**HARMONICS SUPPRESSION OF VACUUM CHAMBER EDDY CURRENT
INDUCED FIELDS WITH APPLICATION TO THE SUPERCONDUCTING
SUPER COLLIDER (SSC) LOW ENERGY BOOSTER (LEB) MAGNETS***

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Harmonics Suppression of Vacuum Chamber Eddy Current Induced Fields with Application to the Superconducting Super Collider (SSC) Low Energy Booster (LEB) Magnets

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Abstract

This memo presents the formulation of an expression for eddy currents induced in a thin-walled conductor due to a time-dependent electromagnet field excitation. Then follows an analytical development for prediction of vacuum chamber eddy current induced field harmonics in iron-core electromagnets. A passive technique for harmonics suppression is presented with specific application to the design of the Superconducting Super Collider (SSC) Low Energy Booster (LEB) Magnets.

1 Summary

In Section 2 an expression, Eqn. (11), is derived relating the current density per unit length along the cross section of a conductor (e.g. a vacuum chamber) in a magnetic field to the vector potential of the field over that conductor's cross section.

Section 3 uses a Schwarz-Christoffel transformation to formulate an expression, Eqn. (21) or Eqn. (24), for field harmonics in an iron-core dipole electromagnet induced by arbitrarily located current filaments.

Section 4 shows that the chosen 'alternative' LEB dipole excitation pattern produces eddy current induced harmonics that are less than half that of a slightly different 'strawman' excitation pattern. A closed-form expression, Eqn. (33), for vacuum chamber eddy current induced harmonics in an iron-core dipole electromagnet is formulated using results from Sections 2 and 3. Analytical results of field harmonics from Code EDDY (Appendix C) for an elliptical strawman dipole magnet vacuum chamber shape are shown to match those of a lengthy two-step pair of POISSON runs. An expression, Eqn. (36), for the field correction (i.e. harmonics suppression) provided by a pair of passive compensating coils of given location and size in the dipole magnet field is developed. A candidate vacuum chamber shape, # D of Figure 8, is shown to reduce both the eddy current induced dipole component and the required maximum current in passive compensating coils by a factor of 4.6 over that of the strawman vacuum chamber design. The induced sextupole component is similarly reduced by a factor of 3.5. Recommended size and positioning of passive compensating coils capable of nulling both dipole and sextupole components are listed in Section 4.2.5. A 1 mm error in the positioning of the compensating coils gives rise to a

field quality error of $1 \cdot 10^{-4}$ over $r=2.5$ cm. A coil positioning tolerance of ~ 0.25 mm (10 mils) would put dipole magnet vacuum chamber eddy current induced harmonics in the noise with respect to the desired field quality.

In Section 5 a quadrupole magnet and vacuum chamber are mapped to dipole geometry, after which a closed-form expression, Eqn. (42), for vacuum chamber eddy current induced harmonics is formulated using results from Sections 2 and a Schwarz-Christoffel technique analogous to that in Section 3. Analytical results of field harmonics from Code EDDY (Appendix C) for an elliptical strawman quadrupole vacuum chamber shape are shown to match those of a lengthy two-step pair of POISSON^[1] runs. A candidate circular vacuum chamber shape, # F of Figure 14, featuring a 4.0 cm ID circular vacuum chamber is shown to give a maximum quadrupole harmonic of $0.5 \cdot 10^{-3} B_{I_{coil}}$ at $|r| = 2.5$ cm ($= 0.8 \cdot 10^{-3} B_{I_{coil}}$ at $|r| = 4.0$ cm). All higher order harmonics are negligible. The magnitude of the vacuum chamber eddy current induced quadrupole term for these vacuum chamber shapes may be sufficiently low depending on the final dipole-quadrupole tracking design specification and tolerance to risk; correction coils may not be needed in the quadrupole magnet.

In Section 6 a generalized closed-form expression for vacuum chamber eddy current induced harmonics in an m^{th} order multipole of 'symmetrical' vacuum chamber shape is formulated [Eqn. (44) or Eqn. (47)]. Generalized passive eddy current induced harmonics suppression in an m^{th} order multipole is discussed. Pertinent generalized vacuum chamber shapes in combination with m^{th} order multipole magnets are summarized along with allowed harmonics. Correction coils are not needed in the LEB sextupole magnets.

2 Eddy Currents in a Thin-walled Conductor

If in an electromagnet with a sinusoidal time-dependent excitation

$$I_{coil} = I_{DC} + I_{\Delta} \cos \omega t \quad (1)$$

the iron yoke remains far from saturation, then, neglecting hysteresis, the two-dimensional magnetic vector potential A in the good field region is very nearly linear with current:

$$A(I_{coil}, x, y) = A(I_{DC}, x, y) + A(I_{\Delta}, x, y) \cos \omega t \quad (2)$$

and the rate of change of the vector potential with time is

$$\frac{dA(I_{coil}, x, y)}{dt} = -A(I_{\Delta}, x, y) \omega \sin \omega t \quad (3)$$

Since $B \equiv \nabla \times A$, we can relate the electric field E to the vector potential via Maxwell:

$$-\nabla \times E = \frac{dB}{dt} = \nabla \times \frac{dA}{dt} \quad (4)$$

from which it follows

$$-E = \frac{dA}{dt} + c_0 \quad (5)$$

Eddy currents will be induced in any conducting medium present (such as a vacuum chamber), which alter the magnetic field in the region of interest. The current density in the conducting medium is given by:

$$j = \sigma E = -\sigma \left(\frac{dA}{dt} + c_0 \right) \quad (6)$$

For a vacuum chamber of thickness D , the current per unit length along the arc s of the conducting chamber is

$$I'_{vac}(s \in z) = jD = -\sigma D \left(\frac{dA}{dt} + c_0 \right) \quad (7)$$

Since there is no net current flowing over the cross section of the conducting medium,

$$\int_s -\sigma D \left(\frac{dA}{dt} + c_0 \right) ds = 0 \quad (8)$$

where ds is the incremental arc length of the perimeter of the vacuum chamber. Thus c_0 is given by:

$$c_0 = \frac{-\int_s \sigma D \frac{dA}{dt} ds}{\int_s \sigma D ds} \quad (9)$$

and we have for a uniformly thick and uniformly conductive vacuum chamber:

$$\begin{aligned} I'_{vac}(s \in z) &= -\sigma D \omega \sin \omega t \left[-A(I_\Delta, s) + \frac{\int_s \sigma D A(I_\Delta, s) ds}{\int_s \sigma D ds} \right] = \\ &\sigma D \omega \sin \omega t \left[A(I_\Delta, s) - \frac{1}{S} \int_s A(I_\Delta, s) ds \right] \end{aligned} \quad (10)$$

where S is the perimeter of the vacuum chamber.

For a given magnet geometry, POISSON need be run for only one arbitrary magnet excitation, say $I_{coil} = I_1$, resulting in a tabulated vector potential distribution $A(I_1, x, y)$. Since the problem is linear, $\frac{I_\Delta}{I_1} = \frac{A(I_\Delta, x, y)}{A(I_1, x, y)} = \frac{B(I_\Delta, x, y)}{B(I_1, x, y)}$ and $I'_{vac}(s \in (x, y))$ can be expressed in terms of the tabulated vector potential $A(I_1, s)$ along the vacuum chamber contour, for any specified I_Δ (or equivalently, B_Δ):

$$I'_{vac}(s \in (x, y), t) = \sigma D \omega \sin \omega t \left[\frac{I_\Delta}{I_1} \right] \left[A(I_1, s) - \frac{1}{S} \int_s A(I_1, s) ds \right] \quad (11)$$

Figure 1a. Electromagnet dipole
z plane geometry

Figure 1b. Schwarz-Christoffel transformed
t plane geometry

3 Harmonic Components of Field Perturbations Induced by Arbitrarily Located Current Filaments

Lee^[2] has developed expressions for a multipole expansion of the field in a dipole magnet from vacuum chamber eddy currents. Here we formulate analytical expressions for field harmonics in a dipole electromagnet induced by arbitrarily located current filaments. Analogous expressions for higher order multipole magnets are obtainable after conformally mapping the multipole geometry to a dipole geometry.

Figure 1a shows the gap portion of a wide [in the x -direction] electromagnet dipole exhibiting midplane symmetry. Assuming the vacuum chamber also exhibits midplane symmetry, then current filaments of magnitude I can be thought to occur in pairs at z_0 and z_0^* . The return paths for the currents will be $-I$ at $-z_0$ and $-z_0^*$ if the vacuum chamber is also symmetric about the y -axis.

The x -direction width of the iron pole face yoke is assumed effectively infinite in the analytical model. Exponential decay of field errors as one moves towards the origin from the corner of the iron pole insures that this approximation will not result in large errors of calculated harmonic components near the beam axis (i.e. the origin in Figure 1a).

We shall use a Schwarz-Christoffel transformation to conformally map the gap region between $z = (x, \pm h)$ of Figure 1a to the upper half t plane shown in Figure 1b.¹ Choosing $t = \infty$ at $z = +\infty$ results in the transformation

$$\frac{dz}{dt} = \frac{1/k}{t-a} \quad (12)$$

Choosing $t = a = 0$ at $z = -\infty$ makes $1/k$ take on the value $\frac{2h}{\pi}$ since at $t = 0$, $Im(\Delta z) = Im(\int_{t=0}^{\frac{1}{kt}} dt)$, $\rightarrow 2h = Im(\frac{\pi i}{k}) = \frac{\pi}{k}$. Integrating with respect to t gives

¹ Alternatively, since the dipole exhibits midplane symmetry, mapping the half-gap region between $y = 0$ and $y = h$ of Figure 1a to the upper half t plane yields the same result.

$$z = \frac{2h}{\pi} \ln t + c, \longrightarrow t = e^{\frac{\pi(z-c)}{2h}} \quad (13)$$

Choosing $t = 1$ at $z = -ih$ makes c take on the value $-ih$. The complete transformation is thus

$$z = \frac{2h}{\pi} \ln t - ih, \longrightarrow t = ie^{\frac{\pi z}{2h}} = ie^{kz} = ie^{\alpha} \quad (14)$$

where $k \equiv \frac{\pi}{2h}$ and $\alpha \equiv kz$.

For a filament of magnitude $+I$ at z_0 and its image in the t plane about the real axis, which makes the magnetic field perpendicular to the air-iron interface, the complex potential $F(t)$ is given by

$$F(t) = \frac{-I}{2\pi} \ln(t - t(z_0))(t - t^*(z_0)) \quad (15)$$

The magnetic field is related to the complex potential by $H^*(z) = i \frac{dF}{dz}$. For the case under consideration we have

$$H^*(z) = i \frac{dF/dt}{dz/dt} = \frac{Ik e^{\alpha}}{2\pi i} \left[\frac{1}{e^{\alpha} - e^{\alpha_0}} + \frac{1}{e^{\alpha} + e^{\alpha_0^*}} \right] \quad (16)$$

For current filaments of magnitude $+I$ at z_0 and z_0^* , and their images in the t plane about the real axis, we have for $H^*(z)$:

$$H^*(z) = i \frac{dF/dt}{dz/dt} = \frac{Ik}{2\pi i} e^{\alpha} \left[\frac{1}{e^{\alpha} - e^{\alpha_0}} + \frac{1}{e^{\alpha} + e^{\alpha_0^*}} + \frac{1}{e^{\alpha} - e^{\alpha_0^*}} + \frac{1}{e^{\alpha} + e^{\alpha_0}} \right] \quad (17)$$

where the relations $t(z_0) = ie^{\alpha_0}$, $t^*(z_0) = -ie^{\alpha_0^*}$, $t(z_0^*) = ie^{\alpha_0^*}$, and $t^*(z_0^*) = -ie^{\alpha_0}$ have been used. Simplifying, we have

$$H^*(z) = \frac{Ik}{2\pi i} \left[\frac{e^{\alpha - \alpha_0}}{\sinh(\alpha - \alpha_0)} + \frac{e^{\alpha - \alpha_0^*}}{\sinh(\alpha - \alpha_0^*)} \right] \quad (18)$$

We now decompose these expressions for $H^*(z)$ into harmonic components. Define $G(z) \equiv \frac{e^z}{\sinh(z)} - 1 = \frac{1}{\tanh(z)}$. Then, expanding G in a power series,

$$G(\alpha - \alpha_0) = \frac{1}{\tanh(\alpha - \alpha_0)} = \sum_0 \alpha^n a_n = a_0 + a_1 \alpha + a_2 \alpha^2 + \dots \quad (19)$$

where, since $G(z)$ is an odd function, we have

$$n!a_n = \frac{\partial^n}{\partial \alpha^n} \left(\frac{1}{\text{Tanh}(\alpha - \alpha_0)} \right) \Big|_{\alpha=0} = (-1)^{n+1} \frac{\partial^n}{\partial \alpha_0^n} \left(\frac{1}{\text{Tanh}(\alpha_0)} \right) = (-1)^{n+1} \frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n}$$

Therefore,

$$\begin{aligned} G(\alpha - \alpha_0) + G(\alpha - \alpha_0^*) &= \Sigma_0 \alpha^n (a_n + c_n) = \Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} \left[\frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n} + \frac{\partial^n G(\alpha_0^*)}{\partial \alpha_0^{*n}} \right] = \\ &\Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} 2 \text{Re} \left[\frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n} \right] \end{aligned} \quad (20)$$

Thus, the multipole components for a pair of current filaments of magnitude $+I$ at z_0 and z_0^* are given by (dropping the uniform constant field):

$$H^*(z) = \frac{Ik}{2\pi i} [G(\alpha - \alpha_0) + G(\alpha - \alpha_0^*)] = \frac{I}{2hi} \Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} \text{Re} [b_n] \quad (21)$$

where

$$\begin{aligned} b_0 &= \left(\frac{1}{T} \right), \quad b_1 = \left(\frac{-1}{S^2} \right), \quad b_2 = \left(\frac{2C}{S^3} \right), \\ b_3 &= (-2) \left(\frac{2S^2 + 3}{S^4} \right), \quad b_4 = \left(\frac{8C}{S^5} \right) (S^2 + 3), \quad b_n = \frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n} \end{aligned}$$

and where $S \equiv \text{Sinh}(\alpha_0)$, $C \equiv \text{Cosh}(\alpha_0)$, $T \equiv \text{Tanh}(\alpha_0)$, $\alpha \equiv \frac{\pi z}{2h}$, and $\alpha_0 \equiv \frac{\pi z_0}{2h}$.

If additionally, the vacuum chamber is symmetric about the y -axis, the return paths for the currents will be $-I$ at $-z_0$ and $-z_0^*$.

We have for the z location $-\alpha_0$:

$$G(\alpha + \alpha_0) = \frac{1}{\text{Tanh}(\alpha + \alpha_0)} = \Sigma_0 \alpha^n a_n = a_0 + a_1 \alpha + a_2 \alpha^2 + \dots \quad (22)$$

where

$$n!a_n = \frac{\partial^n}{\partial \alpha^n} \left(\frac{1}{\text{Tanh}(\alpha + \alpha_0)} \right) \Big|_{\alpha=0} = \frac{\partial^n}{\partial \alpha_0^n} \left(\frac{1}{\text{Tanh}(\alpha_0)} \right) = \frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n}$$

Therefore,

$$\begin{aligned} G(\alpha + \alpha_0) + G(\alpha + \alpha_0^*) &= \Sigma_0 \alpha^n (a_n + c_n) = \Sigma_0 \frac{\alpha^n}{n!} \left[\frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n} + \frac{\partial^n G(\alpha_0^*)}{\partial \alpha_0^{*n}} \right] = \\ &\Sigma_0 \frac{\alpha^n}{n!} 2 \text{Re} \left[\frac{\partial^n G(\alpha_0)}{\partial \alpha_0^n} \right] \end{aligned} \quad (23)$$

Figure 2. SSC Low Energy Booster strawman dipole magnet geometry.

Thus, for this special case with current filaments of magnitude $+I$ at z_0 and z_0^* and $-I$ at $-z_0$ and $-z_0^*$, the multipole components are given by

$$H^*(z) = \frac{Ik}{\pi i} \sum_0 \frac{-2\alpha^n}{n!} \text{Re}[b_n] = \frac{I}{hi} \sum_{n=\text{even}} \frac{-\alpha^n}{n!} \text{Re}[b_n] \quad (24)$$

where the b_n are as given above and where the $n = \text{odd}$ terms drop out.

4 SSC Low Energy Booster Dipole Magnet Design

The strawman geometry for the SSC low energy booster dipole magnet is shown in Figure 2. The near on-axis field has components due to both the magnet coil excitation and the eddy current excitation in the vacuum chamber:

$$B_{tot}^*(z) = B_{I_{coil}}^*(z) + B_{I_{vac}(s)}^*(z) \quad (25)$$

each of which may be decomposed into harmonics² as expressed by:

$$iB^*(z) = B_0 \sum_{n=0} (b_n + ia_n) z^n \quad (26)$$

The strawman field quality specification for these magnets are that $\frac{\Delta B}{B} \leq 10^{-4}$ at $|z| = 2.5$ cm (later changed to $\frac{\Delta B}{B} \leq 10^{-4}$ at $|z| \leq 3.25 \cos \theta + i2.5 \sin \theta$ cm.) The

²Here we use SSC's harmonics nomenclature; POISSON code results are generated using the convention $B^*(z) = \sum_{n=1} \ln \frac{(a_n + ib_n)}{r} \left(\frac{z}{r}\right)^{n-1}$, thus for the k^{th} harmonic ($k = 0$ being the dipole) SSC's $B_0 b_k$ and $B_0 a_k$ are equivalent to POISSON's (see Figure 4) $\frac{-(k+1)a_{k+1}}{r} \frac{1}{r^k}$ and $\frac{-(k+1)b_{k+1}}{r} \frac{1}{r^k}$, respectively.

excitation is to produce a field strength that cycles between 0.13 T and 1.3 T. The half gap is 2.86 cm.

Maintaining the dipole quality of the first term of Eqn. (25) consists of designing the pole so as to avoid magnetic saturation and shaping the tip so as to compensate for the finite shape of the pole. The LEB dipole magnet design and harmonics tabulations for various magnet coil excitations are discussed elsewhere^[3,4]. Specifications are summarized in Appendix A.

Maintaining the dipole quality of the second term of Eqn. (25) consists of either shaping the vacuum chamber or adding compensating eddy current carrying conductor filaments so as to null harmonic components due to the eddy current distribution in the vacuum chamber conductor. Eddy currents in the vacuum chamber are insensitive to magnet design changes improving the quality of near on-axis field, thus field quality improvement modifications associated with the two contributions to the dipole given by Eqn. (25) may be decoupled.

4.1 Suppression of Pole Magnet Geometry and Iron Saturation Induced Harmonics

Harmonics suppression via designing the pole so as to avoid magnetic saturation and shaping the tip so as to compensate for the finite x -direction pole width is outside the principal thrust of this memo and is discussed elsewhere^[3,4].

4.2 Suppression of Vacuum Chamber Eddy Current Induced Harmonics

4.2.1 Effect of Magnet Excitation Pattern on Maximum $\frac{\Delta B}{B}$ ^[5]

The field quality goal with respect to the eddy current induced field perturbations is to minimize not the field perturbations themselves, but rather the ratio of the field perturbations to the total field B_{tot} . For a magnet of given geometry, it is interesting to see how minor changes of the excitation pattern can dramatically change the magnitude of the eddy current induced maximum $\frac{\Delta B}{B}$. Here excitation modification effects on harmonics are illustrated as an aide for those situations when one has such a freedom.

It should be noted that the ratio of the field harmonics (e.g dipole:quadrupole:sextupole) due to the eddy current distribution in the vacuum chamber is constant for a given chamber shape and does not change with magnet excitation. Thus when altering the excitation pattern for a given magnet and vacuum chamber geometry, the various harmonic components of eddy current induced field perturbations are altered proportionately. Also note that $|B_{I_{coil}}| \gg |B_{I'_{vac}}|$ and thus $|B_{tot}| \simeq |B_{I_{coil}}|$.

The magnitude of $B_{I'_{vac}}$ is proportional to the time rate of change of the field due to the magnet excitation:

$$B_{I'_{vac}}(s) \propto I'_{vac}(s) \propto j \propto E \propto \frac{dA}{dt} \propto dB_{I_{coil}}/dt \quad (27)$$

Figure 3. $\left| \frac{dB_{I_{coil}}/dt}{B_{I_{coil}}} \right|$ and B versus ωt for two magnet excitation functions.

Thus, $\left| \frac{B_{I_{vac}}}{B_{tot}} \right|$ is largest when $|dB_{I_{coil}}/dt/B_{I_{coil}}|$ is maximized.

For the current excitation given by Eqn. (1), the I_{coil} induced magnetic flux density in the good field region and its rate of change with time are given by the numerator and denominator, respectively, of Eqn. (28):

$$\frac{dB_{I_{coil}}/dt}{B_{I_{coil}}} = \frac{-B_{\Delta}\omega \sin \omega t}{B_{dc} + B_{\Delta} \cos \omega t} \quad (28)$$

At its maximum, we have:

$$\left| \frac{dB_{I_{coil}}/dt}{B_{I_{coil}}} \right|_{\cos \omega t = -B_{\Delta}/B_{dc}} = \frac{\omega}{\sqrt{\left(\frac{B_{dc}}{B_{\Delta}}\right)^2 - 1}} \quad (29)$$

Assume a 'strawman' dipole excitation design range were 0 – 13 kG, with injection at 1.3 kG. Thus $B_{dc} = B_{\Delta} = 6.5$ kG. $\left| \frac{dB/dt}{B} \right|$ versus ωt for these values is plotted in Figure 3 along with a plot for the for an 'alternative' excitation pattern with a range 1.3 – 13.0 kG, and with injection at 1.3 kG (i.e. $B_{dc} = 7.15$ kG and $B_{\Delta} = 5.85$ kG).

For the strawman excitation ($B_{dc} = B_{\Delta} = 6.5$ kG), the maximum $\left| \frac{dB/dt}{B} \right|$ that the protons see is at injection, where the field is 1.3 kG (see Figure 3). Here $\omega t \simeq 143.1^\circ$ and $\left| \frac{dB/dt}{B} \right| = 3.00\omega$. For the alternative excitation ($B_{dc} = 7.15$ kG, $B_{\Delta} = 5.85$ kG), the maximum $\left| \frac{dB/dt}{B} \right|$ that the protons see is at $\omega t \simeq 144.9^\circ$ (see Figure 3). Here the field is $B = 2.364$ kG and $\left| \frac{dB/dt}{B} \right| = 1.423\omega$. Thus, maximum $\frac{\Delta B}{B}$ due to eddy currents is reduced

by $\sim$ half when the alternative excitation pattern is employed. The proposed excitation pattern for the LEB dipole magnets is that of the 'alternative' excitation pattern above.

4.2.2 Numerical Adaptation of Vacuum Chamber Eddy Current Effects to POISSON

It is convenient to model the continuously varying current sheet $I'_{vac}(s)$ of Eqn. (11) as discretized currents at N mesh points along the vacuum chamber arc s :

$$dI_{s_j} = I'_{s_j} ds_j \quad (30)$$

where $ds_j = 0.5(|\vec{z}_{j+1} - \vec{z}_j| + |\vec{z}_j - \vec{z}_{j-1}|)$ for $j = 2, \dots, N-1$, $ds_1 = 0.5|\vec{z}_2 - \vec{z}_1|$, and $ds_N = 0.5|\vec{z}_N - \vec{z}_{N-1}|$.

If we have Dirichlet vector potential symmetry about the y -axis, as in Figure 2, $A(x, y) = -A(x, -y)$ and the integral term in Eqn. (11) vanishes. If we also have Neumann vector potential symmetry about the x -axis, as in Figure 2, $A(x, y) = A(x, -y)$ and we can use the midplane symmetry plane default (`con(22) = 1`) in POISSON.

Normalization of Eqn. (11) with respect to $\sigma D \frac{dB_{I_{coil}}}{dt}$ and using $\frac{I_A}{I_1} = \frac{B_A}{B_1}$ gives for the input vacuum chamber current filaments in POISSON:

$$\frac{dI_{s_j}}{\sigma D \frac{dB_{I_{coil}}}{dt}} = \frac{dI_{s_j}}{-\sigma D B_A \omega \sin \omega t} = \left(-A(I_1, s_j) + \frac{1}{S} \int_s A(I_1, s) ds \right) \frac{ds_j}{B_1} \quad (31)$$

The strawman LEB vacuum chamber is an ellipse with x and y major half-axes $a_x = 5.5$ cm and $b_y = 2.75$ cm, respectively. For this symmetry about the y -axis, with $A(0, y) = 0$ the integral term in Eqn. (31) is zero. The eddy current induced harmonic components from a POISSON run with current filaments I_{s_j} input according to Eqn. (31), with $\sigma D \frac{dB_{I_{coil}}}{dt} = 1$ A/cm² and with the $A(I_1, s_j)$, $B_1 \equiv iB^*(z) = B_y$, and ds_j from the dipole magnet excitation POISSON run with I_{coil} set equal to an arbitrary value I_1 , as discussed above Eqn. (11), are tabulated in Figure 4. Sample POISSON files are found in Appendix B. POISSON results for eddy current induced harmonics for the LEB dipole magnet are summarized in Figure 5. The quadrupole, octupole, etc. terms are zero for this vacuum chamber exhibiting symmetry with respect to the y -axis.

4.2.3 Closed Form Analytical Prediction of Eddy Current Induced Harmonics for LEB Dipole Magnet

For a dipole magnet with a vacuum chamber symmetric about the y axis, $\frac{A(I_1, x, y)}{B_1} = -x$. Thus from Eqn. (11) or (31), we have for the current sheet along the vacuum chamber:

$$I'_{vac}(s, t) = x \sigma D \frac{dB_{I_{coil}}}{dt} \quad (32)$$

Using the value for $I'_{vac}(s, t)$ given by Eqn. (32) in the expression for eddy current induced harmonics Eqn. (21) yields

TABLE FOR FIELD COEFFICIENTS

NORMALIZATION RADIUS = 2.50000

$$(BX - I BY) = I * \text{SUM } N * (AN + I BN) / R * (Z/R) ** (N-1)$$

N	N(AN)/R	N(BN)/R	ABS(N(CN)/R)
1	8.5375E+00	0.0000E+00	8.5375E+00
3	-1.1488E+00	0.0000E+00	1.1488E+00
5	5.6757E-02	0.0000E+00	5.6757E-02
7	7.6619E-03	0.0000E+00	7.6619E-03
9	-2.2467E-03	0.0000E+00	2.2467E-03

Figure 4. Vacuum chamber eddy current induced magnetic field harmonics for strawman LEB dipole magnet design (POISSON output)

<ul style="list-style-type: none"> • input $I_{s,i}$ per Eqn. (31), • LEB dipole magnet geometry per Figure 2, half gap $h = 2.86$ cm • elliptical vacuum chamber with major half-axes of 5.5 cm and 2.75 cm 						
At $ r = 2.5$ cm:						
harmonic component	$\frac{iB'_{vac}}{\sigma D \frac{dB}{dt}} \left(\frac{G}{\frac{A}{cm^2}} \right)$	$\frac{iB'_{vac}}{dB/dt} (s)$ $\frac{1}{\sigma} = 47 \mu\Omega \text{ cm},$ $D = 0.03 \text{ cm}$	$iB'_{vac} (G)$ at condition #1: $\sigma D \frac{dB}{dt} = 2.346 \frac{A}{cm^2}$		$iB'_{vac} (G)$ at condition #2: $\sigma D \frac{dB}{dt} = 1.349 \frac{A}{cm^2}$	
			(G)	(% of B_{Icoil})	(G)	(% of B_{Icoil})
dipole:	-8.5400	$-54.5 \cdot 10^{-6}$	-20.00	-.2800	-11.50	-.4865
sextupole:	1.1500	$7.34 \cdot 10^{-6}$	2.7030	.0377	1.550	.0656
decapole:	-0.0567	$-.362 \cdot 10^{-6}$	-.1330	-.0019	-.0765	-.0032
where conditions #1 and #2 are defined by						
<ul style="list-style-type: none"> • an excitation pattern: $\omega = 20\pi/s$, $B_{dc} = 7.15$ kG, $B_{\Delta} = 5.85$ kG, and • a vacuum chamber conductivity σ, $\frac{1}{\sigma} = 47 \mu\Omega \text{ cm}$ and thickness $D = 0.03$ cm 						
condition #	condition	ωt	$B_{Icoil} (G)$	$dB_{Icoil}/dt (G/s)$	$\frac{dB_{Icoil}/dt}{B_{Icoil}} (s)$	$\sigma D \frac{dB_{Icoil}}{dt} \left(\frac{A}{cm^2} \right)$ at $D = 0.03 \text{ cm},$ $\frac{1}{\sigma} = 47 \mu\Omega \text{ cm}$
1	B_{Ivac} maximized	90°	7150	367,566	51.4	2.346
2	$\frac{B'_{vac}}{B_{Icoil}}$ maximized	144.9°	2364	211,368	89.4	1.349

Figure 5. LEB dipole magnet eddy current induced harmonics from POISSON

$$\frac{B_{I_{vac}}^*(z)}{\sigma D \frac{dB_{I_{coil}}}{dt}} = \frac{\mu_0}{2hi} \Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} \int_s \text{Re}[b_n] \frac{dI_s}{\sigma D \frac{dB_{I_{coil}}}{dt}} ds = \frac{\mu_0}{2hi} \Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} \int_s \text{Re}[b_n] x ds \quad (33)$$

where the b_n are as defined in Eqn. (21) with $z_0 \in s$ and integration is along the vacuum chamber arc in the upper half z -plane (which for our elliptical strawman vacuum chamber shape is defined by $s(z) = a_x \cos \phi + ib_y \sin \phi$, with $a_x = 5.5$ cm, $b_y = 2.75$ cm, and $0 < \phi < \pi$). Eqn. (33) is valid only when the vacuum chamber exhibits midplane symmetry.

For $h = 2.86$ cm, (the LEB dipole magnet half gap), harmonics at $|r| = 2.5$ cm as given in closed form by Eqn. (33) are $\frac{iB_{I_{vac}}^*(z)}{\sigma D (dB_{I_{coil}}/dt)} = -8.65 + 1.14 - 0.0709 \frac{G}{A/cm^2}$ for the dipole, sextupole, and decapole components, respectively (See Figure 6). These results for an ideal dipole field are virtually identical to those obtained numerically by POISSON for the LEB dipole magnet geometry in the previous section (Figure 5, column 2). The driver program EDDY for evaluation of the harmonic components given by Eqn. (33) for input vacuum chamber dimensions in a background time dependent multipole (dipole for this case) field is found in Appendix C.

4.2.4 Passive Eddy Current Induced Harmonics Suppression

Compensating eddy current carrying conductor filaments can provide passive nulling of harmonic components. For a filament of uniform cross section at z_0 and its return at z_∞ , both parallel to the beam axis, the eddy current density induced in the filament is (cf. Eqn. (11) or (31)):

$$\begin{aligned} \frac{j_{z_0}}{dB_{I_{coil}}/dt} &= \frac{\sigma}{B_1} \left(-A(I_1, z_0) + \frac{1}{a_{tot}} \int_a A(I_1, z) da \right) = \\ \frac{\sigma}{B_1} \left(-A(I_1, z_0) + \frac{A(I_1, z_0)a_{z_0} + A(I_1, z_\infty)a_{z_\infty}}{a_{z_0} + a_{z_\infty}} \right) &= \frac{\sigma(A(I_1, z_\infty) - A(I_1, z_0))}{2B_1} \end{aligned} \quad (34)$$

since conductor area $a_{z_\infty} = a_{z_0}$. For a dipole magnet $\frac{(A(I_1, z_\infty) - A(I_1, z_0))}{B_1} = -(x_\infty - x_0)$. The compensating current flowing in the conductor at z_0 is then:

$$\frac{I_{z_0}}{dB_{I_{coil}}/dt} = \frac{-\sigma a_{z_0}(x_\infty - x_0)}{2} \equiv \frac{-\sigma a_{z_0} \Delta x}{2} \quad (35)$$

The return current location z_∞ can be positioned where it does not contribute to the near on-axis field.

Using a pair of compensating coils at z_0 and z_0^* of magnitude $+I_{z_0}$ given by Eqn. (35) in Eqn. (21) (with their returns at z_∞), we have for the magnitude of the field correction:

$$\frac{B_{I_{z_0}}^*(z)}{dB_{I_{coil}}/dt} = \frac{-\mu_0 \sigma a_{z_0} \Delta x}{4hi} \Sigma_0 \frac{-1^{n+1} \alpha^n}{n!} R_c[b_n] \quad (36)$$

where the $b_n = b_n(\alpha_0(z_0))$. As an example, say we wish to null both the vacuum chamber eddy current induced dipole and sextupole terms with four compensating conductors attached somewhere to the vacuum chamber (with their returns located far from the region of interest). From Figure 6 (or Figure 5), the ratio of the $B_{I_{vac}}$ dipole to sextupole terms is $\frac{-8.65}{1.14} \simeq -7.5$. From Figure 6, filaments on the vacuum chamber at $(x, y) \simeq (\pm 3.41, \pm 2.16)$ provides this same dipole to sextupole ratio ($\frac{-210}{.0280} \simeq -7.5$). A maximum current magnitude in each of four conductors $I_{z_0} = I_{z_0^*} = -I_{-z_0} = -I_{-z_0^*} = 47.5 \text{ A} = (\frac{1}{2})(\frac{20.0G}{0.210G/Amp})$, corresponding to $\frac{dB_{I_{coil}}}{dt}|_{max} = 367,566 \text{ G/s}$ (see Fig. 5, condition #1), would null both the dipole and sextupole vacuum chamber eddy current induced components. Two conductors of maximum current $I_{z_0} = I_{z_0^*} = 95 \text{ A} = (\frac{20.0}{0.210})$ would also null both these harmonics, but would create a quadrupole component. From Eqn. (35), a copper wire conductor with conductivity $1/\sigma = 1.74\mu\Omega \text{ cm}$ (~ 27 times that of the vacuum chamber), of cross section $a_{z_0} = 0.00750 \text{ cm}^2$ (diameter = 0.0977 cm) would be required for return filaments z_{∞} positioned in a dipole field such that $\Delta x = 6 \text{ cm}$. In practice, the copper filament cross section should be several times larger and tuned with a series resistor. Then the exact location of the return filaments is not important; they can be positioned anywhere beyond the corner of the tip, away from the gap so that they do not directly influence the good field region.

The eddy current conducting copper filaments cannot occupy the exact same physical location as the vacuum chamber; code EDDY (Appendix C) can be used to calculate the dipole/sextupole suppression ratio for an arbitrarily positioned conductor.

4.2.5 Dipole Vacuum Chamber Shaping

The strawman dipole vacuum chamber design results in maximum eddy current induced dipole and sextupole components of $\sim 50 \cdot 10^{-4}$ and $\sim 7 \cdot 10^{-4} B_{I_{coil}}$ at $|r| = 2.5 \text{ cm}$, respectively (see Figure 5, condition #2). While the uniform dipole component does not degrade the field quality, tracking with the quadrupole is adversely affected. The sextupole component is $\sim$ an order of magnitude larger than the field quality specification allows. The cleanest solution is to null both these components, as outlined above. First, however it is desirable to alter the vacuum chamber shape so as to suppress the initial (uncorrected) harmonics level itself as much as possible.

For any given vacuum chamber shape, code EDDY allows rapid evaluation of vacuum chamber eddy current induced harmonics as well as determination of compensating passive harmonics suppression coil locations without resorting to lengthy POISSON code runs.

Figure 7 shows candidate dipole vacuum chamber shapes. For these candidate shapes, in Figure 8 are tabulated eddy-current induced dipole/sextupole ratio, wall thickness, maximum eddy current induced field, and maximum current required in each of four compensating coils along with their approximate locations along the vacuum chamber so as to null both sextupole and dipole vacuum chamber eddy current induced harmonics. (See Appendix C for 'dipole vacuum chamber options' EDDY output files.)

Option D reduces both the eddy current induced (dipole) harmonic and the maximum required corrector coil current by a factor of ~ 4.6 over that of the strawman design A. The sextupole harmonic is also a factor of ~ 3.5 smaller. Part of the improvement is due

* Inner full aperture: 52 mm x 107 mm, elliptical

* Wall thickness: 3.0 mm

A

C

LEB-1

* Wall thickness: 1.5 mm

* elliptical

* Inner full aperture:

52 mm x 70 mm

* Inner full aperture: 52 mm x 82 mm, elliptical

* Wall thickness: 1.5 mm

B

D

LEB-2

* Wall thickness: 1.5 mm

* stretched circle

* Inner full aperture:

52 mm x 62 mm

Figure 7. Candidate LEB dipole vacuum chamber shapes.

#	label	a_x	b_y	horiz. stretch length	Dip/Sext frm vac ch eddy curs.	D mm	$iB_{I_{vac}}^*$ (Dip) max % $B_{I_{coil}}$ (cond. #2)	I_{max} (A) to correct (cond. #1)	corrector location on vac. ch.
A	straw	5.5	2.75	-	-7.59	3.0	0.487	47.5	(3.41,2.16)
B	1 st mod	4.2	2.7	-	-6.04	1.5	0.156	15.2	(2.99,1.89)
C	leb #1	3.575	2.675	-	-5.72	1.5	0.122	11.8	(2.74,1.72)
D	leb #2	2.675	2.675	1.0	-5.70	1.5	0.106	10.3	(2.62,1.62)

Figure 8. Eddy current induced harmonics (at $r = 2.5$ cm) and required maximum compensating coil currents and location for the candidate vacuum chamber shapes of Figure 7 and excitation parameters in Figure 5

Figure 9. Contours of constant dipole/sextupole at $r = 2.5$ cm in a dipole with half gap $h = 2.86$ cm.

to the decreased thickness and part to the improved contour of the vacuum chamber.

For the conductor conductivity above and $\Delta x = 6$ cm in a dipole field, the required corrector coil diameter for option D is $d = 0.0455$ cm, ($a_{z0} = 0.00163$ cm²).

Let's make the copper conductor with $d = 0.1$ cm (a cross-sectional safety margin of ~ 5) and assume an insulating/protection sheath of outer diameter 0.5 cm. Figure 9 can be used to position the corrector coil as shown near the vacuum chamber along the dipole/sextupole = -5.70 contour at the location (2.86, 1.84). (Alternatively, see 'coil positioning' EDDY output files in Appendix C for exact positioning). The coil returns should be placed greater than 6 cm away horizontally around the tip corner, as shown.

Precise placement of the compensating coils (as well as tuning of the series resistor) is necessary because we are nulling two harmonic components. Figures 8 and 9 show that a 1 mm error in compensating coil position can lead to as much as a 0.01% $B_{I_{coil}}$ step in the sextupole component at $r = 2.5$ cm. Ideally, coil positioning tolerances should be such that the remaining sextupole field dwarves the field quality spec; a tolerance ~ 0.25 mm (10 mils) achieves this. Alternatively, field quality errors due to coil mispositioning can be reduced at the expense of dipole/quadrupole magnet tracking by tuning the correction coil resistor so as to completely null the sextupole term at the expense of partial dipole nulling.

The decapole component for option D is $0.5 \cdot 10^{-4} B_{I_{coil}}$ at $|r| = 2.5$ cm; it's within the strawman field quality specification, but not 'in the noise'.

5 SSC Low Energy Booster Quadrupole Magnet Design

The strawman geometry for the SSC low energy booster quadrupole magnet is shown in Figure 10. The near on-axis field has components due to both the magnet coil excitation and the eddy current excitation in the vacuum chamber as given by Eqns. (25) and (26).

The strawman field quality specification for these magnets are that $\frac{\Delta B}{B} \leq 10^{-3}$ at $|z| = 2.0$ cm, (later changed to $\frac{\Delta B}{B} \leq 10^{-3}$ at $|z| = 4.25$ cm.) The excitation is to produce a field strength that cycles between ~ 1.6 and ~ 16 T/m. (Recall that the quadrupole and dipole are powered together and thus the exact quadrupole field strength is determined by the current required to meet the dipole extraction field strength). The quadrupole aperture is 5 cm.

Maintaining the quadrupole quality of the first term of Eqn. (25) consists of designing the pole so as to avoid magnetic saturation and shaping the tip so as to compensate for the finite shape of the pole. The LEB quadrupole magnet design and harmonics tabulations for various magnet coil excitations are discussed elsewhere^[3,4]. Specifications are summarized in Appendix A.

Maintaining the quadrupole quality of the second term of Eqn. (25) consists of either shaping the vacuum chamber or adding compensating eddy current carrying conductor filaments so as to null harmonic components due to the eddy current distribution in the vacuum chamber conductor. Eddy currents in the vacuum chamber are insensitive to magnet design changes improving the quality of near on-axis field, thus field quality

Figure 10. SSC Low Energy Booster quadrupole magnet geometry.

improvement modifications associated with the two contributions to the quadrupole given by Eqn. (25) may be decoupled.

5.1 Suppression of Pole Magnet Geometry and Iron Saturation Induced Harmonics

Harmonics suppression via designing the pole so as to avoid magnetic saturation and shaping the tip so as to compensate for the truncated hyperbolic pole shape is outside the principal thrust of this memo and is discussed elsewhere^[3,4].

5.2 Suppression of Vacuum Chamber Eddy Current Induced Harmonics

5.2.1 Mapping Quadrupole Geometry to Dipole Geometry

To facilitate calculation of eddy current induced harmonics in the region of a time dependent multipole field it is useful to first map the multipole z -plane geometry, including the vacuum chamber to a dipole w -plane geometry, calculate an analytic variable of interest such as $B^*(w)$ in the w -plane geometry, and then map back to the original z -plane geometry. For the quadrupole geometry, we use the transformation $w \equiv u + iv = z^2/r_p$, where r_p is the quadrupole aperture radius. The elliptical vacuum chamber described by $z = a_x \cos \phi + ib_y \sin \phi$ is transformed to an ellipse in w : $w = c_u + a_u \cos \gamma + ib_v \sin \gamma$, where $c_u = (a_x^2 - b_y^2)/2r_p$, $a_u = (a_x^2 + b_y^2)/2r_p$, and $b_v = (2a_x b_y)/2r_p$.

The quadrupole magnet induced complex magnetic potential

$$F(w) \equiv A + iV = -wB + c_2$$

from which it follows $B^*(w) = i \frac{dF}{dw} = iB(t)$. The transformation back to the z -plane gives for $B^*(z)$:

$$B^*(z) = i \frac{dF}{dw} / \frac{dz}{dw} = iB(t)2z/r_p \quad (37)$$

5.2.2 Magnet Excitation

For the LEB quads $\frac{2B_{max}(t)}{r_p} \simeq 1.6$ kG/cm and $\frac{2B_{min}(t)}{r_p} \simeq 0.16$ kG/cm. Thus we have

$$\frac{2B(t)}{r_p} = \frac{2B_{dc}}{r_p} + \frac{2B_{\Delta}}{r_p} \cos \omega t \quad (38)$$

where $\frac{2B_{\Delta}}{r_p} = 0.72$ kG/cm and $\frac{2B_{dc}}{r_p} = 0.88$ kG/cm.

