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Sound Velocity Estimation: A System Theoretic Approach

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Abstract—A system-theoretic approach is proposed to investigate the feasibility of reconstructing a sound velocity profile (SVP) from acoustical hydrophone measurements. This problem is based on a state-space representation of the normal-mode propagation model. It is shown that this representation can be utilized to investigate the so-called observability of the SVP from noisy measurement data. A model-based processor is developed to extract this information and it is shown that even in cases where limited SVP information is available, the SVP can be estimated using this approach.

1 Introduction

The incorporation of ocean acoustic propagation models into signal processing schemes has long been the approach of many techniques with the hope of improving overall processor performance. Current applications are concerned with the localization/detection of an acoustic source and the proposed methods are based on bringing the predictions of a propagation model in concert with pressure measurements from an array. This problem is called the matched-field problem and much work has been accomplished [1-4].

In this paper we are concerned with an oceanographic tomography problem. More specifically, we are concerned with whether or not it is possible to estimate the SVP from noisy hydrophone measurements. There is presently a growing literature on oceanographic tomography, since it can deal with the estimation of many different parameters relative to ocean acoustics. For more information on these issues, see [5] and references therein. In a recent work [6], it was shown that with sound speed measurements available, both pressure field and modal information could simultaneously be estimated using a model-based approach *after* the associated boundary value problem has been solved. It was also shown that the problem could be cast into the state-space framework. Associated with the state-space representations are many useful and powerful concepts/theorems from the discipline of Systems Theory initiated by Kalman in mid-sixties [7]. There is a basic principle of *observability* which is applicable to the problem of SVP estimation. This concept deals with the ability of a system to reconstruct internal states from its measurements. In our problem, the system is, in fact, the normal mode propagation model, the internal states are the modes, ranges and SVP parameters and the measurements are those provided by the array.

To answer our observability question, we will first construct a processor that satisfies this property, then re-

move various measurements and test the resulting system for continued observability. In this approach, we hope to invoke a deeper understanding of ocean acoustic systems utilizing normal mode models. In the noisy case, the question of whether or not the system is observable is related to the question of whether or not a model-based processor can be constructed to estimate the SVP from noisy data. Our approach will be to apply the observability criterion to the deterministic normal mode representation develop the processor and then apply it to simulated data.

We briefly review the state-space representation of the normal mode model which will be used in this paper in Sec. II. The application of systems theory to the SVP estimation problem is discussed in the next section where it is shown that the normal mode model observability is clearly dependent on the SVP. Finally, in Sec. IV we apply the results of this study to design model-based processors to estimate the SVP from noisy simulated measurements of a shallow water problem.

2 State-Space Propagation Models

In this section we briefly review the development of state-space propagation models from the corresponding normal-mode solutions to the wave equation. The state-space models are "forward" propagators used in model-based signal processing schemes [6].

We assume a horizontally-stratified ocean of depth h with a *known* source position (x, y) and assume that the acoustic energy from a point source can be modeled as a trapped wave governed by the cylindrical wave equation. We follow the approach of Pekeris yielding the using the separation of variables technique leading to a set of ordinary differential equations [8]. The resulting depth relation of Eqn. 4 is an eigenvalue equation in z with

$$\frac{d^2}{dz^2} \phi_m(z) + \kappa_z(m) \phi_m(z) = 0, \quad m = 1, \dots, M \quad (1)$$

whose eigensolutions $\{\phi_m(z)\}$ are the so called *modal functions* and κ_z is the wavenumber in the z -direction. These solutions depend on the sound velocity profile, $c(z)$, and the boundary conditions at the surface and bottom as well as the corresponding *dispersion* relation given by

$$\kappa^2 = \frac{\omega^2}{c^2(z)} = \kappa_r^2(m) + \kappa_z^2(m), \quad m = 1, \dots, M \quad (2)$$

where κ_r, κ_z are the respective wave numbers in the r and z directions with c the depth-dependent sound velocity profile and ω the harmonic source frequency.

For our purpose we are concerned with the estimation of the pressure field, therefore we remove the time dependence, normalize units, and obtain the *acoustic pressure propagation model*,

$$p(r, z) = \sum_{m=1}^M q \phi_m(z_s) \phi_m(z) \frac{e^{-\alpha_r(m)r}}{\sqrt{\kappa_r(m)r}} e^{j\kappa_r(m)r} \quad (3)$$

where p is the acoustic pressure; q is the source intensity; ϕ_m is the m^{th} modal function at z and z_s ; $\kappa_r(m)$ is the horizontal wavenumber associated with the m^{th} mode; and r is the horizontal range.

