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Parallel Mathematical Software

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MASTER

Parallel Software for Nonlinear Systems of Equations

Mathematical background for homotopy algorithms.

The nonlinear systems of equations arising in circuit simulation, structural optimization, closed loop optimal control, chemical engineering of distillation systems, combustion chemistry, CAD/CAM modelling, robotics, computer vision, and orbital mechanics have several properties that make them especially amenable to homotopy methods. Even so, the homotopy zero curves are not trivial to track, and sophisticated curve tracking techniques are sometimes required. The size of typical engineering problems also presents some interesting numerical linear algebra challenges, and the proposed work would develop parallel sparse matrix techniques specifically tailored to the sparsity structures corresponding to the mentioned problem areas, in the context of homotopy algorithms.

The original mathematical model, after some sort of discretization, approximation, or reduction, ultimately leads to a nonlinear system of equations

$$F(x) = 0,$$

where $F : E^n \rightarrow E^n$ is assumed to be a C^2 map. Suppose there exists a C^2 map

$$\rho : E^m \times [0, 1) \times E^n \rightarrow E^n$$

such that

- 1) the $n \times (m + 1 + n)$ Jacobian matrix $D\rho(a, \lambda, x)$ has rank n on the set

$$\rho^{-1}(0) = \{(a, \lambda, x) \mid a \in E^m, 0 \leq \lambda < 1, x \in E^n, \rho(a, \lambda, x) = 0\},$$

and for any fixed $a \in E^m$, letting $\rho_a(\lambda, x) = \rho(a, \lambda, x)$,

- 2) $\rho_a(0, x) = 0$ has a unique solution x_0 ,
- 3) $\rho_a(1, x) = F(x)$,
- 4) $\rho_a^{-1}(0)$ is bounded.

Then the supporting theory [4, 25, 31] says that for almost all $a \in E^m$ there exists a zero curve γ of ρ_a , along which the Jacobian matrix $D\rho_a$ has rank n , emanating from $(0, x_0)$ and reaching a zero $\bar{x}$ of F at $\lambda = 1$. γ does not intersect itself and is disjoint from any other zeros of ρ_a . The globally convergent algorithm is to pick $a \in E^m$ (which uniquely determines x_0), and then track the homotopy zero curve γ .

There are many different algorithms for tracking the zero curve γ ; the previous proposal discussed three such algorithms: ordinary differential equation based, normal flow, and augmented Jacobian matrix. The descriptions of these algorithms are now in the literature for the software package HOMPACK [32], so will not be repeated here. The development of sparse homotopy algorithms within HOMPACK specifically tailored for various parallel machines (e.g., hypercube, shared memory, and vector) and problem areas (e.g., circuit simulation, structural optimization, optimal control, and combustion chemistry) remains a major goal of this research.

Accomplishments under DOE Grant DE-FG05-88ER25068.

Nonlinear systems of equations can be roughly classified into one of three types: 1) small polynomial systems for which all solutions are sought; 2) small systems of order up to several hundred, where the Jacobian matrix can be treated as full; and 3) large systems whose Jacobian matrices are sparse, and the sparse matrix structures must be dealt with explicitly. Other possibilities such as large polynomial systems or large dense systems are intractable, and will not be considered further. There are always intermediate size problems, and whether these are treated as type 2) or type 3) is frequently a matter of algorithmic choice or available computer technology. HOMPACK contains several homotopy algorithms for problems in each of classes 1), 2), and 3).

The serial homotopy algorithms for types 1) and 2) are well known and generally accepted. Sparse matrix technology is an active research area, however, so the HOMPACK sparse matrix codes for type 3) problems are by no means the best available, although there is evidence that the HOMPACK serial sparse codes are competitive. Progress under DOE Grant DE-FG05-88ER25068 on parallel code for each of the three problem types will be described in turn.

Polynomial systems.

