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Damage from Pulses with Arbitrary Temporal Shapes

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June 6, 1994

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To: Damage Devotees
From: John B. Trenholme
Subject: Damage from Pulses with Arbitrary Temporal Shapes

Summary

Not yet.

Introduction

In fusion laser designs, the laser pulse has a complicated temporal shape which undergoes significant change as it passes through the laser. Our damage data, however, was taken with pulses whose temporal shapes were (more or less) Gaussian. We want to determine the damage propensity of a material exposed to a pulse of arbitrary temporal shape, given data taken with Gaussian pulses of different pulse widths. To do so, we must adopt a physical model of damage. This model will contain some number of parameters that depend on material properties, geometry, and so forth. We determine the parameters of the model appropriate to each material by fitting the model to the Gaussian data for that material. The resulting normalized model is then applied, using the appropriate pulse shape, to find the damage level for a specific material subjected to a specific pulse.

The model we shall assume is related to diffusion, although (as we shall see) the experimental results do not fit any simple diffusion model. Initially, we will discuss simple diffusion models. We then examine some experimental data, and then develop a modified

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diffusive model from that data. That modified model is then used to predict damage levels in various portions of the NIF laser design.

Simple Diffusion as a Damage Source

In a simple diffusion model, we assume that damage is caused by the accumulation of a substance we refer to as "damageonium," or D for short. This substance is most probably heat, but it might be electrons (or holes) in a plasma, or some other quantity that causes damage if it exceeds a local density limit. The results of the model do not depend on the exact nature of D , but only on its diffusion properties, so we need not be concerned with D 's exact identity. We assume that D is produced when laser light is absorbed in the volume of local structures of some kind (defects, particles, or whatever), and that it diffuses away from the volume of production. This causes the density of D to rise near the absorption volume, but because of the diffusion the rise is not linearly proportional to the energy density per unit area (fluence) in the pulse. The absorption is supposed to remove only a small fraction of the laser beam, so that the deposition is near-uniform in the absorption volume. That is to say, the absorption sites are supposed to be light gray, not black.

We also assume that once the density of D rises above some local damage threshold, damage occurs. This second assumption means that we are ignoring the possibility of damage due to the effects of a quantity such as 3-D stress which effectively integrates D over a volume, and which therefore sees some spatial average of D . Instead, we suppose (for example) that the local concentration of D reaches the melting point of the material, or that a critical concentration of D leads to higher material absorption or decreased diffusion, leading to a runaway increase of D .

We presume that there are one or more (perhaps many more) absorption sites within the laser beam footprint, on the surface or in the bulk of a material, and that the damage threshold corresponds to the weakest of these exceeding its local threshold. This means that the damage threshold depends on the area of the laser beam, and on the extreme-value statistics of the potential damage sites.

Green's Function for Diffusion

If the diffusion depended on only one coordinate (in a pure planar, cylindrical or spherical geometry) and started from a very small source in a large region, then the diffusion would proceed according to the universal diffusion law. For a large homogeneous and isotropic region where the deposition has dimensionality d (which must be 1, 2 or 3), the Green's function for the diffusion equation is given (in, for example, Morse & Feshbach, METHODS OF THEORETICAL PHYSICS, section 7.4) by the equation

$$g(\vec{r}, t; \vec{s}) = \frac{e^{-\frac{(\vec{r}-\vec{s})^2}{4kt}}}{(4\pi kt)^{d/2}} \quad (t > 0)$$

$$= 0 \quad (t < 0)$$

where $\vec{r}$ is the observation point, t is the time after deposition, $\vec{s}$ is the source point, k is the diffusion constant (thermal conductivity, divided by specific heat times density), and the initial condition is a unit-weight delta-function at point $\vec{s}$ at time zero (note that I have normalized M&F's result so that its integral over all space is unity). After a time t , this Green's function is spread around the deposition location by a distance $\pm 2\sqrt{kt}$. For example, if heat is diffusing in laser glass with a typical room-temperature value for k of $0.005 \text{ cm}^2/\text{sec}$, the diffusion distance is ± 0.045 microns after 1 nanosecond and ± 0.14 microns after 10 nanoseconds. The spatial behavior of this Green's function for various times is shown in Fig. 1.

We find the density of D as a function of time and space by convolving the source due to deposition (in time and space) with the Green's function appropriate to the dimensionality of the problem, with a time lag equal to the time which has elapsed since deposition. For very short times compared to the diffusion time, the Green's function is spatially localized and this is equivalent to convolution of the source with a spatial delta function, so D isn't smeared out and at each point we get the temporal integral of the deposition at that point. For very long times, the deposition from each point is smeared out over a Gaussian shape much larger than the deposition region, and the profile of D is Gaussian with width of about $4\sqrt{kt}$ and with its spatial integral equal to the total D that has deposited. For intermediate times, the details depend on the geometry of absorption and diffusion, and on the temporal history of the pulse. We shall investigate these details with a series of more and more complicated examples.

