GENERAL ENGINEERING LABORATORY

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SOME CALCULATIONS PERTAINING TO THE EXPECTED
OPERATIONAL CHARACTERISTICS OF A SMALL
IONIZATION CHAMBER FOR S. A. R.

By

E. E. Goodale
D. H. Morley

Report No 57GL181

May 14, 1957

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SCHENECTADY, NEW YORK

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INTRODUCTION:

In the course of developing a small ionization chamber to detect boiling in a reactor core, a number of calculations were made having to do with the operational characteristics of such a device. Although these calculations apply specifically to this detector and its application, it was felt that some of the results would be of interest in other similar developments. No attempt has been made to make the following a complete report, but the information that seemed worth preserving is presented in individual sections; each with its own heading.

The requirements for the detector are as follows:

1. Physical size - small enough to pass through a 1/8" I.D. instrument tube having a 2" minimum bend radius. A cable shall be an integral part of the chamber and shall be between 20 and 50 feet long. See Fig. No. 1 and No. 2
2. Ambient temperature + 600°F.
3. Maximum neutron flux 10^{14} nv.
4. Maximum gamma flux - 5×10^9 R/hr.

The signal from the chamber will be an a.c. signal representative of a change in neutron flux caused by boiling.

To meet these requirements we are constructing development samples of various small ionization chambers. Some of the characteristics of these chambers can only be determined by test but some predictions can be made on a theoretical basis; these constitute the remainder of this report.

FIGURE #1

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FIGURE #2

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Section I

MAXIMUM CHAMBER SIZE

A. Will the chamber touch in one or two places? If in one place, we can use plane geometry to figure out the maximum chamber length. First we see what length comes from the plane case, then we check whether it touches in one or two places.

Plane Case.

Fig. 1

Chamber length $l = 2x$

$$\text{where } x = \sqrt{\left(B + \frac{D}{2}\right)^2 - \left(B - \frac{D}{2} + d\right)^2}$$

$$\text{So } l = 2 \sqrt{\left(B + \frac{D}{2}\right)^2 - \left(B - \frac{D}{2} + d\right)^2}$$

This is easily calculated for any value of d we might choose.

SECTION A-A

For the circle $\frac{\partial^2 y}{\partial x^2} = -\frac{1}{R}$
($x \rightarrow 0$)

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For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $y = b \left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}$

$$\frac{\partial y}{\partial x} = \frac{b}{2} \frac{-\frac{2x}{a^2}}{\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}} = -\frac{b}{a^2} \frac{x}{\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}}$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{b}{a^2} \left[\frac{1}{\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}} + \frac{x \frac{2x}{a^2}}{2 \left(1 - \frac{x^2}{a^2}\right)^{\frac{3}{2}}} \right]$$

For $x = 0$

$$\frac{\partial^2 y}{\partial x^2} = \frac{-1}{\rho} = -\frac{b}{a^2}$$

Or $\rho = \frac{a^2}{b}$

In this equation

$$a = \frac{D}{2}$$

$$b = \frac{D}{2 \cos \theta}$$

where θ is the angle shown in Fig. 1

$\cos \theta$ was shown to be $= \frac{B - \frac{D}{2} + d}{B + \frac{D}{2}}$

$$h = \frac{D \left(B + \frac{D}{2}\right)}{B - \frac{D}{2} + d} \cdot \frac{1}{2}$$

$$\rho = \frac{\frac{D^2}{4} \left[B - \frac{D}{2} + d\right] 2}{D \left(B + \frac{D}{2}\right)} = \frac{D \left(B - \frac{D}{2} + d\right)}{2 \left(B + \frac{D}{2}\right)}$$

If $\rho > \frac{d}{2}$, the chamber touches in one point.

Before we put in numbers we would like to choose a diameter which will give us the most total boron coating. The amount of boron is proportional to ld .

$$X = ld = 2d \sqrt{\left(B + \frac{D}{2}\right)^2 - \left(B - \frac{D}{2} + d\right)^2}$$

Let us write

$$X = 2d \sqrt{A^2 - (B+d)^2} \quad \text{where } A = \left(B + \frac{D}{2}\right)$$

$$\frac{2x}{2d} = 2 \left[\sqrt{A^2 - (B+d)^2} - \frac{d}{2} \frac{2(B+d)}{\sqrt{A^2 - (B+d)^2}} \right] \quad B = \left(B - \frac{D}{2}\right)$$

For max.

$$\sqrt{A^2 - (B+d)^2} = \frac{d(B+d)}{\sqrt{A^2 - (B+d)^2}}$$

$$A^2 - (B+d)^2 = d(B+d)$$

$$A^2 - B^2 - 2Bd - d^2 = Bd + d^2$$

$$A^2 - B^2 - 3Bd - 2d^2 = 0$$

$$2d^2 + 3Bd + (B^2 - A^2) = 0$$

$$d = \frac{-3B \pm \sqrt{9B^2 - 8(B^2 - A^2)}}{4} = \frac{-3B \pm \sqrt{B^2 + 8A^2}}{4}$$

In our case

$$A = \left(B + \frac{D}{2}\right) = 2.06''$$

$$B = \left(B - \frac{D}{2}\right) = 1.94''$$

$$d = \frac{-5.82 \pm \sqrt{3.7636 + 33.9484}}{4} = \frac{-5.82 \pm 6.141}{4}$$

$$d = \frac{.321}{4} = 0.08''$$

If $d = .08"$

$$l = 2 \sqrt{\left(B + \frac{D}{2}\right)^2 - \left(B - \frac{D}{2} + d\right)^2}$$

$$l = 2 \sqrt{(2.06)^2 - (2.02)^2}$$

$$l = 2 \sqrt{.1632} = 2 \times .404 = .808"$$

This does not allow for cable, however.

