Conf-921072-13

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SEP 07 1993
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APERTURE DETERMINATION OF RHIC92 FROM RANDOMLY GENERATED INITIAL COORDINATES*

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ABSTRACT

Results obtained by tracking 100 particles for 1000 turns when initial coordinates are selected randomly, with the requirement that the total emittance be constant, are compared to results from 1000-turn and 10^6 -turn runs when initial coordinates satisfy $\epsilon_x(i) = \epsilon_y(i)$ and $X'_i = Y'_i = 0$. For studies of ten distributions of magnetic field errors, the 100-particle results give apertures equivalent to those from 10^6 -turn runs, have an aperture distribution of considerably less width, and yet require only one tenth the computer time.

1 INTRODUCTION

Aperture determinations are intended to probe phase space in a direction defined by the initial coordinates. In general, most investigators have used initial coordinates defined by $\epsilon_x(i) = \epsilon_y(i)$ and $X'_i = Y'_i = 0$. In the following text this set of coordinates is denoted by the expression "single-particle launch."

Tracking on lattices, such as the AGS Booster and RHIC at BNL and various SSC lattices, indicates there is repetitive transfer of emittance between the horizontal and vertical planes throughout a tracking run. This transfer depends on sextupoles used for chromaticity correction and random field errors and can be complete to either or both planes.

The relation between ϵ_x and ϵ_y changes throughout the run; Figure 1 shows normalized phase plots from a 300-turn run at $\beta^* = 2$ m when seed 9 is used to generate random field errors, and the initial coordinates are those defined for the single-particle launch. The variation of $\sqrt{\epsilon_x}$ and $\sqrt{\epsilon_y}$ is shown by the smear plot of Figure 2(a), where the solid line indicates the locus of $\sqrt{\epsilon_x + \epsilon_y} = \sqrt{\epsilon_t(0)}$. Had there been no coupling from magnetic field errors and sextupoles used for chromaticity correction, all turns would have been represented by a single point at (1.0,1.0). The time dependences of ϵ_x and ϵ_y are shown in Fig. 2(b); ϵ_x and ϵ_y are unequal most of the time, and hence the particle is not probing in the direction defined by its initial coordinates. Figures 1 and 2 show results from the first 300 turns of the worst case run for which the particle survived 10^6 turns. It is an extreme case in that the emittance transfer is nearly complete in both directions.

A more representative example is shown in Figure 3(a). There is complete emittance transfer from the horizontal to the vertical plane. Figure 3(b) gives the time dependence of ϵ_x and ϵ_y . The emittances become unequal within the

* Work performed under the auspices of the U.S. Department of Energy.

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Figure 1: RHIC92, $\beta^* = 2$ m, seed 9: worst case of Figure 4(b). Phase plots from 300-turn run with $\epsilon_{x(0)} = \epsilon_{y(0)} = 0.871 \pi$ mm mrad. Emittance transfer is nearly 100% between the horizontal and vertical planes.

Figure 2: RHIC92, $\beta^* = 2$ m. seed 9. First 300 turns of tracking run: (a) relation between $\sqrt{\epsilon_x}$ and $\sqrt{\epsilon_y}$; (b) time dependence of ϵ_x and ϵ_y .

Figure 3: RHIC92, $\beta^* = 6$ m. seed 4. First 300 turns of tracking run: (a) relation between $\sqrt{\epsilon_x}$ and $\sqrt{\epsilon_y}$; (b) time dependence of ϵ_x and ϵ_y .

first 20 turns and are only approximately equal about 25% of the time. Again, the intent of probing in a prescribed direction is not realized.

In this paper a method that is patterned after the dependence shown in Figure 2(a) is used for selecting the initial coordinates of the test particles. The total emittance $\epsilon_t(0)$ can be distributed in any way that satisfies the relation $\epsilon_t(0) = \epsilon_x(i) + \epsilon_y(i)$, and also X'_i and Y'_i can be nonzero. One hundred particles having randomly selected $\epsilon_x(i)$, $\epsilon_y(i)$, X_i , X'_i , Y'_i and Y_i are tracked for 1000 turns. The term “multiparticle launch” is used to denote this method of selecting initial coordinates.