The depth relation of Eqn. 1 is a linear, space-varying coefficient (for each layer) differential equation and is easily placed in state-space form by defining the state-vector as $\underline{x}_m := [\phi_m(z) \quad \frac{d}{dz}\phi_m(z)]^T$. Examining mode propagation in more detail, we see that each mode is characterized by a set of second order, ordinary differential equations and the overall system including all of the M -modes is given by

$$\frac{d}{dz}\underline{x}(z) = A(z)\underline{x}(z) + B(z)\underline{u}(z) \quad (4)$$

where A is block diagonal with each 2×2 block given by

$$A_m(z) = \begin{bmatrix} 0 & 1 \\ -\kappa_z^2(m) & 0 \end{bmatrix}, \quad B_m(z) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (5)$$

and $\underline{u}(z)$ is the source driving function.

This constitutes a deterministic representation of the normal-mode model in state-space form. However, since propagation in the ocean is effected by inhomogeneities in the water, slow time variations in the SVP and motion of the surface, the state-space model must be extended in order to include these effects and uncertainties. One approach is to place the model into a linear space-varying Gauss-Markov representation. This can be accomplished by assuming that the horizontal range of the source r_s is known *a-priori*. In this case we use the Hankel function $H_0(\kappa_r r_s)$ which is the source range solution. Therefore, we reduce the state-space model to that of "depth only" and the Gauss-Markov model for this constrained problem as well as the deterministic (or noise free variant) is given by

$$\frac{d}{dz}\underline{x}(z) = A(z)\underline{x}(z) + B(z)\underline{u}(z) + \underline{w}(z) \quad (6)$$

with the pressure field measurement given by

$$p(z) = \underline{C}^T(r_s, z)\underline{x}(z) + \underline{v}(z) \quad (7)$$

where the matrices have been defined previously and

$$\underline{C}^T(r_s, z) = [\beta_1(r_s, z) \quad 0 \cdots \beta_M(r_s, z) \quad 0] \quad (8)$$

with $\beta_i(r_s, z) = \frac{q\phi_i(z_s)}{\int_0^L \phi_i^2(z) dz} H_0(k_r(i)r_s)$. The random noise vectors $\underline{w}$ and $\underline{v}$ are assumed gaussian, zero-mean with respective covariance matrices, R_{ww} and R_{vv} .

Before we close this section, let us consider a more practical case of the normal-mode model. Since our array spatially samples the pressure-field, we choose to discretize the differential state equations using a finite (first) difference approach, that is, for the m^{th} -mode use

$$\frac{d}{dz}\underline{x}_m(z_\ell) \approx \frac{\underline{x}_m(z_{\ell+1}) - \underline{x}_m(z_\ell)}{\Delta z_\ell} \quad (9)$$

where $\Delta z_\ell = z_{\ell+1} - z_\ell$. Therefore, substituting into Eqn. 14, we obtain

$$\underline{x}_m(z_{\ell+1}) = [I + \Delta z_\ell A_m(z_\ell)] \underline{x}_m(z_\ell) + \Delta z_\ell B(z_\ell) \underline{u}(z_\ell) + \underline{w}(z_\ell) \quad (10)$$

Assuming we have a vertical line sensor array to measure the pressure-field, the measurement model for the m^{th} mode becomes

$$p_m(r_s, z_\ell) = \underline{C}_m^T(r_s, z_\ell) \underline{x}_m(z_\ell) + v_m(z_\ell) \quad (11)$$

The vertical wave numbers are functions of the sound velocity profile through the dispersion relationship of Eqn. 2 and can be further analyzed through knowledge of the SVP. Since our processor will be sequential, it is recursing over depth. Let us assume that the SVP is a piecewise constant function at "each" sensor location z_ℓ enabling us to investigate the observability of Eqn. 4 — a space-invariant system depending only on Δz_ℓ . In between each sensor we assume that the SVP is a function of z , $c(z) = c(z_{\ell-1})$ $z_{\ell-1} \leq z < z_\ell$.