Do the ends of the chamber touch the thimble wall at one or two points?

We have
$$f = \frac{D \left(B - \frac{D}{2} + d\right)}{2 \left(B + \frac{D}{2}\right)}$$

One point touches if $f > \frac{d}{2}$

$$f = \frac{0.12 (2.02)}{2 \times 2.06} = 0.588$$

$$\frac{d}{2} = .04$$

So $f > \frac{d}{2}$ and the chamber touches at only one point.

Section II

Lifetime of Boron at 10^{14} nv.

$$\frac{dN}{dt} = -N\sigma nv$$

$$\frac{dN}{N} = -nv\sigma dt$$

$$N = N_0 e^{-nv\sigma t}$$

$$nv\sigma = \frac{10^{14} \times 3.5 \times 10^3 \text{ cm}^2 \times 10^{-24}}{\text{cm}^2 \text{ sec}}$$

$$nv\sigma = 3.5 \times 10^{-7} / \text{sec.}$$

when do we lose 10%

$$\frac{N}{N_0} = 0.9 = e^{-nv\sigma t}$$

$$\text{then } nv\sigma t \approx 0.1$$

$$t = \frac{0.1 \text{ sec}}{3.5 \times 10^{-7}} = 0.285 \times 10^7$$

$$t = 2.85 \times 10^5 \text{ sec}$$

There are about 3.6×10^4 sec. per day, so we would expect a 10% burn up of boron at 10^{14} nv in about 3.3 days.

Analysis of Induced Activity in Ion Chamber

This is a rough approximation to obtain an order of magnitude of the induced activity per unit length of chamber and cable assembly when exposed to 10^{14} nv until saturation is reached. Only the copper and aluminum are considered.

1. Cu center wire

$$V = \pi r^2 l$$

$$\frac{V}{l} = \pi \times 6.45 \times 10^{-4} \text{ cm}^2$$

$$\frac{V}{l} = 3.03 \times 10^{-3} \text{ cm}^2$$

2. Al chamber body

$$\frac{V}{l} = \pi (r_1^2 - r_2^2) \text{ cm}^2$$

$$\frac{V}{l} = \pi (.010 - .0058) \text{ cm}^2$$

$$\frac{V}{l} = 1.316 \times 10^{-2} \text{ cm}^2$$

3. Al cable sheath

$$\frac{V}{l} = \pi (r_1^2 - r_2^2) \text{ cm}^2$$

$$\frac{V}{l} = \pi (.00793 - .00505) \text{ cm}^2$$

$$\frac{V}{l} = 9.05 \times 10^{-3} \text{ cm}^2$$

4. Al_2O_3 insulation

$$\frac{V}{l} = \pi (r_1^2 - r_2^2) \text{ cm}^2$$

$$\frac{V}{l} = \pi (.00505 - .000645) \text{ cm}^2$$

$$\frac{V}{l} = 1.38 \times 10^{-2} \text{ cm}^2$$

Al_2O_3 contains 53% Al

$$\text{So } .53 \times 1.38 \times 10^{-2} \text{ cm}^2 = 7.32 \times 10^{-3} \text{ cm}^2$$

The effective $\frac{v}{e}$ for chamber body = $1.318 \times 10^{-2} \text{ cm}^2$.

The effective $\frac{v}{e}$ for cable = $7.32 \times 10^{-3} + 9.05 \times 10^{-3} = 1.635 \times 10^{-2} \text{ cm}^2$

So for our rough estimate we will use $1.635 \times 10^{-2} \text{ cm}^2$ for

Al and $2.03 \times 10^{-3} \text{ cm}^2$ for Cu.

1. Cu

$$\frac{N}{e} = \frac{a \rho A}{A_w}$$

where

a : area

ρ : density

A : avogadro's number

A_w : Atomic weight

$$\frac{N}{e} = \frac{2.03 \times 10^{-3} \text{ cm}^2 \times 8.89 \text{ gms/mole} \times .6 \times 10^{24} \text{ atoms/mole}}{29 \text{ gms/cm}^3}$$

$$\frac{N}{e} = 3.735 \times 10^{20} \text{ atoms/cm}$$

2. Al

$$\frac{N}{e} = \frac{1.635 \times 10^{-2} \times 2.7 \times .6 \times 10^{24}}{13} \text{ atoms/cm}$$

$$\frac{N}{e} = 2.04 \times 10^{21} \text{ atoms/cm}$$

Now let us look into the various reactions that can take place and calculate the induced activity of each reaction. First let us consider the $Al^{27}(n, \gamma)Al^{28}$ reaction, which has a cross section of .25 barns, a half life of 2.3 min. and emits a 1.78 Mev gamma 100% of the time.

