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INVERSE COMPTON CONVERSION

—Final Report—

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INVERSE COMPTON CONVERSION

1.0 INTRODUCTION

Inverse Compton conversion has been proposed as an alternative to the bremsstrahlung conversion process as a method of transforming the kinetic energy of an electron beam into a directed beam of photons. The concept is illustrated in Figure 1. An electron beam with incident electron kinetic energy E_0 enters a volume of dimension L containing a photon gas, which is characterized by a blackbody temperature E_{bb} and a density ρ_γ . The electrons will inverse-Compton scatter with individual photons in the photon gas. In this process, energy is transferred to the photons, which are then emitted in the forward direction. The resultant photon beam could be used to deliver a radiation dose to a distant target. This report discusses the theoretical formulation of the problem, presents sample results, and describes the computer code developed to analyze this concept.

2.0 THEORY AND METHODOLOGY

2.1 Formulation of the Problem

In order to calculate the output photon fluence from this device, we consider a single incident electron and evaluate the probabilities of three events that must occur for the emission of a photon from the device with energy E_γ and angle θ :

- Probability of an inverse Compton (IC) interaction occurring between depths x and $x + dx$ into the photon gas:

$$P_{IC}(x) dx = \Sigma_{IC} e^{-\Sigma_{IC}x} dx \quad , \quad (1)$$

where Σ_{IC} is the macroscopic inverse Compton cross section.

- Probability that the scattered photon from the inverse Compton event will have energy between E_γ and $E_\gamma + dE_\gamma$ and angle between θ and $\theta + d\theta$:

$$P_p(E_\gamma, \theta) dE_\gamma d\Omega = N_s(E_\gamma, \theta) dE_\gamma d\Omega \quad , \quad (2)$$

Figure 1. Concept of Inverse Compton converter.

where $N_s(E_\gamma, \theta)$ is the normalized source distribution, and $d\Omega$ is the increment of solid angle defined by θ and $\theta+d\theta$.

- Probability that the scattered photon will escape from the cavity:

$$P_e(E_\gamma, x) = \exp[-(L-x)/\ell_{\gamma\gamma}] \exp[-t\Sigma_B] \quad , \quad (3)$$

where $\ell_{\gamma\gamma}(E_\gamma)$ is the attenuation length of a photon with energy E_γ propagating in the photon gas, Σ_B the macroscopic cross section of the bounding case material, and t the thickness of the case material. (The angle θ is small, and will have negligible effect on the path lengths in Equation 3.)

The contribution to the emission spectrum from a slab of thickness dx located at depth x is then given by the product:

$$\begin{aligned} \delta N_\gamma(E_\gamma, \theta) dE_\gamma d\Omega &= P_{IC}(x) dx P_p(E_\gamma, \theta) dE_\gamma d\Omega P_e(E_\gamma, x) \\ &= \Sigma_{IC} \exp[-L/\ell_{\gamma\gamma} - t\Sigma_B] \exp[-(\Sigma_{IC} - 1/\ell_{\gamma\gamma})x] N_s(E_\gamma, \theta) dE_\gamma d\Omega dx \quad . \end{aligned} \quad (4)$$

The total emission is determined by integrating over the volume of the photon gas:

$$\begin{aligned} N_\gamma(E_\gamma, \theta) dE_\gamma d\Omega &= \Sigma_{IC} \exp[-L/\ell_{\gamma\gamma} - t\Sigma_B] N_s(E_\gamma, \theta) dE_\gamma d\Omega \int_0^L e^{-(\Sigma_{IC} - 1/\ell_{\gamma\gamma})x} dx \\ &= \Sigma_{IC} \exp[-L/\ell_{\gamma\gamma} - t\Sigma_B] N_s(E_\gamma, \theta) dE_\gamma d\Omega \frac{1 - \exp[-(\Sigma_{IC} - 1/\ell_{\gamma\gamma})L]}{\Sigma_{IC} - 1/\ell_{\gamma\gamma}} \\ &= \left\{ \frac{\Sigma_{IC}}{\Sigma_{IC} - 1/\ell_{\gamma\gamma}} e^{-t\Sigma_B} \left[e^{-L/\ell_{\gamma\gamma}} - e^{-\Sigma_{IC}L} \right] \right\} N_s(E_\gamma, \theta) dE_\gamma d\Omega \end{aligned}$$

$$= f(E_\gamma) N_s(E_\gamma, \theta) dE_\gamma d\Omega \quad . \quad (5)$$

To proceed with this evaluation, we must determine the macroscopic inverse Compton cross section Σ_{IC} , the source term $N_s(E_\gamma, \theta)$, and the photon attenuation length $\ell_{\gamma\gamma}$.

2.2 Macroscopic Inverse Compton Cross Section

The macroscopic inverse Compton cross section is given by

$$\Sigma_{\text{IC}} = \rho_\gamma \bar{\sigma}_{\text{KN}} \quad , \quad (6)$$

where ρ_γ is the density of the photon gas (in photons per cm^3), and $\bar{\sigma}_{\text{KN}}$ is the Klein–Nishina cross section averaged over the photon energy distribution as seen by the electron in its rest frame.

The Klein–Nishina cross section for an electron at rest may be expressed as

$$\sigma_{\text{KN}} = 2\pi r_0^2 \left\{ \frac{1+\alpha}{\alpha^3} \left[\frac{2\alpha(1+\alpha)}{1+2\alpha} - \ln(1+2\alpha) \right] + \frac{1}{2\alpha} \ln(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \quad . \quad (7)$$

Here $r_0 = e^2/mc^2 = 2.82 \times 10^{-13} \text{ cm}$ is the classical radius of an electron, and α is the initial photon energy as seen in the electron rest frame (E'_i) divided by the rest mass energy of an electron (mc^2):

$$\alpha = E'_i / mc^2 \quad . \quad (8)$$

E'_i may be related to the lab frame parameters E_i , the initial photon energy, and θ_i , the angle of incidence of the photon with respect to the electron path (see Figure 3), by the relativistic transformation

$$E'_i = E_i \gamma (1 - \beta \cos \theta_i) \quad , \quad (9)$$

where $\beta = v/c$ (v = electron velocity in lab frame) and $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$.

A Monte Carlo approach was used to evaluate $\bar{\sigma}_{\text{KN}}$. First, E_i was randomly selected from the blackbody photon energy distribution, and θ_i was selected from the distribution

$$f(\mu) = \frac{1}{2}(1 - \beta\mu), \quad \mu \equiv \cos\vartheta_i \quad . \quad (10)$$

This latter distribution is a kinetic factor that enhances head-on collisions over tail-on collisions, and mathematically represents the fact that the electron "sees" a greater density of approaching photons than receding ones (Reference 1). These quantities were then used in Equations 7-9 to determine a sample value of $\bar{\sigma}_{\text{KN}}$. Repeating this procedure and averaging the results of many trials produced the values for $\bar{\sigma}_{\text{KN}}$ shown in Figure 2.

The photon gas density, ρ_γ , was determined by simply dividing the energy density in the cavity by the mean photon energy in a blackbody spectrum:

$$\rho_\gamma = \frac{E/V}{2.7E_{\text{bb}}} \quad . \quad (11)$$

In this expression, E is the total energy in the cavity, V the volume of the cavity, and the denominator is the mean energy of blackbody photons having temperature E_{bb} .

2.3 Development of Source Term

The next step is the development of the source term $N_s(E_\gamma, \theta_\gamma)$, giving the coupled energy and angle distributions in the lab frame of photons emerging from inverse Compton interactions. The kinematics of an inverse Compton scattering event are illustrated in Figure 3, which also serves to define the quantities of interest. Generally, unprimed quantities refer to the lab frame of reference, while primed quantities are in the electron rest mass frame. The subscript "i" refers to incident (pre-scattered) photon quantities, and subscript " γ " to final (post-scattered) photon values. We proceed as follows:

1. Randomly sample the blackbody energy distribution to determine a value of incident photon energy E_i , and sample the distribution of Equation 10 to obtain a value of incidence angle θ_i .
2. Transform these quantities to the rest frame of the electron, using the relativistic

Figure 2. Average Klein-Nishina cross sections for electrons injected into blackbody photon gas.

- Lab Frame

- Electron Rest Frame

Figure 3. Compton scattering event as observed in the lab and electron rest frames.

transformations

$$E_i' = E_i \gamma (1 - \beta \cos \theta_i) \quad (12)$$

$$\theta_i' = \cos^{-1} \left[\frac{\cos \theta_i - \beta}{1 - \beta \cos \theta_i} \right] \quad (13)$$

3. Assign a weighting factor

$$w = \sigma_{\text{KN}}(E_i') / \bar{\sigma}_{\text{KN}} \quad (14)$$

to this event, where $\sigma_{\text{KN}}(E_i')$ is obtained from Equations 7 and 8. This factor weights each event by its relative probability of occurrence, and is required to account for the energy dependence of the total Klein–Nishina cross section.

4. Apply the Klein–Nishina scattering formulation in the electron rest frame. With E_i' as input, sample from the distribution of final photon energies defined by the differential Klein–Nishina cross section

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \left[x^3 + \left[\frac{2}{\alpha} + \frac{1}{\alpha^2} \right] x^2 + \left[1 - \frac{2}{\alpha} - \frac{2}{\alpha^2} \right] x + \frac{1}{\alpha^2} \right], \quad (15)$$

where α is defined in Equation 8 and $x = E_\gamma' / E_i'$, to obtain a value of E_γ' , the energy of the scattered photon. This sampling is accomplished by utilizing an algorithm developed by Kahn (Reference 2), as implemented in subroutine SCATT of the ITS coupled electron–photon transport code (Reference 3).

