeV}^2$. Each machine has its partisans and whether either machine will be built is quite uncertain; but one thing is perfectly clear. Either machine is at a high enough energy with enough luminosity so that high energy weak interactions should occur copiously. Hopefully we will then get some hint as to what nature does to make weak interactions finite. Whether the result is intermediate bosons, heavy leptons, neutral current or an undreamed-of phenomenon, the solution to the problem of high energy weak interactions must be intimately related to the questions of whether the muon and the electron have any internal structure, why the muon is so much more massive than the electron and how the neutrinos can distinguish "mu-ness".

In the year 1987, we will celebrate the 50th anniversary of the discovery of the muon. With luck, at that time we will be able to explain why the muon is a heavy electron.

References and Footnotes

1. G. Masek and W. Panofsky, Phys. Rev. 101, 1094 (1956)
2. V. Fitch and J. Rainwater, Phys. Rev. 92, 789 (1953); also see J. Wheeler, Rev. Mod. Phys. 21, 133 (1949).
3. G. Danby, J. M. Gaillard, K. Goulianos, L. M. Lederman, N. B. Mistry, M. Schwartz and J. Steinberger, Phys. Rev. Letters 9, 36, (1962).
4. I. Y. Kobzarev and L. B. Okun, J.E.T.P. 41, 1205 (1961).
5. S. D. Drell, Ann. Phys. (N.Y.) 4, 75 (1958); T. D. Lee and G. C. Wick, Phys. Rev. D 2, 1033 (1970).
6. R. Cool, A. Maschke, L. M. Lederman, M. Tannenbaum, R. Ellsworth, A. Melissinos, J. H. Tinlot and T. Yamanouchi, Phys. Rev. Letters 14, 724 (1965); R. W. Ellsworth, et al, Phys. Rev. 165, 1449 (1968); L. Camilleri, J. H. Christenson, M. Kramer, L. M. Lederman, Y. Nagashima and T. Yamanouchi, Phys. Rev. Letters 23, 149 (1969); L. Camilleri, et al, Phys. Rev. Letters 23, 153 (1969); W. T. Toner, et al, SLAC-PUB-1009, 1972; Martin L. Perl, Phys. Today 24, 34 (1971).
7. Proceedings of the International Conference on Electron and Photon Interactions at High Energies 1971, Cornell University; Alles Borelli, et al, Nuovo Cimento Letters 2, 376 (1971).

8. Note that in terms of R.M.S. charge radii

$$\langle r_{\mu}^2 \rangle - \langle r_e^2 \rangle = 6\Lambda^{-2} = 0.07 \pm 0.30 \times (10^{-14} \text{ cm})^2$$

9. Note that the storage ring result does not involve hadron while the scattering experiments do. Thus it is conceivable that muons could have some special hadronic interaction which would explain the discrepancy. I find this hard to swallow because it is well known that muons of tens of TeV penetrate deep inside the earth. I do not understand how this could happen if muons had a hadronic reaction.
10. For a complete review of the early references see A. Petermann, Fortschritte der Physik 6, 505 (1958).
11. J. Bailey, W. Bartl, G. von Bochmann, R.C.A. Brown, F.J.M. Farley, H. Jostlein, E. Picasso and R. W. Williams, Phys. Letters 28B, 287 (1968); J. Bailey, W. Bartl, G. von Bochmann, R.C.A. Brown, F.J.M. Farley, M. Giesch, H. Jostlein, S. van der Meer, E. Picasso and R. W. Williams, Nuovo Cimento 9A, 369 (1972).
12. Gourdin and DeRafael, Nucl. Phys. B 10, 667 (1969); H. Terazawa, Phys. Rev. 177, 2159 (1969).
13. See L. M. Lederman and M. J. Tannenbaum in Advances in High Energy Physics, edited by R.E. Marshak and R.L. Cool (Interscience Publishers, Inc., New York, 1968), for the early references. This process has been discovered recently by the theorists to play an important role at the storage