At equilibrium, the birth rate B equals the decay rate D.

$$B = N \sigma n v$$

$$B = 2.04 \times 10^{21} \times .25 \times 10^{-24} \times 10^{14}$$

$$B = 5.10 \times 10^{10}$$

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$$1 \text{ Curie} = 3.7 \times 10^{10} \text{ d/sec.}$$

$$\text{So } \frac{5.10 \times 10^{10}}{3.7 \times 10^{10}} = 1.38 \text{ Curie/unit length of chamber (cm)}$$

For cu, we have $\text{Cu}^{63} (\text{n}, \gamma) \text{Cu}^{64}$ with a cross-section of 4.7 barns, half life of 12.8 hrs., and emits a 1.34 Mev gamma approx. 0.5% of the time.

$$B = 3.735 \times 10^{20} \times .69 \times 4.7 \times 10^{-24} \times 10^{14}$$

$$B = 12.1 \times 10^{10}$$

$$12.1 \times 10^{10} \times .005 = .0605 \times 10^{10}$$

$$\frac{.0605 \times 10^{10}}{3.7 \times 10^{10}} = .0163 \text{ curies}$$

A positron is also emitted in this reaction 19% of the time.

$$12.1 \times 10^{10} \times .19 = 2.3 \times 10^{10}$$

$$\frac{2.3 \times 10^{10}}{3.7 \times 10^{10}} = .622 \text{ curies}$$

The other cu reaction is $\text{Cu}^{65} (\text{n}, \gamma) \text{Cu}^{66}$ with a cross section of 2.2 barns, half life of 5 min. and emits a 1.04 Mev gamma approximately 7% of the time.

$$B = 3.735 \times 10^{20} \times .31 \times 2.2 \times 10^{-24} \times 10^{14}$$

$$B = 2.55 \times 10^{10}$$

$$2.55 \times 10^{10} \times .07 = .1785 \times 10^{10}$$

$$\frac{.1785 \times 10^{10}}{3.7 \times 10^{10}} = .0483 \text{ curies}$$

now adding, we obtain the total activity at $T = 0$

Al	1.3800
Cu	.0163
Cu	.0483
B+ from Cu	<u>.622</u>
	2.0666 curies per unit length (cm)

Since the bulk of this activity is from the 2.3 min Al, we would expect that most of this would be decayed after several half lives thus reducing the activity appreciable. This is only an order of magnitude estimate as no account was taken of the activity of any impurities in the metals or of the gas filling.

Section IV

NEUTRON INDUCED CURRENTS IN CHAMBER

We would like to estimate the currents in the chamber, with both argon and helium filling. One method of calculation if it is applicable, is based on the cavity principle; namely that the flux density in a small cavity equals that in the surrounding material if the particle range in the cavity is long with respect to the cavity dimensions. It is also useful to realize that the flux density in a volume is equal to the total length of the particle tracks formed per cm^3 per unit time.

We will first calculate the response of a typical chamber using these principles, and see if it checks with the measured response of the chamber. If so, we can feel safe in using cavity principles with our small chamber. If not, we should attempt to find out why not. In any case this type of calculation can serve us as a useful guide in our design.

$$\text{FLUX DENSITY OF 2 PARTICLES } F_2 = F_n \sigma n l$$

WHERE F_n = neutron flux density
 σ = cross-section
 n = atoms/cc
 l = track length per event.

$$n = \frac{\rho N}{A}$$

WHERE ρ = density of boron
 N = Avogadro's number
 A = Atomic Weight.

$$l = \frac{rA}{\rho}$$

WHERE r = moles/unit area

$$nl = \frac{\rho N r A}{A \rho} = Nr$$

$$F_2 = F_n \sigma n l = F_n \sigma Nr$$

For a boron lined chamber,

$$G = 7 \times 10^2 \times 10^{24} \text{ cm}^2/\text{atom} \quad (\text{normal boron})$$

$$N = .6 \times 10^{24} \text{ atoms/mole}$$

$$r = 10^{-4} \text{ moles/cm}^2$$

So: $F_\alpha = 4.2 \times 10^{-2} F_n$

The current (I) in the chamber will be given by: $I = F_\alpha S e V$

where V = chamber volume

S = specific ionization

e = electronic charge

The specific ionization for the alpha energy involved will be about 7×10^4 ion pairs/cm. in an argon filled chamber.

From this we can estimate the current in a Mk II chamber, for example, provided it can be regarded as a cavity chamber. For the Mk II chamber $V = 100 \text{ cm}^3$

So $I = F_\alpha 7 \times 10^4 \times 1.6 \times 10^{-19} \times 10^2 \frac{\text{cm}^3 \text{ Coulombs}}{\text{cm}}$

$$I \approx 10^{-12} F_\alpha \text{ cm}^2 \text{ coulombs}$$

And $F_\alpha = F_n \times 4.2 \times 10^{-2}$

$$I = 4.2 \times 10^{-14} F_n \text{ cm}^2 \text{ Coulombs} \quad (\text{OR } \frac{\text{Amperes}}{\text{nv}})$$

In actuality, our Mk II chambers had a measured sensitivity of only about 6×10^{-15} amps/nv. Some of this difference might be caused by other effects, but one is led to suspect that the Mk II chamber is not a "good" cavity chamber.

To check we will make two calculations, both somewhat simplified compared to the actual case. First we will estimate the average track length of an α particle emitted from a range layer thickness of boron. Then we will estimate the average path that is available in the chamber. Unless the α track length turns out to be large with respect to the average path available in the chamber we will have confirmed our suspicions that the Mk II chamber is not a good cavity chamber.