For this excitation pattern the maximum $\left| \frac{dB/dt}{B} \right|$ that the protons see is at $\omega t \simeq 144.9^\circ$ (see Figure 3 or Eqn. (29)). Here $2B(t)/r_p = 0.2909$ kG/cm and $\left| \frac{dB/dt}{B} \right| = 1.423\omega$.

5.2.3 Numerical Adaptation of Vacuum Chamber Eddy Current Effects to POISSON

The eddy current induced harmonic components for the 'strawman' LEB vacuum chamber geometry (see Figure 8) from a POISSON run with current filaments I_{sj} input according to Eqn. (31), with $\sigma D \frac{dB_{I_{coil}}}{dt} = 1$ A/cm² and with the $A(I_1, s_j)$, $B_1 \equiv \left| \frac{B^*(z)r_p}{i2z} \right|$ (see Eqn. (37)), and ds_j from the quadrupole magnet excitation POISSON run with I_{coil} set equal to an arbitrary value I_1 , as discussed above Eqn. (11), are tabulated in Figure 11. Sample POISSON files are found in Appendix B. POISSON results for eddy current induced harmonics for the LEB quadrupole magnet are summarized in Figure 12.

5.2.4 Closed Form Analytical Prediction of Eddy Current Induced Harmonics for LEB Quadrupole Magnet

For a quadrupole magnet mapped to dipole geometry according to the z - w transformation described above, we have from Eqn. (11) or (31):

$$I'_{vac}(s_z \in z, t) = \sigma D \frac{dB_{I_{coil}}}{dt} \left(\frac{u(s_z)B_1 - \text{Re}[c_2]}{B_1} - \frac{1}{S} \int_{all} \frac{u(s_z)B_1 - \text{Re}[c_2]}{B_1} ds_z \right) \quad (39)$$

where $' \equiv \frac{d}{ds_z}$ and s_z is the arc location in the z -plane.

Choosing³ $c_2 = 0 \implies A_1(u=0) = 0$ and $\int_{s_z} \frac{u(s_z)B_1 - c_2}{B_1} ds_z = \int_{s_z} u(s_z) ds_z \equiv \hat{u}S$. Thus we have for the current sheet density:

³The choice of c_2 is arbitrary; Eqn. (40) results for any choice.

TABLE FOR FIELD COEFFICIENTS

NORMALIZATION RADIUS = 2.50000

$$(BX - I BY) = I * \text{SUM } N * (AN + I BN) / R * (Z/R) ** (N-1)$$

N	N (AN) / R	N (BN) / R	ABS (N (CN) / R)
2	2.0267E+00	0.0000E+00	2.0267E+00
4	-3.9091E-02	0.0000E+00	3.9091E-02
6	-7.7138E-03	0.0000E+00	7.7138E-03
8	-1.9802E-02	0.0000E+00	1.9802E-02

Figure 11. Vacuum chamber eddy current induced magnetic field harmonics for strawman LEB quadrupole magnet design

<ul style="list-style-type: none"> • input $I_{s,j}$ per Eqn. (31), • LEB quadrupole magnet geometry per Figure 10, aperture $r_p = 5.0$ cm • elliptical vacuum chamber with major half-axes of 5.5 cm and 2.75 cm 						
At $ r = 2.5$ cm:						
harmonic component	$\frac{iB'_{vac}}{\sigma D (dB/dt)}$ (G/ $\frac{A}{cm^2}$)	$\frac{iB'_{vac}}{dB/dt}$ (s) $\frac{1}{\sigma} = 47\mu\Omega$ cm, $D = 0.03$ cm	iB'_{vac} (G) at condition #1: $\frac{2}{r_p} \sigma D \frac{dB}{dt} = 2.887 \frac{A}{cm^3}$		iB'_{vac} (G) at condition #2: $\frac{2}{r_p} \sigma D \frac{dB}{dt} = 1.660 \frac{A}{cm^3}$	
			(G)	(% B_{Icoil})	(G)	(% B_{Icoil})
quadrupole:	-2.0267	$-12.94 \cdot 10^{-6}$	-1.463	-.0665	-0.8411	-.1156
octupole:	0.0391	$0.2496 \cdot 10^{-6}$	0.02822	.0013	0.01623	.0022
12-pole:	0.0077	$0.0491 \cdot 10^{-6}$	0.00556	.0003	0.00320	.0004
where conditions #1 and #2 are defined by						
<ul style="list-style-type: none"> • an excitation pattern: $\omega = 20\pi/s$, $2B_{dc}/r_p = 8.8$ kG/cm, $2B_{\Delta}/r_p = 7.2$ kG/cm, • a vacuum chamber conductivity σ, $\frac{1}{\sigma} = 47\mu\Omega$ cm and thickness $D = 0.03$ cm 						
condition #	condition	ωt	$\frac{2B_{Icoil}}{r_p}$ (G/cm)	$\frac{2(dB_{Icoil}/dt)}{r_p}$ (G/s-cm)	$\frac{dB_{Icoil}/dt}{B_{Icoil}}$ (s)	$\frac{2\sigma D}{r_p} \frac{dB_{Icoil}}{dt}$ ($\frac{A}{cm^3}$) at $D = 0.03$ cm, $\frac{1}{\sigma} = 47\mu\Omega$ cm
1	B_{Ivac} maximized	90°	880.0	45,239	51.4	0.2887
2	$\frac{B'_{vac}}{B_{Ivac}}$ maximized	144.9°	290.9	26,015	89.4	0.1660

Figure 12. LEB quadrupole magnet eddy current induced harmonics from POISSON

Figure 13. LEB quadrupole magnet eddy current induced harmonics (analytical)

```

eddy_out.dat      Wed Oct 30 17:10:20 1991      1
program eddy_str

Input:  multipole # (0=dipole, 1=quad, etc) m = 1 ellipse axes: (a,b)=( 5.50, 2.75) cm, str = 0.00 cm, h = 5.00 cm, harmonics evaluated at r= 2.50 cm

Output:
|- z-plane harmonic components iB*(z) (G) for I=1 A |--- components weighted by u(l) - Int_s u/S ds (G-cm) ---|
|- at z(l)e^(i2j pi/m+1), j=0,...,m, and conjugates, -| (S = 6.66042),
|----- cf. Eqn. (21) or (43) -----|

harmonic component mn+m+n (n=0,1,2,...):          1          3          5          7          9          Ff(l)          9          7          5          3          1          w_Fn(l)          w_Sc(l)          w_Td(l)          w_Fr(l)          w_Ff(l)
i  x(l)  y(l)  ssrc(l)  u(l)  v(l)  Ff(l)  Sc(l)  Td(l)  Fr(l)  Ff(l)  9          7          5          3          1          w_Fn(l)          w_Sc(l)          w_Td(l)          w_Fr(l)          w_Ff(l)
1  5.500  0.000  0.051  6.050  0.000  -1.31E+00  -4.61E-02  -1.89E-02  -5.41E-03  -1.25E-03  -5.76E+00  -2.02E-01  -8.31E-02  -2.37E-02  -5.47E-03
2  5.496  0.103  0.103  6.039  0.226  -1.31E+00  -4.59E-02  -1.88E-02  -5.37E-03  -1.23E-03  -5.75E+00  -2.01E-01  -8.24E-02  -2.35E-02  -5.39E-03
3  5.485  0.206  0.104  6.008  0.451  -1.31E+00  -4.55E-02  -1.85E-02  -5.24E-03  -1.19E-03  -5.70E+00  -1.97E-01  -8.04E-02  -2.27E-02  -5.16E-03
4  5.465  0.308  0.105  5.955  0.673  -1.31E+00  -4.42E-02  -1.79E-02  -5.01E-03  -1.11E-03  -5.63E+00  -1.90E-01  -7.70E-02  -2.15E-02  -4.77E-03
5  5.439  0.410  0.106  5.882  0.892  -1.31E+00  -4.26E-02  -1.71E-02  -4.69E-03  -1.00E-03  -5.53E+00  -1.80E-01  -7.22E-02  -1.98E-02  -4.22E-03
6  5.404  0.511  0.108  5.789  1.105  -1.31E+00  -4.04E-02  -1.60E-02  -4.25E-03  -8.52E-04  -5.39E+00  -1.67E-01  -6.60E-02  -1.75E-02  -3.52E-03
7  5.362  0.612  0.110  5.676  1.312  -1.30E+00  -3.75E-02  -1.45E-02  -3.69E-03  -6.61E-04  -5.23E+00  -1.50E-01  -5.83E-02  -1.48E-02  -2.65E-03
8  5.313  0.712  0.113  5.543  1.513  -1.30E+00  -3.39E-02  -1.27E-02  -2.97E-03  -4.20E-04  -5.04E+00  -1.31E-01  -4.92E-02  -1.15E-02  -1.63E-03
9  5.256  0.811  0.116  5.393  1.704  -1.30E+00  -2.93E-02  -1.04E-02  -2.07E-03  -1.24E-04  -4.83E+00  -1.09E-01  -3.86E-02  -7.71E-03  -4.63E-04
10  5.191  0.908  0.119  5.225  1.886  -1.29E+00  -2.36E-02  -7.48E-03  -9.64E-04  0.234E-04  -4.59E+00  -8.40E-02  -2.66E-02  -3.43E-03  0.835E-04
11  5.120  1.005  0.122  5.041  2.058  -1.28E+00  -1.66E-02  -3.92E-03  0.383E-04  0.661E-04  -4.33E+00  -5.59E-02  -1.32E-02  0.129E-03  0.223E-03
12  5.041  1.103  0.125  4.841  2.217  -1.27E+00  -7.91E-03  0.428E-04  0.201E-03  0.116E-03  -4.04E+00  -2.51E-02  0.136E-03  0.638E-03  0.369E-03
13  4.955  1.193  0.129  4.626  2.365  -1.26E+00  0.266E-03  0.572E-03  0.395E-03  0.174E-03  -3.73E+00  0.788E-03  0.169E-02  0.117E-02  0.514E-03
14  4.863  1.285  0.132  4.399  2.499  -1.25E+00  0.155E-02  0.121E-02  0.625E-03  0.238E-03  -3.41E+00  0.424E-02  0.331E-02  0.171E-02  0.650E-03
15  4.763  1.375  0.136  4.159  2.620  -1.23E+00  0.311E-02  0.198E-02  0.892E-03  0.307E-03  -3.07E+00  0.776E-02  0.494E-02  0.223E-02  0.765E-03
16  4.657  1.463  0.140  3.909  2.725  -1.21E+00  0.499E-02  0.290E-02  0.120E-02  0.376E-03  -2.71E+00  0.112E-01  0.650E-02  0.269E-02  0.845E-03
17  4.544  1.549  0.144  3.650  2.816  -1.18E+00  0.725E-02  0.398E-02  0.154E-02  0.441E-03  -2.35E+00  0.144E-01  0.790E-02  0.306E-02  0.875E-03
18  4.425  1.633  0.148  3.383  2.891  -1.16E+00  0.994E-02  0.525E-02  0.191E-02  0.488E-03  -1.99E+00  0.171E-01  0.902E-02  0.329E-02  0.839E-03
19  4.300  1.715  0.151  3.110  2.949  -1.12E+00  0.131E-01  0.672E-02  0.230E-02  0.502E-03  -1.62E+00  0.190E-01  0.971E-02  0.332E-02  0.726E-03
20  4.169  1.794  0.155  2.832  2.991  -1.08E+00  0.169E-01  0.840E-02  0.266E-02  0.458E-03  -1.26E+00  0.198E-01  0.980E-02  0.311E-02  0.534E-03
21  4.032  1.870  0.159  2.551  3.017  -1.03E+00  0.214E-01  0.103E-01  0.296E-02  0.321E-03  -9.15E-01  0.189E-01  0.911E-02  0.262E-02  0.284E-03
22  3.889  1.945  0.163  2.269  3.025  -9.76E-01  0.265E-01  0.123E-01  0.311E-02  0.475E-04  -5.89E-01  0.160E-01  0.744E-02  0.188E-02  0.287E-04

Sum_1 w_zz(l)*sarc(l), where zz=Ff,Sc,Td,Fr,Ff:  -9.56E+00  0.219E-02  0.385E-02  0.213E-02  0.627E-03
23  3.741  2.016  0.166  1.986  3.017  -9.11E-01  0.325E-01  0.145E-01  0.300E-02  -4.12E-03  -2.93E-01  0.105E-01  0.465E-02  0.963E-03  -1.13E-03
24  3.588  2.084  0.170  1.705  2.991  -8.37E-01  0.394E-01  0.166E-01  0.247E-02  -1.11E-02  -3.38E-02  0.159E-02  0.670E-03  0.998E-04  -4.47E-04
25  3.429  2.150  0.173  1.427  2.949  -7.52E-01  0.473E-01  0.184E-01  0.132E-02  -2.05E-02  -3.78E-02  0.178E-01  -4.38E-02  -3.13E-03  0.487E-03
26  3.266  2.213  0.176  1.154  2.891  -6.55E-01  0.561E-01  0.197E-01  -6.99E-03  -3.20E-02  0.334E-01  -2.87E-01  -1.01E-01  0.357E-03  0.163E-02
27  3.098  2.272  0.180  0.887  2.816  -5.44E-01  0.659E-01  0.199E-01  -3.82E-02  -4.38E-02  0.423E-01  -5.12E-01  -1.55E-01  0.297E-02  0.340E-02
28  2.926  2.328  0.183  0.628  2.725  -4.17E-01  0.764E-01  0.184E-01  -8.18E-02  -5.19E-02  0.432E-01  -7.92E-01  -1.91E-01  0.848E-02  0.538E-02
29  2.750  2.382  0.185  0.378  2.620  -2.73E-01  0.873E-01  0.143E-01  -1.37E-01  -4.95E-02  0.351E-01  -1.12E+00  -1.84E-01  0.176E-01  0.637E-02
30  2.570  2.431  0.188  0.139  2.499  -1.09E-01  0.980E-01  0.669E-02  -1.97E-01  -2.72E-02  0.167E-01  -1.50E+00  -1.02E-01  0.301E-01  0.416E-02
31  2.386  2.478  0.191  -0.089  2.365  0.765E-02  0.107E+00  -5.36E-02  -2.49E-01  0.246E-02  -1.34E-01  -1.88E+00  0.939E-02  0.437E-01  -4.31E-02
32  2.199  2.521  0.193  -0.303  2.217  0.286E-01  0.114E+00  -2.23E-01  -2.69E-01  0.109E-01  -5.63E-01  -2.25E+00  0.439E-01  0.530E-01  -2.15E-01
33  2.009  2.560  0.195  -0.503  2.058  0.521E-01  0.117E+00  -4.36E-01  -2.25E-01  0.214E-01  -1.13E+00  -2.53E+00  0.946E-01  0.488E-01  -4.63E-01
34  1.817  2.596  0.197  -0.688  1.886  0.779E-01  0.112E+00  -6.73E-01  -8.48E-02  0.298E-01  -1.83E+00  -2.64E+00  0.158E+00  0.199E-01  -6.94E-01
35  1.621  2.628  0.199  -0.855  1.704  0.106E+00  0.993E-01  -8.92E-01  0.167E-01  0.282E-01  -2.67E+00  -2.50E+00  0.225E+00  -4.21E-01  -7.11E-01
36  1.424  2.656  0.200  -1.006  1.513  0.136E+00  0.754E-01  -1.03E+00  0.501E-01  0.102E-01  -3.62E+00  -2.01E+00  0.275E+00  -1.34E+00  -2.72E-01
37  1.224  2.681  0.202  -1.138  1.312  0.166E+00  0.400E-01  -1.01E+00  0.822E-01  -6.69E-01  -4.46E+00  -1.12E+00  0.282E+00  -2.30E+00  0.721E-01
38  1.023  2.702  0.203  -1.251  1.105  0.196E+00  -5.84E-02  -7.64E-01  0.974E-01  -2.57E-01  -5.72E+00  0.170E-01  0.223E+00  -2.84E+00  0.195E+00
39  0.820  2.719  0.204  -1.345  0.892  0.224E+00  -5.86E-01  -2.81E-01  0.803E-01  -8.73E-01  -6.75E+00  0.176E+00  0.845E-01  -2.42E+00  0.263E+00
40  0.616  2.733  0.205  -1.418  0.673  0.249E+00  -1.112E+00  -0.380E-01  0.380E-01  -0.257E-01  -7.67E+00  -1.76E+00  0.346E+00  -1.17E+00  0.190E+00
41  0.411  2.742  0.205  -1.470  0.451  0.268E+00  -1.159E+00  0.107E+00  -5.20E-01  0.943E-02  -8.40E+00  0.499E+00  -3.36E+00  0.163E-01  -2.96E-01
42  0.206  2.748  0.206  -1.502  0.226  0.280E+00  -1.191E+00  0.160E+00  -1.22E+00  0.887E-01  -8.86E+00  0.606E+00  -5.07E+00  0.385E+00  -2.81E+00
43  0.000  2.750  0.103  -1.513  0.000  0.284E+00  -1.203E+00  0.180E+00  -1.49E+00  0.123E+00  -9.03E+00  0.644E+00  -5.72E+00  0.475E+00  -3.92E+00

Sum_ssrc(l):  6.660, Sum_1 w_zz(l)*sarc(l), w/ zz=Ff,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):  -2.08E+01  0.347E-01  0.136E-01  0.188E-02  -1.47E-03

```

$$\frac{I'_{vac}(s_z \in z, t)}{\sigma D(dB/dt)} = u - \frac{1}{S} \int_{all\ s_z} u(s_z) ds_z \equiv u - \hat{u} \quad (40)$$

where integration is along the entire vacuum chamber arc. Using this value for $I'_{vac}(s_z, t)$ in the expression for eddy current induced harmonics Eqn. (21) yields

$$\frac{B_{I_{vac}}^*(w)}{\sigma D \frac{dB_{I_{coil}}}{dt}} = \frac{\mu_0}{i2r_p} \sum_0 \frac{-1^{n+1} \alpha^n}{n!} \int_{s_z} \text{Re}[b_n] \frac{dI_s}{\sigma D \frac{dB_{I_{coil}}}{dt}} ds_z = \frac{\mu_0}{i2r_p} \sum_0 \frac{-1^{n+1} \alpha^n}{n!} \int_{s_z} \text{Re}[b_n] (u - \hat{u}) ds_z \quad (41)$$

where the b_n are as defined in Eqn. (21), $\alpha \equiv \frac{\pi w}{2r_p}$, $\alpha_0 \equiv \frac{\pi w_0}{2r_p}$, $u = u(s_z)$, and integration in Eqn. (41) is along the vacuum chamber arc in the the first quadrant in the z -plane (equivalently, upper half w -plane).

Finally, making the transformation back to the z -plane, $B_z^* = B_w^* / \frac{dz}{dw} = \frac{2z}{r_p} B_w^*$ and we have for B_z^* :

$$\frac{B_{I_{vac}}^*(z)}{\sigma D \frac{dB_{I_{coil}}}{dt}} = \frac{\mu_0}{ir_p} \sum_{n=0} \frac{-1^{n+1} \left(\frac{z}{2}\right)^n \left(\frac{z}{r_p}\right)^{2n+1}}{n!} \int_{s_z} \text{Re}[b_n] (u - \hat{u}) ds_z \quad (42)$$

where the $b_n = f(\alpha_0)$, $u = u(w(z))$, $\alpha_0 = \alpha_0(w_1(z_0))$, $\hat{u} \equiv \frac{1}{S} \int_{s_z} u(s_z) ds_z$, $ds_z \in z$, and integration is along the vacuum chamber arc in the first quadrant in the z -plane. Eqns. (41) and (42) are valid only for a quadrupole vacuum chamber exhibiting symmetry about both $y = 0$ and $x = 0$.

For $r_p = 5.0$ cm, (the LEB quadrupole magnet aperture), harmonics for the 'strawman' LEB vacuum chamber geometry (see Figure 8) at $|r| = 2.5$ cm as given in closed form by Eqn. (42) are $\frac{iB_{I_{vac}}^*(z)}{\sigma D(dB_{I_{coil}}/dt)} = -2.08 + 0.0347 + 0.0136 \frac{G}{A/cm^2}$ for the quadrupole, octupole, and 12-pole components, respectively (See Figure 13). These results for an ideal quadrupole field are virtually identical to those obtained numerically by POISSON for the LEB quadrupole magnet geometry in the previous section (Figure 12, column 2). The driver program EDDY for evaluation of the harmonic components given by Eqn. (42) for input vacuum chamber dimensions in a background time dependent multipole (quadrupole for this case) field is found in Appendix C.

5.2.5 Quadrupole Vacuum Chamber Shaping

The strawman quadrupole vacuum chamber design (A) results in maximum eddy current induced quadrupole and octupole components of $\sim 11.5 \cdot 10^{-4}$ and $\sim 0.2 \cdot 10^{-4} B_{I_{coil}}$, respectively (see Figure 12, condition #2). While the quadrupole component does not degrade the field quality, tracking with the dipole magnet is adversely affected; the goal is for dipole-quadrupole tracking to within $1 \cdot 10^{-3}$. The octupole and 12-pole components are well within the field quality specification. If an altered vacuum chamber shape can

#	label	a_x	b_y	horiz. stretch length	Quad/Oct frm vac ch eddy curs.	Quad/12-pole frm vac ch eddy curs.	D (mm)	$iB_{I_{vac}}^*$ (Quad) (max % $B_{I_{coil}}$, i.e., cond. #2)
A	straw	5.5	2.75	-	-60	-153	3.0	0.119
B	1 st mod	4.2	2.7	-	-26	-178	1.5	0.040
C	leb #1	3.575	2.675	-	-27	-236	1.5	0.033
D	leb #2	2.675	2.675	1.0	-32	+67.5	1.5	0.030
E	circ 4.5	4.575	4.575	-	∞	-89	1.5	0.065
F	circ 4.0	4.075	4.075	-	∞	-122	1.5	0.050
G	circ 3.5	3.575	3.575	-	∞	-183	1.5	0.039

Figure 14. Maximum eddy current induced harmonics (at $r = 2.5$ cm) for various candidate quadrupole vacuum chamber shapes and the excitation pattern of Figure 12

sufficiently suppress the quadrupole so that tracking with the dipole is not greatly affected, we can do without correction coils for the quadrupole magnet altogether.

Figure 14 shows tabulated maximum eddy current induced harmonics for various candidate quadrupole vacuum chamber shapes.

Any of the design options except A, which has a thicker vacuum chamber wall, will give maximum quadrupole harmonics of less than the strawman design specification $1 \cdot 10^{-3} B_{I_{coil}}$ at $|r| = 2.0$ cm. In particular, option F, featuring a 4.0 cm ID circular vacuum chamber gives a maximum quadrupole harmonic of $0.5 \cdot 10^{-3} B_{I_{coil}}$ at $|r| = 2.5$ cm. At $|r| = 4.0$ cm, however, option F gives a maximum quadrupole harmonic of $0.8 \cdot 10^{-3}$ of $B_{I_{coil}}$. All higher order harmonics are negligible at this radius, thus quadrupole field quality is not significantly degraded by vacuum chamber eddy currents.

The magnitude of the vacuum chamber eddy current induced quadrupole term for these vacuum chamber shapes may be sufficiently low depending on the final dipole-quadrupole tracking design specification and tolerance to risk; correction coils may not be needed in the quadrupole magnet.

6 Generalization of Analytical Expressions for an m^{th} Order Multipole of ‘Symmetrical’ Vacuum Chamber Shape

6.1 Generalization of Analytical Expressions for Vacuum Chamber Eddy Current Induced Harmonics in an m^{th} Order Multipole of ‘Symmetrical’ Vacuum Chamber Shape

Let m be the order of the multipole magnet ($m = 0, 1, 2, \dots$ for dipole, quadrupole, sextupole, etc.) Mapping the multipole magnet in the z -plane onto a dipole geometry in w according to the prescription $w \equiv u + iv = z^{m+1}/r_p^m$, where r_p is the multipole

aperture ($r_p \rightarrow h$ for a dipole magnet), determining the value of $B^*(w)$ in w , and then mapping back to z , yields for $m+1$ pairs of current filaments of magnitude $+I$ at $z_0 e^{\frac{i2j\pi}{m+1}}$ and $(z_0 e^{\frac{i2j\pi}{m+1}})^*$, where $j = 0, \dots, m$:

$$iB^*(z) = iB_x + B_y = \frac{(m+1)\mu_0 I}{2r_p} \sum_{n=0} \frac{-1^{n+1} \left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+n+m}}{n!} \text{Re}[b_n] \quad (43)$$

where the $b_n = f(\alpha_0)$ and $\alpha_0 \equiv \frac{\pi w_0}{2r_p}$.

The harmonic components induced by eddy currents in vacuum chambers restricted to be symmetric⁴ with respect to the lines in the z -plane $\theta = \frac{j\pi}{m+1}$, $j = 0, 1, \dots, 2m+1$, are given by:

$$iB_{I'_{vac}}^*(z) = \frac{(m+1)\mu_0}{2r_p} \sum_{n=0} \frac{-1^{n+1} \left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+n+m}}{n!} \int_{s_z} \text{Re}[b_n] I'_{vac}(s_z) ds_z \quad (44)$$

where the kernel $I'_{vac}(s_z)$ is given by

$$\frac{I'_{vac}(s_z \in z, t)}{\sigma D(dB/dt)} = u - \frac{1}{S} \int_{all \ s_z} u(s_z) ds_z \quad (45)$$

where the $b_n = f(\alpha_0)$ are w -plane variables and the integration in Eqn. (44) is along the vacuum chamber arc: $0 \leq \theta(s_z(z)) \leq \pi/(m+1)$ (along the entire arc for Eqn. (45)).

The code EDDY of Appendix C evaluates the expressions given by Eqns. (43) and (44) for a multipole of arbitrary order with a specified 'symmetric' (as defined above) vacuum chamber shape. Note that Eqns. (21) and (33) are just the dipole specific versions of Eqns. (43) and (44), respectively.

If we have an additional $m+1$ pairs of current filaments of magnitude $-I$ located at $z_0 e^{\frac{i\pi(2j+1)}{m+1}}$ and $(z_0 e^{\frac{i\pi(2j+1)}{m+1}})^*$, where $j = 0, \dots, m$, analogous to Eqn. (43) we obtain:

$$iB^*(z) = iB_x + B_y = \frac{(m+1)\mu_0 I}{r_p} \sum_{n=even} \frac{-\left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+n+m}}{n!} \text{Re}[b_n] \quad (46)$$

Thus, harmonic components induced by a multipole magnet's vacuum chamber which exhibits the additional symmetry about the z -plane lines $\theta = \frac{(1+2j)\pi}{2(m+1)}$, $j = 0, \dots, 2m+1$ is:

$$iB_{I'_{vac}}^*(z) = \frac{(m+1)\mu_0}{r_p} \sum_{n=even} \frac{-\left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+n+m}}{n!} \int_{s_z} \text{Re}[b_n] I'_{vac}(s_z) ds_z \quad (47)$$

⁴Note that a non-circular, elliptical vacuum chamber in a sextupole magnet does not exhibit this symmetry, and thus Eqn. (44) does not apply. This interesting case is treated separately in Appendix D.

where the kernel $I'_{vac}(s_z)$ is given by Eqn. (45). Note that Eqn. (24) is just the dipole specific version of Eqn. (47).

6.2 Generalization of Passive Eddy Current Induced Harmonics Suppression in an m^{th} Order Multipole

Compensating eddy current carrying conductor filaments can provide passive nulling of harmonic components in any m^{th} order multipole. Mapping to dipole (w) geometry, for a filament of uniform cross section at z_0 and its return at z_∞ , both parallel to the beam axis, the eddy current density induced in the filament is (cf. Eqns. (11) and (31)):

$$\begin{aligned} \frac{j_{z_0}}{dB_{I_{coil}}/dt} &= \frac{\sigma}{B_1} \left(-A(I_1, w_0) + \frac{1}{a_{tot}} \int_a A(I_1, w) da \right) = \\ \frac{\sigma}{B_1} \left(-A(I_1, w_0) + \frac{A(I_1, w_0)a_{z_0} + A(I_1, w_\infty)a_{z_\infty}}{a_{z_0} + a_{z_\infty}} \right) &= \frac{\sigma(A(I_1, w_\infty) - A(I_1, w_0))}{2B_1} \quad (48) \end{aligned}$$

since $a_{z_\infty} = a_{z_0}$. In the mapped dipole geometry, $\frac{A(I_1, w_\infty) - A(I_1, w_0)}{B_1} = -(u_\infty - u_0)$, where $B_1 \equiv \left| \frac{B^*(z)r_p^m}{i(m+1)z^m} \right|$. The compensating current flowing in the conductor at w_0 is then:

$$\frac{I_{z_0}}{dB_{I_{coil}}/dt} = \frac{-\sigma a_{z_0}(u_\infty - u_0)}{2} = \frac{-\sigma a_{z_0} \Delta u}{2} \quad (49)$$

The return current location w_∞ can be positioned where it does not contribute to the near on-axis field.

Figure 15 summarizes the eddy current induced harmonics and suppression discussion of this section.

Using the magnitude of compensating current I_{z_0} given by Eqn. (49) in Eqn. (45), the magnitude of the field correction in an m^{th} order multipole with $m+1$ pairs of current filaments of magnitude $+I$ at $z_0 e^{\frac{j2j\pi}{m+1}}$ and $(z_0 e^{\frac{j2j\pi}{m+1}})^*$, where $j = 0, \dots, m$ (and with their returns at z_∞), is

$$\frac{B_{I_{z_0}}^*(z)}{dB_{I_{coil}}/dt} = \frac{-(m+1)\mu_0\sigma a_{z_0}\Delta u}{4r_p i} \sum_{n=0}^{\infty} \frac{-1^{n+1} \left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+m+n}}{n!} Re[b_n] \quad (50)$$

where the $b_n = b_n(\alpha_0 \equiv \frac{\pi w_0}{2r_p})$ are w -plane variables.

One can null both the vacuum chamber eddy current induced fundamental and next allowed harmonic term with $m+1$ pairs of compensating conductors attached somewhere to the vacuum chamber at the appropriate locations and with their returns located far from the region of interest. (See Figure 15a,c,e.)

For the case of additional vacuum chamber symmetry discussed above Eqn. (47), using the value of compensating current I_{z_0} given by Eqn. (49) in Eqn. (46), the magnitude

Figure 15. Summary of eddy current induced harmonics and passive suppression for multipole magnets with various generic vacuum chamber shapes

allowed harmonics (POISSON notation) DIPOLE: $m=0$, QUAD $m=1$, etc.

for design rotatable by $\theta^\circ = 360^\circ/m'$: where $m' = m+1$, $n' = n+1$

m'	cur. change sign. $n' = m'(2M-1)/2$	cur. same sign $n' = m'M$
1		1 2 3 4 5
2	1 3 5 7 9	2 4 6 8 10
3		3 6 9 12 15
4	2 6 10 14 18	4 8 12 16 20
5		5 10 15 20 25
6	3 9 15 21 27	6 12 18 24 30

$M = 1, 2, 3, 4, \dots$

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS

MULTIPOLE MAGNET	VAC. CH SHAPE	ALLOWED HARMONICS	CORRECTOR COILS (APPROX. LOCATED)	REMAINING HARMONICS
DIPOLE		n'		n'
a)		1 2 3 4 5 $r(1)S$ rotate m' same sign		$x \times 3 \ 4 \ 5$
b)		1 3 5 7 9 $r(2)C$		$x \times 5 \ 7 \ 9$
QUAD.				
c)		2 4 6 8 10 $r(2)S$		$x \times 6 \ 8 \ 10$
d)		2 6 10 14 18 $r(4)C$		$x \times 10 \ 14 \ 18$
SEXT.				
e)		3 6 9 12 15 $r(3)S$		$x \times 9 \ 12 \ 15$
f)		3 9 15 21 27 $r(6)C$		$x \times 15 \ 21 \ 27$
g)		1 3 5 7 9 $r(2)C$		$x \times 5 \ 7 \ 9$

of the field correction in an m^{th} order multipole with $m + 1$ pairs of current filaments of magnitude $+I$ at $z_0 e^{\frac{i2j\pi}{m+1}}$ and $(z_0 e^{\frac{i2j\pi}{m+1}})^*$ and $m + 1$ pairs of current filaments of magnitude $-I$ at $z_0 e^{\frac{i(2j+1)\pi}{m+1}}$ and $(z_0 e^{\frac{i(2j+1)\pi}{m+1}})^*$, where $j = 0, \dots, m$ is

$$\frac{B_{I_{z_0}}^*(z)}{dB_{I_{coil}}/dt} = \frac{-(m+1)\mu_0\sigma a_{z_0}\Delta u}{2r_p i} \sum_{n=\text{even}} \frac{-1 \left(\frac{\pi}{2}\right)^n \left(\frac{z}{r_p}\right)^{nm+m+n}}{n!} \text{Re}[b_n] \quad (51)$$

where the $b_n = b_n(\alpha_0 \equiv \frac{\pi\omega_0}{2r_p})$ are w -plane variables.

One can null both the vacuum chamber eddy current induced fundamental and next allowed harmonic term with $2(m+1)$ pairs of compensating conductors attached to the vacuum chamber at the appropriate locations, and with their returns located far from the region of interest. (See Figure 15b,d,f.)

6.3 Vacuum Chamber Eddy Current Induced Harmonics in the LEB Sextupole Magnets

The strawman field quality specification for these magnets are that $\frac{\Delta B}{B} \leq 10^{-3}$ at $|z| = 2.0$ cm, (later changed to $\frac{\Delta B}{B} \leq 10^{-3}$ at $|z| = 4.0$ cm.) The excitation is to produce a field strength that cycles between 6 and 60 T/m². The sextupole aperture is 5 cm.

6.3.1 Mapping Sextupole Geometry to Dipole Geometry

For the sextupole geometry, we use the transformation $w \equiv u + iv = z^3/r_p^2$, where r_p is the sextupole aperture radius. The sextupole magnet induced complex magnetic potential

$$F(w) \equiv A + iV = -wB + c_2$$

from which it follows $B^*(w) = i\frac{dF}{dw} = iB(t)$. The transformation back to the z -plane gives for $B^*(z)$:

$$B^*(z) = i\frac{dF}{dw} \frac{dz}{dw} = iB(t)3z^2/r_p^2 \quad (52)$$

6.3.2 Magnet Excitation

For the LEB sextupoles $\frac{3B_{max}(t)}{r_p^2} \simeq 60$ G/cm² and $\frac{3B_{min}(t)}{r_p^2} \simeq 6$ G/cm². Thus we have

$$\frac{3B(t)}{r_p^2} = \frac{3B_{dc}}{r_p^2} + \frac{3B_{\Delta}}{r_p^2} \cos \omega t \quad (53)$$

where $\frac{3B_{\Delta}}{r_p^2} = 27$ G/cm² and $\frac{3B_{dc}}{r_p^2} = 33$ G/cm².

- input $I_{s,j}$, per Eqn. (44),
- ideal sextupole magnet geometry, aperture $r_p = 5.0$ cm
- circular vacuum chamber with radius 4.075 cm

At $|r| = 2.5$ cm:

harmonic component	$\frac{iB'_{vac}}{\sigma D(dB/dt)}$ (G/ $\frac{A}{cm^2}$)	$\frac{iB'_{vac}}{dB/dt}$ (s) $\frac{1}{\sigma} = 47\mu\Omega$ cm, $D = 0.015$ cm	iB'_{vac} (G) at condition #1: $\frac{3}{r_p^2}\sigma D \frac{dB}{dt} = .00541 \frac{A}{cm^2}$		iB'_{vac} (G) at condition #2: $\frac{3}{r_p^2}\sigma D \frac{dB}{dt} = .00311 \frac{A}{cm^2}$	
			(G)	(% of B_{Icoil})	(G)	(% of B_{Icoil})
sextupole:	-0.794	$-2.534 \cdot 10^{-6}$	-.0358	-.0130	-8.411	-.0226
18-pole:	0.0012	$0.0038 \cdot 10^{-6}$	.00005	.00002	.0127	.00003

where conditions #1 and #2 are defined by

- an excitation pattern: $\omega = 20\pi/s$, $3B_{dc}/r_p^2 = 33$ G/cm², $3B_{\Delta}/r_p^2 = 27$ G/cm²,
- a vacuum chamber conductivity σ , $\frac{1}{\sigma} = 47\mu\Omega$ cm and thickness $D = 0.015$ cm

condition #	condition	ωt	$\frac{3B_{Icoil}}{r_p^2}$ (G/cm ²)	$\frac{3(dB_{Icoil}/dt)}{r_p^2}$ (G/s-cm ²)	$\frac{dB_{Icoil}/dt}{B_{Icoil}}$ (s)	$\frac{3\sigma D}{r_p^2} \frac{dB_{Icoil}}{dt}$ ($\frac{A}{cm^2}$) at $D = .015$ cm, $\frac{1}{\sigma} = 47\mu\Omega$ cm
1	B'_{vac} maximized	90°	33.00	1696	51.4	.00541
2	$\frac{B'_{vac}}{B_{tot}}$ maximized	144.9°	10.91	975	89.4	.00311

Figure 16. Ideal sextupole magnet eddy current induced harmonics (analytical)

For this excitation pattern the maximum $\left|\frac{dB/dt}{B}\right|$ that the protons see is at $\omega t \simeq 144.9^\circ$ (see Figure 3 or Eqn. (29)). Here $3B(t)/r_p^2 = 10.91$ G/cm² and $\left|\frac{dB/dt}{B}\right| = 1.423\omega$.

6.3.3 Closed Form Analytical Prediction of Eddy Current Induced Harmonics for LEB Sextupole Magnet

For a vacuum chamber in a sextupole magnet field that is symmetric with respect to the lines in the z -plane $\theta = \frac{2\pi}{3}$, $j = 0, 1, \dots, 5$, (as is the case for a circular vacuum chamber) the eddy current induced harmonics are given by Eqn. (44) with $m = 2$.

For $r_p = 5.0$ cm, (the LEB sextupole magnet aperture), harmonics for a circular 'strawman' LEB sextupole vacuum chamber of radius $r = 4.075$ cm and thickness $D = 0.015$ mm, at $|r| = 2.5$ cm as given in closed form by Eqn. (44) are $\frac{iB'_{vac}(z)}{\sigma D(dB_{Icoil}/dt)} = -0.794 + 0.0012 \frac{G}{A/cm^2}$ for the sextupole and 18-pole components, respectively (See Figure 16, column 2). The driver program EDDY for evaluation of the harmonic components

given by Eqn. (44) for input vacuum chamber dimensions in a background time dependent multipole (sextupole for this case) field is found in Appendix C.

The strawman sextupole vacuum chamber design results in maximum eddy current induced sextupole and 18-pole components of $\sim 2.3 \cdot 10^{-4}$ and $0.003 \cdot 10^{-4} B_{I_{coil}}$ at $r = 2.5$ cm, respectively (See Figure 16, condition #2). Eddy current induced harmonics do not significantly degrade the sextupole field quality; passive correction coils are not needed on the LEB sextupole magnets.

7 Acknowledgements

We are happy to acknowledge the helpful insight of Gerry Fischer into various issues related to the work described herein. The discussion of magnet excitation patterns (Section 4.2.1) and their effect on eddy current percentage contributions follows his original observations.

8 Literature Cited

1. *Reference Manual for the Poisson/Superfish Group of Codes*, LA-UR-87-126, Los Alamos National Laboratory, Los Alamos, NM, 1987.
2. Lee, S.Y., *A Multipole Expansion for the Field of Vacuum Chamber Eddy Currents*, Brookhaven National Laboratory Internal Report AD/AP/T-12.
3. Schlueter, R., *LEB Dipole and Quadrupole Magnetics Design Summary*, #1, #2, #3, #4, April-July 1991.
4. Thiagarajan, et.al., *Design of Main Dipoles and Quadrupoles for the SSC Low Energy Booster*, SSC Report, Dec. 1991.
5. Fischer, G.E., personal communication.

Appendix A: Suppression of Harmonics due to Magnet Geometry and Iron Saturation

9 Appendix A: Suppression of Harmonics due to Magnet Geometry and Iron Saturation

See the series of memos^[3] entitled *LEB Dipole and Quadrupole Magnetics Design Summary*, #1, 2, 3, 4, 4/91-7/91. Final magnet designs yield:

- a dipole magnet with
 - a good field region with $\Delta B/B \leq 1 \cdot 10^{-4}$ within the ellipse $z \leq 3.25 \cos \theta + i2.5 \sin \theta$ at injection, where $B \simeq 1300$ G,
 - a good field region with $\Delta B/B \leq 1 \cdot 10^{-4}$ within the ellipse $z \leq 1.40 \cos \theta + i1.40 \sin \theta$ at extraction, where $B = 13000$ G
- a quadrupole magnet with
 - a good field region with $\Delta B/B \leq 1 \cdot 10^{-3}$ within the ellipse $z \leq 4.25 \cos \theta + i4.25 \sin \theta$ at injection, where $dB/dz \simeq 1600$ G/cm,
 - a good field region with $\Delta B/B \leq 1 \cdot 10^{-3}$ within the ellipse $z \leq 4.00 \cos \theta + i4.00 \sin \theta$ at extraction, where $dB/dz = 16000$ G/cm,
- The dipole and quadrupole magnets track to within $1 \cdot 10^{-3}$ over the entire range of operation, injection to extraction.

Appendix B: POISSON Files for Eddy Current Induced Harmonics of SSC LEB Magnet Designs

1. LEB001: LEB dipole magnet excitation
2. LEBV001: LEB dipole magnet vacuum chamber eddy current excitation
3. LEBQUAD: LEB quadrupole magnet excitation
4. LEBQUADv: LEB quadrupole magnet vacuum chamber eddy current excitation

```

LEB001, h-magnet *6 -2 *46 6 *110 5 16 2.5 90. 2.5 0. *43 13 *45 8 S
$REG NREG=5, DX=.5, XMAX=26.7, YMAX=21.5, NPOINT=5$
$PO x= 0.00 , y= 0.00$
$PO x= 26.70 , y= 0.00$
$PO x= 26.70 , y=21.500$
$PO x= 0.00 , y=21.500$
$PO x= 0.00 , y= 0.00$
$REG mat=2, NPOINT=9$
$PO x= 0.00 , y= 4.000$
$PO x= 8.800 , y= 4.000$
$PO x= 8.800 , y=12.060$
$PO x=17.500 , y=12.060$
$PO x=17.500 , y= 0.00$
$PO x=26.700 , y= 0.00$
$PO x=26.700 , y=21.500$
$PO x= 0.00 , y=21.500$
$PO x= 0.00 , y= 4.000$
$REG mat=1, NPOINT=5 , CUR=2308.$
$PO x= 9.300 , y= 2.860$
$PO x=17.000 , y= 2.860$
$PO x=17.000 , y=11.500$
$PO x= 9.300 , y=11.500$
$PO x= 9.300 , y= 2.860$
$REG mat=2, NPOINT=6$
$PO x= 0.00 , y= 2.860$
$PO x= 8.500 , y= 2.860$
$PO x= 8.800 , y= 3.200$
$PO x= 8.800 , y= 4.000$
$PO x= 0.00 , y= 4.000$
$PO x= 0.00 , y= 2.860$
$REG mat=1, NPOINT=12$
$PO x= 5.500 , y= .000$
$PO x= 5.000 , y= 1.145$
$PO x= 4.500 , y= 1.581$
$PO x= 4.000 , y= 1.887$
$PO x= 3.500 , y= 2.121$
$PO x= 3.000 , y= 2.305$
$PO x= 2.500 , y= 2.449$
$PO x= 2.000 , y= 2.562$
$PO x= 1.500 , y= 2.646$
$PO x= 1.000 , y= 2.704$
$PO x= .500 , y= 2.739$
$PO x= .000 , y= 2.750$

```

PROB. -LEB001, h-magnet K6 0 K46 6 K11 CYCLE - 560

LEAST SQUARES EDIT OF PROBLEM , CYCLE 560

'H' MAG SYMMETRY TYPE

STORED ENERGY = 1.3849E+01 JOULES / METER OR RADIAN

XJFACT= 1.000000

calc. $\frac{-A_i \alpha S_i}{B_i}$ along vac. ch.