Diffusion from Plate-shaped, Rod-shaped and Ball-shaped Regions: Impulse Deposition

First, consider instantaneous, uniform deposition in geometries of exactly 1, 2 and 3 dimensions. If a value D_0 is instantly produced uniformly throughout a region extending from the origin for a distance A in all directions, it is possible to write down the maximum value of D as a function of time in analytic form. To do so, we carry out a spatial integral of the Green's function with an exponent in the denominator appropriate to the dimensionality of the diffusion. The result of this integration is the temporal Green's function for the maximum D value.

In one dimension ($d = 1$), we assume instantaneous, uniform deposition in a plate-shaped region of thickness $2A$ along one axis (say x). The deposition extends to $\pm\infty$ along the other two axes (say y and z). The center-plane value of D as a function of time is given by

$$\frac{D_1(\text{center}, t)}{D_0} = G_1(t) = \text{erf}\left(\frac{A}{2\sqrt{kt}}\right)$$

which has unit value and zero derivative at zero time (in fact, **all** derivatives are zero when t is zero). The center value stays near unity until diffusion has time to move D from the center to the boundaries at $\pm A$. This center-diffusion time t_c is roughly equal to $A^2/4k$. We shall see that any geometry has this initial period of constant D at the center; we refer to the near-constant- D period as "diffusion in 0 dimensions," since little diffusion effect is seen near the center during this period.

At times long compared to t_c the one-dimensional center D value is approximately given by

$$\frac{D_1(\text{center}, t)}{D_0} \approx \frac{A}{\sqrt{\pi kt}} \quad t \gg t_c = \frac{A^2}{4k}$$

In two dimensions ($d=2$), we assume instantaneous, uniform deposition within the volume of a cylinder of radius A which extends to $\pm\infty$ along the axis of the cylinder. The center-line value of D as a function of time is given by

$$\frac{D_2(\text{center}, t)}{D_0} = G_2(t) = 1 - e^{-A^2/4kt}$$

which again has unit value and zero derivatives at zero time. At long times, the center D value is approximately given by

$$\frac{D_2(\text{center}, t)}{D_0} \approx \frac{A^2}{4kt} \quad t \gg t_c = \frac{A^2}{4k}$$

In three dimensions ($d=3$), we assume instantaneous, uniform deposition within the volume of a sphere of radius A . The center-point value of D is given by

$$\frac{D_3(\text{center}, t)}{D_0} = G_3(t) = \text{erf}\left(\frac{A}{2\sqrt{kt}}\right) - \frac{Ae^{-A^2/4kt}}{\sqrt{\pi kt}}$$

which also has unit value and zero derivatives at zero time. At long times, the center D value is approximately given by

$$\frac{D_3(\text{center}, t)}{D_0} \approx \frac{A^3}{6\sqrt{\pi} k^{3/2} t^{3/2}} \quad t \gg t_c = \frac{A^2}{4k}$$

The center-value temporal Green's functions G_1 , G_2 and G_3 are plotted in Fig. 2.

Diffusion from Plate-shaped, Rod-shaped and Ball-shaped Regions; Rectangular Pulse

To find the center value of D as a function of time for an arbitrary temporal pulse shape deposited uniformly throughout any of the simple regions above, we convolve the temporal shape with the delta-function response (temporal Green's function). If the temporal shape of the deposition is $P(t)$, then the center value of D is given by

$$D_d(\text{center}, t) = Q \int_{-\infty}^t P(u) G_d(t-u) du = Q \int_0^{\infty} P(t-v) G_d(v) dv$$

for d values of 1, 2 and 3. Q is an absorption constant which converts laser power density per unit area to the volumetric rate of production of D . Recall we are assuming weak absorption resulting in uniform deposition.

Initially, we will use an especially simple pulse shape that is rectangular in time. This pulse jumps to a constant power P_0 at time zero and remains at that power until the end of the pulse. The fluence (energy per unit area) in this pulse is

$$\Phi(t) = \int_{-\infty}^t P(u) du = P_0 \int_0^t du = P_0 t$$

where P is the incident laser power per unit area.