Parallel algorithms for polynomial systems have been extensively studied by the PI's, their colleagues, and their students. Computer code has been developed and tested for the parallel shared memory machines Encore Multimax, Alliant FX/8, Elxsi 6400, Sequent Symmetry S81, and the distributed memory Intel iPSC/1 hypercube. This work resulted in a Ph.D. thesis

Amal Chakraborty, "Parallel homotopy curve tracking on a hypercube", Ph.D. thesis, Dept. of Computer Sci., VPI & SU, Blacksburg, 1990;

and the conference and journal papers

- A. P. Morgan and L. T. Watson, "A globally convergent parallel algorithm for zeros of polynomial systems", *Nonlinear Anal.*, 13 (1989) 1339-1350;
- W. Pelz and L. T. Watson, "Message length effects for solving polynomial systems on a hypercube", *Parallel Comput.*, 10 (1989) 161-176;
- A. P. Morgan, A. J. Sommese, and L. T. Watson, "Finding all isolated solutions to polynomial systems using HOMPACK", *ACM Trans. Math. Software*, 15 (1989) 93-122;
- D. C. S. Allison, S. Harimoto, and L. T. Watson, "The granularity of parallel homotopy algorithms for polynomial systems of equations", *Internat. J. Comput. Math.*, 29 (1989) 21-37;
- D. C. S. Allison, A. Chakraborty, and L. T. Watson, "Granularity issues for solving polynomial systems via globally convergent algorithms on a hypercube", *J. Supercomputing*, 3 (1989) 5-20;
- L. T. Watson and A. P. Morgan, "Polynomial programming using multi-homogeneous polynomial continuation", *J. Comput. Appl. Math.*, 42 (1992) .
- D. C. S. Allison, A. Chakraborty, and L. T. Watson, "Granularity issues for solving polynomial systems via globally convergent algorithms on a hypercube", in *Proc. Third Conf. on Hypercube Concurrent Computers and Applications*, G. C. Fox (ed.), ACM, Pasadena, CA, 1988, 1463-1472;
- D. C. S. Allison, S. Harimoto, and L. T. Watson, "The granularity of parallel homotopy algorithms for polynomial systems of equations", in *Proc. 1988 Internat. Conf. on Parallel Processing*, Vol. III, D. H. Bailey (ed.), St. Charles, IL, 1988, 165-168;

S. Harimoto and L. T. Watson, "The granularity of homotopy algorithms for polynomial systems of equations", in *Parallel Processing for Scientific Computing*, G. Rodrigue (ed.), SIAM, Philadelphia, PA, 1989, 115-120.

For the case of polynomial systems, the theory, algorithms, and serial computer code (POLSYS in HOMPACk) are all in good shape. Reasonably good parallel computer codes now exist for both shared memory (Sequent Symmetry) and distributed memory multiprocessors (iPSC/1 - an iPSC/860 version is planned). The original intent was to develop machine independent production quality parallel homotopy code, comparable to the serial HOMPACk code. However, the absence of a truly portable parallel language, or even an emerging standard, makes it clear that this goal is premature. Currently, the most prudent course would appear to be two pronged: (1) to develop parallel mathematical algorithms independent of any machine architecture, yet guided by the general constraints of the shared memory or distributed memory paradigm, and (2) to develop and distribute application-specific and machine-specific code based on HOMPACk. (Subsets of HOMPACk tailored to circuit simulation and control design have been delivered to AT&T and Harris Corporation, respectively.)

Small dense nonlinear systems.

Parallelism for small dense nonlinear systems of equations is much more difficult to exploit. For homotopy curve tracking, there are only three potential sources of parallelism: the algorithm logic, the linear algebra, and the function/derivative evaluations. The parallelism in the algorithm logic appears negligible. Both the linear algebra and function evaluation questions are tough. On the Sequent S81, the overhead cost of setting up a DOACROSS for a matrix multiply is greater than the cost of just doing it serially for small matrices ($n \approx 10$), and on the iPSC/1 the communication cost outweighs the savings until $n > 50$. Similarly function/derivative evaluations must be sufficiently costly to justify splitting them across processors. There are subtle interactions too—on the iPSC, if it pays to parallelize the Jacobian matrix evaluation, then the linear algebra should *also* be done in parallel, even if the linear algebra done serially is faster. The reason is that the cost of *collecting* the matrix components into a single processor for serial linear algebra, and then doing that linear algebra serially, is (usually) more expensive than leaving the matrix components distributed and doing the linear algebra in parallel. For the iPSC hypercube, a comprehensive set of experiments was conducted comparing data decomposition strategies, different linear algebra algorithms, the effect of function evaluation cost and probabilistic distribution, parallel paradigms, and the interaction of all these factors. The results have been reported in

Amal Chakraborty, "Parallel homotopy curve tracking on a hypercube", Ph.D. thesis, Dept. of Computer Sci., VPI & SU, Blacksburg, 1990;