$B_i = -1014 G$

K	L	A (VECTOR)	X	Y	BX (GAUSS)	BY (GAUSS)	BT (GAUSS)	DE
1	1	0.000000E+00	0.00000	0.00000	0.000	-1014.049	1014.049	
2	1	5.108502E+02	0.50377	0.00000	0.000	-1014.042	1014.042	
3	1	1.021692E+03	1.00755	0.00000	0.000	-1014.016	1014.016	
4	1	1.532515E+03	1.51132	0.00000	0.000	-1013.966	1013.966	
5	1	2.043304E+03	2.01509	0.00000	0.000	-1013.875	1013.875	
6	1	2.554031E+03	2.51887	0.00000	0.000	-1013.715	1013.715	
7	1	3.064650E+03	3.02264	0.00000	0.000	-1013.439	1013.439	
8	1	3.575083E+03	3.52642	0.00000	0.000	-1012.959	1012.959	
9	1	4.085192E+03	4.03019	0.00000	0.000	-1012.117	1012.117	
10	1	4.594734E+03	4.53396	0.00000	0.000	-1010.670	1010.670	
11	1	5.103309E+03	5.03774	0.00000	0.000	-1008.177	1008.177	
12	1	5.568549E+03	5.50000	0.00000	0.000	-1004.304	1004.304	
13	1	6.114276E+03	6.04528	0.00000	0.000	-996.601	996.601	
1	2	0.000000E+00	0.00000	0.43878	0.000	-1014.055	1014.055	
2	2	3.070387E+02	0.30278	0.44057	0.008	-1014.052	1014.052	
3	2	7.877885E+02	0.77687	0.44290	0.022	-1014.038	1014.038	
4	2	1.293525E+03	1.27562	0.44475	0.043	-1014.005	1014.005	
5	2	1.806382E+03	1.78140	0.44700	0.079	-1013.944	1013.944	
6	2	2.322652E+03	2.29060	0.45066	0.140	-1013.833	1013.833	
7	2	2.839386E+03	2.80032	0.45518	0.246	-1013.641	1013.641	
8	2	3.353274E+03	3.30737	0.45620	0.428	-1013.310	1013.310	
9	2	3.864874E+03	3.81239	0.44626	0.734	-1012.724	1012.724	
10	2	4.381410E+03	4.32270	0.42331	1.223	-1011.663	1011.664	
11	2	4.901304E+03	4.83708	0.39733	2.012	-1009.770	1009.772	
12	2	5.401667E+03	5.33333	0.38167	3.285	-1006.579	1006.584	
13	2	5.894869E+03	5.82443	0.39850	5.695	-1001.423	1001.439	
1	3	0.000000E+00	0.00000	0.87755	0.000	-1014.070	1014.070	
2	3	5.435944E+02	0.53605	0.88417	0.025	-1014.066	1014.066	

```

LEBV001, h-magnet *6 -2 *46 6 *110 5 16 2.5 90. 2.5 0. *43 13 *45 8 *49 13 S
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$PO x= 26.70 , y=21.500$
$PO x= 0.00 , y=21.500$
$PO x= 0.00 , y= 0.00$
$REG mat=2, NPOINT=9$
$PO x= 0.00 , y= 4.000$
$PO x= 8.800 , y= 4.000$
$PO x= 8.800 , y=12.060$
$PO x=17.500 , y=12.060$
$PO x=17.500 , y= 0.00$
$PO x=26.700 , y= 0.00$
$PO x=26.700 , y=21.500$
$PO x= 0.00 , y=21.500$
$PO x= 0.00 , y= 4.000$
$REG mat=1, NPOINT=5$
$PO x= 9.300 , y= 2.860$
$PO x=17.000 , y= 2.860$
$PO x=17.000 , y=11.500$
$PO x= 9.300 , y=11.500$
$PO x= 9.300 , y= 2.860$
$REG mat=2, NPOINT=6$
$PO x= 0.00 , y= 2.860$
$PO x= 8.500 , y= 2.860$
$PO x= 8.800 , y= 3.200$
$PO x= 8.800 , y= 4.000$
$PO x= 0.00 , y= 4.000$
$PO x= 0.00 , y= 2.860$
$REG mat=1, NPOINT=12$
$PO x= 5.500 , y= .000$
$PO x= 5.000 , y= 1.145$
$PO x= 4.500 , y= 1.581$
$PO x= 4.000 , y= 1.887$
$PO x= 3.500 , y= 2.121$
$PO x= 3.000 , y= 2.305$
$PO x= 2.500 , y= 2.449$
$PO x= 2.000 , y= 2.562$
$PO x= 1.500 , y= 2.646$
$PO x= 1.000 , y= 2.704$
$PO x= .500 , y= 2.739$
$PO x= .000 , y= 2.750$

```

PROB. -LEBV001, h-magnet x6 -2 x46 6 x CYCLE - 580

LEAST SQUARES EDIT OF PROBLEM , CYCLE 580

'H' MAG SYMMETRY TYPE

CURRENT FILAMENT AT	12	1, (	5.50	0.00), HAS	3.4300 AMPS
CURRENT FILAMENT AT	11	4, (	5.00	1.14), HAS	4.7800 AMPS
CURRENT FILAMENT AT	8	5, (	3.75	2.00), HAS	1.0350 AMPS
CURRENT FILAMENT AT	9	5, (	4.00	1.89), HAS	1.7250 AMPS
CURRENT FILAMENT AT	10	5, (	4.50	1.58), HAS	2.8120 AMPS
CURRENT FILAMENT AT	7	6, (	3.00	2.31), HAS	1.5800 AMPS
CURRENT FILAMENT AT	8	6, (	3.50	2.12), HAS	1.4160 AMPS
CURRENT FILAMENT AT	1	7, (	0.00	2.75), HAS	0.0000 AMPS
CURRENT FILAMENT AT	2	7, (	0.50	2.74), HAS	0.2500 AMPS
CURRENT FILAMENT AT	3	7, (	1.00	2.70), HAS	0.5030 AMPS
CURRENT FILAMENT AT	4	7, (	1.50	2.65), HAS	0.7580 AMPS
CURRENT FILAMENT AT	5	7, (	2.00	2.56), HAS	1.0200 AMPS
CURRENT FILAMENT AT	6	7, (	2.50	2.45), HAS	1.2910 AMPS

STORED ENERGY = 3.2568E-04 JOULES / METER OR RADIAN

XJFACT= 1.000000

K	L	A (VECTOR)	X	Y	BX (GAUSS)	BY (GAUSS)	BT (GAUSS)	DB
1	1	0.000000E+00	0.00000	0.00000	0.000	-8.539	8.539	
2	1	4.293969E+00	0.50377	0.00000	0.000	-8.492	8.492	
3	1	8.540224E+00	1.00755	0.00000	0.000	-8.352	8.352	
4	1	1.269386E+01	1.51132	0.00000	0.000	-8.125	8.125	
5	1	1.671217E+01	2.01509	0.00000	0.000	-7.816	7.816	
6	1	2.055655E+01	2.51887	0.00000	0.000	-7.435	7.435	
7	1	2.419223E+01	3.02264	0.00000	0.000	-6.987	6.987	
8	1	2.758604E+01	3.52642	0.00000	0.000	-6.474	6.474	
9	1	3.070296E+01	4.03019	0.00000	0.000	-5.886	5.886	
10	1	3.351033E+01	4.53396	0.00000	0.000	-5.279	5.279	
11	1	3.613195E+01	5.03774	0.00000	0.000	-4.798	4.798	
12	1	3.951679E+01	5.50000	0.00000	0.000	-1.074	1.074	
13	1	3.744845E+01	6.04528	0.00000	0.000	1.655	1.655	
1	2	0.000000E+00	0.00000	0.43878	0.000	-8.575	8.575	
2	2	2.594663E+00	0.30278	0.44057	0.050	-8.558	8.558	

TABLE FOR FIELD COEFFICIENTS

NORMALIZATION RADIUS = 2.50000

$$(BX - I BY) = I * \text{SUM } N * (AN + I BN) / R * (Z/R) ** (N-1)$$

N	N(AN) / R	N(BN) / R	ABS (N(CN) / R)
1	8.5375E+00	0.0000E+00	8.5375E+00
3	-1.1488E+00	0.0000E+00	1.1488E+00
5	5.6757E-02	0.0000E+00	5.6757E-02
7	7.6619E-03	0.0000E+00	7.6619E-03
9	-2.2467E-03	0.0000E+00	2.2467E-03


```

LEBQUAD *24 1 *110 4 16 4.5 90. 2.5 0. *46 4 *43 25 *45 14 S
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&PO X= 0. ,Y= 0. ,&END
&PO X=20. ,y= 0. ,&END
&PO X=20. ,Y=20. ,&END
&PO X= 0. ,Y=20. ,&END
&PO X= 0. ,Y= 0. ,&END
&REG MAT=2,NPOINT=21, &END
&PO X=2.31, Y=6.75, &END
&PO X=1.88, Y=6.50, &END
&PO X=1.76, Y=6.25, &END
&PO X=1.98, Y=6.00, &END
&PO X=2.14, Y=5.75, &END
&PO X=2.2727, y=5.5, &END
&PO X=3.5355, y=3.5355, R=5.0, &END
&PO NT=3, X=5.5, Y=2.2727, R=5.0, &END
&PO X=5.75, Y=2.14, &END
&PO X=6.00, Y=1.98, &END
&PO X=6.25, Y=1.76, &END
&PO X=6.50, Y=1.88, &END
&PO X=6.75, Y=2.31, &END
&PO X=16.75, Y=12.31, &END
&PO X=16.75, Y= 0.00, &END
&PO X=20.00, Y= 0.00, &END
&PO X=20.00, Y=20.00, &END
&PO X= 0.00, Y=20.00, &END
&PO X= 0.00, Y=16.75, &END
&PO X=12.31, Y=16.75, &END
&PO X=2.31, Y=6.75, &END
&REG MAT=1,CUR= 15000.,NPOINT=5, &END
&PO X=15.,y=5., &END
&PO X=16.,Y=5., &END
&PO X=16.,Y=6., &END
&PO X=15.,Y=6., &END
&PO X=15.,Y=5., &END
&REG MAT=1,CUR=-15000.,NPOINT=5, &END
&PO X=5.,y=15., &END
&PO X=5.,Y=16., &END
&PO X=6.,Y=16., &END
&PO X=6.,Y=15., &END
&PO X=5.,Y=15., &END
&REG mat=1, NPOINT=12, &END
&PO x= 5.500 , y= .000, &END
&PO x= 5.000 , y= 1.145, &END
&PO x= 4.500 , y= 1.581, &END
&PO x= 4.000 , y= 1.887, &END
&PO x= 3.500 , y= 2.121, &END
&PO x= 3.000 , y= 2.305, &END
&PO x= 2.500 , y= 2.449, &END
&PO x= 2.000 , y= 2.562, &END
&PO x= 1.500 , y= 2.646, &END
&PO x= 1.000 , y= 2.704, &END
&PO x= .500 , y= 2.739, &END
&PO x= .000 , y= 2.750, &END

```

PROB. -LEBOURD $\kappa 24$ 1 $\kappa 110$ 4 16 4.5 90. CYCLE - 830

LEAST SQUARES EDIT OF PROBLEM , CYCLE 830

SYMM QUA SYMMETRY TYPE

STORED ENERGY = 1.1441E+03 JOULES / METER OR RADIAN

$$\left| \frac{B^*(z) \cdot r_p}{(2z)} \right| = 3730 G = B_z$$

XJFACT= 1.000000

K	L	A(VECTOR)	X	Y	BX (GAUSS)	BY (GAUSS)	BT (GAUSS)	DB
1	1	1.581940E+03	0.00000	0.00000	0.000	0.000	0.000	
2	1	1.630515E+03	0.25000	0.00000	0.000	-373.036	373.036	
3	1	1.770382E+03	0.50000	0.00000	0.000	-744.133	744.133	
4	1	2.002833E+03	0.75000	0.00000	0.000	-1115.688	1115.688	
5	1	2.328308E+03	1.00000	0.00000	0.000	-1488.121	1488.121	
6	1	2.746965E+03	1.25000	0.00000	0.000	-1861.135	1861.135	
7	1	3.258947E+03	1.50000	0.00000	0.000	-2234.747	2234.747	
8	1	3.864412E+03	1.75000	0.00000	0.000	-2609.036	2609.036	
9	1	4.563541E+03	2.00000	0.00000	0.000	-2984.091	2984.091	
10	1	5.356534E+03	2.25000	0.00000	0.000	-3359.980	3359.980	
11	1	6.243606E+03	2.50000	0.00000	0.000	-3736.732	3736.732	
12	1	7.224970E+03	2.75000	0.00000	0.000	-4114.310	4114.310	
13	1	8.300819E+03	3.00000	0.00000	0.000	-4492.572	4492.572	
14	1	9.471295E+03	3.25000	0.00000	0.000	-4871.272	4871.272	
15	1	1.073646E+04	3.50000	0.00000	0.000	-5250.039	5250.039	
16	1	1.209628E+04	3.75000	0.00000	0.000	-5628.312	5628.312	
17	1	1.355052E+04	4.00000	0.00000	0.000	-6005.197	6005.197	
18	1	1.509865E+04	4.25000	0.00000	0.000	-6379.128	6379.128	
19	1	1.673963E+04	4.50000	0.00000	0.000	-6747.384	6747.384	
20	1	1.847151E+04	4.75000	0.00000	0.000	-7105.525	7105.525	
21	1	2.029095E+04	5.00000	0.00000	0.000	-7446.651	7446.651	
22	1	2.219244E+04	5.25000	0.00000	0.000	-7760.056	7760.056	
23	1	2.416712E+04	5.50000	0.00000	0.000	-8030.212	8030.212	
24	1	2.620199E+04	5.75000	0.00000	0.000	-8234.813	8234.813	
25	1	2.827732E+04	6.00000	0.00000	0.000	-8351.120	8351.120	
1	2	1.549517E+03	0.00000	0.21739	-327.655	0.000	327.655	
2	2	1.567763E+03	0.15055	0.21789	-325.990	-225.160	396.190	
3	2	1.660277E+03	0.38595	0.21863	-324.904	-574.262	659.803	

```

LEBQUADv *24 1 *110 4 16 4.5 90. 2.5 0. *46 221 *49 13 S plebq.dat
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&PO X=20. ,Y=20. ,&END
&PO X= 0. ,Y=20. ,&END
&PO X= 0. ,Y= 0. ,&END
&REG MAT=2,NPOINT=21, &END
&PO X=2.31, Y=6.75, &END
&PO X=1.88, Y=6.50, &END
&PO X=1.76, Y=6.25, &END
&PO X=1.98, Y=6.00, &END
&PO X=2.14, Y=5.75, &END
&PO X=2.2727, y=5.5, &END
&PO X=3.5355, y=3.5355, R=5.0, &END
&PO NT=3, X=5.5, Y=2.2727, R=5.0, &END
&PO X=5.75, Y=2.14, &END
&PO X=6.00, Y=1.98, &END
&PO X=6.25, Y=1.76, &END
&PO X=6.50, Y=1.88, &END
&PO X=6.75, Y=2.31, &END
&PO X=16.75, Y=12.31, &END
&PO X=16.75, Y= 0.00, &END
&PO X=20.00, Y= 0.00, &END
&PO X=20.00, Y=20.00, &END
&PO X= 0.00, Y=20.00, &END
&PO X= 0.00, Y=16.75, &END
&PO X=12.31, Y=16.75, &END
&PO X=2.31, Y=6.75, &END
&REG MAT=1,NPOINT=5, &END
&PO X=15.,y=5., &END
&PO X=16.,Y=5., &END
&PO X=16.,Y=6., &END
&PO X=15.,Y=6., &END
&PO X=15.,Y=5., &END
&REG MAT=1, npoint=5, &END
&PO X=5.,y=15., &END
&PO X=5.,Y=16., &END
&PO X=6.,Y=16., &END
&PO X=6.,Y=15., &END
&PO X=5.,Y=15., &END
&REG mat=1, NPOINT=12, &END
&PO x= 5.500 , y= .000, &END
&PO x= 5.000 , y= 1.145, &END
&PO x= 4.500 , y= 1.581, &END
&PO x= 4.000 , y= 1.887, &END
&PO x= 3.500 , y= 2.121, &END
&PO x= 3.000 , y= 2.305, &END
&PO x= 2.500 , y= 2.449, &END
&PO x= 2.000 , y= 2.562, &END
&PO x= 1.500 , y= 2.646, &END
&PO x= 1.000 , y= 2.704, &END
&PO x= .500 , y= 2.739, &END
&PO x= .000 , y= 2.750, &END

```

PROB. -LEBOURD ✓ M24 1 M110 4 16 4.5 90 CYCLE - 850

LEAST SQUARES EDIT OF PROBLEM , CYCLE 860

EDIT ITYPE 221

CURRENT FILAMENT AT	23	1, (	5.50	0.00),	HAS	2.7709 AMPS
CURRENT FILAMENT AT	21	6, (	5.00	1.14),	HAS	3.0119 AMPS
CURRENT FILAMENT AT	19	8, (	4.50	1.58),	HAS	1.2112 AMPS
CURRENT FILAMENT AT	16	10, (	3.75	2.00),	HAS	0.1045 AMPS
CURRENT FILAMENT AT	17	10, (	4.00	1.89),	HAS	0.3722 AMPS
CURRENT FILAMENT AT	15	11, (	3.50	2.12),	HAS	-0.0344 AMPS
CURRENT FILAMENT AT	11	12, (	2.50	2.45),	HAS	-0.8184 AMPS
CURRENT FILAMENT AT	13	12, (	3.00	2.31),	HAS	-0.4739 AMPS
CURRENT FILAMENT AT	5	13, (	1.00	2.70),	HAS	-1.4442 AMPS
CURRENT FILAMENT AT	7	13, (	1.50	2.65),	HAS	-1.2979 AMPS
CURRENT FILAMENT AT	9	13, (	2.00	2.56),	HAS	-1.0904 AMPS
CURRENT FILAMENT AT	1	14, (	0.00	2.75),	HAS	-0.7805 AMPS
CURRENT FILAMENT AT	3	14, (	0.50	2.74),	HAS	-1.5311 AMPS

STORED ENERGY = 4.5703E-05 JOULES / METER OR RADIAN

XJFACT= 1.000000

K	L	A(VECTOR)	X	Y	BX(GAUSS)	BY(GAUSS)	BT(GAUSS)	DB
1	1	-3.775190E+00	0.00000	0.00000	0.000	0.000	0.000	
2	1	-3.748710E+00	0.25000	0.00000	0.000	-0.204	0.204	
3	1	-3.672503E+00	0.50000	0.00000	0.000	-0.405	0.405	
4	1	-3.545983E+00	0.75000	0.00000	0.000	-0.607	0.607	
5	1	-3.369107E+00	1.00000	0.00000	0.000	-0.808	0.808	
6	1	-3.142082E+00	1.25000	0.00000	0.000	-1.008	1.008	
7	1	-2.865236E+00	1.50000	0.00000	0.000	-1.206	1.206	
8	1	-2.539022E+00	1.75000	0.00000	0.000	-1.403	1.403	
9	1	-2.164057E+00	2.00000	0.00000	0.000	-1.596	1.596	
10	1	-1.741158E+00	2.25000	0.00000	0.000	-1.786	1.786	
11	1	-1.271398E+00	2.50000	0.00000	0.000	-1.971	1.971	
12	1	-7.561750E-01	2.75000	0.00000	0.000	-2.150	2.150	
13	1	-1.972908E-01	3.00000	0.00000	0.000	-2.320	2.320	
14	1	4.029561E-01	3.25000	0.00000	0.000	-2.480	2.480	
15	1	1.041664E+00	3.50000	0.00000	0.000	-2.627	2.627	

TABLE FOR FIELD COEFFICIENTS

NORMALIZATION RADIUS = 2.50000

$$(BX - I BY) = I * \text{SUM } N * (AN + I BN) / R * (Z/R)^{(N-1)}$$

N	N(AN)/R	N(BN)/R	ABS(N(CN)/R)
2	2.0267E+00	0.0000E+00	2.0267E+00
4	-3.9091E-02	0.0000E+00	3.9091E-02
6	-7.7138E-03	0.0000E+00	7.7138E-03
8	-1.9802E-02	0.0000E+00	1.9802E-02

Appendix C: Code EDDY files for Eddy Current Induced Harmonic Analytical Calculations

1. Code EDDY
2. Code EDDY_STR
3. dipole vacuum chamber options
4. dipole coil positioning
5. vacuum chamber thickness and tolerance study
6. dipole coil positioning
7. quadrupole vacuum chamber options
8. sextupole vacuum chamber
9. miscellaneous EDDY results
10. Code EDDYCONT


```

c*****
c
c      Program eddy:  given vacuum chamber contour, e.g. ellipse with axes
c                     (a,b), [vac. ch. midplane symmetry required for any
c                     magnet, additionally quarter plane vac. ch. symmetry
c                     required for quadrupole, 1/6 plane vac. ch. symmetry
c                     required for sextupole, etc.] and magnet half gap or
c                     aperture radius (h), determines:
c                     (2) total harmonic components of field in
c                         multipole magnet due to vac. ch. eddy currents
c                     (1) harmonic components of eddy current induced field
c                         in multipole magnet due to current filament pairs
c                         (useful passive harmonics nulling information)
c
c      created:   Feb. 1991      Ross Schlueter
c
c      modified:  May 1991 - see Eddy_str version of this program when
c                        ellipse is to have a stretched straight
c                        section symmetric w.r.t. x=0.
c
c*****
c
c SET UP
c
c      dimension x_0(200), y_0(200), sarc(200), w_0(200),
c      &          u_0(200), v_0(200),
c      &          Fn(200), Sc(200), Td(200), Fr(200), Ff(200),
c      &          w_Fn(200), w_Sc(200), w_Td(200), w_Fr(200), w_Ff(200),
c      &          p2(5), zh(5)
c      complex w_0, zi, C, S, alpha_0, wm, wp, zm, zp
c      real k, mu, mpl, mu2h, mu2hmpl
c      open (unit=2, file='eddy_out.dat', status='old',
c      &      carriagecontrol='fortran')
c      pi=4.*atan(1.)
c      zi=(0,1)
c
c
c INTERACTIVE INPUT/WRITE HEADINGS
c
c      write(5,100) ! multipole #, vac. ch. geom., eval. point
9      read *,m,a,b,h,r
c      if(h.lt..1) h=2.86
c      write(2,101)
c      write(2,102)m,a,b,h,r
c      x_shift=0.0
c      y_shift=0.0
c
c
c GET VACUUM CHAMBER CONTOUR IN W-PLANE
c
c      if(m .eq. 0) then
c          w = r      ! r=|z| the eval. radius
c          a_w = a
c          b_w = b
c      else if(m .eq. 1) then
c          w = r**2/h
c          a_w = (a**2+b**2)/(2.*h)
c          b_w = a*b/h
c          c_w = (a**2-b**2)/(2.*h) ! (c_w is an alternate name for u_hat)
c      else if(m .eq. 2) then
c          ellipse transforms to irregular shape
c      end if
c
c
c EQUATION 43 (or 21 for special case of a dipole magnet)
c
c      alpha = pi*w/(2.*h)
c      k=pi/(2.*h)
c      imax=42

```

```

mu = .4*pi
mu2h=mu/(2.*h)
mpl = float(m+1)
mu2hmpl=mu2h*mpl
do 1 npl= 1,5
    p2(npl)=(pi/2.)*(npl-1)
    zh(npl)=(r/h)**((m+1)*(npl-1)+m)
1  continue
c
do 12 j=1,1 ! m+1
do 11 i=1,imax+1
    ij=i + (j-1)*(imax+1)
    angle=(pi/float(m+1))*(float(i-1)/float(imax)) + (j-1)*pi/(m+1)
    x_0(ij) = a*cos(angle) + x_shift
    y_0(ij) = b*sin(angle) + y_shift
    u_0(ij) = real(cmplx(x_0(ij),y_0(ij))**(m+1))/h**m
    v_0(ij) = aimag(cmplx(x_0(ij),y_0(ij))**(m+1))/h**m
c      for m=0, u=x              = a*cos(angle),
c              v=y              = b*sin(angle)
c      for m=1, u=(x**2-y**2)/h = a_w*cos(2.*angle) + c_w
c              v=(2*x*y)/h      = b_w*sin(2.*angle)
c      for m=2, u=Re[(x+iy)^3]/h^2
c              v=Aimag[(x+iy)^3]/h^2
c
    w_0(ij)=cmplx(u_0(ij),v_0(ij))
    alpha_0=k*w_0(ij)
    C = (exp(alpha_0)+exp(-alpha_0))/2.
    S = (exp(alpha_0)-exp(-alpha_0))/2.
    Fn(ij)=mu2hmpl*p2(1)*zh(1)*Real(-C/S)
    Sc(ij)=mu2hmpl*p2(2)*zh(2)*Real(-1./S**2)
    Td(ij)=mu2hmpl*p2(3)*zh(3)*Real((1./2.)*(-2.*C)/(S**3))
    Fr(ij)=mu2hmpl*p2(4)*zh(4)*Real((1./6.)*(-2/S**4)*(2*S**2+3))
    Ff(ij)=mu2hmpl*p2(5)*zh(5)*Real((1./24.)*(-8.*C/S**5)*(S**2+3))
c*****0*****0*****0*****0*****0*****0*****0*****012
11  continue
12  continue
c
c GET STOT, SARC, AND CENTER OF GRAVITY OF VAC. CHAMBER ARC IN W-PLANE
c
    u_hat=0.
    Stot=0.
    do 22 j=1,1 ! m+1
    do 21 i=1,imax+1
        ij=i + (j-1)*(imax+1)
        if(i.gt.1 .and. i .lt. imax+1)then
            zm=cmplx(x_0(ij)-x_0(ij-1),y_0(ij)-y_0(ij-1))
            zp=cmplx(x_0(ij+1)-x_0(ij),y_0(ij+1)-y_0(ij))
            sarc(ij) = .5*sqrt(Real(zm)**2+Aimag(zm)**2)
            & .5*sqrt(Real(zp)**2+Aimag(zp)**2)
        else if(i.eq.1) then
            zp=cmplx(x_0(ij+1)-x_0(ij),y_0(ij+1)-y_0(ij))
            sarc(ij) = .5*sqrt(Real(zp)**2+Aimag(zp)**2)
        else if(i.eq.imax+1) then
            zm=cmplx(x_0(ij)-x_0(ij-1),y_0(ij)-y_0(ij-1))
            sarc(ij) = .5*sqrt(Real(zm)**2+Aimag(zm)**2)
        end if
        u_hat = u_hat + u_0(ij)*sarc(ij)
        Stot=Stot+sarc(ij)
21  continue
22  continue
    u_hat=u_hat/Stot
c
c EQUATION 44 (or 33 for special case of a dipole magnet)
c
    Fn_tot=0.
    Sc_tot=0.

```

```

Td_tot=0.
Fr_tot=0.
Ff_tot=0.
do 32 j=1,1 ! m+1
  if(j.ne.1)write(2,107)
  write(2,108)Stot, u_hat,
  &      ((m+1)*npl-1,npl=1,5),((m+1)*npl-1,npl=1,5)
  do 31 i=1,imax+1
    ij=i + (j-1)*(imax+1)
    F = u_0(ij) - u_hat
    w_Fn(ij) = F*Fn(ij)
    w_Sc(ij) = F*Sc(ij)
    w_Td(ij) = F*Td(ij)
    w_Fr(ij) = F*Fr(ij)
    w_Ff(ij) = F*Ff(ij)
    write(2,111)i,x_0(ij),y_0(ij),sarc(ij),u_0(ij),v_0(ij),
    &      Fn(ij),Sc(ij),Td(ij),Fr(ij),Ff(ij),
    &      w_Fn(ij),w_Sc(ij),w_Td(ij),w_Fr(ij),w_Ff(ij)
    Fn_tot=Fn_tot+w_Fn(ij)*sarc(ij)
    Sc_tot=Sc_tot+w_Sc(ij)*sarc(ij)
    Td_tot=Td_tot+w_Td(ij)*sarc(ij)
    Fr_tot=Fr_tot+w_Fr(ij)*sarc(ij)
    Ff_tot=Ff_tot+w_Ff(ij)*sarc(ij)
    if(i .eq. imax/2 +1)
      &      write(2,112)Fn_tot, Sc_tot, Td_tot, Fr_tot, Ff_tot
  31 continue
  write(2,113)Stot, Fn_tot, Sc_tot, Td_tot, Fr_tot, Ff_tot
32 continue
c
c FORMATS
c
100 format(' input multipole # (0=dipole, 1=quad, etc),',/,
  &' vac cham axes "a,b", half gap or aperature "h", eval rad "r"')
101 format(' program eddy',/)
102 format('Input:      multipole # (0=dipole, 1=quad, etc) m = ',i1,
  &      3x,'ellipse axes: (a,b) = (',
  &      f5.2,',',f5.2,') cm,', 3x,'h =',f5.2,') cm,',3x,
  &      'harmonics evaluated at r=',f5.2,') cm',/)
107 format(///)
108 format('Output:',36x,
  &'|- z-plane harmonic components iB*(z) (G) for I=1 A --',
  &'|-- components weighted by u(i) - Int_s u/S ds (G-cm) --|',/,
  &16x,
  &'|- at z(i)e^(i2j pi/m+1), j=0,...m, and conjugates, -|',
  &' (S =',f8.5,')',',16x,['',f8.5,']',/,
  &43x,'|----- cf. Eqn. (21) or (43) -----|',/,
  &' harmonic component mn+m+n (n=0,1,2,...):',10(4x,i2,5x),/,
  &' i      x(i)      y(i)      sarc(i)      u(i)      v(i) ',
  &' Fn(i)      Sc(i)      Td(i)      Fr(i)      Ff(i) ',
  &' w_Fn(i)      w_Sc(i)      w_Td(i)      w_Fr(i)      w_Ff(i)',/,
  &' |-- I(x(i),+/-y(i)) = 1 Amp, cf. Eqn. (21) or (43) --|',
111 format(i2,5(2x,f6.3),2x,10(e9.3,2x))
112 format(/,47x,
  &' Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff: ',
  &' 5(e9.3,2x),/)
113 format(/,7x,'Sum_sarc(i):',1x,f6.3,',',1x,
  &'Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn,Sc,Td,Fr,Ff',
  &' (cf. Eqn. (33) or (44)): ', 5(e9.3,2x))
c
c CLOSE
c
  if(3 .lt. 2)go to 9
  close (unit=2, status='keep')
end
c
c*****0*****0*****0*****0*****0*****0*****0*****012

```

```

C*****
C
C Program eddy_str: given vacuum chamber contour, e.g. ellipse with axes
C                   (a,b), [vac. ch. midplane symmetry required for any
C                   magnet, additionally quarter plane vac. ch. symmetry
C                   required for quadrupole, 1/6 plane vac. ch. symmetry
C                   required for sextupole, etc.] and magnet half gap or
C                   aperture radius (h), determines:
C                   (2) total harmonic components of field in
C                   multipole magnet due to vac. ch. eddy currents
C                   (1) harmonic components of eddy current induced field
C                   in multipole magnet due to current filament pairs
C                   (useful passive harmonics nulling information)
C
C created: Feb. 1991 Ross Schlueter
C
C modified: May 1991 - this Eddy_str version accomodates an added
C                   vac. ch. straight section symmetric w.r.t. x=0.
C                   FOR USE ONLY IN DIPOLE AND QUADRUPOLE MAGNETS;
C                   the equations below are not valid for this
C                   asymmetry condition in higher order multipoles.
C                   See eddy_elsex for special case of elliptical
C                   sextupole.
C*****
C
C SET UP
C
C dimension x_0(200), y_0(200), sarc(200), w_0(200),
C & u_0(200), v_0(200),
C & Fn(200), Sc(200), Td(200), Fr(200), Ff(200),
C & w_Fn(200), w_Sc(200), w_Td(200), w_Fr(200), w_Ff(200),
C & p2(5), zh(5)
C complex w_0, zi, C, S, alpha_0, wm, wp, zp, zm
C real k, mu, mpl, mu2h, mu2hmpl
C open (unit=2, file='eddy_out.dat', status='old',
C & carriagecontrol='fortran')
C pi=4.*atan(1.)
C zi=(0,1)
C
C
C INTERACTIVE INPUT/WRITE HEADINGS
C
C write(5,100) ! multipole #, vac. ch. geom., eval. point
9 read *,m,a,b,str,h,r ! str added to accomodate stretched ellipse shapes
C if(h.lt..1) h=2.86
C write(2,101)
C write(2,102)m,a,b,str,h,r ! str
C if( m .gt. 1) then
C write(5,999)
999 format('invalid symmetry condition for this code')
C stop
C end if
C x_shift = 0.
C y_shift = 0.
C
C
C GET VACUUM CHAMBER CONTOUR IN W-PLANE
C
C if(m .eq. 0) then
C w = r ! r=|z| the eval. radius
C a_w = a
C b_w = b
C else if(m .eq. 1) then
C w = r**2/h
C a_w = (a**2+b**2)/(2.*h)
C b_w = a*b/h
C c_w = (a**2-b**2)/(2.*h) ! (c_w is an alternate name for u_hat)

```

```

c      else if(m .eq. 2) then
c          ellipse transforms to irregular shape
c      end if
c
c EQUATION 43 (or 21 for special case of a dipole magnet)
c
c      alpha = pi*w/(2.*h)
c      k=pi/(2.*h)
c      imax=42 ! make it even
c
c      istr=0
c      if(str .ne. 0.) then
c          imax=imax-4
c          istr=4
c      end if
c
c      mu = .4*pi
c      mu2h=mu/(2.*h)
c      mp1 = float(m+1)
c      mu2hmp1=mu2h*mp1
c      do 1 np1= 1,5
c          p2(np1)=(pi/2.)**(np1-1)
c          zh(np1)=(r/h)**((m+1)*(np1-1)+m)
1      continue
c
c      do 12 j=1,1 ! m+1
c      do 11 i=1,imax+1+istr
c          ij=i + (j-1)*(imax+1)
c
c          if(istr.eq.0) then
c              angle=(pi/float(m+1))*(float(i-1)/float(imax))+(j-1)*pi/(m+1)
c              x_0(ij) = a*cos(angle) + x_shift
c              y_0(ij) = b*sin(angle) + y_shift
c          else if(i .le. imax/(2-m) + 1) then
c              angle=(pi/float(m+1))*(float(i-1)/float(imax))+(j-1)*pi/(m+1)
c              x_0(ij) = a*cos(angle) + str/2. + x_shift
c              y_0(ij) = b*sin(angle) + y_shift
c          else if(i .ge. imax/(2-m) + 6) then
c              angle=(pi/float(m+1))*(float(i-5)/float(imax))+(j-1)*pi/(m+1)
c              x_0(ij) = a*cos(angle) - str/2. + x_shift
c              y_0(ij) = b*sin(angle) + y_shift
c          else
c              x_0(ij) = x_0(imax/(2-m) + 1)
c              y_0(ij) = y_0(imax/(2-m) + 1) + x_shift
c          end if
c
c          u_0(ij) = real(cmplx(x_0(ij),y_0(ij))**(m+1))/h**m
c          v_0(ij) = aimag(cmplx(x_0(ij),y_0(ij))**(m+1))/h**m
c          for m=0, u=x = a*cos(angle),
c                      v=y = b*sin(angle)
c          for m=1, u=(x**2-y**2)/h = a_w*cos(2.*angle) + c_w
c                      v=(2*x*y)/h = b_w*sin(2.*angle)
c          for m=2, u=Re[(x+iy)^3]/h^2
c                      v=Aimag[(x+iy)^3]/h^2
c
c          w_0(ij)=cmplx(u_0(ij),v_0(ij))
c          alpha_0=k*w_0(ij)
c          C = (exp(alpha_0)+exp(-alpha_0))/2.
c          S = (exp(alpha_0)-exp(-alpha_0))/2.
c          Fr(ij)=mu2hmp1*p2(1)*zh(1)*Real(-C/S)
c          Sc(ij)=mu2hmp1*p2(2)*zh(2)*Real(-1./S**2)
c          Td(ij)=mu2hmp1*p2(3)*zh(3)*Real((1./2.)*(-2.*C)/(S**3))
c          Fr(ij)=mu2hmp1*p2(4)*zh(4)*Real((1./6.)*(-2/S**4)*(2*S**2+3))
c          Ff(ij)=mu2hmp1*p2(5)*zh(5)*Real((1./24.)*(-8.*C/S**5)*(S**2+3))
c*****0*****0*****0*****0*****0*****0*****0*****0*****012

```

```

11  continue
12  continue
c
c GET STOT, SARC, AND CENTER OF GRAVITY OF VAC. CHAMBER ARC IN W-PLANE
c
  u_hat=0.
  Stot=0.
  do 22 j=1,1 ! m+1
  do 21 i=1,imax+1+istr
    ij=i + (j-1)*(imax+1)
    if(i.gt.1 .and. i .lt. imax+1+istr)then
      zm=cmlpx(x_0(ij)-x_0(ij-1),y_0(ij)-y_0(ij-1))
      zp=cmlpx(x_0(ij+1)-x_0(ij),y_0(ij+1)-y_0(ij))
      sarc(ij) = .5*sqrt(Real(zm)**2+Aimag(zm)**2)+
&      .5*sqrt(Real(zp)**2+Aimag(zp)**2)
    else if(i.eq.1) then
      zp=cmlpx(x_0(ij+1)-x_0(ij),y_0(ij+1)-y_0(ij))
      sarc(ij) = .5*sqrt(Real(zp)**2+Aimag(zp)**2)
    else if(i.eq.imax+1+istr) then
      zm=cmlpx(x_0(ij)-x_0(ij-1),y_0(ij)-y_0(ij-1))
      sarc(ij) = .5*sqrt(Real(zm)**2+Aimag(zm)**2)
    end if
    u_hat = u_hat + u_0(ij)*sarc(ij)
    Stot=Stot+sarc(ij)
21  continue
22  continue
  u_hat=u_hat/Stot
c
c EQUATION 44 (or 33 for special case of a dipole magnet)
c
  Fn_tot=0.
  Sc_tot=0.
  Td_tot=0.
  Fr_tot=0.
  Ff_tot=0.
  do 32 j=1,1 ! m+1
    if(j.ne.1)write(2,107)
    write(2,108)Stot, u_hat,
&    ((m+1)*np1-1,np1=1,5),((m+1)*np1-1,np1=1,5)
  do 31 i=1,imax+1+istr
    ij=i + (j-1)*(imax+1)
    F = u_0(ij) - u_hat
    w_Fn(ij) = F*Fn(ij)
    w_Sc(ij) = F*Sc(ij)
    w_Td(ij) = F*Td(ij)
    w_Fr(ij) = F*Fr(ij)
    w_Ff(ij) = F*Ff(ij)
    write(2,111)i,x_0(ij),y_0(ij),sarc(ij),u_0(ij),v_0(ij),
&    Fn(ij),Sc(ij),Td(ij),Fr(ij),Ff(ij),
&    w_Fn(ij),w_Sc(ij),w_Td(ij),w_Fr(ij),w_Ff(ij)
    Fn_tot=Fn_tot+w_Fn(ij)*sarc(ij)
    Sc_tot=Sc_tot+w_Sc(ij)*sarc(ij)
    Td_tot=Td_tot+w_Td(ij)*sarc(ij)
    Fr_tot=Fr_tot+w_Fr(ij)*sarc(ij)
    Ff_tot=Ff_tot+w_Ff(ij)*sarc(ij)
    if(i .eq. imax/2 +1 + istr/2)
&    write(2,112)Fn_tot, Sc_tot, Td_tot, Fr_tot, Ff_tot
31  continue
  write(2,113)Stot, Fn_tot, Sc_tot, Td_tot, Fr_tot, Ff_tot
32  continue
c
c FORMATS
c
100  format(' input multipole # (0=dipole, 1=quad, etc),',/,
&' axes "a,b", "str", half gap or aperature "h", eval rad "r"')
101  format(' program eddy_str',/)

```

```

102  format('Input:      multipole # (0=dipole, 1=quad, etc) m = ',i1,
&        2x,'ellipse axes: (a,b)=( ',
&        f5.2,',',f5.2,') cm,', 2x,'str =',f5.2,' cm,',
&        2x,'h =',f5.2,' cm,', 2x,
&        'harmonics evaluated at r=',f5.2,' cm',/)
107  format(////)

108  format('Output:',36x,
&'|- z-plane harmonic components iB*(z) (G) for I=1 A -',
&'|-- components weighted by u(i) - Int_s u/S ds (G-cm) --|',/,
&16x,' ',
&'|- at z(i)e^(i2j pi/m+1), j=0,...m, and conjugates, -|',
&' (S =',f8.5,')',',16x,[' ',f8.5,']',/,
&43x,'|----- cf. Eqn. (21) or (43) -----|',/,
&' harmonic component mn+m+n (n=0,1,2,...):',10(4x,i2,5x),/,
&' i      x(i)      y(i)      sarc(i)      u(i)      v(i) ',
&' Fn(i)      Sc(i)      Td(i)      Fr(i)      Ff(i) ',
&' w Fn(i)      w Sc(i)      w Td(i)      w Fr(i)      w Ff(i)',/)
c    &'|-- I(x(i),+/-y(i)) = 1 Amp, cf. Eqn. (21) or (43) --|',
111    format(i2,5(2x,f6.3),2x,10(e9.3,2x))
112  format(/,49x,
&' Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff: ',
&    5(e9.3,2x),/)
113  format(/,7x,'Sum_sarc(i):',1x,f6.3,',',1x,
&'Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn,Sc,Td,Fr,Ff',
&' (cf. Eqn. (33) or (44)): ', 5(e9.3,2x))

c
c CLOSE
c
    if(3 .lt. 2)go to 9
    close (unit=2, status='keep')
end

c
c*****0*****0*****0*****0*****0*****0*****012

```

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (5.50, 2.75) cm, h = 2.86 cm, harmonics evaluated at r = 2.50 cm