We would like our processor to improve the estimation of the SVP "in-between" sensors. With a state-space processor we can employ two spatial increments in z simultaneously: one for the measurement system Δz_ℓ and one for the modal state-space $\Delta z_j = \frac{\Delta z_\ell}{N_\Delta}$ where N_Δ is an integer. Therefore in order to propagate the states (modes) at Δz_j and measurements at Δz_ℓ we must have values of the SVP at each Δz_j (subinterval) as well. Suppose we expand $c(z)$ in a Taylor series about a nominal depth, say z_0 , then we obtain

$$c(z) = c(z_0) + (z - z_0) \frac{\partial c(z_0)}{\partial z} + \frac{(z - z_0)^2}{2!} \frac{\partial^2 c(z_0)}{\partial z^2} + O^3(z - z_0) \quad (12)$$

or

$$c(z) \approx \theta_0(z_0) + \theta_1(z_0)(z - z_0) + \cdots + \theta_N(z_0) \frac{(z - z_0)^N}{N!} \quad (13)$$

where

$$\theta_i(z_0) := \left. \frac{\partial^i c(z)}{\partial z^i} \right|_{z=z_0} \quad i = 0, \dots, N$$

If we expand this function at each previous sensor depth using z_ℓ as the nominal value, $z_0 = z_{\ell-1}$ and assume that uncertainties in the measured SVP can also be modeled as additive noise with v_c assumed gaussian, zero-mean with variance, $R_{v_c v_c}$ we obtain

$$\underline{c}(z_\ell) = \underline{\Delta}_N^T(z_\ell) \underline{\theta}_N(z_\ell) + v_c(z_\ell) \quad (14)$$

We note that we now have essentially a polynomial model (in z) of $c(z)$ “in-between” the measurement (sensor) outputs. Simply stated, as the state-space processor propagates it uses the measured value of $c(z_{\ell-1})$ available at the $(\ell-1)^{\text{st}}$ -sensor and the polynomial model to predict in-between sensor measurements until the next measurement $c(z_\ell)$ becomes available at ℓ^{th} sensor.

If we further characterize the coefficients of the SVP series as random and gaussian as well, then a Gauss-Markov model of the SVP evolves as

$$\begin{aligned} \underline{\theta}_N(z_{\ell+1}) &= \underline{\theta}_N(z_\ell) + \Delta z_\ell \underline{w}_\theta(z_\ell) \\ c(z_\ell) &= \underline{\Delta}_N^T(z_\ell) \underline{\theta}_N(z_\ell) + v_c(z_\ell) \end{aligned} \quad (15)$$

where defining the notation “ $\sim$ ” to mean “distributed as” and “ $N(\cdot, \cdot)$ ” to mean a normal distribution with specified mean and variance we have $w_\theta \sim N(0, R_{w_\theta w_\theta})$, $v_c \sim N(0, R_{v_c v_c})$, $\underline{\theta}_N(z_0) \sim N(\underline{\theta}_N(0), P_\theta(0))$, and $\underline{\Delta}_N^T(z_\ell) = [1 \ \Delta z_{\ell-1} \ \frac{\Delta z_{\ell-1}^2}{2!} \ \dots \ \frac{z_{\ell-1}^N}{N!}]$.

It is this model that we will use in our processor to estimate the SVP. We can now “augment” this SVP representation into our normal-mode/pressure-field propagation model to obtain an overall system model. The augmented *Gauss-Markov (approximate) model* for M -modal functions is given by

$$\begin{aligned} \begin{bmatrix} \underline{x}(z_{\ell+1}) \\ \underline{\theta}_N(z_{\ell+1}) \end{bmatrix} &= \begin{bmatrix} A(z_\ell, \theta) & | & 0 \\ 0 & | & I_{N+1} \end{bmatrix} \begin{bmatrix} \underline{x}(z_\ell) \\ \underline{\theta}_N(z_\ell) \end{bmatrix} \\ &+ \begin{bmatrix} B(z_\ell) \\ 0 \end{bmatrix} \underline{u}(z) + \begin{bmatrix} \underline{w}(z_\ell) \\ \underline{w}_\theta(z_\ell) \end{bmatrix} \end{aligned} \quad (16)$$

with measurement model

$$\begin{aligned} \begin{bmatrix} p(r_s, z_\ell) \\ c(z_\ell) \end{bmatrix} &= \begin{bmatrix} \underline{C}^T(r_s, z_\ell) & | & 0 \\ 0 & | & \underline{\Delta}_N^T(z_\ell) \end{bmatrix} \times \\ &\begin{bmatrix} \underline{x}(z_\ell) \\ \underline{\theta}(z_\ell) \end{bmatrix} + \begin{bmatrix} v_p(z_\ell) \\ v_c(z_\ell) \end{bmatrix} \end{aligned} \quad (17)$$

This model corresponds to the case where *both* acoustics (pressure-field) and sound velocity measurements are available at each sensor. If we were to further assume that there were no SVP measurements but only acoustic pressure-field measurements available at each sensor location, that is, no $c(z_\ell)$ measurements, then the measurement system model consists only of the first row above.