- rings. See A-Romero, Jaccarini and Kessler, Comptes Rendus B 269, 153, 1129 (1969); Balakin, Budnev, Ginsburg, JETP Lett. 11, 559 (1970) [English Translation 71, 388 (1970)]; Brodsky, Kinoshita and Terazawa, Phys. Rev. Letters 25, 972 (1970).
14. J.D. Bjorken and M.C. Chen, Phys. Rev. 154, 1335 (1967); G.R. Henry, Phys. Rev., 154, 1534 (1967); B. Carlin, G. Frielingsdorf and G. Peressutti, Arkiv För Fysik, 37, 601 (1968).
 15. e.g., see J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, New York, 1965, p. 170.
 16. S. Kamefuchi and J. Strathdee, Nucl. Phys. 42, 166 (1963).
 17. M.J. Tannenbaum, Phys. Rev. 167, 1308 (1968).
 18. J.J. Russell, R.C. Sah, M.J. Tannenbaum, W.E. Cleland, D.G. Ryan and D.G. Stairs, Phys. Rev. Letters 26, 46 (1971).
 19. M.L. Morris and R.O. Stenerson, Nuovo Cimento 53B, 494 (1968).
 20. J.C. Barton and I.W. Rogers in Proceedings of the 11th International Conference on Cosmic Rays, Budapest, August 25 to September 4, 1969, Academy Press, Budapest, 1970.
 21. J. H. Christenson, G. Hicks, P. Limon, L.M. Lederman, B. Pope and E. Zavattini, Phys. Rev. Letters 25, 1523 (1970).
 22. Columbia, CERN, Rockefeller Collaboration: R.L. Cool, L. DiLella, L.M. Lederman, E. Zavattini, et al.

23. I.Y. Kobzarev and L.B. Okun, Nucl. Phys. 35, 311 (1962);
see also M. J. Tannenbaum, 1968 NAL Summer Study, Vol. 2
24. J.D. Bjorken and C.H. Llewellyn-Smith, SLAC-PUB-1107
(1972), submitted to Phys. Rev.
25. CERN Report, CERN/ECFA/72/4 Vol. 1, July 1972.
26. BNL Report, BNL-16716, May 1972.
27. SLAC Report 146, April 1972.

Figure Captions

Figure 1 - Feynmann diagrams for photopair production.

Figure 2 - Kinematics for scattering. The incident particle of mass m scatters from target particle of mass M . If the target particle breaks up, the scattering is inelastic.

Figure 3 - Modification of cross section for scattering by a hypothetical heavy photon of mass Λ

$$\sigma \rightarrow \frac{\sigma_{\text{QED}}}{(1 + q^2/\Lambda^2)^2}$$

Figure 4 - Typical data of a muon scattering experiment. The muon data lie consistently below the electron curve so that the discrepancy could simply be a difference in absolute normalization. The graph shown is from Camilleri et al.⁶

Figure 5a - Layout of latest muon scattering experiment now running at Brookhaven. The incident and outgoing muon as well as the recoil proton are all measured with magnetic spectrometers.

Figure 5b - Preliminary results $\frac{d\sigma}{dq^2}$ for muon-proton elastic scattering.

Figure 5c - Preliminary results for $\int_{1.3}^{3.4} dm^2 \frac{d^2\sigma}{dt dm^2}$ which is the t distribution of inelastic scattering integrated over all v and q^2 for the missing mass range specified. Note the similarity to the t distribution of elastic scattering which is drawn in the solid line.

Figure 5d - Preliminary results for the missing mass spectrum of μ^* in the reaction

The quantity plotted is $\int_{0.4}^{2.0} dt \frac{d^2\sigma}{dt dm^2}$

Figure 6 - Colliding electron and positron beams annihilate and produce a pair of muons.

Figure 7 - a) Mass renormalization due to exchange of virtual photon. The effect is logarithmically divergent.

b) Anomalous magnetic moment due to exchange of a virtual photon. The g factor of the muon is changed from $g = 2$ to $g = 2[1 + a]$ where

$$a = \frac{\alpha}{2\pi} + 0.7658 \frac{\alpha^2}{\pi^2} + (21 \pm 1) \frac{\alpha^3}{\pi^3}$$

Figure 8 - Modification of the anomalous magnetic moment due to a heavy photon of mass Λ .

Figure 9 - Muon decay time distribution obtained by the CERN group.¹¹ The modulation is due to the anomalous precession frequency.

Figure 10 - Hadronic contribution to the vacuum polarization in $g-2$. The hadronic intermediate state must have spin-parity 1^- and includes the well-known vector mesons ρ^0 , ω^0 , ϕ^0 .

Figure 11 - Direct pair production of a muon pair by an incident muon in the field of a heavy nucleus. The virtual photon is spacelike.

Figure 12 - Effect of a heavy photon on trident production.

- a) Exchange of a spacelike virtual heavy photon.
- b) Production and decay of a real heavy photon of mass Λ . In this case, the invariant mass of the produced muon pair always has value Λ .

Figure 13 - The four Feynmann diagrams for muon tridents

- (a) and (b) have timelike virtual photons while in (c) and (d), the virtual photon is spacelike.

p^* is the four-momentum of the virtual muon;
 q is the four-momentum of the virtual photon,
 and q_N is the nuclear recoil four momentum.

