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2-15-78

HR. 1843
ORNL/TM-5960

**ORNL Fusion Power Demonstration Study:
Fluid Flow, Heat Transfer, and Stress
Analysis Considerations in the
Design of Blankets for Full-Scale
Fusion Reactors**

MASTER

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OAK RIDGE NATIONAL LABORATORY
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Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy \$6.50; Microfiche \$3.00

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Contract No. W-7405-eng-26

Fusion Energy Division

ORNL FUSION POWER DEMONSTRATION STUDY:
FLUID FLOW, HEAT TRANSFER, AND STRESS ANALYSIS
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FOR FULL-SCALE FUSION REACTORS

A. P. Fraas and A. S. Thompson

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Date Published: February 1978

Prepared by the
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ABSTRACT

The complex and subtle interplay of conditions imposed on fusion reactor blanket designs by heat transfer, coolant flow, thermal stress, pressure stress, fabrication, and maintenance considerations has been examined for a series of representative cases taken from the literature. In view of the difficulties with thermal stress cracking, wall melting, and vaporization that have been experienced in tokamak experiments, particular attention has been given to possible hot spot effects that might stem from aberrant behavior of the plasma. The results of the study indicate that a lithium-cooled niobium blanket structure will withstand ten to twenty times more severe first wall heating conditions than a helium-cooled stainless steel structure. This raises a number of serious problems relative to magnetohydrodynamic effects, and methods for coping with these are outlined.

The blanket design employing a recirculating lithium-cooled niobium structure that appeared most promising from the heat transfer, stress analysis, and coolant flow standpoints is then reviewed from the standpoints of fabricability, cost, and maintenance and found to be competitive with or superior to the several helium-cooled blanket designs considered in the study.

A number of major questions are pointed out and experiments are recommended that should help to resolve the basic uncertainties and provide a sound basis for key design decisions.

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ACKNOWLEDGEMENTS

The work summarized in this report was carried out as part of the ORNL Fusion Power Demonstration Study. Invaluable assistance has been obtained from others working in the field. A particular debt is owed to Denis Breton of Commissariat à L'Energie Atomique of France and to Peter Bonanos of PPPL, who supplied splendid sets of information and glossy prints showing damage that has occurred to limiters and liners in tokamak plasma physics experiments. Don Steiner and Bill Wells have reviewed drafts of this report as it has evolved and made many helpful recommendations. Lee Berry and John Clarke have supplied valuable information and insights, as have John Mitchell of Culham, Fred Tenney of PPPL, and Fred Ribe of LASL. This assistance has been deeply appreciated, for the work could not have been done without it.

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SYMBOLS

c_p = specific heat at constant pressure
 C_1, C_2 = parameters of material properties in thermal fatigue
 d = depth of penetration of melting
 d = thickness of wall
 E = modulus of elasticity
 h_1, h_2 = finite intervals of space
 I = section modulus for bending
 k = thermal conductivity
 K = ratio of vaporization temperature to melting temperature
 L_m = latent heat of melting
 L_v = latent heat of vaporization
 M = bending moment
 N_f = number of cycles of stress to failure
 P = force
 q = finite interval of time
 q = rate of deposition of radiant energy per unit area
 Q = amount of radiant energy deposited per unit area
 s_m = entropy of melting
 s_v = entropy of vaporization
 T = temperature
 T_m = melting temperature
 T_v = vaporization temperature
 $T_s - T_0$ = amount of sudden change in surface temperature
 v = velocity of motion of solid-liquid boundary in melting
 W = dimensionless temperature
 x = distance from origin of coordinate system
 σ = stress
 α = linear coefficient of thermal expansion
 $\beta = h/kd$
 γ = thermal diffusivity, $k/(c_p \rho)$
 δ_m = depth of melting, μ_m
 $\Delta\epsilon$ = total range of strain ($\Delta\epsilon_e + \Delta\epsilon_p$)
 $\Delta\epsilon_e$ = range of elastic strain during cycle
 $\Delta\epsilon_p$ = range of plastic strain during cycle
 η = dimensionless time
 θ = finite interval of time
 λ_m = dimensionless heat of melting
 λ_v = dimensionless heat of vaporization
 μ = Poisson's ratio
 ν = frequency of application of repeated stress
 v_m = dimensionless velocity of solid-liquid boundary
 v_s = dimensionless velocity of liquid-vapor boundary
 ξ = dimensionless distance
 ρ = density
 τ = period of time during which energy is deposited

1. INTRODUCTION

The design of a blanket for a full-scale thermonuclear reactor poses a set of problems even more complex than those posed by the design of the core for a fission reactor. The response to the comprehensive survey of these problems presented in Ref. 1 has indicated that the brief treatment in Ref. 1 of the fluid flow, heat transfer, and stress analysis considerations should be greatly extended and elaborated; this report has been prepared with that objective. In addition to the obvious neutronic and first wall cooling problems under steady-state conditions, it is necessary to consider the possibility of severe local hot spots under transient or off-design conditions and the thermal stress problems associated with complex, three-dimensional temperature distributions inherent in any actual arrangement of blanket cooling passages. If a leak were to develop in the blanket as a consequence of a crack induced by thermal stresses or a hole burned in the first wall, the large size and awkward shapes of the blanket structural components make maintenance so difficult that the time required to replace a section of the blanket has been estimated to be many weeks or months. The life of the blanket and the probability of a failure are highly dependent on the intensity of local stress concentrations; hence it is important indeed to consider the possible superimposed effects of the pressure and thermal stress loadings that may be encountered under both normal and off-design conditions. The thermal stresses can be eased by reducing both the temperature rise in the coolant and the temperature difference between the coolant and the wall. These factors affect the pumping power requirements and pressure drop for the blanket coolant circuit; thus the blanket cooling system design becomes a vital consideration. If a liquid metal is employed, magnetohydrodynamic (MHD) effects become important. The complexity of these problems makes it extremely difficult to evolve a good design for a blanket or even to evaluate the merits of a proposed design.

This report has been prepared in an effort to resolve some of the confusion by delineating the various problems involved, presenting some tables and charts that have been developed to assist in design analyses and to provide quantitative insights into the effects of design parameters, and applying this information to the analysis of a number of typical blanket structures that have been proposed.

2. SUMMARY

Probably the most important points to emerge from the comprehensive set of studies presented in this report are as follows:

- (1) The energy deposition in the blanket under normal operating conditions is fairly well defined for a substantial variety of blanket material compositions and fusion reactor types. This work indicates that 15-20% of the total power output could be deposited in the first wall in tokamaks, about 10% will be deposited in the first wall in mirror reactors, and around 5% will be deposited in the first wall in theta-pinch reactors. The energy deposition rate falls off quite rapidly with distance into the first wall.
- (2) Local hot spots may develop in the first wall. In tokamaks, runaway and trapped electrons have burned holes in the walls of experimental machines. A plasma instability or plasma energy dump to the wall might also produce severe local hot spots. A high level of ion energy deposition might also occur at the inlet to a divertor in a tokamak or in the vicinity of the outlet beam aperture of a mirror machine. (For the latter case, estimates indicate that the local wall heating will be three times the average for a tokamak having the same nominal first wall power loading.)
- (3) Both experiments and analyses indicate that local hot spots are likely to cause thermal stress cracking and may cause

melting or even vaporization of the first wall material.

- (4) Thermal stresses and thermal strain cycling are common causes of failure in structures subject to large temperature differentials. In some of the fusion reactor designs that have been proposed, the temperature differentials between the inner and outer surfaces of the first wall have been such that they would produce severe cracking in only a few hundreds of hours of operation under normal operating conditions. Much more severe thermal strain cycling conditions result from complex, three-dimensional temperature distributions within the blanket structure as a whole which cause differential thermal expansion and stress concentrations at corners, particularly at the junction between the first wall and the bulkheads at the ends of each blanket module.
- (5) With a given wall power loading, the severity of the thermal stresses associated with differential thermal expansion in the structure depends in substantial measure on the choice of coolant and the consequent temperature differential between the wall and the coolant as well as on the temperature rise in the coolant stream.
- (6) The wall power loading that can be tolerated without thermal strain cycling failures with any given blanket structural design is heavily dependent on the physical properties of the structural material. Niobium, for example, would tolerate a wall power loading about five times that permissible with stainless steel.
- (7) Of the various blanket structural design concepts examined in this report, the recirculating lithium blanket built of niobium appears the most promising for giving an acceptable life in a tokamak under normal operating conditions. This blanket appears able to withstand off-design transient hot spot conditions about 20 times more severe than could be tolerated by a helium-cooled stainless steel blanket.
- (8) None of the helium-cooled blankets examined in this report appears to give an acceptable life from the thermal strain cycling standpoint; hence there is a strong incentive to pursue a liquid metal-cooled blanket in spite of the formidable MHD problems inherent in such a blanket. A molten salt, though less effective as a coolant than lithium, might also be suitable if one with a fairly low melting point can be found that will give good materials compatibility characteristics under the severe radiation conditions prevailing near the first wall.
- (9) The seriousness of the embrittling and swelling effects of fast neutron irradiation of the first wall will be heavily dependent on the blanket design concept and the principal design features related to fluid flow, heat transfer, thermal stresses, and pressure stresses under normal, off-design, and accident conditions.
- (10) Depending on the design, failure from thermal and hydraulic stress conditions may occur even before radiation damage effects become significant.
- (11) The sharp peaking of the radial power distribution at the surface of the first wall in fusion reactors is far greater than any power peaking in commercial fission reactors. In fact, the radial power distribution characteristic of a fusion reactor would be ruinous in a fission reactor.
- (12) The pressure drop associated with pumping lithium into and back out of the blanket appears to be excessive, i.e., ~200 psi (14 bar), and the size of the inlet and outlet ducts makes them extremely awkward for any of the arrangements that have been proposed in the literature. These arrangements include the use of iron shielding around the ducts to reduce the magnetic field in the flowing liquid metal stream, the use of an insulating layer in a sandwich-type wall to reduce the flow of electrical eddy currents in the wall, and

the use of a back emf through strategically placed electrodes along the duct wall to give a near-zero pressure rise electromagnetic (EM) pump. The lower volumetric flow rate possible (if potassium is pumped into the blanket, boiled there, and the potassium vapor ducted out of the blanket) leads to an acceptable situation for simple radial inflow of potassium feed into the blanket if either the sandwich construction or the back emf arrangement is employed.

- (13) If the magnet coils are crowded closely together in the region close to the polar axis of a tokamak (so that there is no space there for ducts to penetrate through the magnet region into the blanket along radial paths and hence circumferential flow regions inside the magnet coils must be employed), no arrangement considered appears to give a satisfactory situation with respect to pressure drop in the potassium feed lines.
- (14) The greatest uncertainty associated with the use of a recirculating lithium blanket is the degree to which the magnetic field will suppress eddies in the lithium stream, because even a slight amount of eddying will have a large effect on the first wall temperature distribution in the direction of the flow. This question can be resolved only by an experiment.
- (15) A whole series of small but possibly serious specialized problems must be examined in more detail experimentally to determine whether the recirculating lithium blanket cooled by means of a potassium boiler embedded in the blanket is in fact a viable possibility.

3. EXPERIMENTS NEEDED

The analytical and experimental work carried out at ORNL to date indicates that a recirculating lithium blanket yields an exceptionally attractive system, and good solutions have been found to most of the problems. However, five

major problem areas remain, and analyses indicate that further progress toward their solution will require some experiments. These problems and an indication of the scale of the experiment required in each case may be summarized as follows in roughly the order of priority which it is felt they deserve:

- (1) Fabricate and test a series of structural metal specimens coated first with a thin layer of a ceramic such as yttria, thoria, or zirconia and then coated with a thin protective layer of the structural metal. These should then be exposed to potassium at a series of temperatures from 600-1000°C to determine whether the high temperature liquid potassium causes blistering or other deleterious effects.
- (2) Using the existing 1/6-scale model of a blanket module, investigate the temperature distribution resulting from a local heat input along the outer face of one leg and local cooling along the outer face of the other leg and compare the resulting temperature distributions with those estimated analytically for ideal irrotational slug flow to determine the degree to which there may be a little turbulent mixing.
- (3) Using practical tube bank geometries to simulate banks of potassium boiler tubes in a recirculating lithium blanket module, investigate the effects on flow and pressure drop associated with the MHD flow discontinuities represented by the tube banks.
- (4) Vitrally important insights into MHD effects on liquid metal flow through the inherently complex geometries useful in CTR blanket design could be obtained if there were some means for measuring the velocity distribution in the liquid metal stream. It appears that this might be done through the use of an RF coil employed as a capacitance probe and tuned to give an indication of the local velocity at a specified distance into the stream from the duct wall. Experiments should be carried out

to investigate the feasibility and possible precision of such a device.

- (5) A series of tests should be run with an ion beam to determine the relative resistance of stainless steel and niobium to repeated energy pulses whose magnitude and duration would be chosen to simulate conditions that might be experienced in a tokamak. These tests should be run first with no coolant in back of the simulated first wall, second with static liquid lithium, and third with a static molten salt to simulate the principal blanket cooling situations of interest.

4. NORMAL HEAT LOADS

The energy release per cubic centimeter in some typical blanket systems was worked out by D. Steiner² some years ago and in some additional systems by Santoro³ et al. more recently. This work defines not only the energy deposition as a function of radial distance from the first wall, but some of it also includes calculations of the breeding ratio and of the activation of both the structural material and the coolant for a number of typical geometries. The results of these neutronics calculations provide the basic material essential to any blanket design effort. As a consequence, results of this work for representative geometries are summarized in a set of figures and tables that are presented in Appendix D so that the information will be conveniently available for use in connection with the various problems that will be taken up in the course of this report.

The most crucial cooling problems in the blanket are those associated with the first wall. As shown by Steiner and summarized in Table 1 (from Ref. 4), under normal conditions the bulk of the energy input to the first wall stems from plasma radiation rather than from neutron and gamma heating. In tokamak reactors this energy input will amount to 15-20% of the total power output of the reactor. The energy deposition in the wall may or may not be uniformly distributed, depending on the power density distribution in

the plasma.⁵ Calculations by Danner⁶ indicated that for low beta tokamaks this energy input would be quite uniformly distributed over the entire area of the first wall. However, recent studies of high beta tokamaks indicate that the local heat input to the first wall may run nearly double the average.⁷ If a diverter is employed, near the entrance to the diverter the local energy input from charged particles might be much higher than the average for the wall as a whole.

In mirror reactors, much of the energy from the alpha particles is in the plasma flowing out through the mirrors at the ends. Thus, the average energy input per unit of surface area of the first wall is much less than that for tokamaks, but the local energy input in the vicinity of the plasma discharge ports in the mirrors at the ends appears to be quite high — of the order of three times that for a tokamak.⁸ In theta-pinch reactors, much of the plasma energy would be converted into electricity through the interactions of the expanding plasma with the magnetic field, and hence the average energy input to the first wall for a given power density and the energy input to the first wall per unit of surface area are lower than the average for tokamaks by a factor of three or four. However, the energy input to the first wall is delivered in a pulse which represents only about 1% of the duty cycle, so the overall effect is to give a first wall heating problem much more severe than in tokamaks.

5. TRANSIENT HOT SPOTS IN THE FIRST WALL

In the early experiments with tokamaks in the U.S.S.R., it was found that there was so much difficulty with local hot spots which burned holes in the first wall that the Russians instituted the use of limiters. In effect, these are sharp-edged orifices of tungsten or molybdenum whose bore is a little smaller than that of the first wall. Thus, any conditions that might cause the plasma or a stream of runaway electrons to move locally toward the first wall are likely to cause it to impinge first on the edge of the limiter, which was designed to withstand the damage much more effectively than the first wall.

Table 1. Energy deposition at the first wall of fusion reactor blankets during the plasma burn
(Table excerpted from Ref. 4)

	Mirror (Refs. 8, 9)	Theta-pinch (Ref. 10)	Laser- pellet (Ref. 11)	Tokamak (Refs. 12, 13)
First wall material	Stainless steel	Al_2O_3 + niobium	Lithium-wetted niobium	Stainless steel
First wall thickness (cm)	0.1	0.03 + 0.10	1.0 ^a	0.4
Neutron wall loading (MW/m ²)	1.6	2	2.6	1.25
Time scale for energy deposition	continuous	80 msec	10^{-7} - 10^{-11} sec ^b	continuous
Energy deposition	W/cm ²	J/(cm ² pulse)	J/(cm ² pulse)	W/cm ²
Neutron-induced	0.9	6.6 + 20.7	7.0	5.2
Plasma radiation	5.0	43 + 29	2.8	22.0
Particle fluxes	100.0	negligible	60.6	1.0

^aThe first wall is 60% (by volume) niobium and 40% lithium.

^bDepending on source of energy deposition.

Subsequent experiments with tokamaks in both the U.S. and France have followed the Russian practice of employing limiters. However, even with limiters in the system, in 1973 the French TFR experienced severe damage to the first wall as a consequence of a peculiar plasma behavioral pattern caused by runaway electrons. Similar damage was experienced in the Alcator at MIT early in 1976. These incidents raise a disturbing question — are there yet other modes of plasma behavior in larger scale machines that may lead to types of damage not yet experienced? Thus, it seems very much in order to examine the various types of damage that have been encountered in tokamak experiments and relate them to similar types of damage that have been experienced with neutral beam injection units, plasma guns, lasers, and the like. Analytical models for the various types of damage that might be caused by short, intense bursts of energy input to surfaces were evolved for this report, and these have been related to experimentally observed phenomena. These analytical models can be used to evaluate proposed blanket designs.

5.1 OBSERVATIONS OF DAMAGE IN ORMAK, PPPL-ST, AND TFR

Four major types of damage to the first wall have been observed in the plasma physics experiments with tokamaks. These include cracks, apparently as a result of thermal stresses, local melting (sometimes to a depth of as much as 2 mm), melting accompanied by splattering of droplets from small pools of melted material to nearby surfaces where they adhered and froze, and vaporization of material as evidenced by films on nearby cold surfaces that could have been formed only from the vapor.

5.1.1 Cracking

The first limiters made for ORMAK were made of thick plate and were severely damaged by cracks, apparently as the result of thermal stresses. To avoid this problem, subsequent limiters were fabricated of laminations of thin tungsten plates to give the structure shown in Figs. 1 and 2. This eliminated gross cracking, but there were still small local cracks and spallation of the sort evident in the lamination

shown in the upper right of Fig. 2. These small cracks and spalled areas did not cause any serious difficulty.

5.1.2 Melting

Melting of local areas in limiters has been observed repeatedly. A small melted zone can be seen in the ORNL ORMAK limiter of Fig. 1; a closeup of this zone is shown in Fig. 2. Examination with a magnifying glass indicates that melting occurred to a depth of about 1 mm. Similar examples of melting are shown in the photos (Figs. 3-6) taken of limiters used in the PPPL-ST machine¹⁴ and (Figs. 7-9) of limiters in TFR.¹⁵ Figure 10 shows a hole melted through the 0.5-mm convoluted first wall of IFK at a point midway between magnet coils and at a greater radius than the limiters.

5.1.3 Vaporization

There are several evidences of vaporization of material from limiters. An examination of Figs. 5 and 6, for example, raises questions about the pebbly appearance of the zone in which melting has unquestionably occurred; one explanation for the pebbly appearance is that some vaporization took place. The etched appearance of the region surrounding the pebbly melted zone strongly suggests that the surface damage there occurred as a consequence of vaporization. Analyses of dark deposits found on the inner surface of the torus have disclosed that these were of the same material as the limiter, and the character of the deposits is clearly what one would expect from vapor deposition.^{16,17}

5.1.4 Melting Accompanied by Explosive Vaporization

Examination of some of the melted zones, notably those in Figs. 6-8, gives the impression that, following melting, some of the material in the pool of melted metal was blown out laterally by some sort of a pressure or impact. This impression is supported by the fact that surfaces in the immediate surroundings are peppered with frozen droplets of molten metal that have adhered

to colder base surfaces. There is evidence of this in Figs. 2, 6, 7, and 8. Figure 11 shows some of the splattered material on surfaces in the vicinity of melted spots on the limiter of Fig. 8. Thus, it appears that rapid heating of the surface may proceed to the point where some vaporization occurs, and this may blow off vapor so rapidly that the reaction force acts a bit like a light hammer blow and splatters some of the liquid from the shallow pool of molten metal.

5.2 DAMAGE SPECTRUM

In attempting to relate the evidence of damage presented in the previous section to the design of a full-scale tokamak, one needs to set up analytical models. In doing this one soon reaches the conclusion that the principal parameters involved are the amount of energy deposited per square centimeter per pulse, the duration of the pulse, and the physical properties of the target. The latter include thermal conductivity, specific heat, heat of fusion, melting point, heat of vaporization, vapor pressure as a function of temperature, modulus of elasticity, yield point, and ductility.

5.2.1 Effects of Pulse Duration

Figure 12 has been prepared to help show the effects of pulse duration on the types of damage to be expected. The first damage threshold to be reached as the amount of energy per pulse is increased is cracking caused by thermal stresses. Extensive experience with low level temperature excursions indicates that no damage will occur if the temperature variation can be kept below some limit characteristic of the particular material in question, even for millions of thermal cycles. As the temperature excursion per pulse is increased, the number of thermal cycles required to produce damage is reduced.¹⁸ For Inconel or stainless steel at 600°C, for example, the amplitude of the temperature excursion per pulse required to produce cracks from thermal strain cycling in a relatively short time is of the order of 60°C, with some variation from one alloy to another. A typical example of a surface

cracked in this fashion is shown in Fig. 13. In this instance molten salt streams at two different temperatures were fed into a Y and the flow in the two streams was pulsed so that the wall of the passage downstream of the Y was exposed alternately to fluid at the two different temperatures.¹⁹ Once the temperature difference between the two streams was increased beyond about 50°C, the number of cycles to produce cracking fell off rapidly with a further increase in the temperature difference between the two fluids. This effect is shown in the curve in Fig. 14. Note that the material used in this case, Inconel, is an exceptionally tough and ductile material.

The effect of pulse frequency on the temperature range required to induce thermal strain cracking is not great until the pulse frequency is made greater than about 10 Hz. At higher frequencies the time for relaxation of the thermally induced stresses becomes sufficiently short that there is not time for full plastic flow of the metal to occur; hence somewhat greater temperature amplitudes per cycle can be tolerated for a given number of cycles to damage. At pulse frequencies of 1000 Hz, for example, one might be able to double the temperature amplitude for a given amount of damage from a given number of cycles. This, of course, depends on the particular material and the mean temperature of the test.

5.2.2 Surface Melting

As will be discussed in the next section, the rate at which heat can be conducted away from the surface may be sufficiently great that for long pulse durations surface melting would not occur but rather the entire part would melt. On the other hand, sufficiently short pulse durations would not give time for the heating to penetrate sufficiently to melt a surface layer of significant thickness — the bulk of the heat would go into vaporization of a surface layer. Thus, in Fig. 12 the regime in which surface melting is to be expected lies in the span of pulse durations from 10^{-5} sec to 1 sec.

5.2.3 Vaporization

For the range of energy inputs per pulse for tokamak systems, vaporization from the surface would be unlikely to occur except for pulse durations shorter than about 10^{-1} sec. Note that the range for surface vaporization shown in Fig. 12 overlaps that for surface melting so that melting, explosive vaporization, and splattering can occur over the range from around 10^{-1} to 10^{-5} sec.

5.2.4 Spallation by Shock Waves

Shock waves can be induced in a metal target either by very rapid thermal expansion or by the reaction force from very rapid vaporization. Because of the high velocity of sound in structural metal — of the order of 5000 m/sec — shock waves can be induced only by very short pulse durations, i.e., less than 10^{-6} sec.

5.2.5 Energy per Pulse for Typical Cases

In an effort to provide perspective on the problem, Fig. 15 was prepared to show the energy per pulse in joules per square centimeter associated with some typical controlled experiments. Also shown are several estimates of the energy input per pulse to the wall of typical full-scale magnetically confined thermonuclear reactors.

LASL tests with a plasma gun

In a typical test run at LASL with a plasma gun directed at a copper block calorimeter with a 100- μ sec duration burst of 100 kJ/burst- cm^2 it was found that around 20% of the energy went into heat that remained in the target and 80% went into material that was vaporized or blown off the target.²⁰ Note that these test results are consistent with the range indicated for this type of damage in Fig. 12.

Neutral beam injectors

A photo of the type of damage typically encountered in neutral beam injector test work is

shown in Fig. 16. The damage appears to be similar to that experienced in tokamaks, as evidenced by Figs. 6 and 7, with evidence of both melting and vaporization. Again, the experimental evidence is consistent with the ranges indicated for these types of damage in Fig. 12.

Laser fusion experiments

Although the laser fusion work lies outside the pulse duration range that appears to be of interest for tokamaks, it seems worthwhile to include something on laser fusion tests to put them in perspective. Laser pulses of the order of 10^{-10} sec with energies of the order of 10^4 J/cm² have been obtained and have produced shock wave compression of pellets.