Calculation of the Average Alpha Track Length

Initial range in air R_0

Initial range in boron r_0

$$\frac{R_0}{r_0} = \alpha$$

$$x = \frac{d}{\cos \theta} \quad (r_0 - x)\alpha = \text{range in air} \equiv R$$

so

$$R = (r_0 - x)\alpha = \left(r_0 - \frac{d}{\cos \theta}\right)\alpha$$

If we integrate over a sphere

$$\begin{aligned} \bar{R} &= \frac{\int_0^{\arccos \frac{d}{r_0}} R \cdot 2\pi d^2 \sin \theta d\theta}{2\pi d^2} = \int_0^{\arccos \frac{d}{r_0}} R \sin \theta d\theta \\ \bar{R} &= \int_0^{\arccos \frac{d}{r_0}} \left(r_0 - \frac{d}{\cos \theta}\right)\alpha \sin \theta d\theta \end{aligned}$$

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$$\bar{R} = \alpha r_0 \int_0^{\cos^{-1} \frac{d}{r_0}} \sin \theta d\theta - \alpha d \int_0^{\cos^{-1} \frac{d}{r_0}} \tan \theta d\theta$$

$$\bar{R} = \alpha r_0 (-\cos \theta) \Big|_0^{\cos^{-1} \frac{d}{r_0}} - \alpha d (-\log \cos \theta) \Big|_0^{\cos^{-1} \frac{d}{r_0}}$$

$$\bar{R} = \alpha r_0 \left(\frac{d}{r_0} + 1 \right) - \alpha d \left(-\log \frac{d}{r_0} \right)$$

$$\bar{R}_{(d)} = \alpha d \log \frac{d}{r_0} + \alpha r_0 \left(1 - \frac{d}{r_0} \right)$$

For $\bar{R}$ over various values of d , we must integrate over $d = 0$ to $d = r_0$. The number associated with each element is $n = \rho d(d)$

$$N = \rho \int_0^{r_0} d(d) = \rho r_0$$

$$\bar{R} = \int_0^{r_0} d(d) \left[\alpha r_0 \left(1 - \frac{d}{r_0} \right) \right] + \int_0^{r_0} \alpha d \ln \frac{d}{r_0} d(d)$$

the log part of the integration - $\frac{r_0}{r_0}$

$$\alpha r_0 \int_0^{r_0} \frac{d}{r_0} \ln \frac{d}{r_0} d\left(\frac{d}{r_0}\right)$$

If $x = \frac{d}{r_0}$ Limit $x = 0 \rightarrow 1$

$$\alpha r_0^2 \int_0^1 x \ln x dx = \alpha r_0^2 \left[\frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_0^1$$

$$= \alpha r_0^2 \left[\frac{1}{2} \ln(1) - \frac{1}{4} - \frac{x^2}{2} \ln x \right]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \left(\frac{x^2}{2} \ln x \right) &= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{2(-2x^{-3})}{-17-}} = \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{2(-2x^{-3})} \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{4}{x^3}} = \frac{1}{4} x^2 \lim_{x \rightarrow 0} = 0 \end{aligned}$$

So

$$\begin{aligned} \bar{R} &= \frac{\frac{\alpha r_0^2}{2} - \frac{1}{4} \alpha r_0^2}{r_0} \\ \bar{R} &= \frac{\alpha r_0}{4} = \frac{R_0}{4} \end{aligned}$$

This says that ignoring the other wall, the average track length from a range layer of boron is $\frac{R_0}{4}$, where R_0 is the full range in air.

Calculation of the Average Path Length Available in Chamber

For simplification, we will assume a spherical chamber geometry and let the maximum path length be limited by the chamber curvature.

$$\begin{aligned} r_0 &= 4'' \\ r_1 &= 4'' - \frac{1}{8}'' \\ &= r_0 - t \end{aligned}$$

$$\frac{x}{2} = \sqrt{r_0^2 - r_1^2}$$

$$\frac{x}{2} = \sqrt{r_0^2 - r_0^2 + 2r_0 t - t^2}$$

$$\frac{x}{2} \approx \sqrt{2r_0 t}$$

$$x \approx 2\sqrt{2r_0 t}$$

The integration is given for a parallel plate case by:

$$\bar{r} = S \int_0^{\frac{\pi}{2}} \tan \theta d\theta = S [-\log \cos \theta]_0^{\frac{\pi}{2}}$$

We will now limit the angle to one which produces

an r of $2\sqrt{2BS}$

where B = radius of chamber

S = plate separation

$$\text{we had } r = \frac{S}{\cos \theta} = 2\sqrt{2BS}$$

$$\cos \theta = \frac{S}{2\sqrt{2BS}}$$

so now $\bar{r} = S \left[-\log \cos \theta \right] e^{-1 \frac{S}{2\sqrt{2BS}}}$

$$\bar{r} = S \left[\ln \frac{S}{\sqrt{2BS}} \right] = S \ln \frac{S}{\sqrt{2BS}}$$

For our case, $S = 1/8"$

$$B = 4"$$

$$BS = 1/2$$

$$2BS = 1$$

$$\bar{r} = S \left(- \left[\ln \frac{1}{\sqrt{1}} \right] \right) = S - \ln \frac{1}{\sqrt{1}}$$

$$-\frac{\bar{r}}{S} = \ln \frac{1}{\sqrt{1}}$$

$$e^{-\frac{\bar{r}}{S}} = 0.125$$

$$\frac{\bar{r}}{S} \approx 2.1$$

so the average path length is at least $2.1S$, since some tracks can go down the cylinder further than this.

