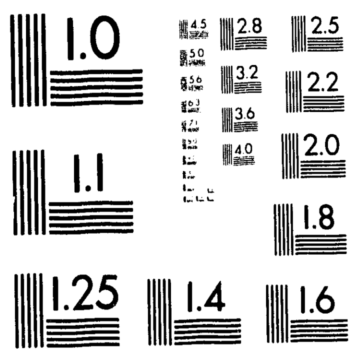
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Scaling of Interceptors for Theater Defense

Gregory H. Canavan

Los Alamos
NATIONAL LABORATORY
Los Alamos, New Mexico 87545

OUTLINE

SCALING OF INTERCEPTORS FOR THEATER DEFENSES

by

Gregory H. Canavan

ABSTRACT

For nominal GBI and SBI cost parameters GBIs are preferred for missile ranges under ≈ 1,000 km; for multiple theaters breakeven ranges decreases to ≈ 500 km. Penalties for using GBIs rather than SBIs for long-range missiles are ≈ factor of 2; penalties for using SBIs for short-range missiles can be larger.

I. INTRODUCTION

This note discusses the overall scaling of ground- and space-based interceptors (GBI and SBIs) in theater defenses. A simple breakeven analysis shows that for nominal GBI and SBI cost parameters, GBIs are preferred for missile ranges under ≈ 1,000 km. For multiple theaters breakeven ranges decrease to ≈ 500 km. Penalties for using GBIs rather than SBIs for long-range missiles are ≈ a factor of 2; penalties for using SBIs for short-range missiles can be larger.

II. GBI ANALYSIS

A "nuclear-armed fanatic in the developing world"[1] who possesses M missiles of range R can threaten a number of targets proportional to R^2. Defenses with range x require about $(R/x)^2$ sites and about $M(R/x)^2$ GBIs to defend all targets fully.

If the GBIs are supported by radars of range y, about $(R/y)^2$ radar sites are required. Radar power-apertures and costs increase roughly as y^4, so the total cost for radars scales as $y^4(R/y)^2 \propto y^2R^2$, which is minimized by using many small radars of range $y \approx x$. If the GBIs have variable costs of about i per GBI and the total radar cost scales as $b \cdot x^2R^2$, then the total cost for defense GBI is about

$$C_G \approx i \cdot M(R/x)^2 + b \cdot x^2R^2, \tag{1}$$

which is minimized by the choice

$$x_o \approx \surd(iM/b), \tag{2}$$

for which the cost is

$$C_{Go} \approx 2\surd(ibM)R^2, \tag{3}$$

split about equally between GBIs and radars. The variation of C_{Go} with R for typical parameters $i \approx \$3M/GBI$ and $b \approx \$100M/(Mm)^4$ is shown in Fig. 1. The curves rising to the right are for M = 10, 30, and 100 missiles. The latter reaches ≈ \$800M by R = 1,500 km. The cost of GBI defense increases as R^2, which could become prohibitive at some range.

The above analysis assumes that the missiles are used just for intimidation. If, instead they are used to some military end, such as penetrating opposing forces, the targets attacked lie along a single radial, the number of interceptors required only increases as M(R/x), and the optimization, which is still analytic, is altered slightly.[2] Thus, the difference between a rational or irrational fanatic can be reduced to the choice of a single geometric exponent. The difference is ignored below, where the optimization of Eqs. (2) and (3) is used.

III. SBI ANALYSIS

Midcourse intercept kinematics for short-range missile trajectories are reviewed elsewhere.[3,4] Ignoring drag and powered flight, for ranges of interest a missile launched on a minimum-energy trajectory to range R requires a velocity $v = \surd(gR)$, reaches an altitude $h = R/4$, and takes a total flight time $T = \surd(2R/g)$, where $g = 0.01$ km/s^2 is the value of gravity at the Earth's surface. Minimum-energy trajectories maximize range and minimize error for given missiles, so they are probably those of most concern for inter-theater launches.[5] For $R = 1{,}000$ km, $v = 3.2$ km/s; $h \approx 250$ km; $T \approx 450$ s; and time to apogee is ≈ 250 s.[6]

If each SBI is capable of reaching missiles with apogees within a radius r, it can defend an area of $\approx \pi r^2$,[7] so the number needed for contiguous coverage is $N \approx (R_e/r)^2$, where R_e is the Earth's radius.[8] SBIs must be able to reach missiles while they are still accessible, which means that the SBI constellation must be dense enough to allow SBIs to reach missiles or weapons before they fall into the atmosphere.