A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "Note on unit tangent vector computation for homotopy curve tracking on a hypercube", *Parallel Comput.*, 17 (1991) 1385-1395;

A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "The parallel complexity of embedding algorithms for the solution of systems of nonlinear equations", *IEEE Trans. Parallel Distrib. Sys.*, to appear;

A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "Parallel orthogonal decompositions of rectangular matrices for curve tracking on a hypercube", in *Proc. Fourth Conf. on Hypercubes, Concurrent Computers, and Applications*, J. L. Gustafson (ed.), ACM, Monterey, CA, 1989, 651-654;

- A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "Parallel unit tangent vector computation for homotopy curve tracing on a hypercube", in *Proc. 1990 ACM Eighteenth Annual Computer Science Conf.*, Washington, DC, 1990, 103-108;
- A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "Low dimensional homotopy curve tracking on a hypercube", in *Proc. 1990 Internat. Conf. on Parallel Processing*, Vol. III, P.-C. Yew (ed.), St. Charles, IL, 1990, 44-51;
- A. Chakraborty, D. C. S. Allison, C. J. Ribbens, and L. T. Watson, "Parallel homotopy curve tracking on a hypercube", in *Parallel Processing for Scientific Computing*, J. Dongarra, P. Messina, D. C. Sorensen, and R. G. Voigt (eds.), SIAM, Philadelphia, PA, 1990, 149-153;
- L. T. Watson, "Numerical analysis of nonlinear equations in computer vision and robotics", in *Numerical Linear Algebra, Digital Signal Processing and Parallel Algorithms*, NATO ASI Series F, G. Golub and P. Van Dooren (eds.), Springer-Verlag, Berlin, 1990, 695-704.

For the code in HOMPACk addressing type 2) problems, a parallel version for the iPSC/1 (the harder case, compared to the Sequent) has been developed, and an iPSC/860 version is planned. The iPSC/1 parallel code for type 2) problems is listed in Intel's Supercomputer Systems Division 1991 Software Resource Catalog. Parallel code for shared memory machines (e.g., Sequent) and vector machines (e.g., Cray 2) is under development, the latter in collaboration with William Thacker of Winthrop College. Dr. Thacker spent 1992-3 on sabbatical leave at VPI&SU, vectorized HOMPACk's POLSYS routine then, and plans to continue vectorizing other parts of HOMPACk.

Large sparse nonlinear systems.

Type 3) problems, and the concomitant sparse matrix technology, have received much of the principal investigators' attention during the current grant. Convergence theory, parallel algorithms, and practical implementation details have all received attention—sparse matrices are likely to remain a research interest of the principal investigators for some time to come. This work has resulted in two M.S. theses, three solid journal articles, and several conference papers:

- Kashmira Irani, "Preconditioned sequential and parallel conjugate gradient algorithms for homotopy curve tracking", M.S. thesis, Dept. of Computer Sci., VPI & SU, Blacksburg, 1990;
- William D. McQuain, "Iterative methods for sparse homotopy curve tracking", M.S. thesis, Dept. of Computer Sci., VPI & SU, Blacksburg, 1992;
- K. M. Irani, M. P. Kamat, C. J. Ribbens, H. F. Walker, and L. T. Watson, "Experiments with conjugate gradient algorithms for homotopy curve tracking", *SIAM J. Optim.*, 1 (1991) 222-251;
- C. deSa, K. M. Irani, C. J. Ribbens, L. T. Watson, and H. F. Walker, "Preconditioned iterative methods for homotopy curve tracking", *SIAM J. Sci. Stat. Comput.*, 13 (1992) 30-46;
- D. C. S. Allison, K. M. Irani, C. J. Ribbens, and L. T. Watson, "High dimensional homotopy curve tracking on a shared memory multiprocessor", *J. Supercomputing*, 5 (1991) 347-366;
- K. M. Irani, C. J. Ribbens, and L. T. Watson, "Parallel HOMPACk: a case study in parallel mathematical software", in *Parallel Processing for Scientific Computing*, J. Dongarra, K. Kennedy, P. Messina, D. C. Sorensen, R. G. Voigt (eds.), SIAM, Philadelphia, PA, 1992, 643-648;
- D. C. S. Allison, K. M. Irani, C. J. Ribbens, and L. T. Watson, "Shared memory parallel algorithms for homotopy curve tracking", in *Proc. 1991 Internat. Conf. on Parallel Processing*, Vol. III, K. So (ed.), CRC Press, Boca Raton, FL, 1991, 17-20;