Output:

|- z-plane harmonic components IB*(z) (G) for I=1 A |--- components weighted by u(l) - Int_s u/s ds (G-cm) ---|
 |- at z(l)e^(i2j pi/m+1), j=0,...,m, and conjugates, -|
----- cf. Eqn. (21) or (43) -----

l	x(l)	y(l)	sarc(l)	u(l)	v(l)	Fn(l)	Sc(l)	Td(l)	Fr(l)	Ff(l)	w_Fn(l)	w_Sc(l)	w_Td(l)	w_Fr(l)	w_Ff(l)
1	5.500	0.000	0.103	5.500	0.000	-2.21E+00	-2.88E-02	-3.98E-02	-3.68E-02	-2.57E-02	-1.21E+01	-1.59E-01	-2.21E-01	-2.02E-01	-1.41E-01
2	5.485	0.206	0.208	5.485	0.206	-2.21E+00	-2.86E-02	-3.94E-02	-3.64E-02	-2.54E-02	-1.21E+01	-1.57E-01	-2.16E-01	-2.00E-01	-1.39E-01
3	5.439	0.410	0.213	5.439	0.410	-2.21E+00	-2.77E-02	-3.82E-02	-3.52E-02	-2.45E-02	-1.20E+01	-1.51E-01	-2.08E-01	-1.92E-01	-1.33E-01
4	5.362	0.612	0.221	5.362	0.612	-2.21E+00	-2.62E-02	-3.60E-02	-3.30E-02	-2.28E-02	-1.18E+01	-1.40E-01	-1.93E-01	-1.77E-01	-1.22E-01
5	5.256	0.811	0.231	5.256	0.811	-2.21E+00	-2.36E-02	-3.23E-02	-2.94E-02	-2.00E-02	-1.16E+01	-1.24E-01	-1.70E-01	-1.55E-01	-1.05E-01
6	5.120	1.005	0.244	5.120	1.005	-2.20E+00	-1.94E-02	-2.64E-02	-2.37E-02	-1.56E-02	-1.13E+01	-9.95E-02	-1.35E-01	-1.21E-01	-7.99E-02
7	4.955	1.193	0.257	4.955	1.193	-2.20E+00	-1.30E-02	-1.73E-02	-1.49E-02	-8.83E-03	-1.08E+01	-6.46E-02	-8.60E-02	-7.37E-02	-4.38E-02
8	4.763	1.375	0.272	4.763	1.375	-2.20E+00	-3.21E-03	-3.46E-03	-1.44E-03	-1.13E-03	-1.05E+01	-1.53E-02	-1.65E-02	-6.88E-03	0.558E-03
9	4.544	1.549	0.287	4.544	1.549	-2.19E+00	0.118E-02	0.176E-02	0.189E-02	0.166E-02	-9.95E+00	0.535E-02	0.801E-02	0.854E-02	0.752E-02
10	4.300	1.715	0.303	4.300	1.715	-2.18E+00	0.345E-02	0.494E-02	0.490E-02	0.386E-02	-9.39E+00	0.148E-01	0.213E-01	0.211E-01	0.166E-01
11	4.032	1.870	0.318	4.032	1.870	-2.17E+00	0.688E-02	0.970E-02	0.935E-02	0.698E-02	-8.76E+00	0.278E-01	0.391E-01	0.376E-01	0.282E-01
12	3.741	2.016	0.332	3.741	2.016	-2.15E+00	0.121E-01	0.168E-01	0.157E-01	0.112E-01	-8.06E+00	0.451E-01	0.627E-01	0.587E-01	0.418E-01
13	3.429	2.150	0.346	3.429	2.150	-2.12E+00	0.198E-01	0.271E-01	0.245E-01	0.164E-01	-7.29E+00	0.679E-01	0.929E-01	0.841E-01	0.561E-01
14	3.098	2.272	0.359	3.098	2.272	-2.08E+00	0.313E-01	0.418E-01	0.361E-01	0.217E-01	-6.45E+00	0.969E-01	0.130E+00	0.112E+00	0.673E-01
15	2.750	2.382	0.371	2.750	2.382	-2.02E+00	0.481E-01	0.621E-01	0.499E-01	0.248E-01	-5.55E+00	0.132E+00	0.171E+00	0.137E+00	0.682E-01
16	2.386	2.478	0.381	2.386	2.478	-1.92E+00	0.720E-01	0.884E-01	0.629E-01	0.205E-01	-4.59E+00	0.172E+00	0.211E+00	0.150E+00	0.490E-01
17	2.009	2.560	0.390	2.009	2.560	-1.78E+00	0.105E+00	0.119E+00	0.681E-01	0.858E-03	-3.57E+00	0.239E+00	0.239E+00	0.137E+00	0.172E-02
18	1.621	2.628	0.397	1.621	2.628	-1.58E+00	0.148E+00	0.148E+00	0.539E-01	-4.11E-01	-2.55E+00	0.239E+00	0.240E+00	0.875E-01	-0.666E-01
19	1.224	2.681	0.403	1.224	2.681	-1.30E+00	0.198E+00	0.162E+00	0.731E-02	-9.88E-01	-1.59E+00	0.242E+00	0.199E+00	0.895E-02	-0.121E+00
20	0.820	2.719	0.408	0.820	2.719	-9.92E-01	0.249E+00	0.146E+00	-1.37E+00	-7.64E-01	-2.04E+00	0.204E+00	0.120E+00	-0.591E-01	-0.112E+00
21	0.411	2.742	0.411	0.411	2.742	-4.90E-01	0.288E+00	0.888E-01	-1.56E+00	-1.05E+00	-2.01E-01	0.118E+00	0.365E-01	-0.639E-01	-0.432E-01
22	0.000	2.750	0.411	0.000	2.750	0.263E-07	0.303E+00	-5.02E-07	-1.92E+00	0.642E-07	-9.87E-14	-1.13E-06	0.188E-13	0.721E-07	-2.40E-13
Sum_i w_zz(lj)*sarc(lj), where zz=Fn,Sc,Td,Fr,Ff:															
23	-0.411	2.742	0.410	-0.411	2.742	0.490E-01	0.288E+00	-8.88E-01	-1.56E+00	0.105E+00	-2.01E-01	-1.18E+00	0.365E-01	0.639E-01	-0.432E-01
24	-0.820	2.719	0.408	-0.820	2.719	0.932E-01	0.249E+00	-1.46E+00	-7.21E-01	0.137E+00	-7.64E-01	-2.04E+00	0.120E+00	0.591E-01	-0.112E+00
25	-1.224	2.681	0.403	-1.224	2.681	0.130E+00	0.198E+00	-1.62E+00	0.731E-02	0.986E-01	-1.59E+00	-2.24E+00	0.199E+00	-8.95E-02	-0.121E+00
26	-1.621	2.628	0.397	-1.621	2.628	0.158E+00	0.148E+00	-1.148E+00	0.539E-01	0.411E-01	-2.55E+00	-2.39E+00	0.240E+00	-8.75E-01	-0.666E-01
27	-2.009	2.560	0.390	-2.009	2.560	0.178E+00	0.105E+00	-1.19E+00	0.681E-01	-8.58E-03	-3.57E+00	-2.11E+00	0.239E+00	-1.37E+00	0.172E-02
28	-2.386	2.478	0.381	-2.386	2.478	0.192E+00	0.720E-01	-8.84E-01	0.629E-01	-2.05E-01	-4.59E+00	-1.72E+00	0.211E+00	-1.50E+00	0.490E-01
29	-2.750	2.382	0.371	-2.750	2.382	0.202E+00	0.481E-01	-6.21E-01	0.499E-01	-2.48E-01	-5.55E+00	-1.32E+00	0.171E+00	-1.37E+00	0.682E-01
30	-3.098	2.272	0.359	-3.098	2.272	0.208E+00	0.313E-01	-4.18E-01	0.361E-01	-2.17E-01	-6.45E+00	-9.69E-01	0.130E+00	-1.12E+00	0.673E-01
31	-3.429	2.150	0.346	-3.429	2.150	0.212E+00	0.198E-01	-2.71E-01	0.245E-01	-1.64E-01	-7.29E+00	-6.79E-01	0.929E-01	-8.41E-01	0.561E-01
32	-3.741	2.016	0.332	-3.741	2.016	0.215E+00	0.121E-01	-1.68E-01	0.157E-01	-1.12E-01	-8.06E+00	-4.51E-01	0.627E-01	-5.87E-01	0.418E-01
33	-4.032	1.870	0.318	-4.032	1.870	0.218E+00	0.688E-02	-1.970E-02	0.935E-02	-6.98E-02	-9.76E+00	-1.48E-01	0.391E-01	-3.76E-01	0.282E-01
34	-4.300	1.715	0.303	-4.300	1.715	0.218E+00	0.345E-02	-4.94E-02	0.490E-02	-3.86E-02	-9.39E+00	-1.48E-01	0.213E-01	-2.11E-01	0.166E-01
35	-4.544	1.549	0.287	-4.544	1.549	0.219E+00	0.118E-02	-1.76E-02	0.189E-02	-1.66E-02	-9.95E+00	-5.35E-02	0.801E-02	-8.54E-02	0.752E-02
36	-4.763	1.375	0.272	-4.763	1.375	0.220E+00	-3.21E-03	0.346E-03	-1.44E-03	-1.13E-03	-1.05E+01	0.153E-02	-1.65E-02	0.688E-03	0.558E-03
37	-4.955	1.193	0.257	-4.955	1.193	0.220E+00	-1.30E-02	0.173E-02	-1.49E-02	0.883E-03	-1.08E+01	0.646E-02	-8.60E-02	0.737E-02	-0.438E-02
38	-5.120	1.005	0.244	-5.120	1.005	0.220E+00	-1.94E-02	0.264E-02	-2.37E-02	0.156E-02	-1.13E+01	0.995E-02	-1.35E-01	0.121E-01	-7.99E-02
39	-5.256	0.811	0.231	-5.256	0.811	0.221E+00	-2.36E-02	0.323E-02	-2.94E-02	0.200E-02	-1.16E+01	0.124E-01	-1.70E-01	0.155E-01	-0.105E-01
40	-5.362	0.612	0.221	-5.362	0.612	0.221E+00	-2.62E-02	0.360E-02	-3.30E-02	0.228E-02	-1.18E+01	0.140E-01	-1.93E-01	0.177E-01	-0.122E-01
41	-5.439	0.410	0.213	-5.439	0.410	0.221E+00	-2.77E-02	0.382E-02	-3.52E-02	0.245E-02	-1.20E+01	0.151E-01	-2.08E-01	0.192E-01	-0.133E-01
42	-5.485	0.206	0.208	-5.485	0.206	0.221E+00	-2.86E-02	0.394E-02	-3.64E-02	0.254E-02	-1.21E+01	0.157E-01	-2.16E-01	0.200E-01	-0.139E-01
43	-5.500	0.000	0.103	-5.500	0.000	0.221E+00	-2.88E-02	0.398E-02	-3.68E-02	0.257E-02	-1.21E+01	0.159E-01	-2.21E-01	0.202E-01	-0.141E-01
Sum_sarc(l): 13.319, Sum_i w_zz(lj)*sarc(lj), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):															
												-1.24E-06	0.114E+01	0.489E-08	-7.09E-01

program eddy

```

Input:  multipole # (0-dipole, l-quadr, etc) m = 0  ellipse axes: (a,b) = ( 4.20, 2.70) cm,  h = 2.86 cm,  harmonics evaluated at r= 2.50 cm

Output:
      l- z-plane harmonic components IB*(z) (G) for I=1 A l- components weighted by u(l) - Int_s u/S ds (G-cm) --l
      | - at z(l)*pi/(m+1), j=0,...m, and conjugates, -| (S=10.96438),
      |----- cf. Eqn. (21) or (43) -----|
      harmonic component mn+mn (n=0,1,2,...):
1  x(l)  y(l)  ssrc(l)  u(l)  v(l)  v(l)  Fn(l)  Sc(l)  Td(l)  Fr(l)  Ff(l)  w_Fn(l)  w_Sc(l)  w_Td(l)  w_Fr(l)  w_Ff(l)
1  4.200  0.000  0.000  0.101  4.200  0.000  0.000  -0.224E+00  -0.122E-01  -0.171E-01  -0.163E-01  -0.120E-01  -0.941E+00  -0.513E-01  -0.718E-01  -0.683E-01  -0.506E-01
2  4.188  0.202  0.202  0.203  4.188  0.202  0.202  -0.224E+00  -0.120E-01  -0.169E-01  -0.160E-01  -0.118E-01  -0.938E+00  -0.505E-01  -0.706E-01  -0.670E-01  -0.494E-01
3  4.153  0.402  0.402  0.205  4.153  0.402  0.402  -0.224E+00  -0.116E-01  -0.161E-01  -0.152E-01  -0.110E-01  -0.930E+00  -0.480E-01  -0.669E-01  -0.630E-01  -0.457E-01
4  4.095  0.601  0.601  0.209  4.095  0.601  0.601  -0.224E+00  -0.107E-01  -0.148E-01  -0.137E-01  -0.922E-02  -0.915E+00  -0.437E-01  -0.605E-01  -0.560E-01  -0.394E-01
5  4.013  0.796  0.796  0.214  4.013  0.796  0.796  -0.223E+00  -0.935E-02  -0.127E-01  -0.115E-01  -0.756E-02  -0.895E+00  -0.375E-01  -0.511E-01  -0.460E-01  -0.303E-01
6  3.910  0.986  0.986  0.220  3.910  0.986  0.986  -0.222E+00  -0.745E-02  -0.985E-02  -0.831E-02  -0.469E-02  -0.870E+00  -0.291E-01  -0.385E-01  -0.325E-01  -0.183E-01
7  3.784  1.171  1.171  0.227  3.784  1.171  1.171  -0.222E+00  -0.478E-02  -0.586E-02  -0.403E-02  -0.888E-03  -0.838E+00  -0.181E-01  -0.222E-01  -0.152E-01  -0.366E-02
8  3.637  1.350  1.350  0.235  3.637  1.350  1.350  -0.220E+00  -0.114E-02  -0.445E-03  -0.165E-02  -0.398E-02  -0.801E+00  -0.414E-02  -0.162E-02  -0.601E-02  -0.145E-01
9  3.470  1.521  1.521  0.243  3.470  1.521  1.521  -0.219E+00  -0.380E-02  -0.677E-02  -0.900E-02  -0.995E-02  -0.758E+00  -0.132E-01  -0.235E-01  -0.312E-01  -0.345E-01
10 3.284  1.683  1.683  0.252  3.284  1.683  1.683  -0.216E+00  -0.105E-01  -0.163E-01  -0.183E-01  -0.169E-01  -0.710E+00  -0.343E-01  -0.534E-01  -0.600E-01  -0.554E-01
11 3.079  1.836  1.836  0.260  3.079  1.836  1.836  -0.213E+00  -0.193E-01  -0.285E-01  -0.295E-01  -0.242E-01  -0.566E+00  -0.595E-01  -0.879E-01  -0.909E-01  -0.746E-01
12 2.857  1.979  1.979  0.268  2.857  1.979  1.979  -0.209E+00  -0.310E-01  -0.441E-01  -0.425E-01  -0.308E-01  -0.596E+00  -0.887E-01  -0.126E+00  -0.121E+00  -0.880E-01
13 2.619  2.111  2.111  0.276  2.619  2.111  2.111  -0.203E+00  -0.463E-01  -0.632E-01  -0.562E-01  -0.345E-01  -0.531E+00  -0.121E+00  -0.166E+00  -0.147E+00  -0.903E-01
14 2.366  2.231  2.231  0.283  2.366  2.231  2.231  -0.195E+00  -0.660E-01  -0.858E-01  -0.686E-01  -0.320E-01  -0.461E+00  -0.156E+00  -0.203E+00  -0.162E+00  -0.757E-01
15 2.100  2.338  2.338  0.290  2.100  2.338  2.338  -0.185E+00  -0.908E-01  -0.111E+00  -0.761E-01  -0.190E-01  -0.388E+00  -0.191E+00  -0.233E+00  -0.160E+00  -0.400E-01
16 1.822  2.433  2.433  0.296  1.822  2.433  2.433  -0.171E+00  -0.121E+00  -0.136E+00  -0.736E-01  -0.813E-02  -0.312E+00  -0.221E+00  -0.247E+00  -0.134E+00  -0.149E-01
17 1.534  2.513  2.513  0.301  1.534  2.513  2.513  -0.154E+00  -0.157E+00  -0.156E+00  -0.547E-01  -0.498E-01  -0.236E+00  -0.240E+00  -0.240E+00  -0.839E-01  -0.764E-01
18 1.238  2.580  2.580  0.306  1.238  2.580  2.580  -0.132E+00  -0.196E+00  -0.166E+00  -0.146E-01  -0.985E-01  -0.163E+00  -0.242E+00  -0.206E+00  -0.181E-01  -0.122E+00
19 0.935  2.632  2.632  0.309  0.935  2.632  2.632  -0.105E+00  -0.236E+00  -0.158E+00  -0.457E-01  -0.82E-01  -0.163E+00  -0.220E+00  -0.148E+00  -0.427E-01  -0.127E+00
20 0.626  2.670  2.670  0.312  0.626  2.670  2.670  -0.734E-01  -0.271E+00  -0.127E+00  -0.115E+00  -0.138E+00  -0.459E-01  -0.169E+00  -0.792E-01  -0.722E-01  -0.861E-01
21 0.314  2.692  0.313  0.314  2.692  -0.378E-01  -0.295E+00  -0.709E-01  -0.173E+00  -0.882E-01  -0.119E-01  -0.926E-01  -0.222E-01  -0.543E-01  -0.277E-01  -0.277E-01
22 0.000  2.700  0.314  0.000  2.700  0.264E-07  -0.304E+00  -0.508E-07  -0.195E+00  -0.656E-07  -0.500E-14  -0.575E-07  -0.960E-14  -0.370E-07  -0.124E-13  -0.569E-01

      Sum_i w_zz(lj)*sarc(lj), where zz=Fn,Sc,Td,Fr,Ff:
23 -0.314  2.692  0.313  -0.314  2.692  0.378E-01  0.295E+00  -0.709E-01  -0.173E+00  0.882E-01  -0.119E-01  -0.926E-01  0.223E-01  0.543E-01  -0.277E-01
24 -0.626  2.670  0.312  -0.626  2.670  0.734E-01  0.271E+00  -0.127E+00  -0.115E+00  0.138E+00  -0.459E-01  -0.169E+00  0.792E-01  0.722E-01  -0.861E-01
25 -0.935  2.632  0.309  -0.935  2.632  0.105E+00  0.236E+00  -0.158E+00  -0.457E-01  -0.82E-01  -0.163E+00  -0.242E+00  0.148E+00  0.427E-01  -0.127E+00
26 -1.238  2.580  0.306  -1.238  2.580  0.132E+00  0.196E+00  -0.166E+00  -0.146E-01  -0.985E-01  -0.163E+00  -0.242E+00  0.206E+00  -0.181E-01  -0.122E+00
27 -1.534  2.513  0.301  -1.534  2.513  0.154E+00  0.157E+00  -0.156E+00  -0.547E-01  -0.498E-01  -0.236E+00  -0.240E+00  -0.240E+00  -0.839E-01  -0.764E-01
28 -1.822  2.433  0.296  -1.822  2.433  0.171E+00  0.121E+00  -0.136E+00  -0.736E-01  -0.813E-02  -0.312E+00  -0.221E+00  -0.247E+00  -0.134E+00  -0.148E-01
29 -2.100  2.338  0.290  -2.100  2.338  0.185E+00  0.908E-01  -0.111E+00  -0.761E-01  -0.190E-01  -0.388E+00  -0.191E+00  -0.233E+00  -0.160E+00  -0.400E-01
30 -2.366  2.231  0.283  -2.366  2.231  0.195E+00  0.660E-01  -0.858E-01  -0.547E-01  -0.320E-01  -0.461E+00  -0.156E+00  -0.203E+00  -0.162E+00  -0.757E-01
31 -2.619  2.111  0.276  -2.619  2.111  0.203E+00  0.463E-01  -0.632E-01  -0.522E-01  -0.345E-01  -0.531E+00  -0.121E+00  -0.166E+00  -0.147E+00  -0.903E-01
32 -2.857  1.979  0.268  -2.857  1.979  0.209E+00  0.310E-01  -0.441E-01  -0.425E-01  -0.308E-01  -0.596E+00  -0.887E-01  -0.126E+00  -0.121E+00  -0.880E-01
33 -3.079  1.836  0.260  -3.079  1.836  0.213E+00  0.193E-01  -0.285E-01  -0.295E-01  -0.242E-01  -0.566E+00  -0.595E-01  -0.879E-01  -0.909E-01  -0.746E-01
34 -3.284  1.683  0.252  -3.284  1.683  0.216E+00  0.105E-01  -0.163E-01  -0.183E-01  -0.169E-01  -0.710E+00  -0.343E-01  -0.534E-01  -0.600E-01  -0.554E-01
35 -3.470  1.521  0.243  -3.470  1.521  0.219E+00  0.380E-02  -0.677E-02  -0.900E-02  -0.995E-02  -0.758E+00  -0.132E-01  -0.235E-01  -0.312E-01  -0.345E-01
36 -3.637  1.350  0.235  -3.637  1.350  0.220E+00  -0.114E-02  -0.445E-03  -0.165E-02  -0.398E-02  -0.801E+00  -0.414E-02  -0.162E-02  -0.601E-02  -0.145E-01
37 -3.784  1.171  0.227  -3.784  1.171  0.222E+00  -0.478E-02  -0.586E-02  -0.403E-02  -0.888E-03  -0.838E+00  -0.181E-01  -0.222E-01  -0.152E-01  -0.366E-02
38 -3.910  0.986  0.220  -3.910  0.986  0.223E+00  -0.745E-02  -0.985E-02  -0.831E-02  -0.469E-02  -0.870E+00  -0.291E-01  -0.385E-01  -0.325E-01  -0.183E-01
39 -4.013  0.796  0.214  -4.013  0.796  0.223E+00  -0.935E-02  -0.127E-01  -0.115E-01  -0.756E-02  -0.895E+00  -0.375E-01  -0.511E-01  -0.460E-01  -0.303E-01
40 -4.095  0.601  0.209  -4.095  0.601  0.224E+00  -0.107E-01  -0.148E-01  -0.137E-01  -0.922E-02  -0.915E+00  -0.437E-01  -0.605E-01  -0.560E-01  -0.394E-01
41 -4.153  0.402  0.205  -4.153  0.402  0.224E+00  -0.116E-01  -0.161E-01  -0.152E-01  -0.110E-01  -0.930E+00  -0.480E-01  -0.669E-01  -0.630E-01  -0.457E-01
42 -4.188  0.202  0.203  -4.188  0.202  0.224E+00  -0.120E-01  -0.169E-01  -0.160E-01  -0.118E-01  -0.938E+00  -0.505E-01  -0.706E-01  -0.670E-01  -0.494E-01
43 -4.200  0.000  0.000  -4.200  0.000  0.224E+00  -0.122E-01  -0.171E-01  -0.163E-01  -0.120E-01  -0.941E+00  -0.513E-01  -0.718E-01  -0.683E-01  -0.506E-01

      Sum_ssrc(l): 10.964, Sum_i w_zz(lj)*sarc(lj), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):
      -0.314  2.692  0.313  -0.314  2.692  0.378E-01  0.295E+00  -0.709E-01  -0.173E+00  0.882E-01  -0.119E-01  -0.926E-01  0.223E-01  0.543E-01  -0.277E-01
      -0.626  2.670  0.312  -0.626  2.670  0.734E-01  0.271E+00  -0.127E+00  -0.115E+00  0.138E+00  -0.459E-01  -0.169E+00  0.792E-01  0.722E-01  -0.861E-01
      -0.935  2.632  0.309  -0.935  2.632  0.105E+00  0.236E+00  -0.158E+00  -0.457E-01  -0.82E-01  -0.163E+00  -0.242E+00  0.148E+00  0.427E-01  -0.127E+00
      -1.238  2.580  0.306  -1.238  2.580  0.132E+00  0.196E+00  -0.166E+00  -0.146E-01  -0.985E-01  -0.163E+00  -0.242E+00  0.206E+00  -0.181E-01  -0.122E+00
      -1.534  2.513  0.301  -1.534  2.513  0.154E+00  0.157E+00  -0.156E+00  -0.547E-01  -0.498E-01  -0.236E+00  -0.240E+00  -0.240E+00  -0.839E-01  -0.764E-01
      -1.822  2.433  0.296  -1.822  2.433  0.171E+00  0.121E+00  -0.136E+00  -0.736E-01  -0.813E-02  -0.312E+00  -0.221E+00  -0.247E+00  -0.134E+00  -0.148E-01
      -2.100  2.338  0.290  -2.100  2.338  0.185E+00  0.908E-01  -0.111E+00  -0.761E-01  -0.190E-01  -0.388E+00  -0.191E+00  -0.233E+00  -0.160E+00  -0.400E-01
      -2.366  2.231  0.283  -2.366  2.231  0.195E+00  0.660E-01  -0.858E-01  -0.547E-01  -0.320E-01  -0.461E+00  -0.156E+00  -0.203E+00  -0.162E+00  -0.757E-01
      -2.619  2.111  0.276  -2.619  2.111  0.203E+00  0.463E-01  -0.632E-01  -0.522E-01  -0.345E-01  -0.531E+00  -0.121E+00  -0.166E+00  -0.147E+00  -0.903E-01
      -2.857  1.979  0.268  -2.857  1.979  0.209E+00  0.310E-01  -0.441E-01  -0.425E-01  -0.308E-01  -0.596E+00  -0.887E-01  -0.126E+00  -0.121E+00  -0.880E-01
      -3.079  1.836  0.260  -3.079  1.836  0.213E+00  0.193E-01  -0.285E-01  -0.295E-01  -0.242E-01  -0.566E+00  -0.595E-01  -0.879E-01  -0.909E-01  -0.746E-01
      -3.284  1.683  0.252  -3.284  1.683  0.216E+00  0.105E-01  -0.163E-01  -0.183E-01  -0.169E-01  -0.710E+00  -0.343E-01  -0.534E-01  -0.600E-01  -0.554E-01
      -3.470  1.521  0.243  -3.470  1.521  0.219E+00  0.380E-02  -0.677E-02  -0.900E-02  -0.995E-02  -0.758E+00  -0.132E-01  -0.235E-01  -0.312E-01  -0.345E-01
      -3.637  1.350  0.235  -3.637  1.350  0.220E+00  -0.114E-02  -0.445E-03  -0.165E-02  -0.398E-02  -0.801E+00  -0.414E-02  -0.162E-02  -0.601E-02  -0.145E-01
      -3.784  1.171  0.227  -3.784  1.171  0.222E+00  -0.478E-02  -0.586E-02  -0.403E-02  -0.888E-03  -0.838E+00  -0.181E-01  -0.222E-01  -0.152E-01  -0.366E-02
      -3.910  0.986  0.220  -3.910  0.986  0.223E+00  -0.745E-02  -0.985E-02  -0.831E-02  -0.469E-02  -0.870E+00  -0.291E-01  -0.385E-01  -0.325E-01  -0.183E-01
      -4.013  0.796  0.214  -4.013  0.796  0.223E+00  -0.935E-02  -0.127E-01  -0.115E-01  -0.756E-02  -0.895E+00  -0.375E-01  -0.511E-01  -0.460E-01  -0.303E-01
      -4.095  0.601  0.209  -4.095  0.601  0.224E+00  -0.107E-01  -0.148E-01  -0.137E-01  -0.922E-02  -0.915E+00  -0.437E-01  -0.605E-01  -0.560E-01  -0.394E-01
      -4.153  0.402  0.205  -4.153  0.402  0.224E+00  -0.116E-01  -0.161E-01  -0.152E-01  -0.110E-01  -0.930E+00  -0.480E-01  -0.669E-01  -0.630E-01  -0.457E-01
      -4.188  0.202  0.203  -4.188  0.202  0.224E+00  -0.120E-01  -0.169E-01  -0.160E-01  -0.118E-01  -0.938E+00  -0.505E-01  -0.706E-01  -0.670E-01  -0.494E-01
      -4.200  0.000  0.000  -4.200  0.000  0.224E+00  -0.122E-01  -0.171E-01  -0.163E-01  -0.120E-01  -0.941E+00  -0.513E-01  -0.718E-01  -0.683E-01  -0.506E-01

```

program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (3.58, 2.67) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:
 |--- z-plane harmonic components 1B*(z) (G) for 1st A |--- components weighted by u(i) - Int s u/S ds (G-cm) ---|
 |--- at z(i) e^(i2j pi/m+1), j=0,...,m, and conjugates, -| (S = 9.86614), { 0.00000 }
 |----- cf. Eqn. (21) or (43) -----|

harmonic component mn+m*n (n=0,1,2,...):																
1	x(i)	y(i)	sarc(i)	u(i)	v(i)	0	1	2	3	4	0	1	2	3	4	
						Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)	
1	3.575	0.000	0.100	3.575	0.000	-229E+00	-247E-01	-353E-01	-349E-01	-277E-01	-817E+00	-884E-01	-126E+00	-125E+00	-989E-01	
2	3.565	0.200	0.201	3.565	0.200	-228E+00	-243E-01	-347E-01	-341E-01	-268E-01	-814E+00	-868E-01	-124E+00	-122E+00	-955E-01	
3	3.535	0.399	0.202	3.535	0.399	-228E+00	-232E-01	-327E-01	-317E-01	-241E-01	-806E+00	-819E-01	-116E+00	-112E+00	-852E-01	
4	3.485	0.595	0.204	3.485	0.595	-227E+00	-211E-01	-293E-01	-275E-01	-196E-01	-792E+00	-736E-01	-102E+00	-958E-01	-684E-01	
5	3.416	0.788	0.207	3.416	0.788	-226E+00	-181E-01	-244E-01	-215E-01	-133E-01	-773E+00	-618E-01	-834E-01	-734E-01	-456E-01	
6	3.328	0.977	0.210	3.328	0.977	-225E+00	-139E-01	-178E-01	-136E-01	-528E-02	-749E+00	-464E-01	-591E-01	-451E-01	-176E-01	
7	3.221	1.161	0.214	3.221	1.161	-223E+00	-847E-02	-912E-02	-360E-02	0.438E-02	-719E+00	-273E-01	-294E-01	-116E-01	0.141E-01	
8	3.096	1.337	0.219	3.096	1.337	-221E+00	-142E-02	0.175E-02	0.842E-02	0.152E-01	-683E+00	-439E-02	0.541E-02	0.261E-01	0.472E-01	
9	2.954	1.507	0.224	2.954	1.507	-218E+00	0.752E-02	0.151E-01	0.224E-01	0.266E-01	-643E+00	0.222E-01	0.446E-01	0.660E-01	0.785E-01	
10	2.795	1.668	0.229	2.795	1.668	-214E+00	0.187E-01	0.311E-01	0.377E-01	0.371E-01	-597E+00	0.523E-01	0.870E-01	0.106E+00	0.104E+00	
11	2.621	1.819	0.234	2.621	1.819	-209E+00	0.325E-01	0.499E-01	0.537E-01	0.451E-01	-547E+00	0.853E-01	0.131E+00	0.141E+00	0.118E+00	
12	2.432	1.961	0.239	2.432	1.961	-202E+00	0.494E-01	0.712E-01	0.689E-01	0.479E-01	-492E+00	0.120E+00	0.173E+00	0.167E+00	0.116E+00	
13	2.229	2.091	0.243	2.229	2.091	-194E+00	0.697E-01	0.944E-01	0.808E-01	0.426E-01	-433E+00	0.155E+00	0.210E+00	0.180E+00	0.094E+00	
14	2.014	2.210	0.248	2.014	2.210	-184E+00	0.937E-01	0.118E+00	0.866E-01	0.264E-01	-371E+00	0.189E+00	0.238E+00	0.174E+00	0.531E-01	
15	1.787	2.317	0.252	1.787	2.317	-172E+00	0.121E+00	0.141E+00	0.826E-01	-243E-02	-308E+00	0.217E+00	0.252E+00	0.148E+00	-435E-02	
16	1.551	2.410	0.256	1.551	2.410	-157E+00	0.153E+00	0.159E+00	0.651E-01	-425E-01	-243E+00	0.237E+00	0.247E+00	0.101E+00	-660E-01	
17	1.306	2.490	0.259	1.306	2.490	-139E+00	0.196E+00	0.170E+00	0.317E-01	-881E-01	-181E+00	0.243E+00	0.222E+00	0.414E-01	-115E+00	
18	1.054	2.556	0.262	1.054	2.556	-117E+00	0.221E+00	0.168E+00	-173E-01	-128E+00	-123E+00	0.232E+00	0.177E+00	-183E-01	-135E+00	
19	0.796	2.608	0.264	0.796	2.608	-918E-01	0.253E+00	0.150E+00	-769E-01	-147E+00	-730E-01	0.201E+00	0.119E+00	-611E-01	-117E+00	
20	0.533	2.645	0.266	0.533	2.645	-633E-01	0.280E+00	0.114E+00	-136E+00	-132E+00	-337E-01	0.149E+00	0.606E-01	-726E-01	-702E-01	
21	0.267	2.668	0.267	0.267	2.668	-324E-01	0.298E+00	0.617E-01	-181E+00	-786E-01	-865E-02	0.797E-01	0.165E-01	-483E-01	-210E-01	
22	0.000	2.675	0.267	0.000	2.675	0.200E-07	0.305E+00	-390E-07	-198E+00	0.517E-07	-306E-14	-467E-07	0.598E-14	0.303E-07	-792E-14	
Sum_i w_zz(i)*sarc(i), where zz=Fn,Sc,Td,Fr,Ff:											-216E+01	0.417E+00	0.378E+00	0.119E+00	-654E-01	
23	-0.267	2.668	0.267	-0.267	2.668	0.324E-01	0.298E+00	-617E-01	-181E+00	0.786E-01	-865E-02	-797E-01	0.165E-01	0.483E-01	-210E-01	
24	-0.533	2.645	0.266	-0.533	2.645	0.633E-01	0.280E+00	-114E+00	-136E+00	0.132E+00	-337E-01	-149E+00	0.606E-01	0.726E-01	-702E-01	
25	-0.796	2.608	0.264	-0.796	2.608	0.918E-01	0.253E+00	-150E+00	-769E-01	0.147E+00	-730E-01	-201E+00	0.119E+00	0.611E-01	-117E+00	
26	-1.054	2.556	0.262	-1.054	2.556	0.117E+00	0.221E+00	-168E+00	-173E-01	0.128E+00	-123E+00	-232E+00	0.177E+00	0.183E-01	-135E+00	
27	-1.306	2.490	0.259	-1.306	2.490	0.139E+00	0.186E+00	-170E+00	0.317E-01	0.881E-01	-181E+00	-243E+00	0.222E+00	-414E-01	-115E+00	
28	-1.551	2.410	0.256	-1.551	2.410	0.157E+00	0.153E+00	-159E+00	0.651E-01	0.425E-01	-243E+00	-237E+00	0.247E+00	-101E+00	-660E-01	
29	-1.788	2.317	0.252	-1.788	2.317	0.172E+00	0.121E+00	-141E+00	0.826E-01	-264E-01	-308E+00	-217E+00	0.252E+00	-148E+00	-435E-02	
30	-2.014	2.210	0.248	-2.014	2.210	0.194E+00	0.937E-01	-118E+00	0.666E-01	-264E-01	-371E+00	-189E+00	0.238E+00	-174E+00	0.531E-01	
31	-2.229	2.091	0.243	-2.229	2.091	0.194E+00	0.697E-01	-944E-01	0.808E-01	-426E-01	-433E+00	-155E+00	0.210E+00	-180E+00	0.949E-01	
32	-2.432	1.961	0.239	-2.432	1.961	0.202E+00	0.494E-01	-712E-01	0.689E-01	-479E-01	-492E+00	-120E+00	0.173E+00	-141E+00	0.118E+00	
33	-2.621	1.819	0.234	-2.621	1.819	0.209E+00	0.325E-01	-499E-01	0.537E-01	-451E-01	-547E+00	-853E-01	0.131E+00	-141E+00	0.118E+00	
34	-2.795	1.668	0.229	-2.795	1.668	0.214E+00	0.197E-01	-311E-01	0.377E-01	-371E-01	-597E+00	-523E-01	0.870E-01	-106E+00	0.104E+00	
35	-2.954	1.507	0.224	-2.954	1.507	0.218E+00	0.752E-02	-151E-01	0.224E-01	-266E-01	-643E+00	-222E-01	0.446E-01	-660E-01	0.785E-01	
36	-3.096	1.338	0.219	-3.096	1.338	0.221E+00	-142E-02	0.175E-02	0.842E-02	-152E-01	-683E+00	0.439E-02	0.541E-02	-261E-01	0.472E-01	
37	-3.221	1.161	0.214	-3.221	1.161	0.223E+00	-847E-02	0.912E-02	-360E-02	-438E-02	-719E+00	0.273E-01	-294E-01	0.116E-01	0.141E-01	
38	-3.328	0.977	0.210	-3.328	0.977	0.225E+00	-139E-01	0.778E-01	-136E-01	0.528E-02	-749E+00	0.464E-01	-591E-01	0.451E-01	-176E-01	
39	-3.416	0.788	0.207	-3.416	0.788	0.226E+00	-181E-01	0.244E-01	-215E-01	0.133E-01	-773E+00	0.618E-01	-834E-01	0.734E-01	-456E-01	
40	-3.485	0.595	0.204	-3.485	0.595	0.227E+00	-211E-01	0.293E-01	-275E-01	0.196E-01	-792E+00	0.736E-01	-102E+00	0.958E-01	-684E-01	
41	-3.535	0.399	0.202	-3.535	0.399	0.228E+00	-232E-01	0.327E-01	-317E-01	0.241E-01	-806E+00	0.819E-01	-116E+00	0.112E+00	-852E-01	
42	-3.565	0.200	0.201	-3.565	0.200	0.229E+00	-243E-01	0.347E-01	-341E-01	0.268E-01	-814E+00	0.868E-01	-124E+00	0.122E+00	-955E-01	
43	-3.575	0.000	0.100	-3.575	0.000	0.228E+00	-247E-01	-353E-01	-349E-01	0.277E-01	-817E+00	0.884E-01	-126E+00	0.125E+00	-989E-01	
Sum_sarc(i):											9.866, Sum_i w_zz(i)*sarc(i), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):	-433E+01	0.233E-07	0.756E+00	0.503E-07	-131E+00

eddy_out.dat Wed Oct 30 16:54:39 1991 1

program eddy_str

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b)=(2.67, 2.67) cm, str = 1.00 cm, h = 2.86 cm, harmonics evaluated at r = 2.50 cm

Output:

!- z-plane harmonic components $IB^*(z)$ (G) for $I=1$ A $-j$ components weighted by $u(i)$ - $\int u/S ds$ (G-cm) $--$
!- at $z(i)e^{(12j\pi/m+1)}$, $j=0,...,m$, and conjugates, $-j$ ($S = 9.40137$),
!----- cf. Eqn. (21) or (43) -----

harmonic component mn+mn (n=0,1,2,...):