5.3 CAUSES OF DAMAGE IN TOKAMAK EXPERIMENTS

The bulk of the damage in tokamak experiments has been thought to stem from runaway or trapped electrons. The energy of the former may reach ~ 10 MeV.^{21,22} Although the depth of penetration of a metal wall by either electrons or ions at normal plasma temperatures of 1-50 keV is so small that the effect is essentially that of a heat input to the surface, high energy runaway electrons might penetrate the surface and deposit their energy at a depth of more than 1 mm. Thus, the character of the damage will depend not only on the parameters cited above but also on the energy of the incident electrons. Figure 17 shows the range of electrons as a function of their incident energy for typical wall materials. The penetration of deuterons is negligible for the entire energy range of interest for tokamaks.

In attempting to correlate the damage observed in TFR, analyses led to the conclusion that holes such as that shown in Fig. 10 (which was melted in the wall of TFR) were caused by interaction between runaway electrons and electrons trapped in the local magnetic mirrors formed by the gaps between adjacent toroidal field coils.²² Subsequent tests to investigate the problem disclosed that electrons left the plasma in bursts which occurred every 200-300 μ sec with an energy of about 4 J per burst and a

total energy deposition per tokamak cycle of as much as 180 J in a zone having a surface area of about 1 cm² with a total duration of the discharge in the main tokamak plasma of about 0.5 sec.²² Note that there are 24 coils in the TFR; hence the total energy deposition in the burned zones represented about 5 kJ, or less than 2% of the total energy in the plasma.²² This type of difficulty with runaway electrons occurred only when the electron density was less than $\sim 10^{13}$ cm⁻³; there has been no trouble in the TFR at the higher densities which approach those characteristic of full-scale reactors.

5.3.1 Prediction of Problems in Full-Scale CTR's

Efforts have been made to estimate from the damage observed in plasma physics experiments the probable severity of local hot spots in the hotter, denser plasmas of full-scale reactors.²³ Unfortunately, the state of the art is not sufficiently firm for one to say more than that even more severe problems could arise than those encountered in experiments made to date.

6. ESTIMATION OF THERMAL STRESSES

Because the calculation of thermal stresses is often a detailed, tedious, and time-consuming task, it is convenient to formulate a procedure for estimating the order of magnitude of the thermal stress to be expected. By use of such a procedure it is sometimes possible to determine rather easily under what conditions a thermal stress problem may become serious enough to warrant detailed calculations and when it may be ignored as not potentially serious. If the preliminary estimates indicate the possibility of a large thermal stress, then it may be necessary to carry out a detailed analysis and perhaps a program of experimentation. Obviously, it is highly desirable to evolve a design with sufficiently low thermal stresses so that such a situation can be avoided.

It is common experience that most materials, when their temperature is raised, undergo an expansion of their dimensions unless they are in some manner constrained. When the material is

heated uniformly and is unconstrained, there is no thermal stress. When the material is constrained by external forces against thermal expansion, thermal stresses result.

The thermal strain, ϵ , which results when the temperature of a constrained body is changed by an amount, ΔT , is given by $\epsilon = \alpha \Delta T$, where α is the linear coefficient of thermal expansion. It is known from the theory of elasticity that, if the expansion is totally constrained, the thermal stress which results in an ideal elastic material is given by $E \epsilon / (1 - 2\mu)$, or

$$\sigma = \frac{E \alpha \Delta T}{1 - 2\mu},$$

where E is the modulus of elasticity and μ is Poisson's ratio. It is noteworthy in this expression that as Poisson's ratio approaches its maximum possible value, $\mu = 0.5$, as is the case with rubber and some plastics, the stress becomes infinite unless the coefficient of thermal expansion, α , simultaneously approaches zero, as it apparently does.

More generally, under partial constraints such as exist in most engineering structures, the maximum thermal stress in the material can be expressed as

$$\sigma_{\max} = K n \frac{E \alpha \Delta T}{1 - a\mu},$$

where K is a stress concentration factor determined by the irregularities in the shape of the body under consideration, n is a factor depending on the amount of constraint imposed, and the factor, a , depends on the directions of the stresses. In general, for the range $0 \leq n \leq 1$, the value unity corresponds to total constraint of thermal expansion, and the value zero corresponds to a complete lack of constraint. The stress concentration factor in practical engineering structures is generally greater than unity, or $1 \leq K \leq \infty$. Experimental and theoretical values of K are available²⁴ in tabulated form for a wide variety of configurations. For instance, a sharp notch in a piece in tension may cause a stress concentration of $K = 10$ or more. A small hole in a specimen under stresses, σ , in two directions

causes stress concentrations for three typical cases as shown in Fig. 18. The sensitivity of a material under stress to a stress concentration varies with the conditions. A ductile material under a single, nonrepetitive applied stress is not very sensitive, whereas a brittle material under a single stress application, any material under repeated stress, and any material under creep stresses will be very sensitive to a stress concentration.

The factor, a , depends on whether the body of material is subjected to constraints in one, two, or three mutually perpendicular directions, and has values as indicated below.

Number of directions constrained	Value of a
1	0
2	1
3	2

For situations of simple tension or compression, $a = 0$. For many problems involving the thermal stress in flat plates, $a = 1$.

It can be shown by the methods of the theory of elasticity that a temperature distribution which is a linear function of rectangular coordinates results in no stress ($n = 0$). Any nonlinearity of the temperature distribution results in thermal stress ($n \neq 0$).

The problem of anticipating failure from thermal stress is complicated by many factors. Among them, a uniform triaxial compressive stress (equivalent to a uniform pressure) cannot cause failure, no matter how high the stress; a uniform triaxial tensile stress (very difficult to obtain in practice) can apparently be very large without causing failure. Combined stresses, rather than simple one-directional stresses, are usual in most engineering problems and often cause failure effects which cannot be anticipated from one-dimensional tests. As is discussed elsewhere, the cyclic stress which can be sustained over many cycles is considerably less than for a single cycle, especially if significant stress concentrations exist.

To know the order of magnitude of the thermal stress it is necessary to evaluate the product $E \alpha \Delta T$. For example, at an operating temperature of 1000°F (538°C) with a temperature difference of 100°F (55°C) in stainless steel [$\alpha = 10 \times 10^{-6}$ per °F (18×10^{-6} per °C) and $E = 21 \times 10^6$ psi (14.5×10^{10} Pa)], this product amounts to 21,000 psi (14.5×10^7 Pa). For niobium [$\alpha = 3.9 \times 10^{-6}$ per °F (7.0×10^{-6} per °C) and $E = 10 \times 10^6$ psi (6.9×10^{10} Pa)], the product amounts to 3900 psi (2.7×10^7 Pa). Thus the thermal stress in stainless steel is 5.4 times that in niobium for these conditions.

6.1 FAILURE CAUSED BY THERMAL STRESS

It has been mentioned previously that thermal stresses may result in failure. A single application of thermal stress may cause a brittle material to fail, whereas many repeated applications may be necessary to cause a ductile material to fail. It is the purpose of this section to attempt to correlate the number of repeated cycles of thermal stress to failure with the repeated temperature difference which causes the failure, particularly for first wall materials subject to both radiation heating and radiation damage — namely stainless steel, niobium, molybdenum, tungsten, vanadium, titanium, and copper. Copper is of interest primarily because it has been widely used in water-cooled targets for ion beams.

Fatigue tests in which the number of repeated cycles of stress to failure is determined provide important experimental information on which to base the design of power apparatus which is subjected to repeated conditions of stress caused by startup and shutdown of the apparatus and by vibrations. For many steels there is a fatigue limit, a stress below which test data indicate that the material will not fail no matter how many times the stressed condition is repeated. The number of cycles commonly associated with this fatigue limit is $\sim 10^6$. The larger the stress above the fatigue limit, the smaller the number of repeated cycles at which the material will fail. Note that many materials, including

copper and aluminum, appear not to have a definite fatigue limit.

Many design problems which are related to thermal stress involve the possibility of failure at a relatively low number of repeated cycles (tens to thousands of repetitions rather than millions). It is this situation with which we are concerned here. It may be observed that the extreme case of failure at a low number of cycles is a single applied stress to failure in rupture (one quarter cycle). Note that for these conditions the material commonly yields and deforms plastically so that the thermal stresses are relieved. Thus, it is more appropriate to speak of "thermal strain" rather than "thermal stress." However, many stress analysts find it convenient to speak in terms of the equivalent thermal stress in an ideal elastic material.

Many factors influence the ability of a material to withstand thermal strain cycling. These include its ductility, the temperature at which it is operated, and the range of plastic strain per cycle. Although good test data are not available for most of the materials and conditions of interest here, a recent paper by Coffin²⁶ reviews an extensive set of experimental data on the low cycle fatigue of materials at high temperature and shows that good correlation can be obtained through the use of an empirical equation (presented in Appendix A). In Appendix A this has been applied with the basic thermal stress equations presented in the immediately preceding pages to derive the curves of Fig. 19 using the physical property data of Table 2.

To find the number of cycles to the initiation of a crack one may enter the ordinate scale in Fig. 19 using (1) the change in surface temperature divided by a constant that is characteristic of the material, or (2) for the energy input per pulse per unit of surface area, Q , divided by the square root of the pulse time, τ . As discussed in the next section the quantity $Q/\sqrt{\tau}$ is the key parameter in transient thermal stress problems of this type.

Table 2. Properties of first wall materials used for estimating low cycle fatigue life

Property	Material						
	Nb-1% Zr 1000°C	TZM 1000°C	SS 600°C	Cu 150°C	W 150°C	V 800°C	Ti-6Al-4V 500°C
c_p , cal/g-°C	0.065	0.066	0.105	0.092	0.032	0.152	0.124
ρ , g/cm ³	8.5	10.0	8.0	8.96	19.3	6.11	4.1
k , cal/sec-cm-°C	0.165	0.250	0.041	0.923	0.476	0.095	0.023
$\alpha \times 10^6$, /°C	7	5	17	16	4.5	10.9	9.6
$E \times 10^{-6}$, psi	10	50	21	17	50	18	11
σ_y , psi	20,000	18,000	10,000	5,000	40,000	38,000	45,000
$\Delta\epsilon_{1/4}$	0.4	0.4	0.4	0.4	0.01	0.2	0.27
C_1 , J/cm ² -sec ^{1/2}	17,200	31,050	4,360	19,040	1,257	5,440	3,038
C_2 , J/cm ² -sec ^{1/2}	172	55.5	10.4	28.0	201	115	92
μ	0.30	0.32	0.30	0.36	0.28	0.30	0.30

7. TEMPERATURE AND STRESS EFFECTS OF PULSED ENERGY INPUTS TO SURFACES

As the temperatures and energy densities in the plasmas of experimental thermonuclear devices have increased toward conditions that would support self-sustained thermonuclear reactions, increasingly severe surface damage to containment walls has been experienced. This damage appears to include thermal stress cracking, melting, and vaporization. Analytically the problem is rendered difficult by several factors. The conditions that cause the damage are generally not design conditions, but rather represent transients caused by stray beams of particles away from their anticipated location. But even if the conditions of energy deposition were adequately known, the analysis would still be difficult because of the complexity of the processes of melting and vaporization going on within the heated wall. It is the purpose of this section to present the preliminary results of an attempt to organize the mathematical analysis of the problem so as to facilitate the drawing of pertinent conclusions of a general nature where possible, and to apply these conclusions to the assessment of

damage to the surfaces of existing and planned containment shells.

7.1 TEMPERATURE DISTRIBUTION AND THERMAL STRESS IF NO MELTING OCCURS

The simplest cases to consider are those in which the magnitude of a short energy pulse is not sufficient to cause melting, and hence the prime consideration is that of thermal stress and the thermal stress cycling failure that may result from severe fluctuations in the surface temperature while the bulk of the material in the first wall remains at a constant temperature. The transient temperature distribution induced in a thick slab by a pulse of energy into its surface has been treated by Jacob.²⁶ The classical relations that he has derived have been applied in Appendix B for the range of conditions of prime interest here to yield the curves of Fig. 20. These curves show the temperature rise over the initial isothermal temperature of the slab as a function of the distance from the heated surface for three typical periods of time as measured from the initiation of the pulse.

The generalized equations for the temperature distribution have been related to the empirical relation for the thermal stress and the number of cycles to failure for thermal strain cracking. The details of the analysis and derivation are presented in Appendix B, and Fig. 21 was constructed from that work to show the temperature rise and thermal stress in an ideal elastic material for the range of energy inputs and pulse durations of prime interest for the first wall of fusion reactors. Note that the effect of the choice of material has been included by presenting characteristic values for the materials of prime interest.

The number of thermal cycles that a first wall might be expected to withstand can be estimated from Fig. 19 using the parameter $Q/\sqrt{\pi}$.

7.1.1 Results

The results of this work as shown in Fig. 21 indicate that both the surface temperature excursion and the thermal stress in an ideal elastic material resulting from a given energy input to an area of the surface in a given time period are much greater for stainless steel than for the other materials considered. As indicated in Fig. 19, failure caused by thermal fatigue at a given number of cycles will occur with a lower energy input for stainless steel than for any of the other materials considered except for tungsten and TZM welds. The poor performance indicated for tungsten results primarily from the low ductility to be expected from unavoidable oxygen contamination. Titanium and vanadium are only slightly better than stainless steel, whereas Nb-1% Zr will withstand over 200 times as many pulses of a given severity as stainless steel. Or, put another way, Nb-1% Zr can withstand over four times as great an energy input to the first wall for a given lifetime as stainless steel. The curve plotted in Fig. 19 for the wrought molybdenum alloy TZM is deceptive in that it makes TZM appear to be the most attractive of all the materials, but note that the curve shown is for the wrought alloy. The material in welds is, in effect, cast and has essentially zero ductility.

The curve plotted in Fig. 19 for TZM (or any molybdenum alloy) in welds actually falls below the curve for tungsten.

7.2 TEMPERATURE DISTRIBUTION IF MELTING AND POSSIBLY VAPORIZATION OCCUR

When radiant energy impinges on a surface, the temperature of the surface is raised and the temperature in the body varies with the distance from the surface as indicated in Fig. 20. If the process of radiant heating is continued long enough and at a sufficiently high rate, the surface comes to the melting point. Then, if the heating is continued, the temperature of the surface increases toward the boiling point, assuming that the melting material stays in place. A moving boundary is then created between the melted and solid material. If the melted material is sloughed off the surface as it is formed, the moving boundary becomes the receding surface of the wall. If the melted material stays in place and reaches the boiling point, a moving face where vaporization occurs becomes one boundary in addition to the moving internal boundary between the liquid and solid. Because of the moving boundaries and the difference between the properties of the liquid and solid states of the same material, the distribution is nonlinear.

Analytically the temperature distribution within a body on whose surface radiant energy is impinging must be considered in parts. First, the temperature of an element of the material must be raised to the melting temperature. That part of the material which has reached the melting temperature must then be treated differently, in part to account for the latent heat which is required to effect the phase change, and in part to account for the fact that the thermal conductivities of the liquid and solid phases are significantly different. (The conductivity of the liquid is usually roughly one-half that of the solid of the same material.) In this report three possible categories of behavior have been considered for the material whose temperature is above the melting temperature. (1) Because the rate of energy input during thermonuclear

processes is sometimes extremely rapid, it is to be expected that there may sometimes be insufficient time for the phase change to occur during the period of the energy burst, and often the phase change would at least not occur under equilibrium circumstances. In the extreme case of rapid energy deposition, it is assumed that the material remains a superheated solid, no latent heat of phase changes being involved. A standard analytical solution is available for the temperature distribution in a thick wall for this case. (2) Some surface material becomes a liquid and is sloughed off the surface as fast as it is formed. As this process continues, the rate of penetration of melting into the material approaches a steady-state value which can be found analytically along with the resulting temperature distribution in the solid. The temperature distribution and the rate of melting during the transient must be found by numerical methods such as finite difference procedures. (3) The material which comes to the melting temperature then melts but stays in place, its temperature rising to the boiling point as more energy arrives at the surface. In this case it is necessary to consider the temperature distribution in both the liquid and the solid material. As in the previous case, the eventual steady-state conditions may be investigated analytically, with resort to numerical methods being required for the transient.

The details of the analysis are complex and tedious and hence have been relegated to Appendix B. The most significant results of the analyses are indicated in Figs. 22, 23, and 24. Figure 22 shows the dimensionless depth of penetration of melting as a function of the parameter ϕ which is a function of the coefficient of thermal expansion, α ; the specific heat and density; the temperature difference between the melting point and the initial temperature of the slab; the energy per burst, Q ; and the burst time, τ . Figure 23 shows two typical generalized temperature distribution curves, one for the case where the surface just reaches the melting point and the other for the case where the surface temperature reaches the boiling point.

7.2.1 Stainless Steel and Niobium Compared

Figure 24 compares the depth of penetration of melting for stainless steel and for niobium. Note that there is a factor of 10 difference in the horizontal scales for stainless steel and niobium in Fig. 24. The maximum depth of penetration with stainless steel is seen to be some 30% higher than for niobium. Figure 24 highlights the much greater ability of niobium to withstand unusual bursts of energy of interest to tokamaks and theta-pinch machines. For example, the damage to TFR cited earlier (see Figs. 9, 10, and 11) was caused by an estimated energy deposition of $\sim 1.8 \text{ J/mm}^2$ ($\sim 180 \text{ J/cm}^2$) in a period of $\sim 30 \text{ msec}$, which gives a value for the parameter τ/Q^2 of ~ 10 [$(\text{msec/J}^2/\text{mm}^4)$]. From Fig. 24 one would expect melting of a stainless steel first wall to a depth of $106 \times 1.8 = \sim 200 \text{ }\mu\text{m}$. In the actual machine the $500\text{-}\mu\text{m}$ thickness of the first wall was melted through, which implies that perhaps the local energy deposition was greater by a factor of two and the duration of the burst was greater by a factor of four than the $\sim 180 \text{ J/cm}^2$ and pulse duration of $\sim 30 \text{ msec}$ indicated by Ref. 22. Note that if the first wall in TFR had been niobium, no melting at all would have been expected.

For very short burst times it might appear from Fig. 24 that stainless steel is superior to niobium in that the depth of melting would be less. For example, for $\tau = 1 \text{ msec}$ and $Q = 1 \text{ J/mm}^2$, $\tau/Q^2 = 1$, and $\delta m/Q = \sim 80 \text{ }\mu\text{m}$ for Nb vs $\sim 25 \text{ }\mu\text{m}$ for stainless steel. The reason for this apparent non sequitur is that a portion of the stainless steel would be vaporized because the thermal conductivity would not have been sufficient for the energy to have been absorbed in the metal wall.

8. EXAMPLES OF THERMAL STRESS IN BLANKET DESIGNS

This section is concerned with the thermal stresses in typical basic structural elements commonly present in blanket designs for thermonuclear power reactors. Representative examples of these may be found in a 1975 EPR design on ORNL,²⁷

the 1974 full-scale Princeton reactor,²⁸ and two ORNL full-scale power reactor designs.^{29,30}

8.1 THERMAL STRESS INDUCED BY THE TEMPERATURE GRADIENT IN THE FIRST WALL

It is probably best to begin with an examination of the first wall thermal stress limitation associated with normal operation in a tokamak. A power cycle will probably last from 100-600 sec. Assuming 10 cycles/hr, 7000 hr of operation per year, and a 10-year life, the first wall ought to withstand 700,000 power cycles. The experimental data obtained by Keyes¹³ on cracking of Inconel 600 in a tube carrying fluid whose temperature fluctuated indicates that to avoid cracking (see Fig. 13) the ideal elastic stress range per cycle ought to be limited to $\sim 15,000$ psi ($1,054$ kg/cm²) (see Fig. 14), which corresponds to a temperature difference through the first wall of 37°C. From the standpoint of thermal strain cycling the properties of type 304 stainless steel at $\sim 600^\circ\text{C}$ are much the same as those of Inconel 600 at 1400°F (760°C).

Note that the temperature distribution through the first wall reaches equilibrium in a few seconds, so that in a tokamak power cycle one is not dealing with the transient hot spot problem discussed in the previous section; the thermal stresses involved in normal power cycling are inversely proportional to the thermal conductivity (not its square root) and are independent of the specific heat.

8.2 EFFECTS OF WALL THICKNESS AND POWER LOADING

The effects of both first wall thickness and the first wall power loading on the temperature drop across the first wall were estimated assuming that 15% of the energy is deposited in the first wall; the results are plotted in Fig. 25. Note that the lines are slightly concave upward because the amount of energy deposited in the first wall increases with its thickness. The energy deposition was taken from a particularly significant set of calculations made in 1972 by Steiner² for a relatively thin wall in a blanket consisting mainly of lithium and graphite. These curves indicate that the wall thickness for 1 MW/m² ought to be less than 5 mm for a 10-year life,

or for 4 MW/m² it should be less than 1.5 mm for a 10-year life. Note that the life depends in part on the ductility of the material, so that radiation embrittlement considerations imply that still thinner walls should be used. Unfortunately, in this thickness range reducing the wall thickness reduces its heat capacity, and hence if it is cooled with helium or a molten salt, thinning the wall will reduce its ability to withstand an abrupt local energy dump of the type discussed in the first section. The reverse would be true if the wall were cooled by lithium instead of helium; the heat capacity and thermal conductivity of lithium are higher than for stainless steel; hence for lithium cooling, thinning the wall would also reduce the temperature excursion and thermal stresses in the wall.

8.3 RADIATION SHIELD WALL OF THE PRINCETON REACTOR

It has been proposed in a number of designs including one prepared at ORNL²⁹ and one at Princeton²⁸ that the main blanket structure be protected by means of a bank of helium-cooled tubes. (See Figs. 26 and 27.) One side of the bank of tubes would be exposed directly to plasma radiation and hence would operate at a higher temperature than the side of the tube bank away from the plasma. Each tube would therefore be exposed to a circumferentially varying temperature and to the tensile and compressive stresses which result.

8.3.1 Calculational Model

For the purpose of this analysis the conditions of Ref. 28 were taken as the point of departure and it was assumed that all of the energy deposited from the plasma would fall uniformly over one-third of the surface of the tube as shown in Fig. 28. The inside of the tube would be cooled over its entire surface by flowing helium, with a uniform heat transfer coefficient, h . As a first approximation the relatively small heat generation in the wall on the side away from the plasma was neglected and the portion of the

wall toward the plasma was assumed to be uniform across its thickness and at a temperature, T , above the temperature of the helium.

For simplicity the circumference of the circular tube has been assumed to be straightened out as shown in Fig. 28, where the pattern of energy deposition on the outer surface and transfer to the helium inside are shown schematically. The distance, Z , is measured from the center of the heated region.

8.3.2 Stresses from Circumferential Temperature Distribution

The details of the analysis are presented in Appendix C. In evaluating the derived equations for a typical case, one may note that to a first approximation the circumferential variation in the tube wall temperature is equal to the film temperature drop at the center of the tube on the side toward the plasma, i.e., where $z = 0$ (see Fig. 28). For example, the anticipated temperature difference across the boundary layer given in Ref. 28 is 220°C. Note that this is much greater than the temperature drop through the wall considered in the previous section and hence is the controlling consideration in the analysis. (The temperature drop through the first wall in this case is only ~12°C, or about 5% of the temperature difference between the hot wall and the helium.) Because roughly one-third of the circumference is hot and two-thirds is at essentially the same temperature as the helium, the estimate of the thermal stress, in compression, in the hot part is approximately

$$\sigma_{\max} = \frac{2}{3} \frac{E \alpha \Delta T}{(1 - \mu)},$$

or, for an ideal elastic material having the properties of stainless steel, $\sigma_{\max} = \sim 80,000$ psi (5600 kg/cm²). At the temperatures involved, this temperature distribution would be expected to cause yielding each cycle, alternately in tension and compression, and eventually failure from thermal cycling after about 1000 power cycles (see Fig. 14). Assuming 300 sec/cycle, the life of the tubes would be only ~80 hr. Note that once a crack was initiated by thermal strain

cycling, the large pressure stresses from high pressure helium would burst the tube.

One reasonable way to reduce the magnitude of the stress in this design would appear to be to raise the heat transfer coefficient by going to a higher velocity or using a better coolant, thereby lowering ΔT_{BL} . It would also be possible to design the module so that the tubes would not be restrained, i.e., they might be anchored at only one end and allowed to flex and thus relieve the bending stress. This presents other problems. If both ends are fixed, severe local bending stresses are likely at the header sheets. If the tubes are made in the form of hairpins, differential expansion between the two legs will induce stresses in the return bend. Further, the structure will be flimsy and easily damaged in handling.

8.4 THERMAL STRESS AT THE JUNCTION BETWEEN A CYLINDER AND A FLAT HEAD

An early design for an EPR (see Fig. 29) employed a construction often used in CTR designs: a set of cylindrical shells fastened to flat headers,²⁷ the shell forming the first wall being at a considerably higher temperature during the power portion of a cycle than the heads because the inner shell surface intercepts far more radiant energy. Because the cylindrical shell temperature rise is much higher than that of the flat head during a power pulse, it tends to expand to a greater radius than the head to which it is fastened. (See Fig. 30.) The result is a localized constraint (a stress concentration) in the cylindrical shell at its junction with the head, resulting in the distorted configuration shown in Fig. 30.