The question now arises as to whether or not the SVP can be estimated for pressure-field measurements alone, but not excluding some a-priori knowledge (historical) of the SVP. In our formulation, this case implies that we have a “model” of the SVP of the form

$$\hat{c}_N(z_\ell) = \underline{\Delta}_N^T(z_\ell) \underline{\theta}(z_\ell) \quad (18)$$

where the set of $\{\theta_i(z_\ell)\}$ are only known at a sparse number of depths, $\ell = j, j+1, \dots, j+N_\theta$. More simply, we have a set of historical values or new measurements of the SVP measured *a-priori* at specific depths — not necessarily corresponding to all sensor locations $\{z_\ell\}$, therefore, we use these values as initial values to the subsequent estimator enabling it to sequentially update the set of parameters $\{\theta_i(z_j)\}$ over the layer $z_{\ell-1} \leq z_j < z_\ell$ until a new value of $\theta_i(z_j)$ becomes available, then we re-initialize the parameter estimator with this value and continue our SVP estimation until we have recursed through each sensor location. In this way we can utilize our historical or measured SVP information in the form of a parameter update and improve the estimates using the processor. Thus, the linear SVP Gauss-Markov model becomes nonlinear when we constrain the SVP parameters to the set $\{\theta(z_\ell)\}$, $z_\ell = z_j, \dots, z_j + N_\theta$, that is,

$$\underline{\theta}_N(z_{\ell+1}) = \begin{cases} \underline{\theta}_N(z_\ell) + \Delta z_\ell \underline{w}_\theta(z_\ell) & z_\ell < z_j < z_{\ell+1} \\ \underline{\theta}_N(z_\ell) \delta(z_\ell - z_j) & z_\ell = z_j \end{cases} \quad (19)$$

$$c(z_\ell) = \underline{\Delta}_N^T(z_\ell) \underline{\theta}_N(z_\ell) + v_c(z_\ell) \quad (20)$$

This completes the section on state-space representation of the normal-mode propagation model both with and without an augmented sound velocity profile model in polynomial form. Next we analytically investigate the observability of these representations wherever possible.

3 A Systems Theory View

In this section we investigate the normal-mode ocean acoustic propagation model from the Systems Theory viewpoint. First, we examine the case of the normal-mode model with an underlying constant sound velocity profile (SVP) measured using a vertical array and analyze the observability of the resulting system. Next, we extend these results to the space-varying case by allowing the sound velocity profile to be a function of depth. From Systems Theory it can be shown [7] that a necessary requirement of a measurement system is that it is observable, that is, measurements of available variables or parameters of interest be able to provide enough information to reconstruct the internal variables or states (modes). Mathematically, a system is said to be *completely observable* if for any initial state, say $\underline{x}(z_0)$, in the state space, there exists a finite $z > z_0$ such that knowledge of the input (source) $\underline{u}(z)$ and the output (pressure-field) $p(z)$ is sufficient to specify $\underline{x}(z_0)$. Recall that the deterministic linear *state-space* representation of a system is defined in Eqn. 4. The solution to the state representation is governed by the *state-transition matrix*, $\Phi(z, z_0)$ [7].

If we assume that the system is space-invariant, then $A(z) \rightarrow A$, $B(z) \rightarrow B$, $C(z) \rightarrow C$, $\Phi(z, z_0) \rightarrow \Phi(z - z_0)$ and from the Cayley-Hamilton theorem of Systems Theory (see [7]), it is possible to show that $\Phi(z - z_0) =$

$e^{A(z-z_0)}$ leading to the $2LM \times 2M$ observability matrix given by

$$\mathbf{O} := [C \mid CA \mid \dots \mid CA^{2M-1}]^T$$

It can be shown that a necessary and sufficient condition for a system to be completely observable is that the rank of $\mathbf{O}$ must be $2M$. Thus, for the space-invariant case, checking that all of the measurements contain the essential information to reconstruct the states for a linear space-invariant system reduces to that of checking the rank of the observability matrix.