2 PROCEDURE

A. Setup

The study was done using the RHIC92(0,0) lattice when $\beta^* = 2$ m and $\beta^* = 6$ m. The tunes were $\nu_x = 28.827$ and $\nu_y = 28.823$. The chromaticity was corrected to zero, and $\Delta P/P = 0$. All dipole, quadrupole, and sextupole elements of RHIC92 were included in the lattice used for tracking. The effects of nonlinear fields were represented by thin-lens kicks located at the center of all quadrupoles and at both ends and the center of all dipoles, where they were given weights of 1/6 and 2/3, respectively. Multipole expansions of random field errors were generated from the rms errors $(\sigma a_n, \sigma b_n)$ of Herrera et al.¹ according to a Gaussian distribution that was truncated at $\pm 3\sigma$. The expansion was made with $2 \leq n \leq 16$ for dipoles and $2 \leq n \leq 10$ for quadrupoles. Aperture determinations were made for ten sets of random field errors generated by using different seeds to initialize the random number generator. No systematic errors were included. Tracking was performed on the NERSC CRAY.C at LLNL using a special version of PATRICIA.² The test particles were always launched at the center of Q9 quadrupole at the beginning of an inner arc. The amplitude of the test particle was checked at every element to assure it remained within the vacuum chamber.

B. Generation of Initial Coordinates

The steps used in generating the coordinates for multiparticle launching are as follows.

1. Define the initial total emittance $\epsilon_t(0)$ of the test particle.
2. Use the randomly generated number RANF to select the initial horizontal emittance:

$$\epsilon_x(i) = \epsilon_t(0) \times \text{RANF}.$$

3. Determine $\epsilon_y(i)$ from $\epsilon_t(0)$ and $\epsilon_x(i)$:

$$\epsilon_y(i) = \epsilon_t(0) - \epsilon_x(i).$$

4. Determine the initial coordinate X_i randomly:

$$(a) \ X_{\max} = \sqrt{(\epsilon_x(i) \times \beta_x)}; \quad (b) \ X_i = X_{\max} (1. - 2. \times \text{RANF}).$$

5. Solve the Courant-Snyder relation for emittance to determine the initial X'_i associated with X_i :

$$(a) \quad X'_i = \left(-\alpha_x \times X_i \pm \sqrt{(\epsilon_x(i) \times \beta_x - X_i^2)} \right) / \beta_x;$$

$$(b) \quad \text{select the sign of } \sqrt{(\epsilon_x(i) \times \beta_x - X_i^2)} \text{ randomly.}$$

6. Repeat steps 4 and 5 when X is replaced by Y .

The random number generator was not reset prior to generating initial coordinates, and hence the one hundred sets of initial coordinates are different for each seed.

C. Tracking

One hundred particles were launched and tracked in sequence. If any particle failed, the motion was considered unstable, and the run was terminated. If no particles failed, the motion was considered stable. The total emittance $\epsilon_t(0)$ was varied until all particles survived the specified number of turns. The results are expressed in terms of an equivalent X defined as $X = \sqrt{(\epsilon_t(0) \times \beta_x / 2)}$ and are thus consistent with the convention used for the single-particle launch for which $\epsilon_x(0) = \epsilon_y(0)$.

3 RESULTS

The results obtained by multiparticle tracking are compared with results obtained using the single-particle launch with tracking times of 1000 and 10^6 turns.

1. Single-particle launch with 1000 turns: With $\epsilon_x(0) = \epsilon_y(0)$, X_i was incremented by 0.2 mm (measured where $\beta_x = 50$ m). The aperture quoted is the smallest X_i of the ten measurements for which the test particle remained within the vacuum chamber at amplitude X_i and hit the vacuum chamber at amplitude $X_i + \Delta X$. The distributions of X_i 's are shown as histograms in Figure 4(a) for $\beta^* = 2$ m and Figure 5(a) for $\beta^* = 6$ m. The number in each cell identifies which seed was used to generate the random field errors; the width of the cell indicates the amplitude at which the particle survived (left side) and failed (right side). The worst case scenario gives an aperture of $X_i = 8.3$ mm at $\beta^* = 2$ m and $X_i = 16.3$ mm at $\beta^* = 6$ m.
2. Single-particle launch with 10^6 turns: With $\epsilon_x(0) = \epsilon_y(0)$, X_i was decreased in increments of $\Delta X = 0.2$ mm (measured where $\beta_x = 50$ m) from a large value until the test particle first stayed within the vacuum chamber for 1 million turns. The results are shown in Figure 4(b) for $\beta^* = 2$ m and Figure 5(b) for $\beta^* = 6$ m.
3. Multiparticle launch: One hundred particles having initial coordinates X_i, X'_i, Y_i, Y'_i satisfying $\epsilon_t(0) = \epsilon_x(i) + \epsilon_y(i)$ were tracked for 1000 turns.