5. Use E_i' and E_γ' in the Compton relation

$$E_\gamma' = \frac{E_i'}{1 + \lambda(1 - \cos \theta')} \quad (16)$$

to determine the scattering angle θ' . Use the angular relationship

$$\cos \theta' = \cos \theta_i' \cos \theta_\gamma' + \sin \theta_i' \sin \theta_\gamma' \cos \phi_\gamma', \quad (17)$$

where azimuthal angle ϕ_{γ}' is randomly selected from a uniform distribution from 0 to 2π , to determine θ_{γ}' , the emergent angle of the scattered photon.

6. Perform an inverse transformation back to the lab frame to determine the energy and angle of the scattered photon, E_{γ} and θ_{γ} , respectively. This is accomplished via the relations

$$\cos \theta_{\gamma} = \frac{\cos \theta_{\gamma}' + \beta}{1 + \beta \cos \theta_{\gamma}'} \quad (18)$$

$$E_{\gamma} = \frac{E_{\gamma}'}{\gamma(1 - \beta \cos \theta_{\gamma}')} \quad (19)$$

7. Repeat this procedure in a Monte Carlo fashion to develop the source distribution $N_{\gamma}(E_{\gamma}, \theta_{\gamma})$. Each trial contributes to this distribution according to its weight w .

2.4 Photon Attenuation Length

After an inverse Compton event has taken place, the upscattered photon must traverse the remaining thickness of the gas-filled cavity to emerge from the device. To evaluate the probability of escape, one must calculate the attenuation length $\ell_{\gamma\gamma}(E_{\gamma})$, as defined in Equation 3. We use a formulation developed by Epstein (References 4,5) to accomplish this. In this approach, the photon attenuation length in a blackbody radiation field of temperature E_{bb} is expressed as

$$\ell_{\gamma\gamma}(E_{\gamma}) = \frac{\ell_c E_{\gamma}^2}{\Theta I} \quad , \quad (20)$$

where E_{γ} is the photon energy in the lab frame (here in units of mc^2), $\ell_c = 2.14 \times 10^{-7}$ cm, $\Theta = E_{bb}/mc^2$, and integral I is given by

$$I = \int_1^{\infty} dE_*(E_*)^3 \left[\frac{\sigma_{\gamma\gamma}(E_*)}{\sigma_T} \right] \left[-\ln(1 - e^{-E_{min}/\Theta}) \right] \quad , \quad (21)$$

$$\text{with } E_{\text{min}} = E_*^2 / E_\gamma .$$

In Equation 21, $\sigma_{\gamma\gamma}(E_*)$ is the cross section for photon annihilation in a gamma-gamma collision with center-of-momentum energy per photon E_* , and σ_T is the Thomson cross section (0.665 barns). The quantity $\sigma_{\gamma\gamma}(E_*)/\sigma_T$ has been expressed by Jauch and Rohrlich (Reference 6) in the form

$$\frac{\sigma_{\gamma\gamma}(E_*)}{\sigma_T} = \frac{3}{8} \phi^2 \left[(2 + 2\phi^2 - \phi^4) \cosh^{-1}(1/\phi) - (1 + \phi^2)(1 - \phi^2)^{\frac{1}{2}} \right] , \quad (22)$$

where $\phi = 1/E_*$.

Attenuation lengths obtained by this procedure are shown in Figure 4 as functions of photon energy E_γ for propagation in 3-keV and 10-keV blackbody photon gases. It is apparent that the attenuation length is significantly longer in a 3-keV blackbody environment than for the 10-keV blackbody, and therefore much less self-absorption should occur at the lower blackbody temperature.

One problem arises in applying this formulation to the concept under consideration here. In developing the theory embodied in Equations 20-22, it was assumed that the blackbody radiation field is in equilibrium. The energy density in a blackbody cavity in equilibrium is given (in units of keV/cm³) by

$$\frac{U}{V} = \frac{\pi^2 (E_{\text{bb}})^4}{15 (hc)^3} = 8.55 \times 10^{22} [E_{\text{bb}}(\text{keV})]^4 . \quad (23)$$

The equilibrium photon density (in units of γ/cm^3) is then

$$\rho_{\text{eq}} = \frac{8.55 \times 10^{22} [E_{\text{bb}}(\text{V})]^4}{2.7 E_{\text{bb}}(\text{keV})} = 3.2 \times 10^{22} [E_{\text{bb}}(\text{keV})]^3 . \quad (24)$$

However, in the present case, we are dealing with a non-equilibrium situation in which the photon density is given by Equation 11. To account for this, we make the plausible assumption that the attenuation length scales with the non-equilibrium density ρ_γ according to

Figure 4. Photon attenuation length l_γ as a function of photon energy.

$$\ell_{\gamma\gamma}' = \ell_{\gamma\gamma} \frac{\rho_{eq}}{\rho_{\gamma}} \quad . \quad (25)$$

The actual equation of state of the photon gas and the proper method of determining the attenuation length in a non-equilibrium situation should be the subjects of further research.

2.5 Electron Beam Divergence

With a slight extension to the geometric formulation considered in Section 2.3 (as displayed in Figure 3), one can include provision for a divergence of the incident electron beam. In that previous derivation, the angle of the emitted photon, θ_{γ} , was measured with reference to the direction of the incident electron. The salient features of this geometry are reproduced in Figure 5a, where the z-axis is along the velocity vector of the incident electron, and $\hat{\gamma}$ is the direction of the emitted photon. To include electron beam divergence, we must allow the electron to have an incident angle θ_e with respect to the normal, and perform a transformation from the coordinate system defined by the electron direction (i.e., that used in the source term derivation) to one defined by a normal direction. Such a transformation is depicted graphically in Figure 5b, where the z' axis is along the normal direction. Using standard coordinate rotation techniques (as in Chapter 4 of Reference 7), we obtain the following relationship between θ_e , θ_{γ} , ϕ_{γ} , and the angle of the emitted photon with respect to the normal direction, θ :

$$\cos\theta = \cos\theta_e \cos\theta_{\gamma} - \sin\theta_e \sin\theta_{\gamma} \sin\phi_{\gamma} \quad (26)$$

If the incident electron beam has a gaussian profile that is characterized by an angular divergence σ_e , then the distribution of incident electron angles θ_e within the beam is given by

$$N(\theta_e)d\theta_e = \frac{\theta_e}{\sigma_e^2} \exp[-\theta_e^2/2\sigma_e^2] d\theta_e \quad . \quad (27)$$

To implement the electron beam divergence algorithm, we perform the following steps:

1. Derive θ_{γ} by the methods discussed in Section 2.3.

a. Geometry for single electron

b. Geometry with relation to normal

Figure 5. Geometry for implementing electron beam divergence algorithm.

$z \equiv$ direction of electron

$z' \equiv$ direction of normal incidence

2. Select the azimuthal angle ϕ_γ from a uniform distribution from 0 to 2π .
3. Select θ_e from the distribution given in Equation 27.
4. Use Equation 26 to calculate θ .

2.6 Verification

A literature search was conducted to identify previous work that may be available to verify the results obtained in the present study. The most useful of such publications appears to be that of Blumenthal and Gould (Reference 8), whose paper includes a theoretical derivation of the photon spectrum produced by the process of Compton up-scattering due to high-energy electrons traversing an arbitrary photon gas. When applied to a gas composed of blackbody photons, their expression (specifically, Equations 2.47–2.49 in their paper) takes the form

$$I(E_\gamma) = \int_{\epsilon_{\min}}^{\infty} \frac{\epsilon}{e^{\epsilon/E_{\text{bb}}} - 1} \left[2q \ln q + (1+2q)(1-q) + \frac{1}{2} \frac{(\Gamma q)^2}{1+\Gamma q} (1-q) \right] d\epsilon , \quad (28)$$

$$\text{where } \Gamma = \frac{4E_o}{(mc^2)^2} \epsilon , \quad q = \frac{E_\gamma}{\Gamma(E_o - E_\gamma)} , \quad \epsilon_{\min} = \frac{(mc^2)^2}{4E_o} \frac{E_\gamma}{E_o - E_\gamma} .$$

Here $I(E_\gamma)$ is the (unnormalized) spectrum for all emitted photons, and is proportional to our source term integrated over all emission directions:

$$I(E_\gamma) \propto \int_{4\pi} N_s(E_\gamma, \theta_\gamma) d\Omega .$$

Comparison of our results with the predictions of Equation 28 provides a verification of our integral spectra, up to an arbitrary normalization constant. Such comparisons are presented in the following section.

3.0 SAMPLE RESULTS

In this section we present sample results obtained with the formulation discussed in Section 2, utilizing the codes described in Section 4.

3.1 Case 1: 500-MeV e^- /3-keV BB

The first representative case considered here is 500-MeV electrons incident normally (i.e., with zero beam divergence) into a 3-keV blackbody photon gas. The characteristics of the source term $N_s(E_\gamma, \theta_\gamma)$ are shown in Figure 6, where we plot the energy spectra for those photons emitted from the inverse Compton events within the specified angles (in radians) from the direction of the incident electrons. It is apparent that most of the photons are emitted with angles less than 0.01 radians, and with energies approaching the kinetic energy of the incident electron. The top curve in this figure ($\theta_\gamma < \pi/2$) can be compared directly with the Blumenthal prediction from Equation 28; this is done in Figure 7. It is seen that the present Monte Carlo result agrees quite well with Blumenthal, differing only slightly in the region near the peak energy. This serves as partial verification of our formulation.

The effects of photon beam attenuation by the cavity (assumed here to have an areal density of 3×10^{24} photons/cm², a representative value) and the containing wall (taken to be 1-cm lead) are displayed in Figure 8. The unattenuated distribution (reproduced from Figure 6) is represented by the lighter lines, the attenuated distribution by the heavier lines. It is seen that the spectral shapes are relatively unaffected by the attenuation, except at low energies. This results from the fact that the photon attenuation length $\ell_{\gamma\gamma}$ (Figure 4) and lead absorption coefficient Σ_B do not change significantly over the energy range of interest. It should be noted that secondary photon production has not been included in these (and the following) analyses.