The average path length in the Mk. II is thus about $1/4"$, while the average alpha track length otherwise is about

$$\frac{R_0}{4} = \frac{1.5 \text{ cm}}{4} \approx .375 \text{ cm} \approx .147 \text{ in.}$$

which is less than the average path length. The Mk II chamber therefore, can certainly not be considered a cavity chamber.

The small ion chamber under consideration will certainly be more of a cavity chamber than the Mk II chamber; the question is however, how to estimate the expected current. Some data reported by K. Watkins in a trip report dated 3/1/55 turns out to be helpful in this respect. Watkins measured the current from a chamber

similar to the Mk II chamber exposed to a constant neutron flux but varying the filling gas pressure. This is the equivalent to varying the electrode spacing in a chamber. At low pressures, the results should approximate those of a cavity chamber and include any other factors that effect the sensitivity. The accompanying graph shows the results, translated into terms useful for our purposes. We have plotted the current per unit volume flux, versus the effective spacing. (This assumes that changing the pressure at constant spacing is equivalent to changing the spacing at constant pressure.)

We may now calculate the current we might expect from the small ion chamber, using the plotted results as a guide. Since the small chamber is not a true parallel plate arrangement we can only make a reasonable assumption as to the equivalent spacing. Since the center wire is fairly small perhaps the figure 0.07 inches, the inside diameter of the counter, is reasonable. So we would expect to draw about 8.3×10^{-17} amps/cm³nv. At 10^{14} nv, this represents about 8 m.a. per cc., an extremely high figure.

The use of Helium in the chamber may be advantageous, since a helium filled chamber should have a lower ionization density, and be easier to saturate.

The atomic stopping^{POWER} for helium is approximately 6 times less than that for argon, and the required energy per ion pair is about 1.2 times greater. So except for the cavity effect a Helium filled chamber should produce about 7 times less current than an argon filled one. However, because of the lower stopping power,

CHAMBER CURRENT PER UNIT VOLUME VS ELECTRODE SPACING.

FILLING = 1 ATM ARGON.

ABSOLUTE VALUES - FROM
CALIBRATIONS OF MK II
CHAMBERS ($\frac{1}{8}$ " SPACING).

RELATIVE VALUES AT DIFFERENT
SPACINGS - FROM DATA
BY L.E. WATKINS.

the chamber will be a better cavity chamber with Helium and, according to the curve, should produce about 1.5 times more current because of this effect. The net result is that we would expect a Helium filled chamber to produce about 0.21 times as much current as one filled with argon, or about 1.8 m.a./cc at 10^{14} nv.

Section V

GAMMA RESPONSE OF CHAMBER

For neutrons, we have estimated (with argon) 8×10^{-17} amps/cc/nv for the small chamber under consideration. The normal response for a chamber to gamma radiation is 1 e.s.u. per cc per R

This is equal to $\frac{1}{3 \times 10^9} \frac{\text{coulombs}}{\text{cc R}}$ for air

For argon, and with some wall effect, we might about double that figure.

$$Q = \frac{2}{3 \times 10^9} \frac{\text{Coulombs}}{\text{cc R}}$$

We have $5 \times 10^9 \text{ R/hr} = \frac{5 \times 10^9 \text{ R hr}}{3.6 \times 10^3 \text{ sec hr}} \quad 1.66 \times 10^6 \text{ R/sec.}$

$$I = \frac{2 \text{ coulombs } 1.66 \times 10^6 \text{ R}}{3 \times 10^9 \text{ cc R sec.}}$$

$$I = 1.1 \times 10^{-3} \text{ amps/cc for } 5 \times 10^9 \text{ R/hr.}$$

Our neutron current at 10^{12} nv would be about 8×10^{-5} amps/cc.

So the gamma current could be more than the neutron current if the 10^{12} nv exists at the same time as the $5 \times 10^9 \text{ R/hr}$. However, since the measurement to be taken concerns fluctuations in the neutron current rather than D.C. measurements, the situation may be tolerable. With a helium filling, one would expect the ratio of the neutron to gamma response to be the same.

Section VI

Saturation

Non saturation in an ion chamber may be caused by any one of several effects, for example, diffusion ("escape" of ions to the walls), columnar or volume recombination, and space charge field depression. We have previously had some success predicting the "knee" of the plateau in our Mk II ion chambers using space charge considerations alone, so we will use that technique to estimate the required voltage for the small ion chamber. If our previous experience is indicative the answer should be fairly accurate.

It is possible, on the other hand, that the other effects mentioned might predominate and make the situation worse than expected. For example, volume recombination rates would vary as the product of the positive and negative charge densities and might suddenly become important at the high current densities under consideration. Unfortunately, the recombination coefficients are not well enough known to make any estimate of this effect reliable. One simply has to hope that, as in the Mk II chambers, the space charge will be the limiting factor.

The following two assumptions are made to simplify the analysis.

1. The electrons are collected so rapidly that their effect on the space charge can be neglected. This, incidently, implies that we should use very pure gas of low electron affinity in the chamber.

2. Charge is produced uniformly throughout the chamber. This will not be strictly true either, but the assumption should not, we believe introduce an appreciable error.