We now uniformly absorb this temporally rectangular pulse of duration t at power level P_0 within a plate-shaped region of width W (W is equal to $2A$). The resulting center value of D at the end of the pulse is given by the temporal convolution integral

$$\begin{aligned} D_1(\text{center}, t) &= QP_0 \int_0^t \text{erf}\left(\frac{W}{4\sqrt{kv}}\right) dv \\ &= QP_0 \left[(t + 2\tau) \text{erf}\left(\sqrt{\frac{\tau}{t}}\right) + 2\sqrt{\frac{\tau t}{\pi}} e^{-\tau/t} - 2\tau \right] \end{aligned}$$

where the characteristic diffusion time τ is given by

$$\tau = \frac{W^2}{16k}$$

The behaviors of D_1 for short and long times (compared to τ) are

$$D_1(\text{center}, t) = \begin{cases} QP_0 t = Q\Phi & t \ll \tau \\ 4QP_0 \sqrt{\frac{\tau t}{\pi}} = \frac{4}{\sqrt{\pi}} \frac{Q\Phi}{\sqrt{t/\tau}} & t \gg \tau \end{cases}$$

We see that D_1 first increases linearly with t , and then smoothly switches to an increase proportional to $\sqrt{t}$. If the center-position, pulse-end value D_1 is equal to the damage level D_D , then the fluence which causes that damage level to be reached is given by

$$\Phi_1(t) = \frac{D_D}{Q} \frac{t/\tau}{\left[\left(\frac{t}{\tau} + 2 \right) \text{erf}\left(\sqrt{\frac{\tau}{t}}\right) + 2\sqrt{\frac{t}{\pi\tau}} e^{-\tau/t} - 2 \right]}$$

The damage fluence, normalized to D_D/Q , is unity for pulse durations short compared to τ , and then smoothly makes a transition to $\sqrt{\pi t/\tau} / 4$ for pulse durations long compared to τ .

If we uniformly absorb a temporally rectangular pulse of duration t at power level P_0 within a rod-shaped region of diameter W , the center value of D at the end of the pulse is given by

$$D_2(\text{center}, t) = QP_0 \int_0^t (1 - e^{-W^2/16kv}) dv$$

$$= QP_0 \left[t(1 - e^{-\tau/t}) + \tau E_1\left(\frac{\tau}{t}\right) \right]$$

where E_1 is the exponential integral. The behaviors of D_2 for short and long times (compared to τ) are

$$D_2(\text{center}, t) = \begin{cases} QP_0 t = Q\Phi & t \ll \tau \\ QP_0 \tau \ln\left(\frac{t}{\tau}\right) = Q\Phi \frac{\tau}{t} \ln\left(\frac{t}{\tau}\right) & t \gg \tau \end{cases}$$

We see that D_2 first increases linearly with t , and then smoothly switches to an increase proportional to $\ln(t)$. If the center-position, pulse-end value D_2 is equal to the damage level D_D , then the fluence which causes the damage level to be reached is given by

$$\Phi_2(t) = \frac{D_D}{Q} \frac{1}{\left[1 - e^{-\tau/t} + \frac{\tau}{t} E_1\left(\frac{\tau}{t}\right) \right]}$$

The damage fluence, normalized to D_D/Q , is unity for pulse durations short compared to τ , and then smoothly makes a transition to $t / [\tau \ln(t/\tau)]$ for pulse durations long compared to τ .

If we uniformly absorb a temporally rectangular pulse of duration t at power level P_0 within a ball-shaped region of diameter W , the center value of D at the end of the pulse is given by

$$D_3(\text{center}, t) = QP_0 \int_0^t \left[\text{erf}\left(\frac{W}{4\sqrt{kv}}\right) - \frac{We^{-W^2/16kv}}{2\sqrt{\pi kv}} \right] dv$$

$$= QP_0 \left[(t - 2\tau) \text{erf}\left(\sqrt{\frac{\tau}{t}}\right) - 2\sqrt{\frac{\tau t}{\pi}} e^{-\tau/t} + 2\tau \right]$$

The behaviors of D_3 for short and long times (compared to τ) are

$$D_3(\text{center}, t) = \begin{cases} QP_0 t = Q\Phi & t \ll \tau \\ QP_0 \tau \left(2 - \frac{8}{3} \sqrt{\frac{\tau}{\pi t}} \right) = Q\Phi \frac{\tau}{t} \left(2 - \frac{8}{3} \sqrt{\frac{\tau}{\pi t}} \right) & t \gg \tau \end{cases}$$

We see that D_3 first increases linearly with t , and then asymptotically approaches the constant value $2QP_0\tau$. This behavior is quite different from the one-dimensional and two-dimensional cases, where the center value of D steadily increased with time for a constant deposition rate. Three-dimensional diffusion, once a steady state is reached, can dissipate a constant power input without additional increase in D . If the center-position, pulse-end value D_3 is equal to the damage level D_D , then the fluence which causes the damage level to be reached is given by

$$\Phi_3(t) = \frac{D_D}{Q} \frac{t/\tau}{\left[\left(\frac{t}{\tau} - 2 \right) \operatorname{erf} \left(\sqrt{\frac{\tau}{t}} \right) - 2 \sqrt{\frac{t}{\pi \tau}} e^{-\tau/t} + 2 \right]}$$

The damage fluence, normalized to D_D/Q , is unity for pulse durations short compared to τ , and then smoothly makes a transition to $t/2\tau$ for pulse durations long compared to τ .

