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Pion-Nucleus Interactions and the STAR Experiment at RHIC

Progress Report

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1 Introduction

This report summarizes the work carried out by personnel from the University of Texas at Austin at the Los Alamos Clinton P. Anderson Meson Physics Facility (LAMPF) during the calendar years 1990-1993 and on the STAR experiment at RHIC under grant DE-FG05-87ER40343 between the University of Texas at Austin and the United States Department of Energy.

One of our faculty and a research faculty member has been on site (at LAMPF) during five months or more of each year. An average of eight graduate students have been associated with this program, most being in residence at LAMPF.

The research activities involved experiments done with the Energetic Pion Channel and Spectrometer (EPICS), the Low Energy Pion Channel (LEP), the Pion and Particle Physics Channel (P³), and the High Resolution Spectrometer (HRS). In addition we have provided data visualization (via an adaptation of STARGL, see below) and simulation support to the development program associated with the new high-resolution Neutral Meson Spectrometer (NMS) at LAMPF.

A brief overview of work supported by this grant is given in Section 2. An account of the study (Experiments #1150, #1154, and #1209) of the double giant resonances in pion double charge exchange forms Section 3. This report contains a list of published papers and preprints in Section 6, invited talks in Section 7, and abstracts in Section 8. These papers summarize experiments involving participants supported by this grant and indicate the work accomplished by these participants in this program of medium energy nuclear physics research. Section 9 contains a list of personnel who have participated in this research program.

The research cited in this report is, in many cases, the collaborative effort of many groups associated with research at LAMPF. Collaboration with the Los Alamos National Laboratory, the University of Minnesota, the University of Colorado, the Louisiana State University, the Ben Gurion University of the Negev, the King Fahd University of Petroleum and Minerals, the Lawrence Livermore Laboratory, the New Mexico State University, Argonne National Laboratory, the University of Colorado, and the University of Pennsylvania is acknowledged.

Two Ph.D. degrees were granted by the University of Texas at Austin during 1992, based on experimental research done at LAMPF, bringing the total number of Ph.D.'s granted to Texas students supported by this grant in meson-nuclear physics to fourteen.

2 Overview

Our program concentrates on pion physics experimental results obtained using the Energetic Pion Channel and Spectrometer (EPICS), Pion and Particle Physics channel (P^3), and the Low Energy Pion physics channel (LEP). These facilities are unique in the world in their intensity and resolution. Two classes of experiments can be done best with this equipment: scattering (elastic and inelastic) and double charge exchange (DCX). Several coincidence experiments are in progress and several are proposed.

Inelastic pion scattering proposal #1025, which has been approved with A⁺ priority, deals with measurements of pion scattering from a polarized ^{13}C target. The asymmetry parameter A_y for $\pi^\pm$ elastic and inelastic scattering has been measured over an angular range of 30° to 100° from a polarized ^{13}C target at EPICS. The elastic-scattering results should be sensitive to spin-dependent effects in the pion-nucleus interaction, such as the Δ -nucleus spin-orbit interaction. The inelastic-scattering results will be sensitive to the relative mixture of spin-flip and non-spin-flip contributions to the transitions. These polarized target experiments will give new information about the pion-nucleus interaction that is not available from experiments with unpolarized targets, because the former measures the interference between amplitudes and the latter measures squares of amplitudes. The target system was assembled principally from polarized-target elements available at LAMPF. The experimental beam time requested is one running cycle, taken to be 1,200 hours.

In experiment #1104 cross-sections to the GQR for the nuclei ^{90}Zr and ^{208}Pb have been measured with π^+ and π^- at energies from 350 to 550 MeV, *i.e.*, above the $\Delta_{3/2,3/2}$ resonance region. The goal of this experiment is more accurate evaluation of neutron and proton transition amplitudes (M_n and M_p) extracted for GQR in those neutron excess nuclei. Those values of M_n and M_p obtained near the $\Delta_{3/2,3/2}$ resonance region have uncertainties in the wave functions involved and the shapes of the neutron and proton transition densities at the nuclear surface due to the short mean free path of the pion near the Δ resonance. The longer mean free path of the pion in these higher energy region may help to eliminate those uncertainties in extraction of the matrix elements.

Regarding proposal #1018, we have analyzed large-angle pion elastic data on ^{12}C and ^{16}O with standard first-order optical models using matter distributions deduced from electron

scattering. We show that these models reproduce only the qualitative features of the data. However, much better agreement can be achieved with the same models provided the matter distributions are modified in a suitable way. At this stage it is not clear whether these corrections are applicable to all nuclei or whether they apply at energies other than those on resonance. To address these questions we propose to measure elastic excitation functions (at 175°) on ^6Li , ^{28}Si , and ^{58}Ni in the energy range from 100 to 250 MeV.

Pion absorption: The LEP and P^3 channels at LAMPF will be used. In the low energy pion channel, we propose to measure the cross sections for π^+ and π^- absorption on ^{12}C , ^{58}Ni , and ^{208}Pb in which the final state includes two or more free protons. Data will be taken for π energies of 50, 100, 150, and 200 MeV. The energy and angular distributions of the outgoing protons are measured using a large solid-angle bismuth germanate BGO detector described in the section of 1991's proposal entitled "BGO BALL".

In the pion particle physics (P^3) channel, we measure the cross sections for π^+ and π^- absorption on ^{12}C , ^{58}Ni , and ^{208}Pb in which the final state includes two or more free protons. Data are taken for π energies of 200, 300, 400, and 500 MeV. The energy and angular distributions of the outgoing protons are measured using a large solid-angle BGO detector.

Among the most exciting results obtained recently at LAMPF in pion physics is the observation of resolved giant-dipole resonances built on the isobaric analog states and the isovector giant dipole resonance built on another giant dipole. The isobaric analog, and the giant dipole were both observed in (π^+, π^-) single charge exchange reaction (with a typical maximum cross section around $0.5 \rightarrow 1.0$ mb/sr). Therefore a combination of these resonances could, in principle, be reached in double charge exchange. However, until recently, none of these double resonances had been found.

The results obtained so far are exciting and have a potential of new and interesting physics. Experiments #1050 and #1118 have demonstrated that pion DCX provides a unique reaction in which one can study the existence of double giant resonances in nuclei. The results indicate that two new different modes of nuclear excitations are observable in pion double charge exchange reactions. These are the giant dipole built on the isobaric analog state and the giant dipole resonance built on the giant dipole resonance. Due to the selectivity of the DCX reaction these exotic states are observable although they are located at high excitation energy where background from DCX to the continuum is large. The data show that the newly discovered resonances are general collective features of all nuclei for the double-dipole, and of all nuclei with $N-Z \geq 1$ for the GDR built on the IAS. In experiment #1118 at the energetic pion channel and spectrometer (EPICS) at LAMPF, we observed a clear splitting of the giant dipole built on the isobaric analog state (GDR@IAS) in the $^{93}\text{Nb}(\pi^+, \pi^-)^{93}\text{Tc}$ reaction at $T_\pi = 295$ MeV. The results obtained in the first two research years of this program have been published (or submitted) in several scientific reports and invited talks at international conferences.

Experiment #1154 (which ran in two phases) was designed to accurately measure the width of the giant dipole built on the isobaric analog state on a wide range of mass and with higher statistics than previously obtained. The results indicate that the width of the GDR built on the IAS increases significantly with mass. For example, the resonance for ^{13}C has a width of 3.0 ± 0.6 MeV, but that for ^{59}Co has a width of 7.0 ± 1.0 MeV (more than twice as

wide as for ^{13}C). The increase of the width with mass is not yet fully understood, and is one of the puzzles arising from this study. An interesting implication was also discovered during this study. The resonance on the $T = 1/2$ targets has a single isospin value ($T = 3/2$) and is the analog of the $T_{\geq}$ GDR state in the target nucleus. Thus pion DCX provides a new general method for an accurate determination of the $T_{\geq} = T + 1$ giant dipole state on any $T = 1/2$ nucleus available as a target for DCX measurements.

Multiparticle final state in pion interactions: We studied the decay protons in the reaction $(\pi, \pi' p)$ as a test of charge symmetry. Comparisons of photo proton and photo neutron breakup of ^4He have provided evidence for a large violation of charge symmetry, caused by forces other than the Coulomb force. However, pion inelastic scattering data show that the deviation from charge symmetry can be explained by the Coulomb potential only. In an attempt to resolve this argument on the existence of charge symmetry breaking forces other than Coulomb interaction, a series of $(\pi, \pi' p)$ coincidence experiments on ^4He (#998), ^{12}C and ^{208}Pb (#1039) were conducted during the 1986-91 time period. The conclusion of these experiments is that there is no other force than Coulomb that breaks up charge symmetry, and the discrepancy between the result using electromagnetic probes and that using pions can be explained by direct-knock-out and semi-direct components, using the widely used Direct-Semidirect Model or Doorway-state Model.

Experiment #1103 aims to measure the ratio of $(\pi^+, \pi^{+'})$ vs $(\pi^-, \pi^{-'})$ on various nuclei to provide information on proton and neutron transition densities at energies around the Roper resonance where the relative strength of pure proton and pure neutron transitions should be reversed as compared to the $\Delta_{3/2,3/2}(1232)$ resonance case.

Experiment #1210 ran at LEP (July 1991). The experiment aimed to study the mechanism for DCX to excite analogs of anti-analog states in heavy nuclei. A previous, high statistics DCX experiment on ^{93}Nb showed for the first time the existence of a resonance at high excitation energy below the double isobaric analog state. The observation of the resonance with such a large cross section was unexpected. In a recent article we suggested a few possible identifications for this resonance. Experiment #1210 confirms the previous observation on ^{93}Nb and gives new results on ^{56}Fe and ^{44}Ca at a lower beam energy. The results have been published, and additional beam time have been approved for this experiment by the PAC.

Experiment #1209, run at EPICS, is designed to measure the energy dependence of the double giant resonance on ^{13}C and ^{51}V . The measurements provide the first attempt to determine the excitation function of a double resonance state. A pair of published papers resulted from this experiment, which show that the cross section of the giant dipole built on the isobaric analog state on ^{13}C and ^{51}V increases with energy as expected from simple sequential model calculations. The results indicate that pion double charge exchange involving the excitation of the closed-shell core proceeds predominantly by a two-step sequential $(\pi^+, \pi^0)(\pi^0, \pi^-)$ mechanism, and therefore should be a unique probe for exciting isotensor double giant resonances in nuclei.

In Section 3 we discuss in detail the results from Experiments #1150, #1151, and #1209 which ran at EPICS in 1991-1992.

3 Review of Double Giant Resonances

3.1 Introduction

Giant resonances are collective excitations of the nuclear medium induced by external fields that interact with the nuclei by the electromagnetic, weak, or hadronic interaction. They are a general feature of all nuclei and are one of the most thoroughly studied characteristics of nuclei[1]. Until recently, one could study giant resonances only as excitations of the nuclear ground state. An exciting development in nuclear physics in the past decade has been the observation of giant-dipole resonances in highly excited or “hot” nuclei, spinning with high angular momenta. These excitations were predicted by the Brink-Axel hypothesis early in the fifties[2]. The hypothesis assumes that the basic features of the giant resonances (*i.e.*, the frequency of oscillation and the strength of the resonance) are fundamental features governed by the nuclear force and are unaffected by any pre-excitation of the nucleus. Figure

Figure 1: Schematic nuclear energy-level diagram illustrating the Brink-Axel hypothesis[2]. The figure shows the ordinary nuclear states (left) and the giant-dipole excitations built on the ordinary nuclear states (right).

1 illustrates the Brink-Axel hypothesis for the case of giant-dipole excitations. The figure shows a schematic energy-level diagram of a nucleus. On the left side, the ground state and the low-lying ordinary states are labeled as $|gs\rangle$, $|1\rangle$, $|2\rangle$, $\dots$, $|n\rangle$. On the right, the giant-dipole excitations built on the gs and the ordinary nuclear states are labeled as $|D;gs\rangle$, $|D;1\rangle$, $\dots$, $|D;n\rangle$, where D is the giant-dipole operator that raises the energy of the state by $\hbar\omega_{dipole}$, and ω_{dipole} is the giant-dipole resonance frequency. Experimental observations to support this hypothesis were made possible only in recent years with the development of heavy-ion accelerators[3]. Results from these studies indicate some remarkable general features of the giant-dipole resonance in excited nuclei when compared with the regular giant dipole built on the nuclear ground state[4].

A special case occurs when the excited state on which the GDR is built is itself a giant resonance. In this case, a collective double giant resonance is excited (*i.e.*, a giant resonance built on another giant resonance). If such excitations do exist, they provide a new test of our understanding of the phenomena of collectivity in nuclei and the basic concepts of giant multipole resonances. The fundamental question is whether a collective mode (GDR or other collective excitations) can still be well formed in special conditions on top of another collective excitation. If they do exist, can we predict their basic features (*i.e.*, excitation energy, total width, and integrated strength) given the known properties of the single giant resonance? The notion of double giant resonances was first introduced theoretically by N. Auerbach, who pointed out the possible existence of these states[5], which provide new types of collective excitations in nuclei. From a nuclear structure point of view, it is important to understand their properties starting from both macroscopic and microscopic approaches. The macroscopic approach should describe the collective picture of the double giant resonances in terms of multi-nucleon states and the microscopic approach gives the wave functions of these exotic resonances in terms of two-particle, two-hole configurations. Double resonances in nuclei have not been observed until very recently because of experimental difficulties in observing a resonance located at very high excitation energy, superimposed on a large background from the continuum, and having a large width. Indeed, a reaction with high selectivity is required in order to observe a double resonance state. In this work, we describe the new experiments performed at LAMPF and other laboratories to search for double giant resonances in nuclei, the puzzles that have been solved, and the new puzzles that have arisen from these studies.

An "ideal" giant resonance state built on an excited state $|n\rangle$ can be written as:

$$|Q_\alpha; n\rangle = Q_\alpha |n\rangle \cdot |\langle n | Q_\alpha^\dagger Q_\alpha | n \rangle|^{-1/2} \quad (1)$$

where Q_α is a one-body operator obtained by taking a coherent sum of one-nucleon operators $q_\alpha(i)$,

$$Q_\alpha = \sum_{i=1}^A q_\alpha(i). \quad (2)$$

One of the fascinating cases is when $|n\rangle$ itself is a giant resonance state built on a ground state $|0\rangle$ then the state in eq. (1) will be a double giant excitation of the type,

$$|Q_\alpha; \alpha'\rangle = Q_\alpha Q_{\alpha'} |0\rangle \cdot N^{-1} \quad (3)$$

where N^{-1} is a proper normalization factor. If the multipole isovector operators, Q_α (and $Q_{\alpha'}$), are of the type:

$$Q_{L,\mu} = \sum_{i=1}^A r_i^L Y_L(\theta_i) t_\mu(i) \quad (4)$$

where $\mu = +, 0$, or $-$, then by taking $\mu = -$ and $L = 1$, we may write the double dipole state:

$$|D_{-1}; D_{-1}\rangle = \sum_{i=1}^A r_i Y_1(\theta_i) t_{-1}(i) |D_{-1}\rangle N^{-1} \quad (5)$$

This is a $\Delta T_z = -2$ double-dipole state observable in a (π^+, π^-) reaction. However, if we take $\mu = +$ and $L = 1$, we may write the model state:

$$|D_{+1}; D_{+1}\rangle = \sum_{i=1}^A r_i Y_1(\theta_i) t_{+}(i) |D_{+1}\rangle N^{-1} \quad (6)$$

This is the $\Delta T_z = +2$ double-dipole state, which can be reached in the (π^-, π^+) reaction and the results compared with the (π^+, π^-) data. In this work, we use the notation $|\text{GDR} \otimes \text{GDR}\rangle$ or $(\text{GDR})^2$ for the double dipole measured in both DCX modes. The simplest but most meaningful example of a double giant resonance is the well-known double isobaric analog resonance DIAS, which is the third ($T_z = T - 2$) member of a nuclear isospin multiplet. The DIAS may thus be characterized as two coexisting IASs. Another interesting mode is when $|n\rangle$ is the isobaric analog state; then the state in eq. (1) will be a giant dipole built on the isobaric analog state $|\text{GDR} \otimes \text{IAS}\rangle$. If we write the isobaric analog state as:

$$|A; \text{parent}\rangle = \sum_{i=1}^A \tau_{-}(i) (N - Z)^{-1/2}, \quad (7)$$

then in terms of the above operators, the giant dipole built on the isobaric analog state can be written as: $|\text{GDR} \otimes \text{GDR}\rangle$ can be written as:

$$|D_{-}; A\rangle = \sum_{i=1}^A r_i Y_1(\theta_i) \tau_{-}|A\rangle N^{-1}, \quad (8)$$

where $|A\rangle$ is given by equation (7). The $|\text{GDR} \otimes \text{GDR}\rangle$ has a multi-polarity of $L = 1$ and therefore for an even-even target nucleus the resonance has a single spin and parity value of 1^- . The DIAS and the $|\text{GDR} \otimes \text{IAS}\rangle$ are observable only in (π^+, π^-) , whereas the double giant dipole states [eqs. (5) and (6)] are in principle, observable in (π^+, π^-) and (π^-, π^+) DCX reactions.

In the simplest picture, $(\pi^\pm, \pi^\mp)$ pion double charge exchange (DCX) reactions are thought to proceed predominantly by a two-step $(\pi^\pm, \pi^0) \otimes (\pi^0, \pi^\mp)$ mechanism, and should be a unique probe for exciting double giant resonances in nuclei. Therefore, the $|\text{GDR} \otimes \text{IAS}\rangle$ and the $|\text{GDR} \otimes \text{GDR}\rangle$ provide an important phenomenon where the double-scattering mechanism can be investigated.

Figure 2 shows a schematic energy-level diagram of the prominent transitions observable in SCX and DCX. The lowest three double resonances expected in DCX are the DIAS, the giant dipole built on the isobaric analog [or vice versa $(\text{GDR} \otimes \text{IAS})$], and the double isovector giant-dipole resonance $(\text{GDR})^2$. Only the lowest isospin component of the higher two resonances is labeled. The Q values listed in the figure refer to the case of ^{93}Nb discussed later. An example of pion DCX as an excellent tool to study double resonances is the observation of the well-known DIAS transition[6, 7], which can be viewed in this context as the simplest double-resonance state. Until recently[8, 9], none of the higher double resonances had been observed. For target nuclei with $J^\pi = 0^+$, the $|\text{GDR} \otimes \text{IAS}\rangle$ is a single state with $J^\pi = 1^-$, where J is the total angular momentum and π is the parity of the state. On the other hand, the double dipole will have two possible spin values, 0^+ and 2^+ . Both resonances also have different isospin components, but for heavy nuclei with a large neutron excess ($N - Z \gg 1$), the lowest isospin component (see Fig. 2) is the dominant one.