The picture, then, is one of positive ions being constantly formed throughout the chamber and drifting under the influence of the field towards the negative electrode. The space charge is thus most dense near the negative electrode. The result is a reduction of the field at the positive electrode as shown below.

If the field at the positive electrode is reduced to zero, the chamber is then space charge limited. It is this point (sometimes called the "throttling point") which can presumably be equated with the knee of the plateau curve for the chamber.

The following terms will be used in the derivation of the necessary equations.

a = diameter of inner electrode

b = diameter of outer electrode

$\vec{r}$ = position vector

α = ionization density (the birthrate, or charge created per unit time per unit volume)

j = current density of positive ions

ρ = charge " " " "

ψ = potential at any point

μ = positive ion mobility

ϵ = permittivity

For any steady state condition the continuity equation

$\nabla \cdot \vec{j} = \frac{\partial \rho}{\partial t} = \alpha$, must hold throughout the volume. In cylindrical coordinates, for radial symmetry ($\vec{j}$ has components only in the $\vec{r}$ direction), this can be written

$$\frac{1}{r} \frac{d}{dr} (r j) = \alpha \quad (1)$$

Also, at any point

$$j = -\mu \rho \frac{d\psi}{dr} \quad (2)$$

And, finally, the Poisson equation in this form is

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) = -\frac{\rho}{\epsilon} \quad (3)$$

$$(\text{In vector form } \nabla^2 \psi = \frac{\rho}{\epsilon})$$

We want to obtain an expression for the potential at any point in the chamber in terms of things that are known, so we must eliminate ρ and j from these equations.

Combining (2) and (3) and eliminating ρ we obtain:

$$\epsilon \mu \frac{d\psi}{dr} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) = r j \quad (4)$$

Combining (1) and (4) and eliminating j
by substituting for (rj) in (1), we obtain

$$\epsilon \mu \frac{d}{dr} \left[\frac{d\psi}{dr} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) \right] = \alpha r \quad (5)$$

For an abbreviation, let us define $R \equiv r \frac{d\psi}{dr}$
Then equation (5) becomes

$$\epsilon \mu \frac{d}{dr} \left[\frac{1}{r} R \frac{dR}{dr} \right] = \alpha r \quad (6)$$

Next we must consider the boundary conditions. There are two, one a general one and one imposed by the throttling condition.

1. At the positive electrodes j must vanish. (There is no positive ion current coming out of the positive electrode.)

In terms of R , from equation (2)

$$j = -\mu \rho \frac{d\psi}{dr} = -\frac{\mu \rho}{r} r \frac{d\psi}{dr} = -\frac{\mu \rho}{r} R$$

and from (3)

$$\rho = -\epsilon \frac{1}{r} \frac{d}{dr} R$$

Combining

$$j = \frac{\mu \epsilon}{r^2} R \frac{dR}{dr}$$

So, at the positive electrode, if $j = 0$, then

$$R \frac{dR}{dr} = 0$$

2. If the operating voltage is such that throttling just sets in, $\frac{d\psi}{dr} = 0$ at the positive electrode, or $R = 0$ at the positive electrode.

There are two cases to consider.

Case 1. Outer electrode is positive.

If we integrate equ.(6) with respect to r , we have

$$\epsilon\mu \frac{R}{r} \frac{dR}{dr} = \frac{\alpha r^2}{2} + C \quad (C = \text{constant of integration})$$

When $r = b$ $R \frac{dR}{dr} = 0$, so $C = -\frac{\alpha b^2}{2}$ and

$$\epsilon\mu \frac{R}{r} \frac{dR}{dr} = \frac{\alpha}{2} (r^2 - b^2) \quad \text{or}$$

$$\epsilon\mu R \frac{dR}{dr} = \alpha (r^3 - b^2 r) \quad (7)$$

If we integrate again:

$$\epsilon\mu R^2 = \alpha \left(\frac{r^4}{4} - \frac{b^2 r^2}{2} + C \right) \quad \text{and, imposing}$$

the throttling condition $R=0$ for $r = b$, we obtain.

$$\begin{aligned} \epsilon\mu R^2 &= \alpha \left(\frac{r^4}{4} - \frac{b^2 r^2}{2} + \frac{b^4}{4} \right) \\ &= \frac{\alpha}{4} (b^2 - r^2)^2 \end{aligned}$$

and finally

$$R = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} (b^2 - r^2) \quad (8)$$

Since $R = r \frac{d\psi}{dr}$ we get

$$\frac{d\psi}{dr} = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} \left(\frac{b^2}{r} - r \right) \quad (9)$$

The critical operating voltage is now given by $V = \int_a^b \frac{d\psi}{dr} dr$

$$\text{so } V = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} \left(b^2 \ln\left(\frac{b}{a}\right) - \frac{b^2 - a^2}{2} \right) \quad (10)$$

Case 2. Inner electrode positive

By the same type of integration as before, we obtain

$$\epsilon\mu R^2 = \alpha \left(\frac{r^4}{4} - \frac{a^2 r^2}{2} + \frac{a^4}{4} \right) \text{ or}$$

$$R = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} (a^2 - r^2) \quad (11)$$

Then $\frac{d\psi}{dr} = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} \left(\frac{a^2}{r} - r \right)$

and $V = - \int_a^b \frac{d\psi}{dr} dr = \frac{1}{2} \sqrt{\frac{\alpha}{\epsilon\mu}} \left(\frac{b^2 - a^2}{2} - a^2 \ln \frac{b}{a} \right) \quad (12)$