The $D(t)$ values (normalized by $QP_0\tau$) as a function of pulse width for one-dimensional, two-dimensional and three-dimensional diffusion are shown in Fig. 3. The D values all start out with a linear increase. They then smoothly transition to $4\sqrt{t/\pi\tau}$, $\ln(t/\tau)$ and constant ($=2$) behavior for long times.

The damage fluences (normalized by D_D/Q) as a function of pulse width for one-dimensional, two-dimensional and three-dimensional diffusion are shown in Fig. 4. The damage fluences all start out at a constant value, with no initial slope. They then smoothly transition to $\sqrt{\pi t/\tau}/4$, $t/[\tau \ln(t/\tau)]$ and $t/2\tau$ behavior for long times.

Absorption of a Temporally Rectangular Pulse in a More General Region

We now want to illustrate how a geometry more complicated than single-coordinate influences the temporal behavior of diffusion, and how this leads to the variation of damage fluence with pulse width. Consider the spatially-uniform absorption of a temporally-rectangular pulse of laser light within the volume of a region shaped like a plank, with width W much less than height H much less than length L . Assume that the diffusion constant is uniform within the plank and within a very large homogeneous region in which the plank is embedded. We use this

shape because, as we shall see, diffusion in this geometry changes smoothly from "0-dimensional" to one-dimensional to two-dimensional to three-dimensional as time goes on.

Consider the point at the center of the plank, since this point will always have the highest D value. We can find the value of D here due to a temporal impulse of deposition D_0 at time t in the past by integrating the three-dimensional Green's function over the volume of the plank. Because of symmetry from the center, we only need to calculate the contribution from one octant and then multiply by eight. The result is the temporal Green's function

$$\frac{D(\text{center}, t)}{D_0} = \frac{8}{(4\pi kt)^{3/2}} \int_0^{W/2} dx \int_0^{H/2} dy \int_0^{L/2} dz e^{-\frac{x^2+y^2+z^2}{4kt}} = \text{erf}\left(\frac{W}{4\sqrt{kt}}\right) \text{erf}\left(\frac{H}{4\sqrt{kt}}\right) \text{erf}\left(\frac{L}{4\sqrt{kt}}\right)$$

We now apply our rectangular temporal pulse to the plank. I have not found any way to do the temporal integral exactly. However, it's easy enough to see what the qualitative behavior must be. Each erf function starts out at unity for short times, and then makes a transition to

$$\frac{S}{2\sqrt{\pi kt}}$$

behavior for long times, where S is the appropriate size (W , H or L). The transition comes around the time

$$t_S = \frac{S^2}{16k}$$

Initially, all of the erf's are unity and the center value of D just increases without any diffusion according to the equation

$$D_0(t) = Q \int_{-\infty}^t P(u) du = Q\Phi(t) \quad t \ll t_w = \frac{W^2}{16k}$$

until there is enough time for D to diffuse from the center to the nearest edge, which is at a distance $W/2$. This time is given by the definition of t_w in the equation above. During this non-diffusing period (or "diffusion in zero dimensions") the fluence required to reach the damage level is independent of pulse width, just as before. This result is true for any pulse shape, not just the rectangular shape we are assuming.

Once diffusion reaches from the center to the nearest edges of the plank, which are in width, diffusion proceeds as though the plank were infinite in height and length, since there has not yet been time for D to diffuse from the center to the edges in height and length. The resulting diffusion is effectively one-dimensional, since only the first erf is different from unity. This

means that around t_W the behavior of the equation for the center value of D switches smoothly from integrating $P(t)$ to integrating

$$P(t) \sqrt{\frac{4t_W}{\pi t}}$$

This means that D begins to increase as $\sqrt{t}$, and the damage fluence also takes on $\sqrt{t}$ behavior. This continues until diffusion reaches the edges in height, at a time

$$t_H = \frac{H^2}{16k}$$

The diffusion now becomes effectively two-dimensional. The temporal integral smoothly changes to integrating the power density times the long-time behavior of the first two erf's, or

$$P(t) \frac{4\sqrt{t_W t_H}}{\pi k t}$$

This means that D begins to increase as $\ln(t)$, and the damage fluence increases as $t/\ln(t)$. This continues until diffusion reaches the ends in length, at a time

$$t_L = \frac{L^2}{16k}$$

The diffusion now becomes effectively three-dimensional. The temporal integral smoothly changes to integrating the power density times the long-time behavior of all three erf's, or

$$P(t) \left(\frac{4}{\pi k t} \right)^{3/2} \sqrt{t_W t_H t_L}$$

This means that D begins an asymptotic approach to a final constant value, and the damage fluence approaches a linear increase with pulse duration.

