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SIMULATION OF THE TIME DEPENDENT FARADAY EFFECT FOR SYNCHROTRON RADIATION

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Synchrotron radiation is simulated by a broad Lorentzian in a calculation of the time-dependent Faraday effect in ^{57}Fe . An extension of the method of Hamermesh to the multi-line case is used.

The Faraday Effect in Mössbauer spectroscopy was calculated by Blume and Kistner [1] for a mixed multiple transition. Many elements of the present treatment are similar, but we are concerned with something additional, namely its development in time. For the case of interest, time is measured from the reception of a broad-band burst of synchrotron radiation, here represented by a broad Lorentzian. Our absorber is ^{57}Fe with magnetic hyperfine splitting including a possible symmetric field gradient. The pure quadrupole case can also be treated. There is a unique sample axis z' , and the (polarized) gamma-rays come in along z , related to z' by a rotation by angle θ about y' . For the Faraday effect, θ is only 0 or π .

The classical Faraday effect is the rotation of the plane of polarized light in passing through a medium in a magnetic field. The rotation angle ξ is related to the field B and the sample length L by $\xi = V B L$, where V , the Verdet constant, depends on the material and the wavelength. In a simple case the medium has a resonance split by the field, say $J = 0 \rightarrow J = 1$. One of the $\Delta J = \pm 1$ transitions can be excited by right circular polarization, and the other by left. In the usual experimental condition, the index of refraction for the two circular parts of the incident linearly polarized light is different, resulting in a phase difference and a rotation of the plane of the reconstituted linear polarization. Well off resonance, the normal case, the real part of the propagation constant which decreases as $1/\Delta\omega$ dominates over the imaginary part ($1/\Delta\omega^2$).

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The time dependent Faraday effect in ^{57}Fe is quite different, especially for the broad sources that we are mainly considering. As a practical matter, the imaginary part is not negligible, and there is a considerable change in the spectrum in the transit.

The electric field vector of a synchrotron source is simulated by a Lorentzian amplitude of width Q_γ . It propagates in the direction of the field B through a magnetically split absorber of thickness L . The absorber nuclei have width γ . The propagation is characterized by a matrix $\tilde{K}$ with elements labeled by polarization in and out. The matrix of the transmitted amplitude $\tilde{A}$ in a circular or a linear polarization basis and the squares of its elements and appropriate graphics are the useful outputs of our calculation. If frequency differences are measured in units of the line width γ and time $\tau = \gamma t$:

$$\tilde{A}(t) = -\left(Q^{1/2} / 2\pi i\right) \int_{-\infty}^{+\infty} dz \left\{ \exp(-iz\tau) / (z + iQ/2) \right\} \exp(i\tilde{K}L) \quad , \quad (1)$$

factors of unit norm common to all amplitude elements are omitted. $\tilde{K}$ or the index of refraction $\tilde{n}$ are simply related to the coherent forward scattering matrix $\tilde{F}$ and the density N of scatterers by [2]:

$$\tilde{n} = \tilde{K} / K_0 = 1 + 2\pi N \tilde{F} / K_0^2 \quad (2)$$

and by use of the elements of $\tilde{F}$ in the circular basis as given by Trammell [3] and by Hannon, Trammell et al. [4], one has:

$$i\tilde{K}_{p'p}L = -i\frac{3}{16}\beta \sum_{R=1}^6 \frac{C_R^2 d_{p'M}^1(\theta) d_{pM}^1(\theta)}{z - z_R + i/2} \quad (3)$$

which with Eq. (1) above yields Eq. (1) of our previous paper [4] (P & P I) -- (editor please insert reference) The symbols p', p ($= \pm 1$, p' incident, p emergent, $+1 = \text{r.h.}$) are circular polarization indices. The $d_{ij}^1(\theta)$ are the dipole rotation matrices as given by Rose [5] and tabulated in Fig. 1a of P & P I, R is the line number and C_R its associated Clebsch-Gordan coefficient. $M = 0, \pm 1$ are photon quantum numbers along the z' axis. The quantity β is the usual Mossbauer effect thickness parameter $N \sigma_0 fL$, and here is the absorption exponent of the intensity at the central frequency of a coalesced spectrum. As

pointed out in P & P I, in an optical description like this, in media as dense as normal samples, summing over the wave front removes the incoherence associated with the two members of the ground state multiplet.

The integral is evaluated numerically by a program written in the Speakeasy language [6].