Applying the equations derived from classical treatments^{31,32} for this case in Appendix C, one finds that large thermal stresses will accompany the large temperature differences induced when the first wall of Fig. 29 is subject to radiation from the plasma; the structure would return to isothermal conditions between cycles unless the time between cycles could be kept to a few seconds or less. The magnitude of this temperature fluctuation — and hence the stresses —

will depend on both the temperature rise in the coolant and the film temperature drop between the first wall and the coolant. The juncture between a long cylinder and a flat head to which it is fastened results in a stress concentration factor $K = 1.86$ in the equation for thermal stress. Less abrupt changes in temperature result in lower stresses. For example, the 300°F (167°C) change in temperature occurring abruptly at the junction between the cylinder and the head estimated for Fig. 29 would result in a thermal stress of 117,000 psi (8200 kg/cm²) for an ideal elastic material having the properties of stainless steel operating at about 1000°F (540°C). From Fig. 19, using a stress concentration factor of 1.86 and temperature difference of 167°C, one can estimate that cracking would be initiated in about 10⁴ power cycles.

8.5 THERMAL STRESS CAUSED BY A TEMPERATURE DIFFERENCE BETWEEN TWO CYLINDERS JOINED TO A COMMON SET OF FLAT HEADS

The blanket structure of Fig. 29 involves two cylinders differing in temperature by an amount, ΔT , and fastened to two heads, as shown schematically in Fig. 31. The two cylinders are shown at radii, r_1 and r_2 , and fastened to two heads separated by a distance, $2L$. The thermal displacement of either head between the junctions with the two cylinders is given by $\delta = L \alpha \Delta T$. As shown in Appendix C the stress concentration factor derived for this geometry would be $K = 45$ for a head thickness of 0.3 cm, a cylinder length of 100 cm, and a radial gap between cylinders of 1 cm. The stress in an ideal elastic material having the properties of stainless steel at about 540°C that would be caused by a temperature difference of only 30°F (14.9°C) is then about 280,000 psi (19,700 kg/cm²).

9. BLANKET COOLING AND STRESS ANALYSIS

It is evident from the preceding sections that blanket cooling and stress analysis problems are tightly interrelated. In an effort to arrive at a reasonably comprehensive treatment, several of the many different blanket designs that have been proposed are examined here to develop a

quantitative indication of the cooling and stress limitations typical of each of the principal approaches that have seemed to be of interest. Hopefully, the treatment here will be such that it will not be too difficult to generalize from the results to appraise the cooling and stress problems of variations from the specific cases chosen.

9.1 HELIUM-COOLED STATIC LITHIUM BLANKET

Helium has been by far the most popular coolant among those attempting to design thermonuclear power reactors because they are thus able to avoid the troublesome problems with MHD effects inherent in the use of lithium, the next most favored coolant. Unfortunately, many boundary conditions must be met, and more often than not some — particularly pressure and thermal stress limitations — are overlooked. Thus, a systematic examination of all of these boundary conditions and their more important implications is in order. The more crucial design parameters are as follows:

- (1) structural material,
- (2) peak operating temperature of the structure,
- (3) choice of thermodynamic cycle,
- (4) blanket geometry,
- (5) amount and distribution of energy deposition through the blanket,
- (6) allowable pressure stresses,
- (7) allowable thermal stresses,
- (8) normal temperature drop through the first wall,
- (9) hot spot effects,
- (10) first wall thickness,
- (11) cooling passage ID (or equivalent),
- (12) coolant pressure,
- (13) coolant outlet temperature,
- (14) coolant temperature rise in the blanket,
- (15) temperature drop between the passage walls and the coolant bulk free stream,
- (16) coolant velocity,
- (17) coolant passage length, and
- (18) the ratio of pumping power to heat removal.

The number of combinations and permutations of the above parameters is so great that it presents a whole set of dilemmas. To bring order out of chaos the field must be narrowed. Note that one cannot vary all of these parameters independently; some are clearly and explicitly related, others are only implicitly or even very subtly related, and, of course, many are independent.

9.1.1 Design Temperature

For the purpose of this paper it appears reasonable to narrow the field drastically by a rationale that fixes the first 12 parameters. First, one finds that materials compatibility considerations limit the peak operating temperature of iron-chrome-nickel alloys containing static lithium to not more than $\sim 600^{\circ}\text{C}$, and that neither Nb-1% Zr nor vanadium can be used with helium at temperatures above $\sim 600^{\circ}\text{C}$ because of the corrosive effects of unavoidable traces of oxygen and moisture. This problem is discussed at some length in Ref. 1. Thus, for helium-cooled static lithium blankets (except for thermal stress considerations) it appears that there is no incentive to go to a material much more exotic than stainless steel, and one can reasonably settle on stainless steel with a peak operating temperature of $\sim 600^{\circ}\text{C}$. This is too low a temperature to give an attractive gas turbine system; hence a steam cycle offers by far the most promising power conversion system.

9.1.2 Blanket Configuration

The most widely used and most easily visualized type of blanket geometry is the wedge-shaped ring shown in Fig. 32. This geometry appears to lend itself to both fabrication and assembly of modules into a complete torus. Thus, it seems to be a reasonable choice for this analysis.

9.1.3 Energy Deposition

The amount and distribution of energy along a minor radius through the blanket depend mainly on the wall power loading and to a much lesser extent on the detailed choice of materials

distributed through the blanket. The fraction of the total fusion energy deposited in the first wall is the most critical factor. In view of radiation damage limitations on the one hand and the minimum acceptable life of a blanket in a commercial power plant on the other, it appears in order to include in this analysis total energy inputs to the blanket of 0.5, 1.0, 2.0, and 4.0 MW/m^2 with the prime emphasis on the 1.0- MW/m^2 case.

9.1.4 Allowable Stresses

Once a structural material and operating temperature are chosen, the allowable pressure and thermal stresses are well defined, preferably by the ASME Pressure Vessel Code. These parameters may then be directly related to the temperature drop through the first wall under normal operating conditions, hot spot effects under off-design conditions, the system pressure, and the cooling passage ID (or its equivalent from the pressure stress standpoint).

To reduce the thermal stresses and thus provide a margin for pressure stresses, it seems best to reduce the maximum wall thickness indicated by Fig. 25 by about 50%. This implies a wall thickness of 2.5 mm for an output of 1 MW/m^2 . If a typical allowable stress for stainless steel at 600°C is taken as 3600 psi (253 kg/cm^2) this gives a permissible helium pressure of 30 atm for a 2.54-cm-OD tube. Thus the first wall structure might consist of a bank of such tubes arranged as shown in Fig. 26, or it might consist of a similar bank of tubes immersed in static lithium as in Fig. 29. In either case note that the permissible system pressure is inversely proportional to the tube diameter and that the higher the system pressure, the lower the helium pumping power requirements. Space does not permit elucidation, but numerous layout studies have yielded tube diameters close to 2.5 cm as being the favored compromise.^{27-29, 33-35}

In a preceding section it was shown that the circumferential variation in the wall temperature of helium-cooled tubes subject to direct radiation from the plasma induced severe thermal stresses, and that for the particular case

considered these thermal stresses would cause a failure in roughly 80 hr of normal operation. In that case the total wall power loading was 3.4 MW/m^2 rather than the 1.0 MW/m^2 considered here. Further, the helium velocity was somewhat less than the upper limit defined by the pumping power-to-heat removal rationale followed here. Extending the discussion to consider the upper limit on first wall heat loading for acceptable stresses in round tube banks used as in Fig. 27, one finds that from this standpoint the upper limit on power density would be about 1.3 MW/m^2 with a surface temperature drop of 68°F (38°C) and a tube length of $\sim 29 \text{ ft}$ ($\sim 8.8 \text{ m}$).

9.1.5 Delineation of Design Options

If one fixes the first twelve design parameters as outlined above, five of the balance can be related by straightforward heat transfer, fluid flow, and heat balance considerations to give the chart shown in Fig. 33. This chart was drawn for a first wall power loading of 1.0 MW/m^2 . If the pumping power-to-heat removal ratio and coolant temperature rise are to be kept constant and the wall power loading varied, the mass flow rate will be directly proportional to the cube root of the power loading and the tube length will be inversely proportional to the two-thirds power of the power loading. For a given mass flow rate, the heat transfer coefficient will be constant and the temperature difference will be directly proportional to the wall power loading and inversely proportional to the heat transfer coefficient.

The field can now be narrowed further by considering the fact that extensive system design work on fission reactor plants has indicated that the pumping power-to-heat removal ratio for the reactor coolant circuit ought not exceed about 3%. If half of this is taken internally in the blanket and half in the external circuit, the pumping power-to-heat removal ratio may then be specified as 0.015. To avoid an excessive number of tube-to-header joints, layout studies favor a tube length of at least 10 m (33 ft); hence Fig. 33 indicates that the helium temperature rise

ought to be at least 222°C (400°F). On this basis, Fig. 33 yields a helium mass flow rate of 1.8 kg/m^2 (40 lb/sec-ft^2) for a wall power loading of 1.0 MW/m^2 . The resulting heat transfer coefficient is $2800 \text{ W/m}^2\text{-}^\circ\text{C}$ ($\sim 500 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$), and the temperature drop from the metal wall through the gas film to the bulk free stream of helium would be 28°C ($\sim 50^\circ\text{F}$), a reasonable value. This in turn means that the helium outlet temperature could run only 28°C below the peak metal temperature of 600°C if there were no need to allow for hot spots. Past experience indicates that ordinary irregularities in flow distribution make it necessary to include a hot spot allowance equal to at least 20% of the film drop plus at least 20% of the temperature rise, which in this case totals $\sim 52^\circ\text{C}$. Thus the helium temperature leaving the blanket ought to be kept to $\sim 520^\circ\text{C}$ for a peak coolant passage wall temperature of 600°C .

9.1.6 Temperature Distribution and Thermal Stress in the Complete Blanket Module of Fig. 29

The design of Fig. 29 was prepared for an experimental power reactor for which a major objective was a minimum bore toroidal field coil with no requirement for tritium breeding; hence there is relatively little lithium, followed by a thick stainless steel region. The first wall power loading for one case considered was 0.25 MW/m^2 . The helium cooling system pressure was 70 atm, the helium temperature rise was 222°C (400°F), and an unusually high pumping power-to-heat removal ratio of 0.036 was accepted in order to get a very high heat transfer coefficient.²⁷

A definitive calculation of thermal stresses and thermal strain cycling effects in a complex structure is extremely tedious and time-consuming, and really is not warranted for a preliminary conceptual design such as that of Fig. 29. Rather, it is more appropriate to make a series of rough estimates of the thermal stresses and strains in the structure to appraise its general suitability for the case at hand. In examining the thermal stress situation in the configuration of Fig. 29, the first point noted was that the

outer perimeter of the blanket structure is massive, and therefore the most severe thermal stresses and strains will occur in thinner sections whose temperatures differ substantially from that of the heavy structure in the outer portion of the blanket.

Stresses in the webs adjacent to the graphite region

One of the most serious sets of thermal stresses in the geometry of Fig. 29 lies in the stainless steel webs adjacent to the graphite region. Thermal conduction calculations showed that the temperature of the graphite in this region would run 344°C (620°F) above the temperature of the tubes in the first wall and 120°C (220°F) above the temperature of the bulk of the stainless steel structure in that vicinity. If the adjacent stainless steel web were to have the same temperature distribution, this would induce severe thermal strains far beyond the elastic limit, and these would probably lead to serious distortion and ultimately cracking of the wall in that vicinity. It might be argued that the stainless steel would not run at the temperature of the graphite, but it must be remembered that the intensity of radiation heating in the stainless steel will actually be higher than that in the graphite and its thermal conductivity will be lower by a factor of 3.5, so that the temperature in the stainless steel web would really be higher than that in the adjacent graphite for the layout of Fig. 29. This condition could be alleviated by providing additional helium cooling passages in the gap between the graphite and the stainless steel side wall, but there would still be serious problems in getting a uniform temperature distribution.

First wall

The energy input to the first wall leads to a large temperature drop through the wall, in this case 67°C (120°F). In addition, there is a further temperature drop of the order of 4°C (9°F) from the first wall through the lithium and the helium tube wall to the surface cooled by the helium, and a film drop between the tube wall and

the bulk free stream of the helium of about 17°C (30°F). The combined effects of this lead to a temperature of the first wall on the side toward the plasma 45°C ($\sim 80^{\circ}\text{F}$) above the temperature of the massive stainless steel structure in the outer portion of the blanket. One way in which this temperature difference could be reduced would be by employing a thinner first wall. If the first wall thickness were made 2.5 mm (0.10 in.), the temperature difference could be cut to about 17°C (30°F) — an acceptable value from the thermal stress standpoint.

Bending in the first wall

The large radial temperature drop through the first wall leads to a mean temperature in the first wall much higher than that in the webs restraining it at either side. As a consequence, the first wall will tend to belly radially outward in much the same manner as the bottom of a frying pan will tend to deflect from a flat to a spherical surface because the center runs hotter than the edges. This deformation will lead to serious bending stresses in the vicinity of the fillet between the first wall and the radial webs, but it is difficult to estimate just how serious these stresses will be. Figure 34 shows the character of the distortion to be expected.

Bending in the ring caused by the circumferential temperature distribution

The temperature rise in the helium coolant flow leads to a variation in the temperature of the ring with position around the circumference as indicated in Fig. 35. This peculiar temperature distribution will lead to distortion of the ring in the plane of the section as shown in Fig. 36. The magnitude of this thermal distortion and the consequent stresses and strains is difficult to estimate, but it would certainly not be trivial. Clearly one effect would be to cause the upper portion of the ring to shear outward away from the lower portion, as indicated in Fig. 36.

A second important effect of the variation in the circumferential temperature distribution is that in the vicinity of the helium coolant inlet and outlet pipes, there will be a temperature

difference of 220°C (428°F) with a steep gradient. This will tend to cause shear between these two regions of massive structure in the plane of the surface of the torus similar to that discussed earlier in connection with the tube bank designed to protect the first wall of the blanket shown in Fig. 27.

General comments

It is clear from the above that the blanket structure of Fig. 29 is unacceptable from the thermal stress standpoint even though the pumping power-to-heat removal ratio is more than double the maximum value for a well proportioned power reactor and the first wall power loading is only 0.25 MW/m².

9.1.7 Temperature Distribution and Thermal Stress in the Helium-cooled Static Li Blanket Module of Fig. 26

The simple structure of Fig. 26 gives a geometry less subject to thermal stresses. Further, the structural material chosen was Nb-1% Zr, a material that will tolerate roughly five times the temperature differential permissible with stainless steel in any given geometry. In attempting to assess the thermal stress situation, note first that the design called for a 410°C (770°F) temperature rise in the helium flowing through the blanket. Inasmuch as the film temperature drop between the structure and the helium would be greater near the first wall and this would be the first portion contacted by the entering helium, the temperature difference within the structure would tend to be somewhat less than the helium temperature rise. On the other hand, there would be a substantial temperature gradient in the static lithium, and this would run about 125°C (257°F) for a first wall power loading of 1 MW/m². (See Fig. C.3 and Fig. 37.) Thus the temperature in the blanket structure would vary by about ±250°C (482°F) from the mean, a possibly tolerable situation. A detailed analysis together with some changes in the design parameters would probably yield acceptable thermal stresses under normal operating conditions. However, as discussed in a previous section, the hot spot

effects associated with high local energy inputs from the plasma would probably lead to serious thermal stresses.

9.1.8 Temperature Distribution and Thermal Stress in the Blanket Module of Fig. 27

A number of severe thermal stress conditions inherent in the design of Fig. 27 were considered and discussed in earlier sections. An additional point implicit in the previous subsection is the temperature variation in the structural wall of the blanket module associated with temperature variations within the static molten salt. The thermal conductivity of Li₂BeF₄ is lower than that of lithium by a factor of 40; hence temperature variations within the static liquid will be 40 times as great as for metallic lithium in a given geometry for a given power density. In appraising any particular design one can use Fig. 37, which was drawn for lithium, and simply multiply the temperature difference obtained for lithium by a factor of 40 to get the corresponding value for Li₂BeF₄.

9.2 TEMPERATURE DISTRIBUTION AND THERMAL STRESSES IN A MOLTEN SALT-COOLED VERSION OF FIG. 26

Several points emerge from a review of the previous sections on helium-cooled blankets. First, helium cooling at best leads to a marginally acceptable design, and then only for a niobium structure with a first wall power loading of no more than 1 MW/m². Second, the relatively simple structure of Fig. 26 is clearly better than the more complex structures of Figs. 27 and 29. Finally, a better coolant is clearly needed. If a liquid metal coolant is tentatively ruled out, the only other candidate appears to be a molten salt. This will not only yield a heat transfer coefficient roughly twice that for helium, but it will also give at least a factor of two reduction in the coolant temperature rise while at the same time giving a lower pumping power-to-heat removal ratio and a far lower coolant pressure. The latter will not only drastically reduce the pressure stresses but it will also avoid the unpleasant possibility of a major

rupture of a blanket module in the event of a small leak of coolant into the static lithium region.

A typical set of values for a molten salt-cooled version of Fig. 26 is presented in Table 3 together with the corresponding values for the original helium-cooled design of the previous subsection. In preparing Table 3 the rationale used for the molten salt case was similar to that followed for helium but advantage was taken of the greater heat capacity and lower pumping power requirements of the molten salt to employ a coolant temperature rise only 25% as great, i.e., 56°C vs the 222°C used for helium. The lower coolant pressure would permit a much thinner tube wall, but it seemed desirable to take advantage of only a part of the potential reduction in coolant passage wall thickness so that the pressure stresses could be reduced to a minor level.

The choice of coolant passage spacing in the static lithium involves a number of compromises. As indicated in the previous subsection (see Fig. 37) the temperature drop involved in heat conduction through the lithium to the cooling passage wall is substantial and will lead to local temperature variations not only in the static lithium but also in the wall of the blanket module. At first thought the latter temperature variations might be reduced by welding the cooling tube to the blanket module wall as in Fig. 38, but for a given coolant passage spacing this will actually increase the local temperature variation in the wall by reducing the minimum value in the temperature "ripple" in the wall. A second boundary condition to be met is that for low pumping power losses and relatively short cooling passages (i.e., <30 m) the wall heat flux ought to be high (of the order of 100 kW/m²) and this in turn means a relatively large coolant passage centerline spacing. If sufficiently large cooling passage spacings are used to give a heat flux of ~100 kW/m², the peak temperature in the static lithium will exceed the coolant passage wall temperature by 400-1000°C, an uncomfortably large amount. This effect can be seen in Fig. 39, which shows the relation between passage spacing and the power density in the lithium

for both a series of constant cooling passage wall heat fluxes and a series of constant differences in temperature between the hottest lithium and the coolant passage wall. Note that for a first wall power loading of 1 MW/m², the heating density in the lithium will drop from ~2 W/cm³ near the first wall to ~0.04 W/cm³ in the outer part of the blanket. Probably the best compromise is to maintain high heat fluxes with large spacings except close to the blanket module wall. This will necessitate a larger pumping power to take care of the greater number of cooling passages close to the wall and also a larger lithium expansion tank so that lithium can flow out of the blanket module when it heats up. Note that the volumetric coefficient of expansion of lithium is large, e.g., increasing the lithium temperature by 500°C will increase its volume by ~8%.

9.3 RECIRCULATING LITHIUM BLANKET COOLED BY BOILING POTASSIUM

The ORNL recirculating lithium blanket concept and the problems associated with it have been covered in a series of reports (see Refs. 30, 36-39). Inasmuch as most people who have been interested in these problems are either familiar with the material in these reports or have the reports readily available, it would be repetitious to recapitulate that material here. Thus, perhaps it will suffice to give just a brief description of the basic blanket module envisioned. A simplified isometric of a section of the torus built up of racetrack-shaped modules is shown in Fig. 40. Graphite would fill the central region between the long legs of the racetrack, while lithium would circulate around the racetrack driven by an electromagnetic pump at one end of the racetrack as indicated in Fig. 40. The favored material for the racetrack is Nb-1% Zr, although stainless steel might be employed. The MHD problems are treated in a later section.

9.3.1 Effects of Proportions of Lithium and Graphite Regions

In selecting the radial thickness of the lithium and graphite regions, one must find a compromise among the various design objectives.

Table 3. Principal heat transfer and thermal stress parameters for the blanket structures of Figs. 26 and 40 for a first wall total power loading of 1 MW/m² with 16% of the energy deposited in the first wall

	Fig. 26 He-cooled		Fig. 26 Salt-cooled		Fig. 40 Li-cooled	
	SS	Nb-1% Zr	Incoloy	Nb-1% Zr	SS	Nb-1% Zr
Coolant pressure, bar	30	30	1	1	0.1	0.1
Coolant temperature rise, °C	222	222	56	56	56	56
Coolant velocity, m/sec	100	100	2	2	0.36	0.36
Heat transfer coefficient, kW/m ² -°C	2.8	2.8	5.05	5.05		
Heat transfer coefficient, Btu/hr-ft ² -°F	500	500	890	890		
First wall heat flux, kW/m ²	160	160	160	160	160	160
First wall heat flux, Btu/hr-ft ²	50,000	50,000	50,000	50,000	50,000	50,000
First wall thickness, mm	2.5	2.5	2	2	2	2
Design temperature drop across first wall, °C	25	5.8	19	4.6	19	4.6
Design temperature drop across first wall, kg/cm ²	600	28	490	23	490	23
Thermal stress across first wall, psi	8,600	400	6,900	320	6,900	320
Film temperature drop at first wall surface, °C	57	57	32	32		
Thermal stress in tubes in first wall caused by circumferential temperature distribution, kg/cm ²	1,420	276	800	155		
Thermal stress in tubes in first wall caused by circumferential temperature distribution, psi	20,500	4,000	11,500	2,240		
Temperature variation in L1 between coolant passages, °C	50	50	50	50		
Temperature differential in blanket module structural envelope, °C	330	330	140	140	140	140
Coolant pressure drop through blanket, kg/cm ²	1.5	1.5	1.5	1.5	0.1	0.1
Pressure stress in structural envelope, kg/cm ²					21	21
Pressure stress in structural envelope, psi					300	300
Pressure stress in coolant tube wall, kg/cm ²	125	125	13	13		
Pressure stress in coolant tube wall, psi	1,760	1,760	185	185		

The tritium inventory will be directly proportional to the volume of lithium in the blanket; hence there is a strong incentive to minimize the radial thickness of the two lithium regions. On the other hand, increasing the thickness of the graphite increases its central temperature, and the degree of radiation damage to the graphite increases rapidly at temperatures above about 1200°C. While these are the two most vital parameters, others that are also important include the breeding ratio, the temperature rise in the lithium flowing along the first wall, the pumping power-to-heat removal ratio for circulating the lithium, and the amount of heat generated directly in the lithium (which does not have to be transferred from the graphite to the lithium). These effects are shown in Fig. 41. Inspection of these curves indicates that a lithium region thickness between 20 and 30 cm gives a good compromise between the various design considerations.

9.3.2 Temperature Distribution

The blanket design concept of Ref. 30 and Fig. 40 was evolved in an effort to obtain a uniform temperature distribution and hence as favorable a situation as possible from the thermal stress and thermal strain cycling standpoints. The design calls for the lithium to be recirculated around the racetrack at a modest velocity³⁶ — of the order of 30 cm/sec — but the large flow passage cross section area and hence large flow rate serve to keep the temperature rise in one transit of the loop down to around 60°C. Further, by cooling the lithium with potassium boiler tube banks which would operate at the boiling point of potassium for the potassium boiler discharge pressure, an exceptionally uniform temperature of the boiler tubes can be maintained. The temperature rise in the lithium flowing through the portion of the loop adjacent to the first wall is equal and opposite to the temperature drop in the return leg. There are no steep local temperature gradients along the wall,³⁷ and there is no large accumulation of differential thermal expansion between the two legs of the loop that could cause serious bending stresses in the return bends at the ends.

The boiling potassium will hold the temperature of the boiler tube walls within 10-20°C of the nominal boiling point, so they should operate at an essentially uniform temperature. The potassium boiler tubes lie along lines that are circles concentric with the polar axis of the torus. As a consequence, as is the case for the flow of liquid lithium, the pressure drop stemming from MHD effects will be quite small. The liquid potassium will enter the boiler tubes at a low velocity, bubbles will form at nucleation sites which have been deliberately introduced,³⁸ a bubbly flow regime will prevail for a distance of perhaps 20 cm, and then an annular flow regime will develop with a liquid film on the wall and vapor largely free of droplets will flow through the center of the tube. In this region, evaporation will take place directly from the free liquid surface of the annular film of potassium. In the last portion of the tube, a dry wall condition will develop and the flow will consist of saturated potassium vapor in which a mist of potassium liquid droplets will be entrained. In short, the situation will be essentially the same as in a conventional, once-through steam generator tube.