The foundation of our investigation is based on the dispersion relation of Eqn. 6. Here we analyze the following cases: (1) The SVP is piecewise constant, that is, $c(z) = c_0$. (2) The SVP is space-varying, that is, $c(z)$ is a function of depth z .

For the first case we let the SVP be a constant and therefore the linear system under investigation becomes space invariant given Eqns. 1 and 2. Using the canonical structure given by the space-invariant system, input and measurement matrices of the above mentioned state equations, we can construct the observability matrix. Since the state-space representation processes the array measurements sequentially, we have a scalar measurement system and therefore the observability matrix is $2M \times 2M$ ($L = 1$) and given by

$$\mathbf{O} = \begin{bmatrix} \beta_1 I_2 & \dots & \beta_M I_2 \\ -\beta_1 \kappa_z^2(1) I_2 & \dots & -\beta_M \kappa_z^2(M) I_2 \\ \vdots & & \vdots \\ (-1)^M \beta_1 \kappa_z^M(1) I_2 & \dots & (-1)^M \beta_M \kappa_z^M(M) I_2 \end{bmatrix}$$

where I_2 is the 2×2 identity matrix. It is clear from its structure that this matrix is always of rank $2M$ as long as $\beta_i \kappa_z^i(i) I_2 \neq \beta_j \kappa_z^j(j) I_2$ which is a reasonable assumption in the real ocean environment.

These results can be extended to the case of a space-varying system by relaxing the space-invariant constraints on the system, input and measurement matrices. In this case we again desire the unique solution for $\underline{x}(z_0)$. This is identical to investigating the null space of the $N_z \times N_x$ -observability Gramian given by

$$\mathbf{O}(z_0, z) = \int_{z_0}^z \Phi^T(\alpha, z_0) C^T(\alpha) C(\alpha) \Phi(\alpha, z_0) d\alpha$$

The space-varying system is *completely observable* if and only if the null space of $\mathbf{O}(z_0, z)$ is $\mathbf{0} \in R^{2M}$ or equivalently $\mathbf{O}(z_0, z)$ is nonsingular. Thus, we must utilize the observability Gramian over the aperture of the array. However, due to the structure of the normal mode model and the measurement vector $C^T(r_s, z_\ell)$, we need only study the state and measurement equations for the m^{th} mode, since all of the modes are decoupled and enter the observability matrix in individual columns. We can simplify the model even further by recognizing that $u(z)$ is a point source given by $u(z) = \delta(z - z_s)$ and therefore the state representation reduces to a homogeneous differential equation. Since an impulse input can be characterized by an initial condition (unforced) response of

the system, we can substitute the impulse source into the solution of the state equation, that is,

$$\underline{x}_m(z) = \Phi_m(z, z_0) \underline{x}_m(z_0)$$

where $\underline{x}_m(z_0) = \Phi_m(z_s, z) B_m(z_s)$. It is well-known that the solution for the state transition matrix of a space varying system can be found as a convergent Neuman series that evolves [9]

$$\underline{x}_m(z) = \sum_{i=0}^{\infty} \Gamma_m^i \underline{x}_m(z_0)$$

with $\Gamma_m = \int_{z_0}^z A_m(z) dz$

Combining these terms and truncating to the first three terms of the series we have that the state transition matrix is given by

$$\Phi_m(z - z_0) = \begin{bmatrix} 1 - \int_{z_0}^z \left[\int_{z_0}^z \kappa_z^2(m) d\alpha \right] dz + & (z - z_0) + \\ - \int_{z_0}^z \kappa_z^2(m) dz + & 1 - \int_{z_0}^z \kappa_z^2(m) (z - z_0) dz + \end{bmatrix}$$

Essential to the observability of the normal-mode model is the a-priori knowledge of the set of vertical wave numbers $\{\kappa_z^2(m)\}$ which are space-varying due to their dependence on the sound velocity profile of the corresponding dispersion relation. Analysis of the space-varying Gramian matrix which requires the state transition matrix leads to little insight into the structure of the observability of normal-mode models, since the development of the observability matrix consists of a sequence of nested integrals of $\kappa_z^2(m)$ with no obvious closed form solution and cannot be solved explicitly except for extremely simple models of the SVP, $c(z)$. However, we have seen that the piecewise constant approximation does lead to some insight into the observability of the normal-mode model indicating a prime dependence on the uniqueness of the vertical wave numbers. This uniqueness property is essentially guaranteed, since the set $\{\kappa_z(m)\}$ are obtained as solutions of the associated boundary value problem and in fact are the eigenvalues associated with the eigenfunction solution.