The output photon beam is further characterized in Figures 9, 10, and 11, which show, in order, the fraction of beam photons contained within a cone of given half-angle θ about the forward direction (abscissa value), the fraction of beam energy contained within θ , and the mean energy of all photons emitted within θ . The latter figure confirms that the more energetic photons are emitted generally in the forward direction. (The cause of the anomalous decrease in the mean energy predicted at very small angles has not yet been resolved.) The beam brightness, shown in Figure 12 in units of MeV per steradian per incident electron, was derived by dividing the photon energy contained within θ (abscissa value) by the solid angle of the cone of half-angle θ [$=2\pi(1-\cos\theta)$]. The constant slope of these curves down to approximately 0.01 radians indicates that essentially all the beam is contained within that angle (the slope is constant in this log-log plot since the solid angle

Figure 6. Inverse Compton source distribution for 500-MeV electrons in 3-keV blackbody gas (Case 1).

Figure 7. Case 1: Comparison with Blumenthal results.

Figure 8. Case 1: Pre- and Post-attenuated photon distributions. Attenuation is through cavity ($\rho L = 3 \times 10^{24} \gamma/\text{cm}^2$) and 1-cm Pb wall.

Figure 9. Case 1: Photon number fraction emitted within given angle.

Figure 10. Case 1: Photon energy fraction emitted within a given angle

Figure 11. Case 1: Mean energy of photons emitted within a given angle.

Figure 12. Case 1: Brightness of unattenuated and attenuated photon beams contained within a given angle.

varies as θ^2 for small angles).

3.2 Case 2: 3-GeV e^- / 3-keV BB

The second case considered in this study is 3 GeV electrons incident normally into a 3-keV blackbody gas. The photon output is characterized in a set of figures similar in format to those presented for Case 1. Figure 13 displays the energy-angle distribution of the source term; Figure 14 indicates excellent agreement of the present Monte Carlo results ($\theta_\gamma < \pi/2$ curve in Figure 13) with the Blumenthal prediction for this case. The effects of beam attenuation by the cavity (again assumed to have an areal density of 3×10^{24} photons/cm²) and a 1/8-inch thick lead containing wall are shown in Figure 15. The photon number fraction, photon energy fraction, and mean photon energy for those photons emitted within an angle θ of the forward direction (abscissa value) are shown in Figures 16, 17, and 18, respectively. Similarly, the brightness levels of the unattenuated and attenuated beams contained within an emission angle θ are presented in Figure 19.

For the results presented above, the areal density ρL of the photon gas has been held fixed at a value of 3×10^{24} photons/cm². The effect of varying the areal density (by varying either the density of the photon gas or the cavity dimensions) is examined in Figure 20. The brightness curves in this figure were obtained for five different areal densities; it is apparent that the beam output could be maximized by selecting an optimum density within the range of values considered. The existence of such an optimum derives from two competing effects: For small areal densities, relatively fewer electrons interact when traversing the photon gas, thus reducing the photon output. For large areal densities, the photons are severely attenuated by $\gamma\gamma$ interactions before they can escape. The subject of cavity optimization is discussed in more detail in the following section.

Finally, we consider the effect of electron beam divergence on photon beam brightness. Brightness plots similar to Figure 19 (for the case of zero divergence) are given in Figures 21 and 22 for incident electron beam divergences of 10 milliradians and 100 milliradians, respectively. These results display the expected trend—the brightness of the beam is essentially constant for angles less than the value of the beam divergence. Thus, the divergence of the electron beam causes the output photon beam to be "smeared out" over an angle comparable to the value of that divergence. If the photon beam must propagate over large distances (e.g., exoatmospheric applications), the electron beam

Figure 13. Inverse Compton source distribution for 3-GeV electrons in 3-keV blackbody gas (Case 2).

Figure 14. Case 2: Comparison with Blumenthal results.

Figure 15. Case 2: Pre- and Post-attenuated photon distributions. Attenuation is through cavity ($\rho L = 3 \times 10^{24} \gamma/\text{cm}^2$) and $1/8''$ Pb wall.

Figure 16. Case 2: Photon number fraction emitted within a given angle.

Figure 17. Case 2: Photon energy fraction emitted within a given angle.

Figure 18. Case 2: Mean energy of photons emitted within a given emission angle.

Figure 19. Case 2: Brightness of unattenuated and attenuated photon beams contained within given emission angle (zero electron beam divergence).

Figure 20. Case 2: Brightness of cavity-attenuated beam as function of blackbody gas thickness.

Figure 21. Case 2: Photon beam brightness for 10 mr electron beam divergence.

Figure 22. Case 2: Photon beam brightness for 100 mr electron beam divergence.

divergence can have a significant impact.

3.3 Cavity Optimization Study

As pointed out above, an optimum areal density exists for the cavity which maximizes photon beam output for given values of electron beam energy (E_o) and blackbody temperature (E_{bb}). A study was performed to identify such optimum conditions as functions of E_o and E_{bb} , and to evaluate the maximum achievable beam brightness. For the purpose of this study, we chose to maximize the brightness of the beam contained within an angle of 0.03 radians of the forward direction, and after propagation through the exit wall and 1 kilometer of air (the optimum conditions thus derived are not sensitive to this choice). The incident electron beam was assumed to have zero divergence. The results of this effort are summarized in Figure 23, where we plot the derived optimum value of areal density ρL as a function of blackbody temperature and electron beam energy. The numbers superimposed on the plot are the values of brightness for the criterion angle and distance, in units of MeV per steradian per incident electron.

From the figure, it is evident that the optimum areal density is a rather strong function of both E_o and E_{bb} . If the designer is constrained to fixed values of these parameters, he can use the present results to guide his selection of physical dimensions and other device features that impact the areal density. However, if the designer has some flexibility in selecting E_o and E_{bb} , he can use the maximum brightness values included in the figure as guidelines to assist in this selection. The maximum achievable brightness is a strong function of electron energy, increasing significantly as one increases E_o from 0.5 GeV to 3 GeV. Conversely, the maximum brightness appears to be only weakly dependent on blackbody temperature over the range of values considered here.

4.0 DESCRIPTION OF INVERSE COMPTON CODES

The codes that have been developed for analysis of the inverse Compton concept are contained in two programs: INVC, which performs the computer-intensive Monte Carlo calculations modeling the basic inverse Compton process and the photon-photon interaction physics; and POSTINVC, which accepts files containing the photon spectral/angular distributions and gamma-gamma attenuation lengths generated by INVC, and post-processes them to determine beam brightness, and to include the effects of attenuation. This structure allows one to run INVC on a mainframe computer (the code

Figure 23. Inverse Compton cavity optimization study.

was developed for the CRAY), then download the files to a PC to run POSTINVC.

The following sections detail the input requirements and output files of these two codes. Complete listings of the codes are included in Appendices A and B.

4.1 INVC

Input to INVC is provided through the file INVCIN. It consists of ten entries (in free format) describing the interaction particles, attenuation parameters, and numerical bounds. These entries must be made in the following order:

1. Temperature of the blackbody gas (in keV).
Any reasonable blackbody temperature is allowable.
2. Incident electron energy (in MeV).
Any value of electron energy is allowable.
3. Divergence of the incident electron beam (in milliradians).
Any reasonable value of beam divergence is allowable.
4. Density of the photon gas (in photons/cm³).
If a zero is entered here, the equilibrium density given by Equation 24 is assumed.
5. Length of the cavity (in cm).
The code output depends only on the areal density ρL (i.e., the product of entries 4 and 5); the user is allowed two separate entries to facilitate parameter studies.
6. Thickness of the bounding wall (in cm of lead).
Any reasonable value acceptable.
7. Air density (in g/cm³).
This is the density of the air through which attenuation is calculated. Nominal air density at sea level is 1.25×10^{-3} g/cm³.
8. Distance in air (in km).
This is the distance of air through which attenuation is calculated.
9. Random number seed.
This may be any number between 0.0 and 1.0.
10. Number of particle histories.
This is the number of trials used in the main Monte Carlo routine to develop the source distribution. We have found little difference in results obtained with 1 million and with 10 million histories; thus it appears that 1 or 2 million histories should be sufficient for most cases.

The output consists of four files (MCGAM, MCE, LGAMATTN, and SORT) that

are input to POSTINVC for post-processing, two files (GAMIN and ENGIN) that are provided for plotting unattenuated photon distributions, and one file (TBBODY) for the blackbody spectrum, and one (NDISTRB) that contains certain diagnostic output. These are described briefly below.

MCGAM: This file contains the complete Monte Carlo photon number distribution generated by INVC. The weighted photon count is sorted by photon energy and emission angle. Units are weighted number of photons (i.e., sum of all photon weights in each bin; see Equation 14). Column one identifies the midpoint of each energy bin, with each subsequent column representing the photon tally occurring with emission angle less than the specified value. Angular bins are defined at 1, 3, and 5 tics within each decade, starting at $3\text{E}-5$ radians. Column headings are provided for clarity.

MCE: This file is the complete Monte Carlo energy distribution generated by INVC. It is similar in format to MCGAM, but contains the amount of energy, in units of MeV, emitted within the specified angular bounds.

LGAMATTN: This file contains the gamma-gamma attenuation lengths for photons propagating in a blackbody gas, as a function of photon energy. The energy range of interest (zero to E_0) is divided into 100 bins, with the midpoints of each bin printed in column one. Column two contains the attenuation length in cm.

SORT: This file contains binning data required by the post-processor to perform integration and averaging of beam statistics. The first row consists of four entries: the number of photon energy bins, the energy bin width in MeV, the number of angular bin boundaries, and the number of main Monte Carlo histories. The second row is the average Klein-Nishina cross section in cm^2 . Rows three through final are the angular bin boundaries in radians.

GAMIN: A subset of MCGAM used for plotting the distribution of scattered photons in energy and angle. Format is similar to that of MCGAM, except that the data is given once per angular decade at $3\text{E}-5$, $3\text{E}-4$, etc. Normalized units of photons/MeV/incident electron are used here.