1	x(i)	y(i)	sarc(i)	u(i)	v(i)	0	1	2	3	4	0	1	2	3	4
						Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)
1	3.175	0.000	0.111	3.175	0.000	-0.234E+00	-0.393E-01	-0.573E-01	-0.590E-01	-0.501E-01	-0.742E+00	-0.125E+00	-0.182E+00	-0.187E+00	-0.159E+00
2	3.166	0.221	0.221	3.166	0.221	-0.233E+00	-0.383E-01	-0.556E-01	-0.567E-01	-0.473E-01	-0.738E+00	-0.121E+00	-0.176E+00	-0.179E+00	-0.150E+00
3	3.139	0.440	0.221	3.139	0.440	-0.232E+00	-0.354E-01	-0.505E-01	-0.498E-01	-0.391E-01	-0.729E+00	-0.111E+00	-0.159E+00	-0.156E+00	-0.123E+00
4	3.093	0.657	0.221	3.093	0.657	-0.231E+00	-0.306E-01	-0.422E-01	-0.388E-01	-0.264E-01	-0.714E+00	-0.946E-01	-0.131E+00	-0.120E+00	-0.817E-01
5	3.030	0.869	0.221	3.030	0.869	-0.229E+00	-0.238E-01	-0.308E-01	-0.243E-01	-0.104E-01	-0.693E+00	-0.722E-01	-0.934E-01	-0.738E-01	-0.316E-01
6	2.950	1.075	0.221	2.950	1.075	-0.226E+00	-0.152E-01	-0.167E-01	-0.114E-02	0.723E-02	-0.668E+00	-0.447E-01	-0.491E-01	-0.211E-01	0.213E-01
7	2.853	1.273	0.221	2.853	1.273	-0.222E+00	-0.457E-02	-0.376E-05	0.117E-01	0.246E-01	-0.634E+00	-0.130E-01	-0.107E-04	0.335E-01	0.702E-01
8	2.739	1.463	0.221	2.739	1.463	-0.218E+00	-0.795E-02	-0.187E-01	0.311E-01	0.397E-01	-0.597E+00	0.219E-01	0.513E-01	0.851E-01	0.109E+00
9	2.611	1.643	0.221	2.611	1.643	-0.213E+00	-0.224E-01	0.390E-01	0.496E-01	0.504E-01	-0.555E+00	0.585E-01	0.102E+00	0.129E+00	0.132E+00
10	2.468	1.812	0.221	2.468	1.812	-0.207E+00	-0.388E-01	0.603E-01	0.658E-01	0.550E-01	-0.510E+00	0.957E-01	0.149E+00	0.162E+00	0.136E+00
11	2.312	1.968	0.221	2.312	1.968	-0.200E+00	-0.571E-01	0.821E-01	0.783E-01	0.520E-01	-0.461E+00	0.137E+00	0.190E+00	0.181E+00	0.120E+00
12	2.143	2.111	0.221	2.143	2.111	-0.191E+00	-0.775E-01	0.103E+00	0.857E-01	0.405E-01	-0.410E+00	0.168E+00	0.222E+00	0.184E+00	0.868E-01
13	1.963	2.239	0.221	1.963	2.239	-0.182E+00	-0.999E-01	0.124E+00	0.864E-01	0.205E-01	-0.357E+00	0.196E+00	0.243E+00	0.170E+00	0.402E-01
14	1.773	2.353	0.221	1.773	2.353	-0.170E+00	-0.124E+00	0.142E+00	0.792E-01	-0.729E-02	-0.302E+00	0.220E+00	0.251E+00	0.140E+00	0.129E-01
15	1.575	2.450	0.221	1.575	2.450	-0.158E+00	-0.151E+00	0.156E+00	0.629E-01	-0.409E-01	-0.248E+00	0.237E+00	0.246E+00	0.990E-01	-0.645E-01
16	1.369	2.530	0.221	1.369	2.530	-0.143E+00	-0.178E+00	0.165E+00	0.366E-01	-0.771E-01	-0.195E+00	0.244E+00	0.226E+00	0.501E-01	-0.106E+00
17	1.157	2.593	0.221	1.157	2.593	-0.125E+00	-0.207E+00	0.167E+00	0.233E-03	-0.111E+00	-0.145E+00	0.239E+00	0.193E+00	0.269E-03	-0.128E+00
18	0.940	2.639	0.221	0.940	2.639	-0.106E+00	-0.235E+00	0.158E+00	-0.446E-01	-0.135E+00	-0.992E-01	0.221E+00	0.199E+00	-0.20E-01	-0.127E+00
19	0.721	2.666	0.221	0.721	2.666	-0.835E-01	-0.261E+00	0.139E+00	-0.941E-01	-0.142E+00	-0.602E-01	0.188E+00	0.100E+00	-0.679E-01	-0.102E+00
20	0.500	2.675	0.236	0.500	2.675	-0.594E-01	-0.282E+00	0.107E+00	-0.142E+00	-0.124E+00	-0.297E-01	0.141E+00	0.534E-01	-0.709E-01	-0.622E-01
21	0.250	2.675	0.250	0.250	2.675	-0.303E-01	-0.299E+00	0.577E-01	-0.182E+00	-0.736E-01	-0.757E-02	0.747E-01	0.144E-01	-0.456E-01	-0.184E-01
22	0.000	2.675	0.250	0.000	2.675	0.200E-07	0.305E+00	-0.390E-07	-0.198E+00	0.517E-07	-0.783E-14	-0.119E-06	0.153E-13	0.774E-07	-0.202E-13
Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff:															
23	-0.250	2.675	0.250	-0.250	2.675	0.303E-01	0.299E+00	-0.577E-01	-0.182E+00	0.736E-01	-0.757E-02	-0.747E-01	0.144E-01	0.456E-01	-0.184E-01
24	-0.500	2.675	0.236	-0.500	2.675	0.594E-01	0.282E+00	-0.107E+00	-0.142E+00	0.124E+00	-0.297E-01	-0.141E+00	0.534E-01	0.709E-01	-0.622E-01
25	-0.721	2.666	0.221	-0.721	2.666	0.835E-01	0.261E+00	-0.139E+00	-0.941E-01	0.142E+00	-0.602E-01	-0.188E+00	0.100E+00	0.679E-01	-0.102E+00
26	-0.940	2.639	0.221	-0.940	2.639	0.106E+00	-0.235E+00	0.158E+00	-0.446E-01	0.135E+00	-0.992E-01	-0.221E+00	0.149E+00	0.420E-01	-0.127E+00
27	-1.157	2.593	0.221	-1.157	2.593	0.125E+00	-0.207E+00	0.167E+00	0.233E-03	0.111E+00	-0.145E+00	-0.239E+00	0.193E+00	-0.269E-03	-0.128E+00
28	-1.369	2.530	0.221	-1.369	2.530	0.143E+00	-0.178E+00	0.158E+00	0.366E-01	0.771E-01	-0.195E+00	-0.244E+00	0.226E+00	-0.501E-01	-0.106E+00
29	-1.575	2.450	0.221	-1.575	2.450	0.158E+00	-0.151E+00	0.156E+00	0.629E-01	-0.409E-01	-0.248E+00	-0.237E+00	0.246E+00	-0.990E-01	-0.645E-01
30	-1.773	2.353	0.221	-1.773	2.353	0.170E+00	-0.207E+00	0.167E+00	0.792E-01	0.729E-02	-0.302E+00	-0.220E+00	0.251E+00	-0.90E-01	-0.129E-01
31	-1.963	2.239	0.221	-1.963	2.239	0.182E+00	-0.999E-01	-0.124E+00	0.864E-01	-0.205E-01	-0.357E+00	-0.196E+00	0.243E+00	-0.170E+00	0.402E-01
32	-2.143	2.111	0.221	-2.143	2.111	0.191E+00	-0.775E-01	-0.103E+00	0.857E-01	-0.405E-01	-0.410E+00	-0.166E+00	0.222E+00	-0.184E+00	0.868E-01
33	-2.312	1.968	0.221	-2.312	1.968	0.200E+00	-0.571E-01	-0.821E-01	0.783E-01	-0.520E-01	-0.461E+00	-0.132E+00	0.190E+00	-0.181E+00	0.120E+00
34	-2.468	1.812	0.221	-2.468	1.812	0.207E+00	-0.388E-01	-0.603E-01	0.658E-01	-0.550E-01	-0.510E+00	-0.957E-01	0.149E+00	-0.162E+00	0.136E+00
35	-2.611	1.643	0.221	-2.611	1.643	0.213E+00	-0.224E-01	0.390E-01	0.496E-01	0.504E-01	-0.555E+00	-0.585E-01	0.102E+00	-0.129E+00	0.132E+00
36	-2.739	1.463	0.221	-2.739	1.463	0.218E+00	-0.795E-02	-0.187E-01	0.311E-01	0.397E-01	-0.597E+00	0.219E-01	0.513E-01	0.851E-01	0.109E+00
37	-2.853	1.273	0.221	-2.853	1.273	0.222E+00	-0.457E-02	0.382E-05	0.117E-01	-0.246E-01	-0.634E+00	-0.130E-01	-0.109E-04	-0.335E-01	0.702E-01
38	-2.950	1.075	0.221	-2.950	1.075	0.226E+00	-0.152E-01	0.167E-01	-0.714E-02	0.723E-02	-0.668E+00	0.447E-01	-0.491E-01	0.211E-01	0.213E-01
39	-3.030	0.869	0.221	-3.030	0.869	0.229E+00	-0.238E-01	0.308E-01	-0.243E-01	0.104E-01	-0.693E+00	0.722E-01	-0.934E-01	0.738E-01	-0.316E-01
40	-3.093	0.657	0.221	-3.093	0.657	0.231E+00	-0.306E-01	0.422E-01	-0.388E-01	0.264E-01	-0.714E+00	0.946E-01	-0.131E+00	0.120E+00	-0.817E-01
41	-3.139	0.440	0.221	-3.139	0.440	0.232E+00	-0.354E-01	0.505E-01	-0.498E-01	0.391E-01	-0.729E+00	0.111E+00	-0.159E+00	0.156E+00	-0.123E+00
42	-3.166	0.221	0.221	-3.166	0.221	0.233E+00	-0.383E-01	0.556E-01	-0.567E-01	0.473E-01	-0.738E+00	0.121E+00	-0.176E+00	0.179E+00	-0.150E+00
43	-3.175	0.000	0.111	-3.175	0.000	0.234E+00	-0.393E-01	-0.573E-01	-0.590E-01	-0.501E-01	-0.742E+00	-0.125E+00	-0.182E+00	-0.187E+00	-0.159E+00
Sum_sarc(i):															
						9.401	Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):								
								-0.377E+01				-0.127E-06	0.661E+00	0.782E-07	-0.167E+00

dipole coil positioning
(5 pages)

program eddy

Input: multipole # (0=dipole, l=quad, etc) m = 0 ellipse axes: (a,b) = (4.00, 2.67) cm, h = 2.86 cm, harmonics evaluated at r = 2.50 cm

Output:

!- z-plane harmonic components $B^n(z)$ (G) for $I=1$ A !- components weighted by $u(l)$ - Int's u/s ds (G-cm) !-
!- at $z(l)e^{i(2j\pi/m+1)}$, $j=0,...,m$, and conjugates, -! (S-10.58614),
!- ----- cf. Eqn. (21) or (43) -----! (0.00000)

Car. Equ. (2.2.01 (4.4.1))																					
harmonic component mn+mt (n=0,1,2,...):		0		1		2		3		4		0		1		2		3		4	
i	x(i)	y(i)	sarc(i)	u(i)	v(i)	Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)	
1	4.000	0.000	0.100	4.000	0.000	-0.225E+00	-0.153E-01	-0.215E-01	-0.207E-01	-0.156E-01	-0.901E+00	-0.611E-01	-0.860E-01	-0.827E-01	-0.623E-01	-0.901E+00	-0.611E-01	-0.860E-01	-0.827E-01	-0.623E-01	
2	3.989	0.200	0.201	3.989	0.200	-0.225E+00	-0.151E-01	-0.212E-01	-0.203E-01	-0.152E-01	-0.898E+00	-0.601E-01	-0.845E-01	-0.809E-01	-0.606E-01	-0.898E+00	-0.601E-01	-0.845E-01	-0.809E-01	-0.606E-01	
3	3.955	0.399	0.203	3.955	0.399	-0.225E+00	-0.144E-01	-0.202E-01	-0.191E-01	-0.141E-01	-0.890E+00	-0.571E-01	-0.799E-01	-0.757E-01	-0.557E-01	-0.890E+00	-0.571E-01	-0.799E-01	-0.757E-01	-0.557E-01	
4	3.900	0.595	0.206	3.900	0.595	-0.225E+00	-0.133E-01	-0.184E-01	-0.172E-01	-0.121E-01	-0.876E+00	-0.520E-01	-0.719E-01	-0.669E-01	-0.473E-01	-0.876E+00	-0.520E-01	-0.719E-01	-0.669E-01	-0.473E-01	
5	3.822	0.788	0.211	3.822	0.788	-0.224E+00	-0.116E-01	-0.158E-01	-0.142E-01	-0.927E-02	-0.856E+00	-0.445E-01	-0.605E-01	-0.542E-01	-0.354E-01	-0.856E+00	-0.445E-01	-0.605E-01	-0.542E-01	-0.354E-01	
6	3.723	0.977	0.216	3.723	0.977	-0.223E+00	-0.925E-02	-0.121E-01	-0.101E-01	-0.537E-02	-0.831E+00	-0.344E-01	-0.452E-01	-0.375E-01	-0.200E-01	-0.831E+00	-0.344E-01	-0.452E-01	-0.375E-01	-0.200E-01	
7	3.604	1.161	0.222	3.604	1.161	-0.222E+00	-0.597E-02	-0.714E-02	-0.456E-02	-0.318E-03	-0.800E+00	-0.215E-01	-0.257E-01	-0.164E-01	-0.114E-02	-0.800E+00	-0.215E-01	-0.257E-01	-0.164E-01	-0.114E-02	
8	3.464	1.337	0.229	3.464	1.337	-0.220E+00	-0.154E-02	-0.482E-03	-0.258E-02	-0.595E-02	-0.764E+00	-0.534E-02	-0.167E-02	-0.892E-02	-0.206E-01	-0.764E+00	-0.534E-02	-0.167E-02	-0.892E-02	-0.206E-01	
9	3.305	1.507	0.236	3.305	1.507	-0.218E+00	-0.435E-02	-0.821E-02	-0.116E-01	-0.134E-01	-0.722E+00	-0.144E-01	-0.271E-01	-0.382E-01	-0.441E-01	-0.722E+00	-0.144E-01	-0.271E-01	-0.382E-01	-0.441E-01	
10	3.127	1.668	0.243	3.127	1.668	-0.216E+00	-0.121E-01	-0.194E-01	-0.225E-01	-0.215E-01	-0.674E+00	-0.379E-01	-0.606E-01	-0.704E-01	-0.673E-01	-0.674E+00	-0.379E-01	-0.606E-01	-0.704E-01	-0.673E-01	
11	2.932	1.819	0.251	2.932	1.819	-0.212E+00	-0.223E-01	-0.335E-01	-0.353E-01	-0.297E-01	-0.622E+00	-0.653E-01	-0.981E-01	-0.104E+00	-0.869E-01	-0.622E+00	-0.653E-01	-0.981E-01	-0.104E+00	-0.869E-01	
12	2.721	1.961	0.258	2.721	1.961	-0.207E+00	-0.354E-01	-0.508E-01	-0.435E-01	-0.361E-01	-0.564E+00	-0.964E-01	-0.138E+00	-0.135E+00	-0.982E-01	-0.564E+00	-0.964E-01	-0.138E+00	-0.135E+00	-0.982E-01	
13	2.494	2.091	0.265	2.494	2.091	-0.201E+00	-0.522E-01	-0.715E-01	-0.635E-01	-0.383E-01	-0.501E+00	-0.130E+00	-0.178E+00	-0.158E+00	-0.956E-01	-0.501E+00	-0.130E+00	-0.178E+00	-0.158E+00	-0.956E-01	
14	2.253	2.210	0.272	2.253	2.210	-0.192E+00	-0.734E-01	-0.950E-01	-0.750E-01	-0.328E-01	-0.433E+00	-0.165E+00	-0.214E+00	-0.169E+00	-0.739E-01	-0.433E+00	-0.165E+00	-0.214E+00	-0.169E+00	-0.739E-01	
15	2.000	2.317	0.278	2.000	2.317	-0.181E+00	-0.993E-01	-0.120E+00	-0.801E-01	-0.155E-01	-0.363E+00	-0.199E+00	-0.240E+00	-0.160E+00	-0.310E-01	-0.363E+00	-0.199E+00	-0.240E+00	-0.160E+00	-0.310E-01	
16	1.736	2.410	0.283	1.736	2.410	-0.167E+00	-0.130E+00	-0.144E+00	-0.736E-01	-0.162E-01	-0.290E+00	-0.226E+00	-0.250E+00	-0.128E+00	-0.282E-01	-0.290E+00	-0.226E+00	-0.250E+00	-0.128E+00	-0.282E-01	
17	1.461	2.490	0.288	1.461	2.490	-0.150E+00	-0.166E+00	-0.162E+00	-0.501E-01	-0.608E-01	-0.219E+00	-0.242E+00	-0.237E+00	-0.732E-01	-0.888E-01	-0.219E+00	-0.242E+00	-0.237E+00	-0.732E-01	-0.888E-01	
18	1.179	2.556	0.292	1.179	2.556	-0.128E+00	-0.204E+00	-0.169E+00	-0.642E-02	-0.109E+00	-0.150E+00	-0.240E+00	-0.199E+00	-0.489E-01	-0.126E+00	-0.150E+00	-0.240E+00	-0.199E+00	-0.489E-01	-0.126E+00	
19	0.890	2.608	0.295	0.890	2.608	-0.101E+00	-0.241E+00	-0.157E+00	-0.550E-01	-0.142E+00	-0.900E-01	-0.215E+00	-0.140E+00	-0.757E-02	-0.126E+00	-0.900E-01	-0.215E+00	-0.140E+00	-0.757E-02	-0.126E+00	
20	0.596	2.645	0.297	0.596	2.645	-0.703E-01	-0.274E+00	-0.124E+00	-0.123E+00	-0.138E+00	-0.419E-01	-0.163E+00	-0.737E-01	-0.730E-01	-0.823E-01	-0.419E-01	-0.163E+00	-0.737E-01	-0.730E-01	-0.823E-01	
21	0.299	2.668	0.299	0.299	2.668	-0.361E-01	-0.297E+00	-0.685E-01	-0.177E+00	-0.865E-01	-0.108E-01	-0.887E-01	-0.205E-01	-0.528E-01	-0.259E-01	-0.108E-01	-0.887E-01	-0.205E-01	-0.528E-01	-0.259E-01	
22	0.000	2.675	0.299	0.000	2.675	-0.267E-07	-0.305E+00	-0.521E-07	-0.198E+00	-0.689E-07	-0.233E-15	-0.267E-08	-0.455E-15	-0.173E-08	-0.603E-15	-0.233E-15	-0.267E-08	-0.455E-15	-0.173E-08	-0.603E-15	
Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff:											-0.591E-01	0.467E+00	0.434E+00	0.152E+00	-0.591E-01						
23	-0.299	2.668	0.299	-0.299	2.668	0.361E-01	-0.297E+00	-0.685E-01	-0.177E+00	-0.865E-01	-0.108E-01	-0.887E-01	-0.205E-01	-0.528E-01	-0.259E-01	-0.108E-01	-0.887E-01	-0.205E-01	-0.528E-01	-0.259E-01	
24	-0.596	2.645	0.297	-0.596	2.645	0.703E-01	-0.274E+00	-0.124E+00	-0.123E+00	-0.138E+00	-0.419E-01	-0.163E+00	-0.737E-01	-0.730E-01	-0.823E-01	-0.419E-01	-0.163E+00	-0.737E-01	-0.730E-01	-0.823E-01	
25	-0.890	2.608	0.295	-0.890	2.608	0.101E+00	-0.241E+00	-0.157E+00	-0.550E-01	-0.142E+00	-0.900E-01	-0.215E+00	-0.140E+00	-0.757E-02	-0.126E+00	-0.900E-01	-0.215E+00	-0.140E+00	-0.757E-02	-0.126E+00	
26	-1.179	2.556	0.292	-1.179	2.556	0.128E+00	-0.204E+00	-0.169E+00	-0.642E-02	-0.109E+00	-0.150E+00	-0.240E+00	-0.199E+00	-0.489E-01	-0.126E+00	-0.150E+00	-0.240E+00	-0.199E+00	-0.489E-01	-0.126E+00	
27	-1.461	2.490	0.288	-1.461	2.490	0.150E+00	-0.166E+00	-0.162E+00	-0.501E-01	-0.608E-01	-0.219E+00	-0.242E+00	-0.237E+00	-0.732E-01	-0.888E-01	-0.219E+00	-0.242E+00	-0.237E+00	-0.732E-01	-0.888E-01	
28	-1.736	2.410	0.283	-1.736	2.410	0.167E+00	-0.130E+00	-0.144E+00	-0.736E-01	-0.162E-01	-0.290E+00	-0.226E+00	-0.250E+00	-0.128E+00	-0.282E-01	-0.290E+00	-0.226E+00	-0.250E+00	-0.128E+00	-0.282E-01	
29	-2.000	2.317	0.278	-2.000	2.317	0.181E+00	-0.993E-01	-0.120E+00	-0.801E-01	-0.155E-01	-0.363E+00	-0.199E+00	-0.240E+00	-0.160E+00	-0.310E-01	-0.363E+00	-0.199E+00	-0.240E+00	-0.160E+00	-0.310E-01	
30	-2.253	2.210	0.272	-2.253	2.210	0.192E+00	-0.734E-01	-0.950E-01	-0.750E-01	-0.328E-01	-0.433E+00	-0.165E+00	-0.214E+00	-0.169E+00	-0.739E-01	-0.433E+00	-0.165E+00	-0.214E+00	-0.169E+00	-0.739E-01	
31	-2.494	2.091	0.265	-2.494	2.091	0.201E+00	-0.522E-01	-0.715E-01	-0.635E-01	-0.383E-01	-0.501E+00	-0.130E+00	-0.178E+00	-0.158E+00	-0.956E-01	-0.501E+00	-0.130E+00	-0.178E+00	-0.158E+00	-0.956E-01	
32	-2.721	1.961	0.258	-2.721	1.961	0.207E+00	-0.354E-01	-0.508E-01	-0.435E-01	-0.361E-01	-0.564E+00	-0.964E-01	-0.138E+00	-0.135E+00	-0.982E-01	-0.564E+00	-0.964E-01	-0.138E+00	-0.135E+00	-0.982E-01	
33	-2.932	1.819	0.251	-2.932	1.819	0.212E+00	-0.223E-01	-0.335E-01	-0.353E-01	-0.297E-01	-0.622E+00	-0.653E-01	-0.981E-01	-0.104E+00	-0.869E-01	-0.622E+00	-0.653E-01	-0.981E-01	-0.104E+00	-0.869E-01	
34	-3.127	1.668	0.243	-3.127	1.668	0.216E+00	-0.121E-01	-0.194E-01	-0.225E-01	-0.215E-01	-0.674E+00	-0.379E-01	-0.606E-01	-0.704E-01	-0.673E-01	-0.674E+00	-0.379E-01	-0.606E-01	-0.704E-01	-0.673E-01	
35	-3.305	1.507	0.236	-3.305	1.507	0.218E+00	-0.435E-02	-0.821E-02	-0.116E-01	-0.134E-01	-0.722E+00	-0.144E-01	-0.271E-01	-0.382E-01	-0.441E-01	-0.722E+00	-0.144E-01	-0.271E-01	-0.382E-01	-0.441E-01	
36	-3.464	1.338	0.229	-3.464	1.338	0.220E+00	-0.154E-02	-0.482E-03	-0.258E-02	-0.595E-02	-0.764E+00	-0.534E-02	-0.167E-02	-0.892E-02	-0.206E-01	-0.764E+00	-0.534E-02	-0.167E-02	-0.892E-02	-0.206E-01	
37	-3.604	1.161	0.222	-3.604	1.161	0.222E+00	-0.597E-02	-0.714E-02	-0.456E-02	-0.318E-03	-0.800E+00	-0.215E-01	-0.257E-01	-0.164E-01	-0.114E-02	-0.800E+00	-0.215E-01	-0.257E-01	-0.164E-01	-0.114E-02	
38	-3.723	0.977	0.216	-3.723	0.977	0.223E+00	-0.925E-02	-0.121E-01	-0.101E-01	-0.537E-02	-0.831E+00	-0.344E-01	-0.452E-01	-0.375E-01	-0.200E-01	-0.831E+00	-0.344E-01	-0.452E-01	-0.375E-01	-0.200E-01	
39	-3.822	0.788	0.211	-3.822	0.788	0.224E+00	-0.116E-01	-0.158E-01	-0.142E-01	-0.927E-02	-0.856E+00	-0.445E-01	-0.605E-01	-0.542E-01	-0.354E-01	-0.856E+00	-0.445E-01	-0.605E-01	-0.542E-01	-0.354E-01	
40	-3.900	0.595	0.206	-3.900	0.595																

eddy_out.dat Thu Aug 22 13:29:36 1991 1
program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (4.50, 2.67) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:
!- z-plane harmonic components $1B^*(z)$ (G) for $I=1$ A --- components weighted by $u(i)$ - Int_s u/S ds (G-cm) ---
!- at $z(1)e^{i(2j\pi/m+1)}$, $j=0, \dots, m$, and conjugates, - (S=11.45083),
!----- cf. Eqn. (21) or (43) -----

harmonic component mn+mn (n=0,1,2,...):																
i	x(i)	y(i)	sarc(i)	u(i)	v(i)	Fn(i)	Sc(i)	Td(i)	Ff(i)	Fr(i)	Ff(i)	0	1	2	3	4

eddy_out.dat

Thu Aug 22 13:29:48 1991

1

Program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (5.00, 2.67) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:

!- z-plane harmonic components IB*(z) (G) for I=1 A !- components weighted by u(1) - Int_s u/S ds (G-cm) --!
!- at z(1)e^(i2j pi/mt+1), j=0,...,m, and conjugates, -! (S=12.33120),
!----- cf. Eqn. (21) or (43) -----! (0.00000)

harmonic component mnt+m (n=0,1,2,...):																			
i	x(1)	y(1)	sarc(1)	u(1)	v(1)	Fn(1)	Sc(1)	Td(1)	Fr(1)	Ff(1)	0	Fn(1)	w_Sc(1)	2	w_Td(1)	3	w_Fr(1)	4	w_Ff(1)
1	5.000	0.000	0.100	5.000	0.000	-222E+00	-501E-02	-594E-02	-645E-02	-458E-02	-111E+01	-111E+01	-251E-01	-347E-01	-323E-01	-323E-01	-229E-01		
2	4.986	0.200	0.202	4.986	0.200	-221E+00	-496E-02	-687E-02	-638E-02	-452E-02	-110E+01	-110E+01	-247E-01	-342E-01	-318E-01	-318E-01	-225E-01		
3	4.944	0.399	0.206	4.944	0.399	-221E+00	-481E-02	-665E-02	-616E-02	-434E-02	-109E+01	-109E+01	-238E-01	-329E-01	-305E-01	-305E-01	-214E-01		
4	4.875	0.595	0.212	4.875	0.595	-221E+00	-454E-02	-625E-02	-576E-02	-400E-02	-108E+01	-108E+01	-221E-01	-305E-01	-281E-01	-281E-01	-195E-01		
5	4.778	0.788	0.221	4.778	0.788	-221E+00	-410E-02	-561E-02	-511E-02	-346E-02	-106E+01	-106E+01	-196E-01	-244E-01	-224E-01	-224E-01	-165E-01		
6	4.654	0.977	0.231	4.654	0.977	-221E+00	-342E-02	-462E-02	-411E-02	-264E-02	-103E+01	-103E+01	-159E-01	-215E-01	-191E-01	-191E-01	-123E-01		
7	4.505	1.161	0.243	4.505	1.161	-221E+00	-239E-02	-315E-02	-262E-02	-143E-02	-994E+00	-994E+00	-108E-01	-142E-01	-118E-01	-118E-01	-64E-02		
8	4.330	1.337	0.255	4.330	1.337	-220E+00	-137E-03	-162E-03	-137E-03	-87E-03	-953E+00	-953E+00	-380E-02	-417E-02	-189E-02	-189E-02	-135E-02		
9	4.131	1.507	0.268	4.131	1.507	-219E+00	0.136E-02	-224E-02	0.272E-02	0.279E-02	-906E+00	-906E+00	0.562E-02	0.924E-02	0.112E-01	0.112E-01	0.115E-01		
10	3.909	1.668	0.281	3.909	1.668	-218E+00	0.464E-02	0.688E-02	0.723E-02	0.820E-02	-852E+00	-852E+00	0.181E-01	0.269E-01	0.283E-01	0.283E-01	0.243E-01		
11	3.665	1.819	0.294	3.665	1.819	-216E+00	0.942E-02	0.136E-01	0.135E-01	0.107E-01	-793E+00	-793E+00	0.345E-01	0.497E-01	0.496E-01	0.496E-01	0.393E-01		
12	3.401	1.961	0.306	3.401	1.961	-214E+00	0.163E-01	0.231E-01	0.221E-01	0.163E-01	-727E+00	-727E+00	0.556E-01	0.784E-01	0.784E-01	0.784E-01	0.556E-01		
13	3.117	2.091	0.318	3.117	2.091	-210E+00	0.263E-01	0.363E-01	0.333E-01	0.225E-01	-655E+00	-655E+00	0.820E-01	0.113E+00	0.104E+00	0.104E+00	0.701E-01		
14	2.817	2.210	0.329	2.817	2.210	-205E+00	0.405E-01	0.542E-01	0.467E-01	0.275E-01	-577E+00	-577E+00	0.114E+00	0.113E+00	0.132E+00	0.132E+00	0.775E-01		
15	2.500	2.317	0.339	2.500	2.317	-197E+00	0.602E-01	0.773E-01	0.608E-01	0.278E-01	-493E+00	-493E+00	0.151E+00	0.193E+00	0.152E+00	0.152E+00	0.695E-01		
16	2.169	2.410	0.348	2.169	2.410	-186E+00	0.871E-01	0.105E+00	0.712E-01	0.173E-01	-404E+00	-404E+00	0.189E+00	0.228E+00	0.154E+00	0.154E+00	0.376E-01		
17	1.827	2.490	0.355	1.827	2.490	-171E+00	0.122E+00	0.135E+00	0.700E-01	-112E-01	-311E+00	-311E+00	0.223E+00	0.246E+00	0.128E+00	0.128E+00	-205E-01		
18	1.474	2.556	0.362	1.474	2.556	-149E+00	0.165E+00	0.159E+00	0.462E-01	-604E-01	-220E+00	-220E+00	0.243E+00	0.234E+00	0.681E-01	0.681E-01	-890E-01		
19	1.113	2.608	0.367	1.113	2.608	-121E+00	0.213E+00	0.165E+00	-857E-02	-117E+00	-135E+00	-135E+00	0.236E+00	0.184E+00	-953E-02	-953E-02	-130E+00		
20	0.745	2.645	0.371	0.745	2.645	-86E+00	0.258E+00	0.142E+00	-887E-01	-144E+00	-643E-01	-643E-01	0.192E+00	0.106E+00	-661E-01	-661E-01	-107E+00		
21	0.374	2.668	0.373	0.374	2.668	-44E+00	0.292E+00	0.838E-01	-165E+00	-103E+00	-168E-01	-168E-01	0.109E+00	0.313E-01	-618E-01	-618E-01	-385E-01		
22	0.000	2.675	0.374	0.000	2.675	-26E+00	0.267E+00	0.521E-07	-198E+00	0.689E-07	-183E-14	-183E-14	0.209E-07	0.358E-14	0.136E-07	0.136E-07	-229E-01		
Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff:																			
23	-0.374	2.668	0.373	-0.374	2.668	0.449E-01	0.292E+00	-838E-01	-165E+00	0.103E+00	-168E-01	-168E-01	-109E+00	0.313E-01	0.618E-01	0.618E-01	-386E-01		
24	-0.745	2.645	0.371	-0.745	2.645	0.862E-01	0.258E+00	-142E+00	-887E-01	0.144E+00	-643E-01	-643E-01	-192E+00	0.106E+00	0.953E-02	0.953E-02	-107E+00		
25	-1.113	2.608	0.367	-1.113	2.608	0.121E+00	0.213E+00	-165E+00	-856E-02	0.117E+00	-135E+00	-135E+00	-236E+00	0.184E+00	0.953E-02	0.953E-02	-130E+00		
26	-1.474	2.556	0.362	-1.474	2.556	0.149E+00	0.165E+00	-159E+00	-462E-01	0.604E-01	-220E+00	-220E+00	-243E+00	0.234E+00	-681E-01	-681E-01	-890E-01		
27	-1.827	2.490	0.355	-1.827	2.490	0.171E+00	0.122E+00	-135E+00	0.700E-01	0.112E-01	-311E+00	-311E+00	-223E+00	0.246E+00	-128E+00	-128E+00	-205E-01		
28	-2.169	2.410	0.348	-2.169	2.410	0.186E+00	0.871E-01	-105E+00	0.712E-01	-173E-01	-404E+00	-404E+00	-189E+00	0.228E+00	-154E+00	-154E+00	-376E-01		
29	-2.500	2.317	0.339	-2.500	2.317	0.197E+00	0.602E-01	-773E-01	0.608E-01	-278E-01	-493E+00	-493E+00	-151E+00	0.193E+00	-152E+00	-152E+00	0.695E-01		
30	-2.817	2.210	0.329	-2.817	2.210	0.205E+00	0.405E-01	-542E-01	0.467E-01	-275E-01	-577E+00	-577E+00	-114E+00	0.153E+00	-132E+00	-132E+00	0.775E-01		
31	-3.117	2.091	0.318	-3.117	2.091	0.210E+00	0.263E-01	-363E-01	0.333E-01	-225E-01	-655E+00	-655E+00	-820E-01	0.113E+00	-104E+00	-104E+00	0.701E-01		
32	-3.401	1.961	0.306	-3.401	1.961	0.214E+00	0.163E-01	-231E-01	0.221E-01	-163E-01	-727E+00	-727E+00	-556E-01	0.784E-01	-752E-01	-752E-01	0.556E-01		
33	-3.665	1.819	0.294	-3.665	1.819	0.216E+00	0.942E-02	-136E-02	0.135E-01	-107E-01	-793E+00	-793E+00	-345E-01	0.497E-01	-496E-01	-496E-01	0.393E-01		
34	-3.909	1.668	0.281	-3.909	1.668	0.218E+00	0.464E-02	-688E-02	0.723E-02	-852E-02	-906E+00	-906E+00	-181E-01	0.269E-01	-283E-01	-283E-01	0.243E-01		
35	-4.131	1.507	0.268	-4.131	1.507	0.219E+00	0.136E-02	-224E-02	0.272E-02	-279E-02	-953E+00	-953E+00	-380E-02	0.924E-02	-112E-01	-112E-01	0.115E-01		
36	-4.330	1.337	0.255	-4.330	1.337	0.220E+00	0.877E-03	-162E-03	-137E-03	-87E-03	-953E+00	-953E+00	-108E-01	-142E-01	0.118E-01	0.118E-01	-64E-02		
37	-4.505	1.161	0.243	-4.505	1.161	0.221E+00	-239E-02	-315E-02	-262E-02	-143E-02	-994E+00	-994E+00	-108E-01	-142E-01	0.118E-01	0.118E-01	-64E-02		
38	-4.654	0.977	0.231	-4.654	0.977	0.221E+00	-342E-02	-462E-02	-411E-02	-264E-02	-103E+01	-103E+01	-159E-01	-215E-01	0.191E-01	0.191E-01	-123E-01		
39	-4.778	0.788	0.221	-4.778	0.788	0.221E+00	-410E-02	-561E-02	-511E-02	-346E-02	-106E+01	-106E+01	-196E-01	-244E-01	-244E-01	-244E-01	-165E-01		
40	-4.875	0.595	0.212	-4.875	0.595	0.221E+00	-454E-02	-625E-02	-576E-02	-400E-02	-108E+01	-108E+01	-221E-01	-305E-01	0.281E-01	0.281E-01	-195E-01		
41	-4.944	0.399	0.206	-4.944	0.399	0.221E+00	-481E-02	-665E-02	-616E-02	-434E-02	-109E+01	-109E+01	-238E-01	-329E-01	0.305E-01	0.305E-01	-214E-01		
42	-4.986	0.200	0.202	-4.986	0.200	0.221E+00	-496E-02	-687E-02	-638E-02	-452E-02	-110E+01	-110E+01	-247E-01	-342E-01	0.318E-01	0.318E-01	-225E-01		
43	-5.000	0.000	0.100	-5.000	0.000	0.222E+00	-501E-02	-594E-02	-645E-02	-458E-02	-111E+01	-111E+01	-251E-01	-347E-01	0.323E-01	0.323E-01	-229E-01		
Sum_sarc(i): 12.331, Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):																			
-731E+01 0.265E-06 0.106E+01 0.577E-07 -830E-01																			

eddy_out.dat Thu Aug 22 13:30:12 1991 1

program eddy

Input: multipole 0 (0-dipole, l=quad, etc) m = 0 ellipse axes: (a,b) = (5.50, 2.67) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:

!- z-plane harmonic components $1B^*(z)$ (G) for $I=1$ A !- components weighted by $u(i)$ - $\int u/s \, ds$ (G-cm) --!
!- at $z(i)e^{i(2j \pi/m + 1)}$, $j=0, \dots, m$, and conjugates, -!
!----- cf. Eqn. (21) or (43) -----!
[0.00000]

harmonic component $m \pm n$ (n=0,1,2,...):

i	x(i)	y(i)	sarc(i)	u(i)	v(i)	Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)
1	5.500	0.000	0.100	5.500	0.000	-0.221E+00	-0.288E-02	-0.398E-02	-0.368E-02	-0.257E-02	-0.121E+01	-0.159E-01	-0.219E-01	-0.202E-01	-0.141E-01
2	5.485	0.200	0.202	5.485	0.200	-0.221E+00	-0.286E-02	-0.395E-02	-0.364E-02	-0.253E-02	-0.121E+01	-0.157E-01	-0.216E-01	-0.200E-01	-0.140E-01
3	5.439	0.399	0.207	5.439	0.399	-0.221E+00	-0.279E-02	-0.384E-02	-0.354E-02	-0.247E-02	-0.120E+01	-0.152E-01	-0.209E-01	-0.193E-01	-0.134E-01
4	5.362	0.595	0.216	5.362	0.595	-0.221E+00	-0.265E-02	-0.365E-02	-0.335E-02	-0.232E-02	-0.119E+01	-0.142E-01	-0.196E-01	-0.180E-01	-0.124E-01
5	5.256	0.788	0.227	5.256	0.788	-0.221E+00	-0.243E-02	-0.333E-02	-0.304E-02	-0.207E-02	-0.118E+01	-0.128E-01	-0.175E-01	-0.160E-01	-0.109E-01
6	5.120	0.977	0.239	5.120	0.977	-0.220E+00	-0.206E-02	-0.281E-02	-0.252E-02	-0.167E-02	-0.113E+01	-0.106E-01	-0.144E-01	-0.129E-01	-0.856E-02
7	4.955	1.161	0.254	4.955	1.161	-0.220E+00	-0.148E-02	-0.199E-02	-0.172E-02	-0.103E-02	-0.103E+01	-0.735E-02	-0.984E-02	-0.853E-02	-0.520E-02
8	4.763	1.337	0.269	4.763	1.337	-0.220E+00	-0.586E-03	-0.712E-03	-0.481E-03	-0.949E-04	-0.103E+01	-0.279E-02	-0.339E-02	-0.229E-02	-0.452E-03
9	4.544	1.507	0.285	4.544	1.507	-0.219E+00	0.801E-03	0.125E-02	0.142E-02	0.133E-02	-0.997E+00	0.364E-02	0.568E-02	0.644E-02	0.612E-02
10	4.300	1.668	0.300	4.300	1.668	-0.219E+00	0.293E-02	0.425E-02	0.429E-02	0.349E-02	-0.940E+00	0.126E-01	0.183E-01	0.184E-01	0.150E-01
11	4.032	1.819	0.316	4.032	1.819	-0.217E+00	0.619E-02	0.879E-02	0.857E-02	0.657E-02	-0.877E+00	0.250E-01	0.355E-01	0.346E-01	0.265E-01
12	3.741	1.961	0.331	3.741	1.961	-0.216E+00	0.112E-01	0.156E-01	0.148E-01	0.108E-01	-0.807E+00	0.417E-01	0.584E-01	0.555E-01	0.405E-01
13	3.429	2.091	0.345	3.429	2.091	-0.213E+00	0.187E-01	0.257E-01	0.237E-01	0.163E-01	-0.730E+00	0.640E-01	0.882E-01	0.812E-01	0.557E-01
14	3.098	2.210	0.358	3.098	2.210	-0.209E+00	0.299E-01	0.403E-01	0.355E-01	0.222E-01	-0.647E+00	0.926E-01	0.125E+00	0.110E+00	0.687E-01
15	2.750	2.317	0.370	2.750	2.317	-0.202E+00	0.465E-01	0.607E-01	0.499E-01	0.262E-01	-0.557E+00	0.128E+00	0.167E+00	0.137E+00	0.721E-01
16	2.386	2.410	0.380	2.386	2.410	-0.193E+00	0.704E-01	0.875E-01	0.640E-01	0.231E-01	-0.460E+00	0.168E+00	0.209E+00	0.153E+00	0.552E-01
17	2.009	2.490	0.390	2.009	2.490	-0.179E+00	0.104E+00	0.119E+00	0.709E-01	0.419E-02	-0.359E+00	0.208E+00	0.239E+00	0.142E+00	0.842E-02
18	1.621	2.556	0.397	1.621	2.556	-0.158E+00	0.147E+00	0.149E+00	0.578E-01	-0.389E-01	-0.257E+00	0.238E+00	0.242E+00	0.938E-01	-0.630E-01
19	1.224	2.608	0.403	1.224	2.608	-0.130E+00	0.198E+00	0.165E+00	0.107E-01	-0.108E+00	-0.160E+00	0.242E+00	0.202E+00	0.938E-01	-0.630E-01
20	0.820	2.645	0.408	0.820	2.645	-0.938E-01	0.250E+00	0.150E+00	-0.716E-01	-0.142E+00	-0.769E+00	0.205E+00	0.123E+00	-0.587E-01	-0.117E+00
21	0.411	2.668	0.410	0.411	2.668	-0.491E-01	0.290E+00	0.911E-01	-0.159E+00	-0.112E+00	-0.203E-01	0.119E+00	0.374E-01	-0.653E-01	-0.455E-01
22	0.000	2.675	0.411	0.000	2.675	0.267E-07	0.303E+00	-0.521E-07	-0.198E+00	0.689E-07	-0.133E-13	-0.152E-06	0.259E-13	0.982E-07	-0.343E-13
Sum_i w_zz(i)*sarc(i), where zz=Fn,Sc,Td,Fr,Ff:															
23	-0.411	2.668	0.410	-0.411	2.668	0.493E-01	0.290E+00	-0.911E-01	-0.159E+00	0.111E+00	-0.203E-01	-0.119E+00	0.374E-01	0.653E-01	-0.455E-01
24	-0.820	2.645	0.408	-0.820	2.645	0.938E-01	0.250E+00	-0.150E+00	-0.716E-01	0.142E+00	-0.769E-01	-0.205E+00	0.123E+00	0.587E-01	-0.117E+00
25	-1.224	2.608	0.403	-1.224	2.608	0.130E+00	0.198E+00	-0.165E+00	0.107E-01	0.108E+00	-0.160E+00	-0.242E+00	0.202E+00	-0.938E-01	-0.630E-01
26	-1.621	2.556	0.397	-1.621	2.556	0.158E+00	0.147E+00	-0.149E+00	0.578E-01	-0.389E-01	-0.257E+00	-0.238E+00	0.242E+00	-0.938E-01	-0.630E-01
27	-2.009	2.490	0.390	-2.009	2.490	0.179E+00	0.104E+00	-0.119E+00	0.709E-01	-0.419E-02	-0.359E+00	-0.208E+00	0.239E+00	-0.142E+00	0.842E-02
28	-2.386	2.410	0.380	-2.386	2.410	0.193E+00	0.704E-01	-0.875E-01	0.640E-01	-0.231E-01	-0.460E+00	-0.168E+00	0.209E+00	-0.153E+00	0.552E-01
29	-2.750	2.317	0.370	-2.750	2.317	0.202E+00	0.465E-01	-0.607E-01	0.499E-01	-0.262E-01	-0.557E+00	-0.128E+00	0.167E+00	-0.137E+00	0.721E-01
30	-3.098	2.210	0.358	-3.098	2.210	0.209E+00	0.299E-01	-0.403E-01	0.355E-01	-0.222E-01	-0.647E+00	-0.926E-01	0.125E+00	-0.110E+00	0.687E-01
31	-3.429	2.091	0.345	-3.429	2.091	0.213E+00	0.187E-01	-0.257E-01	0.237E-01	-0.163E-01	-0.730E+00	-0.926E-01	0.125E+00	-0.110E+00	0.687E-01
32	-3.741	1.961	0.331	-3.741	1.961	0.216E+00	0.112E-01	-0.156E-01	0.148E-01	-0.108E-01	-0.807E+00	-0.417E-01	0.584E-01	-0.555E-01	0.405E-01
33	-4.032	1.819	0.316	-4.032	1.819	0.213E+00	0.619E-02	-0.879E-02	0.857E-02	-0.657E-02	-0.877E+00	-0.250E-01	0.355E-01	-0.346E-01	0.265E-01
34	-4.300	1.668	0.285	-4.300	1.668	0.219E+00	0.293E-02	-0.425E-02	0.429E-02	-0.349E-02	-0.940E+00	-0.126E-01	0.183E-01	-0.184E-01	0.150E-01
35	-4.544	1.507	0.269	-4.544	1.507	0.220E+00	0.801E-03	0.125E-02	0.142E-02	0.133E-02	-0.997E+00	0.364E-02	0.568E-02	0.644E-02	0.612E-02
36	-4.763	1.337	0.254	-4.763	1.337	0.220E+00	-0.586E-03	0.712E-03	-0.481E-03	-0.949E-04	-0.103E+01	-0.279E-02	-0.339E-02	0.229E-02	-0.452E-03
37	-4.955	1.161	0.254	-4.955	1.161	0.220E+00	-0.148E-02	-0.199E-02	-0.172E-02	-0.103E-02	-0.103E+01	-0.735E-02	-0.984E-02	0.853E-02	-0.520E-02
38	-5.120	0.977	0.239	-5.120	0.977	0.220E+00	-0.206E-02	-0.281E-02	-0.252E-02	-0.167E-02	-0.113E+01	-0.106E-01	-0.144E-01	0.129E-01	-0.856E-02
39	-5.256	0.788	0.227	-5.256	0.788	0.221E+00	-0.243E-02	-0.333E-02	-0.304E-02	-0.207E-02	-0.118E+01	-0.128E-01	-0.175E-01	0.160E-01	-0.109E-01
40	-5.362	0.595	0.216	-5.362	0.595	0.221E+00	-0.265E-02	-0.365E-02	-0.335E-02	-0.232E-02	-0.119E+01	-0.142E-01	-0.196E-01	0.180E-01	-0.124E-01
41	-5.439	0.399	0.207	-5.439	0.399	0.221E+00	-0.279E-02	-0.384E-02	-0.354E-02	-0.247E-02	-0.120E+01	-0.152E-01	-0.209E-01	0.193E-01	-0.134E-01
42	-5.485	0.200	0.202	-5.485	0.200	0.221E+00	-0.286E-02	-0.395E-02	-0.364E-02	-0.255E-02	-0.121E+01	-0.157E-01	-0.216E-01	0.200E-01	-0.140E-01
43	-5.500	0.000	0.100	-5.500	0.000	0.221E+00	-0.288E-02	-0.398E-02	-0.368E-02	-0.257E-02	-0.121E+01	-0.159E-01	-0.219E-01	0.202E-01	-0.141E-01
Sum_sarc(i): 13.224, Sum_i w_zz(i)*sarc(i), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):															
												-0.856E+01	-0.285E-06	0.113E+01	0.389E-07
															-0.670E-01