Figure 2: Schematic energy-level diagram of “single” and “double” resonances anticipated in pion single-charge-exchange (π^+ , π^0) and pion double-charge-exchange (π^+ , π^-) reactions, respectively. The numbers to the right are the Q values for the ground state and the three double resonances observed in the DCX experiment on ^{93}Nb .

The purpose of this review article is to gather the various experimental results from pion DCX and from relativistic heavy-ion reactions that support the existence of double giant resonance in atomic nuclei. The identification of double resonances and the interpretation of the data will be emphasized. Theoretical works to predict the properties of these exotic excitations and future applications of this new phenomenon will be mentioned.

3.2 Single charge exchange reactions

3.2.1 The isobaric analog state

Two isovector transitions, the isobaric analog state (IAS) and the giant dipole resonance (GDR), have been strongly excited in pion SCX reactions. The IAS and the GDR have been studied intensively using the (π^+ , π^0) reactions and the GDR also using the (π^- , π^0) reaction, and both were found to be the most prominent features in the SCX spectra.

If the target isospin quantum numbers are T and $T_z = \frac{N-Z}{2} = T_0$, the IAS has quantum numbers $(T, T_0 - 1)$. No spin-space rearrangements in the nuclear wave function occur in the transition, and therefore the cross section is directly related to the hadronic interaction strength. In the simplest picture, the IAS transitions are characterized by a single-step scattering mechanism, and the transition takes place only on the valence neutrons. The most often used transition density for the IAS is:

$$\rho_{IAS}(\vec{r}) = \rho_n(\vec{r}) - \rho_p(\vec{r}) \quad (9)$$

In some cases, an additional simplification is made using:

$$\rho_n(\vec{r}) - \rho_p(\vec{r}) = \frac{N - Z}{A} \rho(\vec{r}), \quad (10)$$

where $\rho = \rho_n + \rho_p$.

Figure 3: Spectra measured for the (π^+, π^0) reaction at 98 MeV on various nuclear targets, and for the (π^-, π^0) reaction on a CH_2 target. The angular range is 0° – 10° [10].

Figure 3 shows typical spectra for the (π^+, π^0) reaction at 98 MeV on various nuclear targets, and for the (π^-, π^0) reaction on a CH_2 target at angular range 0° – 10° . The prominent peaks in the spectra are the isobaric analog transitions, which stand out clearly above the background at forward angles. The arrows indicate the expected locations of the IAS. The measured cross sections are proportional to the valence neutrons and follow the functional form $(N - Z)A^{-4/3}$ (see Ref. [10]).

Figure 4 shows some of the regularities of the $\theta = 0^\circ$ IAS cross sections[11]. The figure shows the $\theta = 0^\circ$ cross section plotted versus T_π for several nuclei. The line represents the center of mass cross section of the free charge-exchange reaction $\pi^+ + n \rightarrow \pi^0 + p$. A surprising feature emerged more recently from SCX data at energies above the $\Delta_{3/2,3/2}$

Figure 4: Excitation function of the $\theta = 0^\circ$ IAS cross section for ${}^7\text{Li}$ (full circles), ${}^{14}\text{C}$ (open circles), and ${}^{90}\text{Zr}$ (full triangles), ${}^{120}\text{Sn}$ (open triangles), and ${}^{208}\text{Pb}$ (open squares). The line represents the center of mass cross section of the free charge-exchange reaction $\pi^+ + n \rightarrow \pi^0 + p$, from Ref. [10].

resonance[11]. The data indicate that the energy dependence of the $\theta = 0^\circ$ cross section to the IAS above the $\Delta_{3/2,3/2}$ resonance is significantly different from the behavior of the free charge exchange process. An explanation to the data was given in terms of the Glauber model with absorption[12].

3.2.2 The giant dipole resonance

The GDR has been measured intensively in (π^+, π^0) and (π^-, π^0) , and found to be a prominent feature in all SCX spectra where giant resonances are measured. Figure 5 presents a comparison between the (π^-, π^0) and (π^+, π^0) spectra for ^{40}Ca at 165 MeV. The arrows represent the expected location of the giant dipole resonance. The data give a strong signal for the GDR[13]. There is a shift of 12 MeV in the energy of the GDR between the two reactions, which is exactly the expected energy shift due to the difference of two Coulomb energies minus the neutron-proton mass difference.

Since pion scattering at forward angles is dominated by $\Delta S = 0$ transitions, the dipole states observed in SCX measurements on even-even target nuclei were interpreted as $\Delta L = 1$, $\Delta S = 0$ transitions to 1^- states. This resonance is closely related (via isospin symmetry) to the electric giant dipole resonance, which is a prominent feature of the photo-nuclear spectra of all nuclei. The existing data from pion SCX at 120, 165, and 230 MeV are summarized in the work of A. Erell *et al.*[14] and Irom *et al.*[15]

3.3 Pion double charge exchange

In the simplest model, pion double charge exchange reactions are viewed as two sequential SCX processes in which two neutrons are changed into protons, or vice versa. For target nuclei with $N - Z \geq 1$, the (π^+, π^0) SCX process strongly excites the GDR and the IAS transitions. Therefore, with the high probability for the single-step excitations a combination of these resonances could, in principle, be reached in DCX. Figure 2 shows the isovector transitions and the energies involved in exciting these new double resonances as well as the previously well-known double isobaric analog state (DIAS). Similarly to the IAS, the DIAS is analogous to the original ground state except for changing two of the excess neutrons in the target nucleus to protons in the same orbitals and leaving all other nucleons unaffected by the reaction. The ground state of the parent nucleus, the IAS, and the DIAS are states that belong to the same isospin multiplet, and differ only in T_Z due to the change of one (IAS) or two (DIAS) neutrons into protons. A simple example is ^{18}O (gs), ^{18}F (1.04 MeV) IAS and ^{18}Ne (gs) DIAS.

The DIAS is a prominent transition at forward angles in the (π^+, π^-) reaction and has been studied intensively in pion DCX[7, 17, 18]. Figure 6 shows an example of the dominance of the DIAS in the $^{93}\text{Nb}(\pi^+, \pi^-)^{93}\text{Tc}$ reaction at $\theta_{lab} = 5^\circ$. This is true even when the DIAS is located highly in the continuum, as shown in the example of Fig. 6, where the DIAS appears at $E_x = 19.8$ MeV ($Q = -21.9$ MeV)[16].

Figure 5: Comparison of the (π^-, π^0) and (π^+, π^0) reactions for ^{40}Ca . The shift of 12 MeV in the GDR energy is the expected Coulomb displacement energy. The solid line is the smoothed 4° spectra contrasted with the 15° data, from Ref. [13].

Figure 6: Doubly differential cross-section spectrum for the (π^+, π^-) reaction on a ^{93}Nb target at $T_\pi = 295$ MeV and $\theta_{lab} = 5^\circ$. The larger peak to the right at $Q = -21.9$ MeV is the double isobaric analog state, and the smaller peak to the left at $Q = -17.4$ MeV is a new resonance[16].

3.4 Measurements

The measurements were performed at the Energetic Pion Channel and Spectrometer (EPICS) at the Los Alamos Meson Physics Facility (LAMPF) using a special set-up for pion DCX[19]. The EPICS spectrometer has superior background rejection and precise energy calibration and therefore was most suitable for these studies. The spectrometer can detect outgoing pions with a kinetic energy range of 100–300 MeV with an excellent energy resolution of about 140 keV, and is considered one of the best pion spectrometers in the world. Electrons and muons (arising from pion decay in the channel or within the spectrometer path length) were identified and rejected using a freon-gas Cherenkov detector and a set of scintillators placed behind a series of graphite blocks, respectively[20]. The choice of the highest beam energy available at EPICS ($T_\pi = 295$ MeV) for most of the measurements has several advantages: a) the background level from the continuum of the DCX reaction is lower since the excitation energy region of interest is pushed away from the inclusive DCX peak, b) a larger outgoing energy range (≈ 50 MeV) is covered by the acceptance of the spectrometer in a single setting, c) cross sections are expected to increase as k^2 or faster, where k is the incident pion momentum, and d) the highest energy was chosen because it is here that sequential processes dominate DCX. This is evidenced by the dominance of the DIAS at $\theta = 5^\circ$.

Elastic-scattering data have been taken at $T_\pi = 295$ MeV to determine the elastic peak shape and the total energy offset. The acceptance of the spectrometer was measured with a ^{12}C target using inelastic scattering to the 4.44-MeV state at an energy $T_\pi = 180$ MeV and a scattering angle of 35° , which corresponds to the peak cross section of the 2^+ state. The spectrometer field was varied to move the 4.44-MeV peak across the focal plane, covering approximately $\pm 10\%$ of the central momentum of the spectrometer. Absolute normalization was obtained using a polyethylene (CH_2) target to measure the hydrogen elastic cross section and comparing the yields with cross sections calculated from π -nucleon phase shifts[21].

3.5 The GDR built on the IAS

3.5.1 Typical Spectra

Figure 7 shows typical Q -value spectra obtained from pion DCX in a recent experiment at LAMPF to search for double resonances. Since π^+ and π^- have exactly the same mass, in the present reaction the negative of the Q value gives the amount of energy put into the nucleus to form a given state. The figure shows the results on three different targets: ^{13}C , ^{27}Al and ^{59}Co . The measurements were made at an incident pion kinetic energy of 295 MeV. All three spectra show clearly the existence of a wide resonance located in the continuum region at about 8.8 and 14.2 MeV above the ground state of ^{13}O and ^{27}P final nuclei, respectively. On the ^{59}Co target the resonance shows up at about 16.8 MeV above the DIAS in ^{59}Cu (27.9 MeV above the ground state of ^{59}Cu). The resonances are identified as the giant-dipole resonances built on the isobaric analog states or vice versa (GDR $\odot$ IAS). The identification is based on their energies, which are very close to the energy where the giant dipoles built on the isobaric analog states are expected to appear, on the characteristic dipole angular distribution of the resonance, and on the cross sections. Similar observations of the

Figure 7: Q -value spectra for (π^+, π^-) pion double-charge-exchange reactions at $T_\pi = 295$ MeV on: (a) ^{13}C at $\theta_{lab} = 18^\circ$, (b) ^{27}Al at $\theta_{lab} = 15^\circ$, and (c) ^{59}Co at $\theta_{lab} = 11^\circ$. The arrows indicate the fitted location of the ground state, the DIAS, and the giant resonance (GDR $\otimes$ IAS). Short vertical lines represent statistical uncertainties of the data. The dashed lines are the fitted background with a polynomial shape, and the solid lines are the fits to the spectra using a line-shape fitting code NEWFIT.

resonance have been confirmed on many other nuclei, indicating that this collective mode is a general feature of all nuclei that have at least one excess neutron[22]. The ground state, the DIAS, and the resonances were fitted with a standard peak shape. The background (dashed lines), which arises from DCX to the continuum, was fitted using a third-order polynomial shape. The energies and widths were varied simultaneously to minimize χ^2 for the entire fit. The figure shows clearly that the width of the GDR $\otimes$ IAS increases significantly with mass. For example, the resonance on ^{13}C has a width of 3.0 ± 0.6 MeV, but on ^{59}Co the resonance has a width of 7.0 ± 1.0 MeV (*i.e.*, more than twice as wide as that on ^{13}C). The increase of the width with mass is not yet fully understood. This is one of the puzzles arising from these studies. Further details on these resonances can be found in Ref. [22], where some systematic features of the GDR $\otimes$ IAS are outlined. The results indicate that in many respects (*e.g.*, energies, cross sections, and variations of the cross sections with mass), the GDR $\otimes$ IAS does behave as a superposition of the two-ingredient resonances, the GDR and the IAS.

3.5.2 Angular distributions

Figure 8 shows the ^{93}Nb Q -value spectra at three angles. In addition to the DIAS at $E_x = 19.3$ MeV ($Q = -21.9$ MeV), the spectra show the existence of a relatively wider peak located in the continuum region at about 13.9 MeV above the DIAS. This peak is labeled GDR $\otimes$ IAS (GR1) in the figure. The DIAS is strong at 5° but very weak at 10° and 20° , whereas GR1 has a maximum around 10° . The peaks were fitted with a Gaussian and a Lorentzian shape of variable widths for the DIAS and GR1, respectively. In Fig. 8 we show a fit to the spectra with $\Gamma_{DIAS} = 0.8$ MeV and $\Gamma_{GR1} = 5.8$ MeV. The spectra can be well fitted also using a Gaussian line shape for the GR, resulting in a smaller cross section by about 50 %. The width of the DIAS arises primarily from resolution energy due to the thick (1.714 g/cm^2) ^{93}Nb target. The spectra have been corrected for the spectrometer acceptance as a function of momentum. The background (dashed line), which arises from DCX to discrete low-lying states and to the continuum, was fitted using a third-order polynomial shape of the form $R(1) + R(2) \times E + R(3) \times E^2 + R(4) \times E^3$ where E is the absolute Q value. It was found that comparable fits and cross sections are obtained if, instead, fourth- or higher-order polynomial shapes are used for the background. The solid line is the resulting fit to the spectrum. The upper resonance in Fig. 8 labeled GDR $\otimes$ GDR will be discussed later.

Figure 9 presents the angular distribution extracted for GR1 on ^{93}Nb at 295 MeV. The maximum cross section is observed at 10° with a smaller cross section at 5° and 15° . At scattering angles of 20° – 25° the resonance is very weak and can hardly be observed, but at 30° the peak shows up again above background. The solid and dashed lines in Fig. 9 are simple calculations in which sequential single charge exchange through the isobaric analog to the giant dipole resonance is evaluated with two different transition densities for the IAS[22] and will be discussed later. The collective model has been used to obtain the radial shape of the transition densities for the dipole. The calculated cross sections using both models reproduce the dipole shape of the data and account surprisingly well for the measured cross sections.

Figure 8: Doubly differential cross-section for the (π^+, π^-) reaction on ^{93}Nb target at $T_\pi = 295$ MeV and $\theta_{\text{lab}} = 5^\circ, 10^\circ$, and 20° . The arrows indicate the three double resonances observable in pion DCX: (1) the DIAS, (2) the $\text{GDR} \otimes \text{IAS}$, and (3) the $\text{GDR} \otimes \text{GDR}$.

Figure 9: Angular distribution for the peak labeled GDR $\otimes$ IAS in the double-charge-exchange spectrum on ^{93}Nb shown in Fig. 8. The cross sections have been extracted with use of a Lorentzian line shape for the giant resonance with constant width of 5.8 MeV and constant $Q = -35.8 \text{ MeV}$ ($E_x = 33.2 \text{ MeV}$). The solid and chain dashed lines are sequential model calculations using volume and surface transition density, respectively.

3.5.3 Energies and widths of the GDR built on the IAS

Table 1: Results from the double-charge-exchange reaction for the GDR built on the IAS at an incident pion energy $T_\pi = 295$ MeV. Cross sections are at the listed laboratory angles.

Target	Experiment ^a				
	Q(DIAS) (MeV)	Q(GR) (MeV)	Γ (GR) (MeV)	θ_{lab} (deg)	$\frac{d\sigma}{d\Omega}$ (GR) ($\mu\text{b/sr}$)
¹³ C	—	$-27.7 \pm .5$	3.0 ± 0.6^b	18	$0.98 \pm .09$
²⁷ Al	—	$-29.6 \pm .5$	4.4 ± 0.9	15	0.59 ± 0.06
⁵¹ V	$-14.0 \pm .1$	$-32.4 \pm .2$	6.2 ± 0.9	11	2.70 ± 0.2
⁵⁶ Fe	$-15.6 \pm .1$	$-34.7 \pm .6$	7.1 ± 2.0	12	$1.15 \pm .19$
⁵⁹ Co	$-15.9 \pm .1$	-32.7 ± 1.1	7.0 ± 1.0	11	$2.11 \pm .17$
⁸⁰ Se	$-18.8 \pm .1$	$-33.7 \pm .6$	5.3 ± 2.5	5	$3.10 \pm .80$
⁹³ Nb	$-21.9 \pm .1$	$-35.8 \pm .5$	5.8 ± 1.0	10	$1.88 \pm .27$
¹¹⁵ In	$-25.0 \pm .1$	$-37.3 \pm .4$	4.6 ± 1.1	9	1.9 ± 0.3
¹³⁸ Ba	$-27.38 \pm .07$	$-38.4 \pm .4$	6.8 ± 1.6	5	$3.44 \pm .37$
¹⁹⁷ Au	$-35.0 \pm .1$	$-44.3 \pm .8$	5.4 ± 2.0	9	$2.90 \pm .3$
²⁰⁸ Pb	$-35.6 \pm .1$	$-45.8 \pm .8$	8.6 ± 1.9	5	8.61 ± 1.1

a) Cross sections, energies and widths for the GDR built on the IAS obtained by fitting the GR with a single Lorentzian peak with the listed widths. Data from Ref. [8] have been refitted under the same conditions.

b) After correcting for the resolution.

Table 1 summarizes the deduced energies, widths, and cross sections for the resonance labeled GDR $\odot$ IAS in Figs. 7 and 8 and on other nuclei studied so far. For $T \geq 1$ target nuclei, the $\Delta T_\pi = -1$ charge exchange isovector giant dipole and the GDR $\odot$ IAS split into three members shown in Fig. 10 (not to scale) with isospin $T-1$, T , and $T+1$. The energy splitting between the members $\Delta E^-(T, T-1)$ and $\Delta E^+(T+1, T)$ is a function of T and A and is determined by the isovector and isotensor energies[23, 24]. Table 1 lists the energies, widths and cross sections of the GR peaks when fitted as a whole with a single Lorentzian peak with the given width. For ¹³C and ²⁷Al the listed cross section is for the $(T+1)$ member, since the lower two components are forbidden from isospin considerations.