Figure 13e - Exact calculation of muon tridents in carbon compared to the Bhabha-Block formula (solid line)

$$\sigma_{\text{trident}}(E) = \frac{\alpha}{2\pi} 1.012 \mu\text{b} \left[\ln^3 \frac{E}{m} - 6.286 \ln^2 \frac{E}{m} + 14.99 \ln \frac{E}{m} - 15.42 \right]$$

and my own formula (dotted line)

$$\sigma_{\text{trident}}(E) \approx \frac{5}{6} \frac{\alpha}{\pi} \left(\ln \frac{E}{m} - 3.40 \right) \sigma_{\text{pair}}(E)$$

The exact calculation has error bars since it is done by a Monte Carlo method. Neither formula fits the exact calculation very well, but my formula doesn't do badly below 100 GeV. For details, see NAL Report SS-177 (1970).

Figure 14 - Layout of muon trident apparatus showing the incident muon spectrometer D_5 and D_6 . The muon beam is incident from the left onto scintillator S_3 .

Figure 15 - Close-up of the trident detector. Elements marked S.C. are spark chambers. Counter banks A, B, C, D, F, and G are vertical hodoscopes on either side of the one-meter-thick iron absorber.

Figure 16 - Energy balance between initial and final states. The -0.298 GeV shift in energy is caused by ionization loss in the lead target.

Figure 17 - Invariant mass distribution of the two identical muons in the final state of the muon trident.

Figure 18 - Results of the Brookhaven experiment for $p + p \rightarrow \mu + \mu + \text{anything}$. The lines represent various theoretical attempts to fit the data.

Figure 19 - Layout of Columbia-CERN-Rockefeller experiment at the ISR.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

cm^2/GeV^2
 $d\sigma/dq^2$

SEMI-LOGARITHMIC
4 CYCLES X 7 1/2 DIVISIONS
KEUFEL & ESSER CO.

FIG. 5b

10^{-30}

$1.63 \times 10^{11} \mu\text{s on } 1.2 \text{ m}$

$\mu + p \rightarrow p + \gamma$
 $\equiv$

$1.63 \times 10^{11} \mu\text{s on } 1.2 \text{ m}$

FIG 5c

$\int_{1.3}^{3.4} dm^2 \frac{d^2\sigma}{d\Omega d\tau}$
 cm^2/GeV^2

KE SEMI-LOGARITHMIC
46 5810
5 CYCLES X 140 DIVISIONS
MADE IN U.S.A.
KEUFFEL & ESSER CO.

0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
 $t (Rm^2)$

$\int_{0.4}^{2.0} \frac{d^2\sigma}{dt d^2m^2} \text{ cm}^2/\text{GeV}^2$

X 10⁻³²

Fig 5d

Fig. 6

Mass renormalization-infinite

Fig. 7a

Anomalous magnetic moment-finite

Fig. 7b

Q.E.D.

+

Heavy photon

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12 a

Fig. 12 b

Fig. 13

TOTAL CROSS SECTION FOR

$$\mu + C \rightarrow \mu + C + \mu^+ + \mu^-$$

NO SCREENING

T_{INCIDENT} IN GeV

FIG 13e

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Fig. 18

FIG 19

T A B L E I

RESULTS OF MUON-ELECTRON SCATTERING COMPARISON

<u>YEAR</u>	<u>GROUP</u>	RESULT WITH N = 1 <u>Λ^{-2} in GeV^{-2}</u>	2 Parameter fit <u>N and Λ^{-2} Free</u>	Cross-section Normalization <u>so that $\Lambda^{-2} = 0$</u>
1965	Columbia Rochester BNL	0.10 ± 0.03	----	0.885 ± 0.038
1969	Columbia Rochester	0.148 ± 0.025	0.064 ± 0.056 [N = 0.95 ± 0.03]	0.920 ± 0.012
1971	SLAC	0.049 ± 0.012	0.021 ± 0.021 [N = 0.946 ± 0.042]	0.917 ± 0.024

STORAGE RING

1971	FRASCATI	0.003 ± 0.013		No Normalization Problem!
------	----------	-------------------	--	------------------------------

TABLE II

Corrected Yields¹⁷

Events Observed		Events Predicted		
Raw data	89 ± 9.4		Fermion	No Exchange
Systematic corrections (events)	-7.1 ± 4.5	Born approximation	101.6 ± 2.4	138.4 ± 2.2
Pseudo-trident correction (events)	-6.7 ± 1.0	Higher order corrections	-12.7 ± 1.0	-17.5 ± 1.4
Corrected events observed	75.2 ± 10.5	Corrected events predicted	88.9 ± 2.6	120.9 ± 2.6