In what follows the angle $\theta = 0$ and the hyperfine field, $B = 330$ kG., serves in lieu of an external magnetic field. We have used a broad source line ($Q = 100$) which overlaps the entire spectrum ($-59.04 \leq z \leq +59.04$). In Fig. 1a we show in the same plot the time-dependent change in phase for incident R and for incident L circular polarization after passing through the absorber. If we write: $A_{RR} = |A_{RR}| \exp(i\xi_{RR})$ and similarly for A_{LL} , we plot in Fig. 1a ξ_{RR} (solid line) and ξ_{LL} (dashed line) vs. τ . The source is centered in the six-line pattern. We note three things: a) the angles are equal and opposite for the two helicities as we expect from the symmetry; b) the patterns have an odd shape, there is clearly interference between the amplitudes for the two levels excitable by each polarization; and c) the phase keeps changing with time, ie the transmitted radiation is different in frequency from that incident. The phenomenon is simplified by setting to zero the cross-sections of the four innermost lines, so that only one resonance is excited for each helicity. In Fig. 6b we see that after a short interval associated with the rapidly decaying source, $\xi_{RR} = -\xi_{LL} = z_1 = -z_6$, the separation of line 1 or line 6 from the center of the pattern. It is mainly an absorptive rather than a dispersive effect as we can demonstrate with a simple model. Let $S(z)$ in the frequency domain be the Fourier transform of $S(\tau)$, the source amplitude in the time domain, and because of the absorption at z_1 subtract a fraction ϕ of unit width. We can write for the transmitted amplitude:

$$A(\tau) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} dz \exp(-iz\tau) S(z) \{1 - \phi \delta(z - z_1)\} \quad (4)$$

$$= S(\tau) - \phi S(z_1) \exp(-iz_1\tau) \quad (5)$$

but $S(\tau) = Q^{1/2} \exp(-Q/2 - iz)\tau$ and decays rapidly so that

$$\xi = \arg\{A(\tau)\} = -z_1\tau + \text{const.} \quad \text{and} \quad \dot{\xi} = -z_1. \quad (6)$$

after the source has decayed.

The odd shape occurs because what is plotted, for example, is $\arg[A_{LL}(1) + A_{LL}(4)]$, the sum of the complex amplitudes for lines 1 and 4, for both of which $M = -1$ and which have opposite signs of ξ_{LL} . The magnitude $|A_{LL}(1)| - |A_{LL}(4)|$ is positive below about $\tau = 1.2$ and negative above. The larger magnitude dominates and the effect of the smaller is to modulate the argument in the neighborhood of the cross-over, as seen in Fig. 1(a).

When as here, $A_{RL} = A_{LR} = 0$, the linear amplitudes are:

$$A_{xx} = (A_{RR} + A_{LL})/2, \quad A_{xy} = i(A_{RR} - A_{LL})/2.$$

From Fig. 1(a) we can write: $A_{RR} = \text{const. exp}(i\xi)$, $A_{LL} = \text{const. exp}(-i\xi)$. The constants are equal by symmetry, A_{xx} and A_{xy} are real, and

$$\cos \xi = A_{xx} / (T_{xu})^{1/2}, \quad \sin \xi = A_{xy} / (T_{xu})^{1/2}. \quad (7)$$

We plot equation (7) in Fig. 1c. The solid line is the cosine and the dashed is the sine. The sine initially lags by $\pi/2$ showing that the initial rotation angle is negative and after the kink the sine leads, showing that the angle is becoming more positive.

In Fig. 1d we plot the rotation angle directly from:

$$\xi = \arctan(A_{xy} / A_{xx}). \quad (8)$$

The angle continues to change with time and only becomes finite because of the decay of the transmitted intensity. All plotted functions of τ are in fact functions of $\tau - \tau_0$, the time since the synchrotron pulse. In the classical Faraday effect, τ_0 is not measured and must be averaged over. The quantity, ξ_{AV} the dashed line, is defined by:

$$\xi_{AV} = \int_0^\tau d\tau \exp(-\tau) \xi(\tau). \quad (9)$$

$\xi(\tau)$ at $\tau \gg 1$ would be the angle assigned as the Faraday rotation angle. It is only approximate because the absorber decay is not a simple exponential. In this case it is about -35 radians.

Some interesting points emerge from these simulations: a) One usually thinks of a transmission experiment as yielding absorption spectra. That is the case here if one looks only during the decay time of the source, and will also be true if one does not measure time at all. But in these simulations, after a short time the lines are present in emission. They have the directional properties appropriate to the collimation of the incident radiation, because the scattering phases are determined by the wave fronts that passed through the medium. b) The rotation angle is strictly proportional to B , because the line spacings are. It does not depend on L (or β) at least not proportionally. (The only such dependence is minor and depends on the location of the kink in Fig. 1a). This is because it is mainly the imaginary part of the refractive index that is causing the effect. If the entire broad incident spectrum were shifted far off resonance, the Faraday rotation would be like the classical effect, and very small.

Siddons, Bergmann and Hastings [7] have recently published an experiment with an analysis that shows some of the features of the present calculations.

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FIGURE CAPTIONS

- Fig. 1. (a) Phase angle change vs. time of flight (full line) or left (dashed line) circularly polarized radiation transmitted through the sample. The source is centered in the hyperfine pattern.
- (b) Same as (a) for a fictitious absorber containing only lines 1 and 6.
- (c) Plot of $A_{xx}/A(T_{xu})^{1/2}$ (solid line), and $A_{xy}/(T_{xu})^{1/2}$ (dashed line).
- (d) Plot of $\arctan(A_{xy}/A_{xx})$ (solid line), and ξ_{AV} which takes into account the decay of the transmitted intensity.

FARADAY ROTATION $\theta = 0$, $\beta = 10$, $Q = 300$, $B_{\text{hyp}} = 330$ kG

Fig. 1

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