Table 2 lists the predicted intensity ratios for the GDR $\odot$ IAS members in DCX, derived from simple isospin arguments[25]. These ratios are quite different from the ratios expected for a GDR in SCX where the dipole is in a $T_\pi = T-1$ nucleus[23]. Simple isospin geometry arguments predict the following strength ratio for the GDR $\odot$ IAS components[22, 25]:

$$\begin{aligned} \sigma(T-1) : \sigma(T) : \sigma(T+1) = \\ (2T-1)(T-1)/T(2T+1) : \\ (2T-1)/T(T+1) : 3/(2T+1)(T+1) \end{aligned} \quad (11)$$

Table 2: Intensity ratios for isovector excitations in pion double charge exchange, assuming equal reduced matrix elements.

Target	$ T, T_z\rangle_i$	$ T, T_z\rangle_f$	Isospin members ^a		
			$ T_i - 1\rangle$	$ T_i\rangle$	$ T_i + 1\rangle$
¹³ C	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{3}{2}, -\frac{3}{2})$	—	—	1.0
²⁷ Al	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{3}{2}, -\frac{3}{2})$	—	—	1.0
⁵¹ V	$(\frac{5}{2}, \frac{5}{2})$	$(\frac{1}{2}, \frac{1}{2})$	0.40	0.46	0.14
⁵⁶ Fe ^b	(2,2)	(0,0)	0.30	0.50	0.20
⁵⁹ Co	$(\frac{5}{2}, \frac{5}{2})$	$(\frac{1}{2}, \frac{1}{2})$	0.40	0.46	0.14
⁸⁰ Se	(6,6)	(4,4)	0.72	0.26	0.03
⁹³ Nb	$(\frac{11}{2}, \frac{11}{2})$	$(\frac{7}{2}, \frac{7}{2})$	0.68	0.28	0.04
¹¹⁵ In	$(\frac{17}{2}, \frac{17}{2})$	$(\frac{13}{2}, \frac{13}{2})$	0.78	0.20	0.02
¹³⁸ Ba	(13,13)	(11,11)	0.85	0.14	0.01
¹⁹⁷ Au	$(\frac{39}{2}, \frac{39}{2})$	$(\frac{35}{2}, \frac{35}{2})$	0.81	0.18	0.01
²⁰⁸ Pb	(22,22)	(20,20)	0.91	0.08	0.003

a) Cross section ratios have been calculated[24] using the relations:

$$\sigma(T-1) : \sigma(T) : \sigma(T+1) = \frac{(2T-1)(T-1)}{T(2T+1)} : \frac{(2T-1)}{T(T+1)} : \frac{3}{(2T+1)(T+1)}$$

The T and $T+1$ members are further suppressed by Pauli blocking, not included in the listed ratios.

b) Using different reduced matrix elements taken from SCX on ⁶⁰Ni[14] (same isospin as ⁵⁶Fe) gives intensity ratios:

$$|T_i - 1\rangle : |T_i\rangle : |T_i + 1\rangle = 0.41 : 0.48 : 0.11.$$

Table 3: Theoretical and experimental peak cross sections for the $|GDR\otimes IAS\rangle$.

Target	θ_{cm}^{peak} (deg)	<i>Theory</i> ^a		<i>Experiment</i> ^b	
		Model I	Model II	$d\sigma/d\Omega_{peak}(GR)$ ($\mu b/sr$)	$\frac{d\sigma/d\Omega_{peak}(GR)}{(N-Z)NZ}$ (nb/sr)
		$d\sigma/d\Omega(GR)$ ($\mu b/sr$)	$d\sigma/d\Omega(GR)$ ($\mu b/sr$)		
¹³ C	18	0.82	0.83	0.98 $\pm$ 0.09	23.3 $\pm$ 2.14
²⁷ Al	15	0.48	0.41	0.59 $\pm$ 0.06	3.24 $\pm$ 0.33
⁵¹ V	11	2.12	1.36	2.70 $\pm$ 0.20	0.84 $\pm$ 0.06
⁵⁶ Fe	11	1.70 ^c	1.28	1.29 $\pm$ 0.21	0.41 $\pm$ 0.07
⁵⁹ Co	11	1.68	1.08	2.11 $\pm$ 0.17	0.49 $\pm$ 0.04
⁸⁰ Se	10	3.13	2.24	4.15 $\pm$ 0.98	0.22 $\pm$ 0.05
⁹³ Nb	10	2.56	1.84	1.88 $\pm$ 0.27	0.10 $\pm$ 0.015
¹¹⁵ In	9	4.08	2.29	1.90 $\pm$ 0.30	0.04 $\pm$ 0.005
¹³⁸ Ba	9	5.95	3.32	4.20 $\pm$ 0.45	0.035 $\pm$ 0.004
¹⁹⁷ Au	9	2.49	1.19	2.90 $\pm$ 0.30	0.008 $\pm$ 0.0008
²⁰⁸ Pb	9	12.93	6.29	10.50 $\pm$ 1.34	0.023 $\pm$ 0.003

- a) Sequential model calculations using volume transition density (Model I) and surface transition density (Model II) for the isobaric analog state, normalized to the pion single-charge-exchange data.
- b) Peak cross sections obtained by normalizing the 5° data points from Table 1 to the calculated sequential model curve using NEWCHOP, and corrected for ⁵⁶Fe and ⁵⁹Co for isospin components.
- c) Corrected for the $|T_i + 1\rangle$ isospin component [12% for ⁵⁶Fe (footnote b of Table 2)]. We assume that the cross sections listed in Table 1 (Column 6) include predominantly the $|T_i - 1\rangle$ and $|T_i\rangle$ components. For the heavy nuclei ($A \geq 80$) the $|T_i + 1\rangle$ component is negligible.

Figure 10: Schematic energy-level diagram of analog and dipole states observed in single and double charge exchange, compared with the E1 resonance in the target from photo-nuclear reactions. The listed Q values refer to the case of DCX on ^{138}Ba .

The different ratios between the $\Delta T_z = -1$ GDR and the $\Delta T_z = -2$ GDR $\otimes$ IAS arise from the fact that the transition induced by the isospin lowering operator are T -dependent and favor the upper two components of the multiplet. Table 2 shows the expected ratios for the GDR $\otimes$ IAS assuming equal reduced matrix elements for the three states. We note, however, that for both SCX and DCX reactions on neutron-rich nuclei, the transition to the lowest isospin member of the multiplet is dominant and the higher two components are generally negligible. The T and $T + 1$ components are further suppressed by Pauli blocking which is not included in the calculated intensity ratios. Table 3 lists the predicted cross sections for the GDR $\otimes$ IAS from a simple sequential model calculation, using the pion coupled-channels impulse-approximation (CCIA) code NEWCHOP[26]. The calculations include the ground state, the GDR, the IAS, and the GDR $\otimes$ IAS. The strength of each SCX transition has been adjusted to reproduce the measured (or extrapolated) SCX cross section to the GDR and the IAS[14]. A collective transition density was used for the dipole. For the IAS we used two different transition densities — one volume (model I) and one surface peaked (model II). For ^{13}C and ^{27}Al the calculations include the isospin factor for $T = 1/2$ nuclei, discussed later. The calculations using both models reproduce the measured cross sections for all targets reasonably well without any normalization factor. The calculations using model II for the IAS, which provide a more realistic shape for ρ_{exc} density, give quantitatively a good description of the measured cross sections. The calculations with model I overpredict, in some cases, the measured cross sections for the GDR $\otimes$ IAS. More details on these calculations are given in Refs. [27] and [22]. The experimental cross sections listed in Table 3 are the measured or the estimated peak cross sections obtained by normalizing the 5° data points to the theoretical curves. They also have been normalized upward to account for the unobserved

T_z isospin component for ^{56}Fe and ^{59}Co using the ratios listed in Table 2.

Figure 11: (a) Giant-dipole resonance energies derived from DCX (corrected for the symmetry energy) multiplied by $A^{1/3}$ plotted versus mass number A . The line is the best constant fit to the data for $A \geq 80$. (b) Same as (a) except for the GDR energies derived from DCX are multiplied by $A^{1/6}$. The line is a fit to all eight data points.

The energy of the $T = T_0 - 1$ member of the $\Delta T_z = -1$ GDR (where T_0 is the isospin of the target nucleus) is related by symmetry energy to the “normal” ($T = T_0$) GDR in the target nucleus observed, for example, in photo-nuclear reactions. Figure 10 illustrates (for target nuclei with $T > 1$) the relation between the dipole energy in inelastic or photo-nuclear reactions and the charge-exchange modes. For $T = 1$ target nuclei the lowest member of the GDR@IAS is forbidden, and for $T = \frac{1}{2}$ only the highest ($T = \frac{3}{2}$) member is allowed. Table 4 gives a comparison between E_{GDR} derived from DCX (Column 6) and the corresponding energies measured or extrapolated from photo-nuclear reactions (Column 9). The table indicates that the excitation energies of the GR above the DIAS plus the symmetry energy (ΔE^-) are in close agreement with the electric giant dipole energies, and can be well represented with either an $A^{-1/3}$ or $A^{-1/6}$ dependence (times a constant 76.6 ± 1.5 and 35.0 ± 0.6 MeV, respectively). These systematics are shown in Fig. 11 and give strong support to the identification of the GR peaks observed in DCX as giant-dipole oscillations built on the isobaric analog as intermediate states. It is known from photo-nuclear reactions that $A^{1/3}$ is too strong and $A^{1/6}$ is too weak an A dependence for E1 energies. Figure 11 shows that the GDR energies derived from DCX are also consistent with this feature. In the cases of ^{13}C and ^{27}Al we deduce the E1 energy by subtracting $2(\Delta E_C - \delta)$ from the measured Q value

Table 4: Giant dipole energies derived from DCX compared with results from photo-nuclear reactions.

Target	$Q_{T_<}(\text{GR})^a$ (MeV)	$\Gamma_{T_<}(\text{GR})^b$ (MeV)	δE_x^c (MeV)	ΔE^-^d (MeV)	$E_{DCX}^{GDR^e}$ (MeV)	$E_{DCX}^{GDR} \times A^{\frac{1}{3}}$ (MeV)	$E_{DCX}^{GDR} \times A^{\frac{1}{6}}$ (MeV)	$E_{photo}^{GDR^f}$ (MeV)
^{13}C	—	—	—	—	24.0 ± 0.5	56.4 ± 1.2	36.8 ± 0.8	23.8^g
^{27}Al	—	—	—	—	21.0 ± 0.5	62.7 ± 1.5	36.2 ± 0.9	21.4
^{51}V	-29.8 ± 0.2	3.0 ± 0.5	19.1 ± 0.4	2.5	19.1 ± 0.4	70.8 ± 1.5	36.8 ± 0.8	18.98
^{56}Fe	-31.8 ± 0.6	3.5 ± 1.4	16.2 ± 0.7	1.8	18.0 ± 0.6	68.8 ± 2.3	35.2 ± 1.2	17.6^h
^{59}Co	-29.8 ± 0.5	3.7 ± 1.0	15.0 ± 0.6	2.1	17.2 ± 0.6	67.0 ± 2.3	33.9 ± 1.2	17.2^h
^{80}Se	-33.3 ± 0.5	4.1 ± 2.2	14.5 ± 0.7	3.5	18.0 ± 0.7	77.6 ± 3.0	37.4 ± 1.5	17.1^i
^{93}Nb	-35.8 ± 0.5	5.8 ± 1.0	13.9 ± 0.6	2.7	16.6 ± 0.6	75.2 ± 2.7	35.3 ± 1.3	16.6
^{115}In	-37.3 ± 0.4	4.6 ± 1.1	12.3 ± 0.4	3.2	15.5 ± 0.4	75.4 ± 1.9	34.2 ± 0.9	15.6
^{138}Ba	-38.4 ± 0.4	6.8 ± 1.6	11.0 ± 0.5	3.6	14.6 ± 0.6	75.4 ± 3.1	33.2 ± 1.4	15.3
^{197}Au	-44.3 ± 0.8	5.4 ± 2.0	9.3 ± 1.5	2.5	11.8 ± 1.5	68.7 ± 8.7	28.5 ± 3.6	13.8
^{208}Pb	-45.8 ± 0.8	8.6 ± 1.6	10.3 ± 0.9	2.2	12.5 ± 0.9	74.1 ± 5.3	30.4 ± 2.2	13.5

a) Energy of the $T_< = T_0 - 1$ component of the GDR $\otimes$ IAS for ^{51}V , ^{56}Fe , ^{59}Co , and ^{80}Se where it is resolved. Energy of the whole resonance for the heavier nuclei (^{93}Nb , ^{115}In , ^{138}Ba , ^{197}Au , and ^{208}Pb).

b) Same as a) except for widths.

c) $\delta E_x = Q(\text{DIAS}) - Q_{T_<}(\text{GR})$, (see Fig. 10).

d) $\Delta E^- = T[E_v - (2T+3)E_t]$; $E_v = 50/A$, and $E_t = -0.003$ (Ref. [25]). We note that for ^{56}Fe this relation gives a smaller value for ΔE^- than that observed in Ref. [22].

e) $E_{DCX}^{GDR} = \delta E_x + \Delta E^-$.

f) E1 energies generally include a combination of $E_<$ and $E_>$ (Ref. [28]).

g) Centroid energy of the $T_>$ component of the GDR (Ref. [29]).

h) E1 on ^{65}Cu (Ref. [28]), scaled by $A^{1/3}$ (see also Ref. [30]).

i) E1 on ^{nat}Rb (Ref. [28]), scaled by $A^{1/3}$.

for the GR, where we have used $\Delta E_C = 1.44 \times (Z_\zeta + \frac{1}{2})/A^{\frac{1}{3}} - 1.13$ MeV for the Coulomb displacement energy[31] and $\delta = m_n - m_p$. This procedure yields $E1(T_\zeta) = 24.0 \pm 0.5$ MeV for ^{13}C . This is in good agreement with the centroid energy of 23.8 MeV for the $T_\zeta = \frac{3}{2}$ component of the GDR on ^{13}C reported in the $^{13}\text{C}(\gamma, p)$ reaction[29]. In ^{27}Al the T_ζ GDR is not well assigned. Our measurements give $E1(T_\zeta) = 21.0 \pm 0.5$ MeV, in agreement with the highest member (at 21.4 MeV) of the structure observed in the GDR region in the $^{27}\text{Al}(\gamma, n)$ reaction[32].

Figure 12: Widths of the lowest member of the giant dipole resonance built on the isobaric analog state (Γ_{T_ζ}) divided by A plotted versus mass number, A (see text).

Figure 12 shows a plot of the ratio Γ_{T_ζ}/A , where Γ_{T_ζ} is the width of the lowest isospin member of the GDR⊗IAS for $A \leq 80$ (in which the T_ζ component is resolved) and the whole GR width for $A > 80$, where the resonance is dominated by the lowest isospin component. These widths are listed in Column 3 of Table 4 as a function of A . The figure indicates that for $A \geq 56$, the width of the lowest component of the GDR⊗IAS is approximately linear with A , and can be represented by $\Gamma_{T_\zeta}(\text{GR}) = (0.048 \pm 0.004)A$ MeV.

3.5.4 The $|\text{GDR} \otimes \text{IAS}\rangle$ on $T = 1/2$ light nuclei

The $T = 1/2$ nuclei provide a special case since in these cases only the upper member ($T = 3/2$) is allowed from isospin arguments, and therefore the $|\text{GDR} \otimes \text{IAS}\rangle$ provides an accurate tool to determine the T_ζ component of the GDR in the target nucleus. Figure 7 shows the missing-mass spectra obtained from the DCX reaction on two $T = 1/2$ nuclei, ^{13}C ,

and ^{27}Al . A suppression of the cross section for the GDR built on the IAS on the $T = 1/2$ nuclei (by a factor of 2—3) is expected due to isospin considerations. Since pion DCX is a probe causing T_z to change by two units, the resonance has a single isospin $T_z = 3/2$ and can be reached only through the IAS and the T_z component of the GDR (Fig. 13). Therefore the coupling through the GDR has only 1/3 of its normal strength. For a heavier nucleus

Figure 13: Schematic energy-level diagram of isospin fragments of dipole and analog-dipole states observed in single and double charge exchange on $T = 1/2$ nuclei compared with E1 from photo-nuclear reactions. The figure shows the $T = 3/2$ multiplet of the giant dipole in the $T_z = 1/2, -1/2, -3/2$ nuclei, and illustrates the absence of the T_z $|GDR \otimes IAS\rangle$ state. The listed Q values refer to the case of DCX on ^{13}C .

with $T \geq 1$ the $GDR \otimes IAS$ will arise by coupling through both the IAS and the full GDR intermediate states. We note, however, that for $T = 1/2$ nuclei the $GDR \otimes IAS$ may gain some extra strength through the IAS coupling, since in these cases the IAS is also the ground state of the intermediate nuclei.

Figure 14 presents the partial angular distributions extracted for the resonances on ^{13}C , ^{27}Al , and ^{59}Co . The maximum cross section is observed at 18° for ^{13}C , 15° for ^{27}Al , and 11° for ^{59}Co . The solid and dashed lines are the sequential model calculations using volume and surface peaked transition density for the $|IAS\rangle$, respectively. The calculations for all three targets are shown without any normalization factor.

The giant dipole in non-self-conjugate nuclei has two isospin components, $T_z = T$ and $T_z = T + 1$, where T is the isospin of the target nucleus (Fig. 13). The energy splitting, $\Delta E = E_z - E_{-z}$, is given approximately by $\Delta E = U(T + 1)/A$, where U is the symmetry energy[33]. For example, if we use $U = 50$ MeV, then $\Delta E = 5.8$ and 2.8 MeV for ^{13}C and ^{27}Al , respectively. Generally, it is difficult to measure the energy centroids of the two isospin components of the $|GDR\rangle$, since the resonances are wide and fragmented (especially in light nuclei). Additional difficulties occur in deformed nuclei, where deformation splitting of the $|GDR\rangle$ may obscure the T_z assignment of the observed structure. Furthermore, the presence of the giant quadrupole in this energy region adds to the complexity of the assignments. A widely used method to determine T_z and T_{-z} is to measure both photo-neutron and photo-proton reactions[29, 33]. If isospin is conserved, both members can decay via proton emission,

Figure 14: Partial angular distributions for the peaks labeled GDR $\oplus$ IAS in the double-charge-exchange spectra on ^{13}C , ^{27}Al , and ^{59}Co shown in Fig. 7. The solid and chain dashed lines are sequential model calculations with volume and surface transition densities, respectively, without any normalization factor.

but in the absence of mixing the neutron decay from the $T_{\frac{1}{2}}$ state to the low-lying $T = 1/2$ states in the residual nucleus is forbidden. Therefore a comparison of the two reactions usually helps in measuring the position of $T_{\frac{1}{2}}$. Thus, for $T = 1/2$ nuclei, pion DCX may provide a new powerful tool in determining the upper-isospin component of the (GDR) by measuring its analog state in the $T_{\frac{1}{2}} = T - 2$ nucleus.

