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ON THE PREDICTION OF TENSILE STRENGTH OF ROCK FROM
FRACTURE TOUGHNESS OR EFFECTIVE SURFACE ENERGY

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ON THE PREDICTION OF TENSILE STRENGTH OF ROCK
FROM FRACTURE TOUGHNESS OR EFFECTIVE SURFACE ENERGY

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ABSTRACT

A tentative and rather speculative model is proposed that enables the direct use of apparent fracture toughness values, K_Q , as measured from fracture specimens with dimensions too small to produce a valid measure of K_{Ic} . The approach is not intended to predict K_{Ic} from K_Q values measured on subsize specimens as has been previously attempted by others, but rather to use K_Q values directly for applications in which the geometry dictates that the use of K_{Ic} would be inappropriate. The assumption made is that K_Q depends only on crack length with parameters such as thickness, remaining ligament, and stress gradient having a second order effect.

This assumption is supported by rather limited results for Indiana limestone. The data is then used with a model for the flaw geometry to predict the tensile strength of this rock. The predicted value is within 30 percent of the measured values which compares to a previous prediction that is six times too high.

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BACKGROUND

An earlier investigation [1] examined the effect of crack length on the apparent fracture toughness of Indiana limestone. Fracture toughness, K_{Ic} , has been used extensively in modeling fracture in brittle metals while the effective surface energy, γ_{eff} , is more common for rock fracture. The two are directly related through the following equation:

$$K_{Ic} = \sqrt{\frac{2E\gamma_{eff}}{1 - \nu^2}} \quad (1)$$

where E is Young's modulus and ν is Poisson's ratio.

Several attempts have been made to predict tensile strengths from effective surface energy with assumptions as to the natural flaw geometry [2,3]. Typically one assumes that the most severe flaw is a "penny-shaped" flaw oriented perpendicular to the stress axis. This provides a prediction of tensile strength that is usually four to ten times higher than the measured value (e.g., six times higher for Indiana limestone as calculated by Hoagland et al. [3]). A likely explanation of this discrepancy was proposed by Hoagland et al. [3] which likened the zone of microcracking that is observed to form at a crack tip in a rock to the plastic zone that forms at the tip of a crack in a metal. This implies that a linear elastic parameter such as K_{Ic} or γ_{eff} may not apply when the zone of inelastic behavior (plasticity or microcracking) is not small when compared to the crack length. This statement has been firmly verified for metals to the extent that ASTM standards have been set on specimen size requirements for fracture toughness testing of metallic materials [4]. The requirement of "small scale yielding" for valid use of K_{Ic} in metals might then simply be restated as "small scale microcracking" in rock.

The variation of apparent fracture toughness, K_Q , with crack length for Indiana limestone as measured by Schmidt [1] and reproduced in Figure 1 is seen to be similar to many aluminum alloys. In fact, the same specimen size requirements as used for metals was shown probably to apply well in determining fracture toughness (the limiting value of apparent fracture toughness) for this rock. This lends additional

verification to the requirement that "small scale microcracking" must predominate for the valid use of fracture toughness values for rock.

The difficulty, then, in using effective surface energy or fracture toughness in predicting tensile strength is simply the fact that the typical flaw size in rock is too small for the resulting behavior to be considered "small scale microcracking." However, rather than abandon all attempts to predict tensile strength a number of alternatives should be investigated. When large scale yielding is encountered in metals, other crack propagation criteria are often used such as COD [5] and the rather promising approach of the J integral [6]. These criteria are known to be in direct agreement with fracture toughness for the special case of small scale yielding.

PRESENT APPROACH WITH EXPERIMENTAL VERIFICATION

Rather than resorting entirely to more complex theories one might wonder whether or not the actual values of apparent fracture toughness might be useful in rough calculations for applications in which the crack lengths are smaller than the required size for use of K_{Ic} . K_Q is seen (from Fig. 1) to decrease with a decreasing crack length and that is at least the right trend to improve the predictions of tensile strength from fracture toughness.

This approach may seem almost heretical since the trend of K_Q vs. crack length is on one hand being accounted for on the basis of large scale inelastic behavior while we propose to make use of a linear-elastic parameter in this regime. But one must recall that the use of K_{Ic} is applied routinely with plastic zones existing at the crack tip and that it is the overall behavior that is governed by this elastic parameter. The only extension being suggested here is to assume that crack length is the dominant parameter affecting the decrease in the apparent fracture toughness in the region where the crack length can no longer be considered large compared to the size of the inelastic zone.

The use of apparent fracture toughness values below K_{Ic} for a given crack length is equivalent to assuming that the K_Q vs. crack length curve is a material curve. K_{Ic} , which is the limiting value of K_Q at long crack lengths, is known to be a material constant [7], but the curve

itself is expected to depend on specimen geometry. However, thickness* effects are expected to be much less severe in rock than in metals since Poisson's ratio for rock is generally low by comparison. This indicates that a condition of plane strain will predominate in the rock samples for smaller thicknesses than for metals. This is supported by a study of thickness effects on apparent fracture toughness of concrete [8].