Summary:

1. Outer electrode positive

$$V_{\text{crit.}} = \frac{1}{4} \sqrt{\frac{\alpha}{\epsilon\mu}} a^2 \left[1 - \left(\frac{b}{a} \right)^2 + \left(\frac{b}{a} \right)^2 \ln \left(\frac{b}{a} \right)^2 \right]$$

2. Inner electrode positive

$$V_{\text{crit.}} = \frac{1}{4} \sqrt{\frac{\alpha}{\epsilon\mu}} a^2 \left[\left(\frac{b}{a} \right)^2 - 1 - \ln \left(\frac{b}{a} \right)^2 \right]$$

$V_{\text{crit.}}$ for case (1) is always larger than for case (2). (This is shown by comparison of the two equations, using a series form for the log functions.)

Therefore, one should use case (2), the inner electrode positive.

Application to our case.

We expect to draw currents of the order of 8 ma/cc, if we use Argon. So $\alpha = 8 \times 10^{-3}$ amps/cm³.

Other typical values.

$$a = 2.5 \times 10^{-2} \text{ cm.}$$

$$b = 8.3 \times 10^{-2} \text{ cm.}$$

The mobility μ , for Argon = $\mu = 1.3 \frac{\text{cm}^2}{\text{volt sec.}}$

Permittivity of free space $\epsilon = .885 \times 10^{-13} \frac{\text{amp sec.}}{\text{cm. volt}}$

For inner electrode positive

$$V_{\text{crit}} = \frac{1}{4} \sqrt{\frac{\alpha}{\epsilon \mu}} a^2 \left[\left(\frac{b}{a} \right)^2 - 1 - \ln \left(\frac{b}{a} \right)^2 \right]$$

$$V_{\text{crit}} = \frac{1}{4} \sqrt{\frac{8 \times 10^{-3} \text{ amps volt cm. unit sec.}}{\text{cm}^3 \cdot .885 \times 10^{-13} \text{ amp sec} \cdot 1.3 \text{ cm}^2}} \times (6.25 \times 10^{-4}) \text{ cm}^2 \times (11.1 - 2.4)$$

$$V_{\text{crit}} \approx 312 \text{ volts}$$

For any other gas, the values of μ , α will be different. We might write it out this way.

$$V_{\text{critical}} = \sqrt{\frac{\frac{\alpha_x}{\alpha_A}}{\frac{\mu_x}{\mu_A}}} 312 \text{ Volts.}$$

where $\alpha_x = \alpha$ of gas $\alpha_A = \alpha$ of argon
 $\mu_x = \mu$ of gas $\mu_A = \mu$ of argon.

For example, Helium: $\mu_{\text{He}} = 5.1 \frac{\text{cm}^2}{\text{volt sec.}}$

$$\frac{\mu_{\text{He}}}{\mu_A} = \frac{5.1}{1.3} = 3.9$$

And we have previously estimated that for our chamber $\frac{\alpha_{He}}{\alpha_A} \approx 0.21$

So: $V_{\text{critical Helium}} = \sqrt{\frac{0.21}{3.9}} \ 312 \text{ volts} = 72 \text{ volts}$

The use of Helium would provide a notable improvement in critical voltage. One should use, of course, a voltage considerably greater than V_{crit} , but this should be possible in the Helium filled chamber.

It should be noted that there would be no particular advantage to switching from Argon to Helium if it were also required that the output current is kept up. This would require an increase in pressure of 4 or 5 times, the mobility would decrease approximately proportionally and V_{crit} would be roughly the same. In our case we wish the number of events to remain high, for statistical reasons, but are not particularly interested in high d.c. levels.

HEATING OF CHAMBER

Section VII

A. HEAT DUE TO NEUTRONS

Maximum flux $10^{14} \text{ nv} = f$

Amount of boron present

$$N = An$$

where A = area

$$n = \text{atoms/cm}^2$$

chamber dimensions approximately
.07" d x .7" l

$$A = \pi d l$$

$$A = .049 \pi = .154 \text{ in}^2$$

$$A = .994 \text{ cm}^2 \approx 1 \text{ cm}^2$$

$$n = .1 \times 10^{-3} \text{ moles/cm}^2 \times .6 \times 10^{24} \text{ atoms/mole}$$

$$n = 6 \times 10^{19} \text{ atoms/cm}^2$$

$$N = An = 6 \times 10^{19} \text{ atoms/cm}^2 \times 1 \text{ cm}^2$$

$$N = 6 \times 10^{19} \text{ atoms}$$

Power Generated $P = N f \sigma \epsilon$

$$P = \frac{6 \times 10^{19} \times 7 \times 10^{-22} \text{ cm}^2 \times 10^{14} \times 2.54 \times 10^6 \text{ ev}}{\text{cm}^2 \text{ sec}}$$

$$P = 1.07 \times 10^{19} \text{ ev/sec}$$

$$1 \text{ ev} = 1.6 \times 10^{-19} \text{ watt sec}$$

$$P = \frac{1.07 \times 10^{19} \text{ ev}}{\text{sec}} \times \frac{1.6 \times 10^{-19} \text{ watt sec}}{\text{ev}}$$

$$P \approx 1.7 \text{ watts}$$

Events per unit time = $R = N \sigma f$

$$R = \frac{6 \times 10^{19} \times 7 \times 10^{-22} \text{ cm}^2 \times 10^{14}}{\text{cm}^2 \text{ sec}}$$

$$R = 4.2 \times 10^{12} / \text{sec}$$

B. GAMMA HEATING

We are given a maximum figure of 20 watts per gram due to gamma heating, or (another way) 5×10^9 R/hr.