3.5.5 Mass dependence of the GDR built on the IAS

We now address the A dependence of the cross section of the GDR@IAS in view of the existing knowledge on the GDR and the IAS from pion SCX reactions. The isobaric analog state has been studied extensively using the (π^+, π^0) charge exchange reaction[10, 11, 34]. Figure 15 shows some of the regularities of the 0° IAS cross sections at 165 and 300 MeV. The data could be well represented[11] by

$$\frac{d\sigma}{d\Omega_{IAS}}(0^\circ) = g_1(N - Z)A^{-\alpha}, \quad (12)$$

where the $(N - Z)$ arises from the fact that the reaction occurs only on valence neutrons. The $A^{-\alpha}$ is due to pion attenuation and is energy dependent. At 165 MeV, α is 1.36, but it decreases as one moves away from the $\Delta_{3/2,3/2}$ resonance region where pion absorption effects are weaker. At 300 MeV, the best representation to the data is given by[11]:

$$\frac{d\sigma}{d\Omega_{IAS}}(0^\circ) = (18.0 \pm 1.7)(N - Z)A^{-1.00} \text{ mb/sr}. \quad (13)$$

The $S = 0$ $\tau_- \{\tau_+\}$ mode of the GDR has been observed to be strongly populated in the (π^+, π^0) $\{(\pi^-, \pi^0)\}$ charge exchange reaction[14, 15]. Figure 16 shows the existing (π^+, π^0) SCX data to the GDR at 165 MeV. Except for the GDR on the ^{40}Ca target, which has only one ($T = 1$) isospin member, the GDR cross sections on the heavier targets ($N \neq Z$) (shown in Fig. 16) include all three isospin members ($T - 1$, T , and $T + 1$) which are not resolved with the π^0 spectrometer[14]. These three members have different reduced matrix elements due to Pauli blocking, which favors the lowest member. Our best fit to the summed GDR cross sections gives:

$$\frac{d\sigma}{d\Omega_{GDR}}(peak) = (1606 \pm 27) \frac{NZ}{A^{1.84}} \text{ } \mu\text{b/sr}, \quad (14)$$

where we have included the expected NZ factor. This relation is in close agreement with calculations for the $T = T_0$, $\Delta S = 0$ GDR in the strong absorption approximation, which gives[35]:

$$\frac{d\sigma}{d\Omega_{GDR}}(\Delta S = 0) \simeq \frac{NZ}{A^2}. \quad (15)$$

Both eq. (14) derived from the data and eq. (15) derived from theory imply a very weak A dependence at the maximum cross section since $NZ \simeq A^2$. Since no SCX data exist for the GDR at 295 MeV, we calculated the GDR at this energy using the code NEWCHOP[26] with dipole transition strengths adjusted to give the experimental peak cross sections at 165 MeV. This method was tested by calculating the energy dependence of the SCX cross section

Figure 15: The A dependence of 0° SCX cross sections for the isobaric analog state at 165 and 300 MeV. The lines represent fits of the function $g_1(N - Z)A^{-\alpha}$ to the data. The data are taken from Refs. [11] and [22].

Figure 16: Normalized GDR peak cross sections obtained in the (π^+, π^0) reaction at 165 MeV as a function of A . The data are taken from Ref. [14]. The solid line is a fit at 165 MeV, eq. (14), and the dashed line is a theoretical calculation for the DGR at 295 MeV, eq. (16) (see text).

for the GDR and is discussed in Ref. [22]. The calculated GDR cross sections at 295 MeV (dashed line) are well represented by:

$$\frac{d\sigma}{d\Omega_{GDR}}(peak) = 4496 \frac{NZ}{A^{1.85}} \mu\text{b/sr}. \quad (16)$$

Thus, except for a scaling factor (of about 2.8), the calculations at 295 MeV predict virtually the same A dependence for the GDR as is experimentally observed at 165 MeV. The increase in the GDR cross section with energy is due to a k^2 factor, where k is the incoming pion momentum, and is discussed in Ref. [15].

If we use the E1 classical sum rule for the dipole[36]:

$$S(E1) = 14.8 \frac{NZ}{A} = |M(E1)|^2 E^{-1} e^2 fm^2 \text{ MeV}, \quad (17)$$

where $|M(E1)|^2$ is the dipole transition strength, then in similarity to the IAS, one can factor the GDR cross section as strength times an attenuation factor:

$$d\sigma/d\Omega_{GDR} \approx |M(E1)|^2 \times A^{-\alpha} = 14.8 \frac{NZ}{AE} A^{-\alpha}. \quad (18)$$

Then, if the dipole energy varies as $A^{-\frac{1}{4}}$, results of eq. (16) together with eq. (18) give $\alpha = 1.18$. This result is in close agreement with the pion attenuation observed at this energy for the IAS, eq. (13).

The existing data on the GDR@IAS consist of the eleven data points listed in Table 1. The experimental peak cross sections (Table 3) are well represented (Fig. 17) by the function[37]:

$$\frac{d\sigma}{d\Omega_{GDR@IAS}}(peak) = 38690(N - Z) \frac{NZ}{A^{2.88}} \text{ nb/sr}. \quad (19)$$

The ^{13}C and ^{27}Al data points are also included in the present fit (solid line). The nuclei ^{13}C and ^{27}Al have $T = 1/2$ and therefore the |GDR@IAS) is expected to be suppressed due to isospin considerations as, illustrated in Fig. 13. However, the measured cross sections for the above nuclei[27] are quite large and are well represented by the global power-law fit displayed in Fig. 17.

The result of eq. (19) is interesting in view of the regularities for the IAS and the GDR pointed out earlier. It is likely that the $(N - Z)$ arises from the IAS transition, eq. (12), and the NZ factor from the GDR form factor, eq. (14). One can similarly break the cross section for the GDR@IAS as the product of the strengths of the first and second step times an overall attenuation factor for DCX:

$$\frac{d\sigma}{d\Omega_{GDR@IAS}} \approx (N - Z) \frac{NZ}{A^{2/3}} A^{-\alpha'}. \quad (20)$$

A comparison of eqs. (19) and (20) yields $\alpha' = 2.21$. This is an interesting result which indicates that for pion DCX, where three pion propagations are involved, α' is about twice larger than for SCX (leading to the GDR or the IAS) in which only two pion propagations

Figure 17: Plot of normalized cross section versus A for the giant dipole built on the isobaric analog state (Table 3, Column 6) measured in DCX reactions at 295 MeV. The line is a power-law fit to the data, eq. (19).

are involved in the reaction. Equation (20) indicates that the GDR $\otimes$ IAS has a simple form for the A -dependence of the cross section which incorporates both the GDR and the IAS systematics. The excitation function measurement (described in Section 7.1) gives further support for the sequential mechanism for the excitation of the GDR $\otimes$ IAS in this energy region. Theoretically, this issue is still open and needs to be addressed.

3.6 The double isovector giant dipole resonance

3.6.1 Experimental results

We now turn to the observation of the double dipole. Figure 8 presents Q -value histograms measured for the $^{93}\text{Nb}(\pi^+, \pi^-)^{93}\text{Te}$ reaction at 295 MeV at three different scattering angles. These spectra were taken in long DCX measurements in order to accumulate good statistics. They also cover a high excitation energy. The DIAS appears at $Q = -21.9$ MeV and is most clear in the 5° spectrum. In addition to the DIAS, two wider peaks are apparent in the spectra. The peaks are labeled GDR $\otimes$ IAS (GR1) and GDR $\otimes$ GDR (GR2). The first appears at about $Q = -35.8$ MeV and the latter at -49.9 MeV. The fit shown uses $\Gamma_{DIAS} = 0.8$ MeV, $\Gamma_{GR1} = 5.8$ MeV, and $\Gamma_{GR2} = 8.8$ MeV. At 20° GR1 vanishes but shows up clearly at 10° . The [GDR $\otimes$ IAS] has been discussed in section 5. The peak labeled GDR $\otimes$ GDR has a different angular distribution. Full angular distributions for the resonances have been measured on ^{40}Ca , ^{93}Nb , and ^{56}Fe (Ref. [38]). Detailed analysis shows that the two resonances have different natures since they have a significantly different angular dependence. GR1 has a dipole shape and corresponds to the GDR $\otimes$ IAS discussed earlier. The higher resonance GR2 has a quadrupole distribution. Based on its energy (which is about twice the energy of the one-step GDR), angular distribution, and cross section, we identify GR2 as the double giant dipole[38]. The double dipole has a width of about 8–10 MeV, which is larger than the width of the “single” dipole by a factor of 1.5 to 2.0. This was found to be in good agreement with theoretical estimates for the width of the double dipole in terms of two phonon states[38, 39, 40]. A unique feature of the pion DCX reaction, discussed later, is the simplicity with which one can study the inverse reaction [*i.e.*, the (π^-, π^+) instead of the (π^+, π^-) reaction] on the same target nucleus.

3.6.2 Energies and transition strengths

Table 5 summarizes the experimental results obtained for the DIAS, the giant dipole built on the IAS and the double giant dipole from several experiments. For heavy nuclei a simple relation is expected between the three double resonances observed in DCX, arising from the addition of one- and two-giant-dipole energies as illustrated in Fig. 2:

$$|Q_{\text{GDR}\otimes\text{IAS}}| - |Q_{\text{DIAS}}| = |Q_{\text{GDR}}| - |Q_{\text{GDR}\otimes\text{IAS}}| \quad (21)$$

or equivalently,

$$|Q_{\text{GDR}\otimes\text{IAS}}| = (1/2)(|Q_{\text{GDR}}| + |Q_{\text{DIAS}}|). \quad (22)$$

Figure 18: (a) Schematic energy-level diagram of dipole and double-dipole states observable in single- and double-charge-exchange reactions. The listed Q values refer to the case of DCX on ^{40}Ca . (b) Same but for ^{27}Al ($T = 1/2$) nucleus. The figure shows also the isospin components reached in the $\Delta T_z = -1$ SCX and $\Delta T_z = -2$ DCX reactions. The listed Q values are the experimental results from the present study.

Table 5: Results from the (π^+, π^-) double-charge-exchange reaction at $T_\pi=295$ MeV.

Target	DIAS	GDR $\oplus$ IAS			GDR $\oplus$ GDR		
	$-Q$ (MeV)	$-Q$ (MeV)	Γ (MeV)	$d\sigma/d\Omega^{peak}$ ($\mu\text{b}/\text{sr}$)	$-Q$ (MeV)	Γ (MeV)	$d\sigma/d\Omega^{peak}$ ($\mu\text{b}/\text{sr}$)
^{12}C	...	...	...	...	48.4 ± 2.0	10.0 ± 3.0	0.86 ± 0.18
^{13}C	...	27.7 ± 0.5	3.0 ± 0.6	0.98 ± 0.9	46.5 ± 2.0	12.0 ± 3.0	3.2 ± 0.5
^{27}Al	...	29.6 ± 0.5	4.4 ± 0.9	0.59 ± 0.06	49.1 ± 0.5	8.4 ± 2.0	2.4 ± 0.2
^{32}S	...	...	...	...	49.6 ± 2.0	9.0 ± 2.0	0.98 ± 0.31
^{40}Ca	...	...	...	...	54.0 ± 0.5	9.0 ± 1.4	2.6 ± 0.2
^{51}V	14.0 ± 0.1	32.4 ± 0.2^a	6.2 ± 0.9	2.7 ± 0.2	48.0 ± 0.4	8.0 ± 2.0	3.4 ± 0.8
^{56}Fe	15.6 ± 0.1	34.7 ± 0.6	7.1 ± 2.0	1.15 ± 0.19	54.4 ± 0.6	10.0 ± 1.5	3.12 ± 0.31
^{60}Co	15.9 ± 0.1	32.7 ± 1.1	7.0 ± 1.0	2.11 ± 0.17	48.6 ± 0.8	8.2 ± 2.0	4.05 ± 0.75
^{80}Se	18.8 ± 0.1	33.7 ± 0.6	5.3 ± 2.5	3.40 ± 8.0	...	...	...
^{93}Nb	21.9 ± 0.1	35.8 ± 0.5	5.8 ± 1.0	1.88 ± 0.27	49.9 ± 0.8	8.8 ± 2.6	3.48 ± 0.47
^{115}In	25.0 ± 0.1	37.3 ± 0.4	4.6 ± 1.1	1.9 ± 0.3	49.6 ± 0.8	8.5 ± 3.0	3.6 ± 0.8
^{138}Ba	27.38 ± 0.7	38.4 ± 0.4	6.8 ± 1.6	3.44 ± 0.37	49.8 ± 0.8	8.5 ± 2.6	4.15 ± 0.52
^{197}Au	35.0 ± 0.1^b	44.3 ± 0.8	5.4 ± 2.0	2.9 ± 0.3	54.9 ± 0.8	9.0 ± 3.0	5.5 ± 1.6
^{208}Pb	35.6 ± 0.1	$45.8 \pm$	8.6 ± 1.9	8.61 ± 1.1	...	...	...

a) Fitting the GDR $\oplus$ IAS on ^{51}V with three peaks gives: $E_1=29.8 \pm 0.2$ MeV and $\Gamma_1=3.0 \pm 0.5$ MeV; $E_2=33.1 \pm 0.4$ MeV and $\Gamma_2=4.0 \pm 1.2$ MeV; and $E_3=36.4 \pm 0.5$ and $\Gamma_3=2.5 \pm 1.5$ MeV.

b) The value for the DIAS is from a separate measurement at $T_\pi=295$ MeV and $\theta_{lab}=5^\circ$, see Ref. [21].

The experimental energies (Table 5) are in close agreement with the above simple relation, which gives additional support for the identification of these phenomena as double resonances. Experimentally, the single-charge-exchange GDR is observed in (π^+, π^0) around $Q = -25$ MeV for a wide range of mass. Thus, the energies of the double dipoles observed in (π^+, π^-) are close to double the energy of the “single” GDR, *i.e.*,

$$Q_{\pi^+, \pi^-}(\text{GDR}^2) \approx 2 \times Q_{\pi^+, \pi^0}(\text{GDR}) \approx -50 \text{ MeV} . \quad (23)$$

This near equality is surprising because it neglects the effect of isospin splitting, which exists in both the (π^+, π^0) and the (π^+, π^-) reactions, but is somewhat different in the two reactions. Table 6 presents a comparison between the deduced energies, widths, and cross sections in (π^+, π^-) and (π^-, π^+) double-charge-exchange reactions on several nuclei compared with the available data from single-charge-exchange on the same or close nuclei. Recent calculations of the double-dipole energies based on the random-phase approximation method generalized to finite temperatures (TDRPA) give a good agreement with the experimental results. These calculations[41] view the double-dipole excitation as a single E1 phonon vibration built on a hot nucleus whose temperature corresponds to the excitation energy of the ground-state GDR.

Table 6: Results from pion double-charge-exchange (π^+, π^-) and (π^-, π^+) reactions at incident pion energy $T_\pi = 295$ MeV and laboratory angle $\theta_{lab} = 5^\circ$ compared with pion single-charge-exchange data.

Double dipole (DCX) ^a				Single dipole (SCX) ^b			
Reaction	Q (MeV)	Γ (MeV)	σ ($\mu\text{b}/\text{sr}$)	Reaction	Q ^c (MeV)	Γ (MeV)	σ ($\mu\text{b}/\text{sr}$)
$^{40}\text{Ca}(\pi^+, \pi^-)$	-54.0 ± 0.5	9.0 ± 1.4	2.6 ± 0.2	$^{40}\text{Ca}(\pi^+, \pi^0)$	-24.7 ± 2.1	6.1 ± 1.9	725 ± 126
$^{40}\text{Ca}(\pi^-, \pi^+)$	-31.1 ± 0.3	9.0 ± 1.0	3.2 ± 0.2	$^{40}\text{Ca}(\pi^-, \pi^0)$	-13.8 ± 1.6	5.7 ± 2.3	856 ± 87
$^{56}\text{Fe}(\pi^+, \pi^-)$	-54.4 ± 0.6	10.0 ± 1.5	3.12 ± 0.31	$^{60}\text{Ni}(\pi^+, \pi^0)$	-25.3 ± 1.5	6.1 ± 1.7	792 ± 190
$^{56}\text{Fe}(\pi^-, \pi^+)$	-31.0 ± 1.0	10.0 ± 1.6	1.2 ± 0.1	$^{60}\text{Ni}(\pi^-, \pi^0)$	-13.5 ± 1.6	4.2 ± 2.0	358 ± 75
$^{93}\text{Nb}(\pi^+, \pi^-)$	-49.9 ± 0.8	8.8 ± 2.6	3.48 ± 0.47	$^{90}\text{Zr}(\pi^+, \pi^0)$	-25.7 ± 1.2	6.0 ± 1.7	912 ± 272
$^{93}\text{Nb}(\pi^-, \pi^+)$	-24.9 ± 1.0	8.5 ± 2.0	0.8 ± 0.3	$^{90}\text{Zr}(\pi^-, \pi^0)$	-10.4 ± 1.8	2.7 ± 1.4	392 ± 61
$^{138}\text{Ba}(\pi^+, \pi^-)$	-49.8 ± 0.8	8.5 ± 2.6	4.15 ± 0.52	$^{120}\text{Sn}(\pi^+, \pi^0)$	-23.9 ± 0.9	3.4 ± 2.1	858 ± 335
$^{197}\text{Au}(\pi^+, \pi^-)$	-54.9 ± 0.8	9.0 ± 3.0	5.5 ± 1.6	$^{208}\text{Pb}(\pi^+, \pi^0)$	(-26.5)	(6)	843 ± 450

- a) Energies, widths, and cross sections for the double giant dipole obtained by fitting the GR with a single Lorentzian peak with the listed widths. The (π^+, π^-) data are from Ref. [38] and the (π^-, π^+) data are from Ref. [42].
- b) Data are taken from Ref. [14].
- c) Mass difference between the ground-state and the charge-exchange GDR.