ENGIN: A subset of MCE used for plotting the energy distribution of scattered photons. Format is similar to that of GAMIN; units are total MeV per bin.

TBBODY: The file containing the cumulative distribution of photons in a blackbody spectrum of the specified (input) blackbody temperature. Column one gives the probability of a blackbody photon falling below the corresponding energy in column two. Note that TBBODY represents the particle, and not the energy, distribution of the spectrum.

NDISTRB: This diagnostic file contains the number of non-physical Monte Carlo histories that are discarded due to machine truncation error (see subroutine SOURCE for the check on the quantity $\text{argsq} = a*a + b*b - d*d$; this number should be small or zero); the unweighted tally of photons by angular bin; and the mean of the weighting factors for all histories (should be close to unity).

The code itself consists of a driver program, eight subroutines, and three functions. The purposes of the various subroutines and functions are described by in-code comments; the complete listing is available in Appendix A

4.2 POSTINVC

The post-processor POSTINVC requires the same input file, INVCIN, as the program INVC. The format of this file is discussed above.

Six output files are provided by the postprocessor:

GAMOUT: A modified subset of MCGAM used for plotting the attenuated distribution of scattered photons as functions of angle and energy. Selected input from MCGAM (one data set per angular decade at $3E-5$, $3E-4$, etc., radians) are normalized per MeV per incident electron, and multiplied by an attenuation factor to account for attenuation through the cavity, cavity wall, and atmosphere. Units are photons/MeV/incident electron.

ENGOUT: A file similar to GAMOUT, but derived from the file MCE to give the attenuated distribution of scattered photon energy as functions of angle and energy. Units are MeV/MeV/incident electron.

PREATTN: A plottable, labelled output file giving a variety of angular characteristics describing the photon emissions prior to gamma-gamma attenuation. Column one lists

angular bin boundaries in radians. Column two gives the average energy (in MeV) for all photons scattered within the corresponding angle. Columns three and four list cumulative photon number and energy fractions within the corresponding angle. Columns five and six are beam brightness in photons/sr/incident electron and MeV/sr/incident electron, respectively, for all photons scattered within the corresponding angle.

POSTGAM: Similar to PREATTN in format, but giving photon beam characteristics after gamma-gamma attenuation.

POSTWALL: Similar to PREATTN in format, but giving photon beam characteristics after gamma-gamma and wall attenuation.

POSTATM: Similar to PREATTN in format, but giving photon beam characteristics after gamma-gamma, wall, and atmospheric attenuation.

The POSTINVC code is composed of a driver program and four subroutines. The functions of these subroutines are described by in-code comments; the full listing is available in Appendix B.

4.3 Sample Problem

Sample input and output are provided in this section. The specific problem involves 3-GeV electrons incident into a 3-keV blackbody gas; the electron beam is assumed to have a divergence of 10 mradians. The input file INVCIN is shown in Table 1. Table 2 displays the messages that are sent to the screen by INVC during the execution of this problem. A history counter is included in the main Monte Carlo loop to assist the reader in monitoring the progress of the run. Similarly, a counter is included in the attenuation length integration routine. These are the two most time-consuming steps in the calculation. Table 3 gives a listing of the post-processed output files; these data are displayed graphically in Figure 21. Finally, Table 4 presents a listing of the diagnostic file NDISTRB.

Table 1. Sample INVCIN file.

3.0	black body temperature (keV)
3000.0	electron beam energy (MeV)
10.0	electron beam divergence (mrad)
3.0E+22	photon density (p/cm ³)
100.0	cavity length (cm)
0.3175	wall thickness (cm)
1.25e-3	aircolumn density (g/cm ³)
1.0	air column length (km)
0.437763067	random number seed
2000000	number of particle histories

Table 2. Sample screen output during INVC run.

```

BEGINNING MONTE CARLO SECTION
FINISHED 200000 HISTORIES
FINISHED 400000 HISTORIES
FINISHED 600000 HISTORIES
FINISHED 800000 HISTORIES
FINISHED 1000000 HISTORIES
FINISHED 1200000 HISTORIES
FINISHED 1400000 HISTORIES
FINISHED 1600000 HISTORIES
FINISHED 1800000 HISTORIES
FINISHED 2000000 HISTORIES
ENDING MONTE CARLO SECTION
INTEGRATING AT 2865. MEV
INTEGRATING AT 2715. MEV
INTEGRATING AT 2565. MEV
INTEGRATING AT 2415. MEV
INTEGRATING AT 2265. MEV
INTEGRATING AT 2115. MEV
INTEGRATING AT 1965. MEV
INTEGRATING AT 1815.0000000001 MEV
INTEGRATING AT 1665.0000000001 MEV
INTEGRATING AT 1515.0000000001 MEV
INTEGRATING AT 1365.0000000001 MEV
INTEGRATING AT 1215.0000000001 MEV
INTEGRATING AT 1065.0000000001 MEV
INTEGRATING AT 915.0000000007 MEV
INTEGRATING AT 765.0000000007 MEV
INTEGRATING AT 615.0000000008 MEV
INTEGRATING AT 465.0000000008 MEV
INTEGRATING AT 315.0000000008 MEV
INTEGRATING AT 165.0000000008 MEV
INTEGRATING AT 15.0000000008 MEV
ATTENUATION LENGTH INFINITE AT 15.0000000008 MEV
STOP

```

Table 3. Sample POSTINVC output.

PREATTN					
ANGLE (RADS)	AVG ENERGY (MEV)	CUM FRACTION GAMMAS	CUM FRACTION ENERGY	BRIGHTNESS (G/SR/E)	BRIGHTNESS (MEV/SR/E)
0.30000E-04	0.17155E-04	0.29549E-05	0.23715E-05	0.11461E-04	0.19661E+07
0.50000E-04	0.24288E-04	0.93134E-05	0.10583E-04	0.13004E-04	0.31585E+07
0.10000E-03	0.22416E-04	0.44301E-04	0.46982E-04	0.15639E-04	0.35056E+07
0.30000E-03	0.22245E-04	0.46297E-03	0.48182E-03	0.17957E-04	0.39943E+07
0.50000E-03	0.21671E-04	0.12872E-02	0.13050E-02	0.17773E-04	0.38950E+07
0.10000E-02	0.21862E-04	0.49653E-02	0.50784E-02	0.17333E-04	0.37892E+07
0.30000E-02	0.21738E-04	0.43106E-01	0.43939E-01	0.16719E-04	0.36428E+07
0.50000E-02	0.21734E-04	0.11457E-00	0.11675E-00	0.15998E-04	0.34849E+07
0.10000E-01	0.21743E-04	0.38480E-00	0.39152E-00	0.13433E-04	0.32214E+07
0.30000E-01	0.21450E-04	0.78531E-00	0.78875E-00	0.38219E-03	0.31799E+06
0.50000E-01	0.21393E-04	0.79918E-00	0.10000E-01	0.13954E-03	0.29852E+06
0.10000E-00	0.21377E-04	0.79982E-00	0.10000E-01	0.34730E-02	0.74673E+05
0.30000E-00	0.21375E-04	0.10000E-01	0.10000E-01	0.39078E-01	0.33531E+04
0.50000E-00	0.21375E-04	0.10000E-01	0.10000E-01	0.14253E-01	0.30475E+04
0.10000E-01	0.21375E-04	0.10000E-01	0.10000E-01	0.37763E-00	0.31157E+03
0.15708E-01	0.21375E-04	0.10000E-01	0.10000E-01	0.17454E-00	0.37308E+03

POSTGAM					
ANGLE (RADS)	AVG ENERGY (MEV)	CUM FRACTION GAMMAS	CUM FRACTION ENERGY	BRIGHTNESS (G/SR/E)	BRIGHTNESS (MEV/SR/E)
0.30000E-04	0.17643E-04	0.28923E-05	0.23628E-05	0.57390E-02	0.10126E+06
0.50000E-04	0.24528E-04	0.93329E-05	0.10622E-04	0.66809E-02	0.16387E+06
0.10000E-03	0.22576E-04	0.44938E-04	0.47015E-04	0.30249E-02	0.18133E+06
0.30000E-03	0.22434E-04	0.46404E-03	0.48201E-03	0.32075E-02	0.20656E+06
0.50000E-03	0.21888E-04	0.12876E-02	0.13047E-02	0.31972E-02	0.20131E+06
0.10000E-02	0.22067E-04	0.49713E-02	0.50774E-02	0.28777E-02	0.19591E+06
0.30000E-02	0.22001E-04	0.43142E-01	0.43947E-01	0.35602E-02	0.18833E+06
0.50000E-02	0.21996E-04	0.11467E-00	0.11679E-00	0.31912E-02	0.18017E+06
0.10000E-01	0.21963E-04	0.38508E-00	0.39159E-00	0.38767E-02	0.15103E+06
0.30000E-01	0.21673E-04	0.78530E-00	0.78875E-00	0.19532E-02	0.42376E+05
0.50000E-01	0.21616E-04	0.79914E-00	0.10000E-01	0.71335E-01	0.15431E+05
0.10000E-00	0.21602E-04	0.79981E-00	0.10000E-01	0.17369E-01	0.38601E+04
0.30000E-00	0.21598E-04	0.10000E-01	0.10000E-01	0.19992E-00	0.43177E+03
0.50000E-00	0.21598E-04	0.10000E-01	0.10000E-01	0.72938E-01	0.15753E+03
0.10000E-01	0.21598E-04	0.10000E-01	0.10000E-01	0.19424E-01	0.41950E+02
0.15708E-01	0.21598E-04	0.10000E-01	0.10000E-01	0.39289E-02	0.19284E+02