The temperature distribution normal to the first wall is difficult to estimate. If normal turbulent flow were occurring, the temperature distribution would be quite flat and the heat transfer coefficient at the first wall would be extremely high — of the order of 85 kW/m²-°C (15,000 Btu/hr-ft²-°F). However, MHD effects will drastically reduce turbulence so that lateral heat transport by eddy diffusivity will be greatly reduced. As a first — and pessimistic — approximation it was assumed for a set of calculations presented in Ref. 37 that the eddy diffusivity would be zero so that the only mechanism for lateral heat flow across the stream would be simple conduction in the lithium. This assumption yielded the temperature distribution shown in Fig. 42 for a drastically simplified case in which the heat removal from the lithium would be accomplished with just two banks of potassium boiler tubes arranged in two planes in the return leg of the racetrack. In the actual plant the potassium boiler tubes would be distributed

in such a way as to yield a fairly uniform temperature distribution across the lithium flow channel.

9.3.3 Thermal Stresses

An examination of Figs. 40 and 42 indicates that the principal source of difficulty with thermal stresses will be the temperature rise in the first wall at the end of the lithium flow pass adjacent to the first wall. If the eddy diffusivity were zero, the first wall temperature would be $\sim 160^\circ\text{C}$ above the base temperature of the blanket. The resulting thermal expansion in a direction transverse to the axis of the module would be accommodated by bowing of the first wall with only minor bending stresses, but axial thermal expansion would produce compression stresses in the first wall which would be balanced by shear and tensile stresses in the radial webs. Even for the pessimistic temperature distribution of Fig. 42 a niobium blanket structure would not be subject to serious thermal stresses at a first wall power loading of 1 MW/m^2 , but a stainless steel blanket structure would probably give difficulty. If there were even a little eddy diffusivity in the lithium — and there is reason to believe that there will be³⁹ — the temperature rise in the first wall would be much reduced and a stainless steel structure ought to be satisfactory from the thermal stress standpoint for first wall total power loadings of $\sim 1 \text{ MW/m}^2$. Values for the principal parameters calculated for the niobium and stainless steel cases are summarized in Table 3.

9.4 THERMAL STRESSES IN THE FIRST WALL OF A THETA-PINCH REACTOR

The energy release in a theta-pinch reactor takes place in a short time interval — $\sim 10 \text{ msec}$ in a 10-sec cycle in the typical case of Ref. 10. (See Table 1.) This leads to a transient temperature excursion and thermal stress in the first wall similar to those considered in an early section of the effects of plasma energy dumps. In the design of Ref. 10 the blanket is made up of numerous modules somewhat similar to those of Fig. 40 but with a circumferential thickness of

the module of only $\sim 4 \text{ cm}$ with a module wall having a thickness of 1.0 mm of Nb-1% Zr coated with 0.3 mm of a ceramic insulator such as alumina. The energy per pulse in this design is $\sim 100 \text{ J/cm}^2$ (see Table 1). Aside from the obvious thermal stress difficulties associated with the radial temperature gradient through the first wall — particularly in the bond between the ceramic coating and the wall — there is a more serious problem in that the first wall will tend to expand relative to the module as a whole during the pulse. The temperature differential across the first wall can be reduced by making it thin (1.0 mm in this case), but the temperature differential between the first wall and the bulk of the structure of the module cannot be eased in this way. From Fig. 21 one can quickly estimate that the temperature rise in the first wall will run about 400°C if one neglects the effects of the Al_2O_3 layer, and the associated thermal stress for the ideal elastic material case would run 2800 kg/cm^2 ($40,000 \text{ psi}$). From Fig. 19 one can deduce that the first wall would fail from thermal strain cycling in a few thousand cycles with no allowances for the effects of the stress concentrations where the first wall is welded to the radial bulkheads. (Note that the value of $Q/\sqrt{t}$ in this case is $380 \text{ J/cm}^2\text{-sec}^{1/2}$.)

It is likely that local buckling may occur in the round nose of the module that constitutes the first wall because the radius-to-thickness ratio is one of the order of 25^{10} . If this should occur, it would represent an additional stress concentration and would induce failure in fewer cycles.

The above appraisal differs from LASL work that has yielded much more favorable results⁴⁰ mainly in that it takes into consideration the axial restraint on the first wall imposed by the balance of the blanket structure, i.e., three-dimensional effects. Other, lesser factors that also make the above analysis more pessimistic than the LASL work are the somewhat less favorable physical properties for Nb-1% Zr used here and the assumption of a square wave for the energy input rather than the more elegant wave form calculated for use in the LASL work. However,

these latter two effects are small compared to the three-dimensional effects.

10. MAGNETOHYDRODYNAMIC PROBLEMS AND CONSIDERATIONS

The previous sections on cooling and thermal stress problems indicate that there is a strong incentive to employ a liquid metal-cooled blanket. A number of different approaches to the design of such a cooling system have been proposed; all of them present difficult problems. This section has been prepared in an effort to indicate the magnitude of these problems, the more promising design approaches, and the uncertainties that should be resolved by experiments.

The first design approach usually considered is to pump lithium through the magnet and magnet shield region into the blanket, where it will absorb heat from the blanket and then be pumped back outward through the magnet shield and magnet region. It has been found that this superficially straightforward approach does not appear to be practicable.^{41,42} A second approach is to employ a recirculating lithium loop within the blanket as indicated in the previous section (see Fig. 40) and cool this loop with another fluid circuit.^{30,36-39} One such approach would be to pump liquid potassium or cesium into the blanket, where it would be boiled; vapor would exit from the blanket, where it could be employed in a Rankine cycle. This approach has the advantage that the quantity of liquid metal that would have to be pumped into the blanket is reduced by about a factor of ten relative to a simple liquid circuit with a comparable temperature rise because of the high heat of vaporization of the liquid metal. A further factor of two can be realized because the vapor leaving the blanket is essentially nonconducting and would not present an MHD pumping problem. Such a system would also have the advantage that it would minimize the temperature loss between the blanket and the inlet to the turbine for the thermodynamic cycle and thus should yield a higher thermal efficiency. However, it still presents some serious MHD problems, and these are discussed in this section. A variation on this approach would be to employ a

molten salt that could be pumped in and out of the blanket to remove heat from the recirculating lithium. The principal uncertainty in this approach from the MHD standpoint is that it may lead to corrosion in the molten salt circuit as a consequence of eddy currents induced in the molten salt coupled with radiation damage to the salt.

The various problems considered and the principal means that have been proposed for coping with them are discussed in the following subsections.

10.1 PRESSURE DROP AND PUMPING POWER IN RECIRCULATING LITHIUM BLANKETS

The first question raised with respect to the recirculating lithium blanket of Fig. 40 was concerned with the validity of the analysis of Ref. 36. To investigate this question a one-sixth scale loop was built and tested in a large solenoid at ORNL.³⁹ The results of these tests indicated that if theoretically derived scale factors⁴³ hold as expected, the pressure drop and pumping power-to-heat removal ratio should be substantially as estimated in the original design analysis, i.e., ~ 0.2 kg/cm² (3 psi) and less than 0.001, respectively.³⁹ Note from Table 4 that an exceptionally high Hartmann number was reached in these tests of the one-sixth scale model, bringing the Hartmann number in the experiment within a factor of five of the full-scale design value. Note further that the original analyses entailed extrapolation by a factor of 50 from the highest Hartmann number test data available prior to the test of the one-sixth scale model, so the bulk of the gap in scale was filled in by the initial model test.

10.2 PUMPING LIQUID METAL INTO THE BLANKET THROUGH THE MAGNET REGION

The most difficult problem envisioned in the recirculating lithium blanket system is carrying liquid metal radially inward to the blanket modules through the magnet and magnet shield regions. The magnitude of the pressure drop, the pumping power requirements, and the pressure stresses in the pipe wall resulting from the pressure drop

Table 4. Comparison of dimensionless parameters applicable to the recirculating lithium blanket of Fig. 40 and Ref. 30

Parameter	Full-scale ³⁰ tokamak	Basic test data obtained by Carlson ⁴⁴	1/6 scale model test data ³⁹
Reynolds number	350,000	17,000	22,000
Magnetic Reynolds number	0.567	0.0409	0.0471
Hartmann number	52,400	960	11,700
Wall-to-channel conductivity ratio	0.008	0.048 to 0.098	0.0158

have been estimated for a typical case by W. M. Wells, Jr.⁴² and are presented in Table 5. Note that two values of feed pipe wall thickness have been given: one the minimum thickness that might conceivably be employed, and the other a substantial thickness intended to give a large safety margin. Two sets of values are given: one for a conventional pipe and the other for a pipe lined with a thin layer of electrical insulation in the form of zirconia, alumina, yttri, thoria, etc., and covered with a coating of the structural metal to prevent attack of the electrical insulator by lithium. Note that with no insulating liner the pressure stress in the tube wall is independent of the wall thickness because the eddy currents between the lithium and the wall increase with wall thickness; hence the pressure drop is directly proportional the pipe wall thickness. On the other hand, with a constant-thickness insulating liner in the pipe, the pressure stress in the tube wall is essentially inversely proportional to the outer wall thickness.

Similar calculations for lithium for the same velocity and pipe diameter give the same pressure drop and pipe wall stress but ten times as many feed pipes entering the magnet region as well as an equal number leaving it, and hence twenty times the pumping power.

The use of an insulating liner was considered for lithium ducts through the magnet shield, but rejected because lithium is so active chemically that it would react with any oxide that might be used as an insulating material. Inasmuch as it is not possible to apply a thin coating of metal over an oxide layer to form a liner of this sort without getting pin holes in the metal coating, one would fully expect that blistering of the liner would occur. It is likely that this would not happen with potassium because it is much less reactive than lithium, and to a first approximation would not react with the stable oxides cited above. However, tests carried

Table 5. Pressure drop, pressure stress, and pumping power-to-heat removal ratio for potassium flowing at 3.05 cm/sec through a 30.5-cm diam Nb-1% Zr duct transverse to a 7-T magnetic field⁴²

With sandwich having inner skin of 0.019	Outer wall thickness, cm	0.19	1.9
	Pumping power-to-heat removal ratio for 6.3-m length	18×10^{-6}	18×10^{-6}
	Pressure drop, kg/cm ² (psi)	0.78 (11.1)	0.78 (11.1)
	Pressure stress, kg/cm ² (psi)	24.6 (350)	2.5 (35)
Without sandwich	Outer wall thickness, cm	0.19	1.9
	Pumping power-to-heat removal ratio for 6.3-m length	0.18×10^{-3}	1.8×10^{-3}
	Pressure drop, kg/cm ² (psi)	7.81 (111)	78.1 (1110)
	Pressure stress, kg/cm ² (psi)	246 (3500)	246 (3500)

out with oxides in potassium under the space power plant program indicated that there is not enough difference in the affinity for oxygen between potassium and such refractory insulating materials as zirconia, magnesia, alumina, and beryllia to eliminate all significant reactions between potassium and the refractory oxide in the electrical insulator. Therefore, a protective metallic coating over the oxide insulator would be required. It seems likely that such a coating operated in potassium would not be sensitive to pin holes and that satisfactory performance might be obtained. Further experiments will be required to determine whether this proves practicable.

10.3 SUPPRESSION OF EDDY DIFFUSIVITY BY THE MAGNETIC FIELD

As mentioned earlier in the section on thermal stresses in a recirculating lithium blanket, the strong magnetic field will suppress turbulence in the lithium stream in the blanket and hence the lateral transport of heat from the first wall across the stream by eddy diffusivity. Even a very small amount of turbulence would help enormously to relieve the hot streak effect along the first wall implicit in Fig. 42, e.g., 180° of rotation of a large-scale eddy in the 10-m length of a blanket module might cut the hot streak effect in half. Experts in MHD problems differ widely in their appraisal of this problem, some being convinced that there will be some eddying while others seem equally convinced that pure, ideal, irrotational flow will prevail. The only way to resolve the problem appears to be an experiment. This could be conducted readily with the one-sixth scale model mentioned in the previous section, and such a test has been proposed. Unfortunately, funds for the test have not been made available up to the time of writing.

10.4 SHIELDING FEED PIPES WITH IRON

It was suggested by D. Steiner of the ORNL Fusion Energy Division that one might shield the potassium feed pipes with massive amounts of iron. This would reduce the magnetic flux traversed by the liquid metal. Inasmuch as any such iron

shield would saturate at about 1.5 T, this approach would not be very effective where the pipes penetrate the magnet and magnet shield region. However, this approach could be used for circumferential manifolds if these were placed 1 or 2 m radially out beyond the envelope for the toroidal field coils. Calculations supplied by Carl Parker of the ORNL Fusion Energy Division (see Fig. 43) indicate that, for a typical case, the field would drop to 0.5 T at 2 m radially outward from the envelope for the toroidal field coils. For the iron to be effective in a field very large relative to the size of the pipe, it is necessary to employ a series of concentric iron sleeves with substantial air gaps between them. The proportions usually can be optimized with a computer program to keep the amount of iron required to a reasonable level. Thus it appears that massive iron shielding could be used to advantage around circumferential manifolds in this low field region.

10.5 USE OF A BACK EMF TO EASE MHD PROBLEMS INVOLVED IN CONVEYING A LIQUID METAL RADIALLY INWARD THROUGH THE MAGNET REGION OF A TOKAMAK

The recirculating lithium blanket system proposed in Refs. 30 and 36 makes use of EM pumps in the return bends of racetrack-shaped loops as a means for circulating liquid lithium in the blanket of a tokamak. A review of this concept suggested that a somewhat similar arrangement might be employed to ease the MHD problems associated with carrying a liquid metal radially inward or outward through the magnet and magnet shield region to facilitate the cooling of a tokamak blanket. The configuration envisioned is shown to scale in Fig. 44 for a typical set of proportions. A blanket similar to that of Fig. 40 and Ref. 30 was assumed as the point of departure. The potassium feed pipes would be carried circumferentially around the torus at a distance of about 2 m radially outward from the magnet coil, where the toroidal field strength would have dropped to about 5 kG. Conductors would be arranged between the radial portions of the feed pipes in such a way that a current would flow circumferentially around the torus and transverse

to the flow in the radial feed pipes. This electric current would be in parallel with the circumferential current flow in the electromagnetic pump regions in the return bends of the racetracks constituting the blanket. (See Fig. 40.) This would provide a back emf to counter the emf generated by the liquid metal flow through the radial feed pipes as it crossed the toroidal field lines.

10.5.1 Delineation of Model

In attempting to assess the possible merit of this concept it seemed best to examine a typical case. Carl Parker kindly supplied a set of calculations for the toroidal field strength as a function of radial position for a radius extending horizontally outward from the polar axis, a radius extending vertically either upward or downward from the centerline of the torus, and a radius extending horizontally inward toward the polar axis. These values were obtained for a set of coils having a coil inner radius of 5 m, a major radius for the torus of 12 m, and a set of 30 toroidal field coils. It was also assumed that the magnet shield region and the blanket region would each have a radial thickness of 1 m. It was further assumed that the reactor would have a mean power output of 1000 MW(t), which gives a first wall power loading of 0.70 MW/m².

There are so many degrees of freedom in selecting the proportions for the radial ducts that it is necessary to impose a number of arbitrary conditions. Those chosen for the initial analysis were as follows:

- (1) The power input per unit area of the first wall will not be uniform but rather will be approximately inversely proportional to the major radius. Thus, inasmuch as each racetrack will have a length which will be directly proportional to the major radius, the power input per racetrack will be essentially independent of its position in the blanket.
- (2) To hold a constant temperature rise in the circulating lithium in each racetrack, the lithium velocity should be the same in all

racetracks. Similarly, the volumetric feed flow rate of the potassium flowing into the blanket should be the same regardless of the circumferential position of the feed pipe.

- (3) The voltage drop across the EM pump region in each racetrack will vary with the circumferential position of the racetrack and will be directly proportional to the local magnetic field. Similar conditions will prevail for the potassium feed pipes. The circumferential flow of electrical current through the feed pipes will be the same for all feed pipes because they are simply hooked in series. The same will be true for the EM pump regions in the racetracks.
- (4) To allow for I^2R losses in the conductors carrying current circumferentially from one potassium feed pipe to the next, the IR drop through each feed pipe will be half the IR drop through the EM pump region for its associated racetrack.
- (5) The circumferential portions of the feed pipes will be clustered to form a ring at a radial distance of about 2 m outward from the magnet coils, and these clusters will be surrounded by magnet iron to reduce the magnetic flux inside the iron shielding to a level of about 2 kG. (At this level the pressure drop per meter of duct length for a liquid metal flow velocity of about 2 m/sec would be only about 20 cm of potassium per meter of pipe length.)
- (6) The feed pipes in the circumferential cluster region would be circular in cross section and about 5 cm in diameter. They would be electrically insulated from each other to minimize fringing currents induced by the circumferential voltage gradient.
- (7) The feed pipes extending radially inward to the blanket would be rectangular in cross section with a large wall thickness on the sides parallel to the magnetic field so that these would readily withstand the internal pressure in the duct.

The sides in planes transverse to the direction of the magnetic field would have a thickness of 1 mm to reduce the electrical current flow bypassing the liquid metal stream. The outer surfaces of these duct walls would be covered with electrical insulation, and this would bear against moderately heavy plates which would carry the pressure load of the potassium inside the duct.

- (8) Electrical current would be carried circumferentially from one duct to the next through copper conductors operating at 700°C, i.e., the temperature of the potassium feed stream. The current density would be sufficiently low that the heat generated from I^2R losses in the conductors could be dissipated by thermal radiation to the magnet shield region and the test cell. All of the ducts and electrical conductors would have to be provided with sturdy structural supports to withstand the electromagnetic forces associated with fluctuations in the poloidal field.

10.5.2 Typical Case

The calculations for a typical case are summarized in Table 6. The first portion of the table gives the basic data for the torus. Note that the numbers used in this instance differ somewhat from those employed in Ref. 36 mainly in that a greater magnetic field strength was considered here (i.e., 4 T rather than 2 T). The next section of the table presents data on the lithium circuit in the blanket. To keep the blanket temperature as uniform as possible, the lithium temperature rise was held to only 56°C. If that rise were allowed to double, as seems reasonable, the electrical power requirements would be reduced accordingly. All of the values presented in this section of the table were calculated from the same basic formulas used in Ref. 36 except for the last item, the current flow through the EM pump region. In this instance the current flow was simply scaled from that given on p. 19 of Ref. 36 on the premise that the current

flow would be directly proportional to the magnetic field and inversely proportional to the lithium velocity in the loop. Calculations for the potassium circuit portion of the table made use of typical values for a potassium vapor cycle. The feed pipe width transverse to the field was chosen to be 15 cm after several scouting calculations indicated this would give a good set of proportions. The back emf was taken as half that for the EM pump region of the racetrack and, from these two values coupled with the 4-T field strength for the central racetracks, a potassium feed flow velocity of 0.40 m/sec was calculated. This in turn defined the potassium feed pipe passage thickness in the direction of the toroidal field at the points where the toroidal field would run 4 T. This calculation yielded a passage thickness of 9.2 cm. The current flow through the 1-mm-thick duct walls of the 3.5-m-long radial ducts was then calculated and found to be about 22,400 A. The last section presents estimates of the size of the bus bars required to connect the radial feed pipes. Heat transfer estimates indicated that a current density of 1000 A/cm² could be employed in these conductors to give a temperature rise of not more than 50°C above the 700°C temperature of the potassium feed stream, assuming that the conductors would be 1 cm square and would be cooled by thermal radiation. Note that only 22.4 cm² of conductor cross section would be required for coupling the radial heat pipes. Note, also, that the total pumping power for both the recirculating lithium and the potassium liquid feed would run about 1.6 MW. This is a quite modest pumping power requirement. Further, the pressure drop through the radial ducts would be negligible.

The thickness of the potassium feed ducts in the direction parallel to the magnetic field would vary with the local field. Table 7 presents estimates for the duct thickness together with the local field strength for ducts traversing the magnet region in both the horizontal and vertical directions. Note that these proportions are reasonable.

Table 6. Summary of design calculations for a potassium feed pipe system employing a back emf to reduce the pressure drop for radial flow through the magnet region

Basic torus	
Thermal power output, MW(t)	1,000
Thermal power output, MW/m ²	0.70
Major radius of torus, m	12
Minor radius of vacuum wall, m	3
Bore of toroidal field coil, m	5
Number of toroidal field coils	30
Number of segments in each sextant of blanket	24
Maximum toroidal field in the coil, kG	75
Circumferential width of blanket segment, cm	82
Lithium channel flow area per segment, m ²	0.2474
Lithium circuit	
Lithium flow rate per segment, liters/sec	60.6
Lithium temperature rise, °C	56
Inner lithium region radial thickness, cm	30
Graphite region radial thickness, cm	45
Outer lithium region radial thickness, cm	25
Lithium velocity in main channel, m/sec	0.245
Lithium velocity in return bend, m/sec	0.146
Back emf at 40 kG (R = 12 m), V	0.4817
Current flow through EM pump region, A	15,087
Potassium circuit	
Potassium density, g/cm ³ at 700°C	0.675
Potassium feed flow (total), liters/sec	793
Potassium feed flow per segment, liters/sec	5.5
Potassium velocity in circumferential pipes, m/sec	3
Potassium feed pipe width transverse to field, cm	15
Potassium feed pipe wall thickness, mm	1.0
Potassium feed pipe back emf, V	0.24
Potassium feed radial velocity in 40-kG field, m/sec	0.40
Potassium feed pipe passage thickness at 40 kG, cm	9.2
Current flow through 3.5-m-long feed pipe wall, A	22,400
Bus bars	
Resistivity of copper at 700°C, μΩ-cm	7
Current density in thermal radiation-cooled, 1-cm-thick bus, A/cm ²	1,000
Conductor cross-sectional area, cm ²	22.4
Total pumping power, MW	1.676

10.5.3 Effects of Duct Divergence

The flow passage area divergence in the radial ducts defined by Table 7 raises a question about possibly adverse effects of the duct divergence on the velocity distribution. Fortunately, as shown in Ref. 36, the imposed emf forces a uniform velocity distribution in the pump region in the return bend; if the velocity in any given region drops below the proper value, the emf generated in that region will be less than that imposed, current will flow, and this will generate a force that will accelerate the liquid stream in that region. Similarly, any tendency toward a local high velocity will generate an emf greater

than the imposed back emf and the resulting induced current will act to brake the flow. Thus, an unusually flat velocity distribution should result.

10.5.4 Interesting Variations

The unusually low field close to the polar axis suggests that it might be best to bring the liquid potassium into the torus region through a set of vertical pipes extending upward or downward along the polar axis of the torus. This would be excellent from the standpoint of symmetry and fabrication of the ducts.

Table 7. Feed pipe channel thickness as a function of radius

Circumferential position Minor radius (m)	Horizontal, pole side		Vertical		Horizontal, outside	
	Field (kG)	Thickness (cm)	Field (kG)	Thickness (cm)	Field (kG)	Thickness (cm)
7.0	0.23		2.5		4.7	1.1
6.5	4.4	1.0	10.9	2.5	9.0	2.1
6.0	66.1	15.2	24.5	5.6	14.8	3.4
5.5	75.0	17.2	34.8	8.0	20.4	4.7
4.0	70.4	16.2	39.1	9.0	25.1	5.8
3.5	65.0	14.9	40.5	9.3	28.2	6.5
3.0	60.0	13.8	40.8	9.4	30.3	7.0

If liquid lithium were conveyed in and out of the blanket rather than feeding in potassium, boiling it, and carrying off potassium vapor, the lithium pipes extending radially through the magnet region would have to have a much larger cross section than is the case for the potassium feed pipes. The flow passage area would have to be increased by a factor of 12 if the lithium temperature rise were held to the 56°C used in this case. It does seem quite reasonable to double that temperature rise, which in turn would give a lithium flow passage area six times that used for the potassium, but both inlet and outlet pipes would have to be provided; thus, 12 times the flow passage area and 12 times the pumping power involved would be required. This might be reduced of two if the pipes were arranged beside each other so that there would be a series of electrical connections. For the larger lithium ducts it would probably be necessary to employ a somewhat larger wall thickness, and this would increase the current flow and power requirement. Also, the lithium duct proportions would be substantially different from those for the potassium feed pipes because as a first approximation the duct width in the direction transverse to the toroidal field would have to be kept constant and the additional flow passage area would be provided by increasing the duct thickness in the direction of the magnetic field. This would lead to a duct thickness for the racetracks on the side toward the polar axis of over a meter, a quite awkward situation. Increasing the flow passage area by increasing the duct width normal to the magnetic field would not help because the lithium velocity would have to be inversely proportional to the duct width for the fixed emf available.