Unfortunately we cannot easily investigate the observability of the augmented system of the previous section to understand (analytically) the dependence of the normal-mode model on the SVP parameters because of the nonlinear set of state equations. This completes the section on observability of the SVP, next we investigate the actual estimation problem and see how these models and concepts can be used to construct a processor.

4 Sound Velocity Estimator Design

In this section we discuss the development of the sound velocity estimator based on the state-space representation of the normal-mode model and a vertical line array of sensors. We briefly discuss the approach, then the algorithm and apply it to a simulated shallow water problem. The *SVP estimation problem* can be defined in terms of our previous models as: GIVEN a set of noisy acoustic (pressure-field) measurements $\{p(r_0, z_\ell)\}$ and a

set of sound speed information $\{\theta(z_\ell)\}$, FIND the "best" (minimum error variance) estimate of the SVP, $\hat{c}(z_\ell)$.

For this problem we consider three cases of interest:

(1) *Both* sets of complete (FULL) measurements available at each sensor location. (2) *Sparse* set of SVP measurements available (PARTIAL) at N_θ depths with complete set of pressure-field measurements. (3) *No* SVP measurements available (NONE) only a set of acoustic measurements and an initial guess at SVP parameters. The solution to the SVP estimation problem can be obtained using the extended Kalman filter (EKF) algorithm or its variants [10] as a joint state/parameter estimator. Here we choose to "discretize" the continuous model first and apply the discrete EKF available in SSPACK [11].

For Case I we see that the estimate of the SVP is produced as a filtered measurement $\hat{c}(z_\ell)$, since we have *both* acoustic and sound velocity measurements available at each depth $c(z_\ell)$, $\ell = 1, \dots, L$. Note that the processor operates in two spaces, the state-space and the corresponding measurement space. The state-space consists of the state model (modal functions), model parameters $\{\{\kappa_z(m)\}, \{\kappa_r(m)\}\}$, $m = 1, \dots, M$, initial conditions and sound speed parameters $\{\theta(z_j)\}$ as well, while the measurement space utilizes the measurement model and set of noisy measurements, $\{\{p(r_s, z_\ell)\}, \{c(z_j)\}\}$. Again let us emphasize that it is assumed that the model parameters are available from other measurements or spatial beamforming, therefore, the vertical wave numbers, source location etc. are known or guessed. In fact, if these parameters are not available, then an iterative approach must be taken. Here our objective is to develop an SVP estimator and we assume all of the parameters are available or can be obtained. So we see that as the estimator sequentially processes the acoustic measurements available at each sensor, it jointly estimates the states (modes and unknown parameters) and measurements (pressure-field and SVP), that is, the processor produces the following set of estimates at each depth: $\{\hat{x}(z_\ell), \hat{\theta}(z_\ell), \hat{p}(r_s, z_\ell), \hat{c}(z_\ell)\}$.

For Cases II and III, we do not have the benefit of FULL sound velocity data at each sensor location, but instead PARTIAL information in the form of parameters $\{\theta(z_j)\}$ or none at all. In these cases, the set of SVP measurements $\{c(z_\ell)\}$ is not available and we merely have historical or sparse measurement data of the sound velocity. For Case II, we utilize the PARTIAL information and a model of the SVP (as in the previous section). Typical partial information consists of SVP sound speed estimates in the form of discrete data pairs $\{c(z_j), z_j\}$ which can be utilized in the estimator for correction as it processes the acoustic data. In the final case we merely make an initial guess, use the same basic sound velocity model and attempt to estimate the SVP with no information (except initial conditions). In this case the processor attempts to find an average (conditional mean) estimate of the parameter set $\{\theta(z_j)\}$ and uses those estimates to construct the "acoustics only" SVP, $\hat{c}_N(z_\ell)$ from the data.