3.6.3 Comparison of (π^-, π^+) and (π^+, π^-) reactions

A unique test for the identification of the double dipole can be provided by measuring the inverse reaction, *i.e.*, the (π^-, π^+) $\Delta T_z = +2$ DCX mode on the same target nuclei. Figure 18 shows a schematic energy-level diagram and isospin splitting of the single-dipole (GDR) and the double-dipole (GDR^2) states anticipated in single- and double-charge-exchange reactions in the two charge-exchange modes on $T = 0$ and $T = 1/2$ targets. Figure 18a illustrates the simplest case of a self-conjugate ^{40}Ca nucleus, where no isospin splitting exists in either direction. The $T = 2$ double-dipole states (labeled GDR^2_- and GDR^2_+ for the $\Delta T_z = -2$ and $\Delta T_z = +2$ modes, respectively) are expected to appear at about the same excitation energy in the final $^{40}\text{Ti} \rightarrow ^{40}\text{Ar}$ mirror nuclei, but at significantly different Q -values corresponding to four Coulomb displacement energies (minus four times the neutron-proton mass difference). Figure 18b shows the states of interest in the case of a $T = 1/2$ nucleus. Now GDR^2_- splits into two isospin components $T = 3/2$ and $T = 5/2$, but GDR^2_+ still has only a single isospin value allowed, *viz.*, $T = 5/2$. The energy difference between GDR^2_+ and GDR^2_- in this case is affected by the symmetry energy in addition to the Coulomb energy and therefore provides new information about the residual interaction of nucleons in nuclei. Also indicated in Figure 18b is the giant-dipole built on the isobaric analog state ($\text{GDR} \odot \text{IAS}$), discussed earlier, which is observable in (π^+, π^-) on non-self-conjugate nuclei[22], but is absent in the inverse reaction and in (π^+, π^-) on $T = 0$ nuclei, where no isobaric analog transition (IAS) is possible.

The observation of the double dipole in the $\Delta T_z = +2$ mode has three clear advantages relative to the inverse process: 1) In the (π^-, π^+) reaction the double dipole is expected (as mentioned above) to appear at a much lower Q -value than in the (π^+, π^-) mode. Therefore, the resonance is expected to appear on a lower background level and may have a smaller width. 2) For non-self-conjugate nuclei the double dipole reached in the (π^+, π^-) splits into two (or up to five for $T \geq 2$ nuclei) isospin components arising from the coupling of two isovector transitions to the ground state, whereas no isospin splitting is possible in the (π^-, π^+) reaction which excites only the upper isospin component. This restriction may also provide a smaller width of GDR^2_+ relative to GDR^2_- . 3) In (π^-, π^+) the spectra are expected to be simpler, since the double isobaric analog state (DIAS) and in particular the overlapping $\text{GDR} \odot \text{IAS}$ that exist in the (π^+, π^-) spectra (on non-self-conjugate nuclei) are now absent, allowing a better determination of the double dipole width and cross section. However, experimentally, the (π^-, π^+) measurements are much more difficult: cross sections for the T_z component of the double dipole are smaller because of the Pauli blocking effect, which is expected to be larger than in SCX reactions[14]. Furthermore, the π^- beam fluxes are generally smaller by about a factor of five relative to the π^+ fluxes at the energies of interest for the present study, making the running time for a single measurement longer.

Figure 19 shows the $^{40}\text{Ca}(\pi^-, \pi^+)^{40}\text{Ar}$ and the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ Q value spectra measured under the same experimental conditions at $\theta_{lab} = 5^\circ$ and incoming pion energy $T_\pi = 295$ MeV. In addition to the weak transitions to the ground state, both spectra contain a wide peak labeled $(\text{GDR})^2$ located in the continuum region. In the (π^+, π^-) reaction the resonance is observed[38] at $Q = -54.0$ MeV, but in the inverse reaction, (π^-, π^+) , the resonance

Table 7: Double-dipole Q values for the (π^-, π^+) reaction compared with data from the (π^+, π^-) reaction measured on the same target nuclei.

Target	(π^+, π^-)		(π^-, π^+)	
	Q_{gs}^a (MeV)	Q_{DD} (MeV)	Q_{gs}^a (MeV)	Q_{DD} (MeV)
^{13}C	-18.96	-46.5 ± 2.0	-32.84	-49.5 ± 0.5
^{27}Al	-15.42	-49.1 ± 0.5	-12.57	-36.2 ± 0.3
^{40}Ca	-24.76	-54.0 ± 0.5	-0.83	-31.1 ± 0.3
^{56}Fe	-5.68	-54.4 ± 0.6	-6.33	-31.0 ± 1.0^b
^{59}Co	-4.85	-48.6 ± 0.8	-7.77	-35.0 ± 0.5
^{93}Nb	-2.58	-49.9 ± 0.8	-3.99	-24.9 ± 1.0

a) Values from the 1988 mass table, Ref. [43].

b) A new measurement of the (π^-, π^+) reaction on ^{56}Fe (including the elastic offset in the DCX data) gives $Q_{DD} = -31.0$ MeV rather than -33.8 MeV reported in Ref. [38].

Table 8: Results from the (π^+, π^-) and (π^-, π^+) double-charge-exchange reactions on ^{13}C , ^{27}Al , ^{40}Ca , ^{56}Fe , ^{59}Co , and ^{93}Nb at $T_\pi = 295$ MeV and $\theta_{\text{lab}} = 5^\circ$.

Target	$(\pi^+, \pi^-)^a$			$(\pi^-, \pi^+)^b$			$\frac{\sigma_{DD}(\pi^-, \pi^+)}{\sigma_{DD}(\pi^+, \pi^-)}$	ΔE_C^c	$\Delta E'_S^d$
	$-Q_{DD}$ (MeV)	$(d\sigma/d\Omega)_{DD}$ ($\mu\text{b/sr}$)	Γ (MeV)	$-Q_{DD}$ (MeV)	$(d\sigma/d\Omega)_{DD}$ ($\mu\text{b/sr}$)	Γ (MeV)			
^{13}C	46.5 ± 2.0	3.2 ± 0.5	12.0 ± 3.0	49.5 ± 0.5	1.3 ± 0.2	9.0 ± 1.4	0.41 ± 0.09	2.71	8.7 ± 2.0
^{27}Al	49.1 ± 0.5	2.4 ± 0.2	8.4 ± 2.0	36.2 ± 0.3	0.95 ± 0.09	6.4 ± 1.0	0.40 ± 0.05	5.19	2.7 ± 0.6
^{40}Ca	54.0 ± 0.5	2.6 ± 0.2	9.0 ± 1.4	31.1 ± 0.3	3.2 ± 0.2	9.0 ± 1.0	1.23 ± 0.12	7.30	1.13 ± 0.6
^{56}Fe	54.4 ± 0.6	3.1 ± 0.3	10.0 ± 1.5	31.0 ± 1.0	1.2 ± 0.1	10.0 ± 1.6	0.39 ± 0.05	8.74	5.7 ± 1.2
^{59}Co	48.6 ± 0.8	4.1 ± 0.8	8.2 ± 2.0	35.0 ± 0.5	1.5 ± 0.2	9.2 ± 1.4	0.37 ± 0.09	8.90	17.0 ± 0.9
^{93}Nb	49.9 ± 0.8	3.5 ± 0.5	8.8 ± 2.6	24.9 ± 1.0	0.8 ± 0.3	8.5 ± 2.0	0.23 ± 0.09	11.9	17.4 ± 1.3

a) Reference [38], except for the ^{27}Al data which is taken from Ref. [27].

b) Reference [42].

c) Sources for the Coulomb energy values are described in Ref. [42].

d) $\Delta E'_S = Q_{DD}(\pi^+, \pi^-) - Q_{DD}(\pi^-, \pi^+) - 4(\Delta E_C - \Delta m_{np})$

Figure 19: (a) Doubly differential cross-section spectrum for the (π^-, π^+) reaction on ^{40}Ca at $T_\pi = 295$ MeV and $\theta_{\text{lab}} = 5^\circ$. The arrows indicate the fitted location of the ground state (g.s.) and the giant resonance $(\text{GDR})^2$. Short vertical lines represent statistical uncertainties of the data. The dashed line is the fitted background with a polynomial shape and the solid line is the fit to the spectrum. (b) Same as (a) except for the inverse $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ reaction.

appears at a much lower energy around $Q = -31.1$ MeV. The GR peaks were fitted with a Lorentzian shape of variable width. The fits shown in Fig. 19 use $\Gamma(\text{GDR})^2 = 9.0$ MeV for both reactions. Fig. 19a shows also the existence of three low-lying states in ^{40}Ar . The strongest of these is observed at an excitation energy of 3.6 ± 0.5 MeV, and may correspond to the single low-lying state observable in the ^{40}Ti mirror nucleus (Fig. 19b). Both transitions are probably the unresolved $0^+, 2^+, 4^+$ triplet of states[44]. The spectra have been corrected for the spectrometer acceptance as a function of momentum. The background (dashed line), which arises from the DCX cross section to discrete low-lying states and to the continuum, was fitted using a third-order polynomial function of the Q -value. In order to reduce uncertainties, the same background form and constraint GR width was used for both DCX modes. The solid lines are the resulting fits to the spectra. The resonance in the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ data was identified[38] as the double isovector giant-dipole resonance based on its energy, characteristic angular distribution and cross section (Fig. 20). The double dipole reached in the (π^-, π^+) reaction appears at almost exactly four Coulomb displacement energies (minus four neutron proton mass differences), as expected. A partial angular distribution at $\theta_{lab} = 5^\circ, 12^\circ$, and 19° (Fig. 21) was measured for the double dipole in the $^{40}\text{Ca}(\pi^-, \pi^+)^{40}\text{Ar}$ reaction. The results exhibit the same characteristic angular distribution as the one observed for the double dipole in the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ reaction[38]. Both angular distributions have a quadrupole shape with some extra strength at forward angles, which may indicate that if the energy difference between the double dipole $J^\pi = 0^+$ and 2^+ states is small, then we actually observe the sum of the two states. This issue is addressed in further detail in Refs. [38] and [45]. The experimental cross section ratio for the double dipole at $\theta_{lab} = 5^\circ$ is $\sigma_{DD}(\pi^-, \pi^+)/\sigma_{DD}(\pi^+, \pi^-) = 1.23 \pm 0.12$. This ratio is consistent with single-charge-exchange data on ^{40}Ca for which the GDR cross section in (π^-, π^0) was found to be 1.18 ± 0.28 times larger than in (π^+, π^0) SCX reaction[14]. Similarly, the overall cross sections of the continuum in the above reactions at the same Q -values are larger in (π^-, π^+) than in (π^+, π^-) and have about the same ratio as that for the double dipole in this energy region. The results for ^{40}Ca , ^{27}Al , and other nuclei studied in both DCX reactions are summarized in Tables 7 and 8.

Figure 22 displays the (π^+, π^-) and (π^-, π^+) Q -value spectra on ^{27}Al . For $T = 1/2$ nuclei the double dipole in the (π^+, π^-) reaction splits into two isospin components ($T_< = 3/2$ and $T_> = 5/2$). However, in the inverse reaction with $\Delta T_z = +2$, only the upper isospin component ($T_> = 5/2$) is allowed from isospin considerations. These states are shown schematically in Fig. 18b. Simple double-isospin coupling arguments for the GDR_+^2 through the GDR_- show that the $T_>$ strength is expected to be weaker by a factor of about seven relative to $T_<$ using equal reduced matrix elements. Pauli blocking will further suppress the $T_>$ component. The observed resonance in the (π^+, π^-) reaction therefore contains both unresolved isospin components and may be slightly wider ($\Gamma = 8.4 \pm 2.0$ MeV) than the double dipole reached in the (π^-, π^+) reaction ($\Gamma = 6.4 \pm 1.0$ MeV). In general, the difference between the centroid energies of the double-dipole resonance in the two reactions can be written as:

$$Q_{DD}(\pi^-, \pi^+) - Q_{DD}(\pi^+, \pi^-) = 4 \times (\Delta E_C - \Delta m_{np}) - \Delta E_S, \quad (24)$$

Figure 20: (a) Angular distribution for the peak labeled GR in the double-charge-exchange spectra on ^{40}Ca shown in Fig. 19b. The cross sections have been extracted with use of a Lorentzian line shape for the giant resonance with constant width of 9.0 MeV and constant $Q = -54.0$ MeV. The solid curve is from a sequential-model calculation for the double-dipole ($J^\pi = 2^+$) state, but renormalized as mentioned in the text. The chain-dashed curve for the double-dipole ($J^\pi = 0^+$) state is shown without any normalization factor. (b) and (c) The same as (a) except for $^{93}\text{Nb}(\pi^+, \pi^-)$ and $^{56}\text{Fe}(\pi^+, \pi^-)$ reactions. The solid and chain-dashed lines are the theoretical calculations for the double dipole and the dipole built on the isobaric analog state, respectively.

Figure 21: Angular distributions for the resonances identified as the double dipoles in the $^{40}\text{Ca}(\pi^\pm, \pi^\mp)$ reactions at $T_\pi = 295$ MeV. The curves are sequential-model calculations.

where ΔE_C is the Coulomb displacement energy, $\Delta m_{np} = 1.29$ MeV is the neutron-proton mass difference, and ΔE_S is the symmetry energy. Using the experimental values listed in Table 8, we obtain $\Delta E_S = 2.7 \pm 0.6$ MeV for the isospin splitting of the double dipole on ^{27}Al . No theoretical calculations have been reported yet on the isospin splitting of the double giant-dipole; however, the above result seems to be in reasonable agreement with the existing information on the isospin splitting of the giant-dipole built on the ground state. For example, if we use the simple expression $\Delta E_S = U(T + 1)/A$, and $U = 50$ MeV for the Lane potential [46, 36] for the symmetry potential, the expected isospin splitting of the giant-dipole on ^{27}Al ($T_z = 1/2$ and $T_z = 3/2$) is $\Delta E_S = E_z - E_{-z} = 2.8$ MeV. Our result indicates that the double dipole on ^{27}Al has a symmetry energy consistent with the single dipole. The same calculations for ^{40}Ca yield ΔE_S close to zero (Table 8) as required for a $T = 0$ nucleus. Additional low-lying, previously unknown, states are also observable in ^{27}Na (Fig. 22a). They appear at excitation energies of 2.6, 5.2, 9.0 and 12.1 MeV above the ground-state transition.

Figure 23 displays the observed Q value for the double dipole as a function of atomic mass. In (π^+, π^-) the double GDR appears around $Q = -50$ MeV and its energy is almost independent of A . The doubly excited GDR in the (π^+, π^-) reaction is reached by adding two E1 vibrational energies and two Coulomb energies to the target nucleus ground state. The E1 energy decreases with A but the Coulomb energy increases with Z , and the two effects almost cancel each other, yielding a nearly constant Q value for the double GDR excitations.

Figure 22: (a) Same as Fig. 19a except for ^{27}Al target. (b) Same as Fig. 19b except for ^{27}Al target. The resonance labeled $\text{GDR} \otimes \text{IAS}$ is the giant dipole built on the $^{27}\text{Si}(\text{g.s.})$ isobaric analog state and shows up more clearly in the $\theta_{\text{lab}} = 15^\circ$ measurement presented in Fig. 7.

Figure 23: Q values versus A for the double dipole in both pion DCX modes. (a) (π^-, π^+) data from Ref. [42]. The curve is to guide the eye. (b) (π^+, π^-) data from Ref. [38]. The solid line is the least-mean-square value.

The double GDR observed in (π^-, π^+) (see Fig. 23a), has a significantly different trend. This state is obtained by adding two E1 vibrational energies to the nucleus and subtracting two Coulomb energies, since two protons are changed into two neutrons. Thus, the Q values in (π^-, π^+) are expected to decrease strongly (in absolute value) with A . The lowering of the magnitude of the double-dipole Q value is very significant for medium and heavy nuclei. For example, the double dipole on ^{93}Nb has $Q = -49.9$ MeV in $^{93}\text{Nb}(\pi^+, \pi^-)^{93}\text{Te}$ but appears as low as $Q = -24.9$ MeV in the $^{93}\text{Nb}(\pi^-, \pi^+)^{93}\text{Y}$ reaction. The solid line drawn in Fig. 23a is merely to guide the eye, while the line in Fig. 23b is a least-square constant fit to the data, which gives $Q = -50.2$ MeV, for $12 \leq A \leq 197$.

Table 9: Comparison between the observed Pauli blocking for the double dipole in DCX, the single dipole in SCX, and theoretical estimates for exciting the isovector giant dipole.

Target	$R(\text{GDR}^2)^a$	$R(\text{GDR})^b$	$R(\text{GDR})_{\text{th}}^c$
^{13}C	0.41 ± 0.09		
^{27}Al	0.40 ± 0.05		
^{40}Ca	1.23 ± 0.12	1.18 ± 0.24	1.37
^{56}Fe	0.39 ± 0.05		
^{59}Co	0.37 ± 0.09	0.45 ± 0.14	0.45^d
^{93}Nb	0.23 ± 0.09	0.43 ± 0.14	0.52^e

- a) $R(\text{GDR}^2) = \sigma_{\text{DD}}(\pi^-, \pi^+) / \sigma_{\text{DD}}(\pi^+, \pi^-)$. The double-dipole cross sections are taken from Table 8.
- b) $R(\text{GDR}) = \sigma_{\text{GDR}}(\pi^-, \pi^0) / \sigma_{\text{GDR}}(\pi^+, \pi^0)$. GDR peak cross sections from single charge exchange (Ref. [14]). For ^{59}Co and ^{93}Nb we give the ratios measured for ^{60}Ni and ^{90}Zr , respectively.
- c) Ref. [47].
- d) Theoretical value for ^{60}Ni (Ref. [47]).
- e) Theoretical value for ^{90}Zr (Ref. [47]).