It would be possible to vary the liquid velocity in the radial ducts and vary the duct width transverse to the magnetic field accordingly. However, this appears to lead to a less favorable set of proportions for the radial ducts and their connections, respectively, to the circumferential legs of the ducts and to the lithium racetracks.

It would be possible to run ducts diagonally in through the magnet region to ease congestion in the blanket region on the side toward the polar axis. If the duct were run diagonally, the duct length would be increased, and, if the liquid metal velocity were kept the same, a greater amount of current would be required. If the duct were increased so that the velocity could be reduced, a smaller back emf would be required and the current available could flow through the greater duct wall conduction path and still give the required back emf.

The simplified analysis presented here has neglected the effects of the poloidal field. However, inasmuch as this field will be of the order of 10% of the toroidal field, it is believed that the modifications required will not be too serious. It might be noted in this connection that a central location for feeding the potassium into the torus region would be better than that shown in Fig. 44 from the standpoint of accommodating the effects of the poloidal field on the potassium feed pipes. This is particularly true because the poloidal field will not fall off rapidly with distance away from the torus as is the case for the toroidal field. Magnetic shielding of the ducts will probably be required.

Table 7. Feed pipe channel thickness as a function of radius

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7.0	0.23		2.5		4.7	1.1
6.5	4.4	1.0	10.9	2.5	9.0	2.1
6.0	66.1	15.2	24.5	5.6	14.8	3.4
5.5	75.0	17.2	34.8	8.0	20.4	4.7
4.0	70.4	16.2	39.1	9.0	25.1	5.8
3.5	65.0	14.9	40.5	9.3	28.2	6.5
3.0	60.0	13.8	40.8	9.4	30.3	7.0

If liquid lithium were conveyed in and out of the blanket rather than feeding in potassium, boiling it, and carrying off potassium vapor, the lithium pipes extending radially through the magnet region would have to have a much larger cross section than is the case for the potassium feed pipes. The flow passage area would have to be increased by a factor of 12 if the lithium temperature rise were held to the 56°C used in this case. It does seem quite reasonable to double that temperature rise, which in turn would give a lithium flow passage area six times that used for the potassium, but both inlet and outlet pipes would have to be provided; thus, 12 times the flow passage area and 12 times the pumping power involved would be required. This might be reduced by a factor of two if the pipes were arranged beside each other so that there would be a series of electrical connections. For the larger lithium ducts it would probably be necessary to employ a somewhat larger wall thickness, and this would increase the current flow and power requirement. Also, the lithium duct proportions would be substantially different from those for the potassium feed pipes because as a first approximation the duct width in the direction transverse to the toroidal field would have to be kept constant and the additional flow passage area would be provided by increasing the duct thickness in the direction of the magnetic field. This would lead to a duct thickness for the racetracks on the side toward the polar axis of over a meter, a quite awkward situation. Increasing the flow passage area by increasing the duct width normal to the magnetic field would not help because the lithium velocity would have to be inversely

proportional to the duct width for the fixed emf available.

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The simplified analysis presented here has neglected the effects of the poloidal field. However, inasmuch as this field will be of the order of 10% of the toroidal field, it is believed that the modifications required will not be too serious. It might be noted in this connection that a central location for feeding the potassium into the torus region would be better than that shown in Fig. 44 from the standpoint of accommodating the effects of the poloidal field on the potassium feed pipes. This is particularly true because the poloidal field will not fall off rapidly with distance away from the torus as is the case for the toroidal field. Magnetic shielding of the ducts will probably be required.

10.6 LIQUID POTASSIUM FLOW IN THE BOILER TUBES

It has been planned to carry the potassium boiler tubes along toroidal field lines so that MHD effects on the liquid flow would be small. However, the poloidal field will inevitably have appreciable effects on that flow, which will be primarily in the form of a liquid film on the wall of the boiler tube. One way to minimize the amount of liquid flowing on the boiler tube wall would be to employ jets which would inject liquid potassium axially down the center of the boiler tubes so that droplets of spray would be scattered on the boiler tube wall over a distance of a meter or more. Flash boilers of this type were used in lightweight boilers employed in motor torpedo boats around the turn of the century, and tests of this type of boiler as a means of removing heat from molten salt when employing feed water at a temperature much below the freezing point of the molten salt were conducted at ORNL. A major problem with this approach is that the spray from a simple, drilled hole type of orifice will give good droplet dispersions at high flow rates but poor and erratic dispersions at low flow rates. Good dispersion over a wide flow range can be obtained with a variable orifice nozzle obtained by employing either a spring-loaded, pintle-type nozzle or a thermostatically controlled pintle. The latter can be made with a bimetallic, temperature-sensitive element to control the pintle much as bimetallic elements are used to control the water flow from the engine block to the radiator of conventional water-cooled automobile engines. If this were done, the potassium feed flow to the boiler tubes in the blanket would be controlled by the blanket operating temperature, i.e., as the blanket temperature increased, the potassium feed flow rate would increase. The bimetallic control element could be designed to give a relatively small temperature range between full and low power conditions. It would also have the advantage that with many boiler tubes operating in parallel, failure of one thermostat element to function as designed would have little effect on the overall performance of either the blanket or the potassium boiler. This would provide a simple, highly

reliable means for controlling the blanket temperature to make it uniform throughout the torus irrespective of local variations in energy input.

10.6.1 Application of a Back emf to Lithium Ducts

The attractive proportions obtained above for a system to convey the liquid potassium boiler feed through the magnet and magnet shield region into the blanket suggests that a similar arrangement might be employed for lithium. To keep thermal stresses in a niobium blanket structure to acceptable levels, the lithium temperature rise probably ought not exceed $\sim 100^\circ\text{C}$; hence the volumetric flow rate for the stream entering the torus would be about ten times that for the potassium feed. If the same match between the potential drop in inlet pipes and the EM pump regions of the return bends were maintained, the velocity would be the same and the flow passage area would be about ten times as great. This would lead to very awkward proportions for the ducts — i.e., at the blanket end on the side toward the polar axis each duct would be 130 cm by 15 cm, with the 130 cm dimension requiring a gap of at least 150 cm (for clearance and thermal insulation purposes) between adjacent toroidal coils in the critical region close to the polar axis. This would probably be unacceptable.

If the lithium were ducted into the blanket, around a sextant of the torus, and then outward instead of mixing with lithium in a recirculating blanket, a more favorable set of duct proportions could be obtained. However, the lower velocity in the blanket and the higher temperature rise than that envisioned for the recirculating blanket might pose problems unless there were some turbulence in the lithium flowing along the first wall. Thus, an experiment to investigate the turbulence problem becomes vitally important if this approach is to be considered.

10.7 COOLING AROUND BEAM HOLES

The recirculating lithium blanket concept of Fig. 40 would not lend itself to provisions for large penetrations for neutral beam injectors if the axes of these beam holes were highly skewed

relative to the surface of the blanket. Fortunately, it appears that the neutral beam injectors need be inclined at only a small angle — e.g., 16° — with respect to planes through the polar axis of the torus. Skewing the straight section of the return bends through a small angle should not give trouble; the injectors could therefore be located in gaps between sextants of the toroidal blanket as shown in Fig. 45 (taken from Ref. 30). Lithium flow through the return bends of the blanket modules should provide good cooling of the walls exposed to stray ions scattered out of the neutral beams.

10.8 CURRENTS INDUCED IN THE RACETRACK BY DECAY OF THE PLASMA CURRENT

J. Powell of BNL pointed out a special problem previously unsuspected by the writers. Induced currents will flow around the racetrack if the poloidal field set up by the plasma current is switched on or off in a time comparable to, or shorter than, the exponential L/R decay time for such induced currents. For the ORNL design of Fig. 40, this L/R time constant is on the order of 10 sec. Powell estimated that for a poloidal field of 4.7 kG, these induced currents would cause an outward pressure on the vacuum wall of ~ 15 psi if the plasma current were switched off in a few seconds, and for a poloidal field of 8 kG, the pulsed pressure would be ~ 40 psi.

The problems are subtle and required some expert help. J. N. Luton, Jr., of the ORNL magnet group, agreed to help with this and estimated that for the conditions given the outward pressure on the Nb-1% Zr vacuum wall would run around 9 psi. In view of the approximations required for a modest analytical effort, this is in reasonably good agreement with the 15 psi estimated by Powell.

The pressure induced by a current of this type will impose a tensile stress in the first wall equal to about 100 times the imposed pressure. Taking the higher value of 40 psi suggested by Powell, the stress in the first wall would run about 4000 psi. This is about equal to the stress for 1% creep in 10,000 hr at 1000°C ,

and is less than 25% of the short time yield point. In view of the fact that such a rapid collapse of the plasma current would probably not occur very often, the stresses are clearly not serious. Even if it were to occur once each cycle, these short-term stresses in the first wall appear to be acceptable.

The hoop tension force in the outer wall of the blanket assembly would be much larger than that in the first wall because of the much larger radius. The original design of Ref. 30 called for an outer wall thickness of 8 mm, which would not be adequate to carry a 40-psi pressure in the blanket. However, the 40 psi could be carried by doubling that wall thickness to 16 mm, and this could be done without any serious increase in the cost or complexity of the assembly.

A second problem raised by Powell has to do with the equal and opposite forces at the ends of the racetracks normal to the plane of the racetrack as a consequence of the induced current circulating in the racetrack when the plasma current collapses rapidly. For the conditions cited above these forces will be roughly equal to the weight of a blanket racetrack module and thus will not be difficult to accommodate in the support structure.

10.9 USE OF TURNING VANES AND ELECTRICALLY INSULATING BAFFLES

In the original design and analysis work in Refs. 30 and 36, a set of insulating baffles and turning vanes was thought to be advantageous to reduce both end leakage current effects and entrance and exit pressure losses in the return bends. In designing the 1/6-scale model experiment described in Ref. 39, it was decided to eliminate both the turning vanes and the insulating baffles in part to avoid their complexity and in part to provide data on a base case that would show to what degree they might be advantageous. The results of the test proved to be favorable indeed — clearly neither the turning vanes nor the insulating baffles are required, so it is currently planned not to employ them.

11. FABRICABILITY, AVAILABILITY, COSTS, AND MAINTENANCE

The heat transfer, fluid flow, and stress analysis as treated above clearly favor a lithium-cooled Nb-1% Zr blanket structure, but most people are afraid that this would be too exotic and expensive. Thus, other considerations and criteria enter the picture. These deserve some attention at this point.

11.1 FABRICABILITY OF Nb-1% Zr

As in any system, there is a strong incentive to employ simple structures with all welds easily accessible for both welding and inspection. The thicknesses of adjacent walls at a weld should be within a factor of two, if possible. Forming operations should be conventional, and tolerances should be well within the capabilities of the fabrication equipment. In general, the fabrication procedures, machinability, weldability, forming characteristics, etc. for Nb-1% Zr are similar to those of type 304 stainless steel except that rigorous cleanliness procedures are required.

In the course of the ANP and space power plant programs, ~300 tons of Nb-1% Zr billets were fabricated into tubing, plate, and forgings at P&W, G.E., and ORNL for use in heat exchangers, pressure vessels, pumps, tanks, valves, and systems. Large quantities of niobium alloy have also been fabricated in the aerospace industry for hypersonic airframes and spacecraft.^{45,46} The bulk of this latter material has been relatively thin sheet, i.e., ~1 mm thick. In addition, a large amount of niobium is being fabricated for special equipment for the Department of Defense. The current production of niobium alloys by the principal source of the refined metal (Wah Chang Albany) is 150,000 kg/year, which represents about 60% of their capacity. Note that the weight of Nb-1% Zr in the blanket design of Fig. 45 is 315,000 kg for a first wall surface area of 1450 m². The design output of the reactor is 1000 MW(t), or 500 MW(e), which gives a cost for the niobium in the blanket structure of $\$28 \times 10^6/\text{kW(t)}$ at \$90/kg, or \$56/kW(e). The principal costs for fabrication would

be for welding, and, for long seams for which automated electron beam equipment would be used, this would be about the same as for stainless steel.

The main objection to the use of niobium seems to be that it is not used in any equipment in commercial production. However, it appears quite feasible to go from the existing specialty fabrication work — for which procedures are well worked out (see Ref. 46, p. ix) — to production items. Indications of this are given by experience with two other refractor metals, titanium and zirconium. The use of titanium rose from 5×10^5 lb/year in 1952 to 25×10^6 lb/year in 1966, when the advantages of titanium for supersonic aircraft became apparent. Metallic zirconium is entirely a development of the fission reactor field; the use of zirconium alloys jumped from 0.45×10^6 lb/year in 1956 to 6.5×10^6 lb/year in 1975.

11.2 FABRICABILITY OF VANADIUM, MOLYBDENUM, AND TUNGSTEN

Vanadium presents a somewhat different situation. Aside from questions about its suitability from the materials compatibility standpoint (particularly its susceptibility to attack by small amounts of oxygen), there is less experience with its fabrication. Although the total amount of vanadium produced in the U.S.A. in 1975 was 25×10^6 lb, essentially none of this was fabricated as metallic vanadium or a vanadium-base alloy. In general, experience indicates that its fabricability is similar to that for niobium.

Molybdenum or tungsten has been specified in some fusion reactor blanket designs. However, over 30 years of research and development effort on molybdenum and over 60 years on tungsten have shown clearly that these metals are so subject to embrittlement in weld zones that there is no reason to hope that they could be used in any structure that must be hermetically sealed and at the same time withstand large thermal stresses. Thus, the use of tungsten and molybdenum must be limited to structural elements that can be bolted or riveted in place.

11.3 AVAILABILITY

The known terrestrial reserves of niobium ore are about the same as those of chromium. As is the case for manganese, nickel, cobalt, chromium, and copper, it will probably be necessary in the 21st century to utilize the vast stores of ore in the metal oxide nodules on the deep sea floor. In some areas these nodules contain attractive concentrations of niobium, and reserves of vanadium are comparable.

11.4 COSTS

An interesting insight into the relative costs of different types of blanket is given by consideration of the cost of candidate materials per unit of volume as well as per unit of weight, as presented in Table 8. Inasmuch as the attenuation length for fast neutrons does not differ greatly from one of these materials to another, it is evident that from the cost standpoint one ought to build the blanket mainly of lithium and graphite and use as little structural material as possible. Note, too, that the salvage value of lithium is essentially equal to its initial cost, whereas the salvage value of graphite, stainless steel, and lead would be less than 10% of the fabricated costs given in Table 9. These latter will, of course, be heavily dependent on the complexity of the structure and the dimensional tolerances. As may be noted from Table 9, the complex structure of Figs. 26 and 28 for stainless steel or Incoloy blanket structures and the large quantities of structural material required actually lead to a higher cost per square meter of blanket than for the two niobium cases of Figs. 25 and 29.

The higher raw material costs for beryllium, niobium, and tungsten would lead to scrap values as much as 25% of the fabricated cost if they could be refabricated with remote handling equipment and reused in a fusion reactor.

11.5 MAINTENANCE

Extensive experience with all types of complex systems has shown that forced outages must be expected and that the ease with which the

consequent maintenance can be carried out is a vitally important consideration. Experience with central stations — both fossil and fission — has forcibly shown the large costs of outages. Clearly, the blanket of a fusion reactor should be designed to minimize outage time. Therefore, the foremost consideration is to design for a high reliability and thus minimize the incidence of forced outages. This in turn implies that both pressure and thermal stresses should be kept to a minimum — really the central theme of this report. Other criteria are, of course, also important. They, too, are both complex and subtle, yet they ought to be major considerations in the design of a blanket structure. John Mitchell of the Culham Laboratory, UKAEA, and one of the writers spent two days in October 1975 drawing up a list of design criteria and precepts in an effort to clarify the situation and put the problems in perspective. The list subsequently served as the point of departure for a meeting at EPRI on February 6, 1976 on the design of fusion reactors for maintenance. The material in Sect. 11.6 was evolved from these two sessions.

Most of the items in Sect. 11.6 do not require elaboration. Many of the detailed operations involved in maintenance of a reactor such as that of Fig. 45 are treated at some length in Refs. 30 and 47. Further, Ref. 48 is an excellent recent paper covering the design of a full-scale tokamak in which maintenance considerations were of primary importance in evolving the designs and shows an application of a set of precepts similar to those of Sect. 11.6. Two of the items of Sect. 11.6 merit some further discussion here. Item 8 is concerned with automatic operation of remote handling equipment and cites the "laser tailor" as an indication of what has been done. This machine, recently put into service in England, ingests a computer card specifying the material to be used in a man's suit and the key dimensions of the man it is to fit. The machine takes the material required from a bolt of cloth, cuts out the pieces required with a laser beam using an arrangement that makes the most economical possible use of the material while still getting a good match in the weave pattern when the

Table 8. Approximate cost of materials in plate form for blanket structures

	Material cost	
	\$/kg	\$/m ³
Graphite	2.5	4,600
Lithium	9	4,500
Li ₂ BeF ₄	15	30,000
Type 304 stainless steel	5	40,000
Incoloy 800	10	80,000
Nb-1% Zr	60	500,000
Vanadium	90	550,000
Tungsten	130	2,500,000
Beryllium	120	200,000
Molybdenum (TZM)	105	1,050,000
Titanium alloy	16	65,000

Table 9. Estimated cost of typical blanket structures

Type of blanket	Material cost \$/m ²	Fabrication \$/m ²	Total cost \$/m ²
He-cooled static Li, massive stainless steel (Fig. 28)	13,500	12,000	25,000
He-cooled static Li, Nb-1% Zr (Fig. 25)	10,000	5,000	15,000
He-cooled Li ₂ BeF ₄ , Incoloy (Fig. 25)	33,000	11,000	44,000
Recirculating Li, Nb-1% Zr (Figs. 39 and 44)	16,000	4,000	20,000
Recirculating Li, stainless steel (Fig. 39)	5,500	3,500	9,000

pieces are assembled, and assembles and sews the pieces together. The secret of the automatic process is that all mens' suits are topologically similar with certain discrete deviations in size and styling that can be included in the programmed instructions. Similarly, for over 20 years automotive engine blocks have been machined in automatic operations that begin with the raw casting and terminate with inspection of the finished block. All of the operations, including transfer of blocks from one machining station to another, are carried out automatically. The cost of designing, building, and shaking down such a complex, automatic collection of machinery is high, but is clearly competitive with doing the same job with a set of skilled operators.

Item 10, regarding the movement of massive sections of the torus on an air cushion when assembling or disassembling the reactor, deserves some discussion. The sextants of the torus of the reactor in Fig. 45 and Ref. 30 would weigh about 900 tonnes, an impressive figure. However, the air cushion approach has become widely used. Examples include the movement of pressure vessels and boilers weighing up to 700 tons from station to station in the Combustion Engineering Company plant in Chattanooga, movement of 1600-tonne sections of Hawaii's Aloha Stadium to give the most favorable configuration for the sporting event of the day,⁴⁹ and precise positioning of massive forgings and castings weighing up to 100 tonnes for machining operations in a Westinghouse Electric Corporation plant.⁵⁰

11.6 DESIGN PRECEPTS AND CRITERIA FOR FUSION REACTOR MAINTENANCE

The following criteria are of importance in the maintenance of fusion reactors:

- (1) The blanket structure in the vicinity of the first wall will become so radioactive after even a short period of operation at power that the equivalent of ~30 cm of lead shielding will be required if personnel are to work within a few meters of the first wall.
- (2) Design for high reliability to minimize the need for unscheduled maintenance is vitally important.
- (3) The blanket and magnet shield region can and should be designed to permit contact maintenance of wiring, etc. around the outside of the torus after shutdown in emergencies. (This will require either space suits or pressurization of the reactor cell.)
- (4) Local gamma shielding will be required around injectors and instrumentation components that will "see" the plasma through beam holes.
- (5) Any operations involving removal and replacement of an element of the blanket structure must be carried out with remote handling equipment.
- (6) Fission reactor experience indicates that it is reasonable to design and build remote handling equipment for operations that can be foreseen; it is the unforeseen operations that lead to long, expensive delays.
- (7) Remote handling processes in the reactor cell must be simple; complex processes should be carried out in separate hot cells. (Carrying out detailed maintenance in hot cells will keep the reactor producing electricity.)
- (8) Where topology is the same, a complex series of operations can be programmed. (The major problem will be coping with deviations from ideality, i.e., tolerances.) (Use of the emissions from the radioactive parts may provide a basis for recognizing deviations and taking corrective action.) (Examples of currently automated production operations are the laser tailor and automotive engine cylinder blocks.)
- (9) Reactor components should be designed so that they will not require cooling during maintenance operations.

- (10) Massive segments of the torus can be moved on air cushions. The handling and automatic welding procedures for 50-ton sections of pressure vessel may be applicable (also steel forging furnace handler).
- (11) Automatic welding equipment is widely used in shop fabrication work. The Culham tokamak liner and TVA piping have been welded in the field with automatic welding equipment. The equipment used is readily applicable to remote operation. The key point is accurate positioning of the parts and the welding equipment. Inspection will be a major problem.
- (12) The number of disconnects for repair and access should be minimized. This should also be advantageous from the standpoints of simplicity and reliability.
- (13) As much of the blanket as possible should be a drainable liquid.
- (14) Scrap pieces to be removed from hot cells should fit into a small shielded volume — probably after crushing.
- (15) Cutting of radioactive parts should be done by shearing rather than with abrasive wheels to avoid generating radioactive dust.
- (16) Stored energy in the toroidal field can be dissipated and restored in ~ 2 hr.
- (17) The heat capacity of the toroidal field magnets and their structure is so large that the cooldown time for the entire reactor is ~ 30 days. Thus magnets should be kept as cool as possible even in the region being maintained.
- (18) One ought to avoid replacing any more of the blanket than necessary at each service period.
- (19) The reactor hall should be at a low pressure, ~ 1 - 10 torr, to control tritium leakage and to help meet cleanliness requirements.
- (20) Service work cells should be under vacuum for tritium control and cleanliness; vacuum technology will be commonplace in fusion.
- (21) Condenser panels in the main reactor hall would help keep the air clean. The

temperature the reactor blanket will assume on shutdown will depend on the design. A clean atmosphere will also help keep the blanket structure clean.

REFERENCES

- 1. A. P. Fraas, *Comparative Study of the More Promising Conversion Systems, and Tritium Recovery and Containment Systems for Fusion Reactors*, ORNL/TM-4999, Oak Ridge National Laboratory, Oak Ridge, Tennessee (November 1975).
- 2. D. Steiner (ORNL), personal communication of data from his calculation book F RT-III, 1976.
- 3. R. T. Santoro, V. C. Baker, and J. M. Barnes, *Neutronics and Photonics Calculations for the Tokamak Experimental Power Reactor*, ORNL/TM-5466, Oak Ridge National Laboratory, Oak Ridge, Tennessee (March 1977).
- 4. D. Steiner, "The Technology Requirements for Power by Fusion," in *Nuclear Science and Engineering*, Vol. 58, No. 2, p. 107 (October 1975).
- 5. W. G. Price, Jr. and D. L. Chapin, *Neutron Wall Load Distributions in a Circular Cross Section Tokamak*, MATT-1102, Princeton University, Princeton, New Jersey (January 1975).
- 6. W. Danner, "Neutron Flux Asymmetry in Toroidal Geometries," *Proc. 7th Symp. on Fusion Technology*, Commission of the European Communities, Luxembourg, EUR 4938e, pp. 107-112 (1972).
- 7. John F. Clarke (Director, Fusion Energy Division, ORNL), private communication, September 21, 1976.
- 8. R. W. Werner et al., *Progress Report No. 2 on the Design Considerations for a Low Power Experimental Mirror Fusion Reactor*, UCRL-74054-2, Lawrence Livermore Laboratory, Livermore, California (1974).
- 9. R. W. Werner, *Materials Problems in Steady State Fusion Reactors with Direct Conversion*, UCRL-75336, Lawrence Livermore Laboratory, Livermore, California (1974).