It is assumed that SBIs are capable of intercepting missiles at any time up to apogee, which gives a maximum fly-in time of $T/2 = \sqrt{(R/2g)}$. Thus, SBIs with average velocity V from ranges less than $r \approx V \cdot T/2 = V\sqrt{(R/2g)}$ with sensors sufficiently sensitive to detect cold bodies could engage missiles on minimum energy trajectories by apogee. The number of SBIs required to have at least one SBI in range of apogee is

$$N_a \approx (R_e/r)^2 \approx (2R_e/VT)^2 \approx 2gR_e^2/V^2R, \quad (4)$$

which gives the apogee "absentee ratio," i.e., the number of SBIs needed on orbit to intercept one missile at its apogee. For a typical divert velocity of $V = 6$ km/s, $R = 1{,}000$ km gives $N_a \approx 23$; $R = 2{,}500$ km gives $N_a \approx 9$, although ranges that long lead to $\approx 10\%$ errors between planar approximations and spherical-earth trajectories.[9]

It can be argued that for the launch of M missiles, the number of SBIs should be increased by a factor of M, but unless the missiles are launched within a time $t \approx 2r/V_o$ of each other, the movement of the SBIs in their constellation brings a new interceptor into position in time for intercept. For $R = 1{,}000$ km, $r \approx V\sqrt{(R/2g)} \approx 1{,}350$ km and $t \approx 2 \cdot 1{,}350$ km/8 km/s ≈ 330 s ≈ 6 minutes, where $V_o \approx 8$ km/s is the SBIs' orbital velocity.

To address M missiles spaced at intervals longer than t, the defense needs N_a SBIs to fill the constellation plus M SBIs for the intercepts. If the SBIs have variable costs s per SBI, the cost for the full constellation is

$$C_S \approx s(M + N_a) \approx s(M + 2gR_e^2/V^2R), \quad (5)$$

which is also shown in Fig. 1 as the curves that fall to the right for M = 10, 30, and 100 missiles and $s \approx \$3M$, i.e., SBIs that cost about as much as short-range missiles. The masses and costs of short-range missiles typically grow as $e^{V/c} \approx e^{\surd(gR)/c}$, where $c \approx 2.5$ km/s is the SBIs' effective exhaust velocity.[10] That refinement is ignored here.

IV. BREAKEVEN ANALYSIS

For $C_{Go} < C_S$ GBIs are preferred on the basis of variable costs; for $C_{Go} > C_S$, SBIs are. From Fig. 1, for M = 10 missiles the breakeven is at $R \approx 1,000$ km; for 30 missiles at 950 km; and for 100 missiles again at $\approx$1,000 km. The crossover doesn't change much with the size of the threat, although from Eq. (3) the costs do increase as $\surd M$. Equating C_{Go} to C_S gives the condition for breakeven, which for N_a large (R small) reduces to

$$R_{BE} \approx [sgR_e^2/V^2\surd(ibM)]^{1/3}, \quad (6)$$

which is insensitive to GBI and SBI cost parameters and scales most strongly on SBI velocity as $V^{-2/3}$. For N_a small (R large)

$$R \approx [s\surd(M/ib)/2]^{1/2}, \quad (7)$$

which scales strongly only on $\surd s$. If for M = 100 and R = 1,500 km GBIs are used rather than SBIs, costs are increased by about a factor of 2, or \$400M. For smaller threats the fractional increases are larger, but the dollar costs are much smaller.

For n separate theaters, each of which has M missiles and all of which must all be protected at the same time, SBI costs are essentially unchanged, but GBI costs are increased by a factor of n. For N_a large (R small), that reduces R_{BE} by a factor of $n^{-1/3}$. For eight theaters that would shift all breakeven ranges to about 500 km, as noted in earlier studies.[11]

For threats with a spectrum of missile ranges, this simple breakeven analysis must be replaced by a true cost optimization. The general result is a mix of GBIs and SBIs for all but the smallest, simultaneous threats in few theaters.[12]

V. SUMMARY AND CONCLUSIONS

This note discusses the overall scaling of ground- and space-based interceptors (GBI and SBIs) in theater defenses. It derives and displays the minimum-cost GBI defense against intimidation attacks and the minimum-cost SBI defense against intimidation or military attacks. For nominal GBI and SBI cost parameters GBIs are preferred for missile ranges under ≈ 1,000 km. For multiple theaters the breakeven range decreases to about 500 km. The penalty for using GBIs rather than SBIs for long-range missiles is about a factor of two. The penalty for using SBIs rather than GBIs for short-range missiles can be larger.