POSTWALL					
ANGLE (RADS)	AVG ENERGY (MEV)	CUM FRACTION GAMMAS	CUM FRACTION ENERGY	BRIGHTNESS (G/SR/E)	BRIGHTNESS (MEV/SR/E)
0.30000E-04	0.17520E-04	0.29030E-05	0.23637E-05	0.38340E-02	0.57174E+05
0.50000E-04	0.24471E-04	0.93320E-05	0.10621E-04	0.44369E-02	0.10858E+06
0.10000E-03	0.22577E-04	0.44825E-04	0.47008E-04	0.53280E-02	0.12013E+06
0.30000E-03	0.22402E-04	0.46250E-03	0.48195E-03	0.51090E-02	0.13685E+06
0.50000E-03	0.21843E-04	0.12843E-02	0.13048E-02	0.51063E-02	0.13338E+06
0.10000E-02	0.22009E-04	0.49615E-02	0.50790E-02	0.58974E-02	0.12980E+06
0.30000E-02	0.21936E-04	0.43071E-01	0.43945E-01	0.56285E-02	0.12473E+06
0.50000E-02	0.21932E-04	0.11443E-00	0.11679E-00	0.54430E-02	0.11938E+06
0.10000E-01	0.21897E-04	0.38447E-00	0.39157E-00	0.45700E-02	0.10007E+06
0.30000E-01	0.21589E-04	0.78468E-00	0.78875E-00	0.13006E-02	0.28073E+05
0.50000E-01	0.21523E-04	0.79894E-00	0.10000E-01	0.47505E-01	0.10224E+05
0.10000E-00	0.21505E-04	0.79976E-00	0.10000E-01	0.11893E-01	0.25577E+04
0.30000E-00	0.21500E-04	0.10000E-01	0.10000E-01	0.13507E-00	0.28609E+03
0.50000E-00	0.21500E-04	0.10000E-01	0.10000E-01	0.43548E-01	0.10632E+03
0.10000E-01	0.21500E-04	0.10000E-01	0.10000E-01	0.12928E-01	0.27776E+02
0.15708E-01	0.21500E-04	0.10000E-01	0.10000E-01	0.59432E-02	0.12773E+02

POSTATTN					
ANGLE (RADS)	AVG ENERGY (MEV)	CUM FRACTION GAMMAS	CUM FRACTION ENERGY	BRIGHTNESS (G/SR/E)	BRIGHTNESS (MEV/SR/E)
0.30000E-04	0.16715E-04	0.29931E-05	0.23859E-05	0.35320E-01	0.59374E+04
0.50000E-04	0.24083E-04	0.92455E-05	0.10619E-04	0.39503E-01	0.95133E+04
0.10000E-03	0.22212E-04	0.44332E-04	0.46960E-04	0.47349E-01	0.10517E+05
0.30000E-03	0.22175E-04	0.45535E-03	0.48155E-03	0.54037E-01	0.11983E+05
0.50000E-03	0.21541E-04	0.12649E-02	0.13045E-02	0.54252E-01	0.11686E+05
0.10000E-02	0.21646E-04	0.49179E-02	0.50765E-02	0.52525E-01	0.11370E+05
0.30000E-02	0.21546E-04	0.42755E-01	0.43930E-01	0.50738E-01	0.10932E+05
0.50000E-02	0.21546E-04	0.11361E-00	0.11674E-00	0.48528E-01	0.10458E+05
0.10000E-01	0.21498E-04	0.38183E-00	0.39145E-00	0.40791E-01	0.87672E+04
0.30000E-01	0.21104E-04	0.78239E-00	0.78871E-00	0.11659E-01	0.24606E+04
0.50000E-01	0.21005E-04	0.79832E-00	0.79999E-00	0.42559E-00	0.89604E+03
0.10000E-00	0.20973E-04	0.79962E-00	0.10000E-01	0.10625E-00	0.32415E+03
0.30000E-00	0.20970E-04	0.10000E-01	0.10000E-01	0.11975E-01	0.25073E+02
0.50000E-00	0.20970E-04	0.10000E-01	0.10000E-01	0.43623E-02	0.71476E+01
0.10000E-01	0.20970E-04	0.10000E-01	0.10000E-01	0.11617E-02	0.24360E+01
0.15708E-01	0.20970E-04	0.10000E-01	0.10000E-01	0.53402E-03	0.11128E+01

Table 4. Sample NDISTRB file.

```

NUMBER OF MC ERRORS: 0
2.999999999999999E-5, 9.
4.999999999999999E-5, 12.
9.999999999999998E-5, 72.
2.999999999999999E-4, 806.
4.999999999999999E-4, 1527.
9.999999999999999E-4, 7217.
3.E-3, 75308.
4.999999999999999E-3, 141132.
9.999999999999999E-3, 535375.
3.E-2, 1203689.
5.E-2, 32184.
1.E-1, 2082.
0.3, 538.
0.5, 33.
1., 13.
0.E-1, 3.
MEAN WEIGHT: 1.096601314447

```

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APPENDIX A:

Listing of INVC Code


```

PROGRAM INVC1(TAPE6=TTY)
*****
C THIS PROGRAM PREDICTS THE DISTRIBUTION OF UP-SCATTERED
C PHOTONS PRODUCED BY INJECTION OF AN ELECTRON BEAM INTO A
C PHOTON GAS OF A GIVEN TEMPERATURE. DISTRIBUTIONS OF THE
C BLACKBODY, THE UNATTENUATED ANGULAR SOURCE OF UP-SCATTERED PHOTONS,
C AND THE TRANSMITTED PHOTONS ARE ALSO GENERATED.
C
C PARAMETER DEFINITIONS:
C
C NT = NUMBER OF BLACK BODY SPECTRAL POINTS
C NAB = MAXIMUM NUMBER OF ANGULAR BINS IN FINAL OUTPUT
C NEB = NUMBER OF ENERGY BINS IN FINAL OUTPUT
C NN2 = NUMBER HISTORIES IN AVERAGE KLEIN-NISHINA CS CALCULATION
C
C INPUT VARIABLES:
C
C TEMP = BLACK BODY TEMPERATURE (KEV)
C TMEV = ELECTRON BEAM ENERGY (MEV)
C ESTG = ELECTRON BEAM DIVERGENCE (MRAD)
C RHOBB = BLACK BODY PHOTON DENSITY (GAMMAS/CM3)
C CAVL = CAVITY DEPTH (CM)
C WALLT = ENCLOSURE WALL THICKNESS (CM)
C RHOAIR = AIR DENSITY (G/CM3)
C AIRLEN = DISTANCE IN AIR (KM)
C RSEED = RANDOM NUMBER SEED
C NN1 = NUMBER OF MAIN MONTE CARLO HISTORIES
C
C *****
C PARAMETER (NT=101, NAB=20, NEB=100, NN2=1000)
C IMPLICIT REAL(A-H,O-Z)
C
C REAL CS(NT),E(NT),DE(NEB)
C REAL EGRID(NEB+1),SUM(NEB,NAB),SUME(NEB,NAB),ANG(NAB+1)
C REAL DIFFN(NEB,NAB),DIFFE(NEB,NAB)
C
C COMMON /ATTEN/ RHOEQ,RHOBB,CAVL
C COMMON /AVEKN/ TOTKN,DE,DIFFN,SUM,SUME,ANG,NANG
C COMMON /INTG/ EGAMMA,TBB
C COMMON R,RSEED
C *****
C
C OPEN APPROPRIATE FILES
C
C INVCIN : UNIT 3 CONTAINS INPUT VARIABLES
C GAMIN : UNIT 7 UNATTENUATED GAMMA POPULATION DISTRIBUTIONS
C          AS FUNCTIONS OF ENERGY AND ANGLE
C ENGIN : UNIT 8 UNATTENUATED GAMMA ENERGY DISTRIBUTIONS
C          AS FUNCTIONS OF ENERGY AND ANGLE
C MCGAM : UNIT 9 MASTER ARRAY OF CUMULATIVE GAMMA POPULATIONS
C          AS FUNCTIONS OF ENERGY
C MCE : UNIT 10 MASTER ARRAY OF CUMULATIVE GAMMA ENERGIES
C          AS FUNCTIONS OF ENERGY
C TBBODY : UNIT 11 INITIAL BLACK BODY PHOTON DISTRIBUTION
C SORT : UNIT 12 CARRY-OVER DATA FOR POST-PROCESSOR
C NDISTRIB : UNIT 14 DISTRIBUTION OF UNWEIGHTED GAMMAS AS A FUNCTION
C          OF ANGLE (DIAGNOSTIC OUTPUT)
C *****
C
C OPEN(UNIT=3, FILE='INVCIN')

```

```

OPEN(UNIT=4, FILE='LGAMATTN')
OPEN(UNIT=7, FILE='GAMIN')
OPEN(UNIT=8, FILE='ENGIN')
OPEN(UNIT=9, FILE='MCGAM')
OPEN(UNIT=10, FILE='MCE')
OPEN(UNIT=11, FILE='TBBODY')
OPEN(UNIT=12, FILE='SORT')
OPEN(UNIT=14, FILE='NDISTRIB')
C
C READ INPUT VARIABLES FROM INVCIN
C
C READ(3,*) TEMP
C READ(3,*) TMEV
C READ(3,*) ESTG
C READ(3,*) RHOBB
C READ(3,*) CAVL
C READ(3,*) WALLT
C READ(3,*) RHOAIR
C READ(3,*) AIRLEN
C READ(3,*) RSEED
C READ(3,*) NN1
C
C SET THE PHOTON DENSITY TO THE EQUILIBRIUM VALUE IN LIEU OF INPUT
C
C RHOEQ = 3.2D+22*TEMP**3
C IF(RHOBB.EQ. 0.0) RHOBB = RHOEQ
C
C PI = ACOS(-1.0)
C *****
C DEFINE ANGULAR BINS FOR OUTPUT DISTRIBUTIONS
C *****
C CALL ANGBIN(PI,NANG,ANG)
C *****
C DETERMINE THE SPECTRAL DISTRIBUTION OF THE BLACK BODY PHOTON GAS
C *****
C *****
C FIX THE UPPER AND LOWER INTEGRATION LIMITS OF THE BLACK BODY DISTRIBUTION
C AT 20*TEMP AND 0.1*TEMP, RESPECTIVELY.
C
C EINF = 20.*TEMP
C ENAUT = 0.1*TEMP
C
C CALL BBODY(EINF,ENAUT,TEMP,CS,E)
C *****
C DETERMINE THE AVERAGE KLEIN-NISHINA CROSS SECTION USING NN2 SAMPLES
C *****
C *****
C CALL KNBAR(TMEV,CS,E,TOTKN)
C *****
C *****
C GENERATE THE UNATTENUATED ANGULAR SOURCE DISTRIBUTION OF
C UP-SCATTERED PHOTONS