10. *An Engineering Design Study of a Reference Theta-Pinch Reactor (RTPR)*, LA-5336, Joint Report by Los Alamos Scientific Laboratory, Los Alamos, New Mexico, and Argonne National Laboratory, Argonne, Illinois (1974).
11. J. Williams et al., "A Conceptual Laser-Controlled Thermonuclear Reactor Power Plant," *Proc. 1st Topical Meeting on the Technology of Controlled Nuclear Fusion*, CONF-740402, Vol. 1, p. 70 (1974).
12. G. L. Kulcinski and R. W. Conn, "The Conceptual Design of a Tokamak Fusion Power Reactor, UMWAC-1," *Proc. 1st Topical Meeting on the Technology of Controlled Nuclear Fusion*, CONF-740402, Vol. 1, p. 165 (1974).
13. G. L. Kulcinski and G. A. Emmert, *Surface Effects in Controlled Fusion*, p. 31, ed. H. Wiedersich et al., North-Holland Publishing Co., Amsterdam, 1974.
14. Peter Bonanos (PPPL), private communication, October 14, 1975.
15. Denis Breton (Commissariat a L'Energie Atomique, France), private communication, December 16, 1975.
16. H. K. Perkins, "Tritium Holdup Due to Coatings on the First Wall of Fusion Reactors," *Proc. Intl. Conf. on Radiation Effects and Tritium Technology for Fusion Reactors*, CONF 750909, Vol. 4, p. 379 (1975).
17. R. J. Colchin et al., "Impurity of Surface Studies in ORMAK," paper presented at the International Surface Effects Conference, San Diego, California, February 16-20, 1976.
18. L. F. Coffin, Jr., "A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal," in *ASME Transactions*, Vol. 76 (1974).
19. J. J. Keyes and A. I. Krakoviak, "High Frequency Surface Thermal Fatigue Cycling of Inconel at 1406°F," in *Nuclear Science and Engineering*, Vol. 9, No. 4 (April 1961).
20. John Marshal (LASL), private communication, December 18, 1975.
21. P. H. Rebut et al., "Plasma Wall Interactions in the TFR Machine," *Journal of Nuclear Materials*, Vol. 53, pp. 16-24, North-Holland Publishing Co., Amsterdam (September 1974).
22. P. Brossier, "On the Plasma-Wall Interactions in the TFR," paper presented at the 7th European Conference on the Tokamak Program, Lausanne, Switzerland, September 1975.
23. E. Hinnov, "Description of Tokamak Discharges in Present and Future Tokamak Devices at Princeton PPL," *Journal of Nuclear Materials*, Vol. 53, pp. 9-15, North-Holland Publishing Co., Amsterdam (September 1974).
24. R. E. Peterson, *Stress Concentration Design Factors*, John Wiley & Sons, Inc., New York, 1953.
25. L. F. Coffin, Jr., "Fatigue at High Temperature," *The Institution of Mechanical Engineers Proceedings*, Vol. 188, pp. 109-127, London (September 1974).
26. Max Jakob, *Heat Transfer*, p. 256, John Wiley & Sons, Inc., New York, 1950.
27. Unpublished design study for an EPR, August 1975.
28. R. G. Mills, ed., *A Fusion Power Plant*, MATT-1050, Princeton Plasma Physics Laboratory, Princeton, New Jersey (August 1974).
29. A. P. Fraas, "Problems of Coupling a Gas Turbine to a Thermonuclear Reactor," ASME Paper 72-GT-98, presented at the 1972 Annual Gas Turbine Meeting, March 1972.
30. A. P. Fraas, *Conceptual Design of the Blanket and Shield Region and Related Systems for a Full-Scale Toroidal Fusion Reactor*, ORNL/TM-3096, Oak Ridge National Laboratory, Oak Ridge, Tennessee (May 1973).
31. S. Timoshenko and Woinowsky Krieger, *Theory of Plates and Shells*, p. 497, McGraw-Hill, 1959.
32. S. Timoshenko, *Strength of Materials*, Part 2, Ch. 1, D. van Nostrand, 1956.
33. J.T.D. Mitchell and J. A. Booth, "Wall Loading Limitations in a Helium-Cooled Fusion Reactor Blanket," CLM-R126, UKAEA Research Group, Culham Laboratory (Paper 13 of Workshop of Fusion Reactor Design Problems, January 29-February 15, 1974).
34. K. Sake, H. Yamato, M. Ohta, Y. Seki, K. Tanaka, N. Asami, S. Matsuda, Y. Ohkubo, T. Takeda, and S. Mori, *Design Study of a*

- Tokamak Reactor*, IAEA-CM-33/G1-5, Japan Atomic Energy Research Institute, Tokai, Ibaraki, Japan.
35. S. Forster, "Closed Helium-Turbine Cycle with a Fusion Reactor," *Mechanical Engineering*, Vol. 95, No. 8, pp. 13-18, (August 1973).
 36. A. P. Fraas, *Analysis of a Recirculating Lithium Blanket Design to Give a Low Magnetohydrodynamic Pumping Power Requirement*, ORNL/TM-3756, Oak Ridge National Laboratory, Oak Ridge, Tennessee (September 1973).
 37. S. J. Leverette, *An Investigation of the Temperature Distribution in the Lithium Blanket of a Fusion Reactor*, ORNL/TM-3701, Oak Ridge National Laboratory, Oak Ridge, Tennessee (May 1972).
 38. A. P. Fraas, D. B. Lloyd, and R. E. MacPherson, *Effects of a Strong Magnetic Field on Boiling of Potassium*, ORNL/TM-4218, Oak Ridge National Laboratory, Oak Ridge, Tennessee (February 1974).
 39. F. J. Young, R. S. Holcomb, and A. P. Fraas, "Magnetohydrodynamics Test of a One-Sixth Scale Model of a CTR Recirculating Lithium Blanket," paper prepared for presentation at the 1975 Annual Meeting of the American Nuclear Society, New Orleans, Louisiana, June 8-13, 1975.
 40. R. A. Krakowski, R. L. Hagenson, and G. E. Cort, *First-Wall Thermal/Mechanical Analysis of the Reference Theta-Pinch Reactor (RTPR)*, LA-UR-76-1225, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (June 1976).
 41. J.C.R. Hunt and R. Hancox, *The Use of Liquid Lithium as Coolant in a Toroidal Fusion Reactor, Parts I and II*, CLM-R115, Culham Laboratory, Abingdon, United Kingdom (1971).
 42. W. M. Wells, *Magnetohydrodynamic Problems and Considerations*, to be published as an ORNL/TM.
 43. F. J. Young, "Magnetohydrodynamic Blanket Scaling in a Toroidal Fusion Reactor," paper presented at the Topical Meeting on Technology on Controlled Nuclear Fusion sponsored by the TGCNF, the Power Division, San Diego Section of ANS and USAEC, San Diego, California, April 16-18, 1974.
 44. M. A. Hoffman and G. A. Carlson, *Calculation Techniques for Estimating the Pressure Losses for Conducting Fluid Flows in Magnetic Fields*, UCRL-51010, Lawrence Livermore Laboratory, Livermore, California (February 1971).
 45. J. A. Houck and E. S. Barlett, *The Roll-Diffusion Bonding of Structural Shapes and Panels*, DMIC Report S-17, Defense Metals Information Center, Battelle Memorial Institute, Columbus, Ohio (October 1967).
 46. *Refractory Metals Fabrication Technology as Applied to Fusion Reactors*, BNWL-2053, Pacific Northwest Laboratories, Richland, Washington (July 1976).
 47. A. P. Fraas and J. J. Tudor, *Mechanical Engineering Design Considerations for a Tokamak Scientific Feasibility Experiment and Plasma Test Reactor*, ORNL/TM-4371, Oak Ridge National Laboratory, Oak Ridge, Tennessee (November 1973).
 48. J.T.D. Mitchell, "A Tokamak Reactor with Servicing Capability," paper FTSG (76)89, presented at the 9th Symposium on Fusion Technology, Garmisch-Partenkirchen, FRG, June 14-18, 1976.
 49. "Movable Grandstands," in *Compressed Air Magazine* (December 1976).
 50. J.T.D. Mitchell (Culham Laboratory, UKAEA), letter to A. P. Fraas, September 1976.

Fig. 1. Photograph of damage to an ORMAK limiter made of tungsten laminations, each of which is 3 mm thick.

Fig. 2. Close-up of the damaged area at the center of Fig. 1.

Fig. 3. Tokamak movable limiter top blade from PPPL-ST — edge view. Installed 8/73, removed 10/73. Limiter material — molybdenum. Job #N670-546. Photograph taken February 27, 1974.

Fig. 4. Tokamak movable limiter from PPPL-ST, top blade looking south. Job #N690-546. Photograph taken June 6, 1974.

Fig. 5. Adjustable plasma limiter from PPPL-ST tokamak. Close-up side 1. Photograph taken June 26, 1970.

Fig. 6. Adjustable plasma limiter from PPPL-ST tokamak. Close-up side 2. Photograph taken June 26, 1970.

MAJOR AXIS

LIMITER RADIUS = 17 CM

Fig. 7. Tungsten blocks (No. 1 and 2) of a TFR limiter eroded by the runaway electrons. The blocks were in the outer part of the limiter.

Fig. 8. The same view as Fig. 7 but with tungsten blocks No. 2 and 3 of the TFR.

Fig. 9. The outer part of the TFR molybdenum limiter eroded by runaway electrons.

Fig. 10. Hole in the Inconel vacuum wall of TFR (scale x 3).

Fig. 11. Molybdenum droplets emitted by the limiter under the runaway electron bombardment and deposited on the wall of TFR.

Fig. 12. Relation between types of surface damage and pulse times for typical cases.

Fig. 13. Photograph of the interior surface of an Inconel tube subjected to thermal strain cycling at 600°C by a temperature variation of 60°C in a stream of molten salt. The number of cycles to failure was $\sim 10^5$ (Ref. 19).

Fig. 14. Correlation of calculated stress, frequency, and exposure time for thermal strain cycling of Inconel at 763°C (1405°F) in a fused salt environment: NaF-ZrF₄-UF₄ (56-30-5 mole %) (Ref. 19).

Fig. 15. Energy input per pulse for typical cases.

Fig. 16. Photograph of a 4-mm-thick, water-cooled copper target employed in ORNL tests of neutral beam injectors. The energy per pulse was typically 700 J/cm^2 with pulse durations of 0.05 sec. The diameter of the melted zone in the center is $\sim 1.5 \text{ cm}$, while that of the heavily fissured region around it is $\sim 3 \text{ cm}$.

Fig. 17. Range of electron penetration in typical structural materials as a function of the energy of the incident electrons. (Data plotted from Berger and Seltzer, "Tables of Energy Losses and Ranges of Electrons and Positrons," NASA SP-3012, 1964.)

Fig. 18. The stress concentration factor K for a hole in a plate for each of three typical plane stress situations.

Fig. 19. Relation between the number of cycles to failure under thermal strain cycling conditions and the pulse energy parameter for seven typical materials as estimated from Eq. (A.3) of Appendix A. The pulse energy parameter on the ordinate, i.e., the unit energy input, Q , divided by the square root of the pulse period, τ , can be related to the surface temperature excursion by the constant K_3 which is tabulated above.

Fig. 20. Transient temperature distribution in a thick slab (latent heat of fusion neglected).

Fig. 21. Relation between energy input per pulse and both the surface temperature rise and the thermal stress in an ideal elastic material for a series of pulse durations. The table at the top gives the constants to be used to obtain values for typical metals.

Fig. 22. Generalized curve for the depth of penetration of melting as a function of the parameter ϕ .

Fig. 23. Temperature distribution in a thick slab with steady-state melting and vaporization.

Fig. 24. Depths of penetration of melting for stainless steel and niobium.

Fig. 25. Relation between the thickness of and temperature drop across a stainless steel first wall for a series of wall power loadings at 600°C (1112°F) with 15% of the total energy deposited in the surface of the first wall.

Fig. 26. Section through a segment of a helium-cooled static lithium blanket with a bank of tubes protecting the main structure (ORNL design of Ref. 29).

SECTION B-B

Fig. 27. Coolant tubes in front of and in the blanket for the Princeton fusion reactor design of Ref. 28.

Fig. 28. Circumferential distribution of the energy flux from the plasma to a circular tubular element of the first wall and from the tube wall to the helium coolant as assumed for a first approximation for an analytical solution of the thermal stress.

Fig. 29. Section through a typical layout for a wedge-shaped ring type of blanket module showing details of the cooling passages and structure. The helium cooling passages follow the first wall for 180° around the minor circumference, then loop back and follow the graphite region so that they exit through a manifold near the inlet (Ref. 27).

Fig. 30. Schematic diagram showing normal and thermally distorted geometries for a simple cylindrical vessel with a thick flat head.

Fig. 31. Schematic diagram of the blanket structure of Fig. 29 showing the distortion, δ , associated with a difference in temperature between two concentric cylindrical shells joined to a common set of heads.

Fig. 32. Wedge-shaped ring type of blanket structural module.

Fig. 33. Effects of helium mass flow rate on the pumping power-to-heat removal ratio (P/H), the tube length, the heat transfer coefficient, and the gas-to-metal temperature difference for a series of helium temperature rises in 1-in.-OD, 0.8-in.-ID tubes with a vacuum wall power loading of 1 MW/m², of which 160 kW/m² would go into the first millimeter of wall. If the power loading were doubled and the tube length halved, the helium temperature rise, mass flow rate, and heat transfer coefficient would be the same, the pumping power-to-heat removal ratio would be halved, and the gas-to-metal temperature difference would be doubled.

Fig. 34. Section through the EPR blanket design of Fig. 29 indicating the deflected form of the first wall as a consequence of thermal distortion when heated during a burn.

Fig. 35. Temperature distribution around the circumference of the blanket structure for the EPR design of Fig. 29.

Fig. 36. Distorting effect of the circumferential temperature distribution during a burn for the EPR blanket structure of Figs. 29 & 35.

Fig. 37. Effects of power density and coolant passage spacing on the difference between the peak temperature in the static lithium and the coolant tube wall temperature for 2.54-cm-OD tubes on equilateral triangular centers.

Fig. 38. Temperature distribution in a uniformly heated flat plate with discrete parallel cooling passages.

Fig. 39. Effects of local power density in the blanket on the cooling passage centerline spacing both for a series of typical heat inputs per unit of coolant passage length assuming a cooling passage array equivalent to that given by an equilateral triangular spacing and for two series of temperature differences within the lithium for 2.54-cm-OD tubes.

Fig. 40. Blanket configuration designed to provide lithium circulation by means of electromagnetic pumps at the ends of sextants of a toroidal blanket.

Fig. 41. Effects of lithium region thickness on major design parameters for a 1-m-thick blanket employing the racetrack configuration of Fig. 40. (Breeding ratio and heat generation data were supplied by D. Steiner.)

Fig. 42. Radial temperature distribution for a 30-cm lithium passage and a 60-sec. lithium transit time around a "racetrack" similar to that for the blanket design of Fig. 40 but with all of the cooling surface in only two planes, i.e., the outer surface of the "racetrack" and the heat sink plane indicated above. The potassium boiler of Ref. 30 would have a nearly uniformly distributed heat sink in the outer leg of the "racetrack."

Fig. 43. Toroidal field strength as a function of radial distance from the plasma centerline in a plane midway between toroidal field coils for a coil bore of 10 m and a torus major radius of 12 m.

Fig. 44. Layout proposed for a low MHD pressure drop in a potassium feed pipe system.

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Fig. 45. ORNL conceptual design of a 1000-MW(t) fusion reactor with a recirculating lithium blanket cooled by boiling potassium.

APPENDIX A

DERIVATION OF ANALYTICAL EXPRESSIONS FOR
THERMAL STRAIN CYCLING EFFECTS IN THE FIRST WALLEMPIRICAL CORRELATION OF TEST DATA
ON LOW CYCLE FATIGUE

A recent paper by Coffin²⁵ reviews the experimental evidence concerning the fatigue of materials at high temperature and correlates the data with the equations,

$$\Delta \epsilon_p v^{\beta(k-1)} = C_2 N_f^{-\beta} \quad (A.1)$$

and

$$\Delta \epsilon_e v^{-k_1} = \frac{A'}{E} N_f^{-\beta'}, \quad (A.2)$$

where $\Delta \epsilon_p$ is the range of plastic strain (the sum of the negative and positive amounts by which the strain goes into the plastic region during a cycle) and $\Delta \epsilon_e$ is the range of the elastic strain (positive and negative). N_f is the number of cycles to failure under the conditions imposed and E is the modulus of elasticity. A' , C_2 , k_1 , k_1' , β , and β' are experimentally determined quantities for each material, and v is the frequency with which the stress application is repeated.

The total range of the strain, $\Delta \epsilon$, is the sum of the plastic and elastic ranges, or

$$\Delta \epsilon = \Delta \epsilon_p + \Delta \epsilon_e$$

On substitution of the expressions from Eqs. (A.1) and (A.2),

$$\Delta \epsilon = C_2 \frac{N_f^{-\beta}}{v^{\beta(k-1)}} + \frac{A'}{E} \frac{N_f^{-\beta'}}{v^{-k_1'}} \quad (A.3)$$

When a thin layer of the surface of a material is heated suddenly by an amount, $T_s - T_0$, the resulting thermal strain is found by the methods of the theory of elasticity to be

$$\Delta \epsilon = \frac{1+\mu}{1-\mu} \alpha (T_s - T_0) \quad (A.4)$$

The thermal stress which accompanies this strain in an ideal elastic material is

$$\sigma = \frac{E \alpha (T_s - T_0)}{1 - \mu} \quad (A.5)$$

The number of cycles of applied surface energy input required to cause failure as indicated by the above assumptions and Table 2 is shown in Fig. 19 in the main body of the report.

RELATION BETWEEN THERMAL STRESS AND PULSE
MAGNITUDE AND DURATION

In Appendix B [see Eq. (B.24)] it is shown that the rise in the temperature, $T_s - T_0$, in an infinite slab whose surface is subjected to a pulse of radiant energy per unit area, Q , during a time period, τ , is given by

$$T_s - T_0 = \frac{2 Q}{\sqrt{\pi} c_p \rho k \tau} \quad (A.6)$$

Where $c_p \rho$ is the heat capacity per unit volume and k is the thermal conductivity.

Combining of Eq. (A.5) and (A.6) to eliminate the temperature difference and to obtain the stress in an ideal elastic material gives

$$\sigma = \frac{2 E \alpha Q}{(1 - \mu) \sqrt{\pi} c_p \rho k \tau} \quad (A.7)$$

Equations (A.3), (A.4), and (A.6) can be combined to eliminate $T_s - T_0$ and $\Delta \epsilon$, giving the surface energy input per unit of area, Q , expended during a period, τ , that would be expected to cause a thermal fatigue failure at the end of N_f cycles, i.e.,

$$Q = \frac{(1 - \mu) \sqrt{\pi} c_p \rho k}{2 (1 + \mu) \alpha} C_2 \frac{N_f^{\beta}}{v^{\beta(k-1)}} + \frac{A'}{E} \frac{N_f^{-\beta'}}{v^{-k_1'}} \sqrt{\tau} \quad (A.8)$$

In order to apply Eq. (A.8) to the prediction of the capacity of a given material to withstand a pulse of energy into its surface, it is necessary to assign values (preferably from experiments) to the constants β , β' , C , A' , k , and k_1' . For most materials of interest here the values of these constants are not well known. For our present purposes the following assumptions are made:

- (1) $\beta = 0.5$ and $\beta' = 0$. For many materials that have been tested, values of β are in the region $0.45 < \beta < 0.6$, and β' is small, so that for a first approximation it can be neglected.
- (2) $k = 1$ and $k' = 0$. Under most conditions of failure in thermal fatigue, the frequency effect appears to be small. The assumptions above drop the frequency effect altogether.
- (3) The constant, A' , is taken to be equal to the yield strength, σ_y .
- (4) The constant, C , is related to the ductility of the material, which of course depends on the temperature and the rapidity of the deformation. Its value here is determined from Eq. (A.1), with the estimated true strain, $\Delta\epsilon_p = \Delta\epsilon_1/4$, corresponding to that of a conventional tensile pull test ($N_f = 1/4$), so that $C = \Delta\epsilon_1/4/2$.

Equation (A.8) can now be written:

$$Q = \left(\frac{C_1}{N_f^{0.5}} + C_2 \right) \sqrt{\tau} \quad , \quad (A.9)$$

where

$$C_1 = \frac{(1 - \mu) \sqrt{\pi} c_p \rho k \Delta\epsilon_1/4}{4 (1 + \mu) \alpha} \quad (A.10)$$

and

$$C_2 = \frac{(1 - \mu) \sqrt{\pi} c_p \rho k \sigma_y}{2 (1 + \mu) \alpha t} \quad (A.11)$$

The values used for the properties of materials and for C_1 and C_2 are shown in Table 2 in the main body of the report.

The surface temperatures and the stresses in an ideal elastic material as calculated from Eqs. (A.6) and (A.7) are shown in Fig. 19 in the main body of the report.

APPENDIX B

DERIVATION OF TEMPERATURE DISTRIBUTIONS
IN THE FIRST WALL FOR TYPICAL CASES

BASES FOR THE ANALYSIS

A burst of energy of the amount, Q , is considered arriving at a unit area of the surface during a time interval, τ . The instantaneous rate of arrival is then $dQ/d\tau$. It is necessary to find the temperature distribution which results from this energy burst and the penetration of the material below the surface.

Let us first consider the temperature, T , in a thick slab of material originally with a uniform temperature, T_0 , whose face is heated by radiant energy at an arbitrary rate starting at a time, $t = 0$. After some time, part of the slab has been heated to temperatures above the melting point, T_m , and the heated face to the vaporization temperature, T_v . As a result of the heating process, a moving boundary is formed between the liquid and solid regions and the receding face of the slab is at the vaporization temperature.

In either the region of melted material or in that of the solid, the distribution of temperature is adequately described for most purposes by the linear partial differential equation,

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = c_p \rho \frac{\partial T}{\partial t}, \quad (\text{B.1})$$

with appropriate local values for the thermal conductivity, k , the specific heat, c_p , and the density, ρ .^{*} It is the provision for the moving boundary between the solid and liquid regions

^{*} If an internal source of energy, S , per unit time and per unit volume exists, Eq. (B.1) becomes

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + S = c_p \rho \frac{\partial T}{\partial t},$$

where S may be a function of both time and location. For the regime of interest here, this additional source term for neutron and gamma heating is sufficiently small that it has only a second order effect. For simplicity no source term is considered here. Its inclusion in a numerical procedure causes relatively little difficulty.

which considerably complicates the solution of the problem, primarily because the equations are rendered nonlinear by this discontinuity.

The boundary between the liquid region (designated region l), and the solid region (designated region s) moves at a rate designated by ds/dt . The conditions at the boundary, $x = b$, are then described by

$$T_l \Big|_b = T_s \Big|_b \quad (\text{B.2})$$

and

$$-k_l \frac{\partial T_l}{\partial x} \Big|_b + k_s \frac{\partial T_s}{\partial x} \Big|_b = L_m \rho \frac{ds}{dt}, \quad (\text{B.3})$$

where L_m represents the latent heat of melting.

At the heated face, if vaporization is not occurring, the slope of the temperature curve is required to match the energy input according to^{*}

$$q + k \frac{\partial T}{\partial x} \Big|_{x=0} = 0, \quad (\text{B.4})$$

where q is the rate of energy input per unit area and per unit time. The conductivity takes on the value $k = k_l$ if the material at the face is liquid, and $k = k_s$ if the material at the face is solid. If vaporization is occurring at the face, which is then moving at a position $x = a$, the boundary condition becomes

$$q + k_l \frac{\partial T_l}{\partial x} \Big|_a = L_v \rho \frac{da}{dt}, \quad (\text{B.5})$$

where L_v is the latent heat of vaporization and da/dt is the velocity with which the face is receding as it vaporizes. For a slab of thickness, t , insulated at the far boundary, $x = t$, the boundary condition to be met is

$$\frac{\partial T_s}{\partial x} \Big|_{x=t} = 0. \quad (\text{B.6})$$

^{*} If the rate of energy input is constant, then q can be expressed by

$$q = \frac{Q}{\tau},$$

where Q is the total energy per unit area impinging on the surface during the time period, τ . If the rate of energy input is not constant, then

$$q = \frac{dQ}{dt}.$$

NONDIMENSIONAL EQUATIONS

Because of the nonlinear nature of the problem, a change in any condition generally requires an entirely new solution because superposition is not applicable. It is therefore particularly desirable to formulate the equations in such a way that any possible generalizations which can be made from a particular solution are apparent. For this purpose the equations are expressed in terms of dimensionless ratios of the physical parameters. In the following analysis, ξ , η , and W are dimensionless variables of distance, time, and temperature, respectively, defined by

$$d\xi = \frac{Q}{k_s (\bar{T}_m - T_0) \tau} dx, \quad (B.7)$$

$$d\eta = \left[\frac{Q}{k_s (\bar{T}_m - T_0) \tau} \right]^2 \gamma_s dt, \quad (B.8)$$

and

$$W = \frac{T - T_0}{\bar{T}_m - T_0}, \quad (B.9)$$

where γ is the thermal diffusivity, defined by

$$\gamma = \frac{k}{c_p \rho}. \quad (B.10)$$

Here γ takes on the value, γ_l , in the liquid region and γ_s in the solid. When these variables are substituted into Eq. (B.1) the result, assuming that no variables depend explicitly on the time, is

$$\frac{\partial^2 W}{\partial \xi^2} = \frac{\partial W}{\partial \eta}. \quad (B.11)$$

In the solid region, $k = k_s$. If the rate of energy input, q , is constant, $\tau q/Q = 1$, and Eq. (B.12) for the solid region becomes

$$\frac{\partial W}{\partial \xi} = -1.$$

The boundary condition of Eq. (B.4) becomes

$$\frac{\partial W}{\partial \xi} \Big|_0 = -\frac{k_s}{k}. \quad (B.12)$$

Equation (B.2) is just

$$W_l \Big|_b = W_s \Big|_b. \quad (B.13)$$

Equation (B.3) becomes

$$\frac{k_l}{k_s} \frac{\partial W_l}{\partial \xi} \Big|_b + \frac{\partial W_s}{\partial \xi} \Big|_b = \lambda_m \frac{ds'}{d\eta}, \quad (B.14)$$

where λ_m is defined by

$$\lambda_m = \frac{L_m}{c_p (\bar{T}_m - T_0)}. \quad (B.15)$$

In harmony with the definition of Eq. (B.7), the dimensionless variable, s' , is given by

$$ds' = \frac{Q}{k_s (\bar{T}_m - T_0) \tau} ds. \quad (B.16)$$

Equation (B.5) becomes*

$$1 + \frac{k_l}{k_s} \frac{\partial W_l}{\partial \xi} \Big|_a = \lambda_v \frac{ds''}{d\eta}, \quad (B.17)$$

where s'' is the distance moved by the vaporizing boundary at $x = a$ in the dimensionless form of Eq. (B.17), and

$$\lambda_v = \frac{L_v}{c_p (\bar{T}_m - T_0)}. \quad (B.18)$$

Equations (B.11) through (B.18) are a system which includes a partial differential equation of the second order and the required boundary conditions for the external and internal boundaries, both stationary and moving, to describe the temperature distribution in a material in which both melting and vaporization are occurring. There are three physical parameters which appear specifically in these equations, namely k_s/k_l , λ_m , and λ_v . If we were so fortunate that these parameters had the same values for all materials, any solution to the equations would apply equally to all materials, with any rate of energy input. We

*Again, for a constant rate of energy input during the time period, τ , the quantity $\tau q/Q = 1$, so that Eq. (B.17) takes the simple form shown.

are not that fortunate, but it is shown in the next section that values of these parameters are surprisingly similar for a wide variety of materials.