Program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (6.00, 2.67) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:

harmonic component mn+m+n (n=0,1,2,...):
 1 - z-plane harmonic components $1B^*(z)$ (G) for $l=1$ A - components weighted by $u(l)$ - $\int_{-s}^s u/S ds$ (G-cm) ---
 2 - at $z(l)e^{i(2j\pi/m+l)}$, $j=0, \dots, m$, and conjugates, - (S = 14.12860),
 3 ----- cf. Eqn. (21) or (43) -----
 4

i	x(l)	y(l)	sarcl(l)	u(l)	v(l)	Fn(l)	Sc(l)	Td(l)	Fr(l)	Ff(l)	0	1	2	3	4
1	6.000	0.000	0.100	6.000	0.000	-0.220E+00	-0.166E-02	-0.229E-02	-0.211E-02	-0.146E-02	-0.132E+01	-0.997E-02	-0.137E-01	-0.126E-01	-0.877E-02
2	5.983	0.200	0.203	5.983	0.200	-0.220E+00	-0.165E-02	-0.227E-02	-0.209E-02	-0.145E-02	-0.132E+01	-0.988E-02	-0.136E-01	-0.125E-01	-0.868E-02
3	5.933	0.399	0.209	5.933	0.399	-0.220E+00	-0.165E-02	-0.223E-02	-0.205E-02	-0.142E-02	-0.131E+01	-0.960E-02	-0.132E-01	-0.121E-01	-0.841E-02
4	5.850	0.595	0.219	5.850	0.595	-0.220E+00	-0.155E-02	-0.213E-02	-0.195E-02	-0.135E-02	-0.129E+01	-0.908E-02	-0.125E-01	-0.115E-01	-0.790E-02
5	5.733	0.788	0.233	5.733	0.788	-0.220E+00	-0.144E-02	-0.197E-02	-0.180E-02	-0.123E-02	-0.126E+01	-0.824E-02	-0.113E-01	-0.103E-01	-0.706E-02
6	5.585	0.977	0.248	5.585	0.977	-0.220E+00	-0.124E-02	-0.169E-02	-0.154E-02	-0.103E-02	-0.123E+01	-0.693E-02	-0.946E-02	-0.857E-02	-0.576E-02
7	5.406	1.161	0.265	5.406	1.161	-0.220E+00	-0.914E-03	-0.124E-02	-0.110E-02	-0.703E-03	-0.119E+01	-0.494E-02	-0.568E-02	-0.592E-02	-0.380E-02
8	5.196	1.337	0.284	5.196	1.337	-0.220E+00	-0.380E-03	-0.486E-03	-0.379E-03	-0.170E-03	-0.114E+01	-0.198E-02	-0.253E-02	-0.197E-02	-0.883E-03
9	4.957	1.507	0.302	4.957	1.507	-0.220E+00	0.483E-03	0.724E-03	0.774E-03	0.683E-03	-0.109E+01	0.240E-02	0.359E-02	0.384E-02	0.339E-02
10	4.691	1.668	0.320	4.691	1.668	-0.219E+00	0.187E-02	0.266E-02	0.261E-02	0.203E-02	-0.103E+01	0.878E-02	0.125E-01	0.122E-01	0.951E-02
11	4.398	1.819	0.339	4.398	1.819	-0.218E+00	0.409E-02	0.573E-02	0.550E-02	0.410E-02	-0.960E+00	0.180E-01	0.253E-01	0.242E-01	0.180E-01
12	4.081	1.961	0.356	4.081	1.961	-0.217E+00	0.763E-02	0.106E-01	0.999E-02	0.720E-02	-0.885E+00	0.311E-01	0.434E-01	0.408E-01	0.294E-01
13	3.741	2.091	0.372	3.741	2.091	-0.215E+00	0.132E-01	0.182E-01	0.168E-01	0.116E-01	-0.804E+00	0.495E-01	0.682E-01	0.628E-01	0.434E-01
14	3.380	2.210	0.387	3.380	2.210	-0.212E+00	0.220E-01	0.299E-01	0.267E-01	0.172E-01	-0.715E+00	0.745E-01	0.101E+00	0.901E-01	0.582E-01
15	3.000	2.317	0.401	3.000	2.317	-0.206E+00	0.357E-01	0.473E-01	0.400E-01	0.229E-01	-0.619E+00	0.107E+00	0.142E+00	0.120E+00	0.686E-01
16	2.603	2.410	0.413	2.603	2.410	-0.198E+00	0.566E-01	0.719E-01	0.558E-01	0.247E-01	-0.516E+00	0.147E+00	0.187E+00	0.145E+00	0.643E-01
17	2.192	2.490	0.424	2.192	2.490	-0.186E+00	0.872E-01	0.104E+00	0.681E-01	0.142E-01	-0.407E+00	0.191E+00	0.228E+00	0.149E+00	0.311E-01
18	1.769	2.556	0.433	1.769	2.556	-0.167E+00	0.130E+00	0.138E+00	0.648E-01	-0.207E-01	-0.295E+00	0.229E+00	0.245E+00	0.115E+00	-0.366E-01
19	1.335	2.608	0.439	1.335	2.608	-0.139E+00	0.163E+00	0.163E+00	0.270E-01	-0.827E-01	-0.185E+00	0.245E+00	0.217E+00	0.361E-01	-0.110E+00
20	0.894	2.645	0.444	0.894	2.645	-0.101E+00	0.240E+00	0.155E+00	-0.549E-01	-0.138E+00	-0.904E-01	0.215E+00	0.139E+00	0.491E-01	-0.123E+00
21	0.448	2.668	0.447	0.448	2.668	-0.536E-01	0.287E+00	0.981E-01	-0.152E+00	-0.117E+00	-0.240E-01	0.129E+00	0.440E-01	-0.682E-01	-0.526E-01
22	0.000	2.675	0.448	0.000	2.675	0.267E-07	0.305E+00	-0.521E-07	-0.198E+00	0.689E-07	-0.868E-14	-0.992E-07	0.169E-13	0.643E-07	-0.224E-13
Sum_i w_zz(ij)*sarc(ij), where zz=Fn,Sc,Td,Fr,Ff:															
23	-0.448	2.668	0.447	-0.448	2.668	0.536E-01	0.287E+00	-0.981E-01	-0.152E+00	0.117E+00	-0.240E-01	-0.129E+00	0.440E-01	0.682E-01	-0.526E-01
24	-0.894	2.645	0.444	-0.894	2.645	0.101E+00	0.240E+00	-0.155E+00	-0.549E-01	0.138E+00	-0.904E-01	-0.215E+00	0.139E+00	0.490E-01	-0.123E+00
25	-1.335	2.608	0.439	-1.335	2.608	0.139E+00	0.163E+00	-0.138E+00	-0.270E-01	0.827E-01	-0.185E+00	-0.245E+00	0.217E+00	0.361E-01	-0.110E+00
26	-1.769	2.556	0.433	-1.769	2.556	0.167E+00	0.130E+00	-0.138E+00	0.648E-01	0.207E-01	-0.295E+00	-0.229E+00	0.245E+00	-0.115E+00	-0.366E-01
27	-2.192	2.490	0.424	-2.192	2.490	0.186E+00	0.872E-01	-0.104E+00	0.681E-01	-0.142E-01	-0.407E+00	-0.191E+00	0.228E+00	-0.149E+00	0.311E-01
28	-2.603	2.410	0.413	-2.603	2.410	0.198E+00	0.566E-01	-0.719E-01	0.558E-01	-0.247E-01	-0.516E+00	-0.147E+00	0.187E+00	-0.145E+00	0.643E-01
29	-3.000	2.317	0.401	-3.000	2.317	0.206E+00	0.357E-01	-0.473E-01	0.400E-01	-0.229E-01	-0.619E+00	-0.107E+00	0.142E+00	-0.120E+00	0.686E-01
30	-3.380	2.210	0.387	-3.380	2.210	0.212E+00	0.220E-01	-0.299E-01	0.267E-01	-0.172E-01	-0.715E+00	-0.745E-01	0.101E+00	-0.901E-01	0.582E-01
31	-3.741	2.091	0.372	-3.741	2.091	0.215E+00	0.132E-01	-0.182E-01	0.168E-01	-0.116E-01	-0.804E+00	-0.495E-01	0.682E-01	-0.628E-01	0.434E-01
32	-4.081	1.961	0.356	-4.081	1.961	0.217E+00	0.763E-02	-0.106E-01	0.999E-02	-0.720E-02	-0.885E+00	-0.311E-01	0.434E-01	-0.408E-01	0.294E-01
33	-4.398	1.819	0.339	-4.398	1.819	0.218E+00	0.409E-02	-0.573E-02	0.550E-02	-0.410E-02	-0.960E+00	-0.180E-01	0.253E-01	-0.242E-01	0.180E-01
34	-4.691	1.668	0.320	-4.691	1.668	0.219E+00	0.187E-02	-0.266E-02	0.261E-02	-0.203E-02	-0.103E+01	-0.878E-02	0.125E-01	-0.122E-01	0.951E-02
35	-4.957	1.507	0.302	-4.957	1.507	0.220E+00	0.483E-03	-0.724E-03	0.774E-03	-0.683E-03	-0.109E+01	-0.240E-02	0.359E-02	-0.384E-02	0.339E-02
36	-5.196	1.338	0.284	-5.196	1.338	0.220E+00	-0.380E-03	-0.486E-03	-0.379E-03	0.170E-03	-0.114E+01	0.198E-02	-0.253E-02	0.197E-02	-0.883E-03
37	-5.406	1.161	0.265	-5.406	1.161	0.220E+00	-0.914E-03	0.124E-02	-0.110E-02	0.703E-03	-0.119E+01	0.494E-02	-0.568E-02	0.592E-02	-0.380E-02
38	-5.585	0.977	0.248	-5.585	0.977	0.220E+00	-0.124E-02	0.169E-02	-0.154E-02	0.103E-02	-0.126E+01	0.693E-02	-0.946E-02	0.857E-02	-0.576E-02
39	-5.733	0.788	0.233	-5.733	0.788	0.220E+00	-0.144E-02	0.197E-02	-0.180E-02	0.135E-02	-0.129E+01	0.824E-02	-0.113E-01	0.103E-01	-0.706E-02
40	-5.850	0.595	0.219	-5.850	0.595	0.220E+00	-0.155E-02	0.213E-02	-0.195E-02	0.135E-02	-0.129E+01	0.908E-02	-0.125E-01	0.115E-01	-0.790E-02
41	-5.933	0.399	0.209	-5.933	0.399	0.220E+00	-0.162E-02	0.223E-02	-0.205E-02	0.142E-02	-0.131E+01	0.960E-02	-0.132E-01	0.121E-01	-0.841E-02
42	-5.983	0.200	0.203	-5.983	0.200	0.220E+00	-0.165E-02	0.227E-02	-0.209E-02	0.145E-02	-0.132E+01	0.988E-02	-0.136E-01	0.125E-01	-0.868E-02
43	-6.000	0.000	0.100	-6.000	0.000	0.220E+00	-0.166E-02	0.229E-02	-0.211E-02	0.146E-02	-0.132E+01	0.997E-02	-0.137E-01	0.126E-01	-0.877E-02
Sum_sarcl(i): 14.129, Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):															
												-0.993E+01	-0.359E-07	0.118E+01	-0.817E-07
															-0.533E-01

vacuum chamber thickness and
tolerance study (4 pages)

eddy_out.dat Thu Aug 22 13:31:07 1991 1

program eddy

Input: multipole # (0-dipole, 1-quadr, etc) m = 0 ellipse axes: (a,b) = (3.65, 2.75) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output: |--- z-plane harmonic components 1B*(z) (G) for I=1 A |--- components weighted by u(1) - Int_s u/s ds (G-cm) ---|
|--- at z(1)e^(12j pi/m+1), j=0,...,m, and conjugates, ---| (S=10.10051), [0.00000]
|----- cf. Eqn. (21) or (43) -----|

harmonic component mn+m+n (n=0,1,2,...):																											
i	x(i)	y(i)	sarc(i)	u(i)	v(i)	Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)	1	2	3	4	1	2	3	4	1	2	3	4
1	3.650	0.000	0.103	3.650	0.000	-0.228E+00	-0.227E-01	-0.323E-01	-0.318E-01	-0.249E-01	-0.832E+00	-0.829E-01	-0.118E+00	-0.116E+00	-0.909E-01												
2	3.640	0.206	0.206	3.640	0.206	-0.228E+00	-0.223E-01	-0.317E-01	-0.310E-01	-0.241E-01	-0.829E+00	-0.813E-01	-0.115E+00	-0.113E+00	-0.877E-01												
3	3.609	0.410	0.207	3.609	0.410	-0.227E+00	-0.212E-01	-0.298E-01	-0.287E-01	-0.212E-01	-0.820E+00	-0.764E-01	-0.108E+00	-0.104E+00	-0.781E-01												
4	3.558	0.612	0.210	3.558	0.612	-0.227E+00	-0.191E-01	-0.268E-01	-0.247E-01	-0.175E-01	-0.806E+00	-0.681E-01	-0.945E-01	-0.881E-01	-0.623E-01												
5	3.488	0.811	0.212	3.488	0.811	-0.226E+00	-0.162E-01	-0.218E-01	-0.191E-01	-0.117E-01	-0.787E+00	-0.565E-01	-0.761E-01	-0.666E-01	-0.410E-01												
6	3.398	1.005	0.216	3.398	1.005	-0.224E+00	-0.121E-01	-0.154E-01	-0.117E-01	-0.439E-02	-0.762E+00	-0.412E-01	-0.523E-01	-0.396E-01	-0.149E-01												
7	3.289	1.193	0.220	3.289	1.193	-0.222E+00	-0.680E-02	-0.711E-02	-0.232E-02	0.439E-02	-0.732E+00	-0.224E-01	-0.234E-01	-0.764E-02	0.144E-01												
8	3.161	1.375	0.224	3.161	1.375	-0.220E+00	0.625E-04	0.329E-02	0.890E-02	0.142E-01	-0.696E+00	0.198E-03	0.104E-01	0.281E-01	0.449E-01												
9	3.016	1.549	0.229	3.016	1.549	-0.217E+00	0.875E-02	0.160E-01	0.219E-01	0.244E-01	-0.655E+00	0.264E-01	0.484E-01	0.659E-01	0.734E-01												
10	2.854	1.715	0.234	2.854	1.715	-0.213E+00	0.196E-01	0.313E-01	0.361E-01	0.337E-01	-0.608E+00	0.560E-01	0.894E-01	0.103E+00	0.962E-01												
11	2.676	1.870	0.239	2.676	1.870	-0.208E+00	0.330E-01	0.492E-01	0.509E-01	0.406E-01	-0.557E+00	0.883E-01	0.132E+00	0.136E+00	0.109E+00												
12	2.483	2.016	0.244	2.483	2.016	-0.202E+00	0.494E-01	0.695E-01	0.648E-01	0.427E-01	-0.502E+00	0.123E+00	0.172E+00	0.161E+00	0.106E+00												
13	2.276	2.150	0.249	2.276	2.150	-0.194E+00	0.691E-01	0.916E-01	0.758E-01	0.375E-01	-0.442E+00	0.157E+00	0.208E+00	0.172E+00	0.853E-01												
14	2.056	2.272	0.253	2.056	2.272	-0.184E+00	0.924E-01	0.114E+00	0.811E-01	0.224E-01	-0.379E+00	0.190E+00	0.235E+00	0.167E+00	0.460E-01												
15	1.825	2.382	0.258	1.825	2.382	-0.172E+00	0.119E+00	0.136E+00	0.773E-01	-0.411E-02	-0.315E+00	0.218E+00	0.249E+00	0.141E+00	-0.750E-02												
16	1.584	2.478	0.262	1.584	2.478	-0.159E+00	0.150E+00	0.154E+00	0.611E-01	-0.409E-01	-0.249E+00	0.237E+00	0.244E+00	0.968E-01	-0.648E-01												
17	1.333	2.560	0.265	1.333	2.560	-0.139E+00	0.183E+00	0.165E+00	0.299E-01	-0.831E-01	-0.186E+00	0.244E+00	0.220E+00	0.398E-01	-0.111E+00												
18	1.076	2.628	0.268	1.076	2.628	-0.118E+00	0.217E+00	0.164E+00	-0.165E-01	-0.121E+00	-0.127E+00	0.234E+00	0.176E+00	-0.177E-01	-0.130E+00												
19	0.812	2.681	0.270	0.812	2.681	-0.927E-01	0.250E+00	0.147E+00	-0.736E-01	-0.139E+00	-0.753E-01	0.203E+00	0.119E+00	-0.598E-01	-0.113E+00												
20	0.544	2.719	0.272	0.544	2.719	-0.641E-01	0.277E+00	0.112E+00	-0.131E+00	-0.126E+00	-0.349E-01	0.151E+00	0.611E-01	-0.715E-01	-0.683E-01												
21	0.273	2.742	0.273	0.273	2.742	-0.328E-01	0.296E+00	0.612E-01	-0.116E+00	-0.757E-01	-0.895E-02	0.808E-01	0.167E-01	-0.479E-01	-0.206E-01												
22	0.000	2.750	0.273	0.000	2.750	-0.198E-07	0.303E+00	-0.377E-07	-0.192E+00	0.481E-07	-0.129E-14	-0.197E-07	0.245E-14	0.125E-07	-0.313E-14												
Sum_1 w_zz(1j)*sarc(1j), where zz=Fn,Sc,Td,Fr,Ff:																0.440E+00	0.396E+00	0.122E+00		0.440E+00	0.396E+00	0.122E+00		0.440E+00	0.396E+00	0.122E+00	
23	-0.273	2.742	0.273	-0.273	2.742	0.328E-01	0.296E+00	-0.612E-01	-0.176E+00	0.757E-01	-0.895E-02	-0.808E-01	0.167E-01	0.479E-01	-0.206E-01					-0.808E-01	0.167E-01	0.479E-01		-0.808E-01	0.167E-01	0.479E-01	
24	-0.544	2.719	0.272	-0.544	2.719	0.641E-01	0.277E+00	-0.112E+00	-0.131E+00	0.166E+00	-0.349E-01	-0.151E+00	0.611E-01	0.715E-01	-0.683E-01					-0.151E+00	0.611E-01	0.715E-01		-0.151E+00	0.611E-01	0.715E-01	
25	-0.812	2.681	0.270	-0.812	2.681	0.927E-01	0.250E+00	-0.147E+00	-0.736E-01	0.139E+00	-0.753E-01	-0.203E+00	0.119E+00	-0.598E-01	-0.113E+00					-0.203E+00	0.119E+00	-0.598E-01		-0.203E+00	0.119E+00	-0.598E-01	
26	-1.076	2.628	0.268	-1.076	2.628	0.118E+00	0.217E+00	-0.164E+00	-0.165E-01	0.121E+00	-0.127E+00	-0.234E+00	0.176E+00	0.177E-01	-0.130E+00					-0.234E+00	0.176E+00	0.177E-01		-0.234E+00	0.176E+00	0.177E-01	
27	-1.333	2.560	0.265	-1.333	2.560	0.139E+00	0.183E+00	-0.165E+00	0.299E-01	0.831E-01	-0.186E+00	-0.244E+00	0.220E+00	-0.398E-01	-0.111E+00					-0.244E+00	0.220E+00	-0.398E-01		-0.244E+00	0.220E+00	-0.398E-01	
28	-1.584	2.478	0.262	-1.584	2.478	0.159E+00	0.150E+00	-0.154E+00	0.611E-01	0.409E-01	-0.249E+00	-0.237E+00	0.244E+00	-0.968E-01	-0.648E-01					-0.237E+00	0.244E+00	-0.968E-01		-0.237E+00	0.244E+00	-0.968E-01	
29	-1.825	2.382	0.258	-1.825	2.382	0.172E+00	0.119E+00	-0.136E+00	0.773E-01	0.411E-02	-0.315E+00	-0.218E+00	0.249E+00	-0.141E+00	-0.750E-02					-0.218E+00	0.249E+00	-0.141E+00		-0.218E+00	0.249E+00	-0.141E+00	
30	-2.056	2.272	0.253	-2.056	2.272	0.184E+00	0.924E-01	-0.114E+00	0.811E-01	-0.224E-01	-0.379E+00	-0.190E+00	0.235E+00	-0.167E+00	0.460E-01					-0.190E+00	0.235E+00	-0.167E+00		-0.190E+00	0.235E+00	-0.167E+00	
31	-2.276	2.150	0.249	-2.276	2.150	0.194E+00	0.691E-01	-0.916E-01	0.758E-01	-0.375E-01	-0.442E+00	-0.157E+00	0.208E+00	-0.172E+00	0.853E-01					-0.157E+00	0.208E+00	-0.172E+00		-0.157E+00	0.208E+00	-0.172E+00	
32	-2.483	2.016	0.244	-2.483	2.016	0.202E+00	0.494E-01	-0.695E-01	0.648E-01	-0.427E-01	-0.502E+00	-0.123E+00	0.172E+00	-0.161E+00	0.106E+00					-0.123E+00	0.172E+00	-0.161E+00		-0.123E+00	0.172E+00	-0.161E+00	
33	-2.676	1.870	0.239	-2.676	1.870	0.208E+00	0.330E-01	-0.492E-01	0.509E-01	-0.406E-01	-0.557E+00	-0.883E-01	0.132E+00	-0.136E+00	0.109E+00					-0.883E-01	0.132E+00	-0.136E+00		-0.883E-01	0.132E+00	-0.136E+00	
34	-2.854	1.715	0.234	-2.854	1.715	0.213E+00	0.196E-01	-0.313E-01	0.361E-01	-0.337E-01	-0.608E+00	-0.560E-01	0.894E-01	-0.103E+00	0.962E-01					-0.560E-01	0.894E-01	-0.103E+00		-0.560E-01	0.894E-01	-0.103E+00	
35	-3.016	1.549	0.229	-3.016	1.549	0.217E+00	0.875E-02	-0.160E-01	0.219E-01	-0.244E-01	-0.655E+00	-0.264E-01	0.484E-01	-0.659E-01	0.734E-01					-0.264E-01	0.484E-01	-0.659E-01		-0.264E-01	0.484E-01	-0.659E-01	
36	-3.161	1.375	0.224	-3.161	1.375	0.222E+00	0.625E-04	-0.329E-02	0.890E-02	0.142E-01	-0.696E+00	0.198E-03	0.104E-01	0.281E-01	0.449E-01					-0.198E-03	0.104E-01	0.281E-01		-0.198E-03	0.104E-01	0.281E-01	
37	-3.289	1.193	0.220	-3.289	1.193	0.222E+00	-0.680E-02	0.711E-02	-0.232E-02	0.439E-02	-0.732E+00	0.224E-01	-0.234E-01	0.764E-02	0.144E-01					0.711E-02	-0.232E-01	0.764E-02		0.711E-02	-0.232E-01	0.764E-02	
38	-3.398	1.005	0.216	-3.398	1.005	0.226E+00	-0.162E-01	-0.154E-01	-0.117E-01	-0.439E-02	-0.762E+00	0.412E-01	-0.523E-01	-0.396E-01	-0.149E-01					0.412E-01	-0.523E-01	-0.396E-01		0.412E-01	-0.523E-01	-0.396E-01	
39	-3.488	0.811	0.212	-3.488	0.811	0.224E+00	-0.121E-01	-0.154E-01	-0.117E-01	-0.439E-02	-0.762E+																

I

program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b) = (3.50, 2.60) cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

```

Output:
|- z-plane harmonic components iB*(z) (G) for I=1 A -|-- components weighted by u(i) - Int s u/s ds (G-cm) --|
|- At z(i)e^(i2 pi/m+1), t=0,...,m, and conjugates, - (S = 9.63183), ( 0.00000 )

```

harmonic component $mn+tn$ ($n=0,1,2,\dots$):

[illegible][illegible]

program eddy_str

Input: multipole # (0=dipole, 1=quad, etc) m = 0 ellipse axes: (a,b)=(2.75, 2.75) cm, str = 1.00 cm, h = 2.86 cm, harmonics evaluated at r= 2.50 cm

Output:

```

|- z-plane harmonic components iB*(z) (G) for I=1 A |--- components weighted by u(i) - Int_s u/S ds (G-cm) ---|
|- at z(i)*e^(i2j pi/mt+1), j=0,...,m, and conjugates, -|
|----- cf. Eqn. (21) or (43) -----|

```

harmonic component mn+n (n=0,1,2,...):																					
1	x(i)	y(i)	sarc(i)	u(i)	v(i)	0	1	2	3	4	0	1	2	3	4						
						Fn(i)	Sc(i)	Td(i)	Fr(i)	Ff(i)	w_Fn(i)	w_Sc(i)	w_Td(i)	w_Fr(i)	w_Ff(i)						
1	3.250	0.000	0.114	3.250	0.000	-0.232E+00	-0.360E-01	-0.523E-01	-0.533E-01	-0.446E-01	-0.755E+00	-0.117E+00	-0.170E+00	-0.173E+00	-0.145E+00						
2	3.241	0.227	0.227	3.241	0.227	-0.232E+00	-0.350E-01	-0.506E-01	-0.512E-01	-0.421E-01	-0.752E+00	-0.114E+00	-0.164E+00	-0.166E+00	-0.136E+00						
3	3.212	0.453	0.227	3.212	0.453	-0.231E+00	-0.323E-01	-0.459E-01	-0.449E-01	-0.347E-01	-0.743E+00	-0.104E+00	-0.147E+00	-0.147E+00	-0.112E+00						
4	3.166	0.675	0.227	3.166	0.675	-0.230E+00	-0.276E-01	-0.380E-01	-0.347E-01	-0.233E-01	-0.727E+00	-0.074E-01	-0.120E+00	-0.110E+00	-0.073E-01						
5	3.101	0.893	0.227	3.101	0.893	-0.228E+00	-0.211E-01	-0.272E-01	-0.214E-01	-0.099E-02	-0.708E+00	-0.055E-01	-0.084E+00	-0.084E+00	-0.027E-01						
6	3.018	1.105	0.227	3.018	1.105	-0.225E+00	-0.128E-01	-0.138E-01	-0.138E-01	0.681E-02	-0.679E+00	-0.386E-01	-0.417E-01	-0.166E-01	0.205E-01						
7	2.919	1.309	0.227	2.919	1.309	-0.221E+00	-0.258E-02	0.191E-02	0.119E-01	0.223E-01	-0.646E+00	-0.754E-02	0.558E-02	0.347E-01	0.651E-01						
8	2.802	1.504	0.227	2.802	1.504	-0.217E+00	0.946E-02	0.196E-01	0.297E-01	0.357E-01	-0.608E+00	0.265E-01	0.549E-01	0.832E-01	0.100E+00						
9	2.670	1.689	0.227	2.670	1.689	-0.212E+00	0.234E-01	0.388E-01	0.467E-01	0.451E-01	-0.566E+00	0.624E-01	0.104E+00	0.125E+00	0.120E+00						
10	2.523	1.863	0.227	2.523	1.863	-0.206E+00	0.392E-01	0.590E-01	0.617E-01	0.490E-01	-0.520E+00	0.988E-01	0.149E+00	0.156E+00	0.124E+00						
11	2.363	2.023	0.227	2.363	2.023	-0.199E+00	0.569E-01	0.797E-01	0.732E-01	0.460E-01	-0.471E+00	0.134E+00	0.188E+00	0.173E+00	0.109E+00						
12	2.189	2.170	0.227	2.189	2.170	-0.191E+00	0.766E-01	0.100E+00	0.801E-01	0.353E-01	-0.419E+00	0.168E+00	0.219E+00	0.175E+00	0.340E-01						
13	2.004	2.302	0.227	2.004	2.302	-0.182E+00	0.984E-01	0.120E+00	0.809E-01	0.170E-01	-0.364E+00	0.197E+00	0.240E+00	0.162E+00	0.774E-01						
14	1.809	2.419	0.227	1.809	2.419	-0.178E+00	0.122E+00	0.137E+00	0.744E-01	-0.854E-02	-0.309E+00	0.221E+00	0.248E+00	0.135E+00	-0.155E-01						
15	1.605	2.518	0.227	1.605	2.518	-0.165E+00	0.148E+00	0.151E+00	0.593E-01	-0.397E-01	-0.254E+00	0.238E+00	0.243E+00	0.048E-01	-0.103E+00						
16	1.393	2.601	0.227	1.393	2.601	-0.153E+00	0.176E+00	0.161E+00	0.348E-01	-0.738E-01	-0.199E+00	0.245E+00	0.224E+00	0.483E-01	-0.103E+00						
17	1.175	2.666	0.227	1.175	2.666	-0.140E+00	0.204E+00	0.163E+00	-0.636E-04	-0.106E+00	-0.148E+00	0.240E+00	0.192E+00	-0.748E-04	-0.124E+00						
18	0.953	2.712	0.227	0.953	2.712	-0.126E+00	0.233E+00	0.158E+00	-0.437E-01	-0.129E+00	-0.101E+00	0.222E+00	0.148E+00	-0.416E-01	-0.123E+00						
19	0.727	2.741	0.227	0.727	2.741	-0.113E+00	0.259E+00	0.136E+00	-0.924E-01	-0.136E+00	-0.608E-01	0.188E+00	0.091E-01	-0.672E-01	-0.989E-01						
20	0.500	2.750	0.239	0.500	2.750	-0.101E-01	0.281E+00	0.104E+00	-0.139E+00	-0.119E+00	-0.295E-01	0.140E+00	0.522E-01	-0.697E-01	-0.593E-01						
21	0.250	2.750	0.250	0.250	2.750	-0.301E-01	0.297E+00	0.056E-01	-0.178E+00	-0.698E-01	-0.752E-02	0.743E-01	0.141E-01	-0.445E-01	-0.175E-01						
22	0.000	2.750	0.250	0.000	2.750	0.198E-07	0.303E+00	-0.377E-07	-0.192E+00	0.481E-07	-0.370E-14	-0.567E-07	0.705E-14	0.360E-07	-0.901E-14						
Sum_i w_zz(i)*sarc(i), where zz=Fn,Sc,Td,Fr,Ff:																					
23	-0.250	2.750	0.250	-0.250	2.750	0.301E-01	0.297E+00	-0.563E-01	-0.178E+00	0.698E-01	-0.752E-02	-0.743E-01	0.408E+00	0.351E+00	0.833E-01						
24	-0.500	2.750	0.239	-0.500	2.750	0.591E-01	0.281E+00	-0.104E+00	-0.139E+00	0.119E+00	-0.295E-01	-0.140E+00	0.522E-01	0.697E-01	-0.593E-01						
25	-0.727	2.741	0.227	-0.727	2.741	0.837E-01	0.259E+00	-0.136E+00	-0.924E-01	0.136E+00	-0.608E-01	-0.188E+00	0.091E-01	0.672E-01	-0.989E-01						
26	-0.953	2.712	0.227	-0.953	2.712	0.108E+00	0.233E+00	-0.156E+00	-0.437E-01	0.129E+00	-0.101E+00	-0.222E+00	0.148E+00	0.416E-01	-0.123E+00						
27	-1.175	2.666	0.227	-1.175	2.666	0.126E+00	0.204E+00	-0.163E+00	-0.636E-04	0.102E+00	-0.148E+00	-0.240E+00	0.192E+00	-0.748E-04	-0.124E+00						
28	-1.393	2.601	0.227	-1.393	2.601	0.143E+00	0.176E+00	-0.161E+00	-0.348E-01	0.136E+00	-0.199E+00	-0.245E+00	0.224E+00	-0.483E-01	-0.103E+00						
29	-1.605	2.518	0.227	-1.605	2.518	0.158E+00	0.148E+00	-0.151E+00	-0.593E-01	0.397E-01	-0.309E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
30	-1.809	2.419	0.227	-1.809	2.419	0.171E+00	0.122E+00	-0.137E+00	0.744E-01	0.854E-02	-0.309E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
31	-2.004	2.302	0.227	-2.004	2.302	0.182E+00	0.098E-01	-0.120E+00	0.809E-01	-0.170E-01	-0.364E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
32	-2.189	2.170	0.227	-2.189	2.170	0.191E+00	0.766E-01	-0.100E+00	0.801E-01	-0.353E-01	-0.471E+00	-0.134E+00	0.188E+00	-0.162E+00	-0.173E+00						
33	-2.363	2.023	0.227	-2.363	2.023	0.209E+00	0.590E-01	-0.590E-01	0.617E-01	-0.490E-01	-0.520E+00	-0.988E-01	0.149E+00	-0.156E+00	-0.124E+00						
34	-2.523	1.863	0.227	-2.523	1.863	0.226E+00	0.392E-01	0.590E-01	0.617E-01	-0.490E-01	-0.520E+00	-0.988E-01	0.149E+00	-0.156E+00	-0.124E+00						
35	-2.670	1.689	0.227	-2.670	1.689	0.212E+00	0.234E-01	-0.388E-01	0.467E-01	-0.451E-01	-0.566E+00	-0.624E-01	0.104E+00	-0.125E+00	0.120E+00						
36	-2.802	1.504	0.227	-2.802	1.504	0.217E+00	0.946E-02	-0.196E-01	0.297E-01	-0.357E-01	-0.608E+00	0.265E-01	0.549E-01	0.832E-01	0.100E+00						
37	-2.919	1.309	0.227	-2.919	1.309	0.221E+00	-0.258E-02	0.191E-02	0.119E-01	0.223E-01	-0.646E+00	-0.754E-02	0.558E-02	0.347E-01	0.651E-01						
38	-3.018	1.105	0.227	-3.018	1.105	0.225E+00	-0.128E-01	-0.138E-01	-0.138E-01	0.681E-02	-0.679E+00	-0.386E-01	-0.417E-01	-0.166E-01	0.205E-01						
39	-3.101	0.893	0.227	-3.101	0.893	0.228E+00	-0.211E-01	-0.272E-01	-0.214E-01	-0.099E-02	-0.708E+00	-0.055E-01	-0.084E+00	-0.084E+00	-0.027E-01						
40	-3.166	0.675	0.227	-3.166	0.675	0.230E+00	-0.276E-01	-0.380E-01	-0.347E-01	-0.233E-01	-0.727E+00	-0.074E-01	-0.120E+00	-0.110E+00	-0.073E-01						
41	-3.212	0.453	0.227	-3.212	0.453	0.231E+00	-0.323E-01	-0.459E-01	-0.449E-01	-0.347E-01	-0.743E+00	-0.104E+00	-0.147E+00	-0.147E+00	-0.112E+00						
42	-3.241	0.227	0.227	-3.241	0.227	0.230E+00	-0.276E-01	-0.380E-01	-0.347E-01	-0.233E-01	-0.727E+00	-0.074E-01	-0.120E+00	-0.110E+00	-0.073E-01						
43	-3.250	0.000	0.114	-3.250	0.000	0.232E+00	-0.360E-01	-0.523E-01	-0.533E-01	-0.446E-01	-0.755E+00	-0.117E+00	-0.170E+00	-0.173E+00	-0.145E+00						
Sum_i w_zz(i)*sarc(i), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):																					
44	-0.250	2.750	0.250	-0.250	2.750	0.301E-01	0.297E+00	-0.563E-01	-0.178E+00	0.698E-01	-0.752E-02	-0.743E-01	0.408E+00	0.351E+00	0.833E-01						
45	-0.500	2.750	0.239	-0.500	2.750	0.591E-01	0.281E+00	-0.104E+00	-0.139E+00	0.119E+00	-0.295E-01	-0.140E+00	0.522E-01	0.697E-01	-0.593E-01						
46	-0.727	2.741	0.227	-0.727	2.741	0.837E-01	0.259E+00	-0.136E+00	-0.924E-01	0.136E+00	-0.608E-01	-0.188E+00	0.091E-01	0.672E-01	-0.989E-01						
47	-0.953	2.712	0.227	-0.953	2.712	0.106E+00	0.233E+00	-0.156E+00	-0.437E-01	0.129E+00	-0.101E+00	-0.222E+00	0.148E+00	0.416E-01	-0.123E+00						
48	-1.175	2.666	0.227	-1.175	2.666	0.126E+00	0.204E+00	-0.163E+00	-0.636E-04	0.102E+00	-0.148E+00	-0.240E+00	0.192E+00	-0.748E-04	-0.124E+00						
49	-1.393	2.601	0.227	-1.393	2.601	0.143E+00	0.176E+00	-0.161E+00	-0.348E-01	0.136E+00	-0.199E+00	-0.245E+00	0.224E+00	-0.483E-01	-0.103E+00						
50	-1.605	2.518	0.227	-1.605	2.518	0.158E+00	0.148E+00	-0.151E+00	-0.593E-01	0.397E-01	-0.309E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
51	-1.809	2.419	0.227	-1.809	2.419	0.171E+00	0.122E+00	-0.137E+00	0.744E-01	0.854E-02	-0.309E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
52	-2.004	2.302	0.227	-2.004	2.302	0.182E+00	0.098E-01	-0.120E+00	0.809E-01	-0.170E-01	-0.364E+00	-0.221E+00	0.248E+00	-0.135E+00	-0.155E-01						
53	-2.189	2.170	0.227	-2.189	2.170	0.191E+00	0.766E-0														

1

program eddy_str

```

Input:      multipole # (0=dipole, 1=quad, etc) m = 0  ellipse axes: (a,b)=( 2.60, 2.60) cm,  str = 1.00 cm,  h = 2.86 cm,  harmonics evaluated at r= 2.50 cm

              | - z-plane harmonic components iB*(z) (G) for I=1 A | - components weighted by u(i) - Int s u/ ds (G-cm) --|
              | - at z(i)e^{i2j pi/m+1}, j=0,...,m, and conjugates, -| (S = 9.16581), { 0.00000 }
Output:

```

Output:

harmonic component $m\Omega + n$ ($n=0, 1, 2, \dots$):

1	3.100	0.000	0.107	3.100	0.000	-0.235E+00	-0.429E-01	-0.629E-01	-0.653E-01	-0.564E-01	-0.728E+00	-0.133E+00	-0.195E+00	-0.203E+00	-0.175E+00
2	3.091	0.215	0.215	3.091	0.215	-0.234E+00	-0.419E-01	-0.611E-01	-0.629E-01	-0.532E-01	-0.725E+00	-0.129E+00	-0.189E+00	-0.194E+00	-0.165E+00
3	3.065	0.428	0.215	3.065	0.428	-0.234E+00	-0.388E-01	-0.557E-01	-0.554E-01	-0.442E-01	-0.716E+00	-0.119E+00	-0.171E+00	-0.170E+00	-0.135E+00
4	3.020	0.638	0.215	3.020	0.638	-0.232E+00	-0.338E-01	-0.469E-01	-0.435E-01	-0.300E-01	-0.701E+00	-0.102E+00	-0.142E+00	-0.131E+00	-0.906E-01
5	2.959	0.844	0.215	2.959	0.844	-0.230E+00	-0.268E-01	-0.349E-01	-0.277E-01	-0.121E-01	-0.680E+00	-0.794E-01	-0.103E+00	-0.821E-01	-0.358E-01
6	2.881	1.044	0.215	2.881	1.044	-0.227E+00	-0.178E-01	-0.199E-01	-0.904E-02	0.769E-02	-0.653E+00	-0.514E-01	-0.572E-01	-0.260E-01	0.222E-01
7	2.787	1.237	0.215	2.787	1.237	-0.222E+00	-0.683E-02	-0.222E-02	0.115E-01	0.273E-01	-0.622E+00	-0.190E-01	-0.618E-02	0.321E-01	0.760E-01
8	2.677	1.422	0.215	2.677	1.422	-0.219E+00	0.616E-02	0.176E-01	0.326E-01	0.443E-01	-0.585E+00	0.165E-01	0.471E-01	0.873E-01	0.119E+00
9	2.552	1.597	0.215	2.552	1.597	-0.213E+00	0.121E-01	0.391E-01	0.528E-01	0.566E-01	-0.544E+00	0.540E-01	0.997E-01	0.135E+00	0.144E+00
10	2.413	1.761	0.215	2.413	1.761	-0.207E+00	0.382E-01	0.616E-01	0.705E-01	0.621E-01	-0.500E+00	0.921E-01	0.149E+00	0.170E+00	0.150E+00
11	2.261	1.913	0.215	2.261	1.913	-0.200E+00	0.572E-01	0.845E-01	0.842E-01	0.592E-01	-0.452E+00	0.129E+00	0.191E+00	0.190E+00	0.134E+00
12	2.097	2.052	0.215	2.097	2.052	-0.191E+00	0.782E-01	0.107E+00	0.922E-01	0.469E-01	-0.401E+00	0.164E+00	0.224E+00	0.193E+00	0.983E-01
13	1.922	2.177	0.215	1.922	2.177	-0.182E+00	0.101E+00	0.128E+00	0.931E-01	0.250E-01	-0.349E+00	0.195E+00	0.246E+00	0.179E+00	0.481E-01
14	1.737	2.287	0.215	1.737	2.287	-0.170E+00	0.126E+00	0.147E+00	0.853E-01	-0.530E-02	-0.296E+00	0.219E+00	0.255E+00	0.148E+00	-0.921E-02
15	1.544	2.381	0.215	1.544	2.381	-0.157E+00	0.153E+00	0.161E+00	0.677E-01	-0.419E-01	-0.243E+00	0.236E+00	0.249E+00	0.105E+00	-0.648E-01
16	1.344	2.459	0.215	1.344	2.459	-0.142E+00	0.181E+00	0.170E+00	0.398E-01	-0.810E-01	-0.191E+00	0.233E+00	0.229E+00	0.534E-01	-0.109E+00
17	1.138	2.520	0.215	1.138	2.520	-0.125E+00	0.209E+00	0.171E+00	0.56E-02	-0.117E-02	-0.142E+00	0.238E+00	0.195E+00	0.177E-02	-0.133E+00
18	0.928	2.565	0.215	0.928	2.565	-0.105E+00	0.237E+00	0.163E+00	-0.001E-01	-0.143E+00	-0.977E-01	0.220E+00	0.151E+00	-0.418E-01	-0.132E+00
19	0.715	2.591	0.215	0.715	2.591	-0.836E-01	0.263E+00	0.143E+00	-0.901E-01	-0.150E+00	-0.598E-01	0.198E+00	0.102E+00	-0.687E-01	-0.107E+00
20	0.500	2.600	0.232	0.500	2.600	-0.605E-01	0.285E+00	0.111E+00	-0.145E+00	-0.133E+00	-0.300E-01	0.142E+00	0.553E-01	-0.727E-01	-0.666E-01
21	0.250	2.600	0.250	0.250	2.600	-0.302E-01	0.302E+00								

$$\text{Sum} \leftarrow w \cdot \text{zz}(1) * \text{sarc}(1), \text{ where } \text{zz} = \text{Fn}, \text{Sc}, \text{Td}, \text{Fr}, \text{Ef};$$
[illegible]

eddy_out.dat Thu Aug 22 13:32:59 1991 1

program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 1 ellipse axes: (a,b) = (5.50, 2.75) cm, h = 5.00 cm, harmonics evaluated at r = 2.50 cm