If we use the latter figure we can more or less check the former one.

From "Isotopic Tracers and Nuclear Radiations" by W.E. Siri we obtain a figure of 2.8×10^3 ergs/cm² for 1R. We will take aluminum for this problem in the form of 1 gram, $1 \text{ cm}^2 \times 1/2.7 \text{ cm}$ thick. We will assume ~ 1 Mev gamma rays.

The total linear absorption coefficient is given as ~ 0.17 per cm. (This energy is not, of course, completely absorbed, so the answer should be on the high side.)

$$P_{\text{abs.}} = P_0 - P = P_0 (1 - e^{-\mu x})$$

$$\text{where } \mu = 0.17 \\ x = \frac{1}{2.7} \quad \mu x = 0.063$$

$$e^{-0.063} \approx 1 - 0.062$$

$$P_{\text{abs.}} \approx P_0 (0.062)$$

$$P_0 = \frac{5 \times 10^9 \text{ R/hr} \cdot 2.8 \times 10^3 \text{ ergs watt sec} (0.062)}{\text{hr} \cdot 3.6 \times 10^3 \text{ sec R erg } 10^7}$$

$$P_0 = 24 \text{ watts for our 1 gm sample.}$$

This is a good check, considering. We will use the 20 watts/gm figure.

We have previously assumed, and checked roughly, the following:

1. Heat transfer lengthwise in the cable is not effective in removing heat from the chamber.
2. The center wire of the cable will not get much hotter than the outside.

Our worst case with the chamber assumes heat loss only radially through the thimble gas. We can calculate the temperature rise in this case.

If q energy is released in the chamber per unit time it must all flow out to the thimble. We are forced to assume that the thimble represents a sink at temperature T_2 , that is, that it is well enough bonded to the surroundings that the heat from the chamber does not raise its temperature appreciably.

The heat flow across a thin element dr is q

where

$$q = \frac{-KAdt}{dr} = \frac{-Kl2\pi r dT}{dr}$$

q is a constant if there is no axial flow.

$$\frac{dr}{r} = \frac{-2\pi K l dT}{q}$$

$$\int_{r_i}^{r_o} \frac{dr}{r} = \frac{-2\pi K l}{q} \int_{T_i}^{T_o} dT$$

$$\ln r \Big|_{r_i}^{r_o} = \frac{-2\pi K l}{q} T \Big|_{T_i}^{T_o}$$

$$\ln \frac{r_o}{r_i} = \frac{-2\pi K l}{q} (T_o - T_i) = \frac{2\pi K l}{q} (T_i - T_o)$$

$$(T_i - T_o) = \frac{q}{2\pi K l} \ln \frac{r_o}{r_i}$$

In our case

$$\frac{r_o}{r_i} = 1.5 \quad \ln \frac{r_o}{r_i} = .405$$

$$(T_i - T_o) = \frac{4.5 \text{ g}}{3\pi k \ell} = 0.0362 \frac{\text{g}}{\text{K cm}}$$

C. Now we will calculate the total q.

Previously we obtained for neutrons $q \approx 1.7$ watts

From gamma's, $q = \frac{20 \text{ watts}}{\text{gram}}$

Chamber shell dimensions $\approx$

ℓ	1.78 cm
r_o	.1143 cm
r_i	.0889 cm

$$\text{volume} = (r_o - r_i) \pi \ell$$

$$\text{volume} = .0288 \text{ cm}^3$$

$$\text{wt} = .0288 \times 2.7 = .07779 \text{ gms}$$

At 20 watts/gm

$$P = 1.55 \text{ watts} \approx 1.6 \text{ watts}$$

Total q from γ 's and n 's ≈ 3.3 Watts.

From "Principles of Physics" by Sears, Thermal conductivity of gases as follows:

$$\text{air} = \frac{8.6}{5.7} \times 10^{-5} \frac{\text{cal cm}}{\text{sec cm}^2 \text{ } ^\circ\text{C}} \quad (\text{at } 600^\circ\text{F})$$

$$\text{He} = \frac{4.4}{3.4} \times 10^{-4} \frac{\text{cal cm}}{\text{sec cm}^2 \text{ } ^\circ\text{C}}$$

$$\text{now, } \Delta T_{\text{air}} = \frac{0.0362 \times 3.3 \text{ watts cm}^2 \text{ } ^\circ\text{C sec cal}}{\text{cm} \times \frac{8.6}{5.7} \times 10^{-5} \text{ cal cm} \times 4.186 \text{ watt sec}} = \frac{498^\circ\text{C}}{330^\circ\text{C}}$$

$$\Delta T_{\text{He}} = \Delta T_{\text{air}} \frac{\frac{8.6}{5.7} \times 10^{-5}}{\frac{4.4}{3.4} \times 10^{-4}} = \frac{83.5^\circ\text{C}}{64.5^\circ\text{C}}$$

Therefore, one can suppose the maximum rise to be of the order of 100°C .
 65°C using He.