- D. Žigić, E. G. Collins, S. Richter, and L. T. Watson, "A homotopy method for solving optimal projection equations for the reduced order model problem", in *Proc. IEEE Southeastcon*, Williamsburg, VA, April, 1991, 1193–1197;
- W. I. Thacker and L. T. Watson, "Vectorization of homotopy algorithms for polynomial systems", in *Parallel Processing for Scientific Computing*, R. F. Sincovec, D. E. Keyes, M. R. Leuze, L. R. Petzold, and D. A. Reed (eds.), SIAM, Philadelphia, PA, 1993, 610–613;
- D. Žigić, L. T. Watson, E. G. Collins, Jr., and L. D. Davis, "A homotopy method for solving Riccati equations on a shared memory parallel computer", in *Parallel Processing for Scientific Computing*, R. F. Sincovec, D. E. Keyes, M. R. Leuze, L. R. Petzold, and D. A. Reed (eds.), SIAM, Philadelphia, PA, 1993, 614–617.

Applications.

Collaboration with Robert Melville and Ljiljana Trajkovic of AT&T Bell Laboratories on the large nonlinear systems of equations arising in circuit simulation has had a significant impact. AT&T developed a large circuit simulation code based on HOMPAC, and supplied the DOE project with realistic large ($n > 10,000$) test problems. These sparse nonlinear systems ranged in size from $n = 20$ to $n > 10,000$, and had essentially random sparsity patterns. Typical matrix iterative methods developed for solving discretizations of PDEs have been woefully inadequate on the circuit simulation problems. It is clear that both parallelism and new sparse matrix algorithms will be required to solve realistic large circuit simulation problems. This collaboration with AT&T is ongoing, and has produced significant results some of which are embodied in

- R. C. Melville, Lj. Trajković, S.-C. Fang, and L. T. Watson, "Globally convergent homotopy methods for the DC operating point problem", *IEEE Trans. Comput. Aided Design*, to appear,
- R. Melville, S. Moinian, P. Feldmann, and L. Watson, "Sframe: An efficient system for detailed DC simulation of bipolar analog integrated circuits using continuation methods", *Analog Integrated Circuits Signal Processing*, to appear;
- W. D. McQuain, C. J. Ribbens, L. T. Watson, and R. C. Melville, "Preconditioned iterative methods for sparse linear algebra problems arising in circuit simulation", *SIAM J. Sci. Stat. Comput.*, submitted.

Collaboration with Emmanuel Collins at Harris Corporation (Melbourne, FL) and Dennis Bernstein at the University of Michigan (Ann Arbor) on optimal control has also influenced our attitude toward parallel algorithms. Controller design problems at Harris are very compute intensive and our homotopy algorithms are now routinely being used at Harris. A significant challenge is to incorporate parallel computing technology into the tinkering MATLAB environment in which practicing engineers work. A problem solving environment for controller design (similar to the prototype for ELLPACK developed by Auvil, Ribbens, and Watson) blending workstation graphics, MATLAB, and algorithm execution on a powerful parallel machine is what is really needed, and is currently being pursued in collaboration with Harris.

This work has resulted in one M.S. thesis

- Yuzhen Ge, "Homotopy algorithms for the H^2 and the combined H^2/H^∞ model order reduction problems", M.S. thesis, Dept. of Computer Sci., VPI & SU, Blacksburg, 1993;

and several journal and conference papers:

- G. Vasudevan, L. T. Watson, and F. H. Lutze, "Homotopy approach for solving constrained optimization problems", *IEEE Trans. Automat. Control*, 36 (1991) 494-498;
- D. Žigić, L. T. Watson, E. G. Collins, Jr., and D. S. Bernstein, "Homotopy methods for solving the optimal projection equations for the H_2 reduced order model problem", *Internat. J. Control*, 56 (1992) 173-191;
- D. Žigić, L. T. Watson, E. G. Collins, Jr., and D. S. Bernstein, "Homotopy approaches to the H_2 reduced order model problem", *J. Math. Systems, Estimation, Control*, 3 (1993) 173-205;
- D. Žigić, L. T. Watson, and C. A. Beattie, "Contragredient transformations applied to the optimal projection equations", *Linear Algebra Appl.*, 188 (1993);
- Y. Ge, E. G. Collins, Jr., L. T. Watson, and L. D. Davis, "An input normal form homotopy for the L^2 optimal model order reduction problem", *IEEE Trans. Automat. Control*, submitted;
- Y. Ge, E. G. Collins, Jr., L. T. Watson, and L. D. Davis, "Minimal parameter homotopies for the L^2 optimal model order reduction problem", *Internat. J. Control*, submitted;
- D. Žigić, L. T. Watson, E. G. Collins, Jr., and D. S. Bernstein, "Effect of initial system on homotopy methods for the H_2 reduced order model problem", in *Proc. First IEEE Conf. on Control Applications*, Vol. 1, IEEE, New York, 1992, 252-257;
- Y. Ge, L. T. Watson, E. G. Collins, Jr., and L. D. Davis, "Computationally efficient homotopies for the H^2 model order reduction problem", in *Linear Algebra for Large Scale and Real-time Applications*, NATO ASI Series E, Applied Sciences, Vol. 232, M. S. Moonen, G. H. Golub, and B. L. R. DeMoor (eds.), Kluwer, Dordrecht, 1993, 385-386;
- Y. Ge, E. G. Collins, L. T. Watson, and D. S. Bernstein, "A homotopy algorithm for the combined H^2/H^∞ model reduction problem", in *Proc. IEEE Southeastcon '93*, IEEE, Piscataway, NJ, 1993, T2B: 22-25.

An important application area for both ELLPACK and HOMPAC is fluid mechanics. Several challenging fluid mechanics problems were solved; they pointed out some weaknesses in both packages, especially the need for new and better sparse matrix algorithms. The fluid mechanics problems are reported in

- C. J. Ribbens, C. Y. Wang, L. T. Watson, and K. A. Alexander, "Vorticity induced by a moving elliptic belt", *Comput. & Fluids*, 20 (1991) 111-119;
- C. J. Ribbens, L. T. Watson, and C. Y. Wang, "Steady viscous flow in a triangular cavity", *J. Comput. Phys.*, submitted;
- W. D. McQuain, C. J. Ribbens, C. Y. Wang, and L. T. Watson, "Steady viscous flow in a trapezoidal cavity", *Comput. & Fluids*, submitted.

Another major application area for ELLPACK and HOMPAC is solid and structural mechanics, especially involving composite materials. In cooperation with structural and materials engineers on campus and at NASA Langley Research Center, we have also applied our mathematical software to the solid mechanics problems reported in

- V. Arun, C. F. Reinholtz, and L. T. Watson, "Application of new homotopy continuation techniques to variable geometry trusses", *J. Mech. Design*, 114 (1992) 422-427;
- L. T. Watson and C. Y. Wang, "Large deformations of rotating polygonal space structures", *Comput. Math. Appl.*, 22 (1991) 51-59;

- C. Y. Wang and L. T. Watson, "Large deformations of a whirling elastic cable", *Acta Mech.*, 87 (1991) 47-57;
- J. Rakowska, R. T. Haftka, and L. T. Watson, "An active set algorithm for tracing parametrized optima", *Structural Optim.*, 3 (1991) 29-44;
- L. T. Watson, "Globally convergent homotopy algorithms for nonlinear systems of equations", *Nonlinear Dynamics*, 1 (1990) 143-191;
- J. Rakowska, R. T. Haftka, and L. T. Watson, "Multi-objective control-structure optimization via homotopy methods", *SIAM J. Optim.*, to appear;
- J. Rakowska, R. T. Haftka, and L. T. Watson, "Tracing the efficient curve for multi-objective control-structure optimization", *Comput. Systems Engrg.*, 2 (1991) 461-472;
- W. I. Thacker, C. Y. Wang, and L. T. Watson, "The nonlinear stability of a heavy rigid plate supported by flexible columns", *Internat. J. Solids and Structures*, submitted.

Parallel Software for Elliptic Partial Differential Equations

In this section we review work under DOE Grant DE-FG05-88ER25068 in three major areas: parallel ELLPACK, domain decomposition algorithms, and parallel scientific computing environments.

Parallel ELLPACK.