To recapitulate, the plank geometry smoothly transitions from behavior corresponding to no diffusion, to behavior corresponding to one-dimensional diffusion, to behavior corresponding to two-dimensional diffusion, to behavior corresponding to three-dimensional diffusion. If two of the dimensions (say W and H) were the same, then there would be a smooth transition from zero to two dimensional behavior without a distinct one-dimensional zone, just as with the rod-shaped region. If all three dimensions were the same (a cube) the transition would be directly from no diffusion to three-dimensional diffusion without either a distinct one-dimensional zone or a distinct two-dimensional zone, just as with the ball-shaped region. We generalize these

observations to expect that for any shape of region the initial behavior will be one of no diffusion (with damage fluence constant) and the final behavior will be three-dimensional (with damage fluence increasing linearly with time), and that there may or may not appear intermediate one-dimensional (square root) and two-dimensional ($t/\ln(t)$) zones. Because the transition times go as the square of the dimensions, a 10-to-1 aspect ratio translates to a 100-to-1 pulse duration difference between different behaviors. Thus the two intermediate zones (one-dimensional and two-dimensional) may last over a wide range of pulse widths.

Actual damage Data

Now that we know what to look for, let's examine some actual damage data. Frank Rainer and Jack Campbell have collected data for six materials over a broad pulse width range. The data are shown in Fig. 5. We find a dependence which is linear in the pulse width to a power near 0.37, over three orders of magnitude in pulse width. In fact, Rainer says the data lie on a straight line for another half-decade pulse width increase, and Mike Perry has data for very short pulse widths that finally become constant around 3 picoseconds, and which increase with a power somewhat less than 0.5 for longer pulses. The power law holds over almost 5 decades in pulse duration! This is very hard to understand in terms of the diffusion picture we have developed. If the power were 0.5 over the whole range, we would argue for one-dimensional diffusion in regions whose transverse dimensions were at least 300 times their thickness. But the power is clearly below 0.5, and we expect such a dependence only in the transition region from no diffusion to one-dimensional diffusion. That transition region cannot possibly last for more than a decade in pulse width, and even over that range the slope has a visible change (see Fig. 4).

What are we to do? I decided to honor the data by using a temporal Green's function which goes as the pulse duration to a fixed power for all pulse widths. No doubt some sort of diffusive process leads to such behavior, but the details are presently unclear. If such a Green's function is used, however, we reproduce the observed behavior of damage fluence as a function of pulse width (see below). No other model I am able to make up has such behavior.

Constant Power Law Model

We now adopt a constant-power-law model of D accumulation as a function of time. At the point of maximum accumulation (the center of the deposition region) the value of D for an arbitrary temporal source is assumed to be given by given by the convolution integral

$$D(t) = A \int_{-\infty}^t \frac{(1-d/2) P(u)}{(t-u)^{d/2}} du = A \int_0^{\infty} \frac{(1-d/2) P(t-v)}{v^{d/2}} dv$$

where $P(t)$ is the incident laser power density (intensity) and A is a coefficient (with unpleasant units) that depends on the absorption and diffusion constants for D produced at this particular site. We will assume that d and A are parameters to be found from the data. The extra factor of $(1-d/2)$ is included to avoid a logarithmic difficulty when $d = 2$, but all results will be in terms of

ratios of convolution integrals and so constants are irrelevant. In any case, no damage data with d greater than one seems to exist.

If the geometry of the source region or the diffusing region is complicated, or if the diffusion constant is not constant in space or time, or if the diffusion constant depends on the density of D , then the temporal convolution integral presumably combines the effects of different d and/or k values to give (approximately) one resulting "average" d . It is also possible that different sites with different A and d values can "cross over" with pulse width, so that one dominates at short pulse widths and another at long pulse widths. Whatever the underlying physics, the observed behavior is that d is constant over a very broad range of pulse widths.

The pulse-width scaling of the damage level in this constant-power-law model can be illustrated by assuming a temporally square input pulse of width τ with fluence Φ . The power level of this pulse is constant with a value of Φ/τ for time τ , which leads (if $d < 2$, as it always is) to a peak $D(t)$ value at the end of the pulse that is equal to

$$D(\tau) = \frac{A\Phi}{\tau} \int_0^\tau \frac{(1 - d/2) du}{(\tau - u)^{d/2}} = \frac{A\Phi\tau^{1-d/2}}{\tau} = \frac{A\Phi}{\tau^{d/2}} = D_D$$

where D_D is the level at which the weakest site damages. The damage fluence as a function of pulse width is then given by

$$\Phi = \frac{D_D \tau^{d/2}}{A}$$

This scaling with pulse width also applies to non-square pulse shapes, as we shall see.