Output:

!- z-plane harmonic components $IB^*(z)$ (G) for $I=1$ A --- components weighted by $u(l)$ - Int_s u/S ds (G-cm) ---
 !- at $z(l)e^{i(2j\pi/m+1)}$, $j=0,...,m$, and conjugates, - (S = 6.66042),
 !- ----- cf. Eqn. (21) or (43) ----- (1.66475)

harmonic component		mn+mN (n=0,1,2,...):		1		3		5		7		9		1		3		5		7		9	
1	x(l)	y(l)	u(l)	v(l)	Fn(l)	Sc(l)	Td(l)	Fr(l)	Fr(l)	Fr(l)	Fr(l)	w_Fn(l)	w_Sc(l)	w_Td(l)	w_Fr(l)	w_Fr(l)							
1	5.500	0.000	0.051	6.050	0.000	-0.131E+00	-0.461E-02	-0.189E-02	-0.541E-03	-0.125E-03	-0.576E+00	-0.202E-01	-0.831E-02	-0.237E-02	-0.547E-03								
2	5.496	0.103	0.103	6.039	0.226	-0.131E+00	-0.459E-02	-0.188E-02	-0.537E-03	-0.123E-03	-0.575E+00	-0.201E-01	-0.824E-02	-0.235E-02	-0.539E-03								
3	5.485	0.206	0.104	6.008	0.451	-0.131E+00	-0.453E-02	-0.185E-02	-0.542E-03	-0.119E-03	-0.570E+00	-0.197E-01	-0.804E-02	-0.227E-02	-0.516E-03								
4	5.465	0.308	0.105	5.955	0.673	-0.131E+00	-0.442E-02	-0.179E-02	-0.501E-03	-0.111E-03	-0.563E+00	-0.190E-01	-0.770E-02	-0.215E-02	-0.477E-03								
5	5.439	0.410	0.106	5.882	0.892	-0.131E+00	-0.426E-02	-0.171E-02	-0.459E-03	-0.100E-03	-0.553E+00	-0.180E-01	-0.722E-02	-0.198E-02	-0.422E-03								
6	5.404	0.511	0.108	5.789	1.105	-0.131E+00	-0.404E-02	-0.160E-02	-0.425E-03	-0.852E-04	-0.539E+00	-0.167E-01	-0.669E-02	-0.175E-02	-0.352E-03								
7	5.362	0.612	0.110	5.676	1.312	-0.130E+00	-0.375E-02	-0.145E-02	-0.369E-03	-0.661E-04	-0.523E+00	-0.150E-01	-0.583E-02	-0.148E-02	-0.265E-03								
8	5.313	0.712	0.113	5.543	1.513	-0.130E+00	-0.339E-02	-0.127E-02	-0.297E-03	-0.420E-04	-0.504E+00	-0.131E-01	-0.492E-02	-0.115E-02	-0.163E-03								
9	5.256	0.811	0.116	5.393	1.704	-0.130E+00	-0.293E-02	-0.104E-02	-0.207E-03	-0.124E-04	-0.483E+00	-0.109E-01	-0.386E-02	-0.0771E-03	-0.462E-04								
10	5.191	0.908	0.119	5.225	1.886	-0.129E+00	-0.236E-02	-0.748E-03	-0.964E-04	-0.234E-04	-0.459E+00	-0.840E-02	-0.266E-02	-0.343E-03	-0.833E-04								
11	5.120	1.005	0.122	5.041	2.058	-0.128E+00	-0.166E-02	-0.322E-03	-0.303E-04	-0.661E-04	-0.433E+00	-0.559E-02	-0.132E-02	-0.129E-03	-0.223E-03								
12	5.041	1.100	0.125	4.841	2.217	-0.127E+00	-0.791E-03	-0.428E-04	-0.201E-03	-0.116E-03	-0.404E+00	-0.251E-02	-0.136E-03	-0.638E-03	-0.369E-03								
13	4.955	1.193	0.129	4.626	2.365	-0.126E+00	-0.266E-03	-0.572E-03	-0.395E-03	-0.174E-03	-0.373E+00	-0.788E-03	-0.169E-02	-0.117E-02	-0.514E-03								
14	4.863	1.283	0.132	4.399	2.499	-0.125E+00	-0.155E-02	-0.121E-02	-0.635E-03	-0.238E-03	-0.341E+00	-0.424E-02	-0.331E-02	-0.171E-02	-0.650E-03								
15	4.763	1.375	0.136	4.159	2.620	-0.123E+00	-0.311E-02	-0.198E-02	-0.892E-03	-0.307E-03	-0.307E+00	-0.776E-02	-0.494E-02	-0.223E-02	-0.765E-03								
16	4.657	1.463	0.140	3.909	2.725	-0.121E+00	-0.499E-02	-0.290E-02	-0.120E-02	-0.376E-03	-0.271E+00	-0.112E-01	-0.650E-02	-0.269E-02	-0.845E-03								
17	4.544	1.549	0.144	3.650	2.816	-0.118E+00	-0.725E-02	-0.398E-02	-0.134E-02	-0.441E-03	-0.235E+00	-0.144E-01	-0.790E-02	-0.306E-02	-0.875E-03								
18	4.425	1.633	0.148	3.383	2.891	-0.116E+00	-0.994E-02	-0.525E-02	-0.191E-02	-0.488E-03	-0.199E+00	-0.171E-01	-0.902E-02	-0.329E-02	-0.839E-03								
19	4.300	1.715	0.151	3.110	2.949	-0.112E+00	-0.131E-01	-0.672E-02	-0.230E-02	-0.502E-03	-0.162E+00	-0.190E-01	-0.971E-02	-0.332E-02	-0.726E-03								
20	4.169	1.794	0.155	2.832	2.991	-0.108E+00	-0.169E-01	-0.840E-02	-0.266E-02	-0.458E-03	-0.126E+00	-0.198E-01	-0.980E-02	-0.311E-02	-0.534E-03								
21	4.032	1.870	0.159	2.551	3.017	-0.103E+00	-0.214E-01	-0.103E-01	-0.236E-02	-0.321E-03	-0.915E-01	-0.189E-01	-0.911E-02	-0.262E-02	-0.284E-03								
22	3.889	1.945	0.163	2.269	3.025	-0.976E-01	-0.265E-01	-0.123E-01	-0.311E-02	-0.475E-04	-0.589E-01	-0.160E-01	-0.744E-02	-0.188E-02	-0.287E-04								
Sum_1_w_zz(l)*aarc(l), where zz=Fn,Sc,Td,Fr,Fc, 0.219E-02 0.385E-02 0.213E-02 0.627E-03																							
23	3.741	2.016	0.166	1.986	3.017	-0.911E-01	-0.325E-01	-0.145E-01	-0.300E-02	-0.412E-03	-0.293E-01	-0.105E-01	-0.465E-02	-0.963E-03	-0.133E-03								
24	3.588	2.084	0.170	1.705	2.991	-0.837E-01	-0.394E-01	-0.166E-01	-0.247E-02	-0.111E-02	-0.338E-02	-0.159E-02	-0.670E-02	-0.998E-04	-0.447E-04								
25	3.429	2.150	0.173	1.427	2.949	-0.752E-01	-0.473E-01	-0.184E-01	-0.132E-02	-0.205E-02	-0.378E-02	-0.187E-02	-0.438E-02	-0.133E-03	-0.487E-03								
26	3.266	2.213	0.176	1.154	2.891	-0.655E-01	-0.561E-01	-0.197E-01	-0.699E-03	-0.320E-02	-0.334E-01	-0.287E-01	-0.101E-01	-0.357E-03	-0.163E-02								
27	3.098	2.272	0.180	0.887	2.816	-0.548E-01	-0.659E-01	-0.199E-01	-0.382E-02	-0.438E-02	-0.423E-01	-0.512E-01	-0.155E-01	-0.297E-02	-0.340E-02								
28	2.926	2.328	0.183	0.628	2.725	-0.417E-01	-0.764E-01	-0.147E-01	-0.818E-02	-0.519E-02	-0.432E-01	-0.792E-01	-0.191E-01	-0.848E-02	-0.538E-02								
29	2.750	2.382	0.185	0.378	2.620	-0.273E-01	-0.873E-01	-0.143E-01	-0.137E-01	-0.495E-02	-0.351E-01	-0.112E+00	-0.184E-01	-0.176E-01	-0.637E-02								
30	2.570	2.431	0.188	0.139	2.499	-0.109E-01	-0.980E-01	-0.669E-02	-0.197E-01	-0.272E-02	-0.167E-01	-0.150E+00	-0.102E-01	-0.301E-01	-0.416E-02								
31	2.386	2.478	0.191	-0.089	2.365	-0.765E-02	-0.107E+00	-0.536E-02	-0.249E-01	-0.246E-02	-0.134E-01	-0.188E+00	-0.939E-02	-0.437E-01	-0.431E-02								
32	2.199	2.521	0.193	-0.303	2.217	-0.286E-01	-0.114E+00	-0.223E-01	-0.269E-01	-0.109E-01	-0.563E-01	-0.225E+00	-0.439E-01	-0.530E-01	-0.215E-01								
33	2.009	2.560	0.195	-0.503	2.058	-0.521E-01	-0.117E+00	-0.436E-01	-0.225E-01	-0.214E-01	-0.113E+00	-0.253E+00	-0.946E-01	-0.488E-01	-0.463E-01								
34	1.817	2.596	0.197	-0.688	1.886	-0.709E-01	-0.112E+00	-0.673E-01	-0.846E-02	-0.295E-01	-0.183E+00	-0.264E+00	-0.158E+00	-0.199E-01	-0.694E-01								
35	1.621	2.628	0.199	-0.855	1.704	-0.106E+00	-0.993E-01	-0.892E-01	-0.167E-01	-0.282E-01	-0.267E+00	-0.350E+00	-0.225E+00	-0.421E-01	-0.711E-01								
36	1.424	2.656	0.200	-1.006	1.513	-0.136E+00	-0.754E-01	-0.103E+00	-0.501E-01	-0.102E-01	-0.362E+00	-0.275E+00	-0.201E+00	-0.134E+00	-0.272E-01								
37	1.224	2.681	0.202	-1.138	1.312	-0.166E+00	-0.400E-01	-0.101E+00	-0.822E-01	-0.257E-01	-0.466E+00	-0.172E+00	-0.282E+00	-0.230E+00	-0.721E-01								
38	1.023	2.702	0.203	-1.251	1.105	-0.196E+00	-0.584E-02	-0.764E-01	-0.974E-01	-0.669E-01	-0.572E+00	-0.170E-01	-0.223E+00	-0.284E+00	-0.195E+00								
39	0.820	2.719	0.204	-1.345	0.892	-0.224E+00	-0.586E-01	-0.281E-01	-0.803E-01	-0.737E-01	-0.745E+00	-0.176E+00	-0.242E+00	-0.263E+00	-0.263E+00								
40	0.616	2.733	0.205	-1.418	0.673	-0.249E+00	-0.112E+00	-0.380E-01	-0.237E-01	-0.617E-01	-0.767E+00	-0.346E+00	-0.117E+00	-0.793E-01	-0.190E+00								
41	0.411	2.742	0.205	-1.470	0.451	-0.268E+00	-0.159E+00	-0.107E+00	-0.159E+00	-0.520E-01	-0.840E+00	-0.499E+00	-0.336E+00	-0.163E+00	-0.296E-01								
42	0.206	2.748	0.206	-1.502	0.226	-0.280E+00	-0.191E+00	-0.160E+00	-0.122E+00	-0.887E-01	-0.868E+00	-0.606E+00	-0.507E+00	-0.385E+00	-0.281E+00								
43	0.000	2.750	0.203	-1.513	0.000	-0.284E+00	-0.203E+00	-0.180E+00	-0.119E+00	-0.123E+00	-0.903E+00	-0.644E+00	-0.572E+00	-0.475E+00	-0.392E+00								
Sum_aarc(l), 6.660 Sum_1_w_zz(l)*aarc(l), w/ zz=Fn,Sc,Td,Fr,Fc, 0.347E-01 0.136E-01 0.188E-02 -0.147E-03																							

Sum_sarc(l): 6.660, Sum_1 w_zz(l)*sarc(l), w/ z=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)): -0.208E+01 0.347E-01 0.136E-01 0.188E-02 -0.147E-03

Input: multipole # (0=dipole, 1=quad, etc) m = 1 ellipse axes: (a,b) = (4.20, 2.70) cm, h = 5.00 cm, harmonics evaluated at r = 2.50 cm

Output:
!- z-plane harmonic components 1B*(z) (G) for I=1 A !- components weighted by u(1) - Int_a u/s ds (G-cm) --!
!- at z(1)e^(12j pi/m*1), j=0,...,m, and conjugates, -! (S = 5.48315), [0.76897]
!----- cf. Eqn. (21) or (43) -----!

harmonic component mn+mn (n=0,1,2,...):																			
1	x(1)	y(1)	sarc(1)	u(1)	v(1)	Fn(1)	Sc(1)	Td(1)	Fr(1)	Ff(1)	w_Fn(1)	w_Sc(1)	w_Td(1)	w_Fr(1)	w_Ff(1)				
1	4.200	0.000	0.050	3.528	0.000	-1.56E+00	-2.71E-01	-1.32E-01	-5.08E-02	-1.80E-02	-4.32E+00	-7.47E-01	-3.65E-01	-1.40E-01	-4.97E-02				
2	4.197	0.101	0.101	3.521	0.169	-1.56E+00	-2.70E-01	-1.31E-01	-5.00E-02	-1.76E-02	-4.30E+00	-7.42E-01	-3.61E-01	-1.38E-01	-4.84E-02				
3	4.188	0.202	0.101	3.500	0.338	-1.56E+00	-2.66E-01	-1.28E-01	-4.77E-02	-1.63E-02	-4.26E+00	-7.25E-01	-3.49E-01	-1.30E-01	-4.44E-02				
4	4.174	0.302	0.102	3.465	0.505	-1.56E+00	-2.59E-01	-1.22E-01	-4.39E-02	-1.41E-02	-4.19E+00	-6.98E-01	-3.29E-01	-1.18E-01	-3.79E-02				
5	4.153	0.402	0.103	3.417	0.669	-1.55E+00	-2.49E-01	-1.14E-01	-3.84E-02	-1.15E-02	-4.10E+00	-6.60E-01	-3.01E-01	-1.02E-01	-2.95E-02				
6	4.127	0.502	0.103	3.356	0.829	-1.54E+00	-2.35E-01	-1.03E-01	-3.14E-02	-7.15E-03	-3.98E+00	-6.11E-01	-2.66E-01	-8.13E-02	-1.85E-02				
7	4.095	0.601	0.104	3.281	0.984	-1.53E+00	-2.20E-01	-8.93E-02	-2.28E-02	-2.53E-03	-3.84E+00	-5.53E-01	-2.25E-01	-5.73E-02	-6.34E-03				
8	4.057	0.699	0.106	3.194	1.134	-1.51E+00	-2.01E-01	-7.32E-02	-1.26E-02	-0.276E-03	-3.67E+00	-4.86E-01	-1.77E-01	-3.30E-02	0.669E-03				
9	4.013	0.796	0.107	3.095	1.278	-1.50E+00	-1.77E-01	-5.38E-02	-8.21E-04	0.856E-03	-3.48E+00	-4.11E-01	-1.25E-01	-1.91E-03	0.199E-02				
10	3.964	0.892	0.109	2.984	1.414	-1.48E+00	-1.49E-01	-3.12E-02	0.124E-02	0.147E-02	-3.27E+00	-3.30E-01	-6.92E-02	0.275E-02	0.325E-02				
11	3.910	0.986	0.110	2.862	1.543	-1.45E+00	-1.16E-01	-5.32E-03	0.270E-02	0.208E-02	-3.05E+00	-2.43E-01	-1.11E-02	0.565E-02	0.436E-02				
12	3.850	1.080	0.112	2.731	1.663	-1.43E+00	-7.77E-02	0.240E-02	0.425E-02	0.266E-02	-2.80E+00	-1.52E-01	0.472E-02	0.834E-02	0.522E-02				
13	3.784	1.171	0.114	2.589	1.773	-1.40E+00	-3.36E-02	0.569E-02	0.586E-02	0.315E-02	-2.54E+00	-6.12E-02	0.104E-01	0.107E-01	0.574E-02				
14	3.713	1.262	0.116	2.439	1.874	-1.36E+00	0.169E-02	0.931E-02	0.747E-02	0.349E-02	-2.27E+00	-2.82E-02	0.156E-01	0.125E-01	0.583E-02				
15	3.637	1.350	0.118	2.281	1.964	-1.32E+00	0.744E-02	0.132E-01	0.898E-02	0.359E-02	-1.99E+00	-0.113E-01	0.200E-01	0.136E-01	0.543E-02				
16	3.556	1.436	0.120	2.117	2.043	-1.27E+00	0.139E-01	0.174E-01	0.103E-01	0.337E-02	-1.71E+00	0.188E-01	0.235E-01	0.138E-01	0.454E-02				
17	3.470	1.521	0.122	1.946	2.111	-1.22E+00	0.213E-01	0.218E-01	0.112E-01	0.272E-02	-1.43E+00	0.250E-01	0.257E-01	0.132E-01	0.320E-02				
18	3.379	1.603	0.124	1.770	2.167	-1.15E+00	0.294E-01	0.262E-01	0.116E-01	0.156E-02	-1.15E+00	0.294E-01	0.262E-01	0.116E-01	0.156E-02				
19	3.284	1.683	0.126	1.590	2.211	-1.08E+00	0.384E-01	0.303E-01	0.111E-01	0.190E-03	-8.89E-01	0.316E-01	0.250E-01	0.915E-02	0.156E-02				
20	3.183	1.761	0.128	1.407	2.243	-1.00E+00	0.483E-01	0.343E-01	0.968E-02	-2.54E-02	-6.40E-01	0.308E-01	0.219E-01	0.617E-02	-1.62E-02				
21	3.079	1.836	0.130	1.221	2.262	-9.15E-01	0.590E-01	0.374E-01	0.692E-02	-5.40E-02	-4.14E-01	0.267E-01	0.169E-01	0.313E-02	-2.44E-02				
22	2.970	1.909	0.132	1.035	2.268	-8.16E-01	0.704E-01	0.393E-01	0.263E-02	-8.56E-02	-2.17E-01	0.187E-01	0.105E-01	0.699E-03	-2.28E-02				
Sum_1 w_zz(1j)*sarc(1j), where zz=Fn,Sc,Td,Fr,Ff:											-6.09E+00	-3.84E-01	-1.21E-03	0.578E-02	0.181E-02				
23	2.857	1.979	0.134	0.849	2.262	-7.05E-01	0.824E-01	0.396E-01	-3.32E-02	-1.16E-01	-5.62E-02	0.657E-02	0.315E-02	-2.65E-03	-9.24E-03				
24	2.740	2.047	0.136	0.663	2.243	-5.81E-01	0.947E-01	0.375E-01	-1.09E-01	-1.38E-01	-0.614E-02	-9.99E-02	-3.96E-02	0.115E-02	0.146E-02				
25	2.619	2.111	0.138	0.480	2.211	-4.45E-01	0.107E+00	0.326E-01	-1.98E-01	-1.44E-01	0.128E-02	-3.09E-01	-9.41E-02	0.572E-02	0.417E-02				
26	2.494	2.172	0.140	0.300	2.167	-2.94E-01	0.119E+00	0.241E-01	-2.83E-01	-1.24E-01	0.138E-01	-5.56E-01	-1.13E-01	0.137E-01	0.580E-02				
27	2.366	2.231	0.142	0.124	2.111	-1.29E-01	0.129E+00	0.117E-01	-3.82E-01	-6.69E-02	0.832E-02	-8.32E-01	-7.52E-02	0.246E-01	0.431E-02				
28	2.235	2.286	0.143	-0.047	2.043	0.514E-02	0.137E+00	-5.04E-02	-4.48E-01	0.312E-02	-4.19E-02	-1.12E+00	0.411E-02	0.366E-01	-2.54E-02				
29	2.100	2.338	0.145	-0.212	1.964	0.247E-01	0.143E+00	-2.58E-01	-4.69E-01	0.166E-01	-2.42E-01	-1.40E+00	0.253E-01	0.460E-01	-1.63E-01				
30	1.963	2.387	0.147	-0.369	1.874	0.457E-01	0.144E+00	-4.97E-01	-4.20E-01	0.318E-01	-5.20E-01	-1.64E+00	0.565E-01	0.478E-01	-3.62E-01				
31	1.822	2.433	0.148	-0.519	1.773	0.681E-01	0.141E+00	-7.50E-01	-2.82E-01	0.449E-01	-8.78E-01	-1.81E+00	0.967E-01	0.364E-01	-5.78E-01				
32	1.680	2.475	0.150	-0.661	1.663	0.917E-01	0.131E+00	-9.93E-01	-4.61E-02	0.506E-01	-1.131E+00	-1.87E+00	0.142E+00	0.659E-02	-7.23E-01				
33	1.534	2.513	0.151	-0.792	1.543	0.116E+00	0.114E+00	-1.19E+00	0.276E-01	0.433E-01	-1.82E+00	-1.78E+00	0.186E+00	-4.31E-01	-6.76E-01				
34	1.387	2.548	0.152	-0.914	1.414	0.141E+00	0.892E-01	-1.30E+00	0.642E-01	0.194E-01	-2.60E+00	-1.50E+00	0.219E+00	-1.08E+00	-3.27E-01				
35	1.238	2.580	0.153	-1.025	1.278	0.167E+00	0.570E-01	-1.28E+00	0.982E-01	-1.56E-01	-2.99E+00	-1.02E+00	0.230E+00	-1.78E+00	0.351E-01				
36	1.087	2.608	0.154	-1.124	1.134	0.191E+00	0.182E-01	-1.10E+00	0.120E+00	-6.50E-01	-3.62E+00	-5.10E-01	0.209E+00	-2.27E+00	0.201E+00				
37	0.935	2.632	0.155	-1.211	0.984	0.231E+00	-2.58E-01	-7.59E-01	0.121E+00	-1.02E+00	-4.26E+00	0.344E-01	0.150E+00	-2.23E+00	0.230E+00				
38	0.781	2.653	0.155	-1.286	0.829	0.237E+00	-7.22E-01	-6.1E-01	0.939E-01	-1.12E+00	-4.86E+00	0.148E+00	0.535E-01	-1.93E+00	0.230E+00				
39	0.626	2.670	0.156	-1.347	0.669	0.256E+00	-1.18E+00	0.340E-01	0.408E-01	-1.12E+00	-5.68E+00	0.249E+00	-7.19E-01	-1.93E+00	0.230E+00				
40	0.470	2.683	0.156	-1.395	0.505	0.272E+00	-1.59E+00	0.961E-01	-2.94E-01	-2.22E-01	-5.88E+00	0.343E+00	-2.08E+00	0.637E-01	0.480E-01				
41	0.314	2.692	0.157	-1.430	0.338	0.284E+00	-1.91E+00	0.151E+00	-1.00E+00	0.568E-01	-6.23E+00	0.420E+00	-3.31E+00	0.221E+00	-1.25E+00				
42	0.157	2.698	0.157	-1.451	0.169	0.291E+00	-2.12E+00	0.188E+00	-1.53E+00	0.123E+00	-6.46E+00	0.471E+00	-4.41E+00	0.341E+00	-2.72E+00				
43	0.000	2.700	0.079	-1.458	0.000	0.293E+00	-2.19E+00	0.201E+00	-1.73E+00	0.148E+00	-6.53E+00	0.489E+00	-4.48E+00	0.385E+00	-3.30E+00				
Sum_sarc(1):										5.483,	Sum_1 w_zz(1j)*sarc(1j), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):	-1.38E+01	0.532E-01	0.774E-02	0.516E-03	-1.36E-03			

4

program eddy_str

```

multipole # (0-dipole, 1-quad, etc) m = 1 ellipse axes: (a,b)=( 2.67, 2.67) cm, str = 1.00 cm, h = 5.00 cm, harmonics evaluated at r= 2.50 cm

Input:

|- z-plane harmonic components IB*(z) (G) for I=1 A |- components weighted by u(i) - Int a u/S ds (G-cm) --|
|- at z(i)e^(i2j pi/m+1), j=0,...,m, and conjugates, -| (S = 4.70158), { 0.19864 }

Output:

```

Output:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86														

	Sum_i w_zz(ij)*sarc(ij), where zz=Fn.Sc,Td,Fr,Ffi	- .488E+00	- .883E-01	- .308E-02	0.119E-01	0.179E-02
Sum_sarc(i):	4.702	Sum_i w_zz(ij)*sarc(ij), w/ zz=Fn.Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):	- .104E+01	0.329E-01	- .154E-01	0.107E-01
2.143	2.111	0.111	0.027	1.810	- .372E-02	0.170E+00
2.054	2.177	0.111	- 0.104	1.789	0.144E-01	0.172E+00
1.963	2.239	0.111	- 0.232	1.758	0.328E-01	0.171E+00
1.869	2.298	0.111	- 0.357	1.718	0.514E-01	0.166E+00
1.773	2.353	0.111	- 0.478	1.669	0.701E-01	0.158E+00
1.675	2.403	0.111	- 0.594	1.610	0.889E-01	0.146E+00
1.575	2.450	0.111	- 0.704	1.543	0.108E+00	0.131E+00
1.472	2.492	0.111	- 0.808	1.476	0.126E+00	0.112E+00
1.369	2.530	0.111	- 0.906	1.385	0.145E+00	0.100E-01
1.263	2.564	0.111	- 0.995	1.296	0.163E+00	0.049E-01
1.157	2.593	0.111	- 1.077	1.200	0.180E+00	0.371E-01
1.049	2.618	0.111	- 1.151	1.098	0.197E+00	0.720E-02
0.940	2.639	0.111	- 1.216	0.992	0.214E-01	- 2.44E-01
0.831	2.654	0.111	- 1.271	0.882	0.229E+00	- .569E-01
0.721	2.666	0.111	- 1.317	0.769	0.247E-01	- .897E-01
0.611	2.673	0.111	- 1.354	0.653	0.257E+00	- 1.22E-01
0.500	2.675	0.118	- 1.381	0.535	0.270E+00	- .153E+00
0.375	2.675	0.125	- 1.403	0.401	0.282E+00	- .183E+00
0.250	2.675	0.125	- 1.419	0.267	0.291E+00	- .208E+00
0.125	2.675	0.125	- 1.428	0.134	0.296E+00	- .223E+00
0.000	2.675	0.063	- 1.431	0.000	0.298E+00	- .228E+00
2.143	2.111	0.111	0.027	1.810	- .372E-02	0.170E+00
2.054	2.177	0.111	- 0.104	1.789	0.144E-01	0.172E+00
1.963	2.239	0.111	- 0.232	1.758	0.328E-01	0.171E+00
1.869	2.298	0.111	- 0.357	1.718	0.514E-01	0.166E+00
1.773	2.353	0.111	- 0.478	1.669	0.701E-01	0.158E+00
1.675	2.403	0.111	- 0.594	1.610	0.889E-01	0.146E+00
1.575	2.450	0.111	- 0.704	1.543	0.108E+00	0.131E+00
1.472	2.492	0.111	- 0.808	1.476	0.126E+00	0.112E+00
1.369	2.530	0.111	- 0.906	1.385	0.145E+00	0.100E-01
1.263	2.564	0.111	- 0.995	1.296	0.163E+00	0.049E-01
1.157	2.593	0.111	- 1.077	1.200	0.180E+00	0.371E-01
1.049	2.618	0.111	- 1.151	1.098	0.197E+00	0.720E-02
0.940	2.639	0.111	- 1.216	0.992	0.214E-01	- 2.44E-01
0.831	2.654	0.111	- 1.271	0.882	0.229E+00	- .569E-01
0.721	2.666	0.111	- 1.317	0.769	0.247E-01	- .897E-01
0.611	2.673	0.111	- 1.354	0.653	0.257E+00	- 1.22E-01
0.500	2.675	0.118	- 1.381	0.535	0.270E+00	- .153E+00
0.375	2.675	0.125	- 1.403	0.401	0.282E+00	- .183E+00
0.250	2.675	0.125	- 1.419	0.267	0.291E+00	- .208E+00
0.125	2.675	0.125	- 1.428	0.134	0.296E+00	- .223E+00
0.000	2.675	0.063	- 1.431	0.000	0.298E+00	- .228E+00
2.143	2.111	0.111	0.027	1.810	- .372E-02	0.170E+00
2.054	2.177	0.111	- 0.104	1.789	0.144E-01	0.172E+00
1.963	2.239	0.111	- 0.232	1.758	0.328E-01	0.171E+00
1.869	2.298	0.111	- 0.357	1.718	0.514E-01	0.166E+00
1.773	2.353	0.111	- 0.478	1.669	0.701E-01	

program eddy

Input: multipole # (0=dipole, l=quad, etc) m = 1 ellipse axes: (a,b) = (4.57, 4.57) cm, h = 5.00 cm, harmonics evaluated at r= 2.50 cm
 Output: l= 2-plane harmonic components 1B*(z) (G) for l=1 A l= components weighted by u(l) - Int_s u/S ds (G-cm) --l
 l= at z(l)e^(i2j pi/m+1), j=0,...,m, and conjugates, -l (S = 7.18597), [0.00000]

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																
harmonic component	x(l)	y(l)	u(l)	v(l)	Fn(l)	Sc(l)	Td(l)	Fz(l)	Ff(l)	w_Fn(l)	w_Sc(l)	w_Td(l)	w_Fz(l)	w_Ff(l)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
mn+tn (n=0,1,2,...):	4.575	0.000	0.086	4.186	0.000	-1.45E+00	-1.65E-01	-7.49E-02	-2.55E-02	-7.72E-03	-6.08E+00	-6.92E-01	-3.14E-01	-1.07E-01	-3.23E-02	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
1	4.572	0.171	0.171	4.174	0.313	-1.45E+00	-1.62E-01	-7.28E-02	-2.44E-02	-7.20E-03	-6.05E+00	-6.75E-01	-3.04E-01	-1.02E-01	-3.00E-02	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
2	4.562	0.342	0.171	4.139	0.624	-1.44E+00	-1.52E-01	-7.63E-02	-2.11E-02	-5.70E-03	-5.96E+00	-6.28E-01	-2.75E-01	-8.73E-02	-2.36E-02	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43		
3	4.546	0.512	0.171	4.081	0.932	-1.42E+00	-1.35E-01	-7.59E-02	-1.59E-02	-4.34E-03	-5.81E+00	-5.51E-01	-2.28E-01	-6.50E-02	-1.40E-02	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43			
4	4.524	0.682	0.171	4.000	1.234	-1.40E+00	-1.12E-01	-7.31E-02	-1.19E-02	-3.12E-03	-5.61E+00	-5.19E-01	-2.08E-01	-6.19E-02	-1.28E-02	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43				
5	4.495	0.851	0.171	3.897	1.529	-1.37E+00	-8.86E-02	-7.00E-02	-1.19E-02	-1.93E-03	-5.35E+00	-4.92E-01	-1.99E-01	-7.53E-02	-1.01E-02	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43					
6	4.460	1.018	0.171	3.772	1.816	-1.34E+00	-8.49E-02	-6.56E-02	-1.04E-02	-1.56E-03	-5.05E+00	-4.62E-01	-1.87E-01	-7.25E-02	-1.12E-02	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43						
7	4.419	1.184	0.171	3.625	2.093	-1.30E+00	-8.11E-02	-5.84E-02	-9.12E-03	-1.26E-03	-4.70E+00	-4.27E-01	-1.76E-01	-6.90E-02	-1.07E-02	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43							
8	4.372	1.349	0.171	3.459	2.358	-1.25E+00	-7.73E-02	-5.12E-02	-8.16E-03	-1.06E-03	-4.32E+00	-3.89E-01	-1.61E-01	-6.44E-02	-9.52E-03	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43								
9	4.318	1.511	0.171	3.273	2.610	-1.20E+00	-7.30E-02	-4.43E-02	-7.00E-03	-9.13E-04	-3.91E+00	-3.48E-01	-1.45E-01	-6.16E-02	-1.23E-02	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43									
10	4.259	1.671	0.171	3.069	2.847	-1.13E+00	-6.83E-02	-3.68E-02	-5.80E-03	-8.21E-04	-3.48E+00	-3.04E-01	-1.26E-01	-5.84E-02	-1.10E-02	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43										
11	4.259	1.671	0.171	3.069	2.847	-1.13E+00	-6.83E-02	-3.68E-02	-5.80E-03	-8.21E-04	-3.48E+00	-3.04E-01	-1.26E-01	-5.84E-02	-1.10E-02	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43										
12	4.193	1.830	0.171	2.847	3.069	-1.07E+00	-6.33E-02	-2.92E-02	-4.58E-03	-7.21E-04	-3.04E+00	-2.59E-01	-1.03E-01	-5.57E-02	-9.65E-03	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43											
13	4.122	1.985	0.171	2.610	3.273	-9.92E-01	-5.75E-01	-2.17E-02	-3.24E-03	-1.19E-03	-2.59E+00	-2.15E-01	-8.73E-02	-4.18E-02	-1.01E-02	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43												
14	4.045	2.138	0.171	2.358	3.459	-9.10E-01	-5.25E-01	-1.40E-02	-2.24E-03	-1.12E-03	-1.72E+00	-1.42E-01	-7.72E-02	-3.55E-02	-1.23E-02	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43													
15	3.962	2.287	0.171	2.093	3.625	-8.21E-01	-4.73E-01	-1.04E-01	-1.12E-02	-5.42E-03	-1.22E+00	-1.07E-01	-6.78E-02	-4.99E-02	-1.49E-02	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43														
16	3.874	2.434	0.171	1.816	3.772	-7.23E-01	-4.30E-01	-9.10E-02	-9.10E-02	-4.80E-03	-1.13E+00	-1.03E-01	-6.32E-01	-5.09E-02	-1.62E-02	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43															
17	3.780	2.577	0.171	1.529	3.897	-6.18E-01	-3.83E-01	-7.30E-02	-8.21E-03	-6.21E-03	-9.46E-01	-8.57E-01	-5.57E-01	-4.16E-02	-2.10E-02	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																
18	3.681	2.717	0.171	1.234	4.000	-5.06E-01	-3.43E-01	-6.03E-02	-5.02E-03	-8.55E-03	-8.24E-01	-7.47E-01	-5.57E-01	-4.16E-02	-2.10E-02	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																	
19	3.577	2.852	0.171	0.932	4.081	-3.86E-01	-2.83E-01	-4.67E-02	-3.12E-02	-1.12E-02	-6.03E-01	-5.40E-01	-4.16E-01	-2.94E-02	-1.70E-02	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																		
20	3.468	2.984	0.171	0.624	4.139	-2.61E-01	-2.07E-01	-3.41E-02	-2.73E-02	-5.87E-03	-5.16E-01	-4.50E-01	-3.16E-01	-2.09E-02	-1.23E-02	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																			
21	3.354	3.112	0.171	0.313	4.174	-1.32E-01	-1.04E-01	-2.43E-02	-3.13E-02	-1.16E-02	-4.12E-01	-3.56E-01	-2.28E-01	-1.45E-02	-8.90E-03	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																				
22	3.235	3.235	0.171	0.000	4.186	0.291E-07	0.577E-01	-5.46E-08	-3.27E-02	0.729E-09	-1.56E-13	-2.82E-07	0.292E-14	0.175E-08	-3.90E-15	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43																					
23	3.112	3.354	0.171	-0.313	4.174	0.132E-01	0.522E-01	-2.13E-02	-3.13E-02	0.316E-03	-4.12E-02	-1.63E-01	0.760E-03	0.978E-03	-9.89E-04	23	24	<																																								

eddy_out.dat Thu Aug 22 13:36:44 1991 1

program eddy

Input: multipole # (0-dipole, 1-quadr, etc) m = 1 ellipse axes: (a,b) = (3.58, 3.58) cm, h = 5.00 cm, harmonics evaluated at r = 2.50 cm

Output:

!- z-plane harmonic components IB*(z) (G) for I=1 A !- components weighted by u(1) - Int_s u/s ds (G-cm) --!
!- at z(1)e^(i2j pi/m+1), j=0,...,m, and conjugates, -! (S = 5.61527),
!----- cf. Eqn. (21) or (43) -----!
[0.00000]

harmonic component mn+m+n (n=0,1,2,...):

1	x(1)	y(1)	sarc(1)	u(1)	v(1)	Fn(1)	Sc(1)	Td(1)	Fr(1)	Ff(1)	W_Fn(1)	W_Sc(1)	W_Td(1)	W_Ff(1)	W_Ff(1)
1	3.575	0.000	0.067	2.556	0.000	-0.189E+00	-0.620E-01	-0.366E-01	-0.184E-01	-0.897E-02	-0.482E+00	-0.158E+00	-0.935E-01	-0.470E-01	-0.229E-01
2	3.573	0.134	0.134	2.549	0.191	-0.189E+00	-0.611E-01	-0.356E-01	-0.176E-01	-0.897E-02	-0.480E+00	-0.156E+00	-0.908E-01	-0.448E-01	-0.213E-01
3	3.565	0.267	0.134	2.528	0.381	-0.187E+00	-0.585E-01	-0.329E-01	-0.152E-01	-0.858E-02	-0.472E+00	-0.148E+00	-0.831E-01	-0.384E-01	-0.166E-01
4	3.553	0.400	0.134	2.492	0.569	-0.184E+00	-0.543E-01	-0.284E-01	-0.115E-01	-0.790E-02	-0.459E+00	-0.135E+00	-0.708E-01	-0.287E-01	-0.972E-02
5	3.535	0.533	0.134	2.443	0.753	-0.181E+00	-0.484E-01	-0.225E-01	-0.678E-02	-0.686E-03	-0.442E+00	-0.118E+00	-0.549E-01	-0.166E-01	-0.168E-02
6	3.513	0.665	0.134	2.379	0.934	-0.176E+00	-0.412E-01	-0.154E-01	-0.146E-02	-0.262E-02	-0.420E+00	-0.980E-01	-0.368E-01	-0.348E-02	0.623E-02
7	3.485	0.796	0.134	2.303	1.109	-0.171E+00	-0.326E-01	-0.745E-02	-0.397E-02	0.556E-02	-0.394E+00	-0.751E-01	-0.172E-01	0.914E-02	0.128E-01
8	3.453	0.925	0.134	2.214	1.278	-0.165E+00	-0.230E-01	-0.876E-03	0.904E-02	0.773E-02	-0.364E+00	-0.508E-01	0.194E-02	0.200E-01	0.171E-01
9	3.416	1.054	0.134	2.112	1.440	-0.157E+00	-0.124E-01	0.921E-02	0.133E-01	0.883E-02	-0.332E+00	-0.262E-01	0.194E-01	0.281E-01	0.186E-01
10	3.374	1.181	0.134	1.998	1.594	-0.149E+00	-0.123E-02	0.171E-01	0.164E-01	0.870E-02	-0.247E+00	-0.247E-02	0.342E-01	0.327E-01	0.174E-01
11	3.328	1.306	0.134	1.874	1.739	-0.140E+00	0.104E-01	0.242E-01	0.180E-01	0.737E-02	-0.263E+00	0.194E-01	0.453E-01	0.336E-01	0.138E-01
12	3.277	1.430	0.134	1.739	1.874	-0.130E+00	0.221E-01	0.301E-01	0.179E-01	0.502E-02	-0.227E+00	0.384E-01	0.524E-01	0.312E-01	0.872E-02
13	3.221	1.551	0.134	1.594	1.998	-0.120E+00	0.337E-01	0.345E-01	0.163E-01	0.197E-02	-0.191E+00	0.538E-01	0.551E-01	0.260E-01	0.313E-02
14	3.161	1.670	0.134	1.440	2.112	-0.109E+00	0.450E-01	0.373E-01	0.132E-01	-0.136E-02	-0.156E+00	0.648E-01	0.537E-01	0.190E-01	-0.196E-02
15	3.096	1.788	0.134	1.278	2.214	-0.965E-01	0.556E-01	0.382E-01	0.894E-02	-0.450E-02	-0.123E+00	0.711E-01	0.488E-01	0.114E-01	-0.575E-02
16	3.027	1.902	0.134	1.109	2.303	-0.840E-01	0.654E-01	0.372E-01	0.385E-02	-0.702E-02	-0.931E-01	0.726E-01	0.412E-01	0.427E-02	-0.779E-02
17	2.954	2.014	0.134	0.934	2.379	-0.709E-01	0.741E-01	0.343E-01	-0.61E-02	-0.857E-02	-0.662E-01	0.692E-01	0.320E-01	-0.15E-02	-0.801E-02
18	2.876	2.123	0.134	0.753	2.443	-0.573E-01	0.815E-01	0.298E-01	-0.695E-02	-0.894E-02	-0.431E-01	0.614E-01	0.224E-01	-0.523E-02	-0.674E-02
19	2.795	2.229	0.134	0.569	2.492	-0.433E-01	0.874E-01	0.237E-01	-0.117E-01	-0.808E-02	-0.246E-01	0.497E-01	0.135E-01	-0.665E-02	-0.460E-02
20	2.710	2.332	0.134	0.381	2.528	-0.290E-01	0.918E-01	0.165E-01	-0.154E-01	-0.610E-02	-0.111E-01	0.350E-01	0.629E-02	-0.587E-02	-0.232E-02
21	2.621	2.432	0.134	0.191	2.549	-0.146E-01	0.944E-01	0.846E-02	-0.178E-01	-0.328E-02	-0.278E-02	0.180E-01	0.162E-02	-0.340E-02	-0.626E-03
22	2.528	2.528	0.134	0.000	2.556	0.205E-07	0.933E-01	-0.121E-07	-0.186E-01	0.476E-08	0.170E-15	0.790E-09	-0.100E-15	-0.154E-09	0.395E-16
23	2.432	2.621	0.134	-0.191	2.549	0.146E-01	0.944E-01	-0.846E-02	-0.178E-01	0.328E-02	-0.278E-02	-0.180E-01	0.162E-02	0.340E-02	-0.626E-03
24	2.332	2.710	0.134	-0.381	2.528	0.290E-01	0.918E-01	-0.165E-01	-0.154E-01	0.610E-02	-0.111E-01	-0.350E-01	0.629E-02	0.587E-02	-0.232E-02
25	2.229	2.795	0.134	-0.569	2.492	0.433E-01	0.874E-01	-0.237E-01	-0.117E-01	0.808E-02	-0.246E-01	-0.497E-01	0.135E-01	0.665E-02	-0.460E-02
26	2.123	2.876	0.134	-0.753	2.443	0.573E-01	0.815E-01	-0.298E-01	-0.695E-02	-0.894E-02	-0.431E-01	-0.614E-01	0.224E-01	0.523E-02	-0.674E-02
27	2.014	2.954	0.134	-0.934	2.379	0.709E-01	0.741E-01	-0.343E-01	-0.161E-02	-0.857E-02	-0.662E-01	-0.692E-01	0.320E-01	0.151E-02	-0.801E-02
28	1.902	3.027	0.134	-1.109	2.303	0.840E-01	0.654E-01	-0.372E-01	0.395E-02	-0.702E-02	-0.931E-01	-0.726E-01	0.412E-01	-0.427E-02	-0.779E-02
29	1.787	3.096	0.134	-1.278	2.214	0.965E-01	0.556E-01	-0.382E-01	0.894E-02	-0.450E-02	-0.123E+00	-0.711E-01	0.488E-01	-0.114E-01	-0.575E-02
30	1.670	3.161	0.134	-1.440	2.112	0.109E+00	0.450E-01	-0.372E-01	0.136E-02	-0.156E+00	-0.648E-01	0.537E-01	0.537E-01	-0.190E-01	-0.196E-02
31	1.551	3.221	0.134	-1.594	1.998	0.120E+00	0.337E-01	-0.345E-01	0.163E-01	-0.197E-02	-0.191E+00	-0.538E-01	0.551E-01	-0.260E-01	0.313E-02
32	1.430	3.277	0.134	-1.739	1.874	0.130E+00	0.221E-01	-0.301E-01	0.179E-01	-0.502E-02	-0.227E+00	-0.384E-01	0.524E-01	-0.312E-01	0.872E-02
33	1.306	3.328	0.134	-1.874	1.739	0.140E+00	0.104E-01	-0.242E-01	0.180E-01	-0.737E-02	-0.263E+00	-0.194E-01	0.453E-01	-0.336E-01	0.138E-01
34	1.181	3.374	0.134	-1.998	1.594	0.149E+00	-0.123E-02	0.171E-01	0.164E-01	-0.870E-02	-0.247E+00	0.247E-02	0.342E-01	-0.327E-01	0.174E-01
35	1.054	3.416	0.134	-2.112	1.440	0.157E+00	-0.124E-01	0.921E-02	0.133E-01	-0.883E-02	-0.332E+00	-0.262E-01	0.194E-01	-0.281E-01	0.186E-01
36	0.925	3.453	0.134	-2.214	1.278	0.165E+00	-0.230E-01	-0.876E-03	0.904E-02	0.773E-02	-0.364E+00	-0.508E-01	0.194E-02	-0.200E-01	0.171E-01
37	0.796	3.485	0.134	-2.303	1.109	0.171E+00	-0.326E-01	0.745E-02	0.397E-02	0.556E-02	-0.394E+00	-0.751E-01	-0.172E-01	-0.914E-02	0.128E-01
38	0.665	3.513	0.134	-2.379	0.934	0.176E+00	-0.424E-01	0.176E-01	-0.146E-02	-0.262E-02	-0.420E+00	-0.980E-01	-0.368E-01	0.348E-02	0.623E-02
39	0.533	3.535	0.134	-2.443	0.753	0.181E+00	-0.484E-01	0.225E-01	-0.167E-02	-0.686E-03	-0.442E+00	0.118E+00	-0.549E-01	0.166E-01	-0.168E-02
40	0.400	3.553	0.134	-2.492	0.569	0.184E+00	-0.543E-01	0.284E-01	-0.115E-01	0.790E-02	-0.459E+00	-0.135E+00	-0.708E-01	0.287E-01	-0.972E-02
41	0.267	3.565	0.134	-2.528	0.381	0.187E+00	-0.329E-01	0.329E-01	-0.152E-01	-0.858E-02	-0.472E+00	0.148E+00	-0.831E-01	0.384E-01	-0.166E-01
42	0.134	3.573	0.134	-2.549	0.191	0.188E+00	-0.611E-01	0.356E-01	-0.176E-01	-0.610E-02	-0.480E+00	0.156E+00	-0.908E-01	-0.213E-01	-0.213E-01
43	0.000	3.575	0.067	-2.556	0.000	0.189E+00	-0.620E-01	-0.366E-01	-0.184E-01	0.897E-02	-0.482E+00	-0.158E+00	-0.935E-01	0.470E-01	-0.229E-01
Sum_sarc(1): 5.615, Sum_1 w_zz(1)*sarc(1), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):															-0.182E-03
Sum_1 w_zz(1)*sarc(1), where zz=Fn,Sc,Td,Fr,Ff:															-0.911E-04

sextupole vacuum chamber

(1 page)

program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 2 ellipse axes: (a,b) = (4.07, 4.07) cm, h = 5.00 cm, harmonics evaluated at r= 2.50 cm