A version of ELLPACK targeted specifically for shared memory parallel machines has been designed and implemented on a Sequent Symmetry S81 computer. This version of ELLPACK contains a representative selection (18 modules) of the problem solving modules in the original ELLPACK system as well as several new modules, including domain decomposition preconditioners and a GMRES iterative solver. The FORTRAN code defining the parallel versions of these modules will be made available on netlib. Relevant publications include:

- C. J. Ribbens, "On parallel ELLPACK for shared memory machines", Department of Computer Science Report TR 90-46, Virginia Polytechnic Institute and State University, 1990.
- K. M. Irani, C. J. Ribbens, and L. T. Watson, "Parallel HOMPACT: a case study in parallel mathematical software," in *Parallel Processing for Scientific Computing*, J. Dongarra, et al. (eds.), SIAM, Philadelphia, 1992, pp 643-648.
- C. J. Ribbens, L. T. Watson, and C. deSa, "Toward parallel mathematical software for elliptic partial differential equations," *ACM Trans. Math. Software*, to appear.
- C. J. Ribbens and G. G. Pitts, "Strategies for parallelizing PDE software," in *Advances in Computer Methods for Partial Differential Equations VII*, R. Vichnevetsky, D. Knight, and G. Richter (eds.), IMACS, New Brunswick, N.J., 1992, pp 615-621.
- C. J. Ribbens, G. G. Pitts, and L. T. Watson, "Parallel ELLPACK for shared memory multiprocessors," *Comput. Systems Engrg.*, submitted.

Domain decomposition.

Our work has concentrated on the implementation and performance of DD algorithms on realistic problems and on real machines. A better understanding of the influence of memory paradigm (shared vs. distributed), of scalability, and of the usefulness of parallel programming tools for such algorithms has been achieved. We have also studied DD methods which use simple adaptive grid techniques in determining a good decomposition. The influence of adaptation on the DD approach, particularly on the performance of preconditioners, has also been studied. Results in this area are described in

- J. Gamble and C. J. Ribbens, "Performance comparison of three parallel implementations of a Schwarz splitting algorithm", Department of Computer Science Report TR 89-37, Virginia Polytechnic Institute and State University, 1989;
- C. J. Ribbens and L. T. Watson, "The parallel performance of Schwarz splitting", Department of Computer Science Report TR 90-34, Virginia Polytechnic Institute and State University, 1990;
- C. J. Ribbens, "A moving mesh scheme for adaptive domain decomposition," in *Unstructured Scientific Computation on Scalable Multiprocessors*, P. Mehrotra, J. Saltz, and R. Voigt (eds.), MIT Press, Cambridge, MA, 1992, pp 205-219.

C. J. Ribbens, "On adaptive domain decomposition with moving subdomains," in *Fifth International Symposium on Domain Decomposition Methods for Partial Differential Equations*, D. Keyes, et al. (eds.), SIAM, Philadelphia, 1992, pp 322-329.

G. G. Pitts, C. J. Ribbens, and L. T. Watson, "Domain decomposition and high order finite differences for elliptic PDEs", in *Parallel Processing for Scientific Computing*, R. F. Sincovec, et al., (eds.), SIAM, Philadelphia, 1993, pp 727-731.

Parallel scientific computing environments.

Two important aspects of effectively using a modern heterogeneous computing environment have been studied: parallel programming languages and interactive user interfaces. Results in this area are reported in:

J. Gamble, "Explicit Parallel Programming", M.S. thesis, Department of Computer Science, Virginia Polytechnic Institute and State University, 1990;

J. Gamble and C. J. Ribbens, "Explicit parallel programming: User's guide", Department of Computer Science Report TR 91-3, Virginia Polytechnic Institute and State University, 1991;

J. Gamble and C. J. Ribbens, "Explicit parallel programming: System description", Department of Computer Science Report TR 91-4, Virginia Polytechnic Institute and State University, 1991;

C. J. deSa, "Distributed Problem-Solving Environments for Scientific Computing," M.S. thesis, Dept. of Computer Science, VPI & SU, Blacksburg, 1991.

L. S. Auvil, "Problem Specific Environments for Parallel Scientific Computing," M.S. thesis, Dept. of Computer Science, VPI & SU, Blacksburg, 1992.

L. S. Auvil, C. J. Ribbens, and L. T. Watson, "Problem specific environments for parallel computing," in *Proc. Scalable High Performance Computing Conference*, R. Voigt (ed.), IEEE Computer Soc. Press, Los Alamitos, CA, 1992, pp 149-152.

L. S. Auvil and C. J. Ribbens, "Problem specific environments for parallel scientific computation," *Intl. J. High Speed Comput.*, submitted.

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