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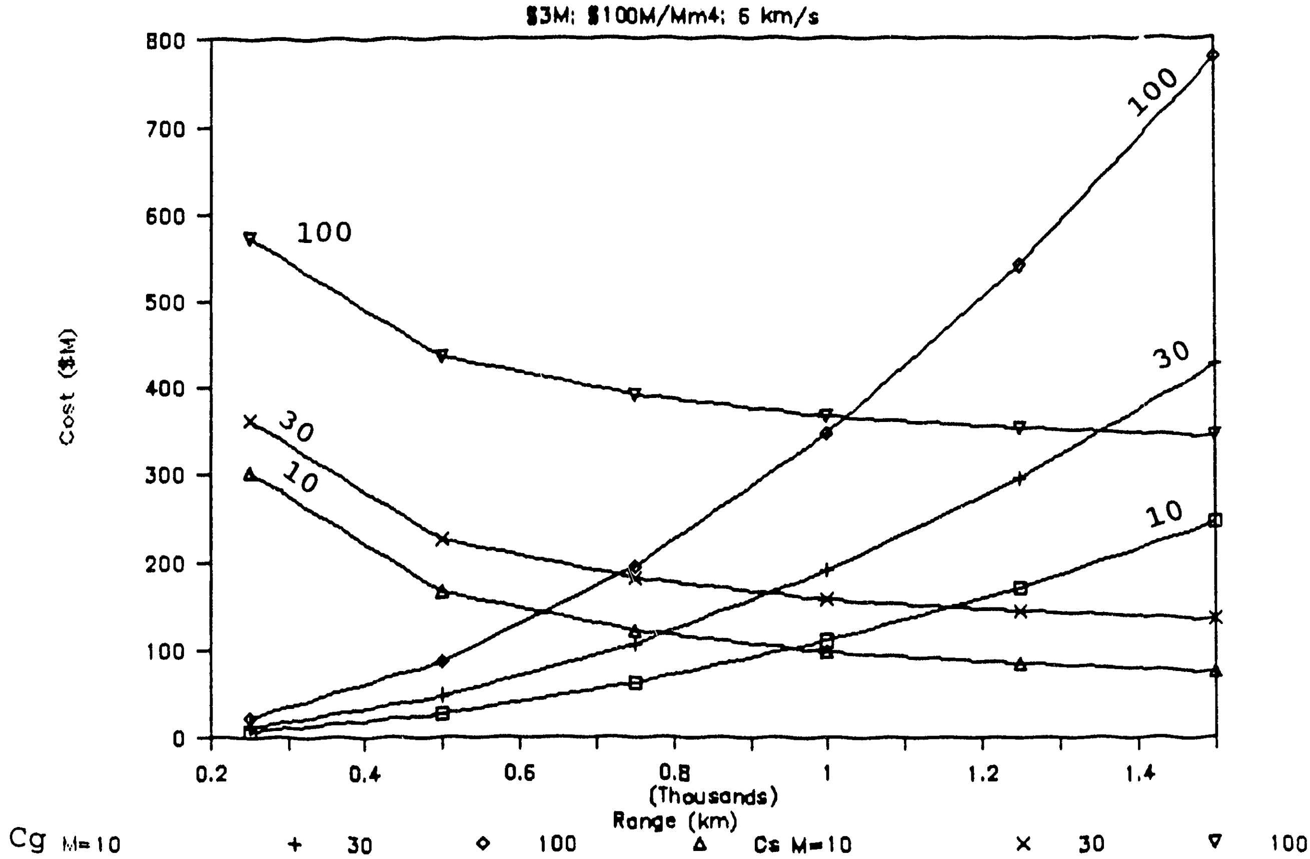
Fig. 1. Cost versus range



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