In contrast to the ^{40}Ca , self-conjugate case, the cross-section ratio $\sigma_{\text{DD}}(\pi^-, \pi^+) / \sigma_{\text{DD}}(\pi^+, \pi^-)$ on ^{27}Al is now less than one (≈ 0.40). The suppression of the double dipole in the (π^-, π^+) reaction probably arises mainly from the Pauli blocking effect for the $T_>$ component (Table 9). This effect is expected to increase with mass and is consistent with results from pion SCX data[14]. The Q -value effect between the double dipole in the two DCX modes arising from their different Q -values is quite negligible and is estimated from DWIA calculations to be only about 1% at $T_\pi = 295$ MeV. An interesting issue is the ratio of the background from the continuum DCX in the two reactions, which will be discussed later. Figure 22 shows that a similar blocking as observed for the double dipole exists also for the continuum in the region of the double dipole resonance. An additional obvious difference between the spectra

shown in Fig. 19 and Fig. 22 is the presence of the giant-dipole built on the isobaric analog state (GDR $\odot$ IAS) in the (π^+, π^-) reaction on ^{27}Al . This resonance is weak in the $\theta_{lab} = 5^\circ$ spectrum but shows up very clearly at $\theta_{lab} = 15^\circ$ (see Fig. 7).

3.6.4 Angular distributions and theoretical calculations

Figure 20a presents the angular distribution extracted for the GR peak on ^{40}Ca . Figs. 20b and 20c show the angular distribution for both GR1 and GR2 on ^{93}Nb and ^{56}Fe , respectively. The two resonances GR1 (lower) and GR2 (upper) shown in Fig. 8 have different angular distributions. GR1 peaks around 10° and has a dipole shape, whereas the higher resonance (GR2) has a quadrupole distribution that is similar in shape and magnitude to the single GR peak observed on ^{40}Ca . We identify GR1 on ^{56}Fe , ^{93}Nb , and several other $T \geq 1$ targets as the giant dipole built on the isobaric analog state, GDR $\odot$ IAS, discussed earlier in Section 5. This resonance is absent on ^{40}Ca (and other self-conjugate targets) because no IAS, and hence no GDR $\odot$ IAS, is possible on a $T = 0$ nucleus.

A simple sequential model has been used to predict the cross section and angular distribution for the double-dipole state. We used the pion coupled-channel impulse approximation code (CCIA) NEWCHOP[26]. The calculations for the double dipole include the ground state (gs), the giant dipole resonance (GDR), and the double dipole (GDR 2). A diagrammatic representation for a $T = 0$ case is shown in Fig. 18. A widely accepted collective Tassie form factor for the transition density, $r^{L-1}\rho'(r)\mathcal{Y}_{LM}(\hat{r})$, with $L = 1$ was used for the GDR[18], where $\rho(r)$ is the nuclear ground-state density. The strength of each SCX has been adjusted to reproduce the measured or extrapolated (π^+, π^0) cross section for the GDR at 165 MeV[14]. The transition strength thus obtained was used at 295 MeV for both SCX processes. This method was tested against SCX data[15] at lower energies. Further details on the calculations for the double dipole can be found in Refs. [38] and [22].

The results of these calculations for the double dipole (solid line) and the giant dipole built on the isobaric analog state (chain-dashed line) shown in Figs. 20 and 21 give strong support to the identification of the lower and upper GR peaks on ^{56}Fe and ^{93}Nb as the giant dipole built on the isobaric analog (GDR $\odot$ IAS) and the double-dipole (GDR 2) states, respectively. Similarly, we identify the single resonance on ^{40}Ca as the $T = 2$ $J^\pi = 2^+$ double-dipole state. If the energy difference between the double dipole $J^\pi = 0^+$ and 2^+ states is small, then we actually observe the sum of the two states. As mentioned earlier, the 5° cross sections are suppressed (by 30% at least) due to Pauli blocking. Our macroscopic impulse-approximation calculations use collective-transition densities for the dipole and do not include this blocking effect. Including this correction to the theory will shift the calculated 5° cross section downward and indicates that we may actually observe both the 0^+ and 2^+ spin members of the double giant dipole. The calculated cross section for the double dipole with $J = 2$ is found to peak near 0° and 22° , as would be expected for coupling of two dipoles, each peaking at this energy around 11° . The calculated solid curves for the double-dipole states shown in Fig. 20 have been multiplied by factors of 2.8 (^{40}Ca), 4.0 (^{93}Nb), and 3.5 (^{56}Fe) to normalize to the data. The chain-dashed curves for the $J^\pi = 0^+$ double dipole state (Fig. 20a) and the GDR $\odot$ IAS (Figs. 20b and 20c) are shown without any renormalization

Figure 21: Comparison between the fitted background for the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ spectra measured at 295 MeV and scattering angles of 5, 15, 25, 32.5, and 40°.

factor.

3.6.5 Background analysis

Table 10 lists the reduced χ^2 and the parameters, $R_i(\theta)$, of the third order polynomial expansion for the background obtained from fitting the individual (π^+, π^-) spectra on ^{40}Ca with two peaks — the ground state and a GR peak with a width of $\Gamma \approx 9.0$ MeV. The fits for the 5° spectrum as shown in Fig. 19b are obtained with a constant energy for the peaks and constrained widths, as listed earlier. Also shown in Table 10, for comparison, are the χ^2 and the background parameters obtained from fitting the 5° spectra with no GR peak. Fitting the spectrum with the unrealistic case of no GR peak gives a significantly larger χ^2 . However, except for an overall upwards shift, the background shape is affected very little by the above assumption.

It is interesting to compare the background in shape and magnitude at different angles using the fitted values of $R_i(\theta)$ listed in Table 10. Using the above values, one can compute the background for each angle using the polynomial expansion:

$$\text{Background}(Q, \theta) = R_1(\theta) + R_2(\theta)Q + R_3(\theta)Q^2 + R_4(\theta)Q^3, \quad (25)$$

where Q is minus the Q value in MeV.

Figure 25 presents a plot of the background computed using eq. (25) for the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ reaction. Except for the 5° spectrum, the background decreases smoothly with scattering

Figure 25: Angular distribution of selective energy ($-Q$ value) bins in the background spectra in the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ reaction at $T_\pi = 295$ MeV shown in Fig. 24. The numbers to the right are the energy bins. The lines are smooth curves to guide the eye.

angle. This feature may arise from the small shift of the whole inclusive DCX spectra toward lower energies with increasing scattering angle[49, 50]. An exception to this behavior is found in the 5° spectrum, where the background level is lower than for 15° , in contrast to the above systematic dependence. The overall lowering (at large missing mass) of the 5° differential cross sections relative to the 15° spectra was also observed in the DCX reaction on ^{93}Nb and may arise from Pauli blocking in changing two inner shell neutrons into protons in the (π^+, π^-) reaction at very small momentum transfer where such transitions are partially blocked.

Figure 26: Widths of the double isovector giant dipole state as a function of A for both DCX modes. The solid lines in (a) and (b) are least-mean-square values. The widths were obtained using NEWFIT and a Lorentzian line shape for the resonances as illustrated in Figs. 19 and 22.

One can choose some selective energy ($-Q$ value) bins around the GR's region in the background spectra of Fig. 21 and plot the background cross section as a function of θ for each of the energy bins. Figure 26 presents such a plot using 5-MeV binning size. The figure shows the absence of isotropy of the non-resonant background in the DCX spectra. The overall feature is the monotonic decrease of the background with scattering angle, except that the 5° cross sections seem to be strongly suppressed — perhaps due to Pauli blocking effects — especially at the higher excitation energies. If we extrapolate the nearly linear curves, we can estimate what the cross section at $\theta_{lab} = 5^\circ$ would have been with no such suppression. Using the 47–52 and 52–57 MeV energy bins (which are located at the GR region) we estimate the Pauli blocking at 5° for this energy region to be larger than 30%. If the cross sections for the GRs are subject to the same amount of suppression as the underlying background, then the calculated cross sections at 5° plotted in Fig. 20 and Fig. 21 should decrease by at least 30%. For a given scattering angle (i.e., 5°) one can also study

the variation of the background with mass. Such analysis indicates that the background shape has only a weak dependence upon A .

3.6.6 Widths of double-dipole resonances

Figure 26 presents the widths of the double-dipole states observed in the (π^-, π^+) and (π^+, π^-) reactions. In (π^+, π^-) the width of the double dipole is around 10 MeV and is larger than the width of the “single” dipole in (π^+, π^0) ($\langle \Gamma_{\text{GDR}} \rangle = 6.3 \pm 1.0$ MeV) by a factor of about 1.5[14]. This disagrees somewhat with theoretical estimates that the width of the double GDR, as a two-phonon state, should be twice the GDR width[38, 39]. The solid lines are the weighted averages of the data, if the widths in a given mode are independent of A . For (π^+, π^-) the weighted average is $\langle \Gamma_{\text{DD}} \rangle = 9.23 \pm 0.76$ MeV, and for (π^-, π^+) $\langle \Gamma_{\text{DD}} \rangle = 8.60 \pm 0.51$ MeV. In (π^-, π^+) our average widths are about twice the average widths measured for the single dipole in (π^-, π^0) ($\Gamma_{\text{GDR}} = 4.2 \pm 1.0$ MeV[14]). The double dipole on ^{27}Al has a width of only 6.4 ± 1.0 MeV and thus is overestimated by the above constant average. In the (π^-, π^+) reaction the double dipole has only a single isospin component, and therefore is expected to be narrower than in (π^+, π^-) where the observed resonance is composed of two to five isospin components. Table 8 summarizes the deduced widths of the double-dipole resonances observed in the two DCX modes.

More than 20 years ago, an expression for the width of the first known double-giant resonance, the DIAS, was derived[51]. The result was that for nuclei with a large neutron excess ($N - Z \gg 1$), the width of the DIAS is twice the width of the “single” isobaric analog state (IAS):

$$\Gamma_{\text{DIAS}} \approx 2 \times \Gamma_{\text{IAS}} . \quad (26)$$

The derivation of this relation is based on the following considerations. The width of the IAS is mainly due to its coupling to the $T = 1$ component of the isovector monopole resonance[51, 52], $|M\rangle$ in the $\Delta T_z = \pm 1$ nucleus. This coupling gives rise to its width. Analogously, the DIAS width results from its coupling to the $T = 1$ component (*i.e.*, $T_-|M\rangle$) of the isovector monopole in the $\Delta T_z = \pm 2$ nucleus member of the isobaric multiplet. Assuming that the analog nucleus has the same density of $T = 1$ states and that the energy difference between the IAS and $|M\rangle$ is the same as the DIAS and $T_-|M\rangle$ (both quite reasonable assumptions), it was possible to express the ratio of the widths of the DIAS and IAS as:

$$\frac{\Gamma_{\text{DIAS}}}{\Gamma_{\text{IAS}}} \approx \frac{2(2T - 2)}{2T - 1} . \quad (27)$$

In the limit of large T (*i.e.*, $N - Z$) one obtains the relation in eq. (26). Taking this limit means, of course, that the IAS and DIAS can be viewed as collective one and two phonons, respectively, of isospin vibrational motion. This idea was generalized recently[39] to any two (or many) independent phonon states.

The width of a collective one-phonon state, $|1; \lambda\rangle$ (but also of any other state), is obtained by coupling it to all other states and writing:

$$\Gamma_{1,\lambda} = 2\pi\rho \sum_{\alpha} |(1; \lambda|V|\alpha)|^2 , \quad (28)$$

where the sum runs, in principle, over all states $|\alpha\rangle$ different from $|1; \lambda\rangle$, and ρ is the density of states α in the vicinity of the phonon state.

In Ref. [39] this expression is treated in the "doorway" state approximation. The sum in Eq. (28) is replaced by a single "doorway", which by itself carries a width, i.e.:

$$\Gamma_{1;\lambda} = \frac{|\langle d|V_{d\lambda}|1; \lambda\rangle|^2}{(h\omega_\lambda - E_d)^2 + \Gamma_d^2/4} \Gamma_d, \quad (29)$$

where $h\omega_\lambda$ is the energy of the phonon state of type λ , and E_d and Γ_d are the energy and width of the doorway. Going over to a state of n independent phonons of type λ , i.e., $|n; \lambda\rangle$, one can now ascribe the width to the coupling of this n phonon state to a state of $(n-1)$ phonons and the state $|d\rangle$. The width of the n phonon state is now given by[39]:

$$\Gamma_{n;\lambda} = \frac{|\langle n-1; \lambda, d|V_{d\lambda}|n; \lambda\rangle|^2}{\{nh\omega_\lambda - (n-1)h\omega_\lambda - E_d\}^2 + \Gamma_d^2/4} \Gamma_d, \quad (30)$$

It is easy to see[36, 39, 53] that the matrix element in the numerator can be written as:

$$\langle n-1; \lambda, d|V_{d\lambda}|n; \lambda\rangle = \sqrt{n}\langle d|V|1; \lambda\rangle. \quad (31)$$

Thus:

$$\Gamma_{n;\lambda} = \frac{n|\langle d|V|1; \lambda\rangle|^2}{(h\omega_\lambda - E_d)^2 + \Gamma_d^2/4} \Gamma_d. \quad (32)$$

Comparing eqs. (29) and (32) we find:

$$\Gamma_{n;\lambda} = n\Gamma_{1;\lambda}. \quad (33)$$

The physical meaning of this result is very simple. Each of the n identical phonons can decay independently into the "doorway" state $|d\rangle$, giving rise to a width which is n times the width of a single-phonon state.

For $n=2$ we have:

$$\Gamma_{2;\lambda} = 2\Gamma_{1;\lambda}. \quad (34)$$

On this basis we should expect the double-dipole states to be twice as wide as the single dipole state.

A different simple relation has been suggested concerning the width and the lifetime of a multiphonon state and single-phonon states assuming that the strength function has a normal distribution[40]:

$$\Gamma_{2;\lambda} = \sqrt{2}\Gamma_{1;\lambda}, \quad (35)$$

i.e., the widths of the single-phonon states are adding quadratically in the above assumption. If additional broadening of the two-phonon multiplet is of the order of 1 MeV[40], then a more realistic prediction is:

$$\Gamma_{2;\lambda} = (\sqrt{2}\Gamma_{1;\lambda} + 1) \text{ MeV}. \quad (36)$$

Indeed, in Table 6, the widths observed in our DCX experiments are about a factor of 1.5 to 2.0 larger than the widths observed in pion SCX reactions[14]. Thus, the existing data lie somewhere between the predictions of eqs. (34) and (36).

Table 10: Parameters of the polynomial background^{a)} obtained from fitting the $^{40}\text{Ca}(\pi^+, \pi^-)^{40}\text{Ti}$ spectra with and without a giant resonance peak assumed in the fit.

No. of ^{b)} peaks	θ (deg)	q^2 ^{c)} (fm ⁻²)	χ^2 ^{d)}	R_1	R_2 $\times 10^1$	R_3 $\times 10^3$	R_4 $\times 10^5$
1	5	0.10	1.19	0.16 ± 0.04	$-.14 \pm 0.04$	0.24 ± 0.11	$.28 \pm 0.09$
1	15	0.33	0.98	0.25 ± 0.05	$-.19 \pm 0.05$	0.31 ± 0.13	$.30 \pm 0.11$
1	25	0.77	1.61	0.17 ± 0.03	$-.14 \pm 0.03$	0.19 ± 0.09	$.32 \pm 0.08$
1	32.5	1.25	1.38	0.07 ± 0.03	$-.03 \pm 0.03$	$-.11 \pm 0.08$	0.52 ± 0.07
1	40	1.82	1.17	0.10 ± 0.03	$-.06 \pm 0.03$	$-.016 \pm 0.08$	0.38 ± 0.07
0	5	0.10	4.31	0.45 ± 0.03	$-.42 \pm 0.03$	1.09 ± 0.07	$-.46 \pm 0.06$

a) $\text{Background}(E, q^2) = R_1(q^2) + R_2(q^2) \times E^1 + R_3(q^2) \times E^2 + R_4(q^2) \times E^3$

b) In addition to the ground state.

c) Momentum transfer at the region of the giant resonance peak ($Q = -54.0$ MeV).

d) Reduced χ^2 for the entire histogram.

Expression (34) is, of course, an approximation. The assumption that the double resonances represent fully independent phonons is violated to a certain degree. Anharmonicity effects leading to all kinds of splittings (because of isospin, deformation, etc.) are well known to alter the simple phonon picture. Also, penetrability effects and, in particular, single-particle fragmentation due to loss of collectivity (the so-called Landau damping) will all affect the expression in eq. (34). Relations 34 and 36 should provide a guideline when one tries to estimate widths of two- (or multi) phonon states and assess the possibilities of observing such states.

A recent paper by Scholtz and Hahne[54] presented an interesting calculation for the expected spectrum and relative strength factors for the two-step excitation of the double GDR states in axially deformed nuclei. The calculations are based on the interacting boson model (IBM) approach[55] and incorporate the p -boson for the GDR[56, 57]. The p -boson interacts with the s - and d -bosons. The deformation of the nucleus is introduced by including an interaction of a quadrupole type between the p -boson and s - and d -bosons in the Hamiltonian. According to these calculations, the doubly excited GDR in deformed nuclei splits into several components. Among these, the strongest states are the lowest 2^+ , the 1^+ , and the 0^+ . The energies of the one- and multiphonon GDR states are given relative to ϵ_p (one p -boson energy), and the splitting energies are given in units of the single-dipole splitting energy of the deformed nucleus.

3.6.7 Mass dependence of the double-giant-dipole resonance

Table 11: Energies and widths of the states observed in the $^{13}\text{C}(\pi^+, \pi^-)^{13}\text{O}$ reaction at $T_\pi = 140\text{--}295$ MeV.^a

State Label	Q (MeV)	E_x (MeV)	Γ (MeV)
gs	-18.9 ± 0.04	0.0	0.46 ± 0.06
22.0	-22.0 ± 0.06	3.1 ± 0.07	0.60 ± 0.40
23.4	-23.4 ± 0.08	4.5 ± 0.09	0.60 ± 0.40
25.0	-25.0 ± 0.08	6.1 ± 0.09	0.60 ± 0.40
GDR $\otimes$ IAS	-27.6 ± 0.2	8.7 ± 0.2	3.1 ± 0.5

a) Ref. [58].

An interesting question is how the cross section of the double-dipole state varies as a function of N , Z , and A , and whether there is a simple relation between the double dipole and the “single” dipole strengths. The results from pion SCX data to the single dipole provide a guide when trying to estimate the strength at which the double dipole is expected to appear. The non-spin t_{-1} [t_{+1}] mode of the GDR has been observed to be strongly populated in the (π^+, π^0) [(π^-, π^0)] charge-exchange reaction[13, 14, 15]. The existing $(\pi^\pm, \pi^0)$ SCX data at 165 MeV are summarized in Ref. [14]. We note that except for the GDR on ^{40}Ca , which has only one ($T = 1$) isospin member, the GDR cross sections on the heavier targets reported in Ref. [14] include all three isospin members ($T - 1$, T , and $T + 1$) which are not resolved with the π^0 spectrometer.