```



```

E(JJ)=FAC*E(J)
CS(JJ)=0.0
ETEM=E(JJ)
DE=(ETEM-ENAUT)/100.
ST=0.
IF(DE .LT. 1.0D-6) GO TO 2
UO=ENAUT/TEMP
UTEM=ETEM/TEMP
DU=DE/TEMP
CALL RUNGE(FN,UO,0.0,UTEM,DU,ANS)
ST=ST+ANS/TEMP
CS(JJ)=ST/RHO
CONTINUE
CS(NT)=0.0
E(NT)=ENAUT
WRITE(11,*)NT
DO 25 I = 1,NT
  WRITE(11,*) CS(I),E(I)
CONTINUE
25
RETURN
END
SUBROUTINE RUNGE(F,X0,Y0,XINF,DX,YINF)
  IMPLICIT REAL (A-H,O-Z)
  C X IS THE INDEPENDENT VARIABLE
  C Y IS THE DEPENDENT VARIABLE
  C DY/DX=F(X)
  C X0,Y0 ARE THE INITIAL CONDITIONS
  C XINF IS THE LAST X VALUE
  C DX IS THE STEP SIZE
  C YINF IS THE RETURNED VALUE OF Y(XINF)

```

```

X=X0
Y=Y0
NSTEP=0
RK1=DX*F(X)
X1=X+DX*.5
Y1=Y+0.5*RK1
RK2=DX*F(X1)
Y1=Y+.5*RK2
RK3=DX*F(X1)
X2=X+DX
Y1=Y+RK3
RK4=DX*F(X2)
RK=(RK1+RK4+2.*(RK2+RK3))/6.
Y=Y+RK
X=X2
NSTEP=NSTEP+1
IF (X.LT.XINF) GOTO 10
YINF=Y
RETURN
END

```

```

FUNCTION FN(X)
  IMPLICIT REAL (A-H,O-Z)
  FN=X*X/(EXP(X)-1.)

```

```

RETURN
END

```

```

C*****
C
C SUBROUTINE KN PRODUCES A DIFFERENTIAL KLEIN-NISHINA ELECTRON CROSS SECTION
C AS A FUNCTION OF BOTH THE INCIDENT PHOTON'S ENERGY AND ANGLE. (TAKEN FROM
C THE INTEGRATED TIGER SERIES OF COMPUTER PROGRAMS WHICH PERFORMS COUPLED
C PHOTON-ELECTRON RADIATION TRANSPORT.)
C*****

```

```

C*****

```

```

SUBROUTINE KN(EP,COM)
  IMPLICIT REAL (A-H,O-Z)

```

```

COMMON R,RSEED
EQUIVALENCE (R,RA)

```

```

CM = 0.511
WL = CM/EP
H = 2./WL
100 RA = STRNUM(RA)
  IF (RA .LE. (1.+H)/(9.+H)) THEN
    RA = STRNUM(RA)
    R1 = 1.+RA*H
    RA = STRNUM(RA)
    IF (RA.GT.4.*(R1-1.)/(R1**2)) GO TO 100
  ELSE
    RA = STRNUM(RA)
    R1 = (1.+H)/(1.+RA*H)
    RA = STRNUM(RA)
    IF (RA.GT.0.5*(WL-R1*WL+1.)**2+1./R1)) GO TO 100
  END IF

```

```

WN = WL*R1
COM = 1.-WN+WL
EP = CM/WN

```

```

RETURN
END

```

```

C*****
C
C SUBROUTINE KNBAR CALCULATES THE AVERAGE KLEIN-NISHINA ELECTRON CROSS*
C SECTION FOR THE PHOTON GAS. THIS AVERAGE ARISES FROM NN2 SAMPLINGS *
C OF THE PHOTON BLACK BODY SPECTRUM AND SUBSEQUENT CROSS SECTION *
C CALCULATIONS VIA FUNCTION XKN. *
C*****

```

```

SUBROUTINE KNBAR(TMEV,CS,E,TOTKN)
  PARAMETER (NT=101, NAB=20, NEB=100, NN2=1000)
  IMPLICIT REAL(A-H,O-Z)
  REAL CS(NT),E(NT)
  COMMON R,RSEED

```

```

XST = 0.0
GAMMA = TMEV/0.511 + 1.0

```

```

      BETA = SORT(1.-1./(GAMMA*GAMMA))
C
C  RANDOM NUMBER SEED
C
      R = RSEED
C
C  PERFORM NN2 SAMPLES OF KLEIN-NISHINA CROSS SECTIONS
C
      DO 1000 M = 1,NN2
        R = STRNUM(R)
C
C  BRACKET RANDOM SELECTION IN THE ENERGY SPECTRUM AND INTERPOLATE
C
        DO 100 I = 1,NT
          IF (R.GT.CS(I)) GO TO 200
          CONTINUE
100
        J = I-1
        OMEGAI = E(I) + (R-CS(I))*(E(J)-E(I))/(CS(J)-CS(I))
C
        R = STRNUM(R)
        COSTHI = -1.0 + 2.*R
        COSTHI = (1./BETA)*(1. - SORT(1.- 2.*BETA*
          & (2.*R - 1. - BETA/2.)))
C
C  TRANSFORM TO REST FRAME
C
        OMIP = OMEGAI*GAMMA*(1 - BETA*COSTHI)
        CTHIP = (COSTHI - BETA)/(1. - BETA*COSTHI)
C
C  CALCULATE TOTAL KLEIN-NISHINA CROSS SECTION BY USING PHOTON ENERGY
C  NORMALIZED TO THE ELECTRON REST MASS ENERGY
C
        ALPHA = OMIP/511.
C
C  SUM ALL CROSS SECTIONS FOR AVERAGING
C
        XST = XST + XKN(ALPHA)
1000 CONTINUE
C
C  DIVIDE BY NUMBER OF SAMPLES TO OBTAIN AVERAGE CROSS SECTION
C
        TOTKN = XST/NN2
C
        RETURN
      END
C*****
C  SUBROUTINE MIDEXP PERFORMS MIDPOINT INTEGRATION ON A SEMI-INFINITE INTERVAL *
C  TO EVALUATE THE EPSTEIN INTEGRAL REQUIRED FOR CALCULATING PHOTON *
C  ATTENUATION LENGTHS. *
C*****
      SUBROUTINE MIDEXP(SIGGG,AA,S,N)
      IMPLICIT REAL (A-H,O-Z)
      FUNC(X)=SIGGG(1./X)/X**2
      B=1./AA
      A=1./BB
      FUNC(X)=SIGGG(-ALOG(X))/X
      B=EXP(-AA)
      A=0.0
      IF (N.EQ.1) THEN
        S=(B-A)*FUNC(0.5*(A+B))

```

```

      IT=1
      ELSE
        TNM=IT
        DEL=(B-A)/(3.*TNM)
        DDEL=DEL+DEL
        X=A+0.5*DEL
        SUM=0.
        DO 11 J=1,IT
          SUM=SUM+FUNC(X)
          X=X+DDEL
          SUM=SUM+FUNC(X)
          X=X+DEL
11 CONTINUE
        S=(S+(B-A)*SUM/TNM)/3.
        IT=3*IT
        ENDIF
        RETURN
      END
C*****
C  FUNCTION SIGGG EVALUATES THE INTEGRAND OF THE EPSTEIN INTEGRAL.
C*****
      FUNCTION SIGGG(ESTAR)
      IMPLICIT REAL (A-H,O-Z)
      COMMON /INTG/ EGAMMA,TBB
C*****
      EMIN = ESTAR*ESTAR/EGAMMA
      PHI = 1./ESTAR
      SIGMA = 3./8.*PHI*PHI*((2.+2.*PHI*PHI-PHI**4.)*
        & ALOG(1./PHI)*SQRT((1./PHI)**2.-1.))-
        & (1.+PHI*PHI)*SQRT(1.-PHI*PHI))
      ARG = 1.0 - EXP(-1.0*EMIN/TBB)
      SIGGG = ESTAR*ESTAR*ESTAR*SIGMA*ALOG(ARG)*(-1.0)
      RETURN
      END
C*****
C  SUBROUTINE SOURCE GENERATES THE UNATTENUATED PHOTON DISTRIBUTIONS CREATED
C  AFTER SCATTERING BY THE INCIDENT ELECTRON BEAM. DISTRIBUTIONS ARE PRODUCED
C  AS FUNCTIONS OF PHOTON ENERGY AND ANGLE FOR GAMMA ENERGIES AND POPULATIONS.
C*****
      SUBROUTINE SOURCE(TMEV,ESIG,NW,CS,E,EGRID)
      PARAMETER (NT=101, NAB=20, NEB=100, NN2=1000)
      IMPLICIT REAL(A-H,O-Z)
      INTEGER I,J,M,NN
      REAL CS(NT),E(NT),EGRID(NEB+1),DE(NEB)
      REAL ANG(NAB+1),SUM(NEB,NAB),SUME(NEB,NAB)
      REAL DIFFN(NEB,NAB),DIFFE(NEB,NAB)
      REAL ANGSUM(NAB)
C
      CHARACTER*10 PCHAR(NEB,NAB),ECHAR(NEB,NAB),ZERO1,ZERO2
C
      COMMON R,RSEED
      COMMON /AVEKN/ TOTKN,DE,DIFFN,DIFFE,SUM,SUME,ANG,NANG
C
      C CHARACTER STRINGS FOR FINAL MASSAGING OF PARTICLE AND ENERGY OUTPUT FILES
      ZERO1 = ' 0.000E+00'