(a) One particle (10^3 turns)

(b) One particle (10^6 turns)

(c) Multipart (100×10^3 turns)

Figure 4: RHIC92, $\beta^* = 2$ m. Aperture determinations for: (a) one particle/seed tracked 1000 turns, (b) one particle/seed tracked 10^6 turns; (c) 100 particles/seed tracked 1000 turns.

Figure 5: RHIC92, $\beta^* = 6$ m. Aperture determinations for: (a) one particle/seed tracked 1000 turns. (b) one particle/seed tracked 10^6 turns; (c) 100 particles/seed tracked 1000 turns.

The total emittance was varied in steps such that $X = \sqrt{(\epsilon_t(0) \times \beta_x/2)}$ was incremented by 0.2 mm (measured where $\beta_x = 50$ m). The total emittance was reduced until all 100 particles remained within the vacuum chamber throughout the run. Histograms of the aperture determinations are shown in Figure 4(c) for $\beta^* = 2$ m and Figure 5(c) for $\beta^* = 6$ m.

4 DISCUSSION

A. Aperture

The apertures, defined by the worst case scenario from Figures 4 and 5, are listed in Table 1. It is seen that:

1. The apertures defined by the worst case values from 10^6 -turn runs and multiparticle runs are essentially equal.
2. The spread in the distribution of results from multiparticle tracking is smaller than that from the single-particle launch.

Table 1: Aperture Determinations for RHIC92

$$X_i \text{ (mm)} = \sqrt{(\epsilon_t(0) \times \beta_x/2)} \text{ where } \beta_x = 50 \text{ m}$$

β^*	Standard launch		Multiparticle launch
	TURNS		TURNS
	1000	10^6	1000
2	8.3 ± 0.1	6.7 ± 0.1	6.5 ± 0.1
6	16.3 ± 0.1	14.3 ± 0.1	13.7 ± 0.1

B. Effectiveness of the Multiparticle Launch

1. Use of 100 particles tracked for 1000 turns requires approximately one tenth the computer time needed for 1-million-turn studies and thus enables more varied studies for a given computer budget.
2. Results for seed 2 in Figure 4(b) and (c) show marked differences determined with the single- and multiparticle launch. The multiparticle launch searches more trajectories and is frequently able to locate the aperture limits in fewer turns than required when using a single trajectory.
3. Not only have recent studies on the single-particle launch shown a correlation between the degree of emittance transfer and the worst and best case aperture, but also the plots of the relation between $\sqrt{\epsilon_x}$ and $\sqrt{\epsilon_y}$ throughout these runs show not much difference between plots for the first 300 turns of a one-million-turn run and those for runs made for 300 turns beyond a million turns. Examples of this are shown in Figure 6 for seed 9, the set of random field errors that gave the smallest aperture value, and in Figure 7 for seed 2, the set of random field errors that gave the largest aperture value. For these seeds it is also found, with initial amplitudes for which runs are stable for 1000 turns or more, that the degree of emittance transfer has only a weak dependence on the initial emittance.

Figure 6: RHIC92, $\beta^* = 2$ m, seed 9: worst case in Fig. 3(b). Plots of ϵ_y vs ϵ_x : (a) first 300 turns of 10^6 -turn run; (b) continuation from 10^6 to $10^6 + 300$ turns. Plots show little change during 10^6 -turn run.

Figure 7: RHIC92, $\beta^* = 2$ m, seed 2: best case of Figure 3(b). Plots of ϵ_y vs ϵ_x : (a) first 300 turns of 10^6 -turn run; (b) continuation from 10^6 to $10^6 + 300$ turns. Plots show little change during 10^6 -turn run.

These results suggest that the use of many sets of randomly generated field errors may just be a method of finding a case for which large emittance transfers occur. In contrast, the multiparticle launch forces exploration of many combinations of (ϵ_x, ϵ_y) that satisfy the relation $\epsilon_x + \epsilon_y = \epsilon_t(0)$. This is thought to be the explanation for the small spread of aperture determination in Figures 4 and 5 obtained with multiparticle tracking.

5 REFERENCES

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