The existing data on the double dipole consist of the twelve (π^+, π^-) data points listed in Table 5. The 5° cross sections listed in the table are for fitting the resonance as a whole with a single peak with the listed width. We note that in the deformed nuclei ^{12}C , ^{13}C , and ^{32}S , the double dipole splits due to deformation corresponding to vibrations along the major and minor axes. In ^{13}C the $(\text{GDR})^2$ is expected to have additional splitting due to isospin. In the above nuclei the double dipole was also fit with a single peak in order to study the global features of the resonance. Following Section 5.5, the 5° cross sections for the $\Delta T_z = -2$ double-dipole resonance are well represented (Fig. 27) by the function[37]:

$$d\sigma/d\Omega_{\text{GDR}^2}(5^\circ) = 14840 \frac{(NZ)^2}{A^{3.70}} \text{ nb/sr} . \quad (37)$$

The fits in Fig. 27 do not include the ^{12}C and ^{32}S deformed nuclei. The result of eq. (37) is interesting in view of the regularities for the GDR pointed out in Section 5.5. It is likely that the $(NZ)^2$ factor arises from the two-step GDR form factor. One can similarly break the cross section for the $(\text{GDR})^2$ as the product of the strength of the first and second step times an overall attenuation factor for DCX:

$$d\sigma/d\Omega_{\text{GDR}^2} \approx \left(\frac{NZ}{A^{2/3}} \right)^2 A^{-\alpha'} . \quad (38)$$

Figure 27: Plot of the double-dipole cross sections at 5° divided by $(NZ)^2$ vs A for the (π^+, π^-) reactions. The filled circles and the $\times$ s are from different measurements. The dashed line is a power-law fit of the data, and the solid line is a refit including the newer data. The ^{12}C and ^{32}S data are not included in either fit[37].

Table 12: Measured and calculated cross sections for the g.s., the states at $Q = -22.0, -23.4$, and -25.0 MeV, and the $|GDR \otimes IAS\rangle$ from DCX on ^{13}C .^a

T_π (MeV)	θ_{lab} (deg)	$d\sigma/d\Omega$ (Experimental)					$d\sigma/d\Omega$ (Theory)	
		g.s.	22.0	23.4	25.0	$ GDR \otimes IAS\rangle$ ^b	$ GDR \otimes IAS\rangle$ ^c Model I	Model II
		($\mu\text{b/sr}$)	($\mu\text{b/sr}$)	($\mu\text{b/sr}$)	($\mu\text{b/sr}$)	($\mu\text{b/sr}$)	($\mu\text{b/sr}$)	($\mu\text{b/sr}$)
295	17.0	0.14 ± 0.02	< 0.04	< 0.06	< 0.08	0.95 ± 0.10	1.32	0.81
260	18.8	0.21 ± 0.02	< 0.04	< 0.06	< 0.09	0.73 ± 0.07	1.30	0.89
220	21.3	0.17 ± 0.02	< 0.04	< 0.06	< 0.09	0.68 ± 0.08	1.16	0.91
180	24.8	0.12 ± 0.02	0.06 ± 0.04	0.08 ± 0.05	0.09 ± 0.05	0.70 ± 0.12	0.88	0.76
140	29.9	0.17 ± 0.02	0.07 ± 0.04	0.10 ± 0.06	0.16 ± 0.09	0.55 ± 0.12	0.51	0.37

a) Ref. [58].

b) Peak cross sections (at the listed energies and angles) for the giant dipole resonance built on the isobaric analog state.

c) Sequential model calculations using volume transition density (model I) and surface transition density (model II) for the isobaric analog state, normalized properly to pion single-charge-exchange data.

A comparison of eqs. (37) and (38) yields $\alpha' = 2.37$. This attenuation factor is slightly larger than $\alpha' = 2.21$, deduced from the systematics of the GDR built on the IAS (Section 5.5). Therefore the average pion attenuation in DCX from the two types of double resonances ($\bar{\alpha}' = 2.29$) is surprisingly close to twice the average attenuation factor for SCX ($\bar{\alpha} = 1.14$) leading to the GDR or to the IAS. This is quite interesting and may be attributed to the fact that pion DCX involves three-pion propagations, whereas only two-pion propagations are involved in the SCX reaction. A more theoretical approach to understand these phenomena is highly desirable.

3.7 Other properties of double giant resonances

3.7.1 Energy dependence of the $|GDR \otimes IAS\rangle$

Figure 28 shows the $^{13}\text{C}(\pi^+, \pi^-)^{13}\text{O}$ missing-mass spectra at five different incoming pion beam energies. The scattering angles have been chosen to correspond to the peak cross section of the giant dipole built on the isobaric analog state. In addition to the ground state (gs) and the three low-lying narrow states reported earlier[59] (at $Q = -22.0$ MeV, $Q = -23.4$ MeV, and $Q = -25.0$ MeV), most spectra show clearly the existence of a wider peak located in the continuum region at $Q = -27.6$ MeV ($E_x = 8.7$ MeV above the gs of ^{13}O). This resonance was previously identified as the $T_> = 3/2$ component of the giant dipole resonance built on the isobaric analog state[27] (see Fig. 7). In Fig. 28 this resonance is labeled $|GDR \otimes IAS\rangle$. For $T = 1/2$ nuclei the $T_< = 1/2$ component of the $|GDR \otimes IAS\rangle$ is forbidden as illustrated in

Figure 28: Double-differential-cross section spectra for the (π^+, π^-) reaction on a ^{13}C target at $T_\pi = 295, 260, 220, 180,$ and 140 MeV at the scattering angles listed in Table 12. The arrows indicate the fitted location of the gs, the three low-lying states and the giant resonance. Short vertical lines represent statistical uncertainty of the data. The dashed lines are the fitted background with a polynomial shape and the solid lines are the fits to the spectra using NEWFIT.

Fig. 13. The ground state and the low-lying narrow states were fitted with a Gaussian, and the resonance with a Lorentzian shape of variable widths. The background (dashed line), which arises from DCX to the continuum, was fitted using a third-order polynomial shape. Figure 28 demonstrates that the cross section for the $[\text{GDR} \oplus \text{IAS}]$ increases significantly with

Figure 29: Excitation function at 140–295 MeV for the gs ($Q = -18.9$ MeV), the three low-lying states ($Q = -22.0$, $Q = -23.4$ and $Q = -25.0$ MeV), and the giant dipole built on the isobaric analog state ($Q = -27.6$ MeV). The dashed lines are constant, $\sin^2(\theta)$ or $\cos^2(\theta)$ curves, normalized arbitrarily to the data. The solid line labeled “CCIA” in section (e) is the result of simple sequential model calculations using the computer code NEWCHOP (see text).

beam energy. The resonance shows up clearly in the 295-MeV spectrum, but almost vanishes in the 140-MeV measurement. The transitions to the three lower excited states at $E_x = 3.1$ MeV ($Q = -22.0$ MeV), $E_x = 4.5$ MeV ($Q = -23.4$ MeV), and $E_x = 6.1$ MeV ($Q = -25.0$ MeV) exhibit a substantially different excitation function. Their cross sections decrease with increasing beam energy. These states have been observed earlier[59], but little is known of their nature. Figure 29 displays the excitation functions for the gs, the states at 3.1, 4.5, and 6.1 MeV, and the $[\text{GDR} \oplus \text{IAS}]$.

Table 11 summarizes the deduced energies and widths for the low-lying states and the giant resonance extracted with the line-shape fitting code NEWFIT[60]. For the strong transitions (the gs and the $[\text{GDR} \oplus \text{IAS}]$), the energies and widths were varied simultaneously to minimize χ^2 for the entire fit. Table 12 lists the observed cross sections for the above states at

all different energies. The uncertainties listed in Tables 11 and 12 come only from the fitting procedure, with no account taken for the uncertainties associated with the lack of knowledge of the overall spectrum shape of the DCX reaction to the continuum. Table 12 also lists the predicted cross sections for the $|\text{GDR} \odot \text{IAS}\rangle$ in a simple sequential-mode calculation, using the pion coupled-channels impulse-approximation (CCIA) code NEWCHOP[26], discussed earlier. The strength of each SCX transition has been adjusted to reproduce the extrapolated SCX cross section to the GDR[14] and the IAS[11]. The same transition strength was used at all energies. Two different transition densities have been used for the IAS: one a volume transition density (model I), and the other a surface-peaked transition density (model II). The calculations (using both models) take into account the suppression of the $|\text{GDR} \odot \text{IAS}\rangle$ on $T = 1/2$ nuclei due to isospin discussed earlier and reproduce the measured cross sections without any additional normalization factor.

3.7.2 The giant Gamow-Teller resonance built on the IAS

In order to understand the observed energy dependence in a simple model, we can use an expression for the energy dependence of the inelastic cross section derived by Siciliano and Walker[61] in a strong absorption model assuming the dominance of the $[3,3]$ resonance:

$$\sigma(\theta) = \alpha(E)[4M^2(q_0)\cos^2(\theta) + S^2(q_0)\sin^2(\theta)] \quad . \quad (39)$$

Here q_0 is a constant momentum transfer near the maximum of the differential cross section, and $\alpha(E)$ contains the energy dependence of the elementary π -N force. In general, only the S form factor can contribute to $\Delta S = 1$ spin transitions. The equation indicates that the $\Delta S = 0$ non-spin-flip transition cross section increases with energy like $\cos^2(\theta)$ at a constant momentum transfer since $q_0 = 2k \sin(\theta/2)$, where k is the beam momentum. Using the same argument, the spin-flip transitions should decrease with energy like $\sin^2(\theta)$. This technique for distinguishing spin-flip from non-spin-flip transitions by examining the constant- q excitation function was successfully used earlier on ^{13}C [62], to identify $\Delta S = 0$ and $\Delta S = 1$ transitions in pion scattering. It should, however, be noted that these relations are only approximate, since the central part of the pion-nucleus interaction may contribute to $\Delta S = 1$ transitions due to current corrections.

Figure 29 indicates that the $|\text{GDR} \odot \text{IAS}\rangle$ can be well fitted with a $\cos^2(\theta)$ dependence, similar to non-spin-flip transitions in inelastic scattering. The three excited states at $E_x = 3.1$, $E_x = 4.5$ and $E_x = 6.1$ MeV[59] are only weakly populated in the this study; however, the data indicate that they can be better described by $\sin^2(\theta)$, the signature expected for spin-flip transitions. This behavior ($\Delta S = 1$), along with the small angle and low-momentum transfer, which suggest a low angular momentum transfer, leads to the conclusion that these states may contain the Gamow-Teller $\odot$ IAS ($|\text{GT} \odot \text{IAS}\rangle$) transition strength.

From simple arguments the expected energy for the $|\text{GT} \odot \text{IAS}\rangle$ in pion DCX is:

$$Q(|\text{GT} \odot \text{IAS}\rangle) = -[E(\text{M1}) + 2(\overline{\Delta E_C} - \delta)] \quad (40)$$

where $\overline{\Delta E_C}$ is the average Coulomb energy, and $\delta = m_n - m_p = 1.29$ MeV is the neutron-proton mass difference. In inelastic scattering a strong isovector M1 transition has been

observed on ^{12}C for the 15.11 MeV ($T = 1$, $J^\pi = 1^+$) state[63, 64]. Using $\overline{\Delta E_C} = 3.16$ MeV yields $Q(|\text{GT} \odot \text{IAS}\rangle) = -18.85$ MeV, surprisingly close to the gs of ^{13}O . In the simplest model, a coupling of the 1^+ Gamow-Teller transition to the $^{13}\text{N}_{\text{gs}}$ (the isobaric analog of $^{13}\text{C}_{\text{gs}}$) gives a pair of states with $1/2^-$ and $3/2^-$. Can we identify these states in ^{13}O in the present study? It is most likely that $^{13}\text{O}_{\text{gs}}$ has $J^\pi = 3/2^-$, the same as the mirror nucleus $^{13}\text{B}_{\text{gs}}$ [59]. From the $A = 13$ isobar diagram it is also obvious[59] that $^{13}\text{O}_{\text{gs}}$ is the analog of the well-known Gamow-Teller states at 15.06 and 15.11 MeV ($T = 3/2$, $J^\pi = 3/2^-$) in ^{13}N and ^{13}C , respectively, and thus is a natural candidate for the $3/2^-$ member of the $|\text{GT} \odot \text{IAS}\rangle$. It is, therefore, puzzling that in Ref. [58] this transition does not show the expected spin-flip character. This may result from the fact that a $\text{gs}(1/2^-) \rightarrow \text{gs}(3/2^-)$ transition may proceed via both $L = 0$, $S = 1$ Gamow-Teller and $L = 2$, $S = 0$ electric quadrupole transitions built on the IAS. This is consistent with the nearly constant angular distribution, with no strong angular dependence, observed for the $^{13}\text{O}_{\text{gs}}$ in an earlier DCX reaction on ^{13}C at two pion energies, 164 and 295 MeV[65]. The two multipoles would mask the characteristic angular distribution and the spin-flip nature of the transition. A good candidate for the $1/2^-$ state in ^{13}O is the 4.5 MeV state. This state (4.21 MeV) was reported in the work of Seidl *et al.*[65] It was tentatively assigned $J^\pi = 1/2^-$ based on its forward-peaked angular distribution, which exhibits a simple diffractive $\Delta J = 0$ shape. This state is well fitted with a $\sin^2(\theta)$ dependence and thus might be the missing $1/2^-$ member of the $|\text{GT} \odot \text{IAS}\rangle$. Since the GT resonance was not observed in any previous pion SCX data, no reliable calculations can be done at this time to quantitatively compare the measured cross sections with the predicted cross sections for the $|\text{GT} \odot \text{IAS}\rangle$, (*i.e.*, a GT transition followed by an IAS transition in the second step or *vice versa*):

$$\sigma(\text{GT} \odot \text{IAS}) \approx \sigma(\text{GT}) \cdot (\sigma(\text{IAS})/\sigma(\text{total})). \quad (41)$$

If we use the isovector M1 on ^{12}C for the GT cross section[63], extrapolated IAS cross section on ^{13}C [11] and total reaction cross section on carbon, the predicted cross section of the $|\text{GT} \odot \text{IAS}\rangle$ around $T_\pi = 180$ MeV is about $0.12 \mu\text{b/sr}$, surprisingly close to the measured cross section for the low-lying states in the work of Ward *et al.*, [58].

The solid line shown in Fig. 29(e) is the result of the simple sequential model analysis discussed earlier. The calculations with a fixed transition strength for the GDR and the IAS give an adequate representation of the measured excitation function for this resonance. This analysis indicates that the excitation of the $|\text{GDR} \odot \text{IAS}\rangle$ in this energy region proceeds by a two-step sequential $(\pi^+, \pi^0)(\pi^0, \pi^-)$ mechanism and, therefore, pion DCX should be a unique probe for exciting isotensor double giant resonances in nuclei.

3.7.3 Isospin splitting of the $|\text{GDR} \odot \text{IAS}\rangle$ and the double dipole

In the SCX studies the measured GDR generally includes more than one isospin member because of the limited (6 to 8 MeV) energy resolution of the old π^0 spectrometer at LAMPF. For example, in $^{60}\text{Ni}(T = 2)$ the SCX GDR splits into three components with ratios of 9:5:1[23]. However, experimentally the GDR shows up as a single peak in SCX with $\Gamma = 6.4 \pm 1.7$ MeV[14]. Previously, useful information on isospin splitting of the charge exchange

GDR was obtained by comparing (π^-, π^0) and (π^+, π^0) yields, since (π^-, π^0) populates only the $(T+1)$ member whereas (π^+, π^0) populates predominantly the $(T-1)$ member[14, 15]. The upper component is expected to have a smaller width relative to the lower two components. This may be attributable to the fact that a $T_{\geq}$ state cannot decay either via proton or via neutron to the low-lying states of the neighboring nuclei. In addition, the spreading width is also expected to be smaller for a $T_{\geq}$ state in comparison with the lower T members. Therefore $T_{\geq} = T+1$ should have a smaller width relative to T and $T-1$ members. Reference [24] reports the isospin splitting of the $[\text{GDR} \odot \text{IAS}]$ on ^{56}Fe and provides the first observation of isospin splitting of the analog GDR.

The splitting of the $[\text{GDR} \odot \text{IAS}]$ can in general arise from three different sources:

Deformation effects: In deformed nuclei with axial symmetry, a dipole splits into two components arising from vibrations along the major and minor axes of the nucleus.

Spin coupling: For nuclei with $J_i \geq 1$, the GDR splits into three members with $J_f = J_i - 1$, J_i , and $J_i + 1$. In the simplest case of even-even target nuclei, the $[\text{GDR} \odot \text{IAS}]$ has a single J^π value of 1^- .

Isospin splitting: For $T \geq 1$ target nuclei, the charge exchange dipole splits into three members with $T-1$, T , and $T+1$, arising from coupling of one unit of isospin to a state with isospin T (see Fig. 10).

For example, $^{56}\text{Fe}_{gs}$ has only a small deformation and $J_i^\pi = 0^+$; therefore, isospin coupling is the predominant source for the observed energy splitting[24].

Simple isospin geometry arguments predict relative strengths for the $[\text{GDR} \odot \text{IAS}]$ members[24, 25]:

$$\begin{aligned} \sigma(T-1) : \sigma(T) : \sigma(T+1) = \\ \frac{(2T-1)(T-1)}{T(2T+1)} : \frac{(2T-1)}{T(T+1)} : \frac{3}{(2T+1)(T+1)} \end{aligned} \quad (42)$$

These ratios are quite different from the ratios expected for a GDR in SCX, where the GDR is in a $T_z = T-1$ nucleus. For $T=2$, eq. (42) gives the ratios 3:5:2, respectively. Therefore the $[\text{GDR} \odot \text{IAS}]$ on ^{56}Fe should split into three components with the middle member about 50% stronger than the lower member. This is in close agreement with the observed ratios (Fig. 5 of Ref. [24]). The intensity ratios obtained from the summed histogram are 3.1:5.0:3.4 from a Gaussian fit and 3.0:5.0:3.5 from a Lorentzian fit. There is, however, a small enhancement of the upper isospin member relative to the above prediction.