```

49

50

```

C      RETURN
      END
C*****
C      C FUNCTION STRNUM GENERATES A PSEUDO-RANDOM NUMBER FROM SEED R
C*****
      FUNCTION STRNUM(R)
      IMPLICIT REAL (A-H,O-Z)
      BB      = 1.0
      P1      = R*317
      STRNUM = MOD(P1,BB)
      RETURN
      END

```

APPENDIX B:

Listing of POSTINVC Code


```

*****
C THIS PROGRAM PERFORMS POST-PROCESSING OF THE GAMMA DISTRIBUTIONS
C PRODUCED BY PROGRAM INVC TO INCLUDE ATTENUATION BY THE PHOTON GAS,
C BY THE CONTAINING WALL, AND BY AIR. THE RESULTS ARE PRESENTED
C AS BEAM BRIGHTNESS AND CUMULATIVE NUMBER AND ENERGY DISTRIBUTIONS.
C *****
C BLOCK DATA
C REAL E(30),PBCX(30),AIRCX(30)
C COMMON / ATTNE / E
C COMMON / XSC / PBCX,AIRCX
C
C GAMMA ATTENUATION DATA FOR LEAD AND AIR
C
C DATA E /
C 1 1.000E+03,8.000E+02,6.000E+02,5.000E+02,4.000E+02,3.000E+02,2.000E+02,1.500E+02,1.000E+02,8.000E+01,
C 2 6.000E+01,5.000E+01,4.000E+01,3.000E+01,2.000E+01,1.500E+01,1.000E+01,8.000E+00,6.000E+00,5.000E+00,
C 3 4.000E+00,3.000E+00,2.000E+00,1.500E+00,1.000E+00,8.000E-01,6.000E-01,5.000E-01,4.000E-01,3.000E-01 /
C
C DATA PBCX /
C 1 1.1452E-01,1.1354E-01,1.1198E-01,1.1082E-01,1.0918E-01,1.0678E-01,1.0250E-01,9.8906E-02,9.4045E-02,8.9632E-02,
C 2 8.4007E-02,8.0384E-02,7.5796E-02,6.9567E-02,6.0690E-02,
C 3 5.143E-02,4.890E-02,4.6249E-02,4.341E-02,4.2121E-02,
C 4 4.1210E-02,4.1374E-02,4.4907E-02,5.0669E-02,6.8230E-02,
C 5 8.452E-02,1.1776E-01,1.5152E-01,2.1744E-01,3.7830E-01 /
C
C DATA AIRCX /
C 1 1.9498E-02,1.9288E-02,1.8989E-02,1.8776E-02,1.8488E-02,
C 2 1.8080E-02,1.7439E-02,1.6971E-02,1.6235E-02,1.5905E-02,
C 3 1.5641E-02,1.5564E-02,1.574E-02,1.5805E-02,1.6727E-02,
C 4 1.7891E-02,2.0305E-02,2.2105E-02,2.5060E-02,2.7356E-02,
C 5 3.0651E-02,3.5723E-02,4.4451E-02,5.1732E-02,6.3550E-02,
C 6 7.0696E-02,8.0505E-02,8.7051E-02,9.5379E-02,1.0652E-01 /
C
C END
C
C PROGRAM POSTIN
C
C PARAMETER (NT=101, NAP=4, NAB=20, NEB=100, NN2=10000)
C
C IMPLICIT REAL(A-H,O-Z)
C
C REAL F(NAP)
C REAL EGRID(NEB+1),ANG(NAB+1)
C REAL SUM(NEB,NAB),SUME(NEB,NAB)
C REAL TSUM(NAB,NAP),TSUME(NAB,NAP),BE(NAB,NAP)
C REAL LATTEN
C
C COMMON / OPT / BE
C COMMON / SUMS / SUM, SUME
C COMMON / ATTEN / TSUM, TSUME,
C & TEMP, RHOBB, RHOEQ, WALLT, SIGB, CAVL, RHOAIR, AIRLEN, NN1
C COMMON / AVEKN / TOTKN, EGRID, DE, ANG, NANG
C COMMON / INTG / EGAMMA, TBB
C *****
C OPEN APPROPRIATE FILES FOR I/O
C
C INVCIN : UNIT 3 CONTAINS INPUT VARIABLES
C *****

```

```

*****
C LGAMATTN: UNIT 4 CONTAINS GAMMA ATTENUATION LENGTHS IN PHOTON GAS
C MCGAM : UNIT 7 UNATTENUATED GAMMA POPULATION DISTRIBUTIONS AS FUNCTIONS
C MCE : UNIT 8 UNATTENUATED GAMMA ENERGY DISTRIBUTIONS AS FUNCTIONS
C GAMOUT : UNIT 9 ATTENUATED GAMMA POPULATION DISTRIBUTIONS AS FUNCTIONS
C ENGOUT : UNIT 10 ATTENUATED GAMMA ENERGY DISTRIBUTIONS AS FUNCTIONS
C SORT : UNIT 11 VARIOUS BINNING DATA NEEDED FOR POST-PROCESSING
C PREATTN : UNIT 12 UNATTENUATED ENERGIES, CUMULATIVE DISTRIBUTIONS
C POSTGAM : UNIT 13 ENERGIES, CUMULATIVE DISTRIBUTIONS AND BEAM BRIGHTNESS
C POSTWALL: UNIT 14 ENERGIES, CUMULATIVE DISTRIBUTIONS AND BEAM BRIGHTNESS
C POSTATM : UNIT 15 ENERGIES, CUMULATIVE DISTRIBUTIONS AND BEAM BRIGHTNESS
C *****
C OPEN(UNIT=3, FILE='INVCIN')
C OPEN(UNIT=4, FILE='LGAMATTN')
C OPEN(UNIT=7, FILE='MCGAM')
C OPEN(UNIT=8, FILE='MCE')
C OPEN(UNIT=9, FILE='GAMOUT.DAT')
C OPEN(UNIT=10, FILE='ENGOUT.DAT')
C OPEN(UNIT=11, FILE='SORT')
C OPEN(UNIT=12, FILE='PREATTN.DAT')
C OPEN(UNIT=13, FILE='POSTGAM.DAT')
C OPEN(UNIT=14, FILE='POSTWALL.DAT')
C OPEN(UNIT=15, FILE='POSTATM.DAT')
C
C C READ INPUT VARIABLES FROM INVCIN (UNIT 3)
C
C READ(3,*) TEMP
C READ(3,*) TMEV
C READ(3,*) ESIG
C READ(3,*) RHOBB
C READ(3,*) CAVL
C READ(3,*) WALLT
C READ(3,*) RHOAIR
C READ(3,*) AIRLEN
C READ(3,*) RSEED
C READ(3,*) NN1
C
C C SET THE PHOTON DENSITY TO THE EQUILIBRIUM VALUE IF NO INPUT IS GIVEN
C
C RHOEQ = 3.2E+22*TEMP**3
C IF(RHOBB .EQ. 0.0) RHOBB = RHOEQ
C
C PI = ACOS(-1.0)
C *****
C READ MASTER MONTE CARLO ARRAYS OF SORTED POPULATION AND
C ENERGY DISTRIBUTIONS.
C *****
C CALL READMC
C *****

```

```

C *****
C MODIFY THE SOURCE DISTRIBUTION BY THE PROBABILITY OF A PHOTON BEING
C ATTENUATED THROUGH THE GAS, WALL, OR AIR.
C *****
CALL GAMMATTN
C *****
C 'USE PRE- AND POST-ATTENUATION OUTPUT TO PERFORM FINAL
C AVERAGING, CUMULATIVE SUMS (ENERGY AND POPULATION) AND CALCULATIONS
C OF BEAM BRIGHTNESS.
C *****
CALL BRIGHT(PI,NANG,ANG)
WRITE(6,*) CAVL,BE(10,4)
STOP
END
C *****
C SUBROUTINE READMC READS THE MASTER ARRAYS OF SORTED MONTE CARLO
C EVENTS ALREADY GENERATED BY THE MAIN MC PROGRAM INVC.
C IT ALSO PICKS UP SORT-RELATED DATA FOR SUBSEQUENT POST-PROCESSING.
C *****
SUBROUTINE READMC
PARAMETER (NT=101, NAB=4, NAB=20, NEB=100, NN2=10000)
IMPLICIT REAL(A-H,O-Z)
REAL EGRID(NEB+1),ANG(NAB+1)
REAL SUM(NEB,NAB),SUME(NEB,NAB)
COMMON /SUMS/ SUM,SUME
COMMON /AVEKN/ TOTKN,EGRID,DE,ANG,NANG
READ(11,*) NE,DE,NANG,NN
READ(11,*) TOTKN
DO 200 J=1,NANG
  READ(11,*) ANG(J)
200 CONTINUE
DO 100 I=1,NE
  READ(7,*) EGRID(I), (SUM(I,J), J=1,NANG)
  READ(8,*) EGRID(I), (SUME(I,J), J=1,NANG)
100 CONTINUE
RETURN
END
C *****
C SUBROUTINE BRIGHT PERFORMS FINAL AVERAGING, CUMULATIVE SUMS
C AND CALCULATIONS OF BEAM BRIGHTNESS. OUTPUT ARRAYS ARE
C GENERATED FOR FOUR CASES:
C 1) NO SOURCE ATTENUATION OF ANY KIND
C 2) GAMMA-GAMMA ATTENUATION ONLY
C 3) GAMMA PLUS REAR WALL ATTENUATION
C 4) GAMMA, REAR WALL AND ATMOSPHERIC ATTENUATION
C *****