The diffusion integral above has the unpleasant property that it has a mathematical singularity at times very close to the observation time ($u = t$). There is (of course) no physical singularity, because the short-time behavior involves diffusion over very short distances, and any physical absorber has some finite size and therefore integrates the power density for short times. Thus at short times the diffusion must have a d value near zero. Rather than deal with this complication, we will just ignore the short-time physics (i.e., assume the absorption takes place in zero volume) and integrate over the resulting singularity. The singularity is integrable, as it must be, but it is numerically inconvenient. The standard trick for dealing with this situation (check out Forman Acton's NUMERICAL METHODS THAT WORK, chapter 4, for examples) is to make a transformation that stretches out the interval of integration in such a way that the singularity is just canceled. We do this here by changing to the variable of integration s , where

$$s = (t - u)^{1-d/2}$$

so that we can change to the better-behaved convolution integral form

$$D(t) = A \int_0^{\infty} P \left(t - s^{1-d/2} \right) ds$$

We will now connect this constant-power-law model with the damage data. That data is usually presented in the functional form $F = CW^\beta$, where F is the damage-level fluence (Joules/cm²), C is a material constant that sets the damage-level fluence, W is the pulse width (full width at half maximum), and β is a pulse-width scaling exponent. Each material (including its preparation and irradiation history), at each wavelength or wavelength combination, has its own particular values of C and β . Note that the units of C are the unusual combination energy*area⁻¹*time^{- β} . Curves fitted to experimental damage fluence as a function of time for NIF materials are shown in Fig. 1. The values of C and β for these materials are as follows:

Material	C	β
Anti-Reflection coatings at 1ω	23.7	0.4
High-Reflectivity coatings at 1ω	17.2	0.3
Laser glass at 1ω	21.8	0.4
Polarizer coating in reflection at 1ω	11.8	0.35
Polarizer coating in transmission at 1ω	10.0	0.35
KDP in Pockels' cell & doubler	24.0	0.5
KDP in tripler at 3ω	7.0	0.5
Anti-Reflection coatings at 3ω	11.2	0.4

Comparing the scalings with fluence of the diffusion integral and the data, we see that the damage exponent β is the same as $d/2$, where d is the effective dimensionality in the temporal convolution integral. This integral can therefore be written as

$$D(t) = A \int_0^{\infty} P \left(t - s^{1/(1-\beta)} \right) ds$$

Actual damage data has β mostly in the range from about 0.3 to about 0.6, so we conclude that the most dangerous absorbers must diffuse with an effective dimensionality from 0.6 to 1.2. We shall therefore restrict our attention to β values that lie between zero and one.

The data is taken with a nominally Gaussian pulse shape in time that has the form

$$G(t) = P_G e^{-\left(\frac{t}{qW}\right)^2}$$

where W is the "width" of the pulse, q is a numeric factor that converts W to the half-width at $1/e$, and P_G is the peak power density during the pulse. Our damage data is quoted with W equal to the full width at half maximum (FWHM), so we can find the numeric value of q from

$$\frac{1}{2} = e^{-\left(\frac{1}{2q}\right)^2} \Rightarrow q = \frac{1}{2\sqrt{\ln 2}} = 0.6005612...$$

and the fluence in the Gaussian is

$$F = \int_{-\infty}^{\infty} P_G e^{-\left(\frac{t}{qW}\right)^2} dt = P_G qW \int_{-\infty}^{\infty} e^{-u^2} du = P_G qW \sqrt{\pi}$$

This means that P_G is given by

$$P_G = \frac{F}{qW\sqrt{\pi}} = \frac{2F}{W} \sqrt{\frac{\ln 2}{\pi}} = \frac{F}{1.064467 W}$$

which means that a temporally square pulse with the same fluence and peak power density as a Gaussian pulse will have a width 6.4% wider than the FWHM of the Gaussian pulse.