A general treatment of isospin splitting in terms of isovector and isotensor potentials[23] gives the following relations for the energy splittings $\Delta E^+(T+1, T)$ and $\Delta E^-(T, T-1)$ for the isovector excitations:

$$\Delta E^+ = (T+1)[E_v + (2T-1)E_t] \quad (43)$$

$$\Delta E^- = T[E_v - (2T+3)E_t] \quad (44)$$

where E_v and E_t are the isovector and the isotensor energies. Therefore, one can use the splitting energy to determine the isovector and the isotensor energies in eqs. (43)–(44). For example, if we use $\Delta E^+ = 3.5 \pm 1.0$ and $\Delta E^- = 4.3 \pm 0.6$ MeV, we obtain[24] $E_v = 1.46 \pm 0.43$ MeV and $E_t = -0.098 \pm 0.014$ MeV. The deduced isovector and isotensor potentials[24] are in agreement with the results from pion SCX data. For ^{60}Ni ($T = 2$, same ^{56}Fe), by comparing (π^+, π^0) and (π^-, π^0) data to the GDR, and arbitrarily setting $E_t = 0$, a value of $E_v = 0.88 \pm 0.67$ MeV was reported in the work of A. Erell *et al.*[14]. The above values are consistent with the results from theoretical RPA calculations, which give $E_v = 1.06$ MeV and $E_t = -0.003$ MeV[46] for ^{40}Ca . We note that these splittings for a collective dipole are only about one half those expected for single-particle excitations[46]. Bohr and Mottelson estimate ΔE^+ to be about $V_1(T_0 + 1)/(2A)$ with $V_1 \approx 130$ MeV, in good agreement with the results[24].

We turn now to the isospin splitting of the double dipole. The double dipole reached in (π^+, π^-) on $T_0 \geq 2$ target nuclei have five isospin members with $T_0 - 2 \leq T \leq T_0 + 2$ arising from coupling of $\Delta T = 2$ to the ground state. For medium and heavy nuclei, most of the double-dipole strength concentrates in the lowest two or three members. For example, for ^{56}Fe ($T = 2$), simple double-isospin coupling arguments through the GDR ($T = 1, T, T + 1$)(assuming all components have the same reduced matrix elements) give the following intensity ratios for the isospin multiplets:

$$\sigma(T = 0) : \sigma(T = 1) : \sigma(T = 2) : \sigma(T = 3) : \sigma(T = 4) = \quad (45)$$

$$0.20 : 0.75 : 0.74 : 0.19 : 0.014 .$$

Therefore, about 90% of the total strength concentrates in the $T = 0, 1$, and 2 components. The reduced matrix elements will further suppress the upper isospin members due to Pauli blocking. Thus, most of the double-dipole strength concentrates in the lowest three members. The splitting between the isospin components of the double dipole are found to be similar in magnitude to the isospin splitting in single excitations[66].

3.7.4 Comparison with relativistic heavy-ion reactions

Recent calculations of electromagnetic excitation indicate that the excitation of multiphonon processes, especially the giant dipole resonance, is a very likely process in peripheral relativistic heavy-ion collisions (RHI)[53, 70]. These exotic excitations with large probabilities are expected from the strong electromagnetic fields that occur for a very short time scale in RHI. In such collisions virtual photons of energy sufficient to excite the GDR and other collective excitations with higher multipoles are produced. With the high probability for single-step excitations, also multistep excitations become very likely at RHI[67, 68]. Evidence for high excitation energy structures in RHI have been reported in many studies in the last decade (see, *e.g.*, the review paper by N. Frascaria[71]). However, it was generally difficult to give a unique interpretation to the observed structures since they may arise from both nuclear excitations and numerous pick-up and break-up reactions. Recently, two successful direct observations of the double giant dipole resonance have been reported in RHI via detecting the double- γ decay[68], and through an exclusive measurement of the neutron

Figure 30: (a) Doubly differential cross-section spectrum for the (π^-, π^+) reaction on ^{40}Ca at $T_\pi = 295$ MeV and $\theta_{lab} = 5^\circ$. The arrow indicates the fitted location of the ground state (g.s.) and the double giant dipole $(\text{GDR})^2$. The dashed line is the fitted background with a polynomial shape and the solid line is the fit to the spectrum[69]. (b) Sum energy of coincident photon pairs with a difference less than 6 MeV for peripheral $^{209}\text{Bi} + ^{208}\text{Pb}$ collisions at 1 GeV/nucleon. The large structure around 26 MeV is assigned to the two- γ decay of the double dipole. The inset show a Lorentzian fit to the difference between the data and the background (dashed line) (from Ref. [68]).

Table 13: Parameters of the double giant dipole resonance deduced from relativistic heavy ion reactions.

Nucleus	$E(\text{GDR}^2)$ (MeV)	$\Gamma(\text{GDR}^2)$ (MeV)	$\sigma_{exp}(\text{GDR}^2)$ (mb)	$\frac{E(\text{GDR}^2)}{E(\text{GDR})}$	$\frac{\Gamma(\text{GDR}^2)}{\Gamma(\text{GDR})}$	$E1(\text{photo})$ (MeV)
^{136}Xe ^a	28.3 ± 0.7	6.3 ± 1.6	215 ± 50	1.86 ± 0.05	1.3 ± 0.4	15.3 ^c
^{208}Pb ^b	25.6 ± 0.9	5.8 ± 1.1	770 ± 220	1.91 ± 0.08	1.4 ± 0.3	13.4 ^d

a) Ref. [67].

b) Ref. [68].

c) E1 on ^{208}Pb from Ref. [28].

d) Ref. [28].

decay of the excited projectile[67]. The results show a clear signal for the double dipole on ^{208}Pb and ^{136}Xe obtained using two substantially different experimental setups. Figure 30 presents a comparison between a typical DCX spectra where the double giant dipole is observed and the recent observation of the double dipole in the system $^{209}\text{Bi} + ^{208}\text{Pb}$ at 1 GeV/u[68]. Figure 30a is the same as Fig. 19a and shows the double dipole [labeled $(\text{GDR})^2$ in the figure] observed in the $^{40}\text{Ca}(\pi^-, \pi^+)^{40}\text{Ar}$ at $T_s = 295$ MeV and $\theta_{lab} = 5^\circ$. Figure 30b shows the sum energy of coincidence photon pairs with a difference less than 6 MeV. A large structure is observable around 26 MeV which is assigned to the two- γ decay of the double giant dipole resonance in ^{208}Pb . The resonance parameters are summarized in Table 13, and are consistent with the value expected from a harmonic oscillator model and the measured γ -decay of 13 MeV, which corresponds to the “single” GDR in the ^{208}Pb target (Ref. [68]). The table indicates that the double-dipole energy is just about twice the energy of the single GDR and the width of the double dipole is about 50% larger than single-dipole excitation, in good agreement with the pion DCX data presented earlier.

We note, however, that the double dipole observed in RHI and pion double charge exchange are not quite the same excitations since they have a different isospin structure. For heavy nuclei with $T_z \gg 1$, the double dipole reached via Coulomb excitation has predominantly isospin T , whereas $(\pi^+, \pi^-)/(\pi^-, \pi^+)$ populates predominantly the $|T - 2\rangle/|T + 2\rangle$ isospin components, respectively. Thus, for heavy nuclei a simple approximate relation expected between the $\Delta T_z = \pm 2$ DCX modes and the $\Delta T_z = 0$ mode in RHI excitation is:

$$[Q_{DD}(\pi^+, \pi^-) + Q_{DD}(\pi^-, \pi^+)]/2 \simeq Q_{DD}(\text{RHI}) \quad . \quad (46)$$

This simple relation provides a check on future RHI and DCX studies of the double giant dipole resonance on the same nuclei.

The unique selectivity of pion DCX for the study of double giant resonances can be briefly summarized as follows:

a) Pion DCX reactions are $\Delta T_z = \pm 2$ nuclear probes, and thus only isotensor ($\Delta T = 2$)

transitions are allowed. The numerous isoscalar and isovector excitations that exist at these high excitation energies are forbidden.

- b) One can measure with simplicity the inverse reaction under precisely the same kinematical conditions by changing polarities of all magnets in the pion channel and the spectrometer.
- c) In (π^+, π^-) one can measure the single dipole built on the IAS and the double dipole is a single measurement, giving an excellent energy calibration. At forward angles the strong DIAS transition provides an additional accurate test on the energy calibration.
- d) Pion DCX gives a good signature of the multipolarity of the resonance due to the diffractive nature of the pion scattering at the $\Delta_{3,3}$ energy.
- e) At resonance energy the pions are especially sensitive to the surface region of the nucleus, where the GDR has a strong component of its transition density. This selectivity is evidenced in the strong signal observed for the GDR above background in pion SCX.
- f) Pions are spin-zero particles and therefore at forward-scattering angles, the background from numerous spin-flip transitions [which dominate, for example, the (p, n) spectra] are strongly suppressed, yielding a lower background in the DCX spectra.

Relativistic heavy-ion reactions also proved to be a good probe of multiphonon excitations, especially the giant dipole resonance discussed in Refs. [67] and [28]. The large cross sections observed for the double dipole in RHI reactions and the relatively narrow widths of the resonance (Table 13) are very promising for further successful studies of these exotic multiphonon collective states in peripheral RHI reactions.

3.8 Conclusions and outlook

We have reviewed the observation of two new double resonances in nuclei. These are the isovector giant dipole built on the isobaric analog state and the double giant-dipole resonance. The identification of the resonances is based on the energies at which they appear, their angular distributions, and their cross sections. The data indicate that the giant dipole built on the isobaric analog is a general collective feature of all nuclei with $N - Z \geq 1$, and the double dipole is a general feature of all nuclei. The observed double resonances behave as a superposition of the ingredient "ordinary" resonances. However, some puzzles have arisen from these studies. Why does the width of the GDR@IAS increase rapidly with mass, and where is the missing strength of the 0^+ member of the double giant dipole? The data is so new that very little theoretical analysis has been made. Further experiments are planned at LAMPF to study these questions and possible future implications of these new phenomena. There is the possibility that other double giant resonances, unobserved so far, could also be detected. The isovector monopole resonance (IVM) was observed in pion SCX, and it is not inconceivable that the IVM built on the IAS or the double IVM could be detected.

The new resonances reviewed in this work include some evidence for the observation of the giant Gamow-Teller transition built on the isobaric analog state on ^{13}C . The giant Gamow-Teller was observed in the middle of the seventies in the (p,n) reaction, and since then has been intensively studied in several laboratories. What are the prospects for detecting the double giant Gamow-Teller (DGT) in nuclei? The existence of this new mode was recently predicted theoretically[25, 72], but attempts at experimental study have not been successful so far. The observation of the DGT will present a new challenging problem for nuclear structure studies, and is a subject of continuing interest. The relation of the DGT to double beta decay is of interest to both particle and nuclear physicists. Theoretically, many questions related to the nuclear structure, deformation effects, isospin splitting, widths, and decay modes of the double resonances are still open and need to be addressed.

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4 Visualization of Time Projection Chamber

4.1 Introduction

In preparation for the Solenoidal Tracker at the Relativistic Heavy Ion Collider, a computer program has been developed for examining the three-dimensional relationships of points and curves of high multiplicity expected in ultrarelativistic heavy ion reactions. The program features "fly-through" capability and is adaptable to any situation involving drawing and examining curves and points in space.

4.2 Discussion

One of the two primary experiments approved for day-one operation at the Relativistic Heavy Ion Collider (RHIC) is the Solenoidal Tracker at RHIC (STAR)[1, 2, 3]. This experiment will use a large time projection chamber (TPC) of cylindrical shape with parallel electric and magnetic fields parallel to the symmetry axis (the beam pipe). An ionizing particle passing through the active volume has from several to several hundred ionizing collisions (hits) along its trajectory. Detectors at the ends of the cylinder will be two-dimensional, and drift time parallel to the axis will provide the third dimension providing a three-dimensional specification of the hits. Software will be used to fit the large number of hits which occur as a result of collision events. This fitting is a necessary step in extraction of physics from the raw data. Without a method for simultaneously viewing large numbers of hits and the associated fits, one cannot have confidence in the fitting software, which is complex and involves decisions at fit crossings and allowance for in-flight decay (which produces a Y-shaped track). We have developed a computer code ("STARGL") for doing this which is among several visualization schemes currently under development by STAR software workers[4]. Figure 31 shows a black-and-white screen dump of STARGL's color display during application to real data[5] from the Equation of State (EOS) experiment at Lawrence Berkeley National Laboratory. This experiment involved nucleus-nucleus collisions (as is planned for STAR,

Figure 31: A screen dump of fits to real data from the EOS experiment showing a fitting error. Track density for the STAR project is expected to be much higher. The menu bar is visible at the top with the menus File, Plotting, View, Help, Drawing, and Subsets.

but at a lower beam energy than STAR). In this figure, a fitting error (confusion of two coplanar tracks) is apparent. Both perspective and hidden surface depth cueing are illustrated in this figure. However, the figure is severely limited compared to the actual color display because of its black-and-white nature.

STARGL uses the special graphics capabilities of Silicon Graphics Inc.'s (SGI) Graphics Library (GL). SGI computers have dedicated hardware for such things as three-dimensional rotation and hidden-surface removal[6] and use of GL in the code allows control of this hardware[7]. At present, the program requires GL and the associated hardware to be available. Because of this, it does not run on an arbitrary X server, but rather requires that an SGI machine or equivalent be used. Since one must therefore have an SGI machine to use the program at all, it is better to compile the program (a single C file) locally than to use a remote host (net delays are particularly irritating during animation and use of the mouse).

We have adopted terminology based on aviation to describe aspects of the program's use. Although informal, these terms are intuitive and mnemonic, and seem to work well.

STARGL allows one to "fly an airplane" through the hits and fits, and thus allows rapid, intuitive, and complete examination of their three dimensional relationships. The "View" menu allows one to choose whether the picture on the screen represents the view from the airplane or from one of the "control towers".

STARGL has an X/Xt/Motif user interface with a menu bar (Fig. 31) across the top of the window and incorporates various functionalities such as piloting the airplane, controlling the view and drawing, selecting subsets of the data, and identifying various components of the picture.

The Input File

The fits/hits are specified in an editable, human-readable, "ASCII" file of simple format which allows use of a selection of seven primary colors (red, pink, yellow, green, blue, purple, and white). The sample file shown in Table 14 is largely self-explanatory. Each color line is for the fit or point whose cartesian coordinates follow. Fits have at least two points specified, while points have only one. Therefore, the red entities below are drawn as single points, while the green one is drawn as a fit (curve) in space. The numbers (optional) after the colors are called track numbers and the same number can be used more than once for grouping. These numbers are used for runtime identification and control.

Description of Major Functionality

The program comes up viewing from the control tower at $x = 6$ meters, $y = 0$, $z = 0$. This control tower is permanently fixed in location. At this time, the airplane appears at the left, and the detector outline (with the fits/hits) appears centered in STARGL's window. One may start the airplane flying forward or backwards and steer it with the four arrow keys. There are also control towers at $x = 0$, $y = 6$, $z = 0$ and $x = 0$, $y = 0$, $z = 6$. The "View" menu contains the four items "View from Control Tower X", "View from Control Tower Y", "View from Control Tower Z", and "View from Airplane". This menu can be used at any time to switch the view presented in the program's window. The above tower coordinates

Table 14: A small sample data input file. The number triplets are cartesian coordinates (in meters for STAR). Each color line may be followed by n such triplets, where $n \geq 1$ and $n <$ limits imposed by static memory allocation and degradation of animation performance.

# comment lines begin with a pound sign.			
green 1			
1.235	0.1153	-0.9409	
1.24	0.12000	-0.9306	
1.25	0.13000	-0.9204	
1.26	0.14000	-0.9102	
1.27	0.15000	-0.9001	
red 2			
1.111111	0.9	0.9998	
red 3			
1.211	0.006	0.9997	
red 3			
1.311	0.003	0.9996	
red 3			
1.411	0.001	0.9995	

have been chosen to fit the STAR detector geometry, but they can be reprogrammed for other applications.

Normally, the arrow keys cause the airplane to pivot about its nose, as shown in Fig. 32(a) (the airplane does not have to be moving). However, the "Pivot on Exterior Point" mode causes the airplane to pivot about a point an arbitrary distance in front of the nose, as in Fig. 32(b). When viewing from a control tower, one may move the airplane and the exterior pivot point independently by dragging with the mouse. Thus, a convenient procedure for examining a point of particular interest is: Begin by viewing from the x control tower. Select the "Pivot on Exterior Point" mode from the "Piloting" menu. Drag the pivot point to the point of interest. Drag the airplane to an appropriate position. Since dragging does not affect the position component perpendicular to the screen, one must repeat the two drags from either the y or z control tower to adjust the x -coordinates. Select "View from Airplane", and then use the arrow keys to pivot the airplane and inspect the point of interest from any direction. We have found close careful inspection of track crossings to be important.

STARGL's major weakness is sluggish animation when dealing with large data sets. Indeed, inter-frame time for a full STAR simulated event is several seconds on our SGI Indigo R3000. We have therefore given considerable attention to methods for viewing subsets of the data. These include multiple selection of fits/hits by clicking, hiding of un-selected fits/hits, selection of a subset of the input file based on fits/hits either being completely contained within or passing through a volume definable in either cartesian or polar coordinates, and elimination of fits which are not close to the pivot point.

Other functionality includes far and near clipping planes for eliminating background,

(a) Pivot on nose.

(b) Pivot on exterior point.

Figure 32: Illustration of the two pivot modes for the airplane. Mode (a) more closely corresponds to piloting a real airplane, while the mode (b) is useful for examining a point of interest from a variety of angles.

runtime color randomization, runtime selection of how thick fits are drawn, use of two input data files and instantly switching back and forth between them, black or white background (for making hardcopy screen dumps), an artificial-horizon/compass, and various mechanisms for examining the fits one by one.

Issues of Interest to Potential Users

The code is available by contacting the authors (ward@utaphy.ph.utexas.edu) via electronic mail. As explained above, you must presently have an SGI computer or equivalent to use the program. You will receive a README file, a UNIX script for compiling, the C file, a small sample data file, and a pair of documents in the form of a postscript files: a formal manual and a "quick-start" page.

Non-STAR users will want to change/disable the function DrawDetector, which draws an outline of the STAR detector. Width of the drawn fits and the size of the points are adjustable through several C #define statements.

We are interested in obtaining suggestions for making STARGL useful and will help export the code to a few groups analyzing real data.

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