```

```

C *****
SUBROUTINE BRIGHT(PI,NANG,ANG)
PARAMETER (NT=101, NAB=4, NAB=20, NEB=100, NN2=10000)
IMPLICIT REAL(A-H,O-Z)
REAL TSUM(NAB,NAB),TSUME(NAB,NAB)
REAL EBAR(NAB,NAB),CUME(NAB,NAB)
REAL CUMGAM(NAB,NAB),BE(NAB,NAB)
REAL BGAM(NAB,NAB),EGRID(NEB+1),ANG(NAB+1)
COMMON /ATTEN/ TSUM,TSUME,
2 TEMP,RHOBB,RHOEQ,WALLT,SIGB,CAVL,RHOAIR,AIRLEN,NN1
COMMON /AVEKN/ TOTKN,EGRID,DE,ANG,NANG
C PUT UP HEADER IN EACH OUTPUT FILE
DO 700 I=1,NAB
  NUNIT = I + 1
  WRITE(NUNIT,851)
700 CONTINUE
C DEFINE UPPER BOUNDARY OF THE LAST ANGULAR BIN NANG
ANG(NANG+1) = PI/2
C GENERATE OVERALL OUTPUT QUANTITIES SUCH AS AVERAGE ENERGY, CUMULATIVE
C NUMBER AND ENERGY FRACTIONS, AND BRIGHTNESS.
DO 850 K=1,NANG
  DO 855 J=1,NAB
    IF(TSUM(K,J) .NE. 0.0) THEN
      EBAR(K,J) = TSUME(K,J)/TSUM(K,J)
    ELSE
      EBAR(K,J) = 0.0
    ENDIF
    CUMGAM(K,J) = TSUM(K,J)/TSUM(NANG,J)
    CUME(K,J) = TSUME(K,J)/TSUME(NANG,J)
    BE(K,J) = TSUME(K,J)/(2*PI*(1. - COS(ANG(K+1))))
    BGAM(K,J) = TSUM(K,J)/(2*PI*(1. - COS(ANG(K+1))))
    NUNIT = I + J
    WRITE(NUNIT,902) ANG(K+1),EBAR(K,J),CUMGAM(K,J),
      & CUME(K,J),BGAM(K,J),BE(K,J)
855 CONTINUE
850 CONTINUE
851 FORMAT(7X,'ANGLE ',3X,' AVG ENERGY ',3X,' CUM FRACTION',3X,
  & 'CUM FRACTION',3X,' BRIGHTNESS ',3X,' BRIGHTNESS ',/,
  & 7X,'(RADS) ',3X,' (MEV) ',3X,' GAMMAS ',3X,
  & 'ENERGY ',3X,' ( G/SR/E )',3X,' ( MEV/SR/E )',/)
902 FORMAT(2X,6(E12.5,3X))
RETURN
END
C *****
C SUBROUTINE GAMMATTN MODIFIES THE SOURCE DISTRIBUTION BY THE
C PROBABILITY OF A PHOTON BEING ATTENUATED WITHIN THE GAS, THE WALL,
C AND THE SUBSEQUENT AIR CHANNEL.
C *****

```

```

SUBROUTINE GAMMATN
  PARAMETER (NT=101, NAP=4, NAB=20, NEB=100, NN2=10000)
  IMPLICIT REAL(A-H,O-Z)

  REAL TSUM(NAB,NAP), TSUME(NAB,NAP)
  REAL EGRID(NEB+1), SUM(NEB,NAB), SUME(NEB,NAB)
  REAL ANG(NAB+1), F(NAP)
  REAL PBCX(30), AIRCX(30)
  REAL EG(NEB), GGALP(NEB)

  CHARACTER*10 PCHAR(NEB,NAB), ECHAR(NEB,NAB), ZERO1, ZERO2

  COMMON /SUMS/ SUM, SUME
  COMMON /ATTEN/ TSUM, TSUME,
    & TEMP, RHOBB, RHOEQ, WALLT, SIGB, CAVL, RHOAIR, AIRLEN, NN1
  COMMON /AVEKN/ TOTKN, EGRID, DE, ANG, NANG
  COMMON /INTG/ EGAMMA, TBB
  COMMON /XSC/ PBCX, AIRCX

  C DEFINE REPLACEMENT CHARACTER STRING FOR .00000E+00 IN OUTPUT DATA
  C
  ZERO1 = '.00000E+00'
  ZERO2 = ''

  C READ ATTENUATION LENGTHS VS ENERGY FROM EXTERNAL FILE LGAMATTN
  C
  DO 50 I=1, NEB
    READ(4,*) EG(I), GGALP(I)
  50 CONTINUE

  DO 800 I = 1, NEB
    EGAMMA = EG(I)

  C INTERPOLATE ON ENERGY FROM BLOCK DATA SUBPROGRAM
  C TO OBTAIN ATTENUATION COEFFICIENT FOR EXIT WALL. IF EGAMMA ABOVE 1 GEV,
  =>
  C USE PBCX AND AIRCX AT 1 GEV.
  C
  IF(EGAMMA .LE. 1000.0) THEN
    CALL XSECT(EGAMMA, PBCX, CXWALL)
    SIGB = CXWALL*11.330+00
  C INTERPOLATE ON ENERGY FROM BLOCK DATA SUBPROGRAM
  C TO OBTAIN ATTENUATION COEFFICIENT FOR AIR OF DENSITY RHOAIR.
  C
  CALL XSECT(EGAMMA, AIRCX, CXATM)
  SIGATH = CXATM*RHOAIR
  ELSE
    SIGB = PBCX(1)*11.330+00
    SIGATH = AIRCX(1)*RHOAIR
  ENDF

  C EVALUATE ATTENUATION COEFFICIENTS FOR SOURCE DISTRIBUTION
  C
  SIGIC = RHOBB*TOTKN
  F1 = SIGIC/(SIGIC - 1./GGALP(1))
  F2GAM = 1.00+00
  F2WLL = EXP(-1.00+00*WALLT*SIGB)
  F2ATH = EXP(-1.00+00*AIRLEN*1.00+05*SIGATH)
  F3 = EXP(-1.00+00*AIRLEN*1.00+05*SIGATH)
  F3 = EXP(-1.00+00*CAVL/GGALP(1))
  F4 = EXP(-1.00+00*SIGIC*CAVL)
  F(1) = 1.00+00
  F(2) = F1*F2GAM*(F3 - F4)

```

```

  F(3) = F1*F2GAM*F2WLL*(F3 - F4)
  F(4) = F1*F2GAM*F2WLL*F2ATH*(F3 - F4)

  C MODIFY PHOTON SOURCE DISTRIBUTION BY ATTENUATION FACTORS.
  C WRITE PHOTON NUMBER AND ENERGY DISTRIBUTIONS AS A FUNCTION
  C OF EXIT ANGLE.
  C
  WRITE(9,901) EGAMMA, (F(4)*SUM(I,K)/(NN1*DE), K=1,NANG,3)
  WRITE(10,901) EGAMMA, (F(4)*SUME(I,K)/NN1/DE, K=1,NANG,3)
  FORMAT(1X,7(1X,E10.5))
  901
  C
  DO 750 K=1,NANG
    DO 740 J=1,NAP
      TSUM(K,J) = TSUM(K,J) + F(J)*SUM(I,K)/NN1
      TSUME(K,J) = TSUME(K,J) + F(J)*SUME(I,K)/NN1
    740 CONTINUE
  750 CONTINUE
  C
  800 CONTINUE
  C
  C READ MODIFIED OUTPUT ARRAYS SUM & SUME AS CHARACTER GROUPS. SEARCH FOR ZERO
  C VALUES AND REPLACE WITH BLANKS. THIS ELIMINATES UNNECESSARY POST-PROCESSING
  C AND FACILITATES PLOTTING OF PARTICLE AND ENERGY DISTRIBUTIONS.
  C
  REWIND(9)
  REWIND(10)
  DO 80 I=1, NEB
    READ(9,3) (PCHAR(I,J), J=1,7)
    READ(10,3) (ECHAR(I,J), J=1,7)
    DO 85 J=1,7
      IF(PCHAR(I,J) .EQ. ZERO1) PCHAR(I,J) = ZERO2
      IF(ECHAR(I,J) .EQ. ZERO1) ECHAR(I,J) = ZERO2
    85 CONTINUE
  80 CONTINUE
  C
  REWIND(9)
  REWIND(10)
  DO 90 I=1, NEB
    WRITE(9,3) (PCHAR(I,J), J=1,7)
    WRITE(10,3) (ECHAR(I,J), J=1,7)
  90 CONTINUE
  C
  1 FORMAT(1X,7(1X,E10.5))
  2 FORMAT(1X,F12.4,16(1X,E10.5))
  3 FORMAT(1X,7(1X,A10))
  C
  RETURN
  END
  C*****
  C
  C SUBROUTINE XSECT PERFORMS LINEAR INTERPOLATION ON ARRAYS OF PHOTON
  C CROSS SECTIONS FOR LEAD AND AIR.
  C*****
  C
  SUBROUTINE XSECT(EGAMMA, ATMTL, CX)
  IMPLICIT REAL(A-H,O-Z)
  REAL E(30), ATMTL(30)
  COMMON / ATNE / E
  DO 100 I=1,24
    IF(EGAMMA .GE. E(I)) THEN
      EFRAC = (E(I-1) - EGAMMA)/(E(I-1) - E(I))

```

```

GO TO 200
ENDIF
100 CONTINUE
C 200 IF(ATTMTL(I-1) - ATTMTL(I) .GE. 0.0) THEN
      CX = ATTMTL(I-1) - EFRAC*(ATTMTL(I-1) - ATTMTL(I))
ELSE
      CX = ATTMTL(I-1) + EFRAC*(ATTMTL(I-1) - ATTMTL(I))
ENDIF
C RETURN
END

```

**DATE
FILMED**

10/27/93

END