In this study, we will use the shallow water SNAP model (see [12]) driven only by initial conditions. We assume that the source horizontal range is known, $r_s = 7500m$ and therefore the measurement model is linear in $\underline{x}(z_\ell)$. The simulation consists of a 100m ocean depth

with a semi-infinite hard bottom of density $2g/cm^3$ and a compressional speed of 1600m/s along with a point source located at $z_s = 25m$ in depth. The SNAP solution to the boundary value problem yields $M = 4$ modes (eight states). We use the first two terms ($N=2$) of the Taylor series expansion of the SVP (piecewise linear) for our model of Eqn. 14 where both θ_0 and θ_1 are simulated as space-varying gaussian random functions with specified means and variances, and therefore, through linearity, so is $c(z_\ell)$. Thus, our Gauss-Markov model for this problem is given by Eqns. 16 and 17. This model corresponds to Case I with both acoustic and sound velocity measurements available. We will use a spatial sampling interval of $\Delta z_j = \frac{\Delta z_\ell}{10}$ in the state propagation equations as discussed previously. The corresponding EKF estimator evolves from the algorithm (see [10]) with all of the appropriate functions and jacobians.

We performed simulations for each case using SNR of (10db, 0db, -10db) with an assumed 10% ($\Delta\theta_m = 0.1$) for the mean SVP parameter error. Next we applied the SVP estimator to the various data sets at each SNR. The estimated SVP parameters, estimated SVP and corresponding *weighted sum-squared residual* (WSSR) statistic for a window length of 25 samples are determined by the processor. The WSSR is a measure of overall global estimation performance for the EKF processor essentially determining the "whiteness" of the innovations sequence. If the WSSR statistic lies beneath the threshold shown, then theoretically the estimator is tuned and converges. For each decrease in SNR the parameter and SVP estimates become noisier, however, the statistical tests reveal a zero mean/white innovations sequence — a necessary and sufficient condition for optimality, that is, an estimator is considered "on track" when these properties hold.

Next we observe the performance of the SVP estimator for PARTIAL (Case II) sound velocity information available. In this case we use only a limited number of sound speed measurements (four) assumed from historical data and employ them in the estimation scheme described above. Here we use only the acoustic measurements. The processor uses the 4 sound velocity data values $\{c(z_j)\}$, $j = 1, \dots, 4$ to set hard constraints on the parameter estimator and force it to meet these values only when the appropriate depth is achieved. The results of the runs are shown in Figure 1. Here we see the estimated SVP parameters and reconstructed SVP. The estimator appears to track the SVP parameters as well as the SVP.

Finally we consider the case of no sound velocity data (NONE) and only acoustics. Runs were made without using any a-priori sound velocity measurements and it appears that the EKF SVP estimator cannot converge rapidly enough to the abrupt changes in the parameters due to the limited number of sensor measurements.

In any case it appears that the SVP estimator can be tuned to produce reasonable estimates, but it does not perform as well as when some or all of the SVP data is utilized. In terms of observability, it still appears that the system remains observable even when constrained to acoustic data only. This completes the section on SVP estimation.

Figure 1: SVP Estimation for PARTIAL SVP Information, Case II, (*true mean, 10db, 0db, -10db*): (a) SVP parameter estimates: constant and slope. (b) SVP estimate and WSSR statistic (Window=40 samples).

5 Discussion

In this paper we have developed a sound velocity profile estimation scheme based on coupling the normal-mode propagation model to a functional model of the SVP which evolved by expanding it in a Taylor series about the most currently available sound velocity measurement. The algorithm employed was the extended Kalman filter which evolved as the solution to the minimum variance estimation problem when the models were placed in state-space form. We examined various cases of the estimator and showed that it performed best when a complete set of acoustic and sound speed data was available simultaneously at each sensor. With the inavailability of complete set of sound velocity measurements, we also showed that the processor performs reasonably even for very sparse sound speed information. For the case of no *a-priori* sound velocity data, the processor requires more sensor measurements in order to track abrupt changes in the SVP.

Returning to the observability issues discussed previously, it is clear that the modal system is completely observable as long as each of the eigenvalues obtained from the solution of the boundary value problem are unique, which was the case in our simulations. We also know that in order for our estimator to converge the system must be observable (and controllable, stable). In the case of *acoustic only* measurements with PARTIAL SVP data, it appears that the SVP estimation can produce reasonable estimates as long as the associated Gauss-Markov model is representative of the SVP. In the case of no (NONE)

a-priori SVP information (no sound speed data), it does not appear that this scheme will track abrupt changes in the SVP due to the limited number of sensors available; however, this situation is not realistic since there is usually current sound velocity measurements available in a given area of operation.

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