Output:

!- z-plane harmonic components $ib^*(z)$ (G) for $l=1$ A !- components weighted by $u(l) - \text{Int}_s u/s ds$ (G-cm) !-
 !- at $z(l)e^{i(2j\pi/m+1)}$, $j=0, \dots, m$, and conjugates, !- (S = 4.26722),
 !- ----- cf. Eqn. (21) or (43) -----

zz=Eqn. (24) or (25)															
1	x(l)	y(l)	sarc(l)	u(l)	v(l)	2	5	8	11	14	2	5	8	11	14
harmonic component	mn+mn					Fn(l)	Sc(l)	Td(l)	Fr(l)	Ff(l)	w_Fn(l)	w_Sc(l)	w_Td(l)	w_Fr(l)	w_Ff(l)
1	4.075	0.000	0.051	2.707	0.000	-136E+00	-202E-01	-574E-02	-137E-02	-316E-03	-369E+00	-547E-01	-155E-01	-371E-02	-855E-03
2	4.074	0.102	0.102	2.699	0.202	-136E+00	-199E-01	-560E-02	-131E-02	-294E-03	-367E+00	-538E-01	-151E-01	-354E-02	-794E-03
3	4.070	0.203	0.102	2.676	0.403	-135E+00	-191E-01	-516E-02	-113E-02	-232E-03	-361E+00	-510E-01	-138E-01	-304E-02	-620E-03
4	4.064	0.305	0.102	2.639	0.602	-133E+00	-176E-01	-445E-02	-858E-03	-138E-03	-351E+00	-465E-01	-118E-01	-227E-02	-363E-03
5	4.055	0.406	0.102	2.586	0.798	-131E+00	-157E-01	-352E-02	-506E-03	-244E-04	-338E+00	-405E-01	-909E-02	-131E-02	-631E-04
6	4.043	0.507	0.102	2.520	0.989	-127E+00	-132E-01	-239E-02	-110E-03	-920E-04	-321E+00	-333E-01	-603E-02	-278E-03	0.232E-03
7	4.029	0.607	0.102	2.439	1.174	-124E+00	-104E-01	-114E-02	0.295E-03	0.196E-03	-301E+00	-253E-01	-279E-02	0.719E-03	0.477E-03
8	4.013	0.708	0.102	2.344	1.353	-119E+00	-712E-02	0.173E-03	0.673E-03	0.272E-03	-279E+00	-167E-01	0.405E-03	0.158E-02	0.637E-03
9	3.994	0.807	0.102	2.236	1.525	-114E+00	-357E-02	0.149E-02	0.990E-03	0.311E-03	-255E+00	-799E-02	0.333E-02	0.221E-02	0.694E-03
10	3.973	0.907	0.102	2.116	1.688	-108E+00	0.190E-03	0.274E-02	0.122E-02	0.306E-03	-229E+00	0.402E-03	0.580E-02	0.258E-02	0.648E-03
11	3.949	1.006	0.102	1.984	1.841	-102E+00	0.409E-02	0.386E-02	0.134E-02	0.259E-03	-202E+00	0.812E-02	0.766E-02	0.265E-02	0.514E-03
12	3.923	1.104	0.102	1.841	1.984	-945E-01	0.805E-02	0.479E-02	0.134E-02	0.176E-03	-174E+00	0.148E-01	0.882E-02	0.246E-02	0.324E-03
13	3.894	1.201	0.102	1.688	2.116	-869E-01	0.120E-01	0.549E-02	0.121E-02	0.688E-04	-147E+00	0.202E-01	0.927E-02	0.203E-02	0.116E-03
14	3.863	1.298	0.102	1.525	2.236	-787E-01	0.158E-01	0.592E-02	0.983E-03	-483E-04	-120E+00	0.240E-01	0.903E-02	0.150E-02	-736E-04
15	3.829	1.394	0.102	1.353	2.344	-701E-01	0.193E-01	0.606E-02	0.663E-03	-159E-03	-949E-01	0.262E-01	0.821E-02	0.897E-03	-215E-03
16	3.793	1.489	0.102	1.174	2.439	-610E-01	0.226E-01	0.590E-02	0.282E-03	-248E-03	-716E-01	0.266E-01	0.693E-02	0.331E-03	-291E-03
17	3.755	1.583	0.102	0.989	2.520	-515E-01	0.256E-01	0.545E-02	-1.26E-03	-3.02E-03	-509E-01	0.253E-01	0.539E-02	-1.24E-03	-2.99E-03
18	3.714	1.676	0.102	0.798	2.586	-416E-01	0.281E-01	0.473E-02	-5.24E-03	-3.15E-03	-332E-01	0.224E-01	0.377E-02	-4.18E-03	-2.52E-03
19	3.671	1.768	0.102	0.602	2.639	-315E-01	0.301E-01	0.377E-02	-8.87E-03	-2.85E-03	-190E-01	0.181E-01	0.227E-02	-5.29E-03	-1.72E-03
20	3.626	1.859	0.102	0.403	2.676	-211E-01	0.316E-01	0.262E-02	-1.16E-02	-1.15E-03	-852E-02	0.127E-01	0.106E-02	-4.66E-03	-8.67E-04
21	3.579	1.949	0.102	0.202	2.699	-106E-01	0.325E-01	0.134E-02	-1.33E-02	-1.15E-03	-214E-02	0.657E-02	0.272E-03	-2.70E-03	-2.33E-04
22	3.529	2.037	0.102	0.000	2.707	0.000E+00	0.328E-01	0.000E+00	-1.39E-02	0.000E+00	0.000E+00	0.378E-08	0.000E+00	-1.61E-09	0.000E+00
23	3.477	2.125	0.102	-0.202	2.699	0.106E-01	0.325E-01	-1.14E-02	-1.33E-02	0.115E-03	-2.14E-02	-657E-02	0.595E-03	0.293E-03	-3.64E-05
24	3.423	2.211	0.102	-0.403	2.676	0.211E-01	0.316E-01	-2.62E-02	-1.16E-02	0.215E-03	-852E-02	-1.27E-01	0.106E-02	0.270E-03	-2.33E-04
25	3.367	2.296	0.102	-0.602	2.639	0.315E-01	0.301E-01	-3.77E-02	-8.78E-03	0.285E-03	-852E-02	-1.81E-01	0.227E-02	0.466E-03	-8.67E-04
26	3.309	2.379	0.102	-0.798	2.586	0.416E-01	0.281E-01	-4.73E-02	-5.24E-03	0.315E-03	-332E-01	-2.24E-01	0.377E-02	0.418E-03	-2.52E-03
27	3.248	2.461	0.102	-0.989	2.520	0.515E-01	0.256E-01	-5.45E-02	-1.26E-03	0.302E-03	-509E-01	-2.53E-01	0.539E-02	0.124E-03	-2.99E-03
28	3.186	2.541	0.102	-1.174	2.439	0.610E-01	0.226E-01	-5.90E-02	0.282E-03	0.248E-03	-716E-01	-2.66E-01	0.693E-02	-3.31E-03	-2.91E-03
29	3.122	2.619	0.102	-1.353	2.344	0.701E-01	0.193E-01	-6.06E-02	0.663E-03	0.159E-03	-949E-01	-2.62E-01	0.821E-02	-8.97E-03	-2.15E-03
30	3.055	2.696	0.102	-1.525	2.236	0.787E-01	0.158E-01	-5.92E-02	0.983E-03	0.483E-04	-120E+00	-2.40E-01	0.903E-02	-1.50E-02	-7.36E-04
31	2.987	2.772	0.102	-1.688	2.116	0.869E-01	0.120E-01	-5.49E-02	0.121E-02	-6.88E-04	-147E+00	-2.02E-01	0.927E-02	-2.05E-02	0.116E-03
32	2.917	2.845	0.102	-1.841	1.984	0.945E-01	0.805E-02	-4.79E-02	0.134E-02	-1.76E-03	-174E+00	-1.48E-01	0.882E-02	-2.46E-02	0.324E-03
33	2.845	2.917	0.102	-1.984	1.841	0.102E+00	0.409E-02	-3.86E-02	0.134E-02	-2.59E-03	-202E+00	-4.02E-03	0.580E-02	-2.58E-02	0.514E-03
34	2.772	2.987	0.102	-2.116	1.688	0.108E+00	0.190E-03	-2.74E-02	0.122E-02	-3.06E-03	-229E+00	-8.12E-02	0.766E-02	-2.65E-02	0.648E-03
35	2.696	3.055	0.102	-2.236	1.525	0.114E+00	-3.57E-02	-1.49E-02	0.990E-03	-3.11E-03	-253E+00	0.799E-02	0.333E-02	-2.21E-02	0.694E-03
36	2.619	3.122	0.102	-2.344	1.353	0.119E+00	-7.12E-02	-1.73E-03	0.673E-03	-2.72E-03	-301E+00	0.253E-01	-2.79E-02	-1.58E-02	0.637E-03
37	2.541	3.186	0.102	-2.439	1.174	0.124E+00	-1.04E-01	0.114E-02	0.295E-03	-1.96E-03	-279E+00	0.253E-01	-2.79E-02	-1.58E-02	0.637E-03
38	2.461	3.248	0.102	-2.520	0.989	0.127E+00	-1.32E-01	0.239E-02	-1.10E-03	-9.20E-04	-321E+00	0.333E-01	-6.03E-02	0.278E-03	0.232E-03
39	2.379	3.309	0.102	-2.586	0.798	0.131E+00	-1.57E-01	0.352E-02	-5.06E-03	-2.44E-04	-338E+00	0.405E-01	-9.09E-02	0.131E-02	-6.31E-04
40	2.296	3.367	0.102	-2.639	0.602	0.133E+00	-1.76E-01	0.445E-02	-8.58E-03	-1.38E-03	-351E+00	-4.65E-01	-1.18E-01	0.227E-02	-3.63E-03
41	2.211	3.423	0.102	-2.676	0.403	0.135E+00	-1.91E-01	0.516E-02	-1.13E-02	0.232E-03	-361E+00	0.510E-01	-1.38E-01	0.304E-02	-6.20E-03
42	2.125	3.477	0.102	-2.699	0.202	0.136E+00	-1.99E-01	0.560E-02	-1.31E-02	0.294E-03	-367E+00	0.538E-01	-1.51E-01	0.354E-02	-7.94E-03
43	2.037	3.529	0.051	-2.707	0.000	0.136E+00	-2.02E-01	0.574E-02	-1.37E-02	0.316E-03	-369E+00	0.547E-01	-1.55E-01	0.371E-02	-8.55E-03
Sum_sarc(l):	4.267	Sum_i w_zz(l)*sarc(l), w/ zz=Fn,Sc,Td,Fr,Ff (cf. Eqn. (33) or (44)):	-794E+00	0.722E-08	0.119E-02	0.480E-09	-7.28E-05								

program eddy

Input: multipole # (0=dipole, 1=quad, etc) m = 1 ellipse axes: (a,b) = (4.07, 4.07) cm, h = 5.00 cm, harmonics evaluated at r= 4.00 cm

Output:

!- z-plane harmonic components IB*(z) (G) for I=1 A -!- components weighted by u(i) - Int s u/s ds (G-cm) --!
!- at z(i)e^(i2j pi/m+1), j=0,...,m, and conjugates, -! (S = 6.40062), [0.00000]

harmonic component mn+mn (n=0,1,2,...):									
i	x(i)	y(i)	sarc(i)	u(i)	v(i)	Fn(i)	Sc(i)	Td(i)	Ff(i)
1	4.075	0.000	0.076	3.321	0.000	-0.258E+00	-0.131E+00	-0.169E+00	-0.174E+00
2	4.072	0.152	0.152	3.312	0.248	-0.257E+00	-0.129E+00	-0.164E+00	-0.166E+00
3	4.064	0.305	0.152	3.284	0.495	-0.255E+00	-0.122E+00	-0.151E+00	-0.153E+00
4	4.049	0.456	0.152	3.238	0.739	-0.252E+00	-0.112E+00	-0.129E+00	-0.129E+00
5	4.029	0.607	0.152	3.174	0.979	-0.248E+00	-0.097E+00	-0.109E+00	-0.109E+00
6	4.004	0.758	0.152	3.092	1.213	-0.242E+00	-0.079E+00	-0.088E+00	-0.088E+00
7	3.973	0.907	0.152	2.992	1.441	-0.235E+00	-0.059E+00	-0.068E+00	-0.068E+00
8	3.936	1.055	0.152	2.876	1.661	-0.227E+00	-0.039E+00	-0.048E+00	-0.048E+00
9	3.894	1.201	0.152	2.744	1.871	-0.218E+00	-0.019E+00	-0.028E+00	-0.028E+00
10	3.846	1.346	0.152	2.597	2.071	-0.207E+00	0.000E+00	0.000E+00	0.000E+00
11	3.793	1.489	0.152	2.435	2.259	-0.195E+00	0.000E+00	0.000E+00	0.000E+00
12	3.735	1.630	0.152	2.259	2.435	-0.182E+00	0.000E+00	0.000E+00	0.000E+00
13	3.671	1.768	0.152	2.071	2.597	-0.168E+00	0.000E+00	0.000E+00	0.000E+00
14	3.603	1.904	0.152	1.871	2.744	-0.153E+00	0.000E+00	0.000E+00	0.000E+00
15	3.529	2.037	0.152	1.661	2.876	-0.137E+00	0.000E+00	0.000E+00	0.000E+00
16	3.450	2.168	0.152	1.441	2.992	-0.119E+00	0.000E+00	0.000E+00	0.000E+00
17	3.367	2.296	0.152	1.213	3.092	-0.101E+00	0.000E+00	0.000E+00	0.000E+00
18	3.279	2.420	0.152	0.979	3.174	-0.082E+00	0.000E+00	0.000E+00	0.000E+00
19	3.186	2.541	0.152	0.739	3.238	-0.062E+00	0.000E+00	0.000E+00	0.000E+00
20	3.089	2.658	0.152	0.495	3.284	-0.041E+00	0.000E+00	0.000E+00	0.000E+00
21	2.987	2.772	0.152	0.248	3.312	-0.020E+00	0.000E+00	0.000E+00	0.000E+00
22	2.881	2.881	0.152	0.000	3.321	0.289E-07	0.271E+00	-0.610E-07	-0.184E+00
Sum_i w_zz(i)*sarc(i), where zz=Fn,Sc,Td,Ff:									
23	2.772	2.987	0.152	-0.248	3.312	0.210E-01	0.268E+00	-0.425E-01	-0.176E+00
24	2.658	3.089	0.152	-0.495	3.284	0.418E-01	0.261E+00	-0.828E-01	-0.153E+00
25	2.541	3.186	0.152	-0.739	3.238	0.621E-01	0.250E+00	-0.119E+00	-0.119E+00
26	2.420	3.279	0.152	-0.979	3.174	0.820E-01	0.234E+00	-0.149E+00	-0.149E+00
27	2.296	3.367	0.152	-1.213	3.092	0.101E+00	0.214E+00	-0.172E+00	-0.205E-01
28	2.168	3.450	0.152	-1.441	2.992	0.119E+00	0.191E+00	-0.186E+00	-0.205E-01
29	2.037	3.529	0.152	-1.661	2.876	0.137E+00	0.166E+00	-0.191E+00	-0.205E-01
30	1.904	3.603	0.152	-1.871	2.744	0.153E+00	0.138E+00	-0.187E+00	-0.205E-01
31	1.768	3.671	0.152	-2.071	2.597	0.168E+00	0.109E+00	-0.174E+00	-0.205E-01
32	1.630	3.735	0.152	-2.259	2.435	0.182E+00	0.079E+00	-0.153E+00	-0.205E-01
33	1.489	3.793	0.152	-2.435	2.259	0.195E+00	0.050E+00	-0.124E+00	-0.205E-01
34	1.346	3.846	0.152	-2.597	2.071	0.207E+00	0.020E+00	-0.089E+00	-0.205E-01
35	1.201	3.894	0.152	-2.744	1.871	0.218E+00	0.000E+00	-0.059E+00	-0.205E-01
36	1.055	3.936	0.152	-2.876	1.661	0.227E+00	-0.039E+00	-0.028E+00	-0.205E-01
37	0.907	3.973	0.152	-2.992	1.441	0.235E+00	-0.059E+00	-0.008E+00	-0.205E-01
38	0.758	4.004	0.152	-3.092	1.213	0.242E+00	-0.079E+00	-0.028E+00	-0.205E-01
39	0.607	4.029	0.152	-3.174	0.979	0.248E+00	-0.097E+00	-0.048E+00	-0.205E-01
40	0.456	4.049	0.152	-3.238	0.739	0.252E+00	-0.109E+00	-0.068E+00	-0.205E-01
41	0.305	4.064	0.152	-3.284	0.495	0.255E+00	-0.122E+00	-0.088E+00	-0.205E-01
42	0.152	4.072	0.152	-3.312	0.248	0.257E+00	-0.129E+00	-0.109E+00	-0.205E-01
43	0.000	4.075	0.076	-3.321	0.000	0.258E+00	-0.131E+00	-0.169E+00	-0.174E+00
Sum_sarc(i):									
6.401,	Sum_i w_zz(i)*sarc(i), w/	zz=Fn,Sc,Td,Ff (cf. Eqn. (33) or (44)):	-0.279E+01	0.119E-06	0.150E+00	0.559E-07	-0.241E-01		


```

C*****
C
C      Program Eddycont: This program takes values of Z over an X-Y plane
C                        and plots Z contours.
C
C      created:   11/25/88      Schlueter
C
C      modified:  12/xx/90 changes to grafh.olb: putword replaced by ptext
C                        pscale(11,xmin,xmax,ymin,ymax,IQUAD)
C      modified:  06/xx/91 specific version for SSC LEB dipole vac. ch.
C                        eddy current induced dip/sext contours
C      modified:  06/xx/91 see Eddycont2 for blowup of good field region
C*****
C
C      DECLARATIONS
C
C      dimension x(201),      y(113),      ! (201,113)= # of points in (x,y) directions
C      &          z(201,113),      ! 26      = # of contours to be plotted
C      &          alev(13), ws(4000), ! 2000   = arbitrary (large) workspace dim.
C      &          w_0(201,113), xp(28), yp(28), xv(22), yv(22)
C      complex w_0, C, S, alpha_0
C      real k
C
C      character ddevice*3
C      character*60,string
C
C      common/xylim/xmin,xmax,ymin,ymax      !for use w/ pcont
C      common/device/ddevice
C
C      ddevice='qms'
C
C*****2
C
C      INPUT DATA AND INITIALIZE
C
C      data xmin,xmax,ymin,ymax/0., 14., 0., 14./ !for use w/ pcont
C      data xv/0.000, 0.250, 0.500, 0.721, 0.940, 1.157, 1.369, 1.575,
C      &      1.773, 1.963, 2.143, 2.312, 2.468, 2.611, 2.739, 2.853,
C      &      2.950, 3.030, 3.093, 3.139, 3.166, 3.175/
C      data yv/2.675, 2.675, 2.675, 2.666, 2.639, 2.593, 2.530, 2.450,
C      &      2.353, 2.239, 2.111, 1.968, 1.812, 1.643, 1.463, 1.273,
C      &      1.075, 0.869, 0.657, 0.440, 0.221, 0.000/
C      data xp/0.00, 1.40, 1.65, 1.90, 2.15, 2.40, 2.65, 2.90, 3.15,
C      &      3.40, 3.65, 3.90, 4.15, 4.40, 4.65, 4.90, 5.15,
C      &      5.40, 5.65, 5.91, 6.17, 6.43, 6.67, 6.85, 7.01,
C      &      7.11, 7.21, 11.9/
C      data yp/2.86, 2.86, 2.86, 2.86, 2.86, 2.86, 2.86, 2.85,
C      &      2.85, 2.87, 2.89, 2.90, 2.88, 2.84, 2.80, 2.81,
C      &      2.76, 2.71, 2.71, 2.75, 2.77, 2.82, 2.90, 3.08,
C      &      3.30, 3.51, 13.3/
C
C      np=28
C      nv=22
C
C      h=2.86
C      r=2.5
C      pi=4.*Atan(1.)
C      k=pi/(2.*h)
C
C      imx=201
C      jmx=113
C      do 10 i=1,imx
C          x(i) = float(i-1)*.025 + 1. ! (cm)
10  continue
C

```

```

do 11 j=1,jmx
  y(j) = float(j-1)*.025      ! (cm)
11 continue
c
do 13 i=1,imx
  do 12 j=1,jmx
    w_0(i,j)=cmplx(x(i),y(j))
    alpha_0=k*w_0(i,j)
    C = (exp(alpha_0)+exp(-alpha_0))/2.
    S = (exp(alpha_0)-exp(-alpha_0))/2.
    p2zh=((pi/2.)*(r/h))**2
    z(i,j)=Real(-C/S)/Real((1./2.)*(-2.*C)/(S**3))/p2zh !z=FnTd
    if(j.eq.1 .or. j.eq.10 .or. j.eq.20 .or. j.eq.113) then
      if(i.eq.1 .or. i.eq.17 .or. i.eq.34 .or. i.eq.201) then
        type *, i,j,x(i),y(j),z(i,j)
      end if
    end if
  end if
12 continue
13 continue
c
ccc      open (unit=1, file='oldfilename.dat', status='old')
c
c*****
c
c BEGIN GRAPHING
c
  call plstart('mac')
c      call plstart('QMS')
  iquad=0
  call pscale('linlin',xmin,xmax,ymin,ymax,IQUAD)
c      call pfont('ROMAN')
  call pgrid('on','on')
  call ptitle('Dipole/Sextupole = Constant Contours (for r=2.5)',
& '      x      (cm)      $',
& 'y      (cm)$', '$')
c
  xlet=(xmax-xmin)*.01/1.6
  ylet=(ymax-ymin)*.015/1.6
  xmm=xmax-xmin
  ymm=ymax-ymin
  call setchar(xlet,ylet)
  call setpos(1.10*xmm,.98*ymm)
c11      call ptext('CONTOURS:')
c
c*****
c
c      graph lines of constant Gamma:
c
  x1=12.3
  y1=3.65
  x2=14.0
  y2=3.65
  call pline(x1,y1,x2,y2)
  x1=12.3
  y1=3.65
  x2=12.3
  y2=12.9
  call pline(x1,y1,x2,y2)
  x1=12.3
  y1=12.9
  x2=14.0
  y2=12.9
  call pline(x1,y1,x2,y2)
  x1=11.9
  y1=13.3
  x2=14.0

```



```

y2=13.3
call pline(x1,y1,x2,y2)
call pcur(xp,yp,np)
call pcur(xv,yv,nv)
ilev=13
alev(1) = - 3.
alev(2) = - 5.
alev(3) = - 7.
alev(4) = - 9.
alev(5) = -11.
alev(6) = -20.
alev(7) = -50.
c
alev(8) = -1.
alev(9) = -2.
c
alev(10) = 20.
alev(11) = 5.
alev(12) = 50.
alev(13) = 10.
c
call pstyle('solid')
call pcont('linlin', x, y, z, imx, imx, jmx,
&      alev, ilev, iquad, '$', '$', '$', '$')
c
xlet=(xmax-xmin)*.01/1.6
ylet=(ymax-ymin)*.015/1.6
call setchar(xlet,ylet)
call setpos(1.06*xmm,.95*ymm)
ccc call ptext('-1/3, -1/4, -1/5, -1/6, -1/7, -1/8, -1/9, -1/10,')
call setpos(1.00*xmm,.95*ymm)
ccc call ptext('1/inf, 1/200, 1/100, 1/50, 1/10')
c
xmuilt=.95
do 30 iter=1,ilev
    xmuilt=xmuilt-.02
    call setpos(1.06*xmm,xmuilt*ymm)
    write(string,'(f5.0)')alev(iter)
c11 call ptext(string)
30 end do
c
c*****
c
c TITLES
c
xlet=(xmax-xmin)*.01
ylet=(ymax-ymin)*.015
xmm=xmax-xmin
ymm=ymax-ymin
call setchar(xlet,ylet)
call setpos(.30*xmm,.97*ymm)
ccc call ptext('put legend here')
call setpos(.32*xmm,.935*ymm)
ccc call ptext('legend continued')
c
c*****
c
c CONSTANTS FOR GRAPH
c
call setpos(1.06*xmm,.65*ymm)
ccc call ptext('input data here')
c
c*****
c
c Contour Line Labels
c

```

```

        call setpos(.64*xmm,.67*ymm)
ccc      call plineh(.64*xmm,.68*ymm,.60*xmm,.64*ymm)
ccc      call ptext('z contour')
c
c*****
c
        call pclose
        close (unit=1, status='keep')
c
        end
c
c*****

```

Appendix D: Elliptical Vacuum Chamber Eddy Current Induced Harmonics and their Suppression for a Sextupole Electromagnet

Figure 17. Sextupole magnet with elliptical vacuum chamber: (a) lab frame z -plane, (b) dipole frame w -plane, (c) upper half t -plane, (d) first quadrant Ω -plane

12 Appendix D: Elliptical Vacuum Chamber Eddy Current Induced Harmonics and their Suppression for a Sextupole Electromagnet

For the sextupole magnet with an elliptical vacuum chamber in the laboratory z -plane (Figure 17a), we use the transformation⁵ $w = z^3/r_p^2$, where r_p is the-sextupole aperture radius, to map to a dipole w -plane geometry (Figure 17b). The Schwarz-Christoffel transformation

$$\frac{dw}{dt} = \frac{(1/k)\sqrt{t}}{(t-a^2)(t-1)} \quad (54)$$

maps the region of interest of the dipole w -plane to the upper half t -plane (Figure 17c). Vacuum chamber eddy current induced magnetic field lines are perpendicular to the t -plane's positive abscissa, but parallel to its negative abscissa. A mapping function $\Omega = \sqrt{t}$ results in field lines in the iron-free Ω -plane perpendicular to the positive abscissa, parallel to the positive ordinate, and thus perpendicular to the negative abscissa as well (Figure 17d). Contributions to the magnetic field of current filaments are readily calculated in

⁵Computationally, one must be careful in transforming from w back to z , since values in the w -plane with negative imaginary parts are handled internally by the computer as having an argument θ : $-\pi < \theta < 0$ (refer to Figure 17b), while transformationally $\pi < \theta < 3\pi/2$. *Caution!*

the iron-free Ω -plane and transformation back to the laboratory z -plane yields the desired solution for eddy current induced $B^*(z)$.

The relationship between w and Ω is

$$\frac{dw}{d\Omega} = \frac{dw}{dt} \frac{dt}{d\Omega} = \frac{(2/k)t}{(t-a^2)(t-1)} = \frac{1}{k(1-a^2)} \left[\frac{-a}{\Omega-a} + \frac{a}{\Omega+a} + \frac{1}{\Omega-1} + \frac{-1}{\Omega+1} \right] \quad (55)$$

Now $Im(\Delta w|_{t=a^2}) = -2r_p = \frac{\pm\pi a}{k(a^2-1)}$ and $Im(\Delta w|_{t=1}) = r_p = \frac{\pm\pi}{k(1-a^2)}$ so we have $a = \pm 2, \frac{1}{k} = \frac{\pm 3r_p}{\pi}$ and

$$\frac{dw}{d\Omega} = \frac{-r_p}{\pi} \left[\frac{-2}{\Omega-2} + \frac{2}{\Omega+2} + \frac{1}{\Omega-1} + \frac{-1}{\Omega+1} \right] \quad (56)$$

Integrating, we have

$$\frac{\pi}{r_p} w = 2 \ln(\Omega-2) - 2 \ln(\Omega+2) - \ln(\Omega-1) + \ln(\Omega+1) - \pi i \quad (57)$$

This equation's format is useful for numerical work since on the computer $-\pi < Im(\ln(f(\Omega))) \leq \pi$ and Ω values are restricted to the first quadrant; combining the terms under just one or two 'ln' signs leads to transformation errors by factors of πi : *Caution!* We have $w = f_1(\Omega)$; obtaining $\Omega = f_2(w)$ comprises solving the complex cubic equation in Ω for the appropriate root. Figure 18 shows the overall z - Ω mapping.

For current filaments in the Ω -plane of magnitudes $+I$ at Ω_1 and $-I$ at $-\Omega_1^*$ and their images about the midplane:

$$F(\Omega) = \frac{-I}{2\pi} \ln \left[\frac{(\Omega - \Omega_1)(\Omega - \Omega_1^*)}{(\Omega + \Omega_1^*)(\Omega + \Omega_1)} \right] \quad (58)$$

and

$$iF'(\Omega) = \frac{I}{2\pi i} \left[\frac{1}{\Omega - \Omega_1} + \frac{1}{\Omega - \Omega_1^*} - \frac{1}{\Omega + \Omega_1^*} - \frac{1}{\Omega + \Omega_1} \right] \quad (59)$$

This current filament distribution in Ω is equivalent to filaments in the z -plane of magnitudes $+I$ at $z_1(w_1(t_1(\Omega_1)))$ and $-I$ at $-z_1^*$ and their images about the midplane in z . So for this current distribution, the magnetic field in the laboratory z -plane is:

$$B^*(z) = B^*(w)/(dz/dw) = \mu_0 i F'(\Omega)/(dw/d\Omega)/(dz/dw) = \frac{-3\mu_0 I z^2 \left[\frac{1}{\Omega - \Omega_1} + \frac{1}{\Omega - \Omega_1^*} - \frac{1}{\Omega + \Omega_1^*} - \frac{1}{\Omega + \Omega_1} \right]}{2r_p^3 i \left[\frac{-2}{\Omega - 2} + \frac{2}{\Omega + 2} + \frac{1}{\Omega - 1} + \frac{-1}{\Omega + 1} \right]} \quad (60)$$

Figure 18. First quadrant of elliptical sextupole in z plane conformally transformed to iron-free first quadrant Ω -plane

Using the value of $I'_{vac}(s_z)$ from Eqn. (45), Eqn. (60) can be integrated over the portion of the elliptical (or any shape symmetrical with respect to both the x and y axes) vacuum chamber arc lying in the first quadrant of the laboratory z -plane to yield the eddy current induced field in a sextupole magnet.

Field values from Eqn. (60) agree beautifully with a test POISSON run on a sextupole magnet.

As $z \rightarrow 0$, we have $w \rightarrow 0$, $\frac{dw}{dt} \rightarrow \sqrt{t}/ka^2$, $w = \frac{r_p}{2\pi} t^{3/2}$, $\frac{dw}{d\Omega} = \frac{dw}{dt} \frac{dt}{d\Omega} = \frac{3r_p w_2}{2\pi} = \frac{3z^2}{(2\pi)^{1/3} r_p}$, $\Omega \rightarrow 0$, $iF'(\Omega = 0) = \frac{-I_2 \text{Re}(\Omega_1)}{\pi i |\Omega_1|^2}$, and thus

$$iB^*(z = 0) = \left(\frac{4}{\pi}\right)^{2/3} \frac{\mu_0 I \text{Re}(\Omega_1)}{r_p |\Omega_1|^2} \quad (61)$$

```

c*****
c
c      Program elsexcont:This program takes values of Z over an Omega plane
c                        and plots Re(Z) and Im(Z) contours (or alternatively,
c                        Mag(Z) and Arg(Z) contours).
c
c      created:   7/91      Ross Schlueter
c
c      this version:  for elliptical sextupole z-to-Omega plane transform
c
c*****
c
c      DECLARATIONS
c
c      dimension rOm(100), AiOm(100),
c      &          rz(100,100), aiz(100,100), radz(100,100), argz(100,100),
c      &          alev(13), ws(40000) ! 2000 = arbitrary (large) workspace dim.
c      complex Om, w, z, zi
c
c      character ddevice*3
c      character*60,string
c
c      common/xylim/xmin,xmax,ymin,ymax      !for use w/ pcont
c      common/device/ddevice
c
c      ddevice='qms'
c
c*****2
c
c      INPUT DATA AND INITIALIZE
c
c      data xmin,xmax,ymin,ymax/0., 4.1, 0., 4.1/ !for use w/ pcont
c      data xmin,xmax,ymin,ymax/0., 20.1, 0., 20.1/ !for use w/ pcont
c
c      rp=5.
c      zi=cmplx(0.,1.)
c      pi=4.*Atan(1.)
c
c      imx=100
c      jmx=100
c
c      do 13 i=1,imx
c          rOm(i) = 0.0001
c          if(i.ne.1) rOm(i) = float(i-2)*.1 +.05 ! (cm)
c          do 12 j=1,jmx
c              if(j.le.5) AiOm(j) = float(j-1)*.025 ! (cm)
c              if(j.gt.5) AiOm(j) = float(j-5)*.1 +.1 ! (cm)
c
c              Om=cmplx(rOm(i),AiOm(j))
c              w=(-rp/pi)*
c              &      (-2.*log(Om-2.)+2.*log(Om+2.)+log(Om-1.)-log(Om+1.)) - rp*zi
c23456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012
c              z=(w*(rp**2))**.3333
c              if(Aimag(w).lt.0.)z=z*exp(zi*2.*pi/3.)
c              rz(i,j) = Real(z)
c              aiz(i,j)= Aimag(z)
c              radz(i,j) = sqrt(aiz(i,j)**2 + rz(i,j)**2)
c              argz(i,j)=atan(aiz(i,j)/(rz(i,j)+.01))
c              radz(i,j)=(sqrt(real(w)**2+aimag(w)**2)*rp**2)**.3333
c              argz(i,j)=(atan(aimag(w)/(real(w)+.01)))/3. ! alternate method not as c.
c              if(argz(i,j).lt.0.)argz(i,j)=argz(i,j)+(2.*pi/3.)
c              rz(i,j) =radz(i,j)*cos(argz(i,j))
c              aiz(i,j)=radz(i,j)*sin(argz(i,j))
c          12 continue
c      13 continue
c

```



```

ccc      open (unit=1, file='oldfilename.dat', status='old')
c
c*****
c
c BEGIN GRAPHING
c
c      call plstart('mac')
c      call plstart('QMS')
c      iquad=0
c      call pscale('linlin',xmin,xmax,ymin,ymax,IQUAD)
c      call pfont('ROMAN')
c      call pgrid('on','on')
c      call pttitle('Mag(z) and Arg(z) contours in Omega plane',
c      call pttitle('Re(z) and Im(z) contours in Omega plane',
c      & '      O_x      (cm)      $',
c      & 'O_y      (cm)$', '$')
c
c      xlet=(xmax-xmin)*.01/1.6
c      ylet=(ymax-ymin)*.015/1.6
c      xmm=xmax-xmin
c      ymm=ymax-ymin
c      call setchar(xlet,ylet)
c      call setpos(0.80*xmm,.98*ymm)
c11      call ptext('CONTOURS:')
c
c*****
c
c      graph lines of POLE TIPS:
c
c      x1=1.
c      y1=0.01
c      x2=1.95
c      y2=0.01
c      call pline(x1,y1,x2,y2)
c      x1=2.05
c      y1=0.01
c      x2=xmax
c      y2=0.01
c      call pline(x1,y1,x2,y2)
c
c      ilev=13
c      alev(1) = 0.00*rp
c      alev(2) = 0.10*rp
c      alev(3) = 0.20*rp
c      alev(4) = 0.30*rp
c      alev(5) = 0.40*rp
c      alev(6) = 0.50*rp
c      alev(7) = 0.60*rp
c      alev(8) = 0.70*rp
c      alev(9) = 0.80*rp
c      alev(10) = 0.90*rp
c      alev(11) = 1.00*rp
c      alev(12) = 1.10*rp
c      alev(13) = 1.20*rp
c
c      call pstyle('solid')
c      call pcont('linlin', rOm, AiOm, rz, imx, imx, jmx,
c      & alev, ilev, iquad, '$', '$', '$', '$')
c
c      call pstyle('solid')
c      call pcont('linlin', rOm, AiOm, aiz, imx, imx, jmx,
c      & alev, ilev, iquad, '$', '$', '$', '$')
c
c      call pstyle('dashed')
c      call pcont('linlin', rOm, AiOm, radz, imx, imx, jmx,
c      & alev, ilev, iquad, '$', '$', '$', '$')

```

```

c
    alev(1) = 0.00*pi/2.
    alev(2) = 0.10*pi/2.
    alev(3) = 0.20*pi/2.
    alev(4) = 0.30*pi/2.
    alev(5) = 0.40*pi/2.
    alev(6) = 0.50*pi/2.
    alev(7) = 0.60*pi/2.
    alev(8) = 0.70*pi/2.
    alev(9) = 0.80*pi/2.
    alev(10) = 0.90*pi/2.
    alev(11) = 1.00*pi/2.
    alev(12) = 1.10*pi/2.
    alev(13) = 1.20*pi/2.

c
    call pstyle('dashed')
cc    call pcont('linlin', rOm, AiOm, argz, imx, imx, jmx,
cc    &          alev, ilev, iquad, '$', '$', '$', '$')
c
    call pclose
    close (unit=1, status='keep')
c
    end
c
c*****

```

```

c program: log
c
c analytical check for elliptical sextupole (1 current filament in 1st quadrant)
c
c input: Omega, Omegal
c output: B*(z(Omega),Omegal)
c
      complex zi, Bst, z, w, Om, z1, w1, Oml, Omlst, fpOmi, dwdOm, dwdz
      open (unit=2, file='log_out.dat', status='old',
& carriagecontrol='fortran')
      rp=5.
      Cur=1000.
      zi=cmplx(0.,1.)
      pi=4.*atan(1.)
      read(5,*)Om,Oml
c23456789012345678901234567890123456789012345678901234567890123456789012
      w=(-rp/pi)*(-2.*log(Om-2.)+2.*log(Om+2.)+log(Om-1.)-log(Om+1.))
& -rp*zi
      z = (w*rp**2)**.3333
      if(Aimag(w).lt.0.)z=z*exp(zi*2.*pi/3.)
      w1=(-rp/pi)*
& (-2.*log(Oml-2.)+2.*log(Oml+2.)+log(Oml-1.)-log(Oml+1.)) - rp*zi
      z1 = (w1*rp**2)**.3333
      if(Aimag(w1).lt.0.)z1=z1*exp(zi*2.*pi/3.)
      type *, Om, w, z
      type *, Oml, w1, z1
      Omlst=cmplx(Real(Oml),-Aimag(Oml))
      fpOmi=(Cur/(2.*pi*zi))*4*pi*
& ((1./(Om-Oml))+1./(Om-Omlst))-(1./(Om+Omlst))-(1./(Om+Oml))
      dwdOm=
& (-rp/pi)*((-2./(Om-2.))+(2./(Om+2.))+(1./(Om-1.))-1./(Om+1.))
      dwdz= 3.*(z**2)/(rp**2)
      Bst=dwdz*fpOmi/dwdOm
      type *, fpOmi, dwdOm, dwdz, Bst
      Bst = (3.*(z**2)/(rp**2))*(Cur/(2.*pi*zi))*4*pi*
& ((1./(Om-Oml))+1./(Om-Omlst))-(1./(Om+Omlst))-(1./(Om+Oml))/
& ((-rp/pi)*((-2./(Om-2.))+(2./(Om+2.))+(1./(Om-1.))-1./(Om+1.))
      type *, Oml, Omlst, Bst
99      end

```

END

**DATE
FILMED**

11 / 12 / 92