As a rough check on the validity of assuming that the K_Q vs. crack length curve is a material curve, four fracture toughness tests were performed [9] using a different specimen configuration and a different testing technique than that of Schmidt [1]. Plates of Indiana limestone 100 mm square and 19 mm thick were mechanically notched to simulate a center cracked plate specimen in contrast to Schmidt's [1] three-point-bend specimens that were precracked in fatigue. The notches were cut with a 0.2 mm diamond wire producing a notch root radius of 0.10-0.12 mm. Aluminum grips were epoxied to the limestone plates and the specimens pulled to failure in a direction perpendicular to the notch.

An assumption was made that the specimen is uniformly stressed on its boundary and the numerical results of Bowie and Neal [10] were used to determine the stress intensity at failure (i.e., apparent fracture toughness). Maximum load was used as the failure load. These results were plotted with those of Schmidt [1] in Figure 1.

DISCUSSION

These data are seen to merge remarkably well with those of the three-point-bend configuration. While this data base is somewhat meager and by no means proves that the curve is a material curve, it is felt that reasonable values of apparent fracture toughness for other configurations that involve small cracks can be obtained. A best fit curve was drawn and extrapolated slightly to include crack lengths on the order of the grain size. The value of toughness to be used in a calculation of tensile strength, then, depends on the maximum flaw size expected in the material.

While the grain size for Indiana limestone was observed to be in the range of 0.5-2.0 mm, in agreement with Hoagland et al. [3], grains

* Thickness is the dimension parallel to the crack front.

and flaws up to 3 mm diameter were observed to occur quite frequently. Assuming that the most severe "penny-shaped" flaw is 1.5 mm in radius, an appropriate value of toughness from a crude extrapolation is $325 \text{ kN m}^{-3/2}$. The stress intensity factor, K , for a "penny-shaped" flaw of radius " a " oriented perpendicular to a uniform stress, σ , in an infinite medium is given by [11] (Figure 2)

$$K = \frac{2}{\pi} \sigma \sqrt{\pi a} . \quad (2)$$

When σ reaches the tensile strength, σ_t , then K reaches K_Q and Equation 2 can be rewritten

$$\sigma_t = 0.886 \frac{K_Q}{\sqrt{a}} . \quad (3)$$

For values of K_Q and " a " given above, the tensile strength of Indiana limestone is calculated to be 7.43 MPa (1078 psi). This is in reasonable agreement with the measured value of 5.39 MPa (781 psi) reported by Schmidt [1]. Six additional direct pull tension tests were performed on "dogbone" specimens of Indiana limestone using great care to obtain precise alignment to eliminate bending. From these tests the average tensile strength was 7.69 MPa (1115 psi) with a standard deviation of 3.4 percent. This strength is only 3 percent higher than predicted but inaccuracies in σ_t , a , and K_Q make such close agreement fortuitous.

The prediction of tensile strength can be carried a step further in that a fracture stress can be predicted for any flaw size or crack length once the K_Q vs. " a " curve is established. Figure 3 shows a plot of the fracture stress versus crack length of Indiana limestone for penny-shaped flaws on the basis of Equation 3 and Figure 1. For reference, another curve was drawn on the same figure assuming K_Q to be constant and equal to K_{Ic} . The error in this assumption is seen to be quite large for cracks less than 20 mm in radius. The curve resulting from Figure 1 is especially interesting in that a plateau level of fracture strength is predicted for crack lengths from about 8 to 18 mm. This is a region in which the fracture strength is predicted to be

rather insensitive to crack length and also indicates that while an especially large flaw in a large specimen* will decrease the tensile strength, the decrease may not be more than about 28 percent for flaws as large as 20 mm in radius.

CONCLUSIONS

Data are presented that indicate a possible unique relationship between apparent fracture toughness, K_Q , and crack length for Indiana limestone regardless of specimen and crack configuration. Vast improvements are then possible for the prediction of tensile strength from fracture toughness and flaw size data when consideration is given to this effect of crack length on K_Q . A calculated tensile strength for Indiana limestone was 28 percent high and 3 percent low when compared with two recently measured values from direct pull tests. Effective surface energy, γ_{eff} , can also be used in this approach instead of fracture toughness since the two are directly related.

* The specimen must be large in diameter when compared to the flaw size in order for Equations 1 and 2 to be valid.

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Fig. 1 Apparent fracture toughness of Indiana limestone as a function of crack length for two specimen configurations.

PENNY-SHAPED CRACK IN A TENSILE SPECIMEN

Fig. 2 Model of critical flaw in tensile specimen responsible for final fracture.

Fig. 3 Tensile strength prediction for Indiana limestone based on the radius of a penny-shaped flaw lying in a plane perpendicular to the stress axis.

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