DISCUSSION OF THE PARAMETERS λ_m AND λ_v , AND THE RATIO k_ℓ/k_s

The parameter

$$\lambda_m = \frac{L_m}{c_p (T_m - T_0)} = \frac{L_m}{c_p T_m} \left(\frac{T_m}{T_m - T_0} \right)$$

is a dimensionless ratio of the latent heat of melting to the sensible heat required to bring the temperature of the material to the melting point from its initial temperature, T_0 . The parameter

$$\lambda_v = \frac{L_v}{c_p (T_m - T_0)} = \frac{L_v}{c_p T_v} \frac{T_v}{T_v - T_0} \frac{T_v - T_0}{T_m - T_0}$$

is a dimensionless ratio of the latent heat of vaporization to the sensible heat required to bring the temperature of the material to the melting temperature. Of particular interest in these expressions are the terms $L_m/c_p T_m$ and $L_v/c_p T_v$, where T_v is the temperature of evaporation of the material and the ratio of temperatures, $(T_v - T_0)/(T_m - T_0)$. Let us consider these terms for high temperature materials, for which the initial temperature, T_0 , may be small compared with T_m and T_v . Then,

$$\frac{T_m}{T_m - T_0} = \frac{T_v}{T_v - T_0} = 1$$

and

$$\frac{T_v - T_0}{T_m - T_0} = \frac{T_v}{T_m} \quad (B.19)$$

There are reasons, both theoretical and experimental, for expecting each of the parameters, λ_m and λ_v , to have values for a wide range of materials which are not too different.

For a wide range of materials, the latent heat of melting is around 3 or 4% of the latent heat of evaporation, and the latent heat of evaporation coincides quite well with the prediction of Trouton's rule: namely,

$$s_v = \frac{L_v}{T_v} = 21 \frac{\text{cal}}{\text{mol} \cdot ^\circ\text{C}} = 21 \frac{\text{Btu}}{\text{mol} \cdot ^\circ\text{F}},$$

where s_v is the latent entropy of evaporation. Also for a wide variety of materials, the melting temperature is of the order of one-half the temperature of evaporation. Therefore we would expect the latent entropy of melting to be of the order of

$$s_m = \frac{L_m}{T_m} = 2.5 \frac{\text{cal}}{\text{mol} \cdot ^\circ\text{C}} = 2.5 \frac{\text{Btu}}{\text{mol} \cdot ^\circ\text{F}}.$$

Finally, the Debye theory predicts that the specific heat at constant volume of simple solids approaches at high temperature the value $c_v = 3R$, where R is the universal gas constant ($R = 1.987 \text{ cal/mol} \cdot ^\circ\text{C} = 1.987 \text{ Btu/mol} \cdot ^\circ\text{F}$). For ordinary solids or liquids under conditions, c_p and c_v have essentially the same value, or

$$c_p = 6 \frac{\text{cal}}{\text{mol} \cdot ^\circ\text{C}} = 6 \frac{\text{Btu}}{\text{mol} \cdot ^\circ\text{F}}.$$

The values to be expected for λ_m and λ_v on this basis are

$$\lambda_m = 0.417$$

and

$$\lambda_v = 7.0.$$

Values for λ_m , λ_v , and T_v/T_m , calculated from data available in the literature on melting and vaporization, are shown in Table B.1. The reliability of the values shown for λ_v is probably rather poor. The data on the temperature and latent heat of vaporization for high temperature materials such as tungsten and molybdenum vary considerably among sources, and data for niobium appear to be unavailable.

There appears to be some merit in carrying out an analysis of the problem of heating with melting and vaporization using the derived values for $\lambda_m = 0.417$ and $\lambda_v = 7.0$ as previously discussed. To the extent that these values are pertinent to various materials, their use enables one to make a generalized study whose results are applicable to all of those materials. That is

Table B.1. Material Properties for $T_0 = 0$

Material	λ_m	λ_v	T_v/T_m
Magnesium	0.280	8.2	1.49
Aluminum	0.455	7.6	2.50
Chromium	0.358	9.0	1.51
Manganese	0.385	8.0	1.60
Iron	0.328	6.4	1.81
Cobalt	0.347	9.0	1.81
Nickel	0.407	5.6	1.84
Copper	0.382	8.8	2.12
Niobium	0.394		
Molybdenum	0.371	10.4	1.75
Silver	0.365	8.6	1.84
Tungsten	0.355	11.0	1.70
Platinum	0.438	8.6	2.28
Gold	0.378	9.0	2.42
Sodium fluoride (NaF)	0.619		
Sodium chloride (NaCl)	0.562		
Lithium nitrate (LiNO_3)	0.428		
Potassium dichromate ($\text{K}_2\text{Cr}_2\text{O}_7$)	0.249		
p-Aminobenzoic acid [$\text{C}_6\text{H}_4(\text{H}_2)\text{COOH}$]	0.276		

applicable to all of those materials. That is the basis of the analysis carried out in the rest of the report. The solution for the temperature distribution and the rate of progress of melting and vaporization is carried out for several situations for the radiant heating of the surface of an infinitely thick slab by an amount of energy per unit area, Q , applied at a uniform rate during a time interval, τ .

The following situations are considered:

- (1) The latent heats of melting and vaporization are taken to negligible, and it is assumed that no material disappears from the surface. For this situation an analytical solution is available in terms of error functions.
- (2) It is assumed that the process of heating has gone on for a long enough time that melting, or melting and vaporization, is

progressing at a uniform rate. For this situation also an analytical solution exists.

- (3) A numerical solution can be obtained for the transient heating of the surface, taking into account the latent heats and providing for the moving boundaries which result. This solution requires the use of finite difference procedures.

The next section considers the situation where latent heats are neglected.

The thermal conductivities are generally not known for both the solid and liquid phases of the same materials near the melting point. The ratio of the thermal conductivity of the liquid phase to that of the solid phase is listed in Table B.2 for a few materials. Note that for these materials this ratio appears to have values around 0.5.

Table B.2. Ratio of the thermal conductivity of the liquid to that of the solid for some typical metals

Material	Ratio of Conductivities, k_l/k_s
Mercury	0.33
Lead	0.54
Zinc	0.52
Sodium	0.62
Potassium	0.45
Aluminum	0.39

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LATENT HEATS NEGLECTED

Jakob²⁶ has presented the solution to the problem of the temperature distribution in an infinitely thick plate of uniform properties, originally at a uniform temperature, whose surface is subjected to the sudden start of heating at a uniform rate, i.e.,

$$T - T_0 = 2 \frac{Q \sqrt{\gamma t} u}{k \tau} \left(\frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du + \frac{e^{-u^2}}{\pi u} - 1 \right), \quad (\text{B.20})$$

where

$$u = \frac{x}{2 \sqrt{\gamma t}} \quad (\text{B.21})$$

and the other variables are as previously defined in this report. In terms of the dimensionless variables of Eqs. (B.7), (B.8), and (B.9), Eq. (B.20) becomes

$$\frac{W}{2 \sqrt{\eta}} = g(u), \quad (\text{B.22})$$

where

$$g(u) = u [f(u) - 1] + \frac{e^{-u^2}}{\sqrt{\pi}}$$

and

$$u = \frac{\xi}{2 \sqrt{\eta}}$$

and

$$f(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du$$

The solution of Eq. (B.22) is shown for three values of the dimensionless time, η , in tabular form in Table B.3 and plotted in Fig. 20, which is in the main body of the report.

Table B.3. Dimensionless temperature distribution parameters for a thick plate whose surface is suddenly heated at a uniform rate from an isothermal condition

u	f(u)	g(u)	$\eta = 1$		$\eta = 2$		$\eta = 3$	
			ξ	W	ξ	W	ξ	W
0.00	0.0000	0.5642	0.00	1.128	0.00	1.596	0.00	1.964
0.10	0.1125	0.4698	0.20	0.940	0.28	1.329	0.35	1.627
0.20	0.222	0.3866	0.40	0.773	0.57	1.093	0.69	1.339
0.30	0.3206	0.3142	0.60	0.620	0.85	0.809	1.04	1.088
0.40	0.4284	0.2522	0.80	0.504	1.13	0.713	1.39	0.874
0.50	0.5205	0.1996	1.00	0.399	1.41	0.565	1.73	0.691
0.60	0.6039	0.1559	1.20	0.312	1.70	0.441	2.08	0.540
0.70	0.6778	0.1201	1.40	0.240	1.98	0.340	2.43	0.416
0.80	0.7469	0.0882	1.60	0.176	2.25	0.253	2.77	0.316
1.00	0.8427	0.0503	2.00	0.101	2.83	0.142	3.46	0.174
1.20	0.9103	0.0261	2.40	0.052	3.39	0.074	4.16	0.090
1.50	0.9661	0.0086	3.00	0.017	4.24	0.024	5.20	0.030
2.00	0.9953	0.0009	4.00	0.002	5.66	0.008	6.93	0.003
3.00	1.0000	0.0001	6.00	0.000	8.49	0.000	10.39	0.000

THE DEPTH OF PENETRATION OF MELTING

For small values of the variable, ξ , Eq. (B.22) becomes approximately linear and can be written:

$$W = 2\sqrt{\frac{\eta}{\pi}} - \xi \quad (B.23)$$

If the melting temperature, $T = T_m$, occurs at a depth, $x = d$, at the end of an energy dump of duration, $t = \tau$, then from their definitions in Eqs. (B.7), (B.8), and (B.9), $W = 1$,

$$\xi = \frac{Q}{k_s (T_m - T_0) \tau} d,$$

and

$$\eta = \left[\frac{Q}{k_s (T_m - T_0)} \right]^2 \frac{Y_s}{\tau}.$$

In terms of real variables, for this case Eq. (B.23) becomes*

$$T_m - T_0 = \frac{Q}{k \tau} (2\sqrt{\frac{Y \tau}{\pi}} - d). \quad (B.24)$$

Eq. (B.24) can be arranged in the dimensionless form

$$\delta_m = \frac{2}{\sqrt{\pi}} \sqrt{\phi} - \phi, \quad (B.25)$$

where

$$\phi = \delta \tau \left[\frac{c_p \rho (T_m - T_0)}{Q} \right]^2. \quad (B.26)$$

and

$$\delta_m = d \frac{c_p \rho (T_m - T_0)}{Q}. \quad (B.27)$$

Inspection of Eq. (B.25) shows that $\delta_m = 0$ when $\phi = 0$ or when $\phi = 4/\pi = 1.2732$. A maximum exists, namely $\delta_{m \max} = 1/\pi = 0.3183$, when $\phi = 1/\pi = 0.3183$. A plot of Eq. (B.25) is shown in Fig. 22 in the main body of the report.

* If the melting temperature just reaches the surface, then $T_m = T_s$ at $d = 0$, so that in this case

$$T_m - T_0 = T_s - T_0 = \frac{2Q}{\sqrt{\pi} c_p \rho k \tau} \quad (B.24a)$$

THERMAL STRESS CAUSED BY ENERGY BURSTS

Even when melting does not occur at a transient hot spot caused by a burst of radiant energy, failure may still occur because of thermal stress from either a single cycle or repeated cycling. In this connection it is interesting to consider the thermal stress which occurs when the surface is brought just to the melting point. The thermal stress, σ , assuming that there is no stress concentration, is given by

$$\sigma = \frac{E \gamma (T_m - T_0)}{1 - \mu}. \quad (B.28)$$

The thermal stresses for incipient melting as given by Eq. (B.28) and the material properties listed are shown in Table B.4 for a representative set of materials. It appears from this table that all of the materials except possibly graphite and niobium could be expected to be in trouble from thermal stress before the onset of melting occurred.

STEADY-STATE MELTING AND VAPORIZATION

Analytically the steady-state melting and vaporization of an infinite slab of material on whose surface radiant energy is impinging must be considered in parts. First, the temperature of an element of the material must be raised to the melting point. That part of the material which has reached the melting temperature must then be treated differently from the material whose temperature is still below the melting point, from which it is separated by a moving boundary. When the surface of the slab reaches the temperature of evaporation, a second moving boundary is formed at the face which is disappearing as it vaporizes. In the steady-state process the two boundaries must move at the same rate.

In this section three possible categories of behavior are considered for the material whose temperature is above the melting point.

- (1) It becomes a liquid, having passed through the phase change, and is sloughed off the

Table B.4. Thermal stress at melting point

Material	$T_m - T_0$ °C	E psi $\times 10^{-5}$	α 1/°F $\times 10^6$	μ	σ psi	σ MPa
Graphite ^a	3700	2	2.5	0.3	26,000	179
Niobium	2400	10	3.9	0.3	160,000	1103
Copper	1000	17	9	0.36	225,000	1551
Vanadium	1700	18	10.9	0.3	476,000	3282
Molybdenum	2500	50	2.8	0.32	515,000	3551
Stainless steel	1400	21	10	0.3	580,000	4000
Alumina	2000	50	4.9	0.3	700,000	4826
Tungsten	3300	50	4	0.28	935,000	6446

^aSublimes T_m = melting temperature T_0 = 20°C E = modulus of elasticity α = coefficient of expansion μ = Poisson's ratio σ = thermal stress

surface as fast as it is melted and is no longer available to intercept energy flowing to the surface.

- (2) It becomes a liquid, but remains in place until it reaches the temperature of evaporation, at which point it leaves the surface. In this case it is necessary to consider the temperature distribution in both the liquid and solid material.
- (3) Its temperature has been raised so rapidly that there has not been time for the phase change to occur and it remains a superheated solid.

Energy is considered to arrive at the surface at a steady rate, $dQ/dt = Q/\tau$, the process having continued long enough to have established steady-state conditions.

BASIS OF THE ANALYSIS

Within the boundaries of the heated system it is assumed that the temperature distribution must satisfy the differential equation of Eq. (B.11) and that the boundary conditions accompanying Eq. (B.11) must be satisfied at the moving boundaries.

For steady-state conditions, solutions to Eq. (B.11) are of the form

$$T = T(x - v t),$$

or, in terms of dimensionless quantities as defined by Eqs. (B.7), (B.8), and (B.9),

$$W = W(\xi - v \eta), \quad (B.29)$$

where v is the dimensionless (in this case constant) velocity defined by

$$v = \frac{\rho c_p (T_m - T_0) \tau}{Q} \quad (B.30)$$

Equation (B.29) is, for the solid region,

$$W_s = e^{-(v\xi - v^2\eta)} \quad (B.31)$$

and for the liquid region,

$$W_l = (1 + \lambda_m) e^{\frac{k_l}{k_s}(v\xi - v^2\eta)} - \lambda_m \quad (B.32)$$

STEADY-STATE MELTING

From Eq. (B.14) for the solid at the surface,

$$1 + \frac{\partial W_s}{\partial \xi} \bigg|_0 = \lambda_m v_m$$

or, with the derivative from Eq. (B.31) and solving for the dimensionless velocity, v_m , of the melting surface,

$$v_m = \frac{1}{1 + \lambda_m} \quad (B.33)$$

For the dimensionless time, $\eta = 0$, Eq. (B.31), for steady-state melting, now becomes

$$W_s = e^{\frac{\xi}{1+\lambda_m}} \quad (B.34)$$

STEADY-STATE VAPORIZATION

When both melting and vaporization are occurring simultaneously, it is necessary to solve Eqs. (B.31) and (B.32) simultaneously. If these equations are adjusted so that vaporization is occurring at the original surface, $x = 0$, it is required that $W_\ell = K$ at $\xi = 0$. A check of the equations shows that this occurs at a time, $\eta = \eta_1$, where

$$e^{v^2 \eta_1} = \left(\frac{K + \lambda_m}{1 + \lambda_m} \right)^{\frac{k_\ell}{k_s}} \quad .$$

Equation (B.31) becomes

$$W_s = e^{-v_s \xi} \left(\frac{K + \lambda_m}{1 + \lambda_m} \right)^{\frac{k_\ell}{k_s}} \quad , \quad (B.35)$$

and Eq. (B.32) becomes

$$W_\ell = (K + \lambda_m) e^{\frac{k_s}{k_\ell} v_s \xi} - \lambda_m \quad . \quad (B.36)$$

The slope of this function at $\xi = 0$ is

$$\left. \frac{\partial W_\ell}{\partial \xi} \right|_0 = -(K + \lambda_m) \frac{k_s}{k_\ell} v_s \quad .$$

Substituting this value for the slope into Eq. (B.18) and solving for the velocity of motion of the vaporizing face, v_s ,

$$v_s = \frac{1}{K + \lambda_s} \quad , \quad (B.37)$$

where

$$\lambda_s = \lambda_m + \lambda_v \quad . \quad (B.38)$$

Equation (B.35) becomes

$$W_s = e^{-\frac{1}{K+\lambda_s} \xi} \left(\frac{K + \lambda_m}{1 + \lambda_m} \right)^{\frac{k_\ell}{k_s}} \quad . \quad (B.39)$$

Equation (B.36) becomes

$$W_\ell = (K + \lambda_m) e^{\frac{k_s}{k_\ell (K+\lambda_s)} \xi} - \lambda_m \quad . \quad (B.40)$$

The ratio of the velocity of melting, v_m , to that of vaporization, v_s , from Eqs. (B.33) and (B.37) is

$$\frac{v_m}{v_s} = \frac{v_m}{v_s} = \frac{K + \lambda_s}{1 + \lambda_m} \quad . \quad (B.41)$$

With the values $K = 2$, $\lambda_m = 0.4$, $\lambda_s = 7.0$, and $k_s/k_\ell = 2$, Eq. (B.39) becomes

$$W_s = 1.309 e^{-.111 \xi} \quad . \quad (B.42)$$

Equation (B.40) becomes

$$W_\ell = 2.4 e^{-.222 \xi} - 0.4 \quad . \quad (B.43)$$

The dimensionless velocity of melting is

$$v_m = 0.714 \quad ,$$

and for vaporization

$$v_s = 0.109 \quad .$$

The results of Eqs. (B.42) and (B.43) are plotted in Fig. 23, which is shown in the main body of the report.

FINITE DIFFERENCE PROCEDURES

In this section finite difference procedures are applied to the solution of Eqs. (B.11)-(B.19) to find the transient distribution of temperature in an infinitely thick slab whose surface is being subjected to a burst of radiant energy and in which both melting and vaporization are occurring.

In one particularly useful finite difference form Eq. (B.11) becomes, in the nomenclature of Fig. B.1,

$$\frac{2 W(\xi + h, \eta)}{h_2 (h_1 + h_2)} - \frac{2 W(\xi, \eta)}{h_2 h_1} + \frac{2 W(\xi - h, \eta)}{h_1 (h_1 + h_2)} = \frac{W(\xi, \eta + \theta) - W(\xi, \eta)}{\theta}, \quad (B.44)$$

where the space interval is in the dimensionless form

$$h = \Delta \xi = \frac{Q}{k_s (T_m - T_0) \tau} \Delta x, \quad (B.45)$$

and the time interval, θ , is in the dimensionless form

$$\theta = \Delta \eta = \gamma_s \Delta t \left[\frac{Q}{k_s (T_m - T_0) \tau} \right], \quad (B.46)$$

For convergence of the numerical solution for $W(\xi, \eta + \theta)$ at the later time, $\eta + \theta$, in terms of values at present time, η , it is necessary that

$$\frac{2}{h_1 h_2} \leq \frac{1}{\theta}. \quad (B.47)$$

The simplest possible equation and procedure result at the limit of convergence when

$$\theta = \frac{h_1 h_2}{2}. \quad (B.48)$$

In regions in which the thermal diffusivity is constant, it is appropriate to consider evenly spaced increments, $h_1 = h_2 = h$. Then Eq. (B.48) becomes

$$\theta = \frac{h^2}{2}, \quad (B.49)$$

and Eq. (B.44) becomes

$$W(\xi, \eta + \theta) = \frac{W(\xi + h, \eta) + W(\xi - h, \eta)}{2}. \quad (B.50)$$

Equation (B.12) for the initial boundary condition becomes

$$W(0, \eta + \theta) = W(h, \eta + \theta) = h \frac{k_s}{k}. \quad (B.51)$$

Equation (B.13) becomes

$$W_1(b, \eta + \theta) = W_2(b, \eta + \theta). \quad (B.52)$$

Equation (B.14) is, in accordance with the symbols defined in Fig. B.2,

$$\frac{k_l}{k_s} \frac{W_c - 1}{c + d} - \frac{1 - W_d}{2h - c - d} = \lambda_m \frac{d}{\theta}, \quad (B.53)$$

where c represents the location along the ξ axis of the melting point at time, η , and d represents the distance the boundary moves during the time interval, θ .

Equation (B.18) can be expressed in accordance with the sketch of Fig. B.3:

$$1 + \frac{k_l}{k_s} \frac{K - W_d}{h - e - f} = \frac{\lambda_v f}{A}, \quad (B.54)$$

where e represents the position of the moving face on which vaporization is taking place at time, η , and f represents the distance the face moves during the time interval, θ .

A graphical procedure for the solution of Eqs. (B.50) and (B.51) in the vicinity of the boundary, $\xi = 0$, is shown schematically in Fig. B.4.

Equation (B.53) is a cubic equation in d . One procedure for its solution is to assign trial values for d in the terms of the left side of the equation and to solve for a new value for d in the term on the right in an iterative procedure.

If melting and vaporization occur within a distance which is small compared with the finite difference interval, h , the scheme of Fig. B.5 can be used to find d and f simultaneously. For the vaporization process,

$$1 - \frac{k_l}{k_s} \frac{K - 1}{c + d - e - f} = \frac{\lambda_s f}{\theta} \quad (B.55)$$

and for the melting process,

$$\frac{k_l}{k_s} \frac{K - 1}{c + d - e - f} = \frac{1 - W_a}{2h - c - d} + \frac{\lambda_m d}{\theta}. \quad (B.56)$$

These equations can be solved for f in the form

$$f = \left(1 - \frac{1 - W_a}{2h - c - d} - \frac{\lambda_m d}{q} \right) \frac{\theta}{\lambda_s} \quad (B.57)$$

and for d as

$$d = \left(\frac{k_\ell}{k_s} \frac{K - 1}{c + d - e - f} - \frac{1 - W_a}{2h - c - d} \right) \frac{\theta}{\lambda_m} \quad (\text{B.58})$$

The simplest procedure for solving these equations appears to be:

- (1) assume a value of d in Eq. (B.57);
- (2) solve Eq. (B.57) for f ;
- (3) substitute f and d into right side of Eq. (B.58);
- (4) solve Eq. (B.58) for a new value of d ; and
- (5) repeat the process with a new value of d .

Example

For an example of the method of calculation of the temperature distribution when melting and vaporization occur, an infinitely thick slab of material is considered, originally at the temperature, T_0 . At time $t = 0$, its face, at $x = 0$, is suddenly subjected to heating at a constant rate, q . The following values are assumed: $k_\ell/k_s = 0.5$, $h = 0.5$, $\theta = 0.125$, $\lambda_m = 0.4$, $\lambda_v = 7$, $(T_v - T_0)/(T_m - T_0) = 2$.

The calculated temperatures and the locations of the moving boundaries are shown in Table B.5. These temperatures are also shown graphically in Fig. B.6.