Equating the fluence F to the damage level CW^β for a Gaussian pulse, we have

$$P_G = \frac{C}{qW^{1-\beta}\sqrt{\pi}}$$

Inserting a Gaussian pulse at the damage level into the diffusion integral which gives the value of D at the absorption point, we have the Gaussian damage integral

$$D_G(t) = \frac{AC}{qW^{1-\beta}\sqrt{\pi}} \int_0^\infty e^{-\left(\frac{t-s^{1/(1-\beta)}}{qW}\right)^2} ds$$

which can be normalized by changing to the (dimensionless) variable $r = (qW)^{\beta-1}s$ to give

$$D_G(t) = \frac{AC}{q^\beta \sqrt{\pi}} \int_0^\infty e^{-\left[\frac{t}{qW} - r^{1/(1-\beta)}\right]^2} dr$$

The behavior of $D_G(t)$ for various β values is shown in Fig. 6. For any specific value of β , there is a value of t/qW which maximizes $D_G(t)$. This value determines the level of D at which damage will occur for a Gaussian pulse. As we might expect, the maximum time comes a bit after the peak of the input power density. This is also the level D_D at which any pulse will damage. We therefore have

$$D_D = AC \max_t \left[\frac{1}{q^\beta \sqrt{\pi}} \int_0^\infty e^{-\left(\frac{t}{qW} - r^{1/(1-\beta)}\right)^2} dr \right] = ACK(\beta)$$

We find the maximum by differentiating the integral and setting the result to zero, since this is faster and more accurate than finding the maximum directly. We therefore solve

$$\int_0^\infty (\sigma - r^{1/(1-\beta)}) e^{-\left(\sigma - r^{1/(1-\beta)}\right)^2} dr = 0$$

by varying σ and evaluate the integral at the resulting σ_{MAX} value to get these results:

β	0	.1	.2	.3	.35	.4	.5	.6	.7	.8	.9	1
σ_{MAX}	∞	1.403	1.082	.8625	.7712	.6882	.5409	.4117	.2956	.1895	.0915	0
$K(\beta)$	1.000	.9074	.8509	.8141	.8015	.7924	.7835	.7867	.8022	.8311	.8757	.9394

The above maximum values of the integral (including the $q^\beta \sqrt{\pi}$ factor) can be numerically approximated to about ± 0.002 by the expression

$$K(\beta) = \max_\sigma \left[\frac{(2\sqrt{\ln 2})^\beta}{\sqrt{\pi}} \int_0^\infty e^{-\left(\sigma - r^{1/(1-\beta)}\right)^2} dr \right] \cong \frac{0.997651 + 0.125449\beta}{1 + (1.218987 - 1.022997\beta)\beta}$$

Suppose that we are given a specified power density as a function of time $P(t)$. The fluence Φ in this pulse is defined as

$$\Phi = \int_{-\infty}^\infty P(t) dt$$

and we define the **fluence-equivalent square-pulse duration** τ by the requirement that a pulse of width τ at the maximum power density during $P(t)$ have the same fluence as $P(t)$:

$$\Phi = \tau P_{MAX} \Rightarrow \tau = \frac{\Phi}{P_{MAX}}$$

We can now define the normalized pulse shape $n(t)$ as

$$n(t) = \frac{P(t)}{P_{MAX}} = \frac{\tau P(t)}{\Phi}$$

The damage convolution integral can be rewritten as

$$D(t) = \frac{A\Phi}{\tau} \int_0^\infty n(t - s^{1/(1-\beta)}) ds$$

which, by changing to the dimensionless variable of integration $r = \tau^{\beta-1}s$, becomes

$$D(t) = \frac{A\Phi}{\tau^\beta} \int_0^\infty n(t - \tau r^{1/(1-\beta)}) dr$$

Equating the maximum value of $D(t)$ reached during the pulse to the damage level D_D , we find the fluence at which the specified pulse shape damages at a given value of τ to be

$$\Phi = \frac{D_D \tau^\beta}{A \max_t \left[\int_0^\infty n(t - \tau r^{1/(1-\beta)}) dr \right]}$$

This illustrates the universal scaling, for any pulse shape, of damage fluence with pulse duration as τ^β which we promised above. We can normalize this via the Gaussian damage, which gave us D_D/A as

$$\frac{D_D}{A} = \frac{C}{q^\beta \sqrt{\pi}} \max_t \left[\int_0^\infty e^{-\left(\frac{t}{qW} - r^{1/(1-\beta)}\right)^2} dr \right]$$

and as a result (finally!) get the shaped-pulse damage level as

$$\Phi = C \tau^\beta \frac{\max_{\sigma} \left[\frac{(2\sqrt{\ln 2})^\beta}{\sqrt{\pi}} \int_0^\infty e^{-\left(\sigma - r^{1/(1-\beta)}\right)^2} dr \right]}{\max_t \left[\int_0^\infty n(t - \tau r^{1/(1-\beta)}) dr \right]}$$

Recall that the upper max in this expression is $K(\beta)$. The determination of the damage level of an arbitrarily-shaped pulse is reduced to the determination of the maximum value of the one-parameter integral

$$H(\beta, n) = \max_t \left[\int_0^\infty n(t - \tau r^{1/(1-\beta)}) dr \right]$$

Note that this integral is normalized so that it is equal to unity for a square pulse of any width.