Plots of the distances moved by each boundary during succeeding time intervals are shown in Fig. B.7. As the time increases, the distances moved during succeeding time intervals approach the same limit, namely the steady-state value found from Eq. (B.37). In this case the limit, as shown on Fig. B.7 is

$$f = \frac{q}{K + \lambda_s} = \frac{0.125}{9} = 0.0139.$$

If the melted material were sloughed off as it formed, then the steady-state value, from Eq. (B.33), would be

$$f = \frac{\theta}{1 + \lambda_m} = \frac{0.125}{1.4} = 0.089.$$

This value also is shown on Fig. B.7.

CURVES OF DEPTH OF MELTING AND VAPORIZATION

The calculated values of the dimensionless depth of melting, c , and of vaporization, e , shown in Table B.6 can be used to generate a design curve relating the depth of penetration to the energy density in the burst, Q , and the period of the burst, τ . By definition,

$$c = \frac{Q}{k_s (T_m - T_0) \tau} s_m, \quad (\text{B.59})$$

where the actual depth of melting is represented by s_m , and

$$e = \frac{Q}{k_s (T_m - T_0) \tau}, \quad (\text{B.60})$$

where the actual depth of vaporization is represented by s_v . The dimensionless time, η , at which measurements are made is defined in terms of the real time, t , by

$$h = \left[\frac{Q}{k_s (T_m - T_0) \tau} \right]^2 \gamma_s t. \quad (\text{B.61})$$

When the time, t , is equated to the end of the period, $t = \tau$, Eq. (B.61) becomes

$$\eta = \left[\frac{Q}{k_s (T_m - T_0) \tau} \right] \frac{\gamma_s}{\tau}. \quad (\text{B.62})$$

The period of the burst, τ , has previously been represented in Eq. (B.26) as

$$\phi = \gamma_s \tau \left[\frac{c_p \rho (T_m - T_0)}{Q} \right]^2. \quad (\text{B.63})$$

Let us define a new dimensionless variable, as in Eq. (B.27), for the depth of penetration of melting, δ_m , independent of the period, τ , as

$$\delta_m = s_m \frac{c_p \rho (T_m - T_0)}{Q} \quad (\text{B.64})$$

Combining Eqs. (B.59), (B.63), and (B.64) to eliminate τ and s_m gives for the dimensionless depth of melting,

$$\delta_m = c \phi. \quad (\text{B.65})$$

Table B.5. Transient temperature distribution in a thick slab with melting and vaporization

$\rightarrow$ ξ $\downarrow \eta$	Dimensionless temperature, W											Depth	
	e	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	c	e
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.125	0.500	0	0	0	0	0	0	0	0	0	0	0	0
0.250	0.500	0.250	0	0	0	0	0	0	0	0	0	0	0
0.375	0.750	0.250	0.125	0	0	0	0	0	0	0	0	0	0
0.500	0.750	0.438	0.125	0.063	0	0	0	0	0	0	0	0	0
0.625	0.938	0.438	0.250	0.063	0.031	0	0	0	0	0	0	0	0
0.750	0.938	0.594	0.250	0.141	0.031	0.016	0	0	0	0	0	0	0
0.875	1.076	0.594	0.367	0.141	0.078	0.016	0.008	0	0	0	0	0.038	0
1.000	1.249	0.729	0.367	0.223	0.078	0.043	0.008	0.004	0	0	0	0.124	0
1.125	1.451	0.814	0.476	0.223	0.133	0.043	0.023	0.004	0.002	0	0	0.225	0
1.250	1.635	0.871	0.518	0.304	0.133	0.078	0.023	0.013	0.002	0.001	0	0.317	0
1.375	1.822	0.938	0.588	0.326	0.191	0.078	0.045	0.013	0.007	0.001	0	0.411	0
1.500	1.991	0.997	0.632	0.389	0.202	0.118	0.045	0.026	0.007	0.004	0	0.495	0
1.625	2.000	1.105	0.693	0.417	0.254	0.124	0.072	0.026	0.015	0.004	0.002	0.559	0.002
1.750	2.000	1.188	0.753	0.473	0.270	0.163	0.075	0.044	0.015	0.008	0.002	0.615	0.005
1.875	2.000	1.246	0.787	0.512	0.318	0.173	0.103	0.045	0.026	0.008	0.005	0.660	0.009
2.000	2.000	1.291	0.819	0.553	0.342	0.211	0.109	0.065	0.027	0.015	0.005	0.699	0.014
2.125	2.000	1.327	0.843	0.581	0.382	0.225	0.138	0.068	0.040	0.016	0.009	0.734	0.019
2.250	2.000	1.358	0.865	0.612	0.403	0.260	0.147	0.089	0.042	0.024	0.009	0.765	0.025
2.375	2.000	1.385	0.883	0.634	0.436	0.275	0.174	0.094	0.057	0.025	0.015	0.793	0.031
2.500	2.000	1.409	0.900	0.660	0.454	0.305	0.184	0.115	0.060	0.036	0.015	0.820	0.038
2.625	2.000	1.430	0.915	0.677	0.482	0.319	0.210	0.122	0.076	0.038	0.022	0.844	0.044
2.750	2.000	1.450	0.928	0.698	0.498	0.346	0.221	0.143	0.080	0.049	0.023	0.867	0.051
2.875	2.000	1.468	0.940	0.713	0.522	0.360	0.245	0.150	0.096	0.052	0.031	0.888	0.058
3.000	2.000	1.484	0.951	0.731	0.536	0.384	0.255	0.170	0.101	0.064	0.033	0.907	0.066
3.125	2.000	1.499	0.960	0.744	0.557	0.396	0.277	0.178	0.117	0.067	0.042	0.925	0.073
3.250	2.000	1.513	0.968	0.759	0.570	0.417	0.287	0.197	0.122	0.079	0.044	0.942	0.080

$$\xi = \frac{Q}{k_s \tau (T_m - T_0)} x, \quad \eta = \left[\frac{Q}{k_s \tau (T_m - T_0)} \right]^2 \gamma_s t, \quad W = \frac{T_1 - T_0}{T_m - T_0}$$

k_l = thermal conductivity of liquid
 k_s = thermal conductivity of solid
 γ_s = thermal diffusivity of solid
 Q = energy deposition per unit area
 τ = period of energy burst
 T_m = melting temperature
 T_0 = initial temperature
 s = depth of penetration, sub. m for melting, v for vaporization
 t = time
 W = dimensionless temperature = 1 at melting, 2 at vaporization
 x = distance

$$c = \frac{Q}{k_s \tau (T_m - T_0)} s_m$$

$$d = \frac{Q}{k_s \tau (T_m - T_0)} s_v$$

Table B.6. Penetration at end of burst for melting and vaporization

Time, η	Depth ^a		Burst period, ϕ	Penetration at end of burst	
	Melting,	Vaporization,		Melting,	Vaporization,
	c	e		δ_m	δ_v
0.750	0	0	1.333	0	0
0.875	0.038	0	1.143	0.043	0
1.000	0.124	0	1.000	0.124	0
1.125	0.225	0	0.889	0.200	0
1.250	0.317	0	0.800	0.254	0
1.375	0.411	0	0.727	0.299	0
1.500	0.495	0	0.667	0.330	0
1.625	0.559	0.002	0.615	0.344	0.001
1.750	0.615	0.005	0.571	0.351	0.003
1.875	0.660	0.009	0.533	0.352	0.005
2.000	0.699	0.014	0.500	0.350	0.007
2.125	0.734	0.019	0.471	0.346	0.009
2.250	0.765	0.025	0.444	0.340	0.011
2.375	0.793	0.031	0.421	0.334	0.013
2.500	0.820	0.038	0.400	0.328	0.015
2.625	0.844	0.044	0.381	0.322	0.017
2.750	0.867	0.051	0.364	0.316	0.019
2.875	0.888	0.058	0.348	0.309	0.020
3.000	0.907	0.066	0.333	0.302	0.022
3.125	0.925	0.073	0.320	0.296	0.023
3.250	0.942	0.080	0.308	0.290	0.025

^aNote: Values of c and e are from Table B.5.

All variables are dimensionless.

$$\phi = \frac{1}{\eta}, \quad \delta_m = s_m \frac{c_p \rho (T_m - T_0)}{Q}, \quad \delta_v = s_v \frac{c_p \rho (T_m - T_0)}{Q}$$

Similarly, the dimensionless depth of vaporization, defined by

$$\delta_v = s_v \frac{c_p \rho (T_m - T_0)}{Q}, \quad (B.66)$$

can be written

$$\delta_v = e \phi. \quad (B.67)$$

The results of solving Eqs. (B.65) and (B.67) are shown in Table B.6 and Fig. B.8 using the

values for c and e from Table B.5.* For comparison, the curve for depth of penetration neglecting the latent heats, from Fig. 22 in the main body of the report, is also shown on Fig. B.8.

*Note: At the point at which the calculations were stopped, the values of c and e reported in Table B.5 had developed an instability, presumably because the convergence requirement of Eq. (B.48) is violated somewhere in the region. To carry these calculations further would require using a value of θ less than that prescribed by Eq. (B.49).

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Fig. B.1. Delineation of symbols used for the finite difference analysis of the temperature distribution for the case where both melting and vaporization occur.

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Fig. B.2. Delineation of symbols used for the finite difference solution in the vicinity of the liquid-solid interface.

4

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Fig. B.5. Graphical indication of the procedure for finding the quantities d and f .

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Fig. B.6. Transient temperature distribution in a thick slab with melting and vaporization.

Fig. B.7. Boundary motion during heating.

Fig. B.8. Depth of penetration of melting and vaporization.

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APPENDIX C

THERMAL STRESSES IN TYPICAL
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TUBE IN THE FIRST WALL

The equation for the temperature distribution in the heated region of a helium-cooled tubular element in the first wall (see Fig. 28) can be expressed in accordance with the assumptions above as

$$\frac{d^2T}{dz^2} - \beta^2 T = \frac{Q'}{kd} \quad , \quad (C.1)$$

where

$$\beta^2 = \frac{h}{kd} \quad .$$

Q' is the rate of energy deposition per unit time and per unit area, k is the thermal conductivity of the wall material, and d is the thickness of the wall. The solution to Eq. (C.1) subject to the appropriate boundary conditions is expressed as

$$T = \frac{Q'}{h} \left[1 - \frac{e^{\frac{\beta w}{6}} \left(e^{\frac{2\beta w}{3}} - 1 \right)}{2(e^{\beta w} - 1)} (e^{\beta z} + e^{-\beta z}) \right] \quad , (C.2)$$

where w is the circumference of the tube, i.e., $w = 2\pi r$. The maximum temperature occurs when $z = 0$, when the quantity in brackets is, for any reasonable set of design numbers, essentially unity, so that

$$T_0 = \frac{Q'}{h} \quad . \quad (C.3)$$

The equation to be solved in Region 2 can be expressed as

$$\frac{d^2T}{dz^2} - \beta^2 T = 0 \quad . \quad (C.4)$$

The solution to Eq. (C.4), subject to the appropriate boundary conditions, is

$$T = \frac{Q'}{h} \left\{ \frac{e^{\frac{\beta w}{6}} \left(1 - e^{\frac{-\beta w}{3}} \right)}{2(e^{\beta w} - 1)} \left[e^{\beta z} + e^{\beta(w-z)} \right] \right\} \quad . (C.5)$$

For any reasonable set of numbers again, when $z = w/2$, the quantity in brackets is essentially zero, so that

$$T_{w/2} = 0 \quad . \quad (C.6)$$

The temperature difference around the circumference of the tube is therefore

$$\Delta T = T_0 - T_{w/2} = \frac{Q'}{h} \quad . \quad (C.7)$$

The maximum temperature difference across the boundary layer also is

$$\Delta T_{BL} = \frac{Q'}{h} \quad . \quad (C.8)$$

The compressive thermal stress in the hot central region is

$$\sigma_{\max} = \frac{2}{3} \frac{E \alpha \Delta T}{(1 - \mu)} \quad . \quad (C.9)$$

THERMAL STRESSES CAUSED BY TEMPERATURE
DIFFERENCES BETWEEN A CYLINDRICAL
SHELL JOINED TO A FLAT HEAD

The thermal stresses arising from a temperature difference between a thin cylindrical shell and a thick flat head (see Figs. 29 and 30) can be estimated by taking the thermal strain in the cylindrical shell at $x = 0$ as

$$\frac{y}{r} \Big|_{x=0} = 0 \quad , \quad (C.10)$$

and at $x = \infty$ is

$$\frac{y}{r} \Big|_{x=\infty} = \alpha \Delta T \quad . \quad (C.11)$$

It is assumed that the thickness, t , of the cylindrical shell is small compared with the thickness of the flat head, a necessary condition if the pressure stresses are to be comparable. The head is held at a temperature, $T = 0$. The temperature in the cylindrical shell varies with the distance from the head according to

$$T = \left(1 - e^{-\frac{x}{L}} \right) \Delta T \quad , \quad (C.12)$$

where ΔT is the temperature of the cylindrical shell at $x = \infty$. The smaller the value of L , the more abrupt is the temperature change ($L = 0$ corresponds to a step change in temperature at the juncture). The character of the temperature distribution described by Eq. (C.12) is shown in Fig. C.1. The deformation and stresses in the cylinder can be analyzed by considering it as a "beam on an elastic foundation."^{31,32} The equation for the deflection of the cylindrical shell is then

$$\frac{d^4 y}{dx^4} = \left(1 - e^{-\frac{x}{L}}\right) A - 4B^4 y, \quad (C.13)$$

where

$$A = \frac{12(1 - \mu^2) \alpha \Delta T}{t^2 r}$$

and

$$B^4 = \frac{3(1 - \mu^2)}{t^2 r^2},$$

and where μ is the Poisson ratio and α is the linear coefficient of thermal expansion. The solution to Eq. (C.13) can be written

$$y = \alpha \Delta T r \left[1 - \frac{1}{1 + \left(\frac{1}{BL}\right)^4} \right] e^{\frac{x}{L}} + e^{Bx} (C_1 \cos Bx + C_2 \sin Bx) + e^{-Bx} (C_3 \cos Bx + C_4 \sin Bx) \quad (C.14)$$

If the cylindrical shell is sufficiently long, then

$$C_1 = C_2 = 0 \dots$$

This amounts to the assumption that, for large values of x ,

$$y = \frac{A}{4B^4} = \alpha \Delta T r$$

Because $y = 0$ when $x = 0$, it is required that

$$C_3 = -\frac{A}{4B^4} \frac{\left(\frac{1}{BL}\right)^4}{4 + \left(\frac{1}{BL}\right)^4} = -\alpha \Delta T r \frac{\left(\frac{1}{BL}\right)^4}{4 + \left(\frac{1}{BL}\right)^4}$$

Because $dy/dx = 0$ when $x = 0$, it is required that

$$C_4 = -\alpha \Delta T r \frac{4 + \left(\frac{1}{BL}\right)^3}{BL \left[4 + \left(\frac{1}{BL}\right)^4 \right]}$$

Substituting these values of the arbitrary constants into Eq. (C.14) gives

$$\frac{y}{\alpha \Delta T r} = 1 - \frac{4}{4 + \left(\frac{1}{BL}\right)^4} e^{-\frac{x}{L}} - \frac{\left(\frac{1}{BL}\right)^4}{4 + \left(\frac{1}{BL}\right)^4} e^{-Bx} \times \cos Bx - \frac{4 + \left(\frac{1}{BL}\right)^3}{BL \left[4 + \left(\frac{1}{BL}\right)^4 \right]} e^{-Bx} \sin Bx \quad (C.15)$$

The moment caused by this deformation is given by

$$M = -\frac{E t^3}{12(1 - \mu^2)} \frac{d^2 y}{dx^2}, \quad (C.16)$$

where E is the modulus of elasticity. This moment has its maximum value when $x = 0$, and is given by

$$M_{\max} = -\frac{E \alpha \Delta T t^2}{2\sqrt{3}(1 - \mu^2)} \frac{1 + 4(BL)^3 - 2(BL)^2}{1 + 4(BL)^4} \quad (C.17)$$

The maximum tensile bending stress is given by

$$\sigma_{\max} = -\frac{6 M_{\max}}{t^2} \quad (C.18)$$

or*

$$\sigma_{\max} = \sqrt{\frac{3}{1 - \mu^2}} E \alpha \Delta T \frac{1 + 4(BL)^3 - 2(BL)^2}{1 + 4(BL)^4} \quad (C.19)$$

For $BL \ll 1$, Eq. (C.19) degenerates to

$$\sigma_{\max} = 1.816 E \alpha \Delta T \quad (C.20)$$

This equation coincides with that for a step change in temperature, ΔT , at the juncture of the cylinder and the head. For $BL \rightarrow \infty$, Eq. (C.19) becomes

* With $\mu = 0.3$,

$$\sqrt{\frac{3}{1 - \mu^2}} = 1.816 \dots$$

$$\sigma_{\max} = 1.412 \frac{(rt)^{1/2}}{L} E \alpha \Delta T \quad (C.21)$$

This equation coincides with that for a ramp change in temperature with a slope, $\Delta T/L$. In general the smaller the value of BL , the more precipitous is the change in temperature in the cylinder and the larger is the resulting thermal bending stress concentration.

Substituting for B its defined value,

$$BL = \sqrt[4]{3(1-\mu^2)} \frac{L}{\sqrt{tr}} = 1.285 \frac{L}{\sqrt{tr}} \quad (C.22)$$

It is interesting that the value of BL is essentially independent of the properties of the material used, depending on the steepness of the temperature gradient as determined by $1/L$ and by the geometry of the configuration as determined by the thickness, t , and radius, r , of the cylinder.

Eq. (C.19) can be written

$$\frac{\sigma_{\max}}{E \alpha \Delta T} = F \quad (C.23)$$

where

$$F = \frac{1 + 4(BL)^3 - 2(BL)^2}{1 + 4(BL)^4} \sqrt{\frac{3}{1-\mu^2}} \quad (C.24)$$

A plot of the values of F as a function of L/tr is shown in Fig. C.2.

THERMAL STRESS CAUSED BY A TEMPERATURE DIFFERENCE BETWEEN TWO CYLINDERS JOINED TO A COMMON SET OF FLAT HEADS

The blanket structure of Fig. 29 involves two cylinders differing in temperature by an amount, ΔT , and fastened to two heads, as shown schematically in Fig. 31. The two cylinders are

shown at radii r_1 and r_2 and fastened to two heads separated by a distance, $2L$. The thermal displacement of either head between the junctions with the two cylinders is given by

$$\delta = L \alpha \Delta T$$

where again α is the linear coefficient of thermal expansion. The force, P , required to cause this displacement is found from

$$\frac{\delta}{2} = \frac{P \left(\frac{\ell}{2}\right)^3}{3EI}$$

where, as before, E is the modulus of elasticity and I is the section moment of inertia. The bending moment, M , in the header is

$$M = P \frac{\ell}{2}$$

and the stress, σ , caused by this moment is

$$\sigma = \frac{Mc}{I}$$

where c is the distance from the neutral axis of bending to the most remote fiber from the neutral axis. Combining these expressions to eliminate M , P , and δ gives for the stress,

$$\sigma = \frac{3tL}{\ell^2} E \alpha \Delta T$$

In this case the equivalent stress concentration is

$$K = \frac{3tL}{\ell^2}$$

For example, with $t = 0.003$ m, $L = 0.5$ m, and $\ell = 0.01$ m, the stress concentration is $K = 45$.

Fig. C.1. Typical axial temperature distribution in a cylindrical shell heated by a constant temperature heat source and cooled by heat conduction into a constant temperature flat head.

Fig. C.2. Thermal stress in the wall of the cylindrical shell of Fig. C.1.

APPENDIX D

ENERGY DEPOSITION IN REPRESENTATIVE BLANKETS

Table D.1. Composition of the blanket of Fig. D.1

The total power loading of the first wall is 0.716 MW/m^2 with 22% of the energy going directly into the first wall.

Distance from first wall (cm)	Material in region
0-0.25	Nb
0.25-39.75	Li-2% Nb
29.75-30	Nb
30-70	Graphite
70-70.25	Nb
60.25-99.75	Li-2% Nb
99.75-100.0	Nb

Table D.2. Composition of the blanket of Fig. D.2.

The total power loading of the first wall is 0.716 MW/m^2 with 22% of the energy going directly into the first wall.

Distance from first wall (cm)	Material in region
0-0.25	Nb
0.25-19.75	Li-2% Nb
19.75-20	Nb
20-80	Graphite
80-80.25	Nb
80.25-99.75	Li-2% Nb
99.75-100.0	Nb

Table D.3. Composition of the blanket of Fig. D.3

The total power loading of the first wall is 0.716 Mw/m^2 with 22% of the energy going directly into the first wall.

Distance from first wall (cm)	Material in region
0-0.25	Nb
0.25-9.75	Li-2% Nb
9.75-10	Nb
10-90	Graphite
90-90.25	Nb
90.25-99.75	Li-2% Nb
99.75-100.0	Nb

Table D.4. Composition of the blanket and shield regions for Figs. D.4, D.5, and D.6 as measured from the first wall

Region	Material	t(cm)
Blanket	Stainless steel	0.31
	Liquid metal ^a	0.635
	Stainless steel	1.0
	Graphite	10.0
	Stainless steel	2.5
	Liquid metal ^a	5.0
	Stainless steel	2.5
	Liquid metal ^a	0.5
	Stainless steel	2.5
	Liquid metal ^a	0.5
	Stainless steel	2.5
	Liquid metal ^a	0.5
	Stainless steel	1.0
	Stainless steel	5.0
Shield	34% H ₂ O, 1% B, 65% stainless steel	30.0
	Lead	11.0
	Stainless steel	5.0

^aLithium, potassium, or sodium

Fig. D.1. Volume heat generation for 1-m blanket with 40-cm lithium passages.

Fig. D.2. Volume heat generation for 1-m blanket with 20-cm lithium passages.

Fig. D.3. Volume heat generation for 1-m blanket with 10-cm lithium passages.

3

Fig. D.5. Energy deposition rate as a function of reactor radius in the water-cooled reference design reactor when potassium plus SS-316, lithium, and potassium only are used in the energy absorber (from Ref. 3).

Fig. D.6. Energy deposition rate vs reactor radius in the sodium hydroxide-cooled reference design reactor with lithium as the energy absorbing material.

Table D.5. EPR reference design blanket assemblies (data for Figs. D.4, D.5, and D.6).

Zone	Reference Design, Water Cooled			Reference Design ^a			Reference Design, NaOH Cooled		
	Material Composition	Outer Radius (cm)	Thickness (cm)	Material Composition	Outer Radius (cm)	Thickness (cm)	Material Composition	Outer Radius (cm)	Thickness (cm)
1	Plasma	224.07	224.07	Plasma	223.70	223.70	Plasma	223.70	223.70
2	Carbon	224.37	0.30	Carbon	224.00	0.30	Carbon	224.00	0.30
3	Water	225.00	0.63	Void	225.00	1.00	NaOH	224.64	0.64
4	SS-316	226.11	1.11	SS-316	226.11	1.11	SS-316	225.28	0.64
5	Absorber	251.11	25.00	Absorber	251.11	25.00	Absorber	227.78	2.50
6	SS-316	252.11	1.00	SS-316	252.11	1.00	SS-316	227.82	0.04
7	Carbon	262.11	10.00	Carbon	262.11	10.00	NaOH	227.92	0.10
8	SS-316	263.38	1.27	SS-316	263.38	1.27	SS-316	227.96	0.04
9	Coolant	265.88	2.50	Coolant	265.88	2.50	Absorber	247.96	20.00
10	SS-316	268.38	3.00	SS-316	268.38	3.00	SS-316	248.00	0.04
11	Coolant	268.88	0.50	Coolant	268.88	0.50	NaOH	248.10	0.10
12	SS-316	271.38	2.50	SS-316	271.38	2.50	SS-316	248.14	0.04
13	Coolant	271.88	0.50	Coolant	271.88	0.50	Absorber	250.64	2.50
14	SS-316	274.38	2.50	SS-316	274.38	2.50	SS-316	251.64	1.00
15	Coolant	274.88	0.50	Coolant	274.88	0.50	Carbon	261.64	10.00
16	SS-316	276.50	1.62	SS-316	276.50	1.62	SS-316	262.64	1.00
17	-	-	-	-	-	-	Absorber	266.64	4.00
18	-	-	-	-	-	-	SS-316	266.68	0.04
19	-	-	-	-	-	-	NaOH	266.78	0.10
20	-	-	-	-	-	-	SS-316	266.82	0.04
21	-	-	-	-	-	-	Coolant	268.00	1.18
22	-	-	-	-	-	-	SS-316	270.50	2.50
23	-	-	-	-	-	-	Coolant	271.00	0.50
24	-	-	-	-	-	-	SS-316	273.50	2.50
25	-	-	-	-	-	-	Coolant	274.00	0.50
26	-	-	-	-	-	-	SS-316	276.50	2.50
27	-	-	-	-	-	-	Coolant	277.00	0.50
28	-	-	-	-	-	-	SS-316	278.16	1.16

a. See Refs. 4 and 5.