In the general case, it is probably best to specify the integral in this form, because of the absence of the singularity in the temporal Green's function. However, in the common case where the power history $P(t)$ is given as a series of straight-line segments joined at their ends (piecewise-linear), we can return to the singular form

$$H(\beta, n) = \max_t \left[(1-\beta) \int_0^\infty \frac{n(t - \tau y)}{y^\beta} dy \right]$$

and do the integration analytically over each straight-line portion (note that y is t/τ). In each straight-line portion, the power is given by

$$P(t) = A_J t + B_J \quad t_J \leq t \leq t_{J+1}$$

where the coefficients A_J and B_J can be expressed in terms of the end-point values as

$$A_J = \frac{P_{J+1} - P_J}{t_{J+1} - t_J} \quad B_J = P_J - t_J A_J$$

The integral is then the sum of terms of the form

$$S_J = (A_J t + B_J) \left(y_2^{1-\beta} - y_1^{1-\beta} \right) - A_J \tau \left(\frac{1-\beta}{2-\beta} \right) \left(y_2^{2-\beta} - y_1^{2-\beta} \right)$$

NIF 3 ω Output Pulse

The shape of the piecewise-linear integral as a function of time is shown in Fig. 7 for the 3 ω pulse shape needed for the outer cone of beamlets, for β values representative of the NIF laser materials. This pulse has an equivalent square pulse width value of 3.443 ns. Based on the maximum values $H(\beta, n)$, we find the damage levels for this pulse shape from the formula

$$\Phi = C \tau^\beta \frac{K(\beta)}{H(\beta, n)} = C R(\beta, n)$$

β	.3	.35	.4	.45	.5
$K(\beta)$	.8141	.8015	.7924	.7864	.7835
$H(\beta, n)$	.8888	.8715	.8540	.8363	.8203
$R(\beta, n)$	1.327	1.418	1.521	1.640	1.772

For example, anti-reflection coatings (ARC) at 3 ω damage as $CW^\beta = 13.7W^{0.4}$, so the outer-cone 3 ω pulse will damage ARC at $13.7 \times 1.521 = 20.84 \text{ J/cm}^2$. If the same fluence were incident in a Gaussian pulse, the FWHM of that pulse would have to be

$$W = \tau \left(\frac{K(\beta)}{H(\beta, n)} \right)^{1/\beta} = R^{1/\beta} = 3.443 \left(\frac{0.7924}{0.8540} \right)^{1.0/0.4} \text{ ns} = 2.855 \text{ ns}$$

in order to damage ARC at the same level as the shaped pulse. For purposes of ARC damage at 3 ω , the shaped pulse is equivalent to a Gaussian pulse with the same fluence, and with a FWHM of 2.855 ns. Given this equivalent pulse width, we can look up the damage in a chart or table of existing experimental ARC 3 ω damage values for Gaussian pulses.

NIF 1 ω Output Pulse

=== Need more here - conversion from Renard & Auerbach ===

NIF 1ω Polarizer Pulse

=== Need more here - Renard ===

NIF 1ω Injection Mirror Pulse

=== Need more here - Renard ===

Rising Exponential Pulse

Another interesting pulse shape to investigate is a rising exponential, since this shape approximates the shape of the laser pulse early in the chain but allows analytic calculation. Write such a pulse as

$$P(t) = e^{t/L} \quad t < 0$$

and find the fluence-equivalent square pulse width as

$$\tau = \frac{\int_{-\infty}^0 e^{t/L} dt}{e^0} = L$$

which leads to the normalized pulse

$$n(t) = e^{t/\tau} \quad t < 0$$

The H integral is therefore given by

$$H_E(\beta, n) = (1 - \beta) \int_0^\infty \frac{e^{-y}}{y^\beta} dy = (1 - \beta) \Gamma(1 - \beta)$$

which leads to the values

β	.3	.35	.4	.45	.5
$K(\beta)$	.8141	.8015	.7924	.7864	.7835
$H_E(\beta, n)$	.9086	.9001	.8935	.8889	.8862

Note that the rising exponential pulse shape is slightly safer than the outer-cone pulse, in the sense that a rising exponential with the same fluence-equivalent pulse width as the outer-cone shape can run at about 1.05 times the fluence of the outer-cone shape. On the other hand, the

rising exponential is about 10% more damage-prone than a square pulse (which we recall has H equal to unity for all values of β).

Conclusions

=== Need more here ===

A handwritten signature in cursive script, reading "John B. Trenholme", is written over a horizontal line.

John B. Trenholme

DATE

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10 / 17 / 94

END